KPAP : Field line & particle transport in synthetic turbulence

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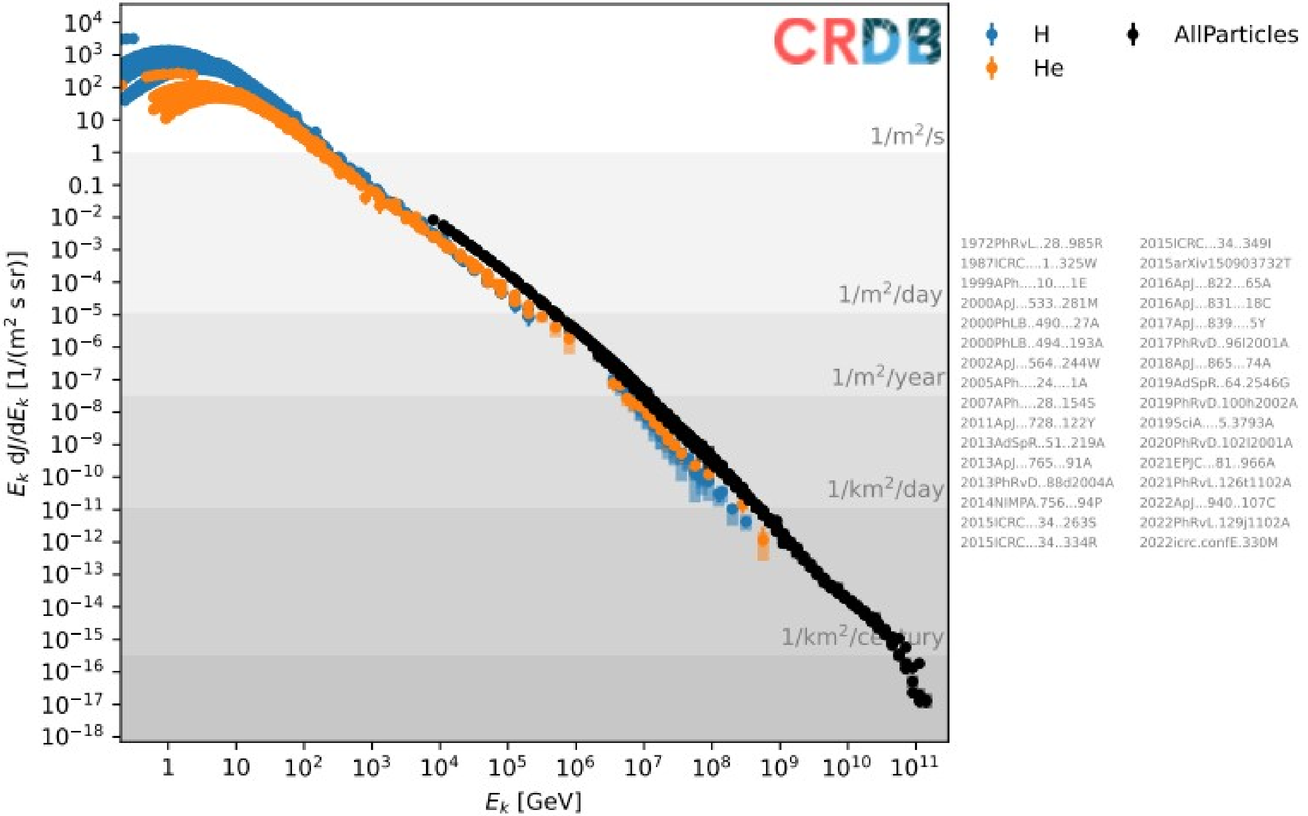
LAPTh

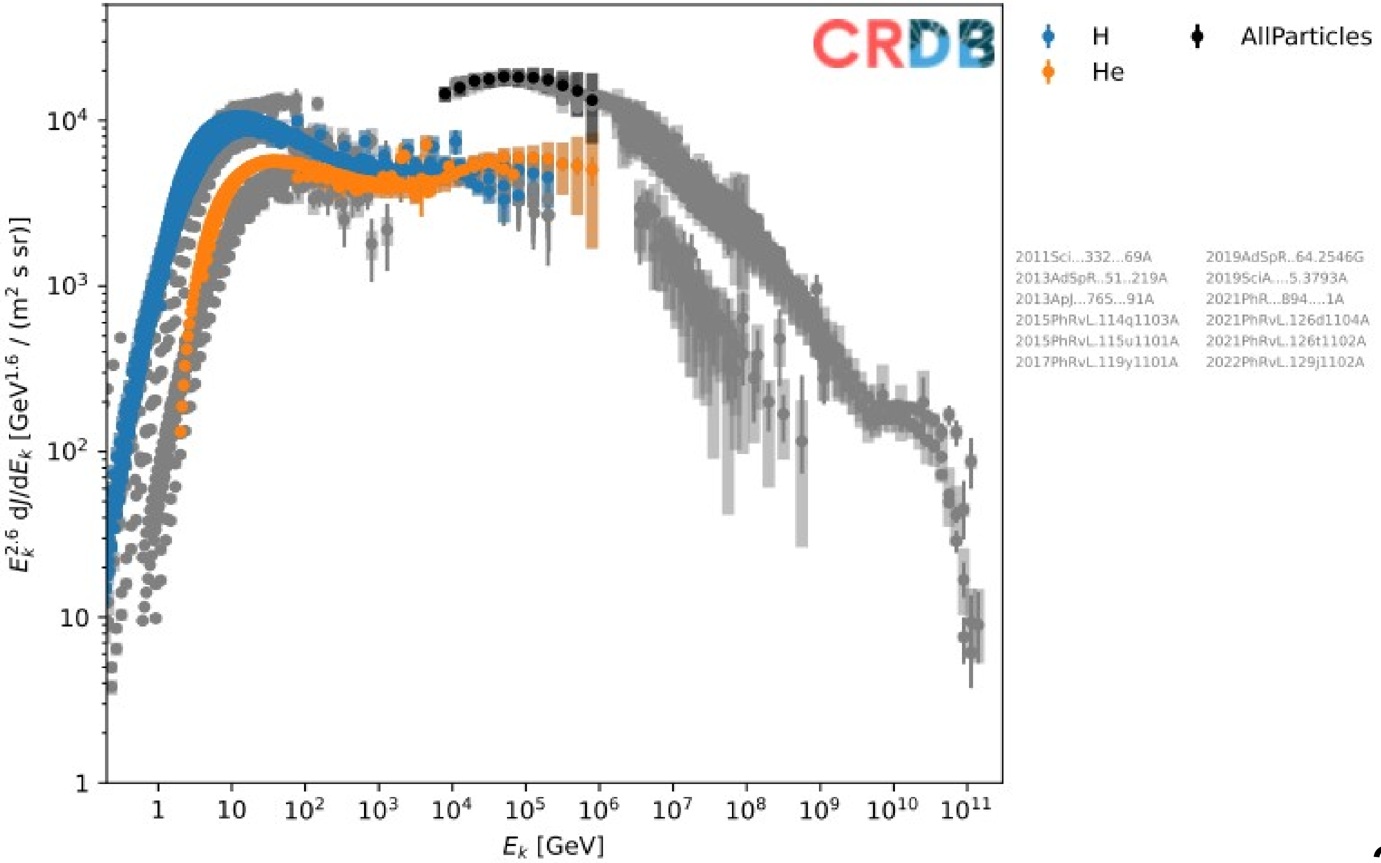
Introduction: phenomenology

- Composition
  - Protons and He nuclei (99%), heavy nuclei and antiparticles
- Observations
  - Energy spectrum follows a power law : injection, diffusive transport



