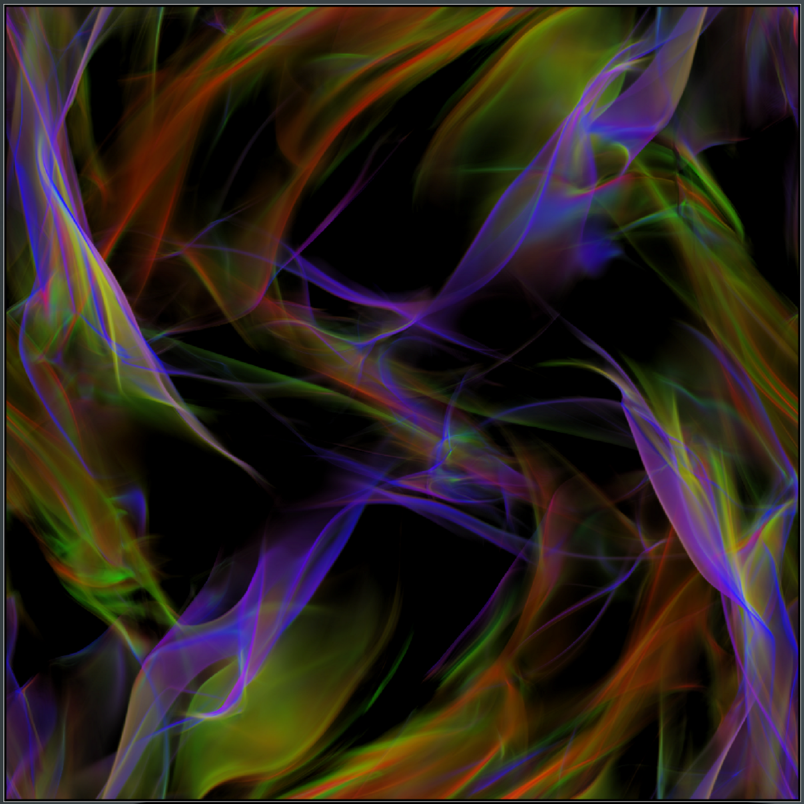
KPAP (2026)

Intermittency in simulations of MHD turbulence

(Pierre Lesaffre, CNRS/LPENS)

MIST
Edith Falgarone

(sea side in the Maldives)



(CSIDEs, Coherent Structures of Intense Dissipation Extrema, in an MHD simulation)

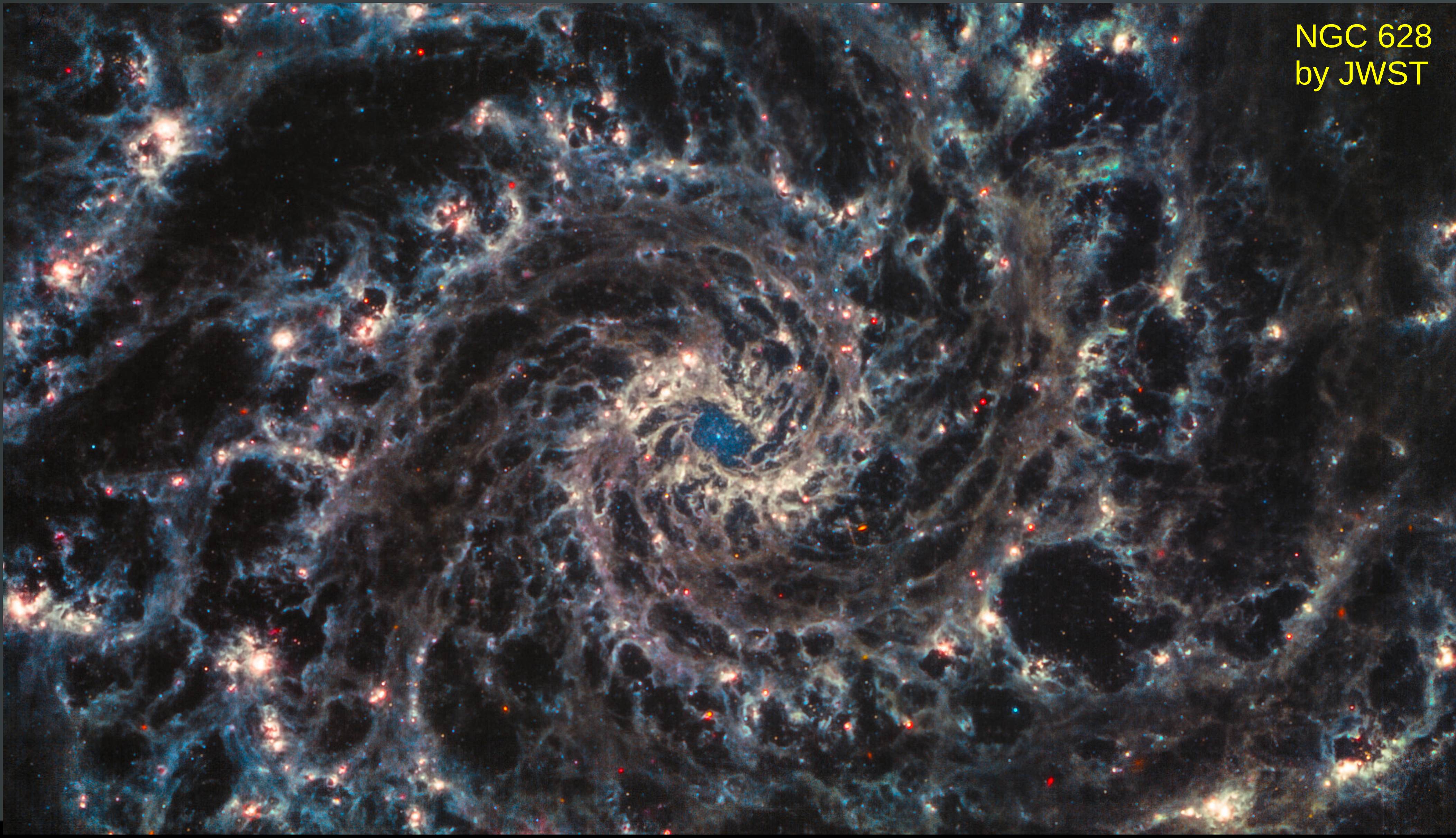
Outline

- 1. Introduction on intermittency
- 2. Coherent structures : CSIDEs and their statistics

  Intermittency revealed by increments of observables
- 3. Infinite Mach number hydrodynamics
- 4. Turbulence synthesis

Disclaimer:

Hydrodynamics, Compressibility, Magnetic fields, heating/cooling, self-Gravity, Cosmic Rays, Dust



1. Characterise turbulence ?

1. Some known statistical properties of 3D incompressible homogeneous turbulence

- Kolmogorov (1941) : ***power spectrum*** $E_u(k) \sim k^{-5/3}$
- Kolmogorov (1962) : ***intermittency*** $P(\log \varepsilon) \sim$ Gaussian

→ lots of measurements and theories on the statistics of increments $\delta_r F = F(x + r) - F(x)$

- Howarth-Karman-Monin equation

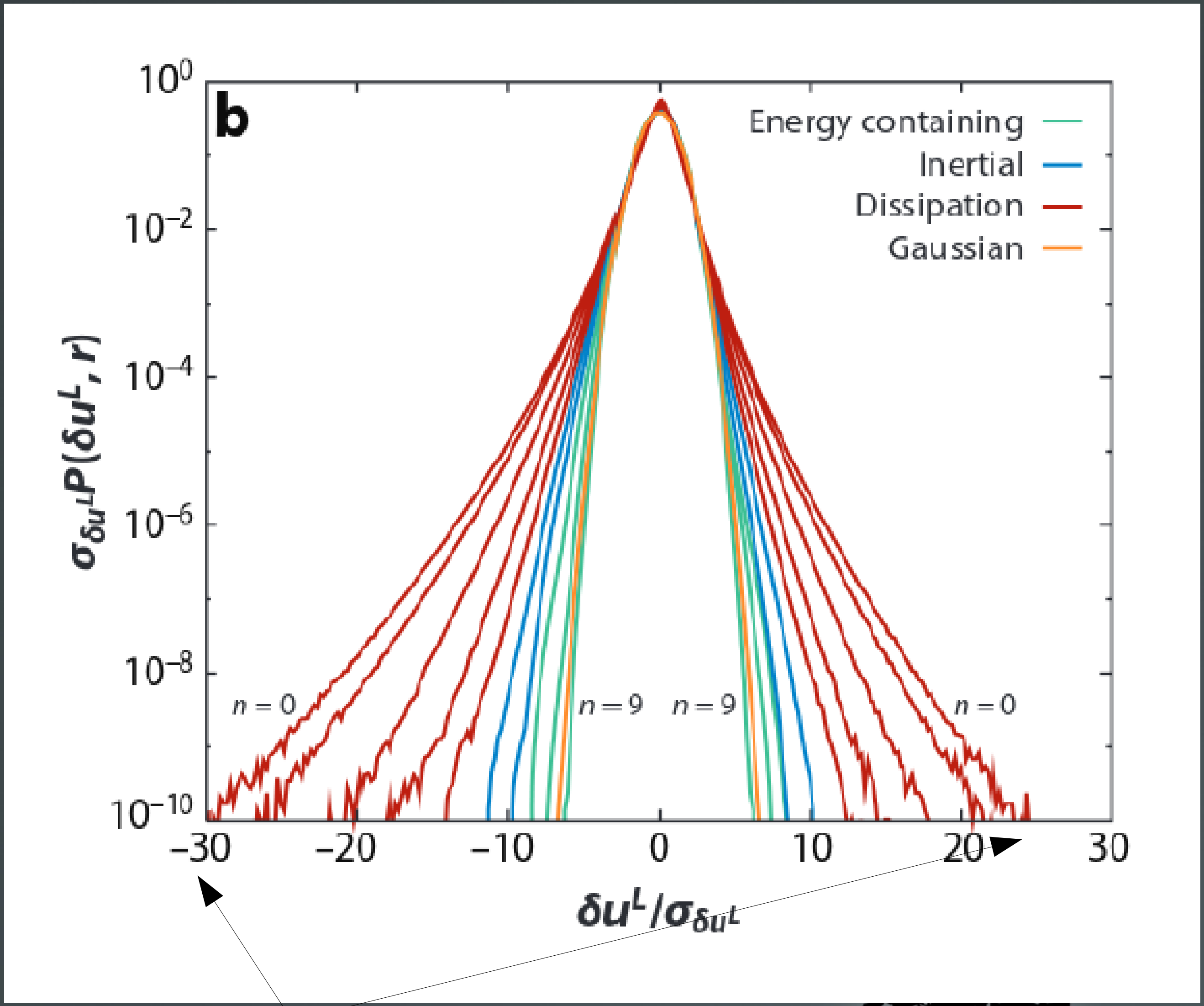
→ ***energy transfer*** function $\langle (\delta_r u_{//})^3 \rangle = -4/5 \langle \varepsilon \rangle r$

for r in the inertial range

1. Statistics of velocity increments

Ishihara et al. (2009)

Incompressible HD
DNS 4096^3



Real HD turbulence" is asymmetric !

This is at the origin of the energy transfer (non zero $< (\delta_r u_{//})^3 >$)

1. Some known statistical properties of 3D MHD turbulence

- The power-law scaling itself is disputed:

  Iroshnikov(1963)-Kraichnan(1965) advocate -3/2

  Goldreich & Sridhar (1995) want -5/3

  - See Schekochihin (2022) for a fair discussion
- Intermittency scaling derived in either case:
  - in the -3/2 case, Politano & Pouquet(1995) Grauer(1994)
  - in the -5/3 case, Boldyrev (2002)

2. CSIDEs

“Coherent Structure of Intense Dissipation Extrema”

(Lesaffre, Falgarone, Hily-Blant 2024 ; Richard et al. 2022 ; Lesaffre et al. 2020 “CHEMSES”)

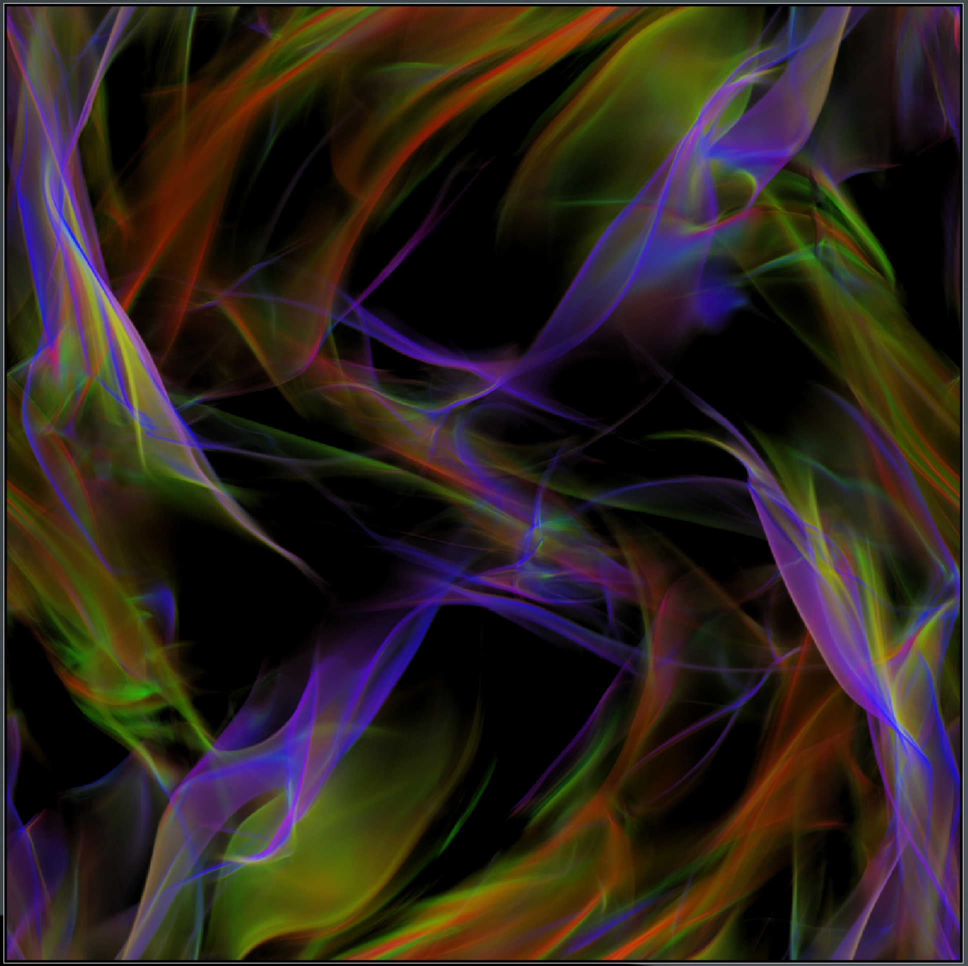
(sea side in the Maldives)

Bioluminescence in waves: plankton highlights
Strong shear change

2. Turbulent dissipation

Strong dissipation structures are on sheets, ribbons and filaments, which project as ridges on the plane of sky.

Lesaffre+2024



Dissipation map:

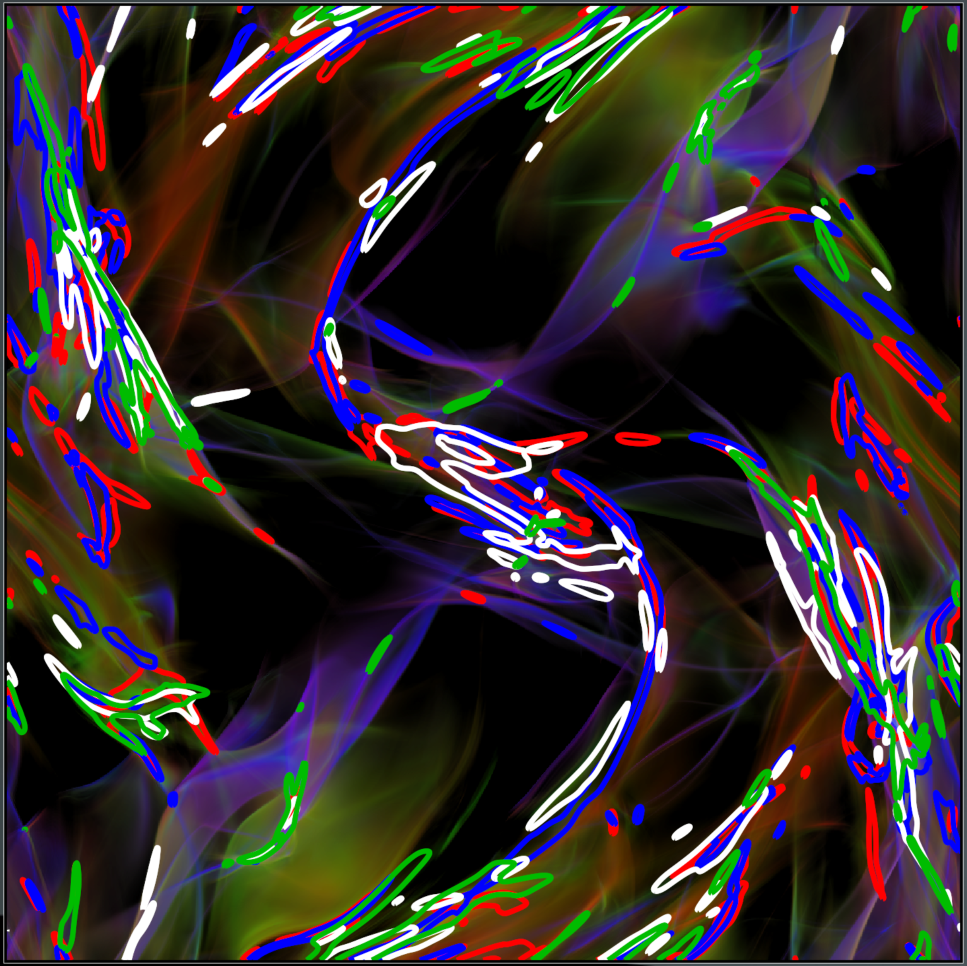
- Ohmic
- Viscous shear
- Viscous compression

Sonic Mach=4
Alfvén Mach=1
Re=9000
Orszag-Tang initial cond.

2. Observable signatures of turbulent dissipation

The ***increments*** of l.o.s. Integrated observables match with dissipation caustics

Lesaffre+2024



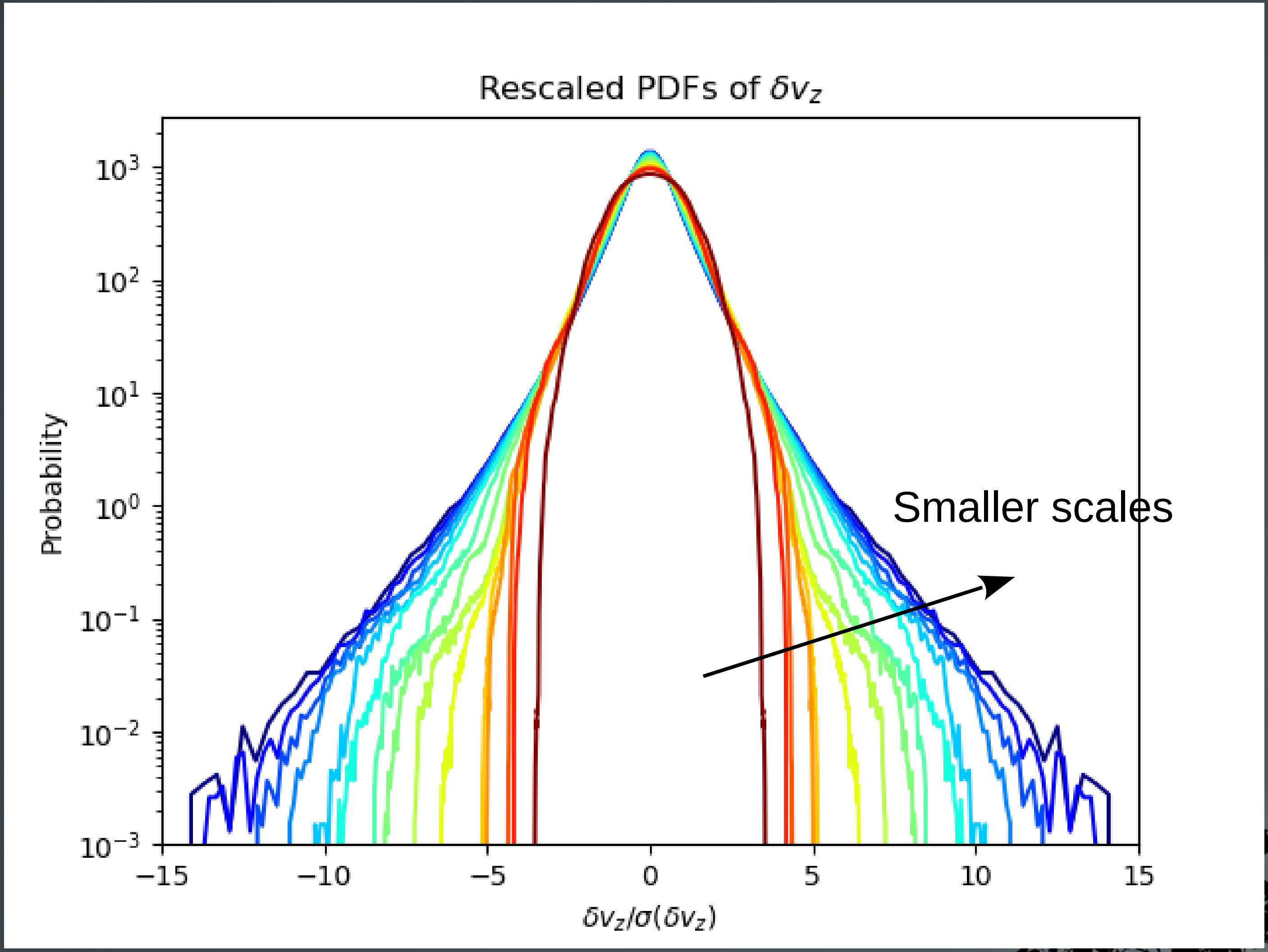
Observables contours:

- Stokes Q
- Stokes U
- Column-Density
- Centroid velocity

Sonic Mach=4
Alfvén Mach=1
Re=9000
Orszag-Tang initial cond.

