Simulating galaxies with cosmic-ray magneto-hydrodynamics

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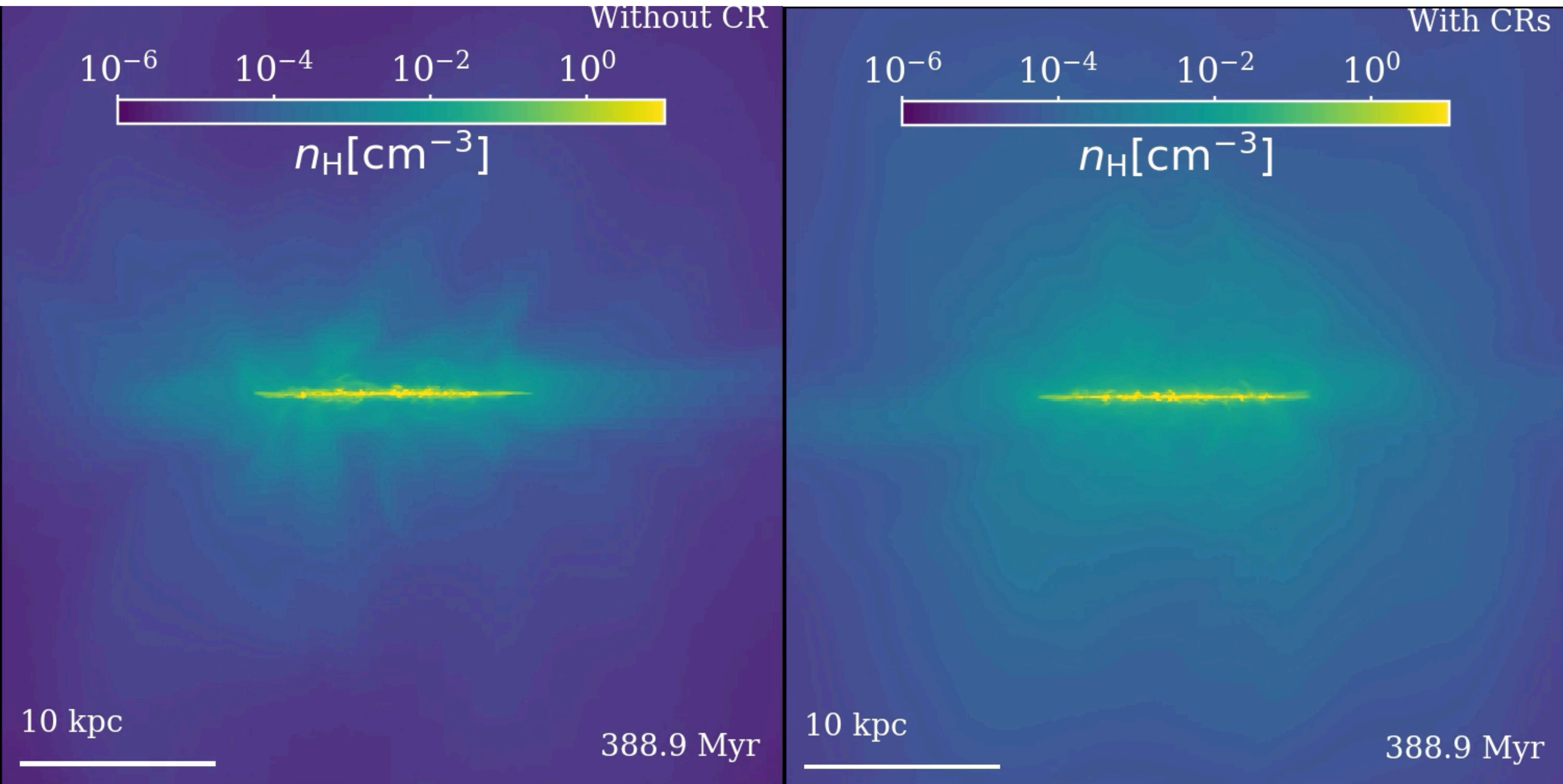
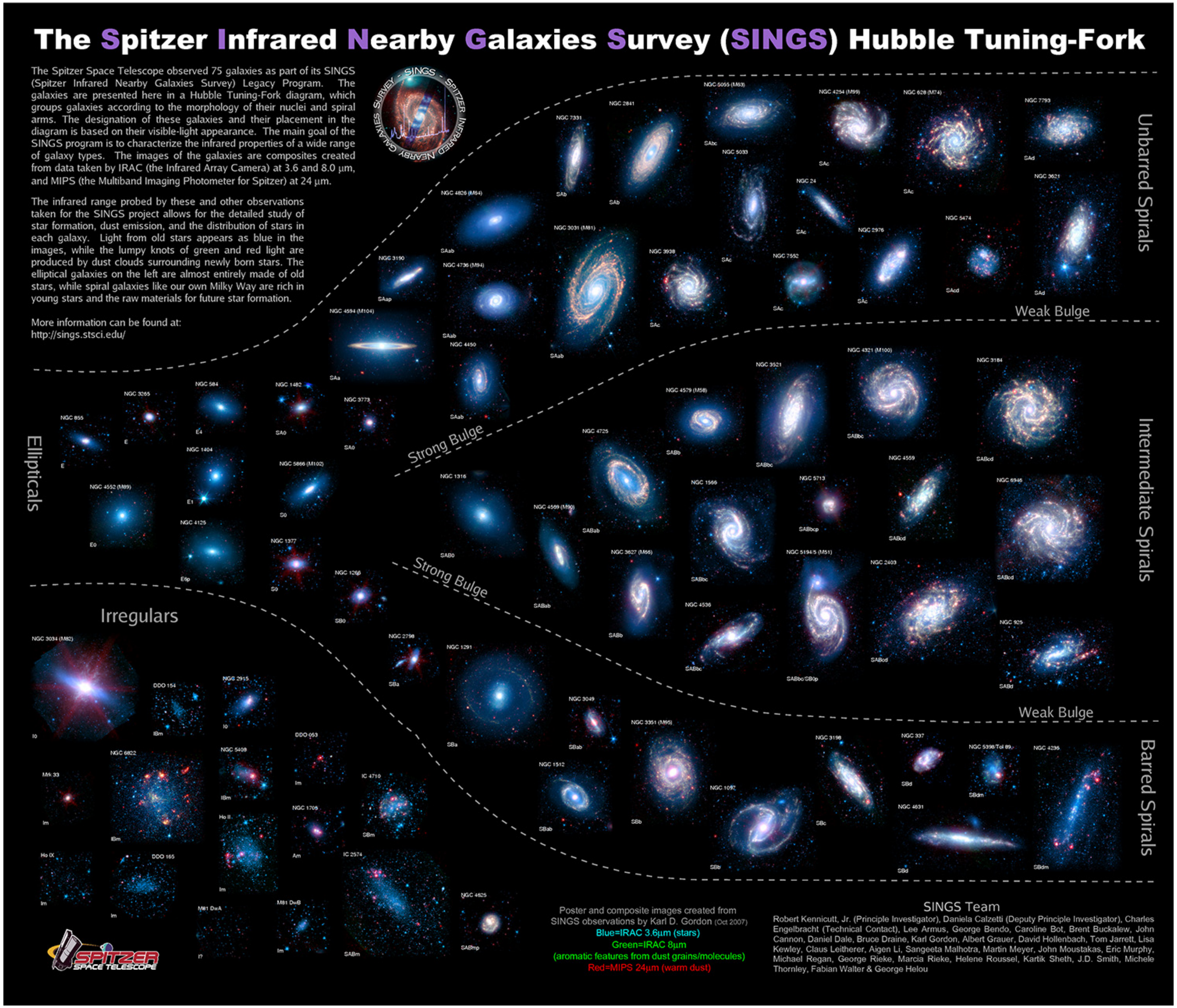