Introduction: phenomenology

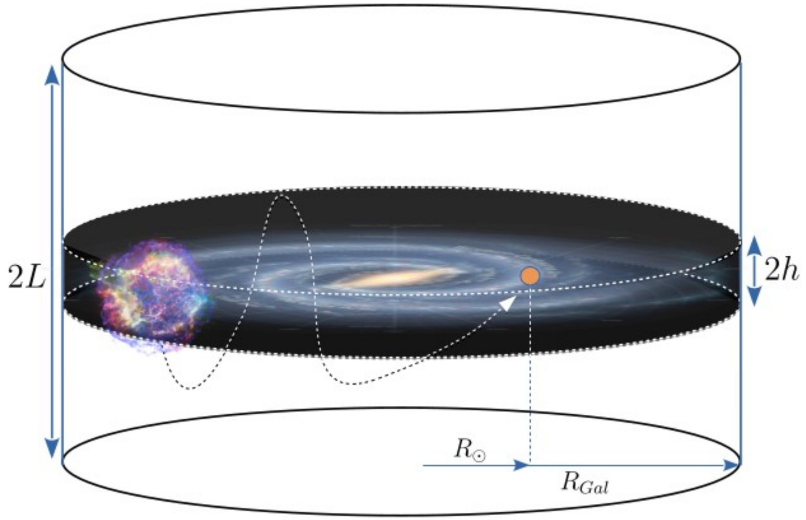
- Composition
  - Protons and He nuclei (99%), heavy nuclei and antiparticles
- Observations
  - Energy spectrum follows a power law : injection, diffusive transport
  - Features : knee, ankle >> change of sources or diffusion regime
- *Galactic confinement*
  - Spallation
  - Radioisotopes
  - Secondary-to-primary ratio



Microphysics of CR transport

Spherical cow galactic diffusion model
→ Quasi-Linear-Theory (QLT)

- Cylindrical diffusion box
- Homogeneous & isotropic diffusion tensor
- Pheno model reproduces the data but does not explain : rigidity dependence of diffusion transport, CR small scales anisotropies, CR spectral hardening towards GC...



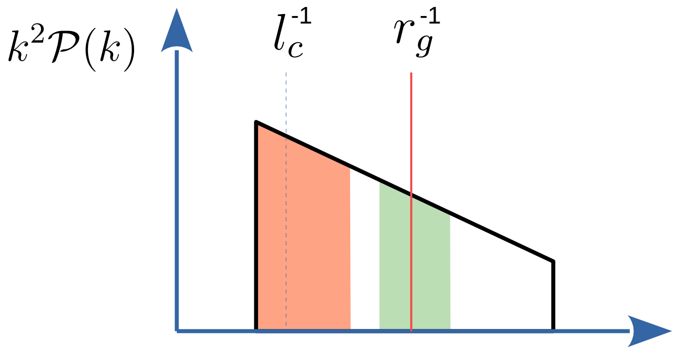
Microphysics of CR transport

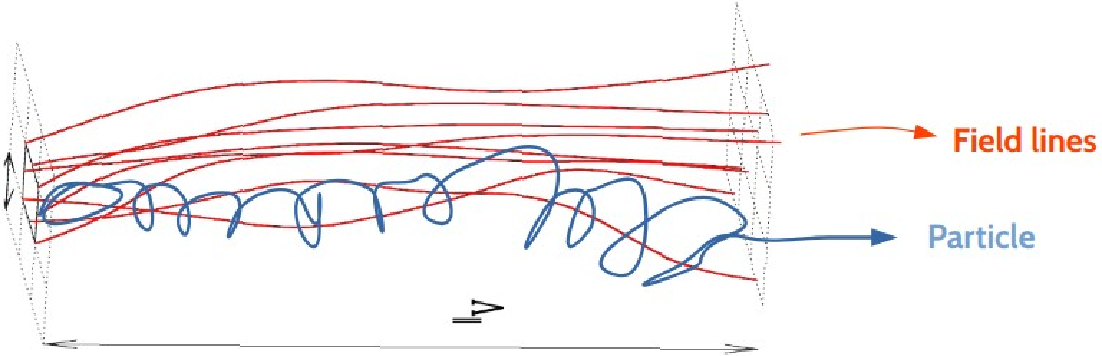
Physics of perpendicular transport

$$D = \begin{bmatrix} D_{||} & 0 & 0 \\ 0 & D_{\perp} & 0 \\ 0 & 0 & D_{\perp} \end{bmatrix}$$

- Parallel transport to the bckg field B driven by pitch angle scattering

- Perpendicular transport driven by :
  - Field line random walk
  - Parallel transport
  - Transverse complexity



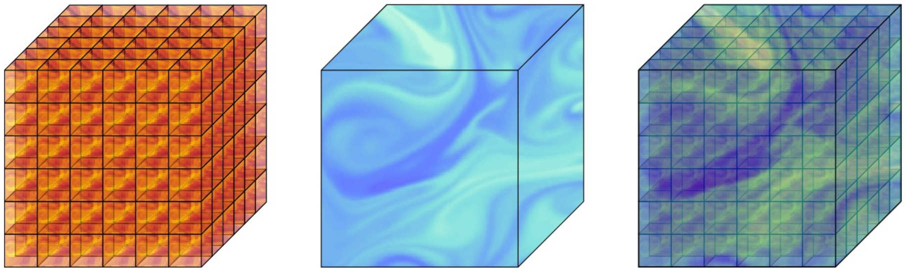


Goldreich+ 2001

Studied in synthetic turbulence simulations

Synthetic turbulence simulations

How to generate synthetic turbulence ?	Stored in memory	Main pros	Main cons
Harmonic method *(see Giacalone 1994)* $\boldsymbol{\delta B}(\boldsymbol{r}) = \sqrt{2}\sum_{n=0}^{N-1} A_n \hat{\boldsymbol{\xi}}_n \cos\left(k_n \hat{\boldsymbol{k}}_n \cdot \boldsymbol{r} + \beta_n\right)$	$\{A_n, \hat{\boldsymbol{\xi}}_n, \hat{\boldsymbol{k}}_n, \beta_n\}$	Dynamical range	Computing time
Nested grid method *(see Mertsch 2020)* $\delta B_j(\boldsymbol{r}_{n_1,n_2,n_3}) = \frac{1}{N^3}\sum_{l_1,l_2,l_3=0}^{N-1} e^{-2\pi i\left(\frac{l_1 n_1}{N}+\frac{l_2 n_2}{N}+\frac{l_3 n_3}{N}\right)} \delta\tilde{B}_j^{l_1,l_2,l_3}$	$\delta B_j(\boldsymbol{r}_{n_1,n_2,n_3})$	Computing time	Memory



Total B field : $\boldsymbol{B} = \boldsymbol{\delta B} + B_0 \boldsymbol{z}$

Kolmogorov spectrum

Turb. level: $\eta = \frac{\delta B^2}{\delta B^2 + B_0^2}$

We study field lines and particle transport in this turbulence

Synthetic polarized turbulence simulations

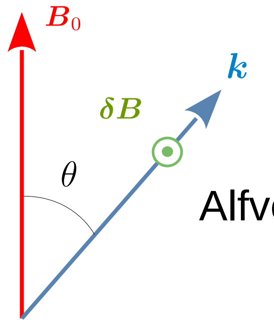
- Polarized synthetic turbulence
  - From ideal linearised MHD equations → eigenmodes

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

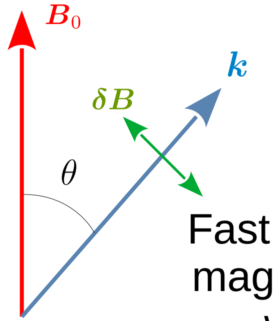
$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\nabla \cdot \mathbf{B} = 0$$



Alfvèn waves



Fast and slow magnetosonic waves

- We generate synthetic isotropic polarised turbulence

$$\boldsymbol{\delta B}(\boldsymbol{r}) = \sum_{n=0}^{N-1} A_n \hat{\boldsymbol{\xi}}_n \cos\left[k_n \hat{\boldsymbol{k}}_n \cdot \boldsymbol{r} + \beta_n \right]$$