2. Increments of projected observables

Lesaffre+2024



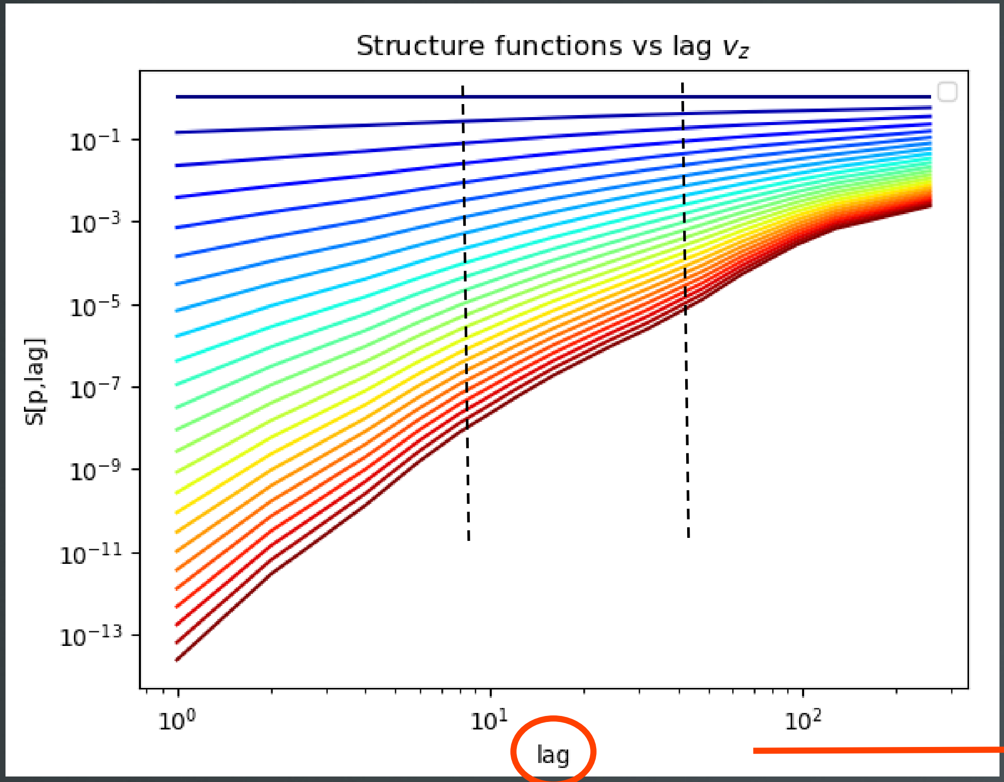
(Mach 4, Orszag-Tang, @ peak dissipation)

2. Extended Self-Similarity ?!

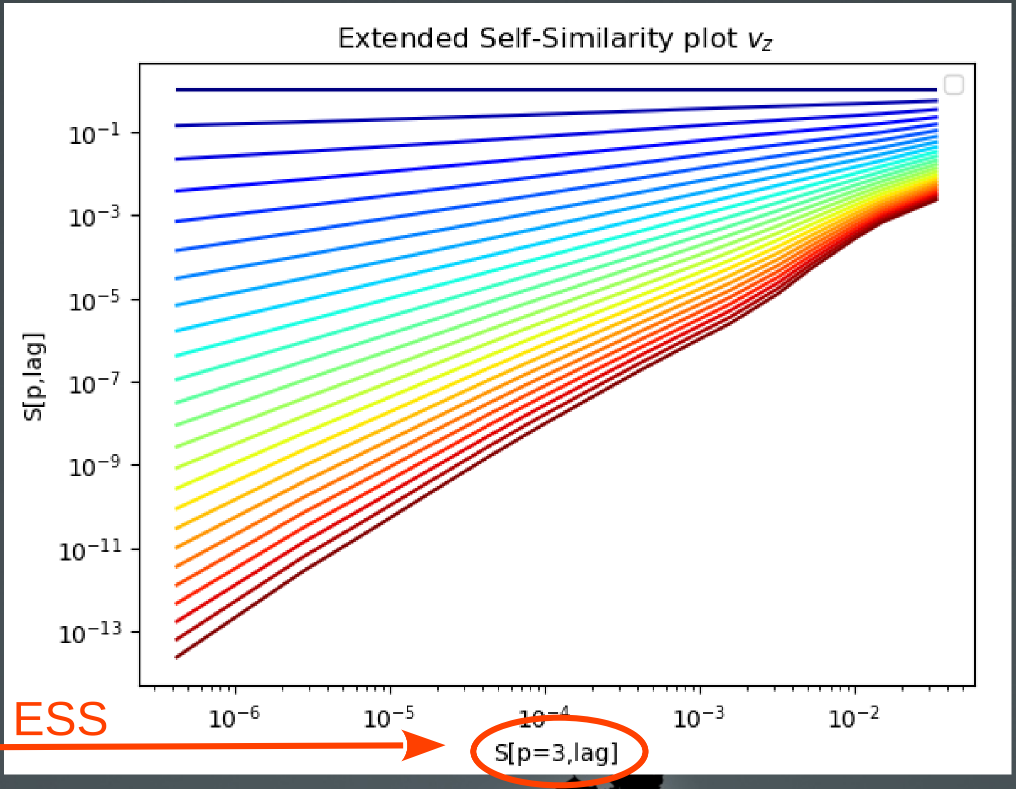
Let’s extend Structure function definition to projections of observables:

$$S(\ell, p) =< |\delta_{\boldsymbol{\ell}} X|^p >_{\ell \leq |\boldsymbol{\ell}| < \ell+1}$$

Lesaffre+2024



ESS



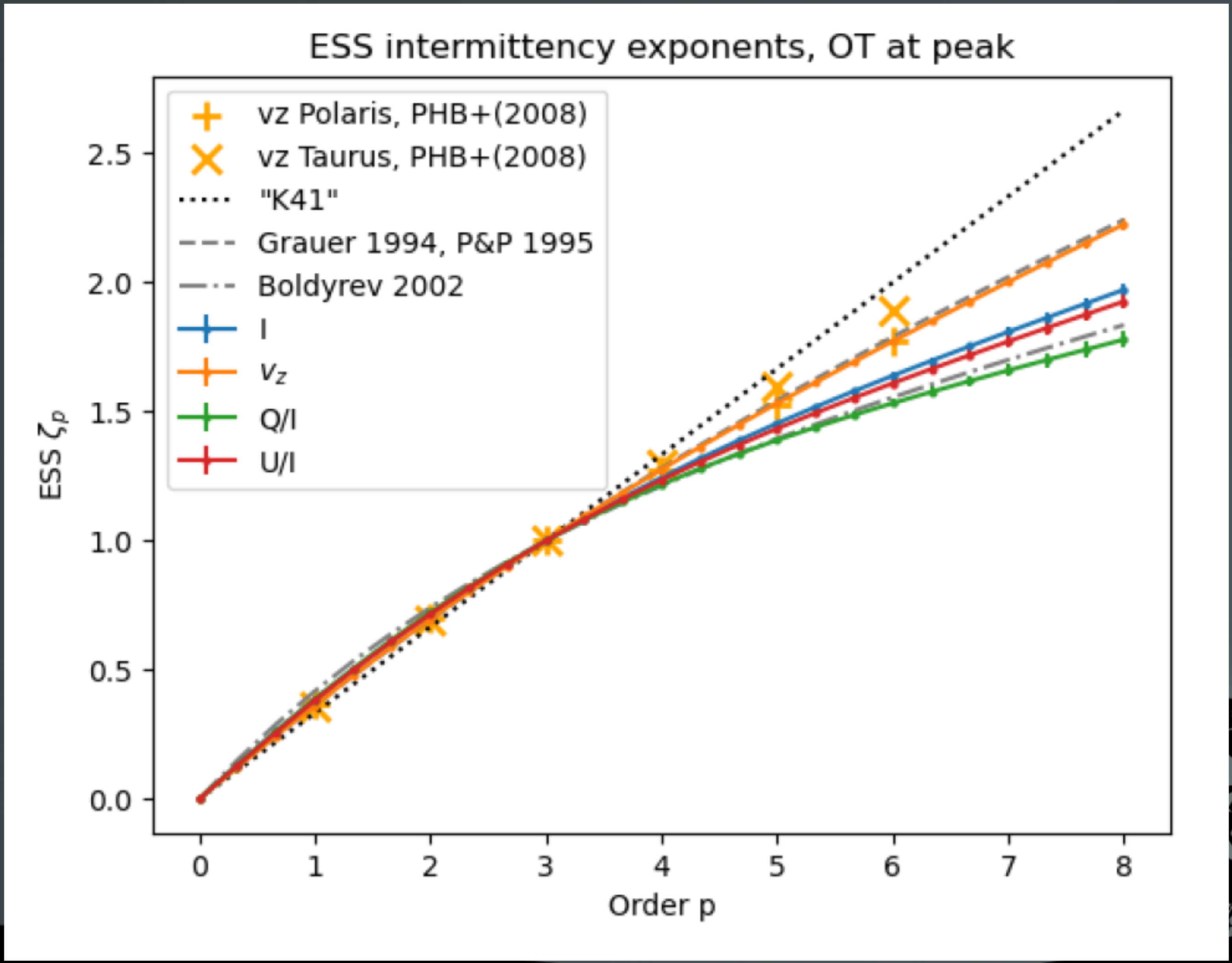
Benzi (1996) discovers that replacing ℓ by S(3,ℓ) improves power law scalings

=> Linear least square fits to get slopes ζ(p) with *error bars* to assess *goodness of fit*

(Mach 4, Orszag-Tang, @ peak dissipation)

2. Increments statistics as (not good) probes of textures

Lesaffre, Falgarone, Hily-Blant (2024) with “ESS”



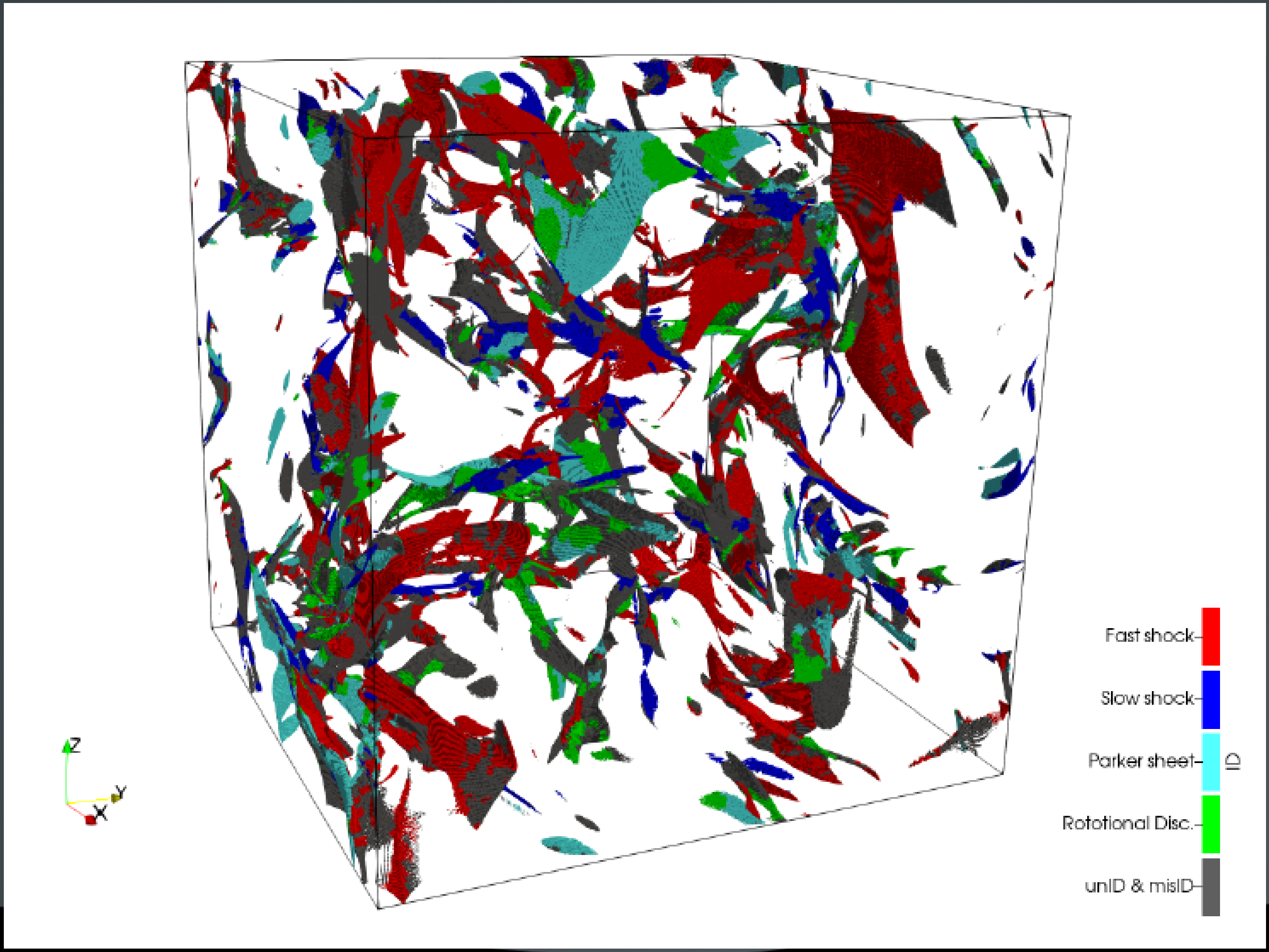
2. The nature of CSIDEs in isothermal MHD turbulence

Regions of intense dissipation heating (4 σ over mean)
Early time (1/3 turnover) snapshot of initial OT flow

Fast shock **Slow shock** **Rotational Discontinuity** **Parker sheet**

Richard+(2022)

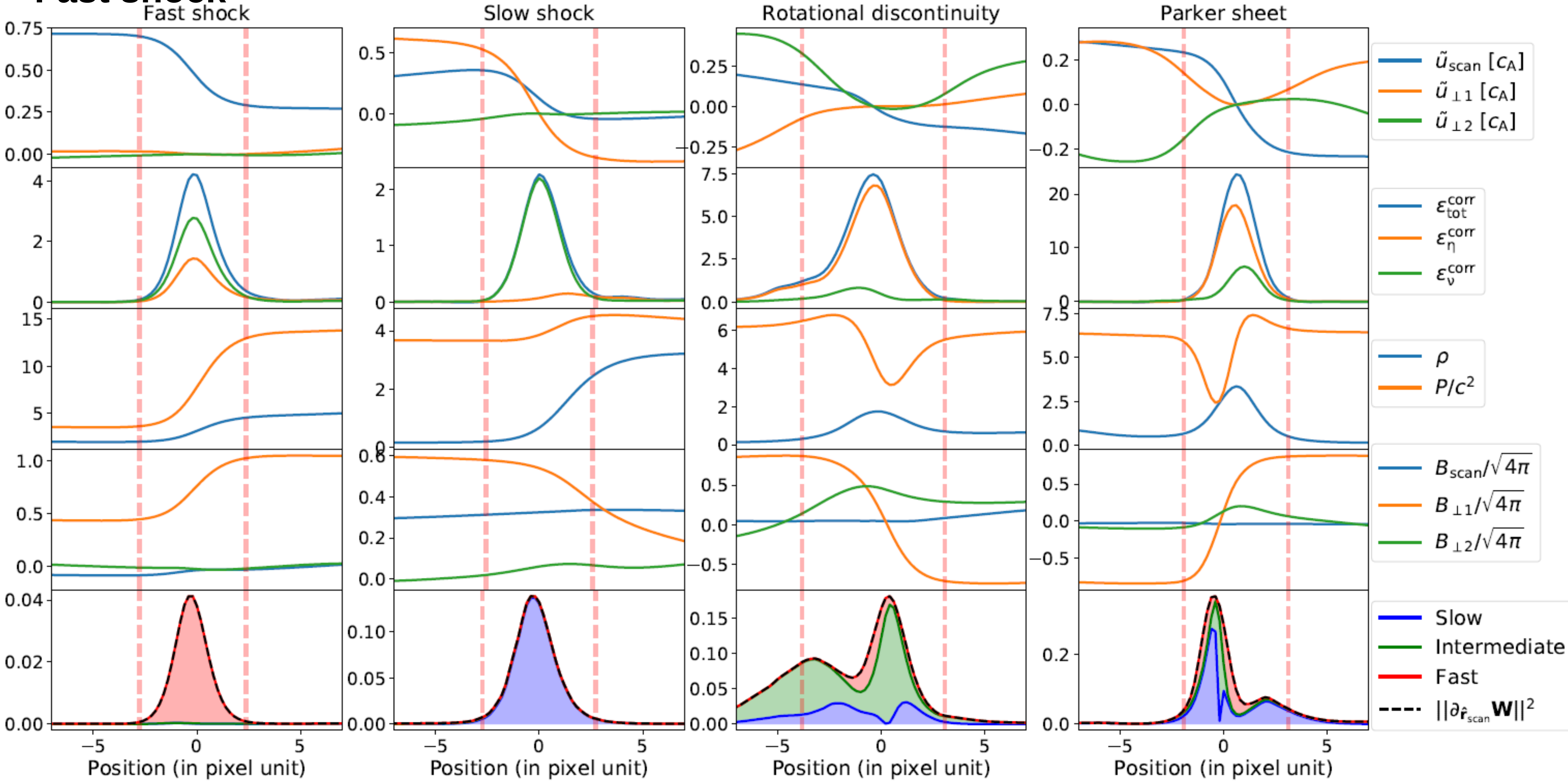
DUMSES isothermal MHD simulations, decaying from
Sonic Mach=4
Alfvén Mach=1
Re=9000, Pm=1
N=1024^3, periodic



CSIDES Identification

See also: Uritsky (2010), Momferratos (2014), Lehmann (2016), Zhdankin (2013,14,15,16)

Fast shock | **Slow shock** | **Rotational disc.** | **Parker sheet**



➡ Heuristic rules to classify structures + Gradient decomposition into wave strength

- Density **step**
- Total pressure **step**
- Transverse magnetic field **rise**

- Density **step**
- Total pressure **step**
- Transverse B field **decrease**

- Density **peak**
- **Trough** in total pressure
- **Trough** in transverse B field

- Density **peak**
- **Trough** in total pressure
- **Trough** in transverse B field

Effect of dissipation coefficients

Dissipation coefficients do not change the statistics of dissipative structures
Rather, initial conditions control their macroscopic parameters.

Richard+ (2022)

Varying I. C.

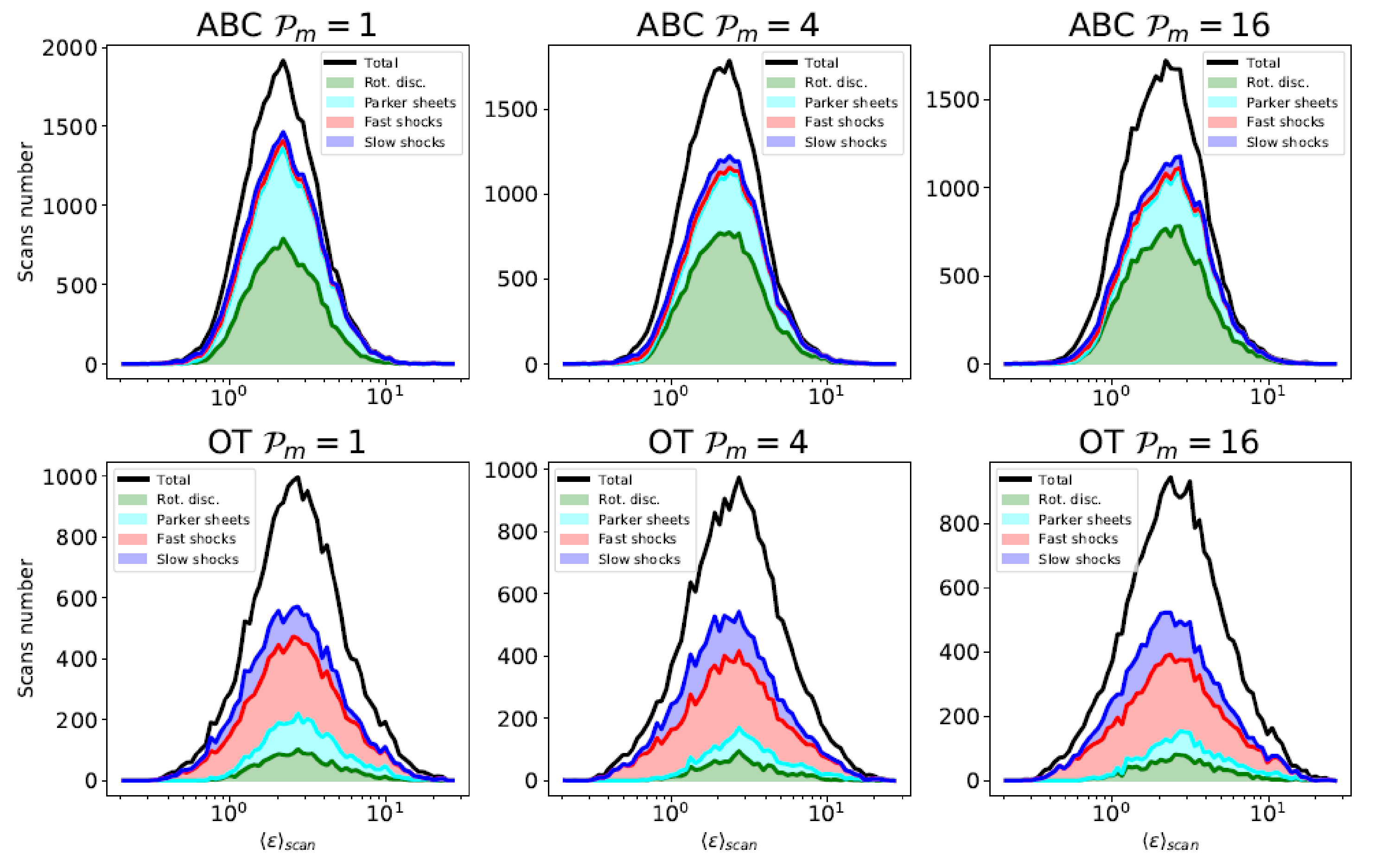


Fig. 13: dissipation structures distributions for our two initial conditions with varying magnetic Prandtl number from $\mathcal{P}_m = 1$ on the left to $\mathcal{P}_m = 16$ on the right. The time step shown here is at early time, near the dissipation peak.

Varying Pm

3. Mach=∞

Ambipolar isothermal MHD equations

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{u}) = 0 \qquad (1.5)$$

$$\partial_t \rho u_i + \partial_j(\rho u_i u_j + p\delta_{ij} - \nu\rho S_{ij}[u]) - \boldsymbol{J} \times \boldsymbol{B} = 0. \qquad (1.6)$$

$$\partial_t \boldsymbol{B} - \boldsymbol{\nabla} \times \left(\boldsymbol{u} \times \boldsymbol{B} + \frac{1}{F_{in}} (\boldsymbol{J} \times \boldsymbol{B}) \times \boldsymbol{B} - \eta \boldsymbol{\nabla} \times \boldsymbol{B} \right) = 0. \qquad (1.7)$$

With the viscous stresses:

$$S_{ij}[u] = \frac{1}{2}(\partial_i u_j + \partial_j u_i) - \frac{1}{3}\partial_k u_k \delta_{ij} \qquad (1.8)$$

And the ion-neutral momentum exchange rate:

$$F_{in} = \rho_c \rho \left\langle \sigma v \right\rangle_{in} / (\mu_c + \mu_n) \qquad (1.9)$$

Global form with a diffusive flux $\boldsymbol{\mathcal{F}}_d$:

$$\partial_t \boldsymbol{W} + \boldsymbol{\nabla}.[\boldsymbol{\mathcal{F}}(\boldsymbol{W}) + \boldsymbol{\mathcal{F}}_d(\boldsymbol{W}, \partial_i \boldsymbol{W})] = 0$$

3. Mach=∞, inviscid

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.(\rho \boldsymbol{u}) = 0 \tag{1.5}$$

$$\partial_t \rho u_i + \partial_j(\rho u_i u_j + \cancel{p\delta_{ij}} - \nu\rho \cancel{S_{ij}}[u]) - \boldsymbol{J} \cancel{\times} \boldsymbol{B} = 0. \tag{1.6}$$

$$\partial_t \boldsymbol{B} - \boldsymbol{\nabla}\times\left(\boldsymbol{u}\times\boldsymbol{B} + \frac{1}{F_{in}}(\boldsymbol{J}\cancel{\times}\boldsymbol{B})\times\boldsymbol{B} - \eta\boldsymbol{\nabla}\cancel{\times}\boldsymbol{B}\right) = 0. \tag{1.7}$$

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3. New solution to inviscid Burger's equation

$$\partial_t u = -u\,\partial_x u$$

Johannes Martinus Burgers

JB Durrive: Taylor expand at t=0 and use Cauchy Formula →
Obtain a closed form at any time which depends on initial conditions :

Jean-Baptiste Durrive

$$u(t,x) = \frac{1}{2i\pi t} \oint \ln\left(1 + \frac{u_0(z)t}{z-x}\right) dz$$

(Note: similar formulae for 2D and 3D can be obtained, for Burgers and for HD at infinite Mach number, too)

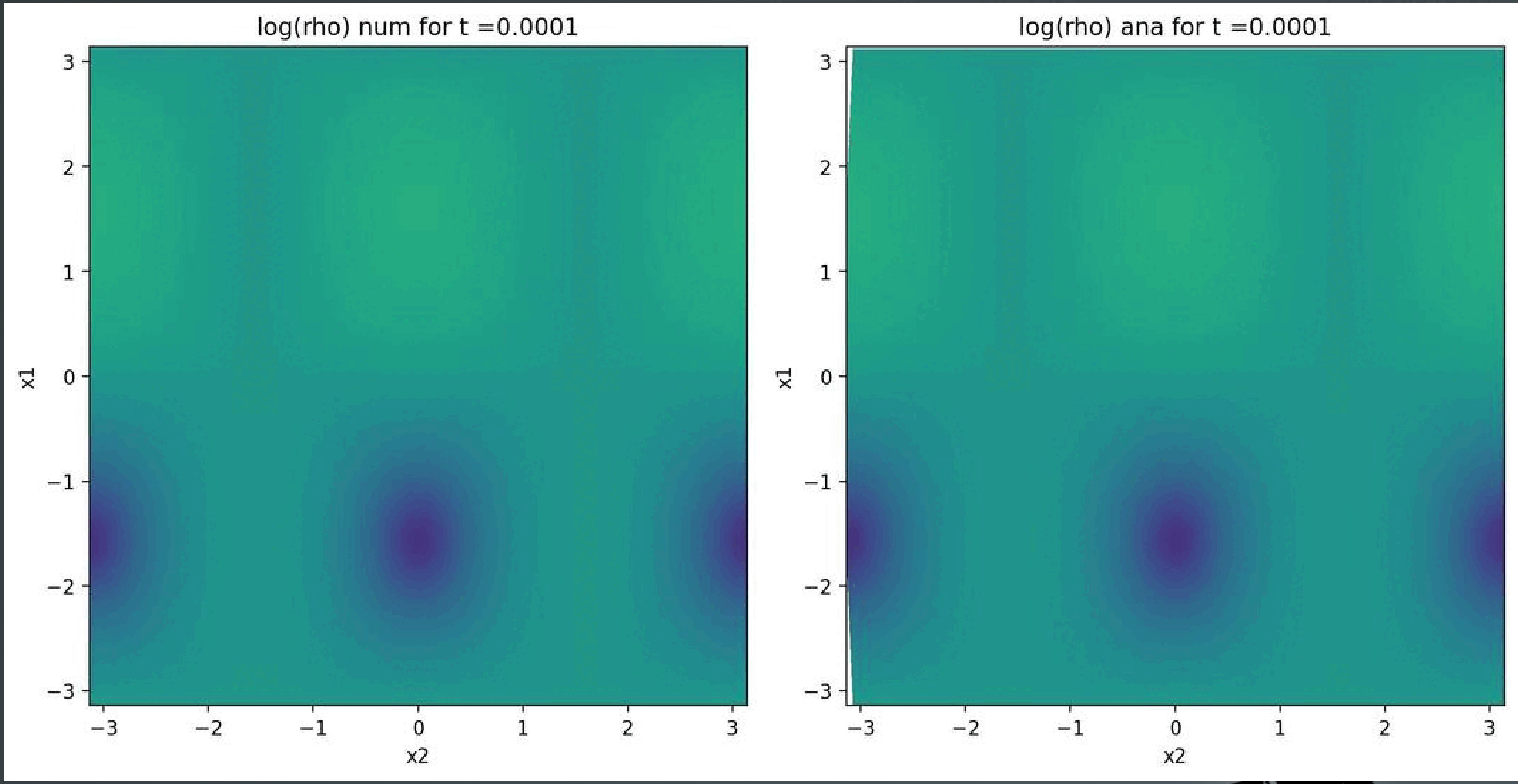
Our agenda:

Taylor expand the physical system of differential equations at time origin,
then use mathematical arsenal to find efficient approximations to the obtained series

Advantages:
no need to compute intermediate times,
no need to compute the solution everywhere