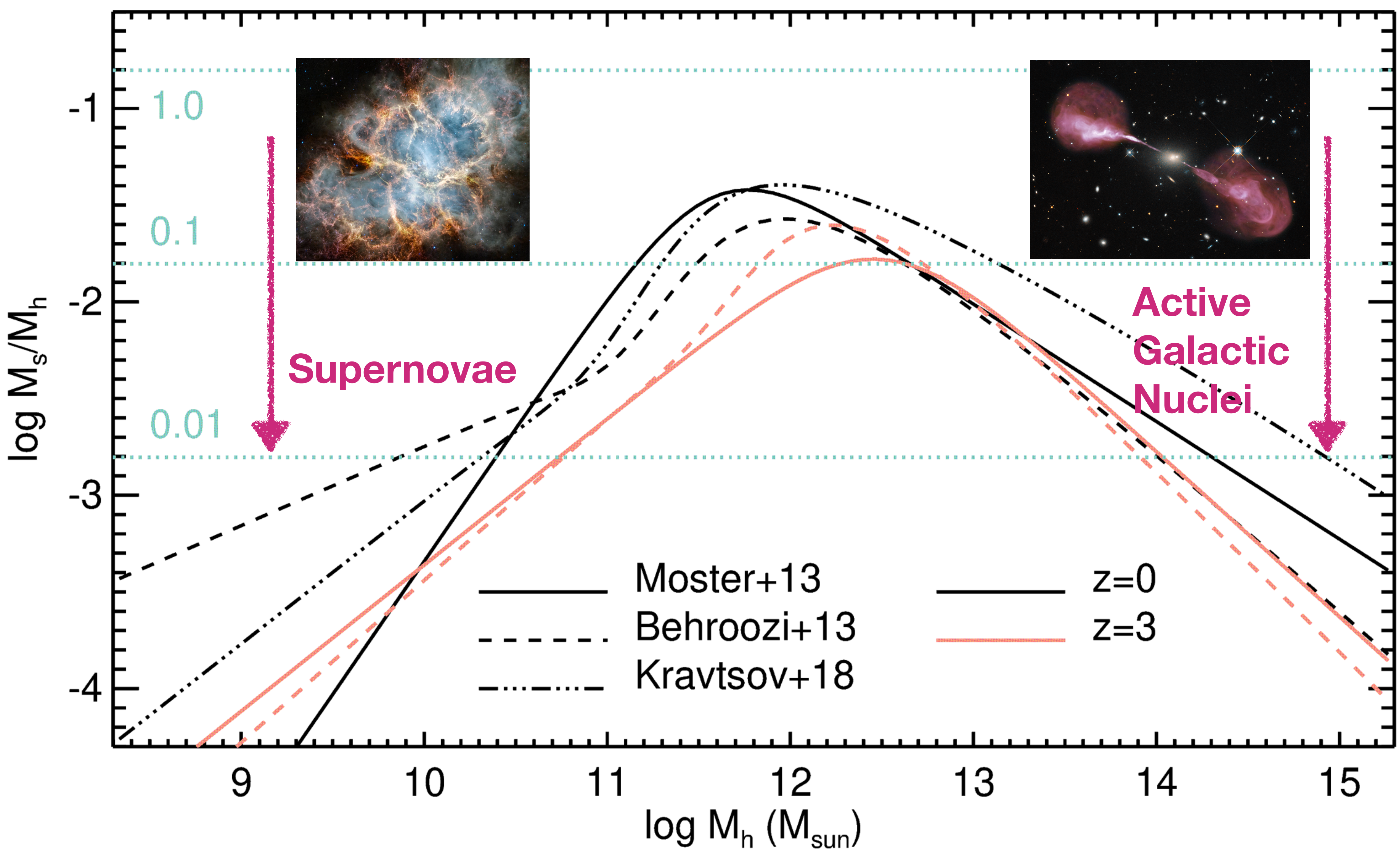


Galaxies: evolution and feedback

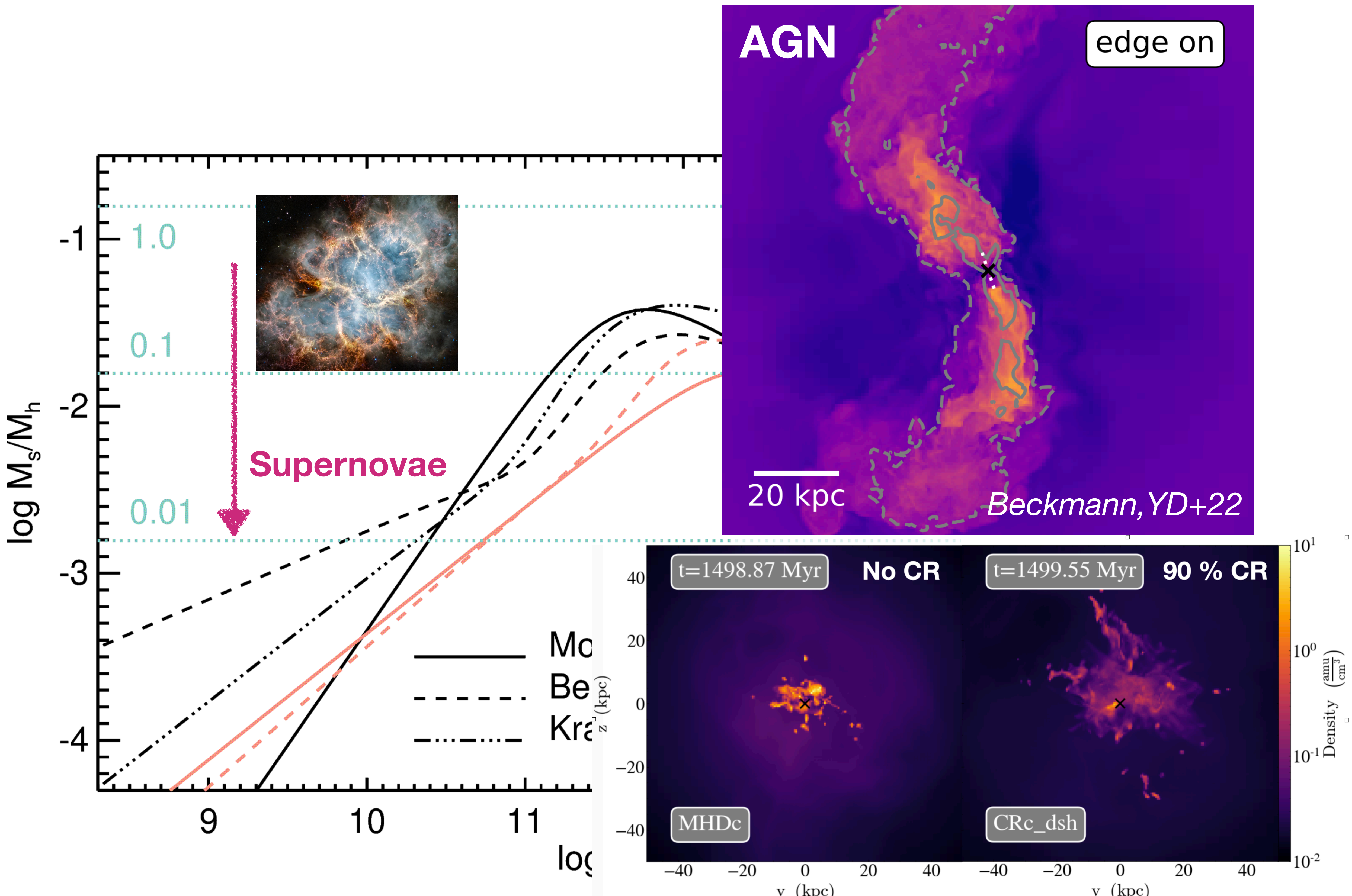