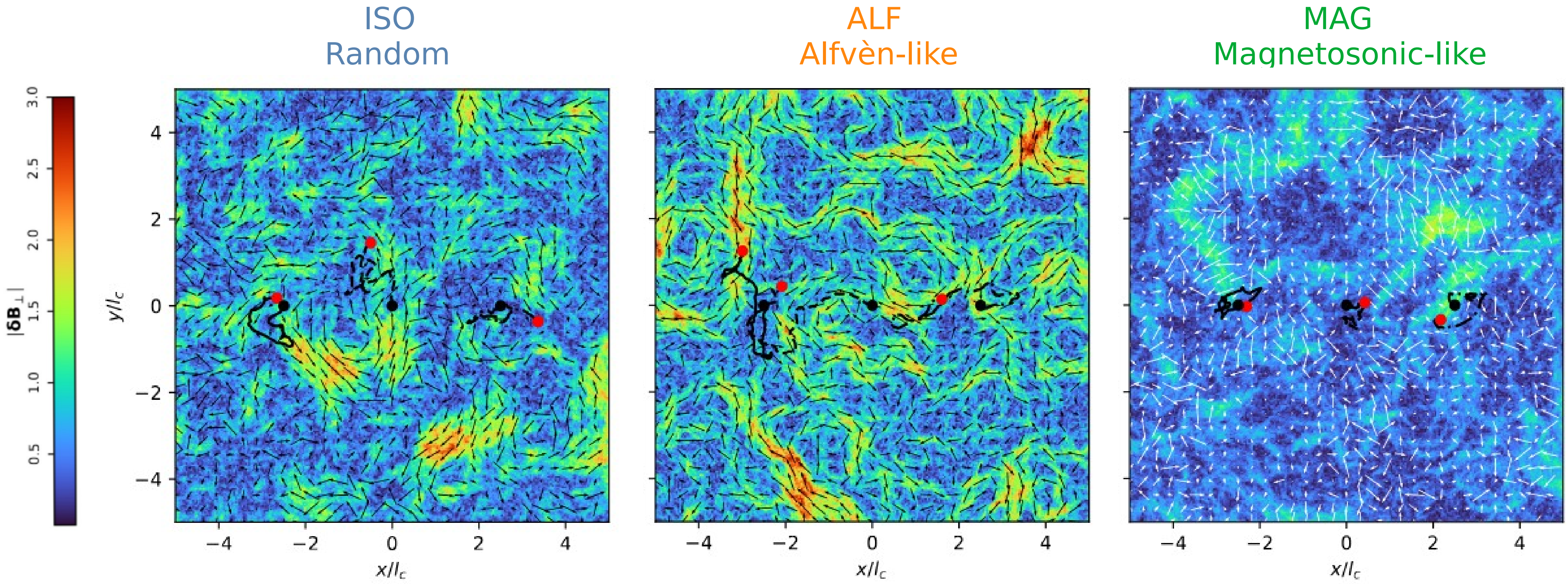
3 cases : Polarisation state

ALF : Alfvèn-like

MAG: Magnetosonic-like

ISO: Random

Synthetic polarized turbulence simulations



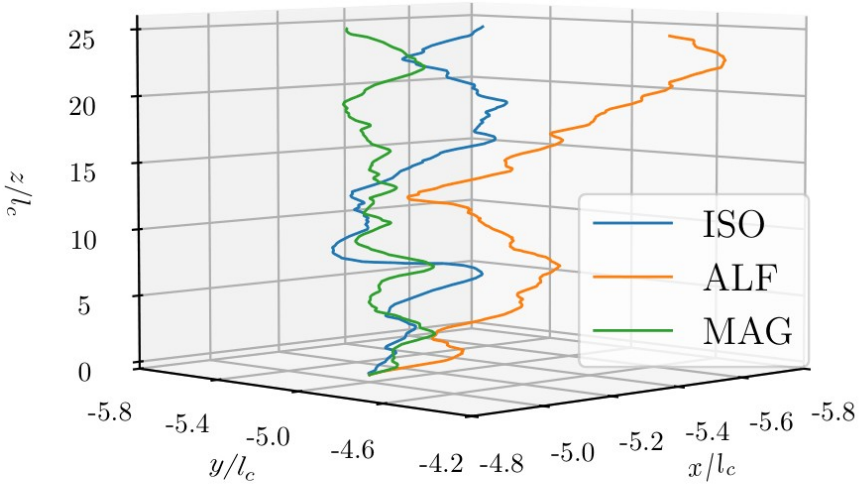
→ We do see some patterns in such synthetic turbulence: filaments, different orientation of B vectors wrt these structures

Bouchet+ (to be sub.)

Field line transport

Solve field line equations :

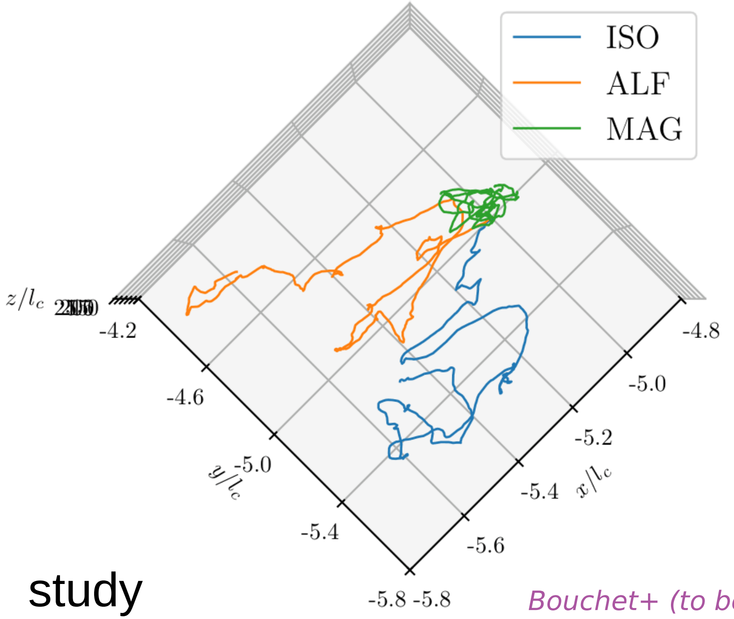
$$\frac{d\mathbf{r}(s)}{ds} = \frac{\mathbf{B}(\mathbf{r}(s))}{|\mathbf{B}(\mathbf{r}(s))|} \equiv \mathbf{b}(\mathbf{r}(s)) \text{ or } \frac{d\mathbf{r}(\tau)}{d\tau} = \mathbf{B}(\tau)\ \tau = s/|\mathbf{B}|$$



- FL in MAG much more confined in (x, y) plane
- After reconstructing thousands of field lines, we study statistical behaviours

Display of field lines for each polarisation config :

ISO Random | ALF Alfvèn-like | MAG Magnetosonic-like



Bouchet+ (to be sub.)

Field line transport

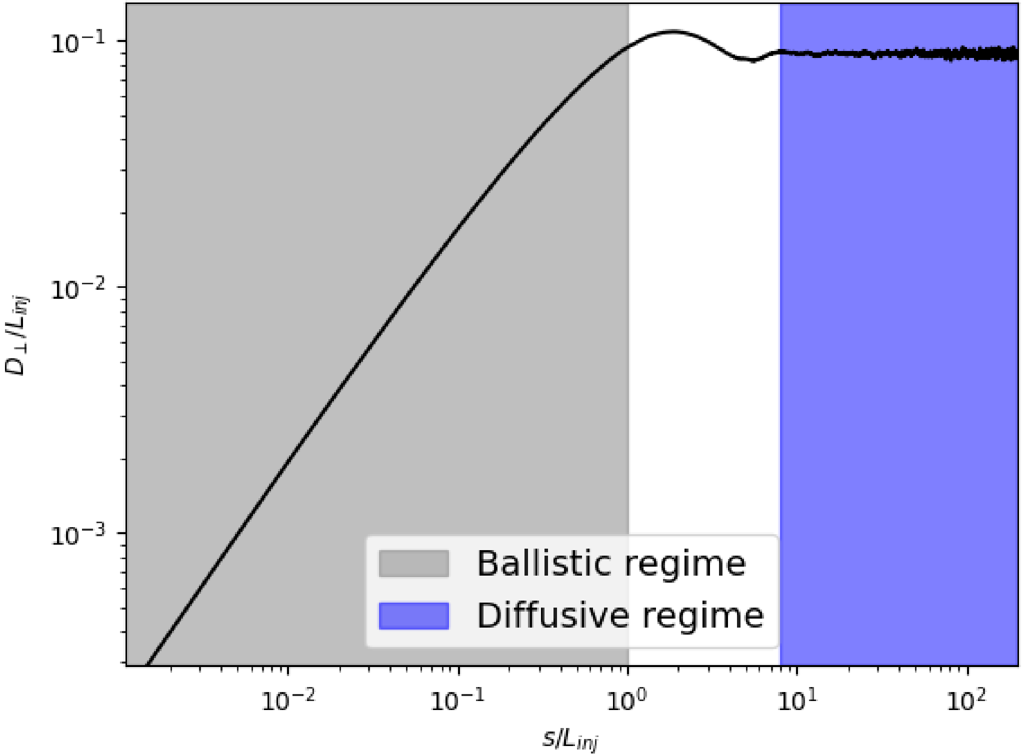
- Define parallel and transverse diffusion coefficients