Drawback:
Choose complex contour integral

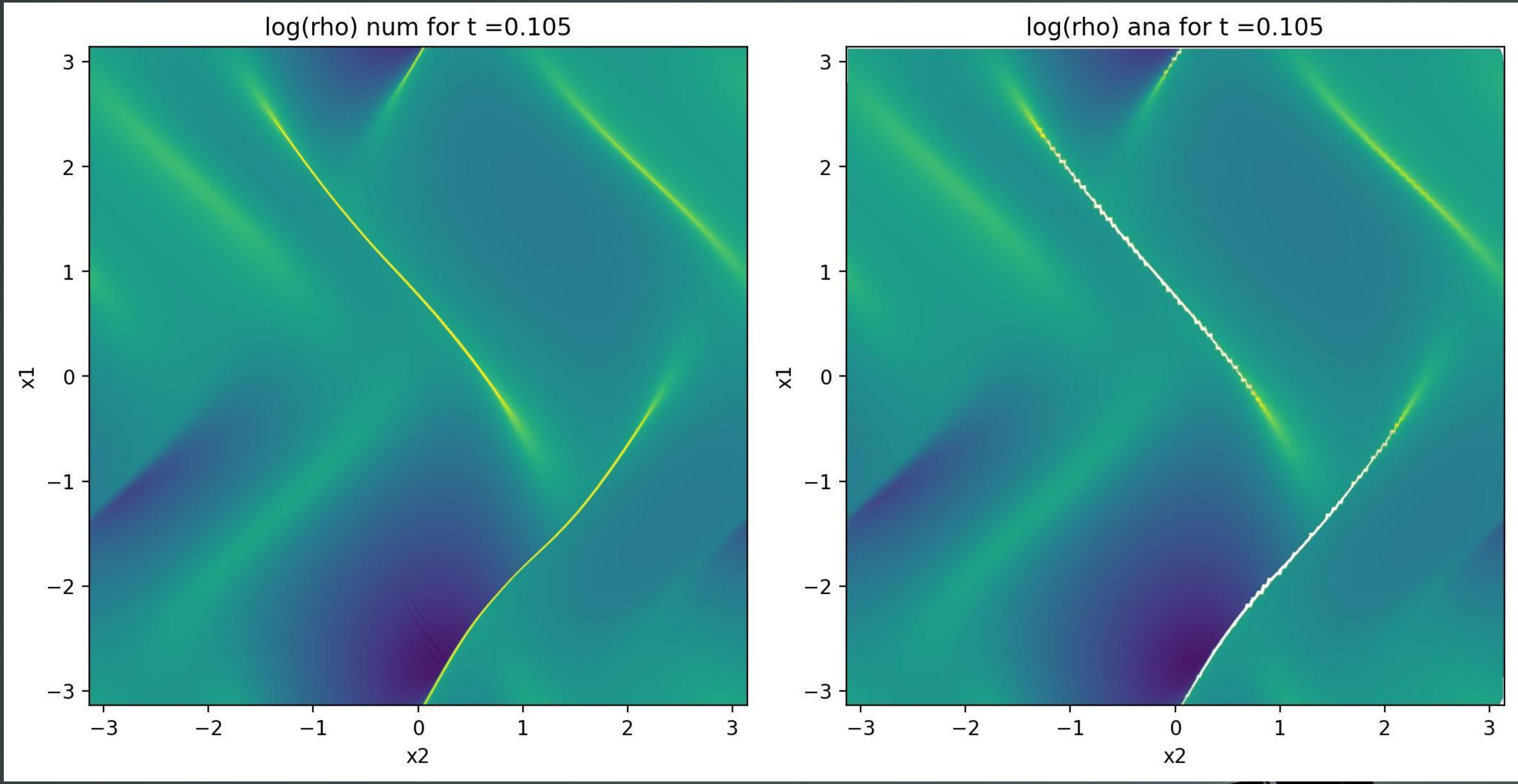
3. Example: 2D HD Mach=∞



2D Godunov with
Lax-Friedrich Riemann solver

Analytical Formula
(involves a double integral)

3. Example: 2D HD Mach=∞



2D Godunov with
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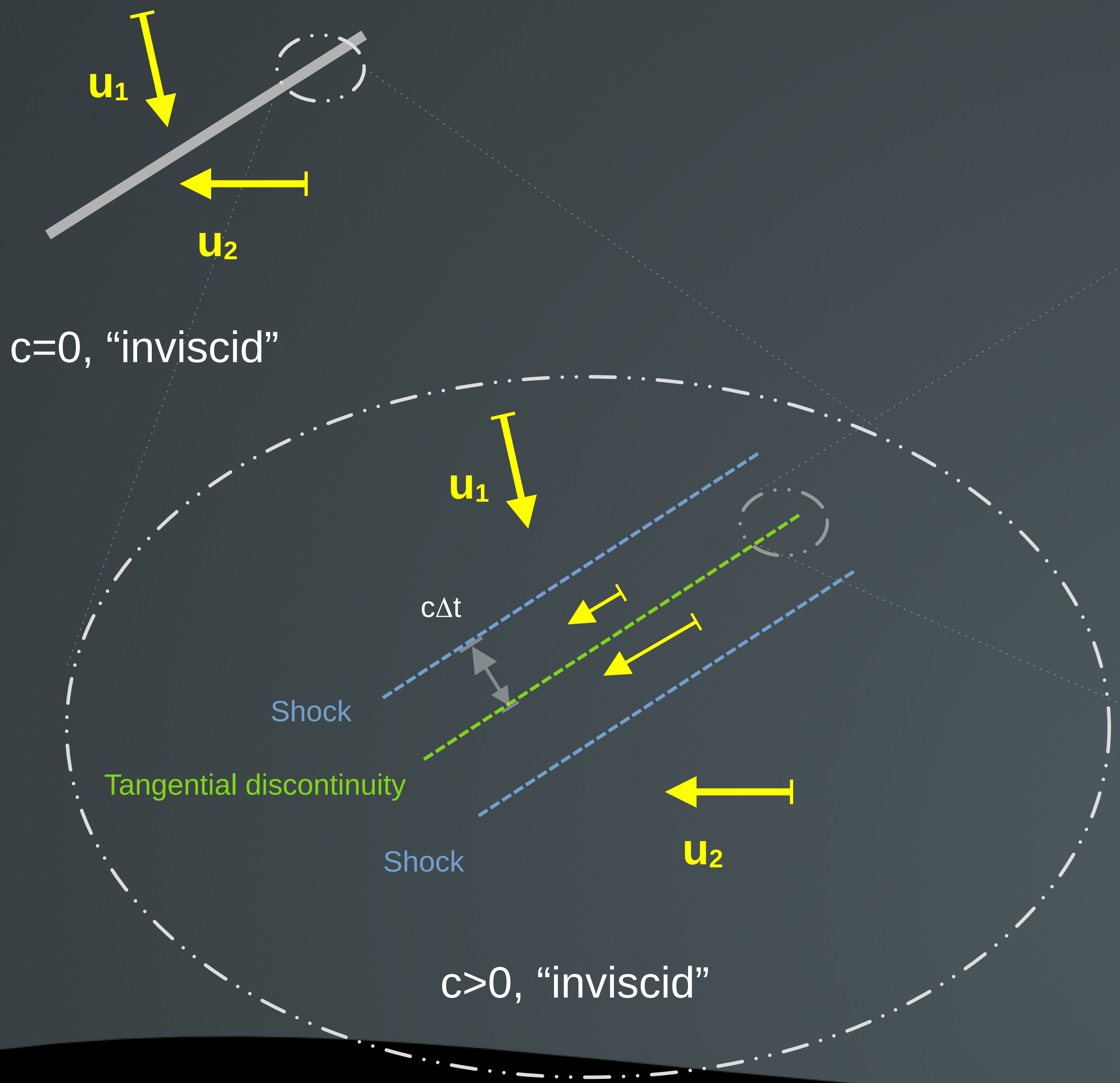
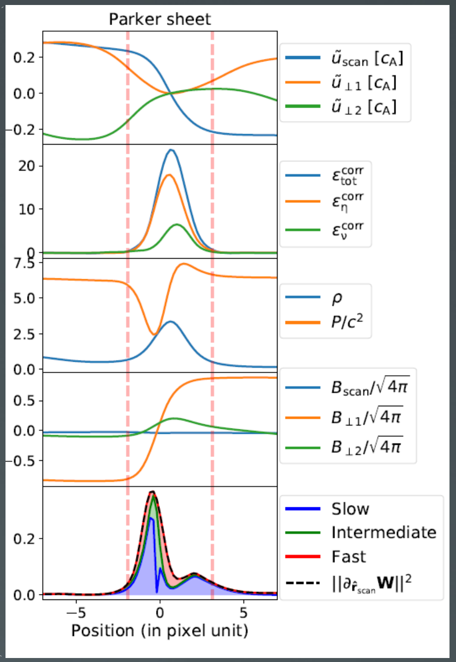
3. Mach=∞, “inviscid”

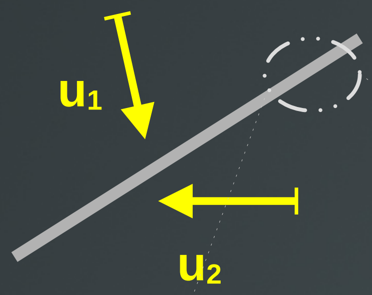


c>0, ν>0 , η>0



3. Mach=∞, “inviscid”

Conservation of
mass, momentum, B
inside the Burgers ”shock” surface
=> **“adhesion” model**



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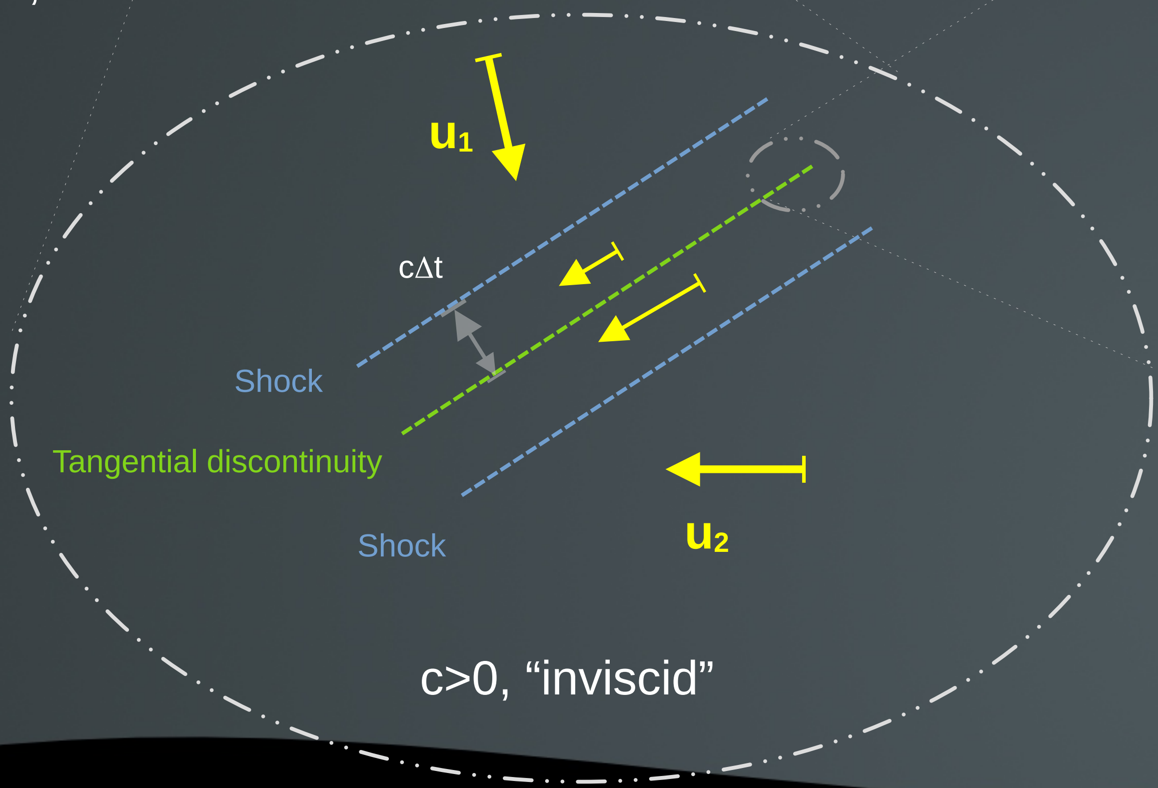

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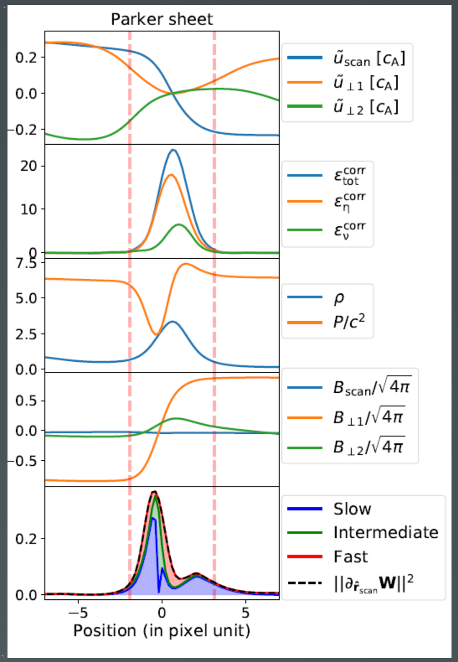

3. Mach=∞ Adhesion model
Ballistics

$$\mathbf{x} = \mathbf{X} + \mathbf{u_0}(\mathbf{X})t$$

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Ballistics

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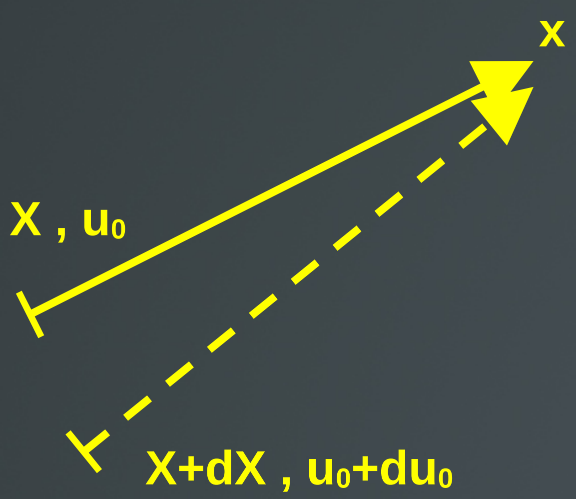


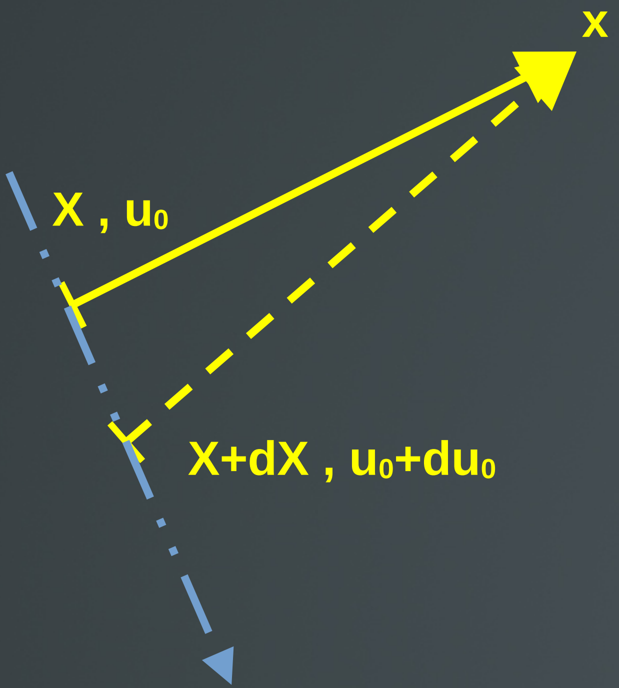
$$\delta\mathbf{x} = (\mathbb{1} + t\mathbf{J})\cdot\delta\mathbf{X}$$

$$\mathbf{J} = \frac{\partial\mathbf{u_0}}{\partial\mathbf{X}}$$

Shock when **t = -1/λ** with λ ***most negative*** eigenvalue of jacobian **J**

3. Mach=∞ Adhesion model
Ballistics

$$\mathbf{x} = \mathbf{X} + \mathbf{u_0}(\mathbf{X})t$$



$$\delta\mathbf{x} = (\mathbb{1} + t\mathbf{J})\cdot\delta\mathbf{X}$$

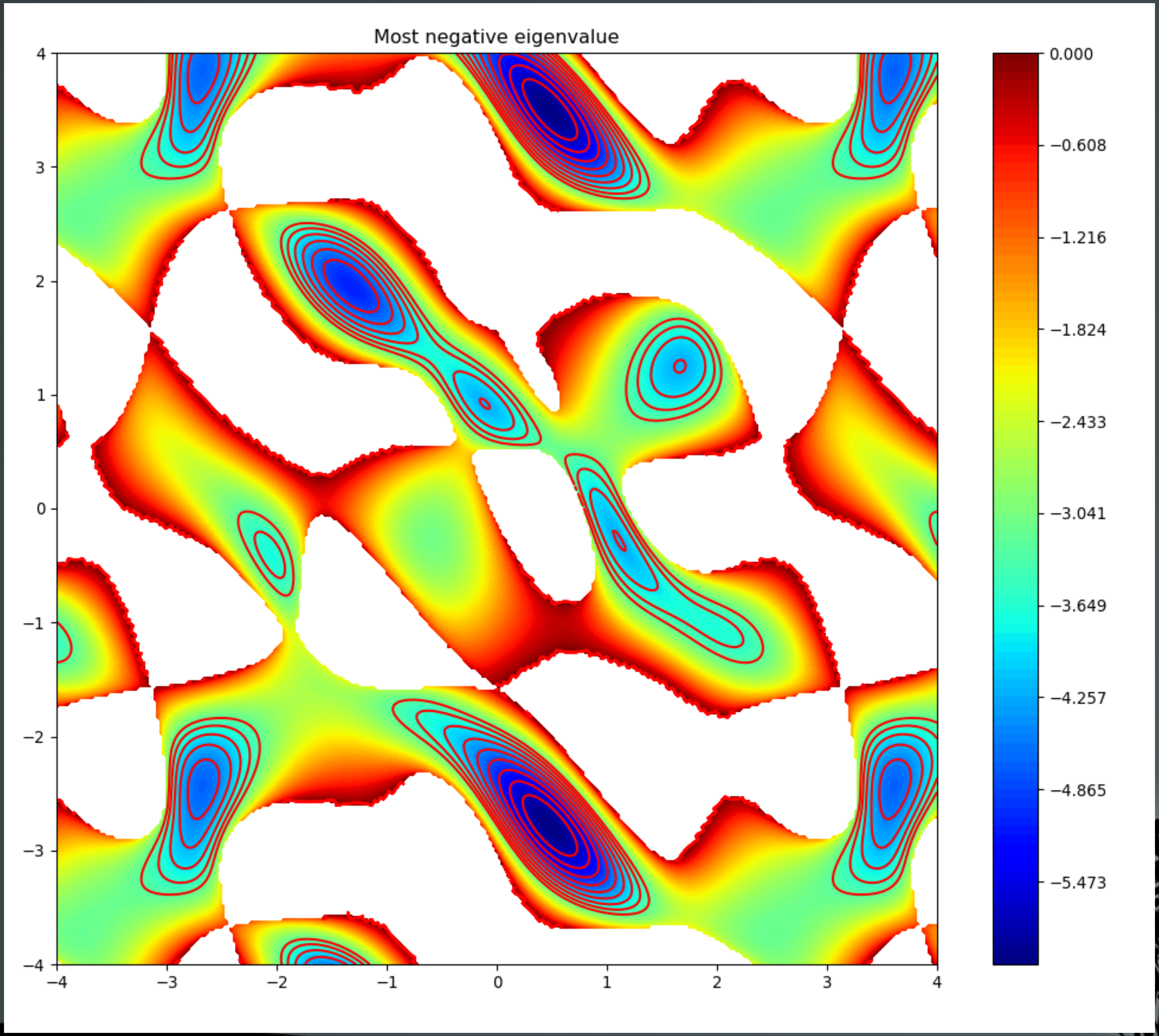
$$\mathbf{J} = \frac{\partial \mathbf{u_0}}{\partial \mathbf{X}}$$

Shock when $t = -1/\lambda$ with λ most negative eigenvalue of jacobian J

Points in the direction of the λ eigenvector end up on the same position.