$$\mathcal{D}_\perp = \frac{1}{4}\frac{d}{ds}\left(\langle (x(s)-x_0)^2\rangle + \langle (y(s)-y_0)^2\rangle\right)$$

$$\mathcal{K}_\perp = \lim_{s\to\infty} \mathcal{D}_\perp$$

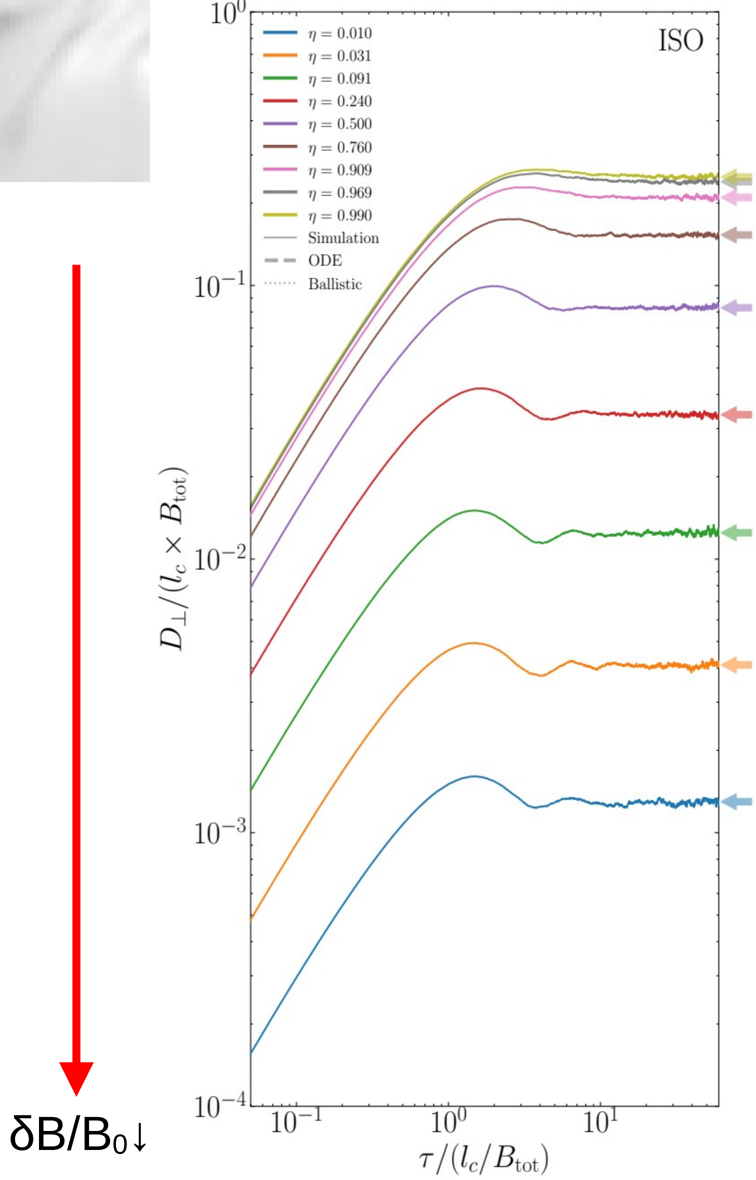
- Alternative parametrization : helpful for analytical modeling (especially at high η)

$$D_\perp = \frac{1}{4}\frac{d}{d\tau}\left(\langle (x(\tau)-x_0)^2\rangle + \langle (y(\tau)-y_0)^2\rangle\right)$$
$$= \frac{1}{2}\left(\langle \delta B_x(\boldsymbol{r}(\tau))(x(\tau)-x_0)\rangle\right.$$
$$\left. + \langle \delta B_y(\boldsymbol{r}(\tau))(y(\tau)-y_0)\rangle\right) \ ,$$
$$D_\parallel = \frac{1}{2}\frac{d}{d\tau}\langle (z-\langle z\rangle)^2\rangle = \langle \delta B_z(\tau)(z-\langle z\rangle)\rangle \ .$$
$$K_\perp = \lim_{\tau\to\infty} D_\perp$$



Here we study FL running diffusion coefficients wrt to turbulence level $\eta = \frac{\delta B^2}{\delta B^2 + B_0^2}$ & polarisation config .

Field line transport



Results from simulations :

- Three phases : ballistic (α=1), transition through subdiffusion (α<0), diffusive plateau (α=0)
- Particular scaling with B_0

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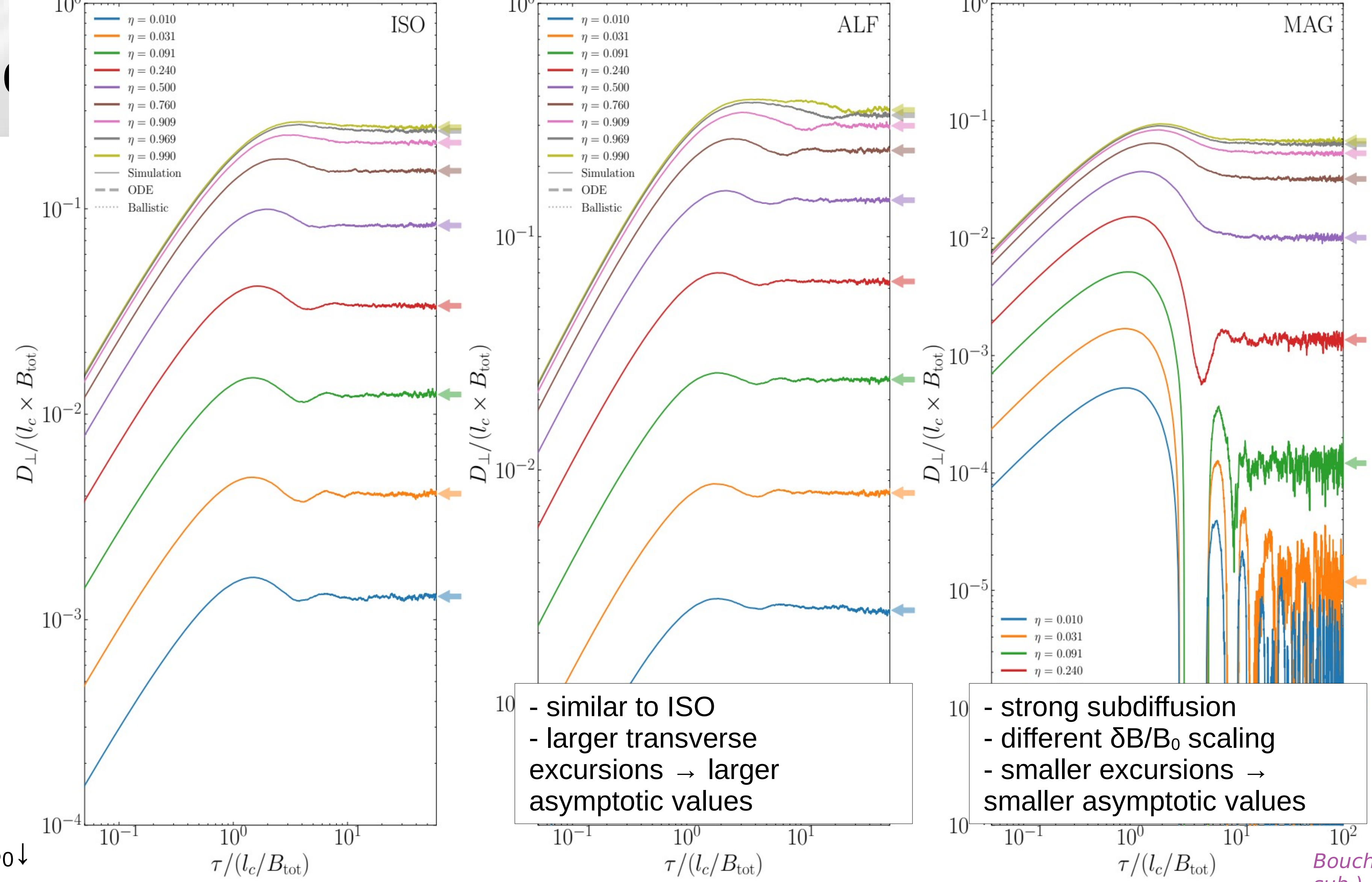


- similar to ISO
- larger transverse excursions → larger asymptotic values

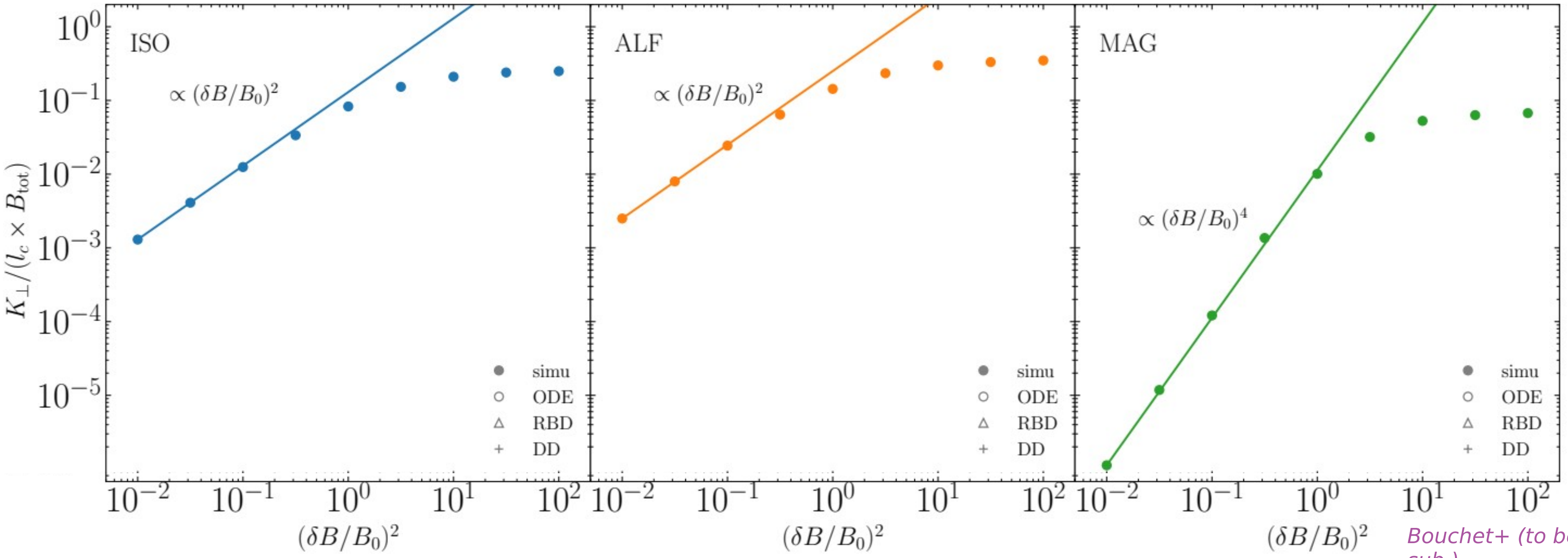
- strong subdiffusion
- different $\delta B/B_0$ scaling
- smaller excursions → smaller asymptotic values

$\delta B/B_0\downarrow$

Bouchet+ (to be sub.)

Field line transport

Field line asymptotic diffusion coefficients



Bouchet+ (to be sub.)

- Different scaling in the MAG case

Field line transport

Correlation length of the total Bfield :

$$l_c = \frac{2}{\delta B^2} \int_0^{\infty} \mathrm{d}l \, \langle \delta \boldsymbol{B}(\boldsymbol{r}) \delta \boldsymbol{B}(\boldsymbol{r} + \boldsymbol{l}) \rangle$$

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Correlation length of the total transverse Bfield :

$$l_{B_\perp}(\theta_*) = \frac{2}{\delta B_\perp^2} \int_0^\infty \langle \delta \boldsymbol{B}_\perp(\boldsymbol{r}) \delta \boldsymbol{B}_\perp(\boldsymbol{r} + \boldsymbol{l}(\theta_*)) \rangle \, \mathrm{d}l$$

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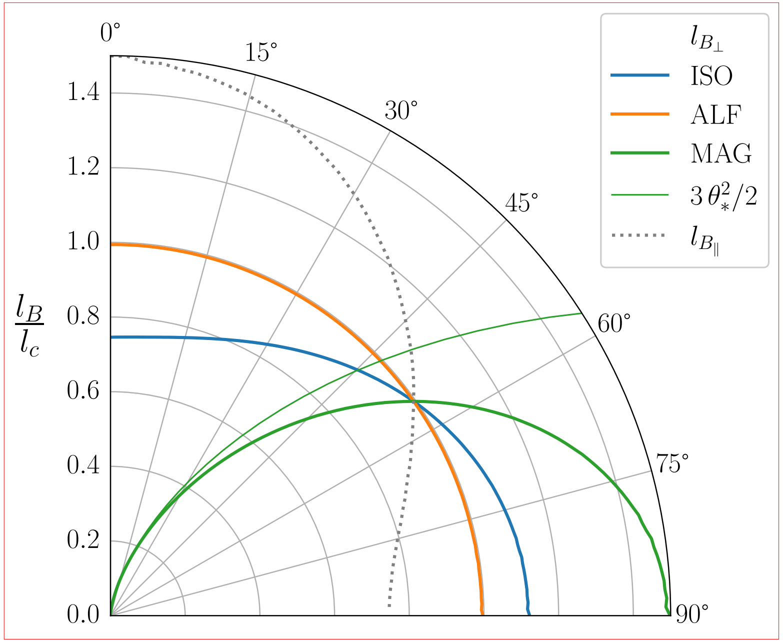
In the MAG case : $l_{B_\perp}^{\mathrm{MAG}}(\theta_* \to 0) \approx \frac{3}{2} l_c \theta_*^2 \approx \frac{3}{2} l_c \left(\frac{\delta B_\perp}{B_0} \right)^2$



Bouchet+ (to be sub.)