3. Mach=∞ Adhesion model

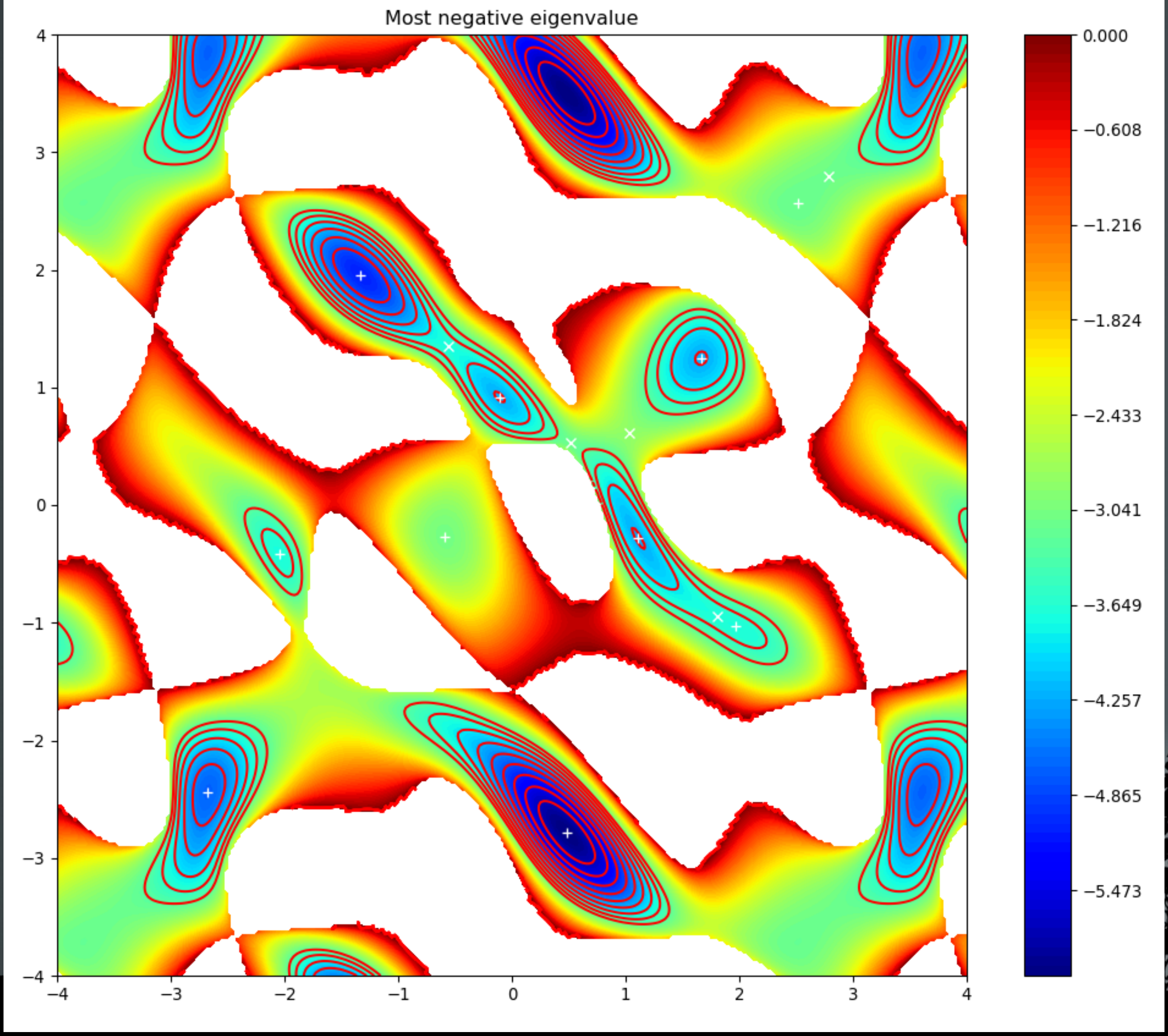
Lagrange



3. Mach=∞ Adhesion model

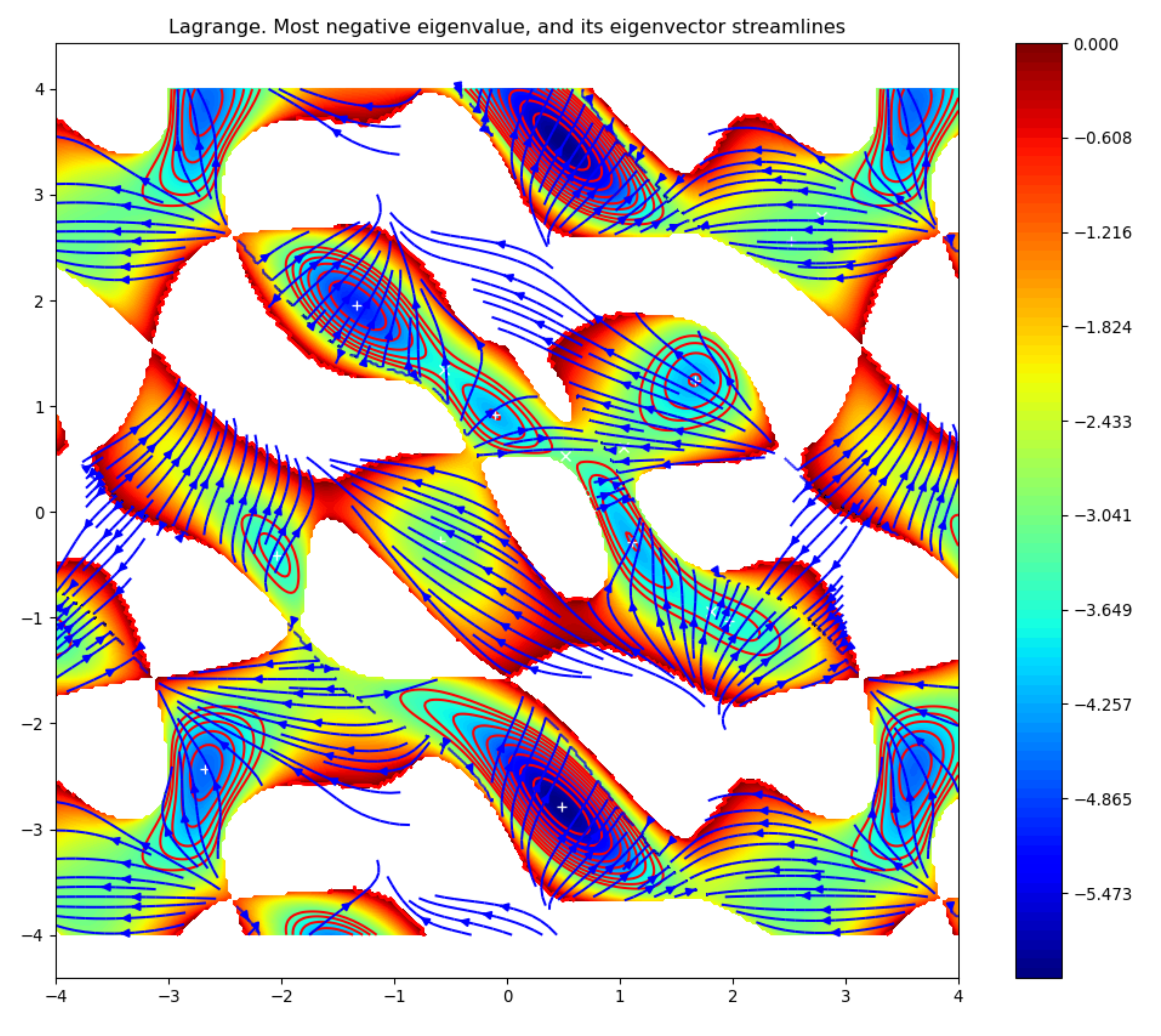
Shocks start at eigenvalue minima

Lagrange



3. Mach=∞ Adhesion model

Edges of the shock where isocontours and eigenvectors are tangent



Lagrange

3. Mach=∞ Adhesion model

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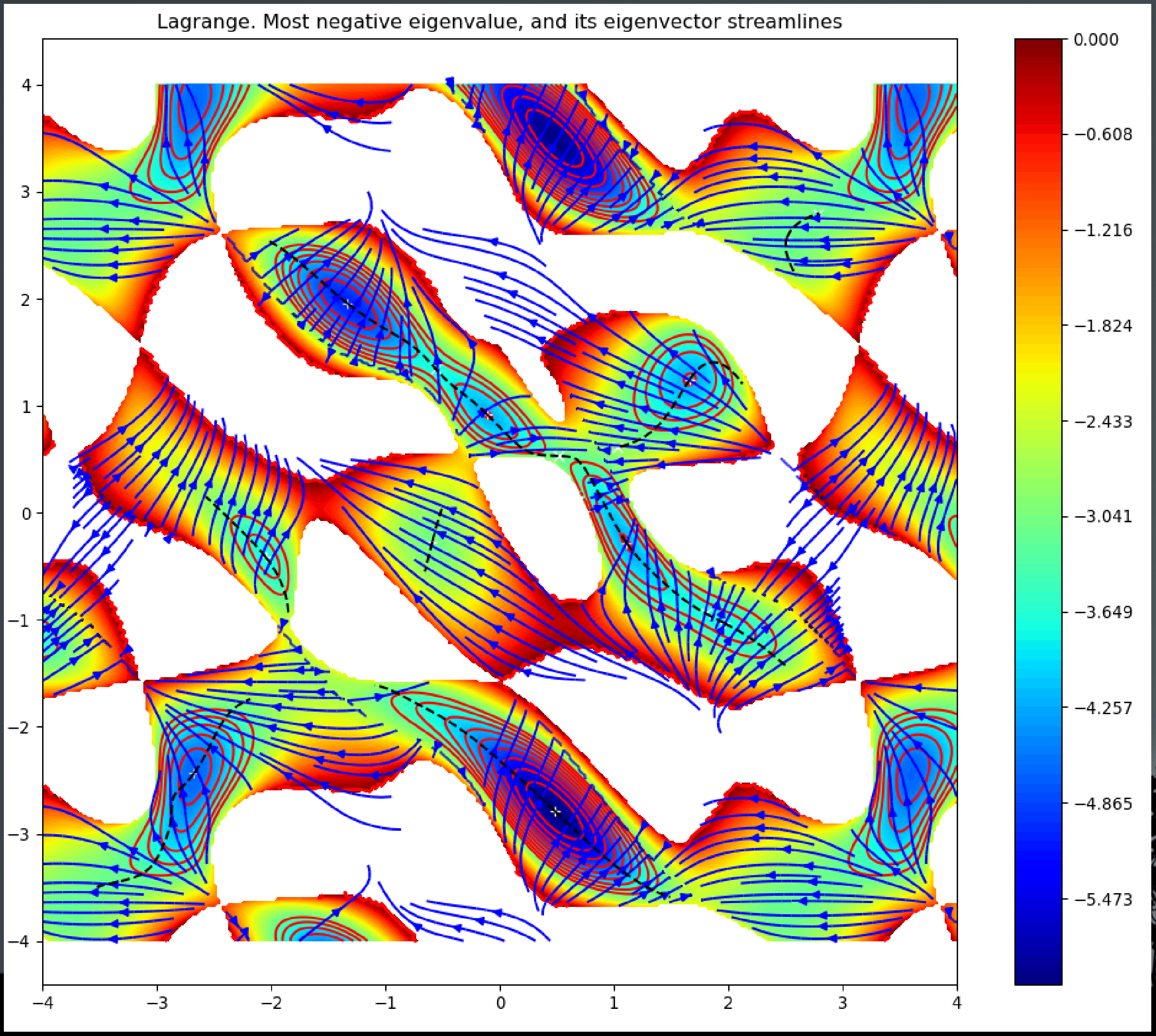

Lagrange

3. Mach=∞ Adhesion model

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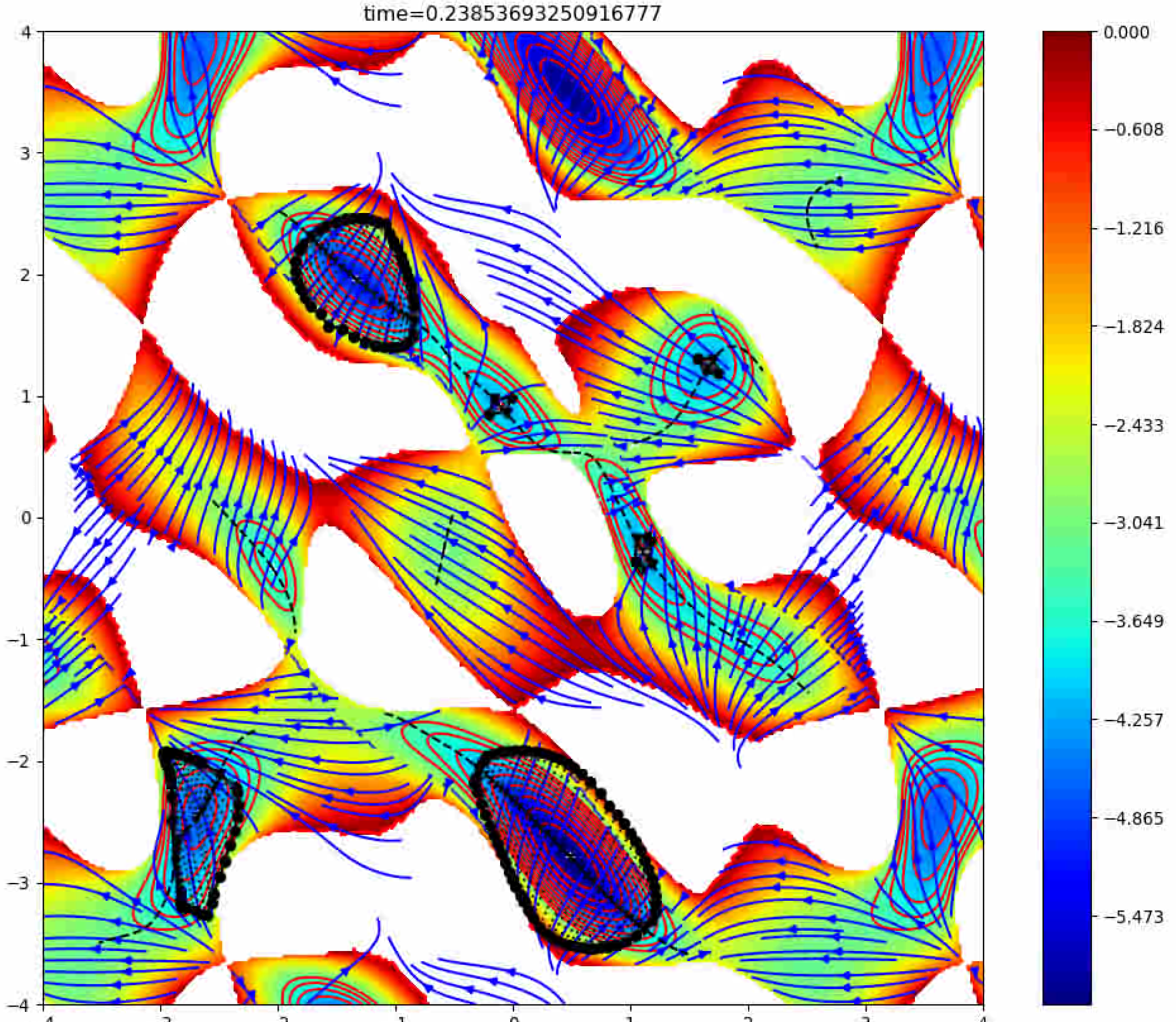
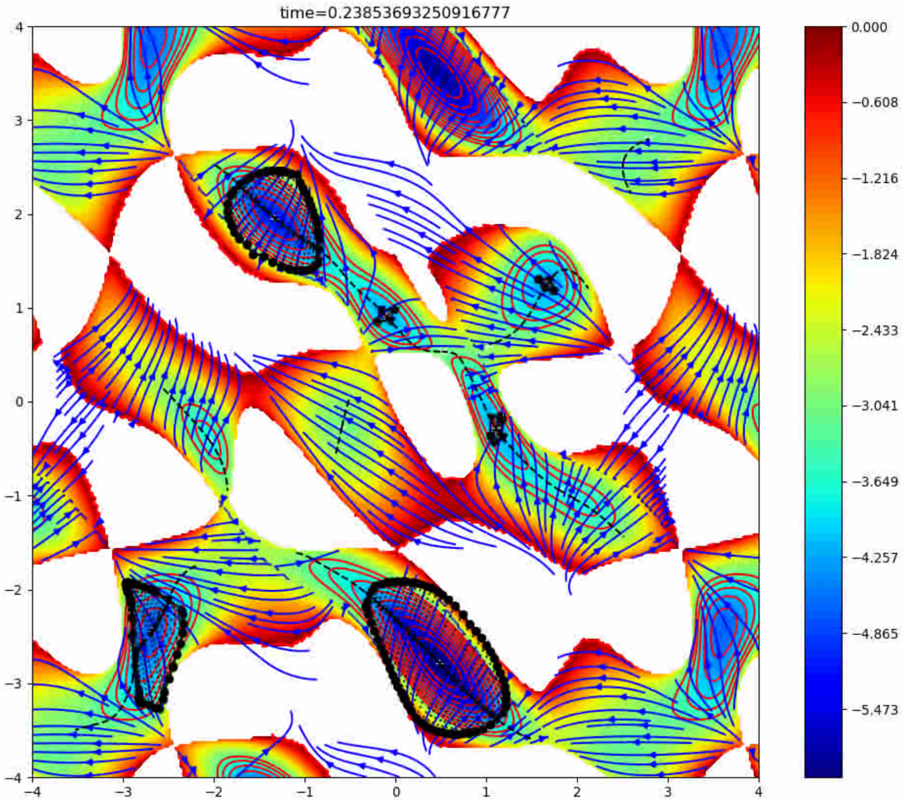
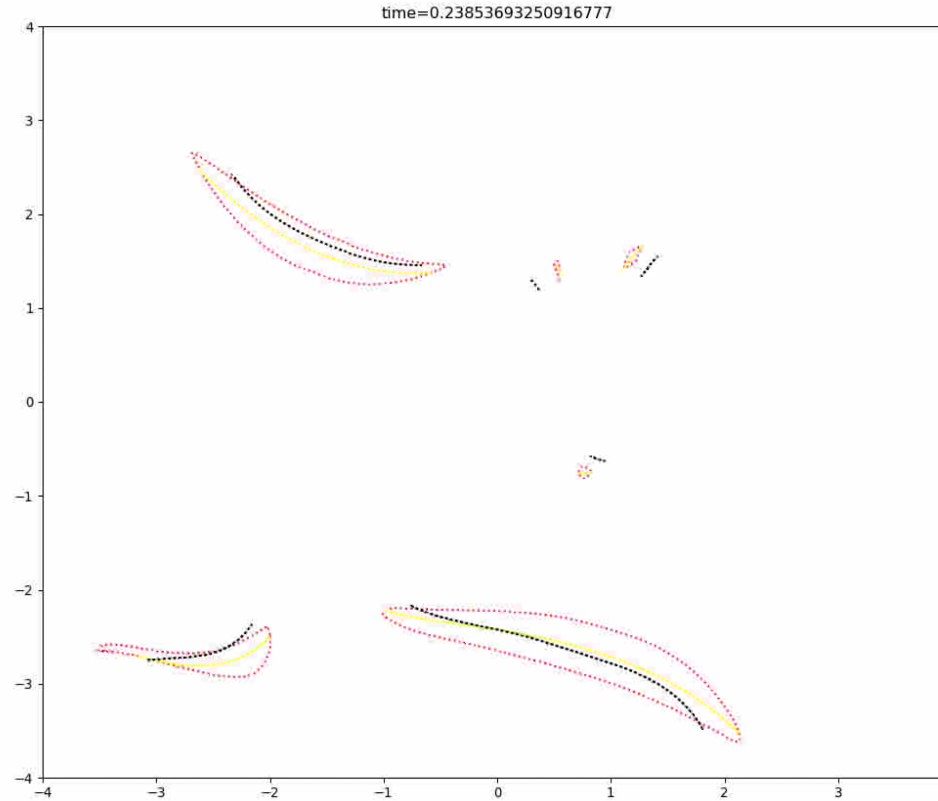


3. Mach=∞ Adhesion model

Lagrange



Euler

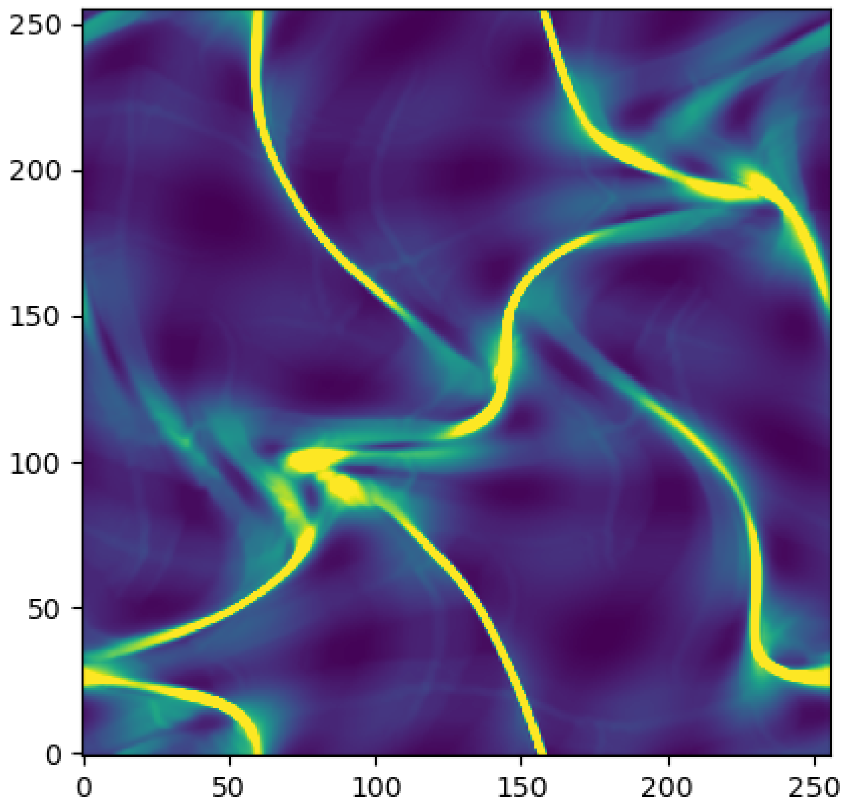


$$\mathbf{x} = \mathbf{X} + \mathbf{u_0}(\mathbf{X})t$$

3. Mach=∞ Adhesion model

Euler

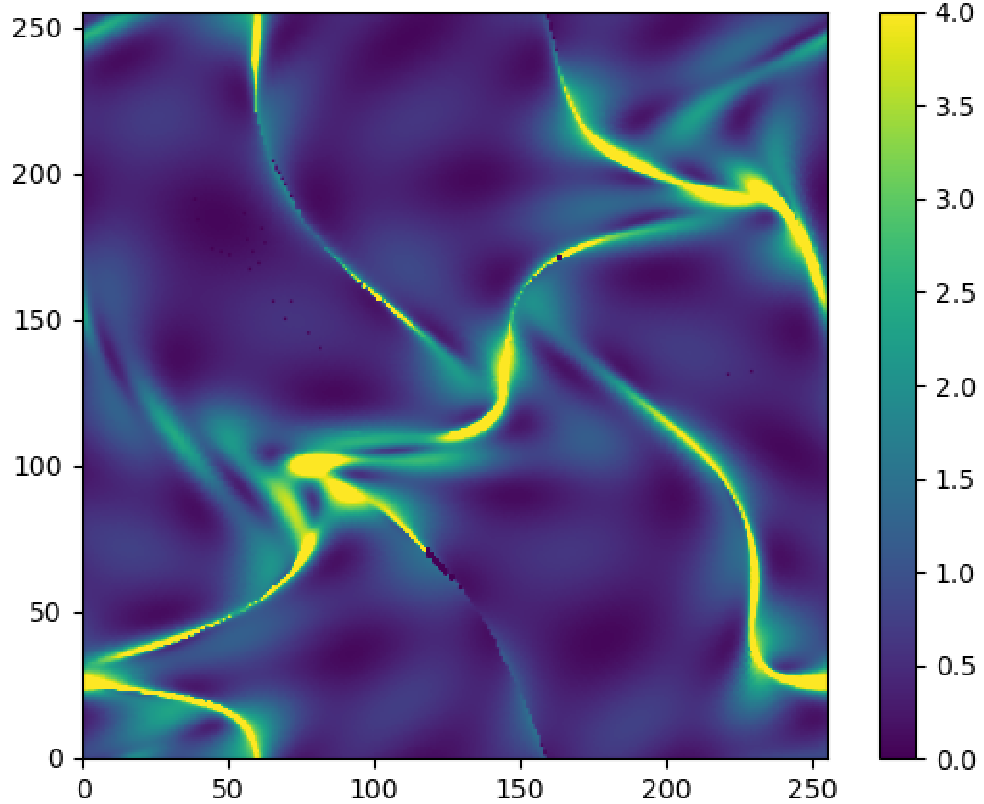
$$\rho(\mathbf{x}) = \rho_0$$



2D Godunov with
Lax-Friedrich Riemann solver

Euler

$$\rho(\mathbf{x}) = \rho_0(\mathbf{X})\frac{1}{|\mathbb{1} + t\mathbf{J}|}$$



Semi-analytical construction
using "adhesion" model

4. Turbulence synthesis

Can we build a random field which bears all known statistical properties of turbulence, at low cost ?

→ the attempt should at least help understand better what characterises turbulence

Previous attempts:

- Fractionated Gaussian Fields, aka fBms (Mandelbrot,1968)
- HD: Chevillard et al. (2006, 2011) / Rosales & Meneveau (2006)
- MHD: Durrive et al. (2020,2022) / Subedi et al. (2014)

  Lühbke et al. (2024)

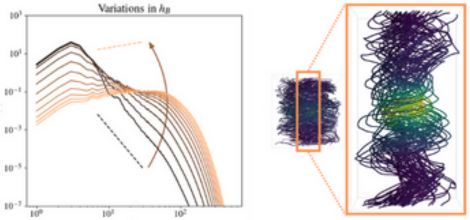
4. "BxC" Durrive+ (2020,2022) Maci+(2024)

Arbitrary spectral index & degree of intermittency, some impact of MHD equations on B,u vectors and their correlations.

The BxC toolkit
A swift generator for 3D MHD turbulence

Home | About | Publications | Features | User Guide

BxC, which stands for **magnetic fields** from **multiplicative chaos**, is a swift generator for 3D turbulent magnetic fields, which allows to generate high-resolution data cubes, **in minutes**, on laptops and desktops.



Capabilities

In addition to having an actual fields also match physical turbu DNS simulations. The relative customization of the power spectrum as well as the inclusion of realistic features such as anisotropy and background structured topologies.

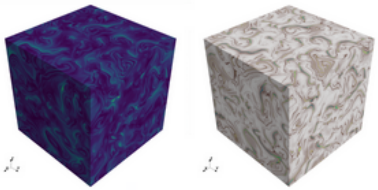
https://bxc.academy

See more

Getting started

BxC is fully implemented in Python. The relatively simple structure of the code makes it extremely user-friendly and easy to use. The Python implementation also facilitates the post-processing of data, for which users can readily use their own routines.

User Guide



Using BxC

The development of BxC took and still takes a lot of time and effort. We kindly ask that the first published peer-reviewed paper from applying BxC is done in co-authorship with at least one of the original authors. Additionally, if you use BxC in a publication we kindly request that you cite the code paper.

Published works

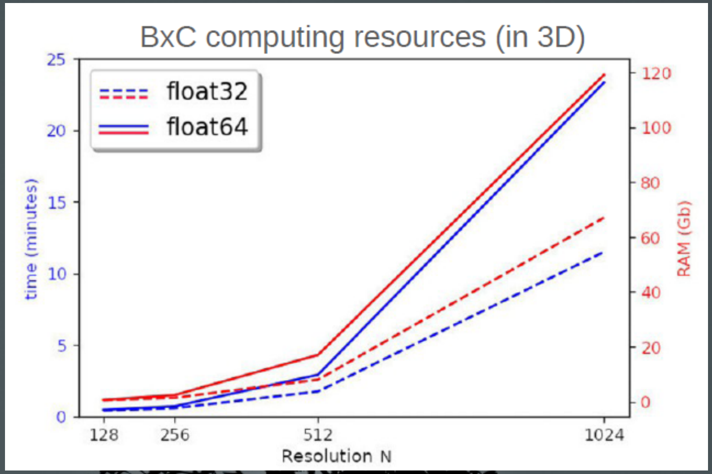
Fundings

BxC is supported by funding from the European Research Council (ERC) under the European Unions Horizon 2020 research and innovation programme, Grant agreement No. 833251 PROMINENT ERC-ADG 2018; the project received funding from the Internal Funds KU Leuven, Project No. C14/19/089 TRACESpace, and Agence Nationale de la Recherche, project BxB:ANR-17-CE31-0022.

erc PROMINENT

https://bxc.academy

- a 3D vector field
- divergence-free
- with current sheets (curl of B)
- controllable power spectrum
- very cheap

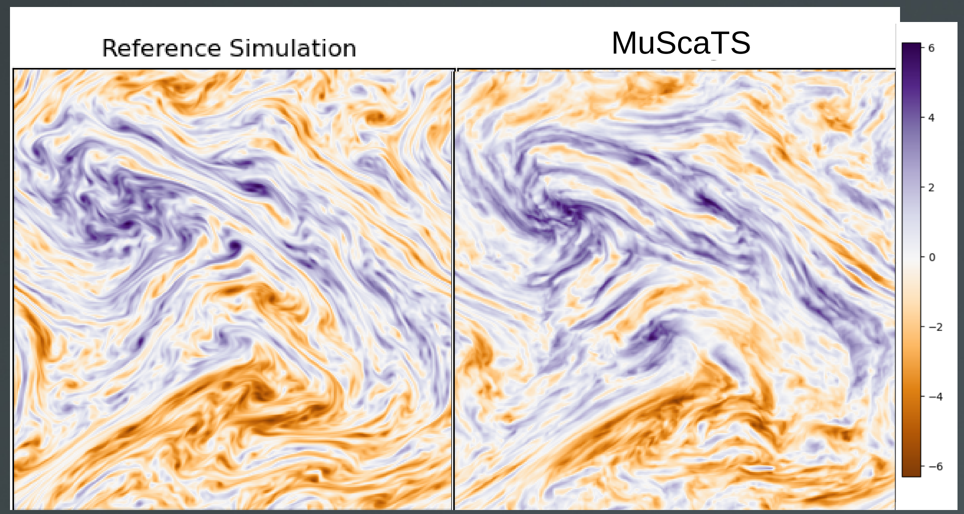


4. MUltiSCAle Turbulence Synthesis

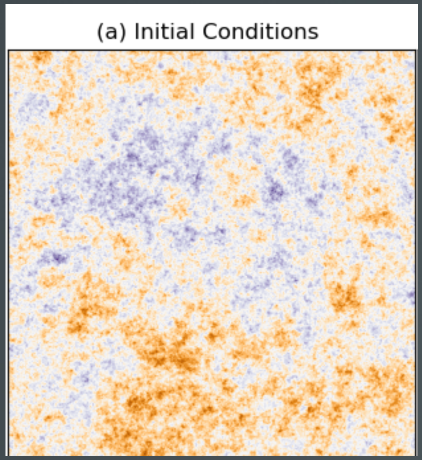
Pierre Lesaffre (CNRS/LPENS), Jean-Baptiste Durrive

S. Poirier, J. Gossaert, P. Richard, E.Allys, S. Colombi, W Béthune

Vorticity maps:



300 standard steps



4 MuScaTS Steps (1/17 CPU)

(1024x1024 2D incompressible hydro)
Initial Conditions are Gaussian
(ie: vorticity is a fractionated Gaussian field)

4. Generic form of evolution equations

$$\partial_t \boldsymbol{W} + (\boldsymbol{v}[\boldsymbol{W}].\boldsymbol{\nabla})\boldsymbol{W} = \boldsymbol{S}[\boldsymbol{W}].\boldsymbol{W} + \boldsymbol{D}[\boldsymbol{W}]$$

advection deformation diffusion

W vector of state variables
[W] means affine in W

examples

* incompressible 2D HD

$$\partial_t w + \boldsymbol{u}.\boldsymbol{\nabla} w = \nu \Delta w$$ $\boldsymbol{W} = w$ u[w] 2D Biot-Savart $\boldsymbol{S} = 0$ $\boldsymbol{D} = \nu \Delta w$

* incompressible 3D HD

$$\partial_t \boldsymbol{w} + \boldsymbol{u}.\boldsymbol{\nabla} \boldsymbol{w} = \boldsymbol{w}.\boldsymbol{\nabla} \boldsymbol{u} + \nu \Delta \boldsymbol{w}$$ $\boldsymbol{W} = \boldsymbol{w}$ $\boldsymbol{S}_{ij} = \partial_i u_j$ $\boldsymbol{D} = \nu \Delta \boldsymbol{w}$

* incompressible 2D HD, on divorticiy $\boldsymbol{B} = \boldsymbol{\nabla} \times (w \boldsymbol{e}_z)$ $\partial_t \boldsymbol{B} + \boldsymbol{u}.\boldsymbol{\nabla} \boldsymbol{B} = \boldsymbol{S}.\boldsymbol{B} + \nu \Delta \boldsymbol{B}$

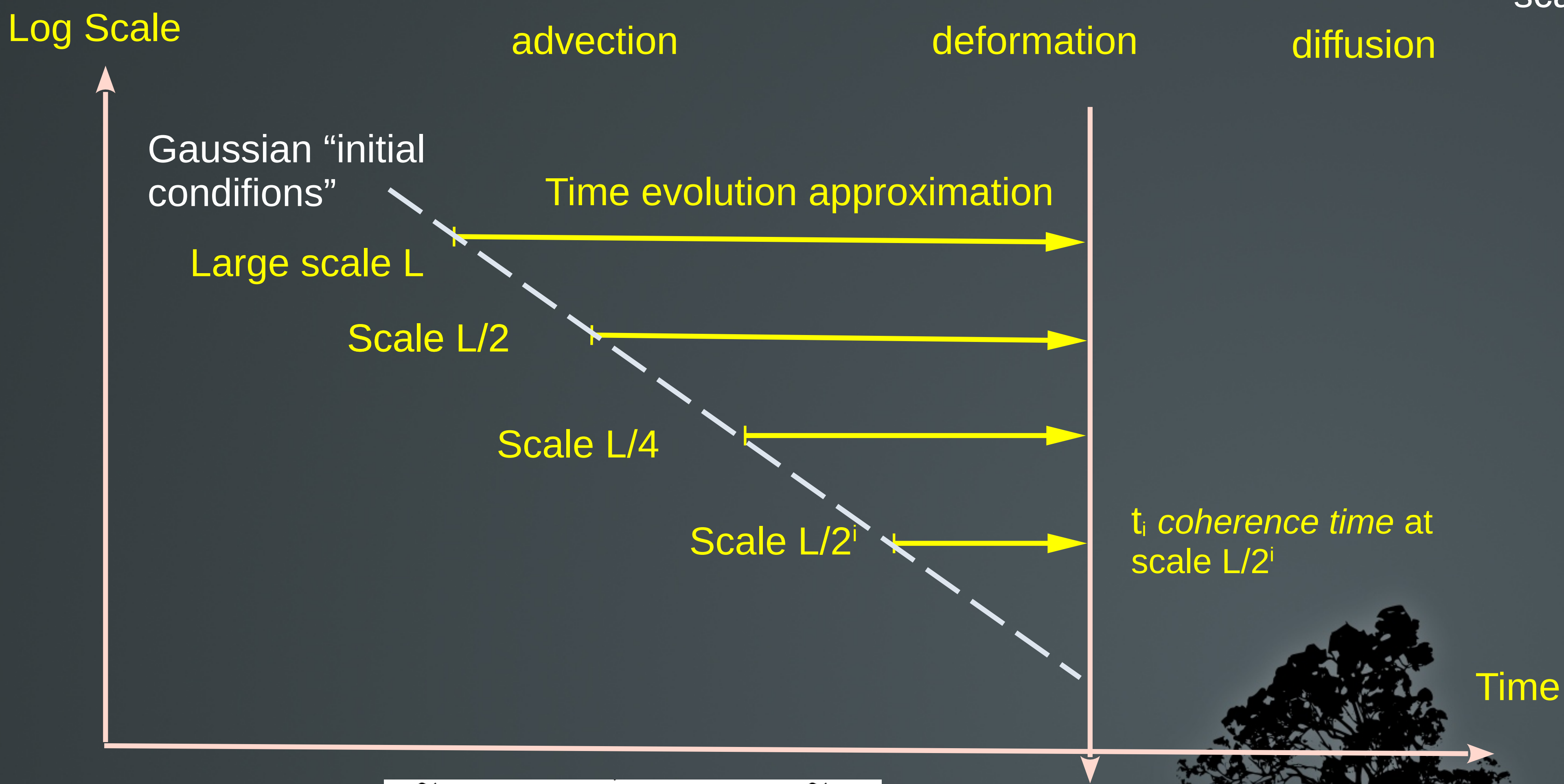
* incompressible 3D MHD, self-gravitating isothermal fluids

4. Filtered form of evolution equations

$$\partial_t \tilde{\boldsymbol{W}}_\ell = -\boldsymbol{v}_{>\ell}.\nabla\tilde{\boldsymbol{W}}_\ell + \boldsymbol{S}_{>\ell}.\tilde{\boldsymbol{W}}_\ell + \boldsymbol{D}[\tilde{\boldsymbol{W}}_\ell]$$

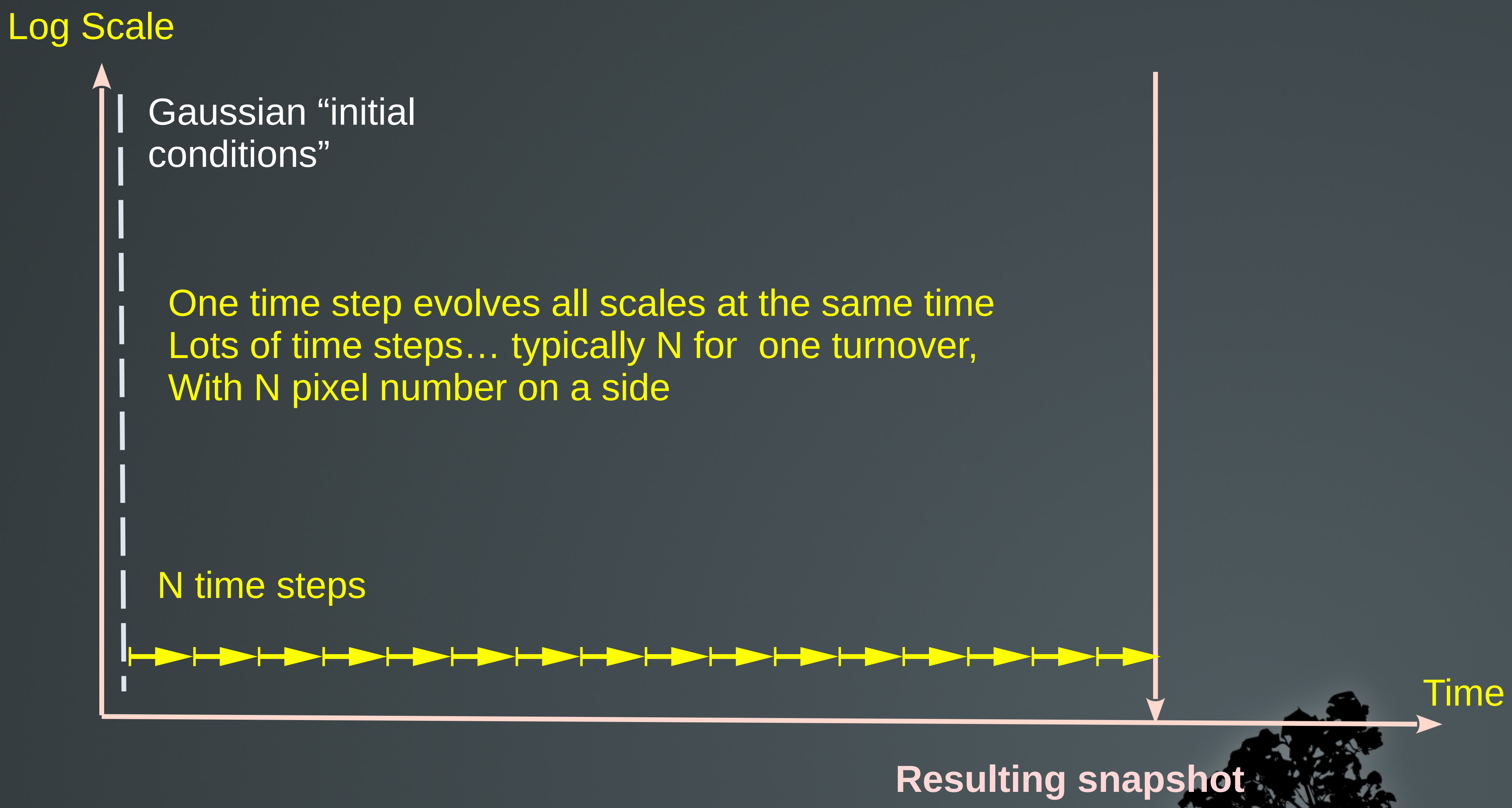
plus “stochastic smaller scales”

advection deformation diffusion



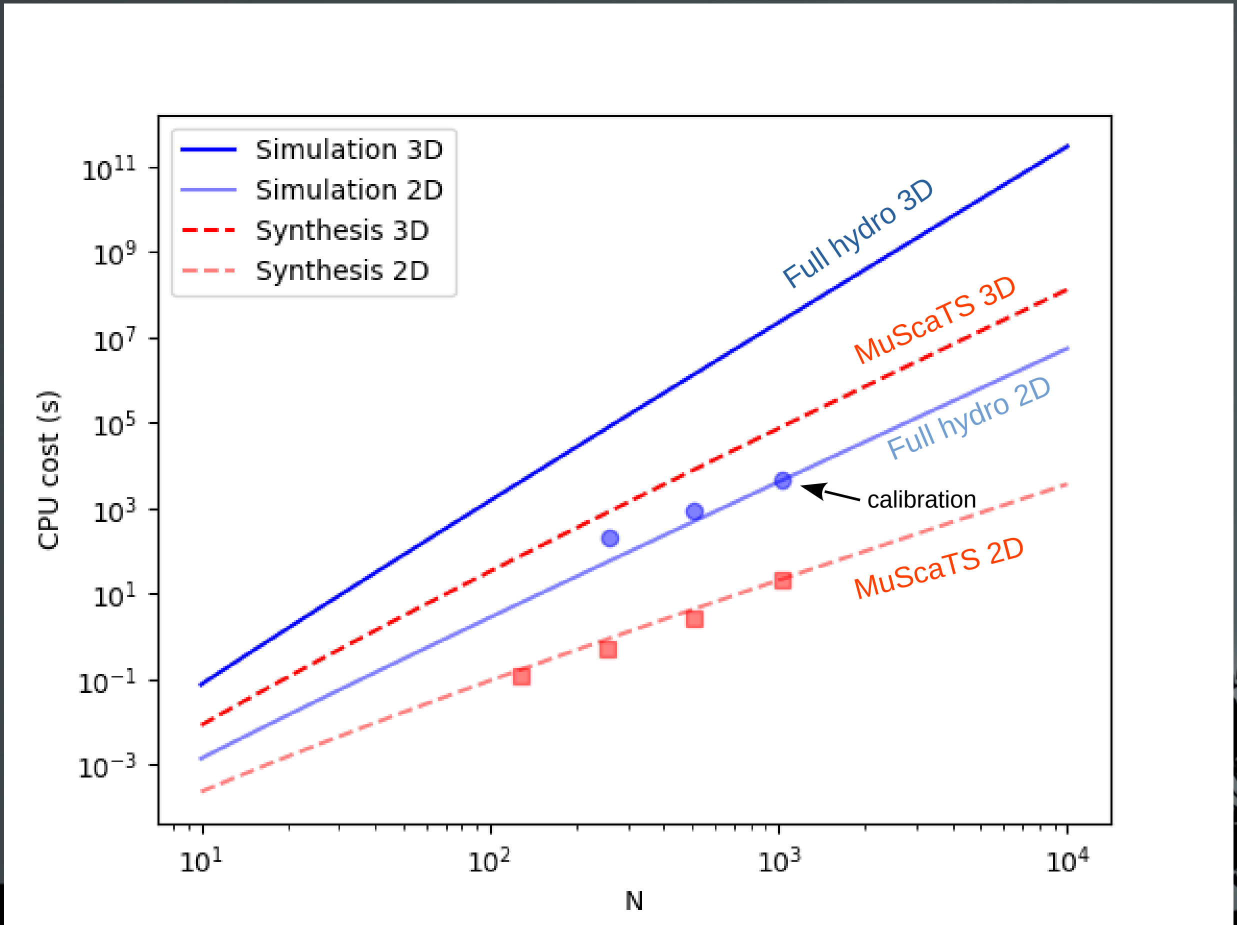
$$\tilde{\boldsymbol{W}} = \int \mathrm{d}\ln\ell\,\tilde{\boldsymbol{W}}_\ell$$

4. Integration scheme in a standard simulation

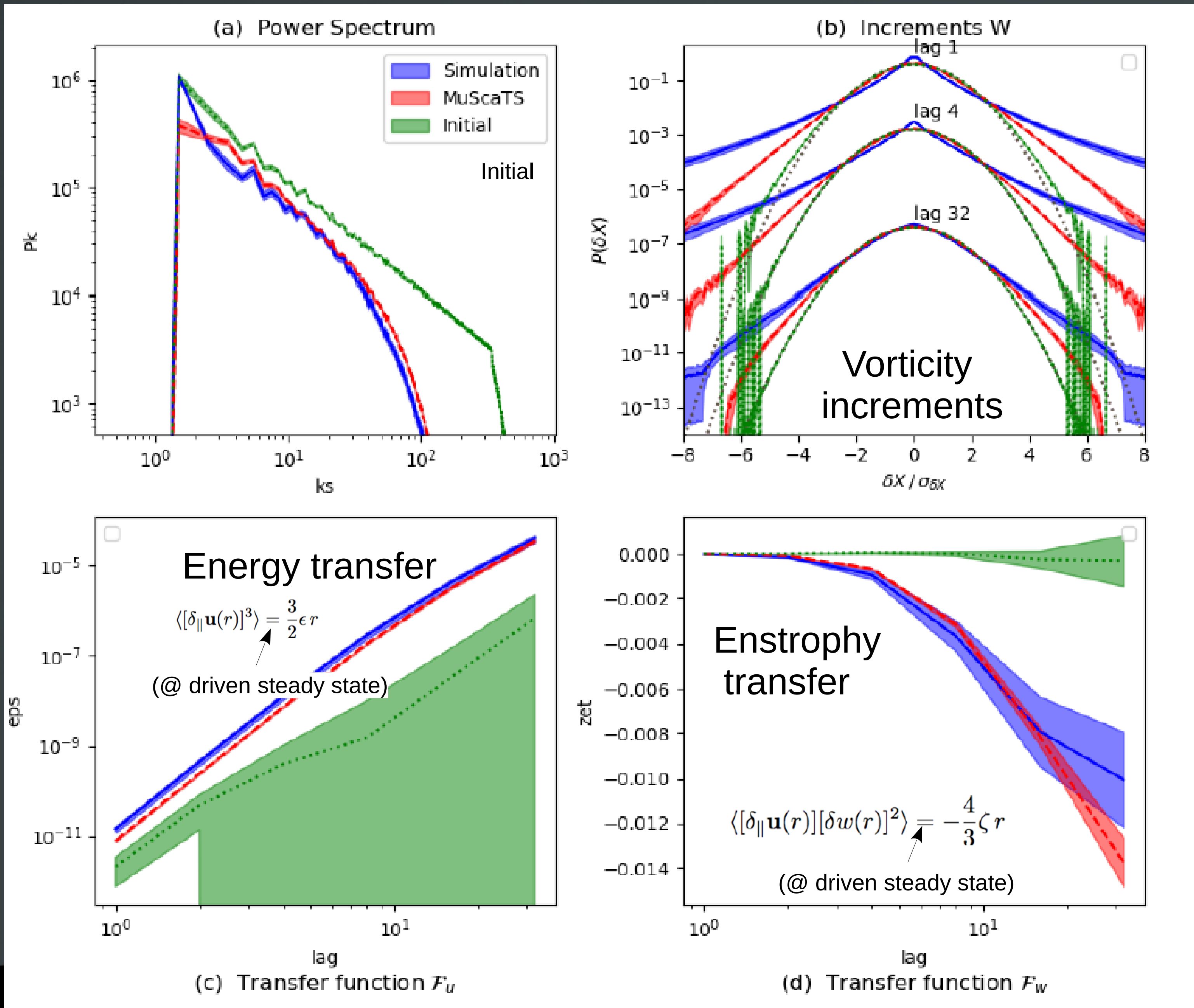


4. Computational cost (hydro)
(assuming 1 turnover time and CFL=1)

- 3D: *sim:* $216N^4.\log(N)$ *synthesis:* $72N^3.\log(N)^2$
- 2D: *sim:* $40N^3.\log(N)$ *synthesis:* $20N^2.\log(N)^2$

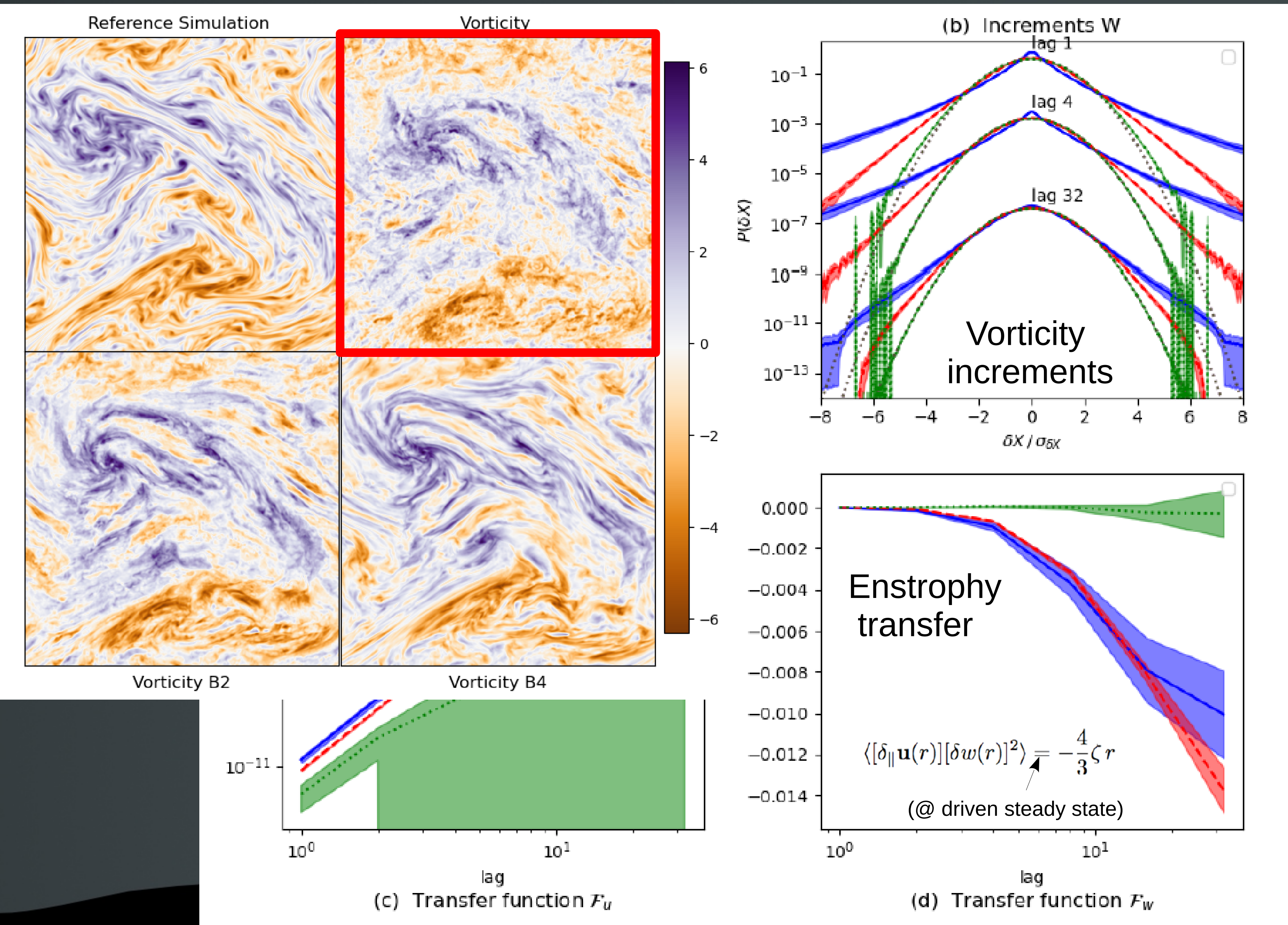


4. “Classical” statistics (@ ~ 1 turnover time)



(Note: More stringent Stats: **WST coeffs** are much better to characterise textures)

4. “Classical” statistics (@ ~ 1 turnover time)



(Note: More stringent Stats: **WST coeffs** are much better to characterise textures)

Thanks !

Image credits:
J-B Durrive
B,v fields in BxC model
(see Durrive+2022)

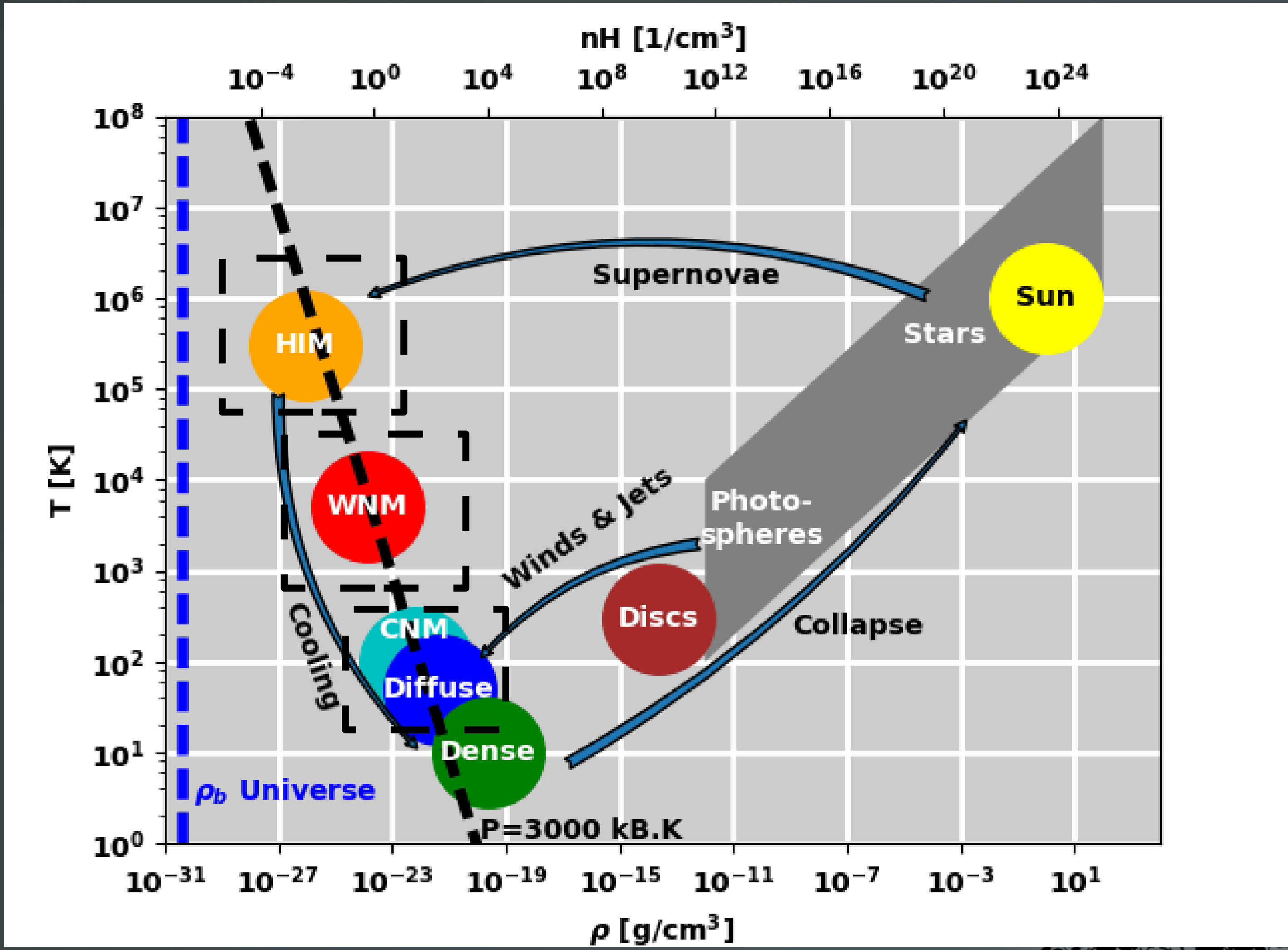
Take home messages

- 1. Intermittent scaling exponents can be generalised even for projections, but poorly characterise coherent structures (CSIDEs).
- 2. Dissipation changes internal profiles, initial conditions constrain CSIDEs position and nature.