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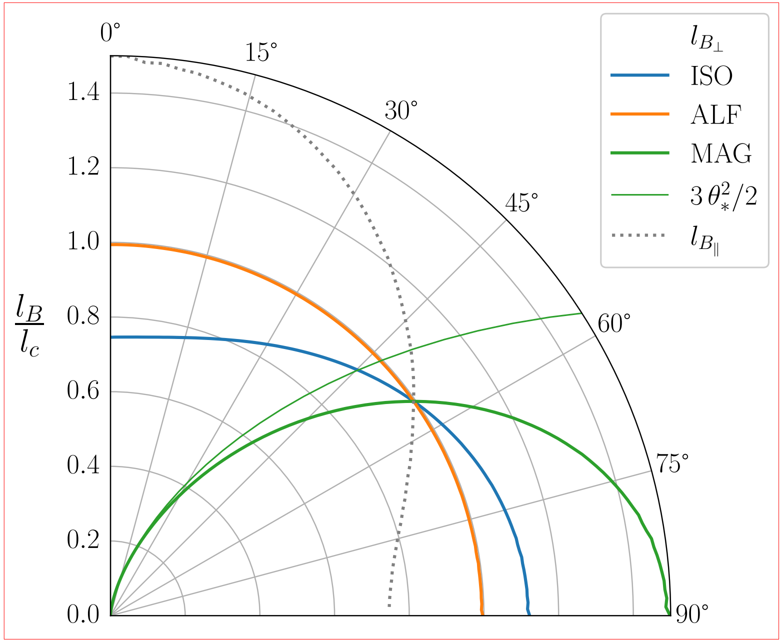
Heuristic derivation of the scalings in the regime : $\delta B / B_0 \ll 1$

- Transverse displacement : $\Delta r_\perp \approx \theta_* \, l_{B_\perp}(\theta_* \to 0)$
- Scaling : $K_\perp \approx \frac{(\Delta r_\perp)^2}{l_{B_\perp}(\theta_* \to 0)} \approx \theta_*^2 \, l_{B_\perp}(\theta_* \to 0)$



Bouchet+ (to be sub.)

ISO & ALF

$K_\perp \propto \theta_*^2 \propto (\delta B / B_0)^2$

MAG

$K_\perp \propto \theta_*^4 \propto (\delta B / B_0)^4.$

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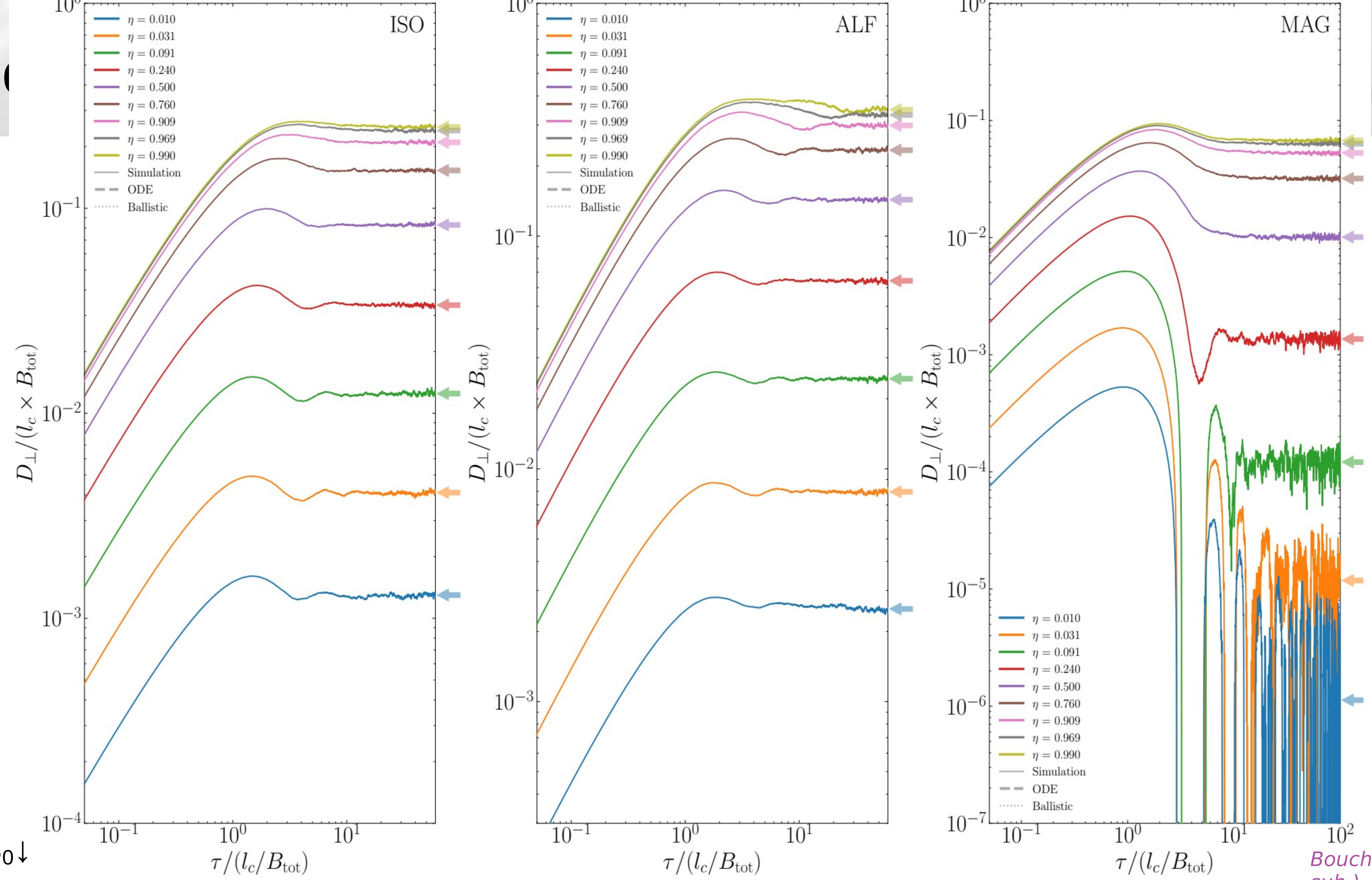
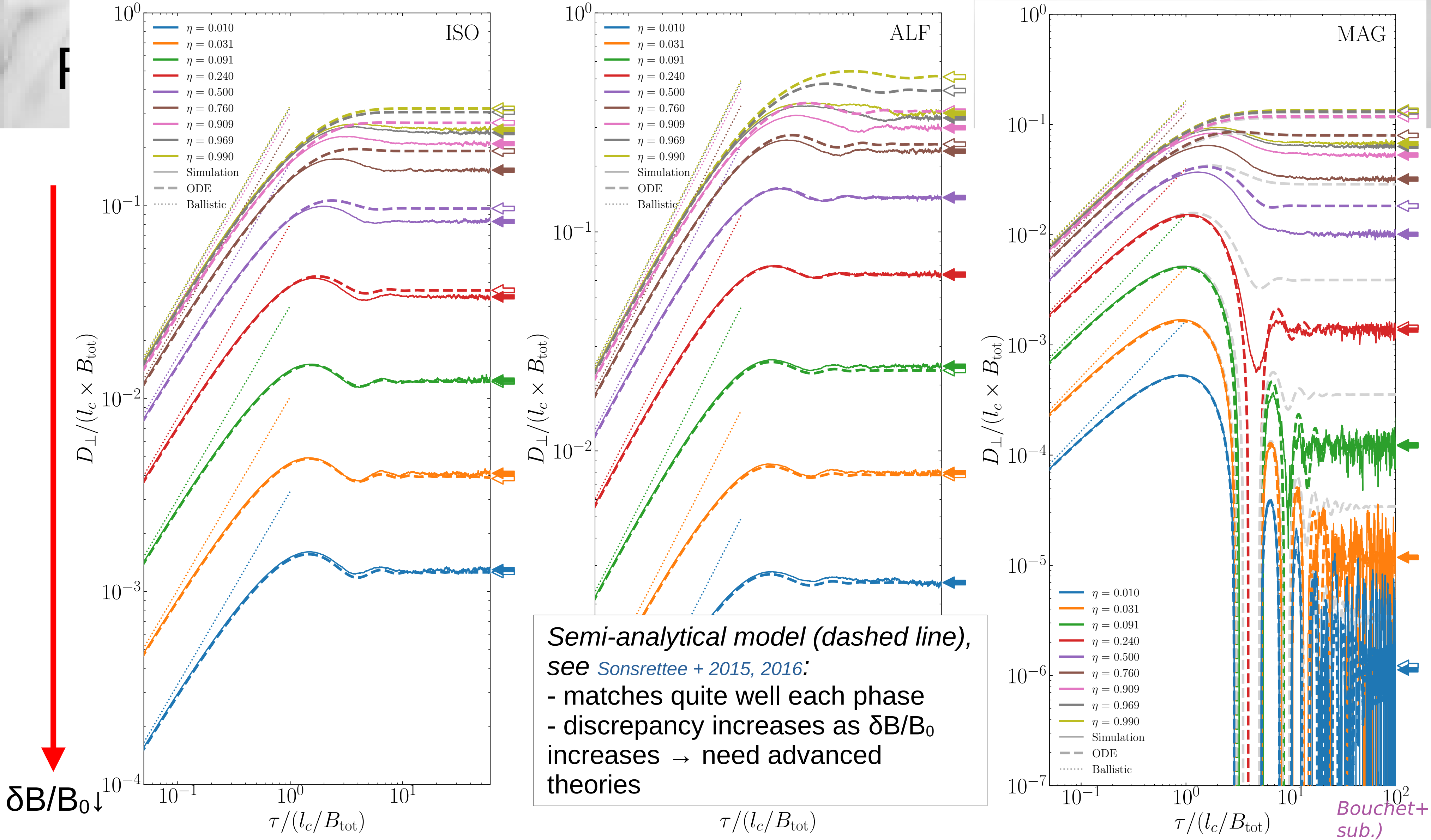