  Turbulence dominated by random collection of CSIDEs ?
- 3. CSIDEs stats @ initial conditions in Mach $= \infty$
- 4. Synthetic fields cheap and help understand what characterises turbulence

The galactic matter cycle

→ isothermal MHD simulations



(HDR P. Lesaffre)

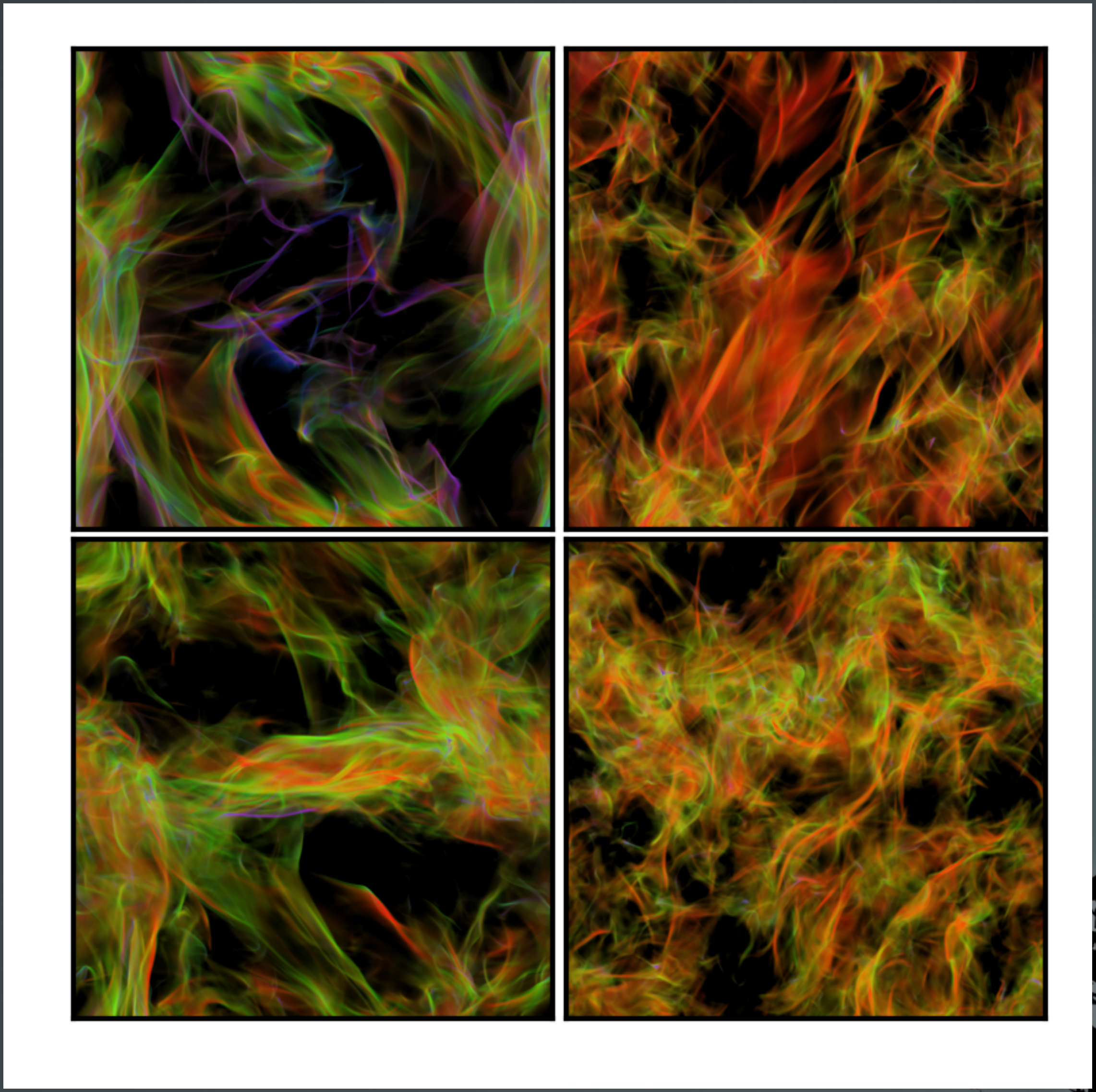
Dissipation projections

Orszag-Tang ABC flow

(simulations from Richard+ 2022)

@ early times
(near peak dissipation)

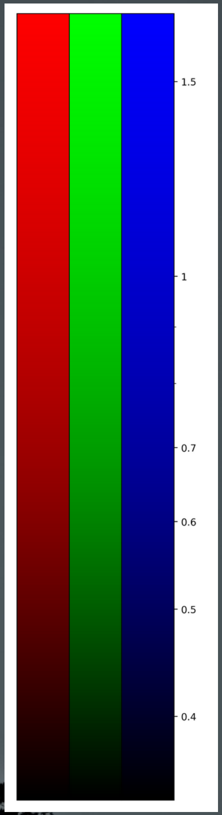
@ “late” times
(at one turnover time)



$\int \varepsilon dz$

in units of <ρ> u_{rms}^3

Ohmic heat $\eta\ j^2$
Viscous shear $\nu\ w^2$
Viscous compression
4/3 ν div $(u)^2$



Dissipation projections,

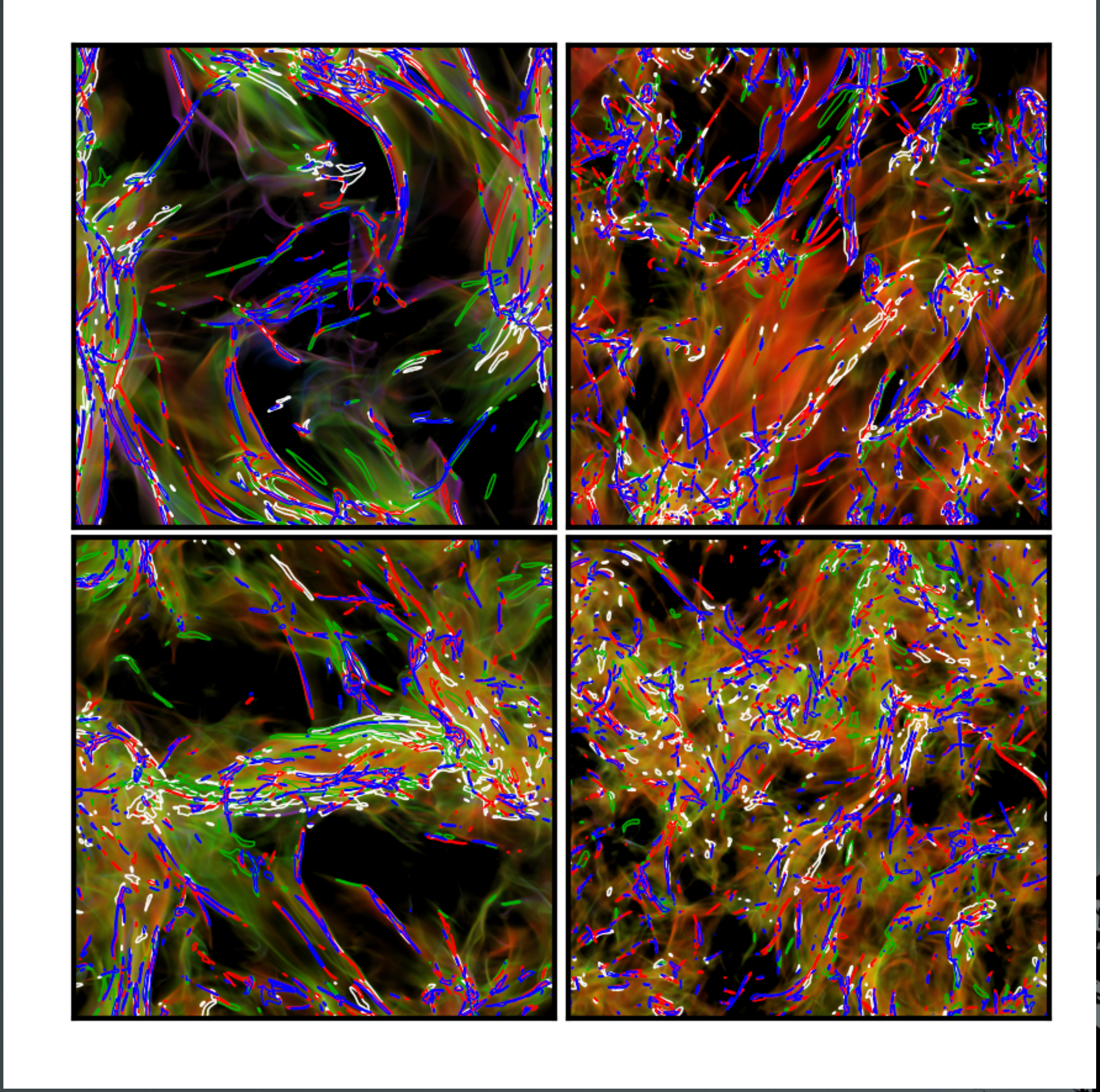
overlayed with 2-σ contours of observable increments (lag=1 pix)

(simulations from Richard+ 2022)

Orszag-Tang ABC flow

@ early times
(near peak dissipation)

@ "late" times
(at one turnover time)



Increments of Observables 2-σ contours:
Column-Density
Centroid velocity
Stokes Q
Stokes U

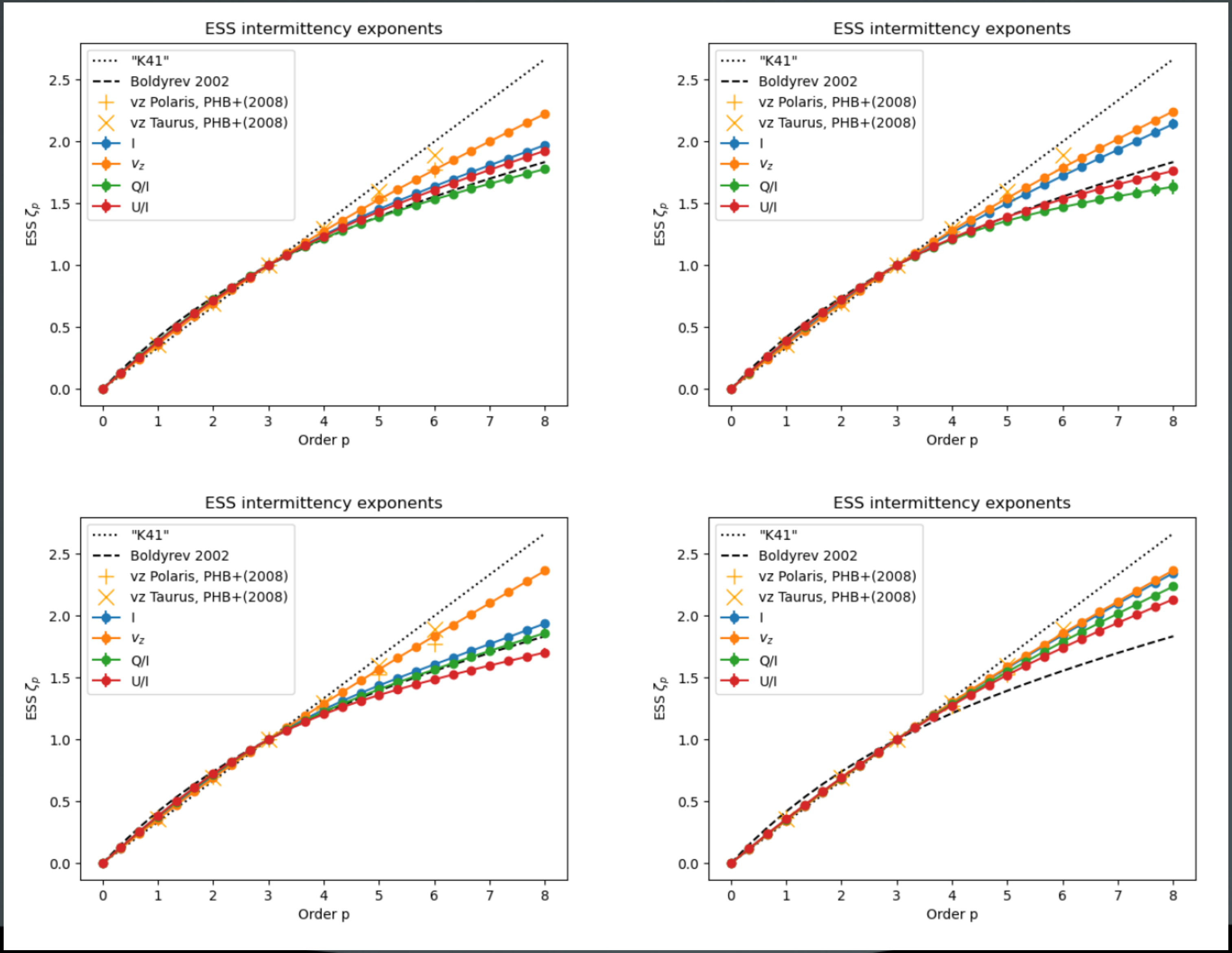
ESS intermittency exponents

Orszag-Tang

ABC flow

@ early times
(near peak dissipation)

@ "late" times
(at one turnover time)

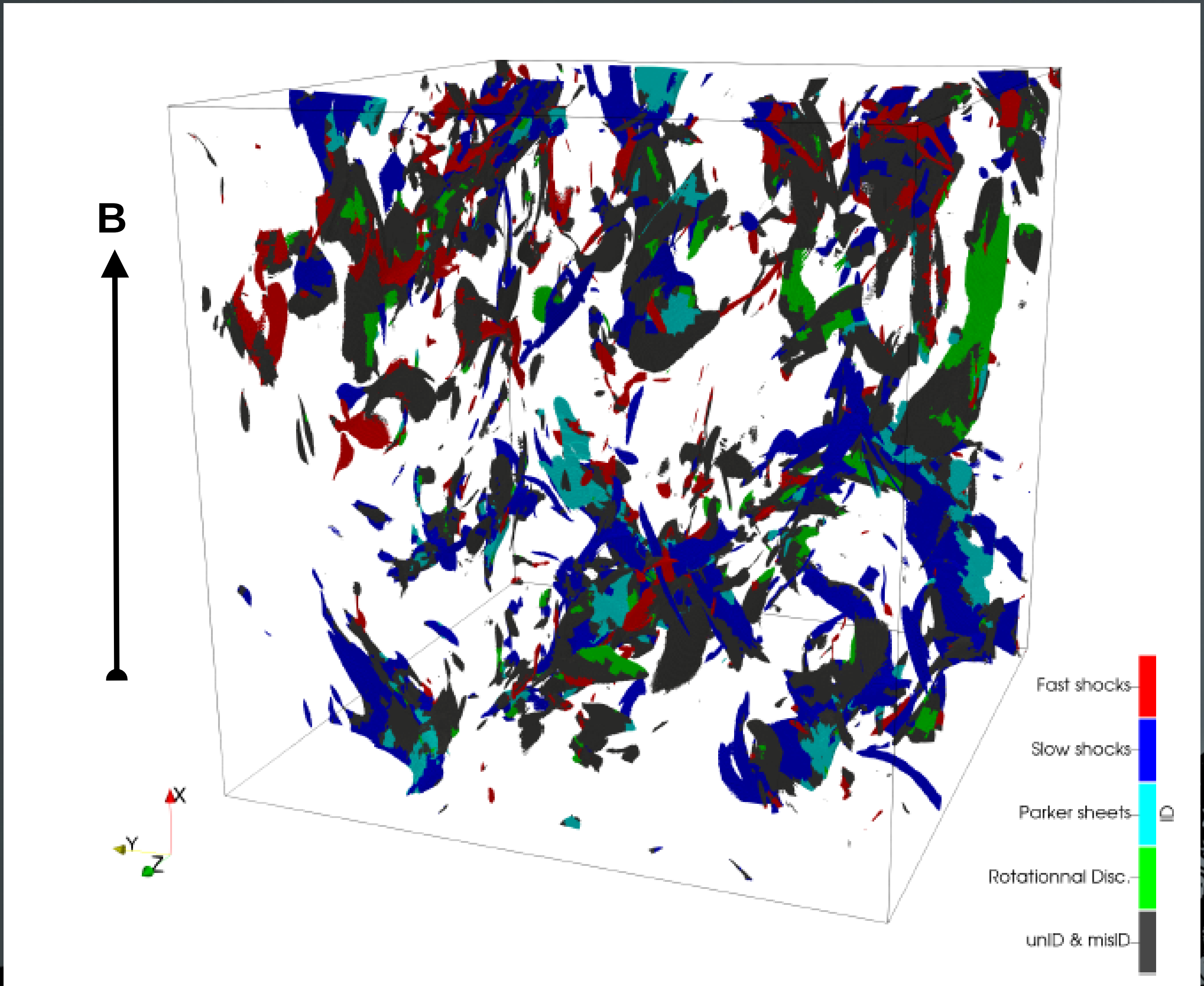


Depolarisation canals / LOFAR

With a strong mean B

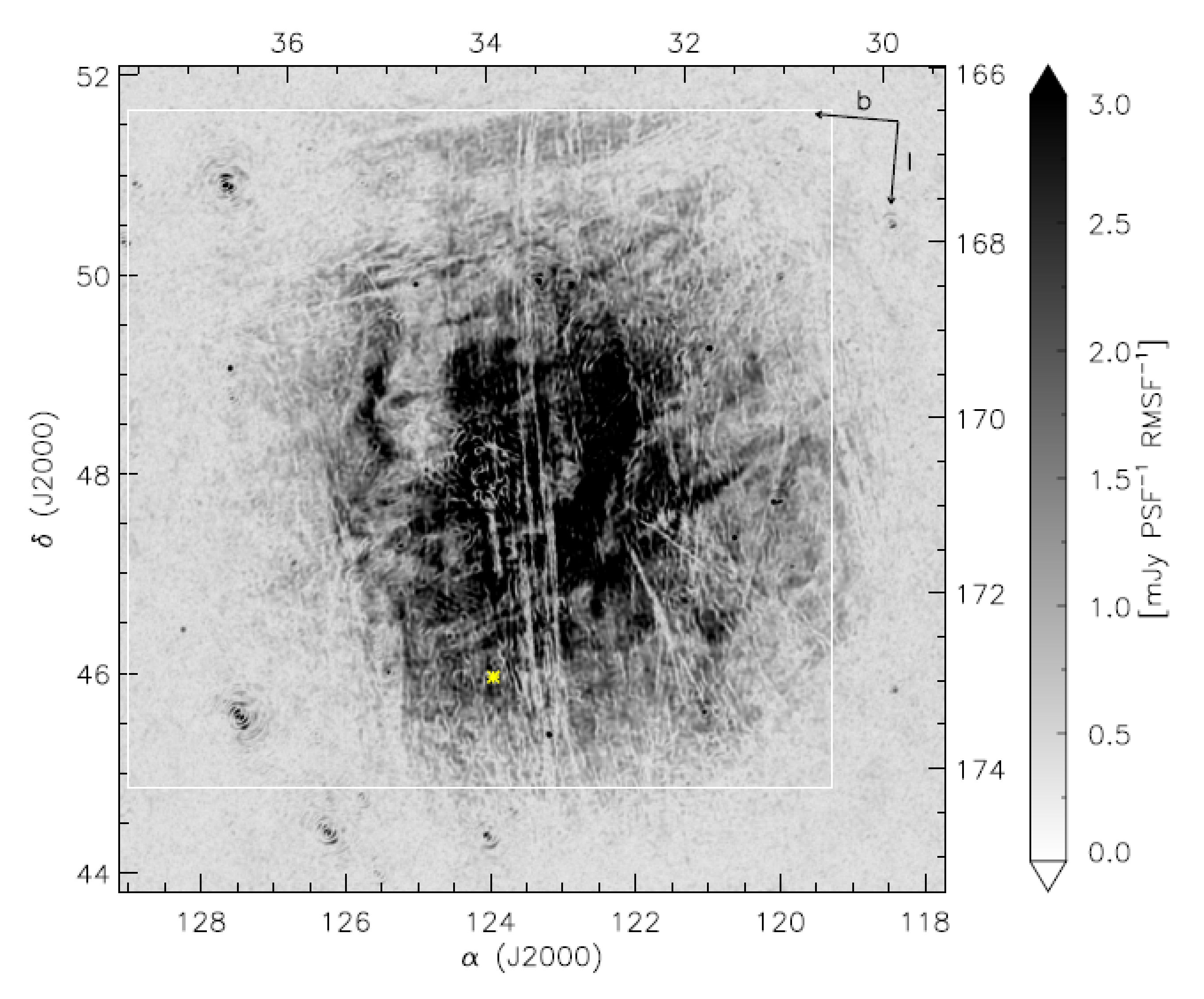
Fast shock Slow shock Rotational Discontinuity Parker sheet

Richard+ (2022)



Strong heating regions (> mean + 4 σ)

LOFAR’s depolarisation canals



Jelic et al. (2015):

Depolarisation canals
Straight and thin

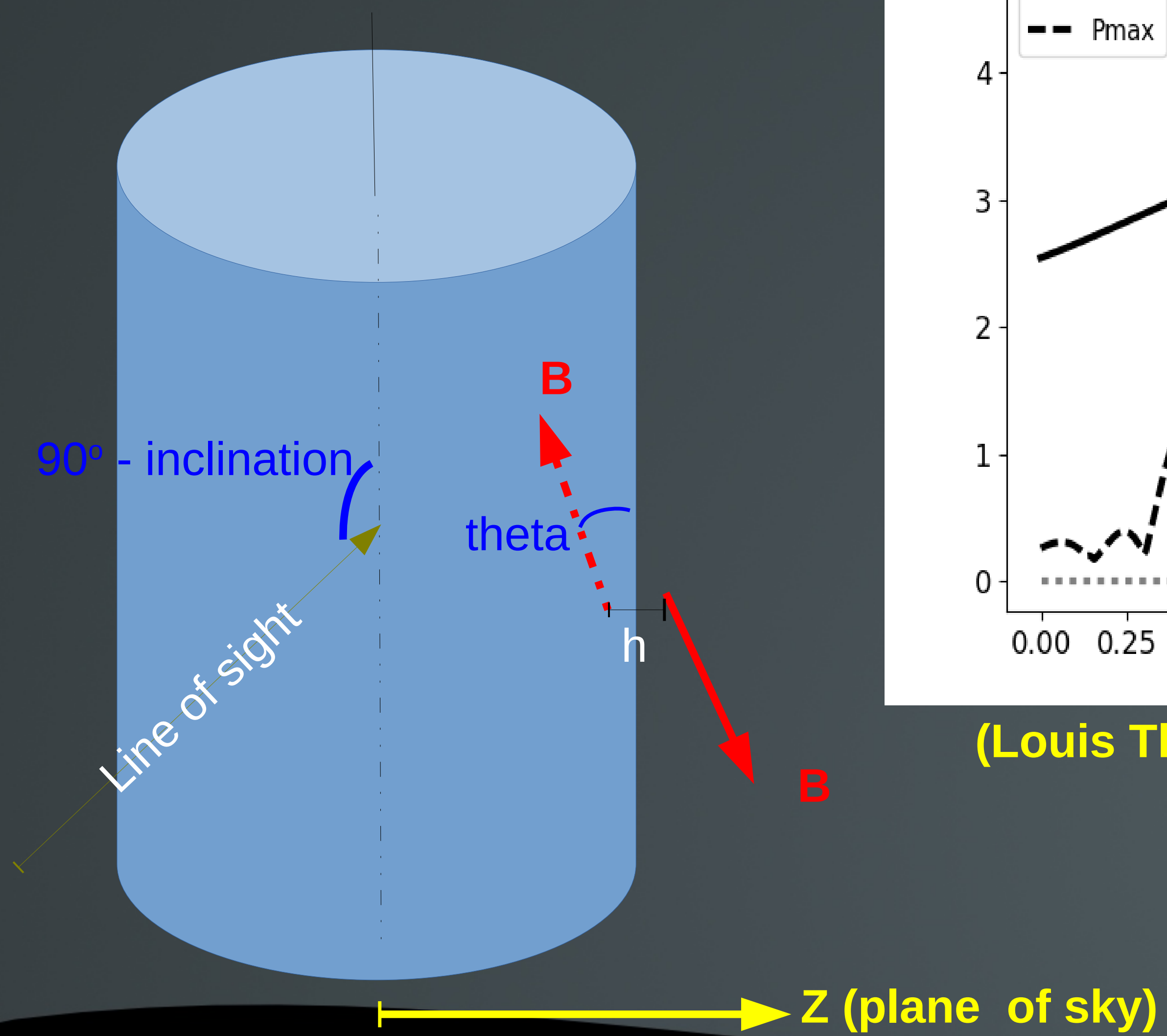
Direction correlated to
B seen by Planck,
And HI structures
(Jelic+2018)

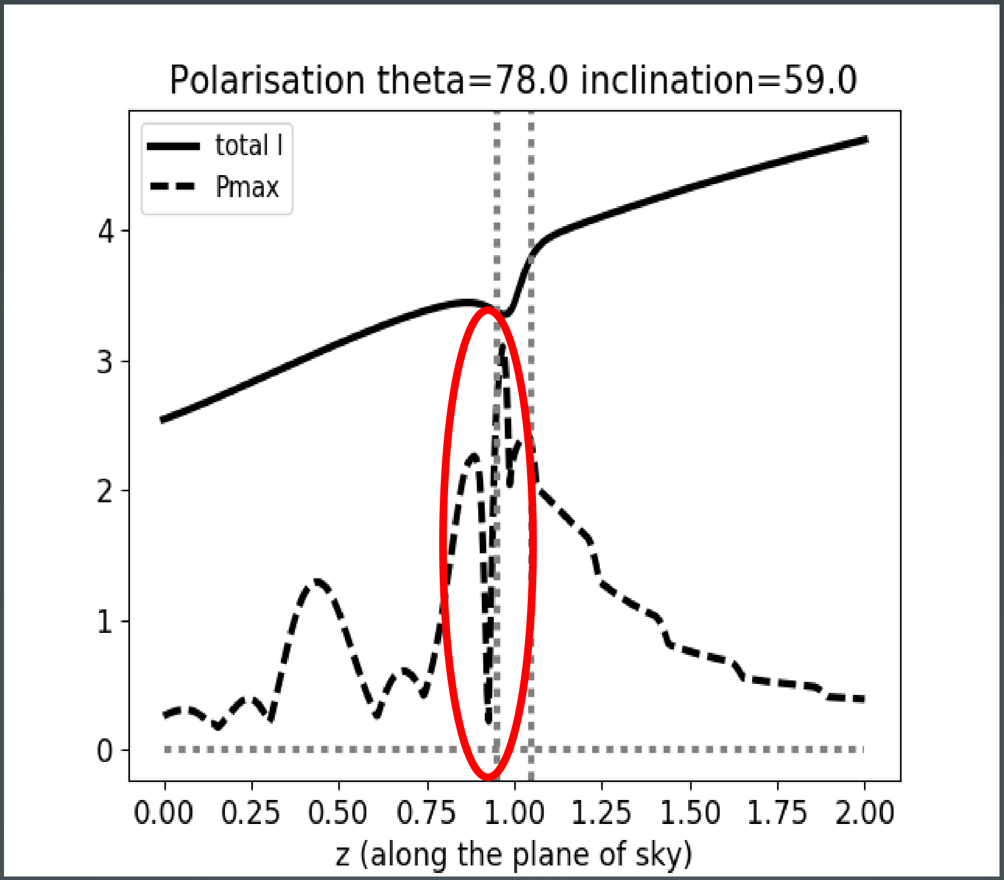
Link to
dissipation sheets ?

Observations LOFAR

LOw Frequency ARray

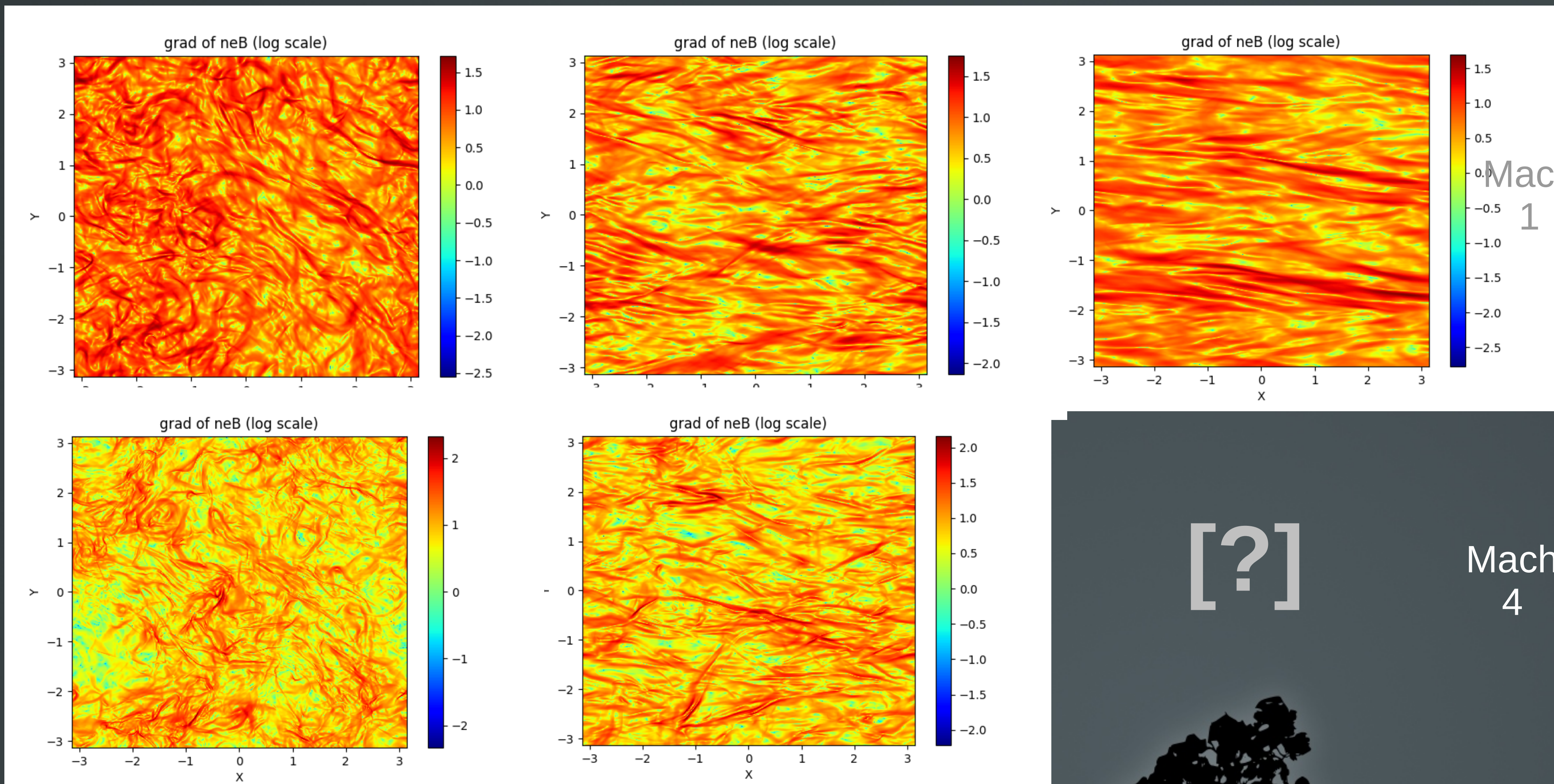
An oversimplified model for curved Parker sheets





(Louis Thouviot internship)

Log |grad(RM)| ABC



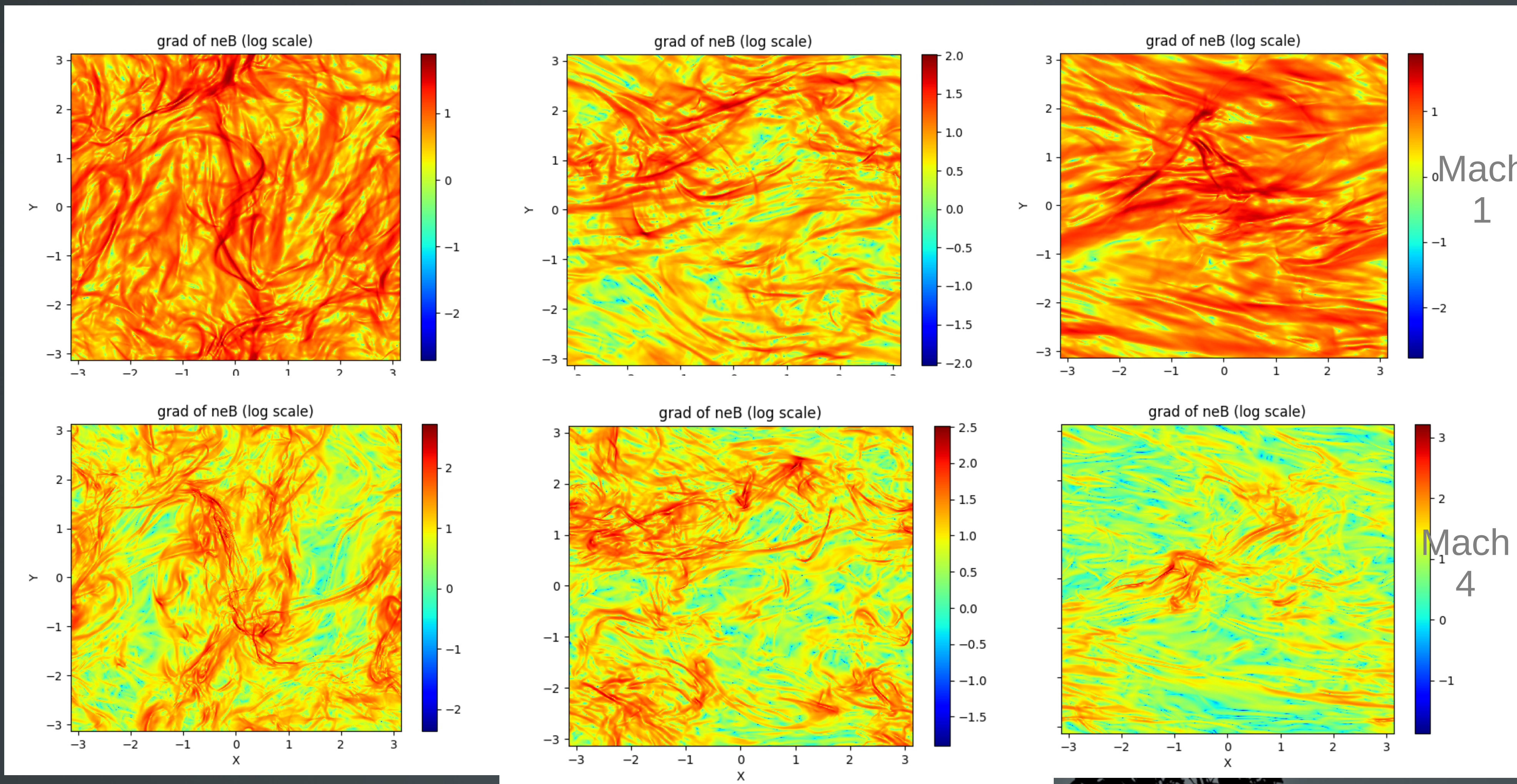
Mach 1

[?]

Mach 4

<B>=0   <Bx>=B.rms   <Bx>= 2 B.rms

Log |grad(RM)| OT



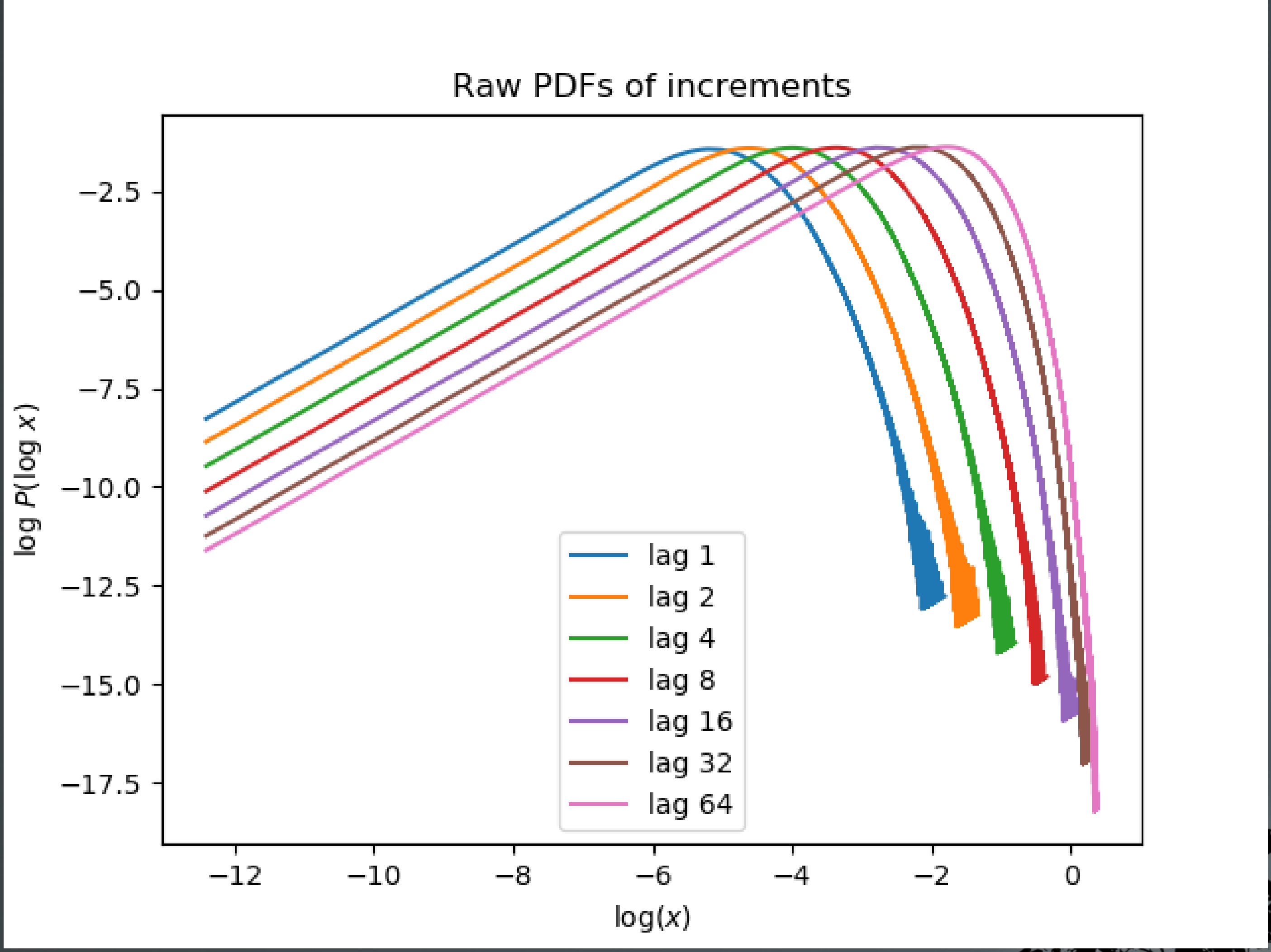
<B>=0

<Bx>= B.rms

<Bx>= 2 B.rms

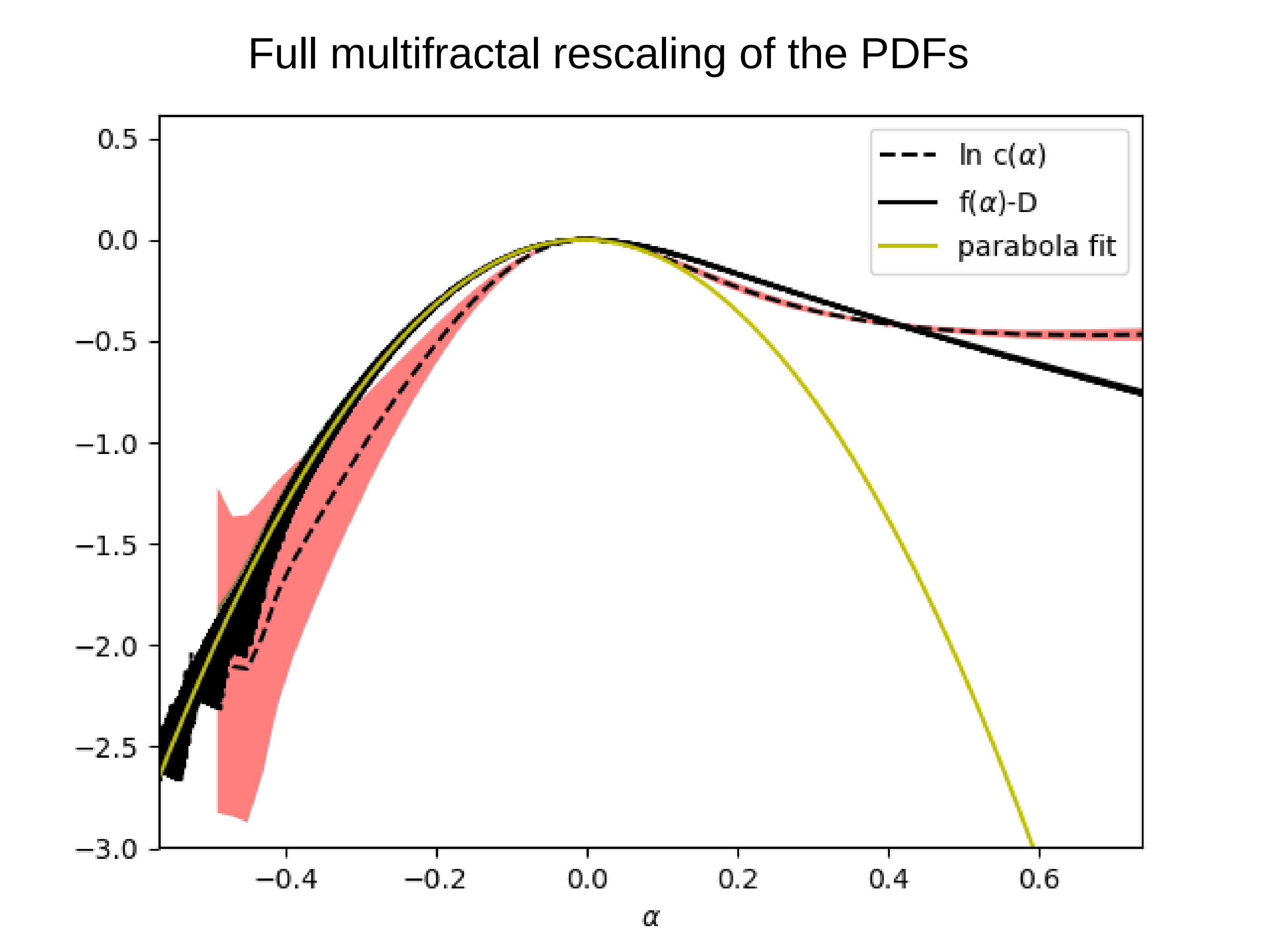
“Extended Self-Similarity” → “Extended Multifractal Scaling”

PDFs of longitudinal increments in MHD decaying turbulence



x: |longitudinal velocity increments at a given lag|

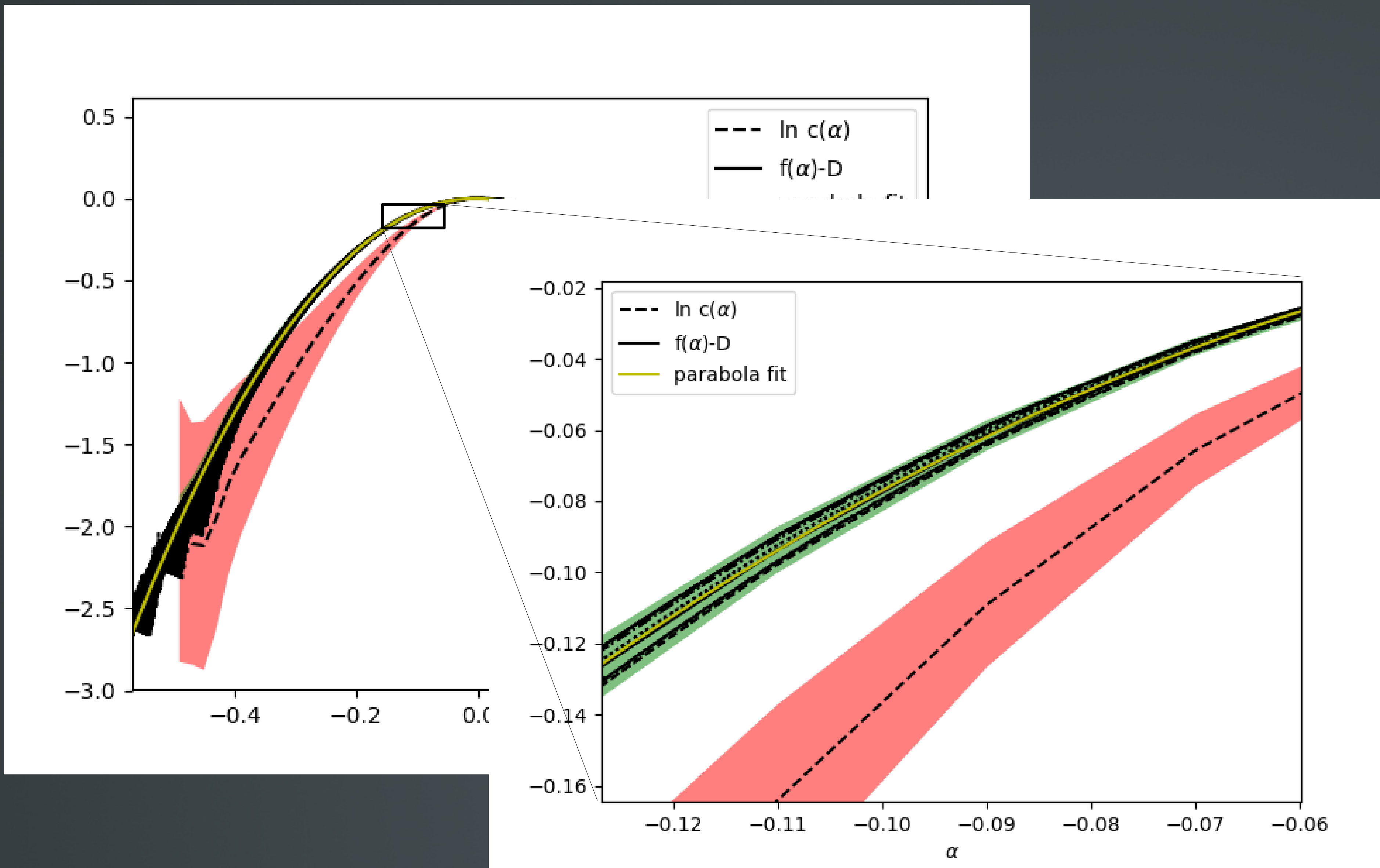
Rescaled PDFs of longitudinal increments in MHD decaying turbulence



$\ln P_\ell(\alpha) = \ln c(\alpha) + [D - f(\alpha)] \,.\, \ln(\ell/\ell_0)$

$\alpha = (\ln|\delta_\ell X| - \ln|\delta_\ell X|_{@max}) / \ln(\ell/\ell_0)$

“Extended Multifracal Scaling”



$\ln P_\ell(\alpha) = \ln c(\alpha) + [D - f(\alpha)] \,.\, \ln(\ell/\ell_0)$

$\alpha = (\ln|\delta_\ell X| - \ln|\delta_\ell X|_{@max}) / \ln(\ell/\ell_0)$

Multifractal scaling holds in many situations

- We tested various variables, averages vs. increments, and their projections
- We tested various Mach numbers, initial conditions, incompressible HD vs. compressible MHD.

→ All display multifracal scaling, especially in the dissipative range of scales.

- Log-normal model always provides a good fit to the large increments end.
- Can we relate projections to 3D ? Can we explain the relation between different variables ?

Wavelet Scattering Transform coefficients

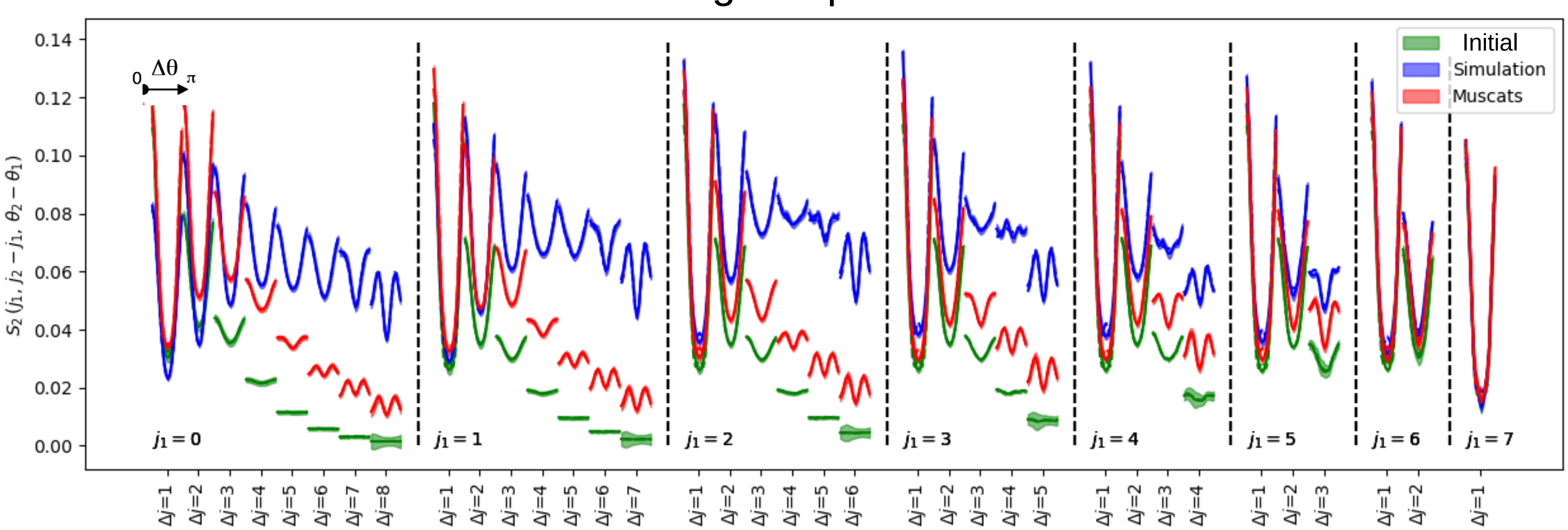
WST coefficients are a very efiicient tool to **characterise textures** (Mallat 2010)

Allys (2020) show that they predict many non-Gaussian quantifiers (increments, Minkowsky functionals, bi-spectrum...)