$\delta B/B_0\downarrow$

Bouchet+ (to be sub.)



Semi-analytical model (dashed line), see Sonsrettee + 2015, 2016*:*
- matches quite well each phase
- discrepancy increases as $\delta B/B_0$ increases → need advanced theories

Bouchet+ (to be sub.)

Particle transport

Now, we propagate particles in these magnetostatic nested grid turbulence configurations :

- Solve Lorentz force + Boris pusher
- Compute running diffusion coefficients

$$d_{\parallel}(t) \equiv \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\langle (z(t)-z(0))^2\rangle\,,$$

$$d_{\perp}(t) \equiv \frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}t}\left(\langle (x(t)-x(0))^2\rangle + \langle (y(t)-y(0))^2\rangle\right)$$

- Extract asymptotic values

$$\kappa_{\parallel} \equiv \lim_{t\to\infty} d_{\parallel}(t) \propto \lambda_{\parallel}$$

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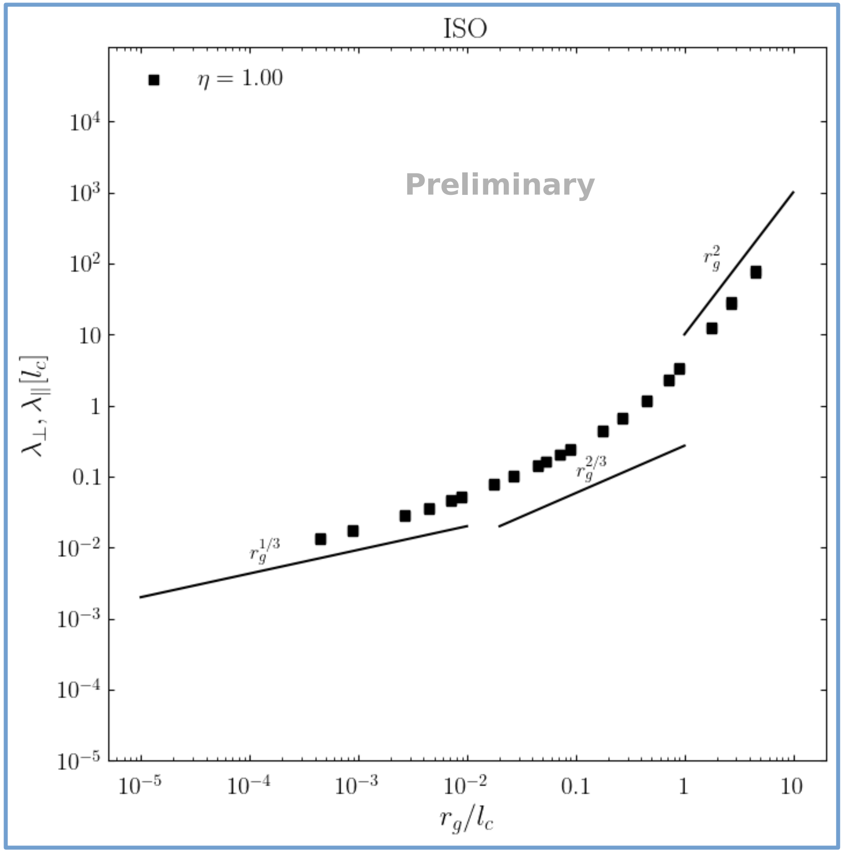
$\lambda_{\parallel}$ Transition from QLT limit to small scattering regime

$\lambda_{\perp}$ ISO : - small r_g recover QLT scaling
- steeper transition before flattening
- is it linked to FL subdiffusion ?



Bouchet+ (in prep)

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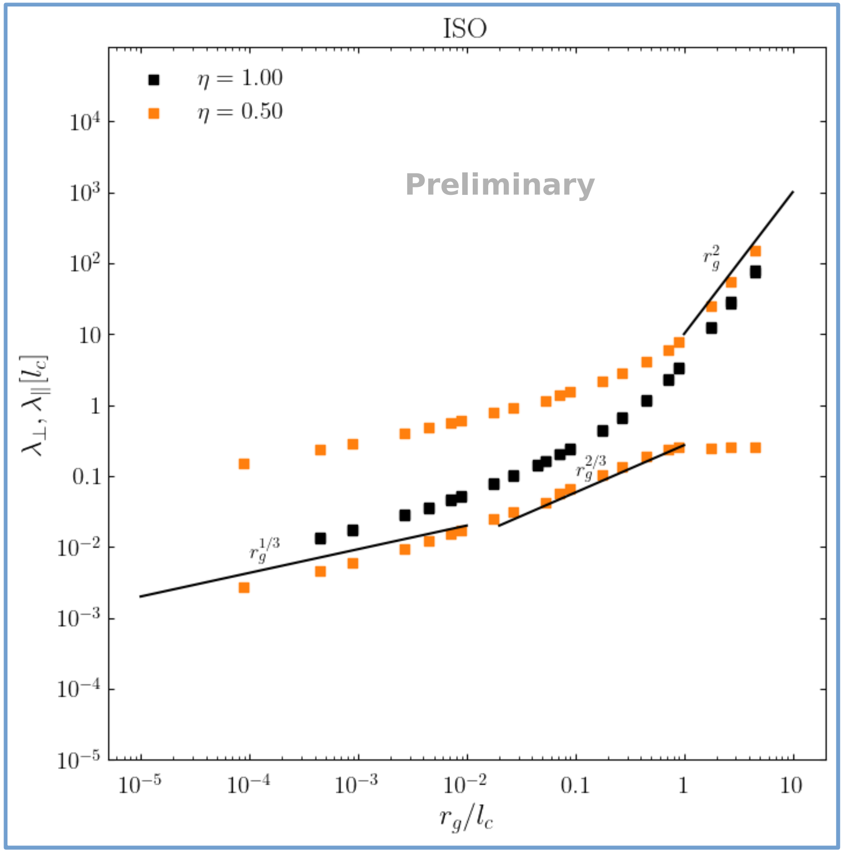