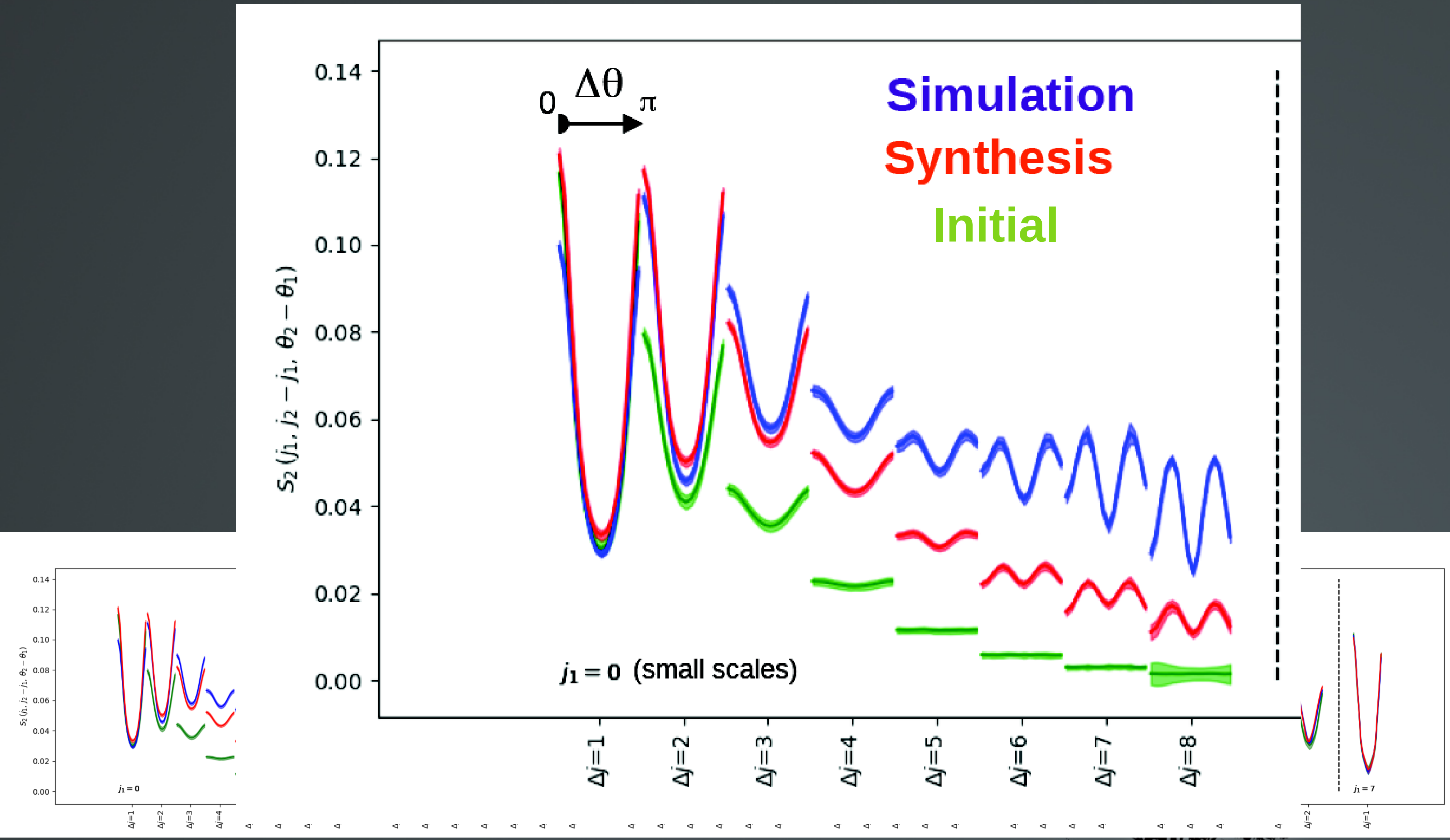
The normalised layer 2 coefficients (S_2) shown here estimate the coupling between two scales $\ell_1 = \ell_0.2^{j1}$ and $\ell_2 = \ell_0.2^{j2}$ and two angles θ_1 and θ_2 in the image, labeled by j_1, θ_1, j_2 and θ_2. Thanks to isotropy, the coefficients depend only on the difference $\Delta\theta = \theta_1 - \theta_2$.

“Seagulls” plot

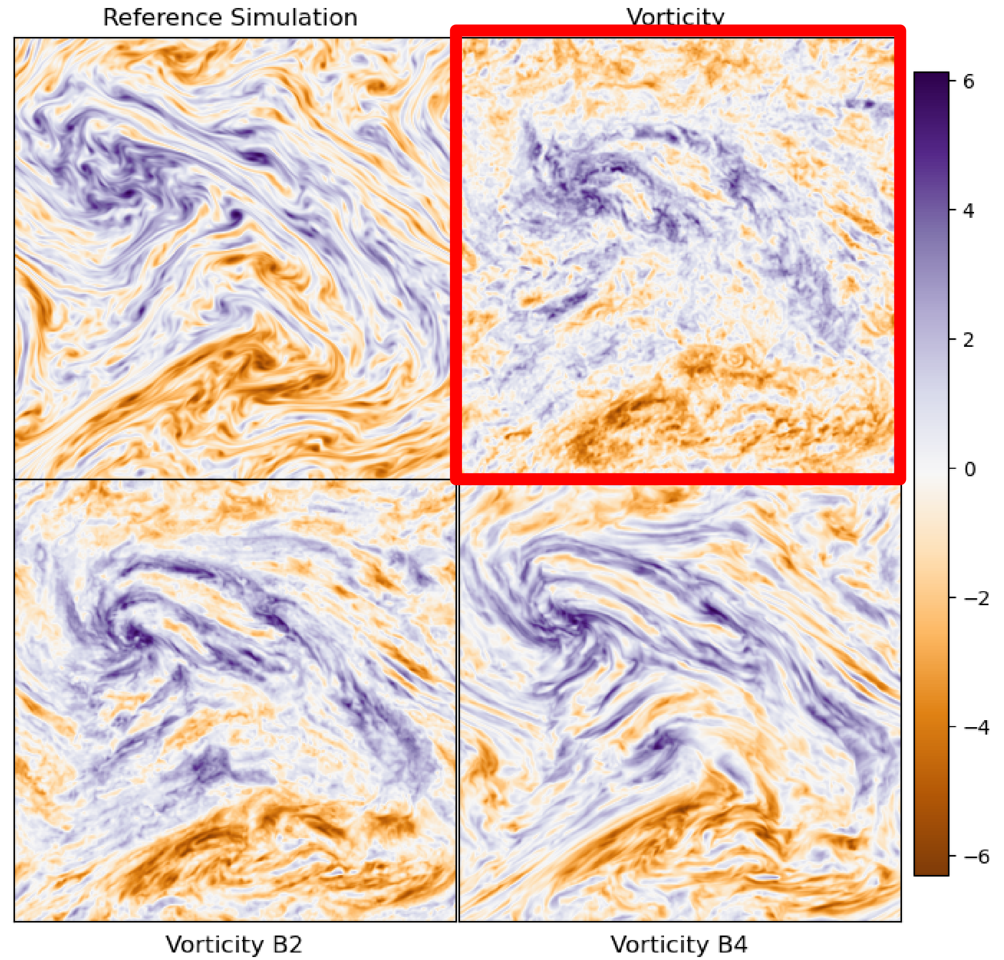
.Amp1.Beta3/000020



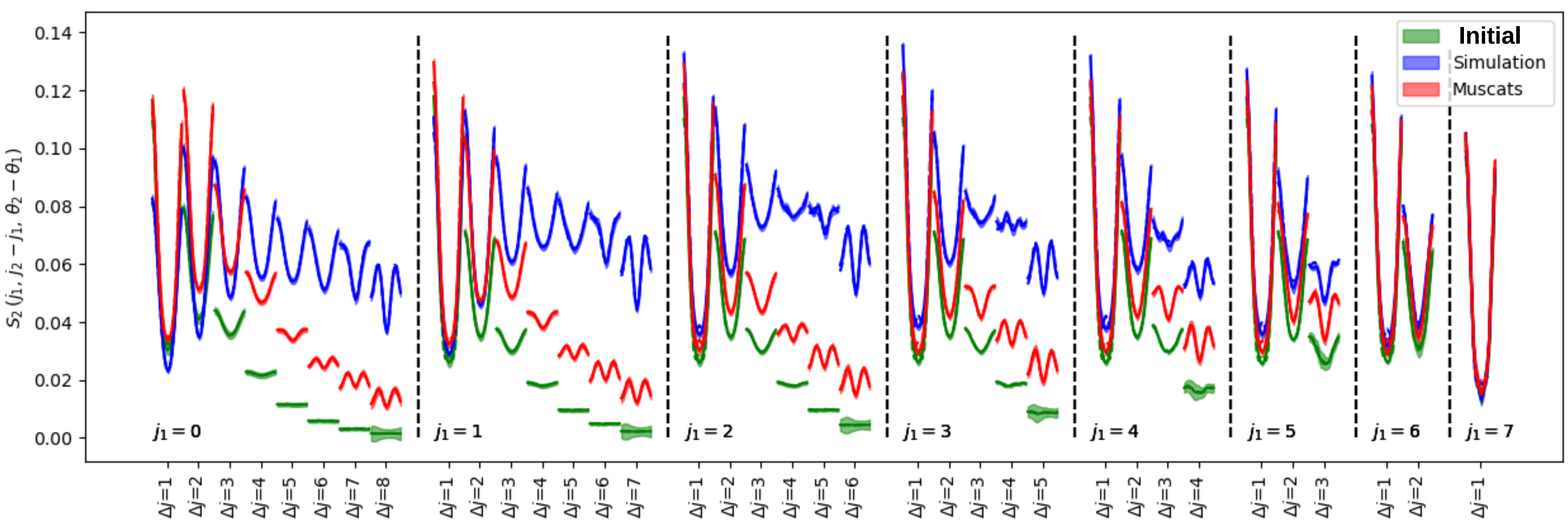
WST coefficients



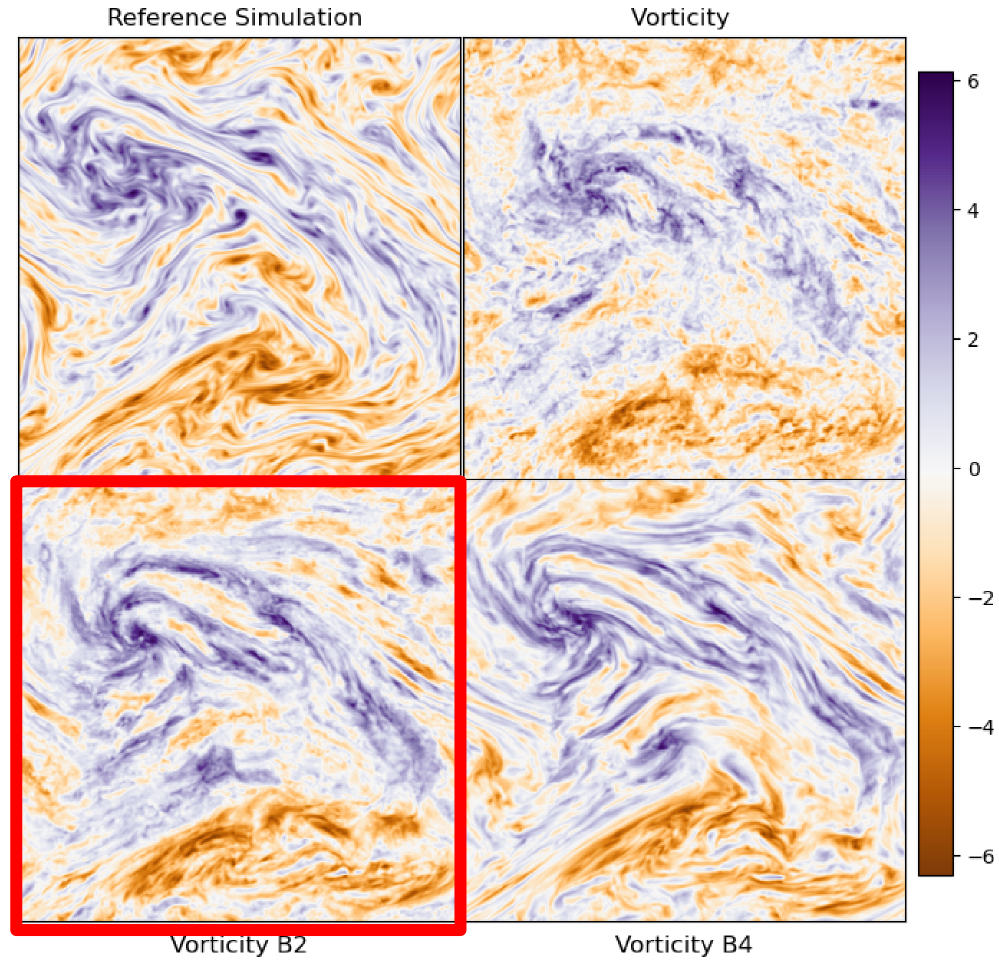
No bootstrap (1 MuScaTS step)



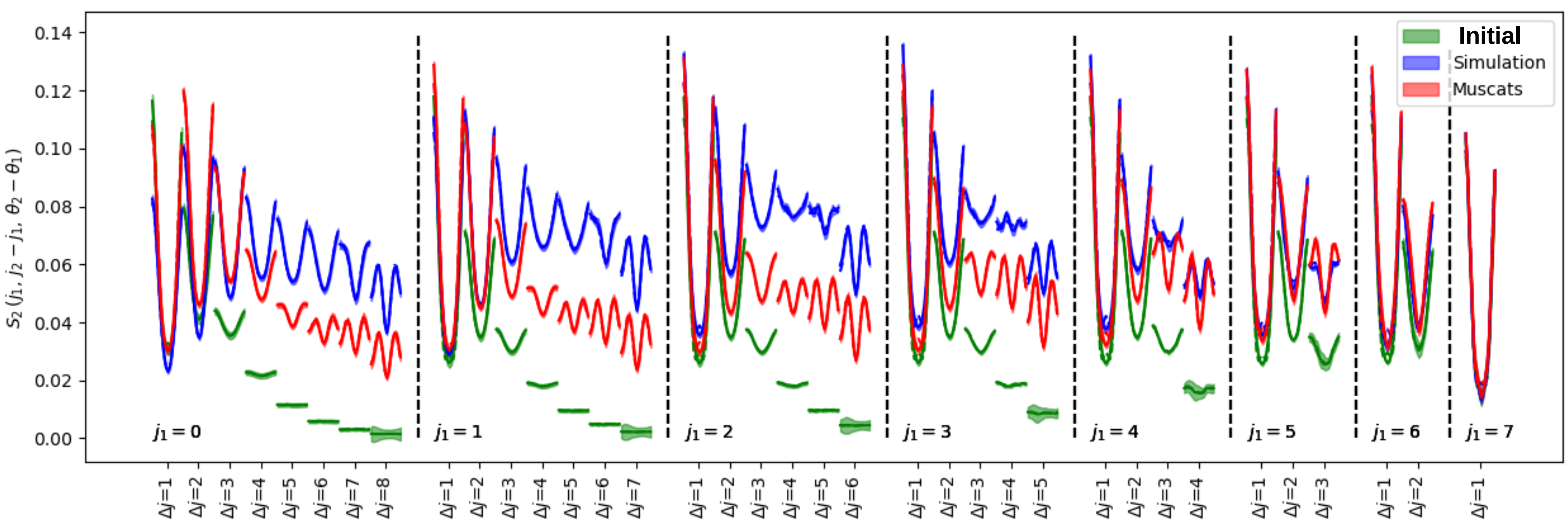
(t=1/3rd turnover time)



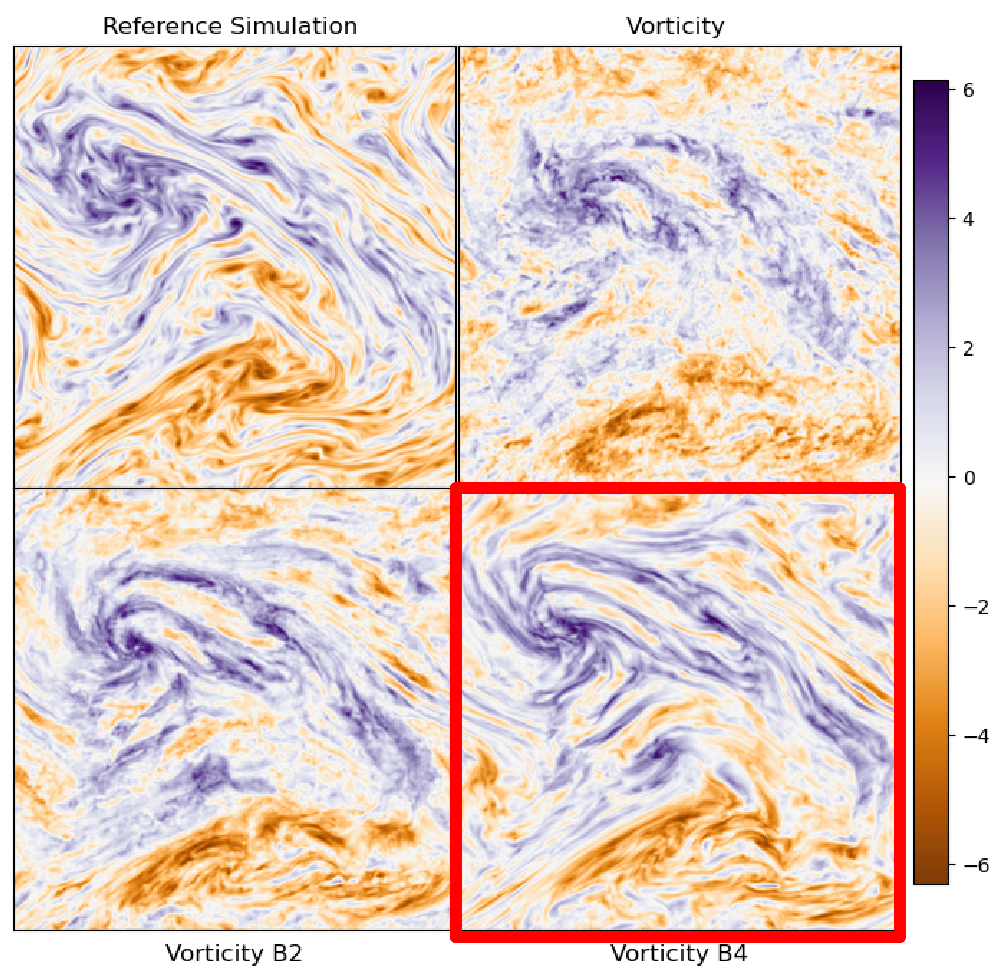
Bootstrap
(2 MuScaTS steps)



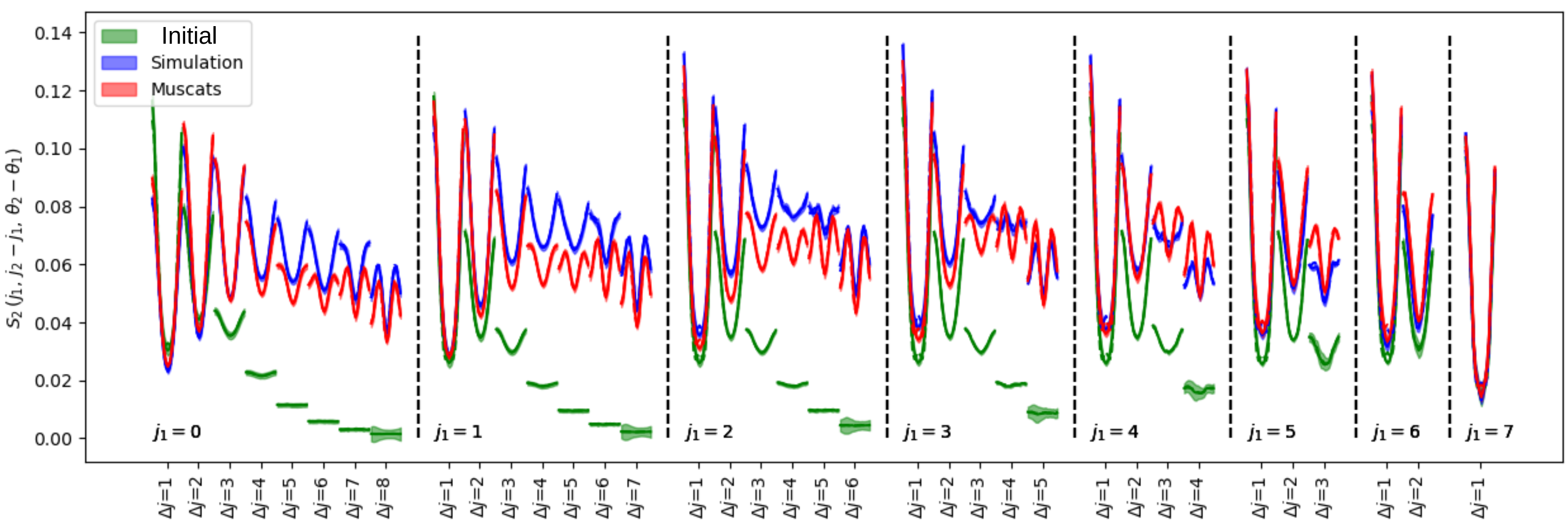
(t=1/3rd turnover time)



Bootstrap (4 MuScaTS steps)



(t=1/3rd turnover time)



Summary

- MuScaTS: filter generic equations from large to small scales and advance them ballistically during a local coherence time. (~ multiscale Zel'dovich approximation)
- Intermittency, inter-scale transfers and viscous cuts off are not prescribed, they emerge naturally
- Caveat: the initial spectrum slope *is prescribed*...
- Scope of the method: compressible, 3D, MHD, gravitating fluids, inhomogeneous turbulence…
- Usage: initial conditions for developped turbulence, parameter space investigation, high resolution exploration

MuScaTS vs. previous work

- Retains *both* advection and deformation, also adds diffusion.
- Is directly linked to governing equations

  => Larger scope and approximations are controlled
- Better realism, incorporates coherent structures

MuScaTS vs. neural networks

- MuScaTS:

  - takes only partial differential equations as inputs

  - is linked to physics

  - cost ~ handful of equivalent simulation steps

- Neural-Networks

  - require large training sets

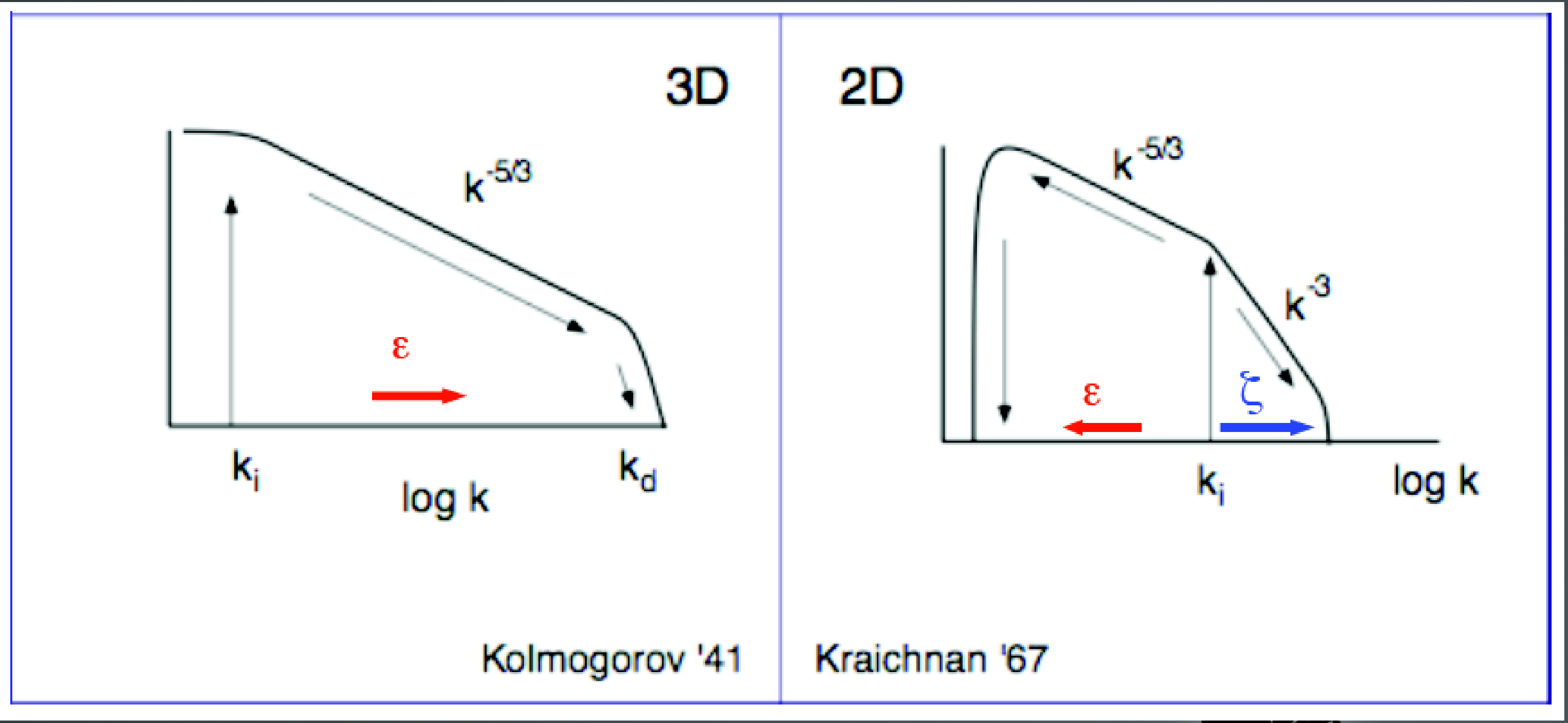
  - are black boxes

  - once trained, are *very* efficient to run

Prospects

- Compressible hydro, 3D, MHD, ...
- Higher order integration
- Try additional sweep from small scales
- Large training set generation for neural networks
- Deprojection (e.g. recovering the actual 3D structure from the position-position-velocity data cube)
- Weather forecast (random clouds generation, velocity inference from sparse data)
- Theoretical framework to understand coherent structure formation and their statistics

3D vs 2D



3D
2D
k-5/3
ε
k_i
log k
k_d
Kolmogorov '41
k-5/3
k-3
ε
ζ
k_i
log k
Kraichnan '67