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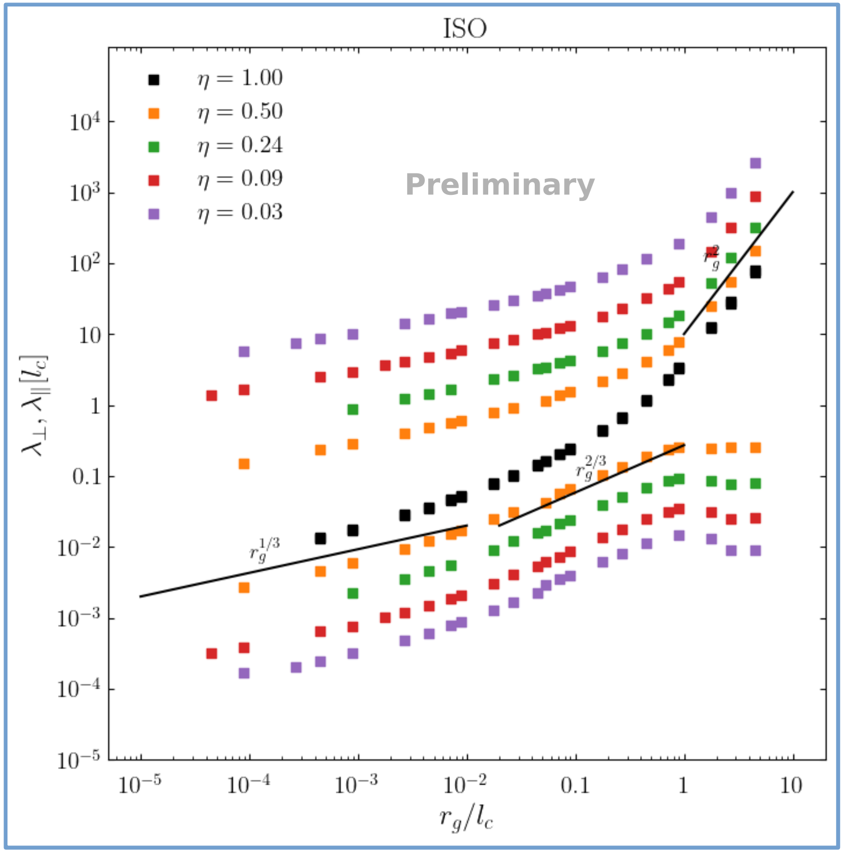

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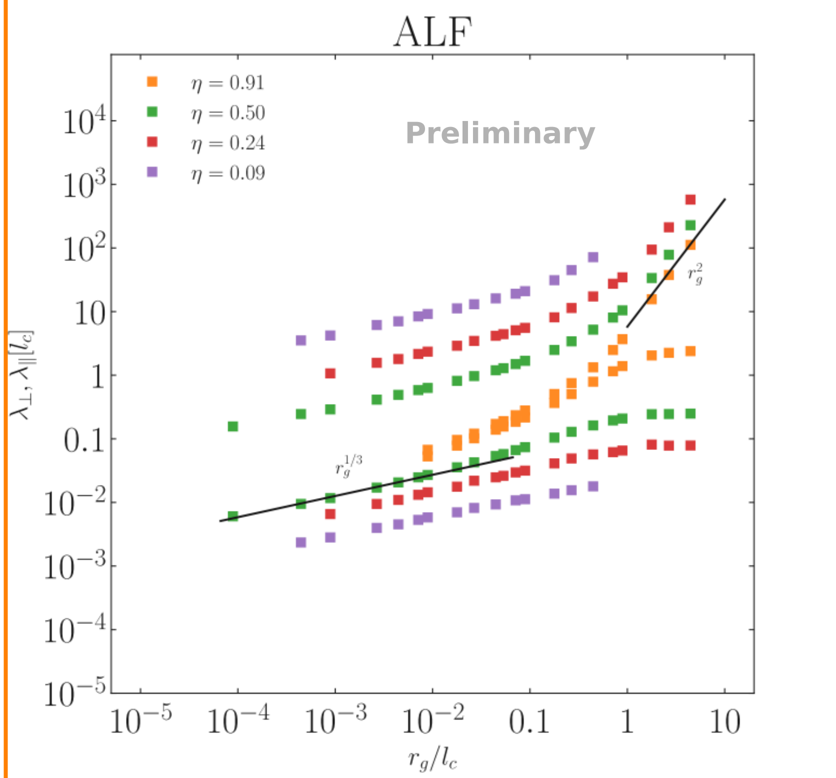
ALF : - QLT scaling and faster transition before flattening



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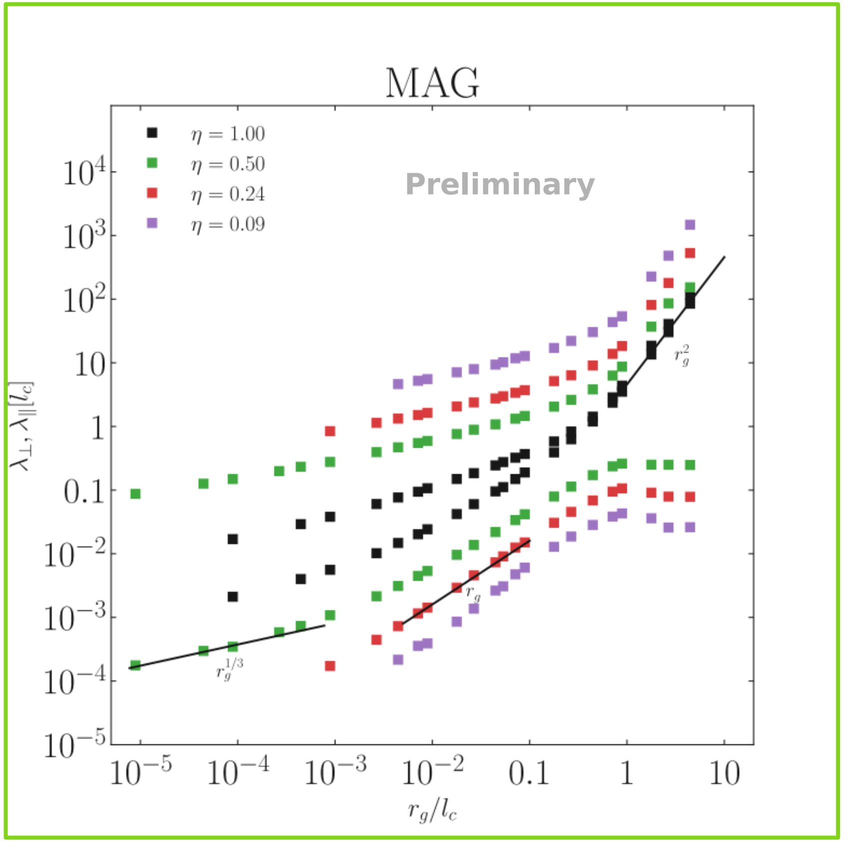
ALF : - QLT scaling and faster transition before flattening

MAG : - intermediate regime propto r_g

We are currently working on predicting these values from FL transport



Bouchet+ (in prep)

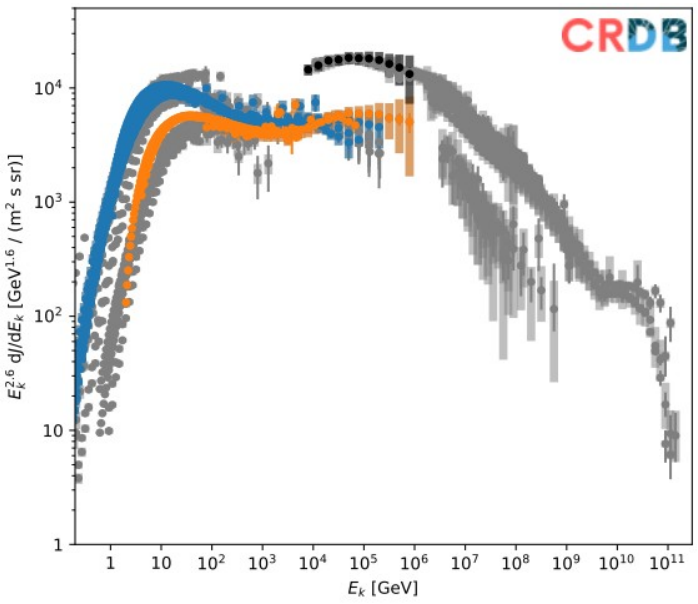
Conclusion

- Context : cosmic ray data from high precision direct and indirect observations → from phenomenolgy to microphysics

- Goal : understand and quantify CR scattering in « simple » synthetic turbulence, still ongoing !

- We found new behaviours for FL & particle transport in magnetosonic-like polarized turbulence : two papers coming soon…

- These findings are small steps towards a general understanding of CR scattering in MHD turbulence.



Thank you !