Morphological mix depends on accreted gas angular momentum, mergers, and feedback

Galaxies: evolution and feedback

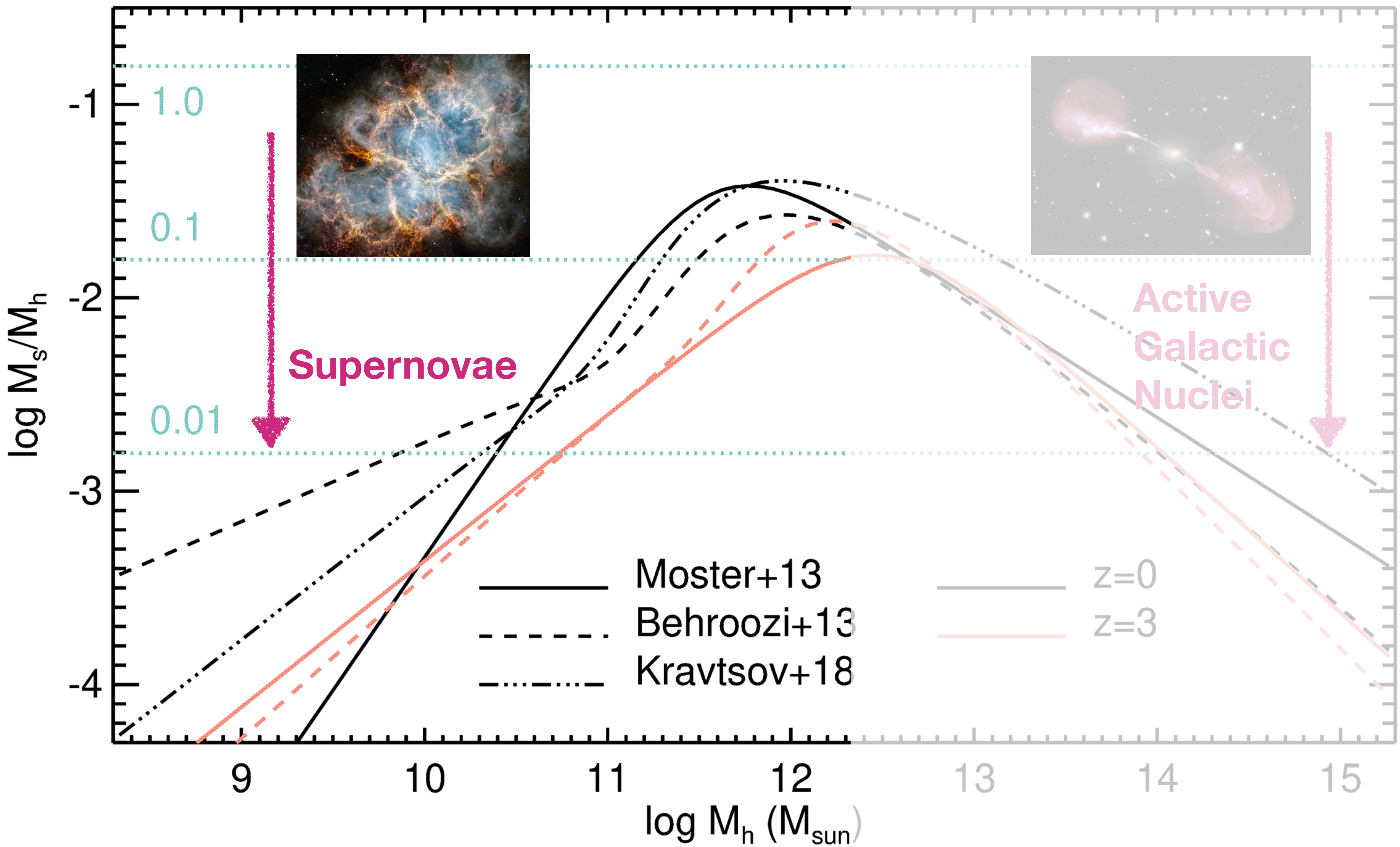


Galaxies: evolution and feedback

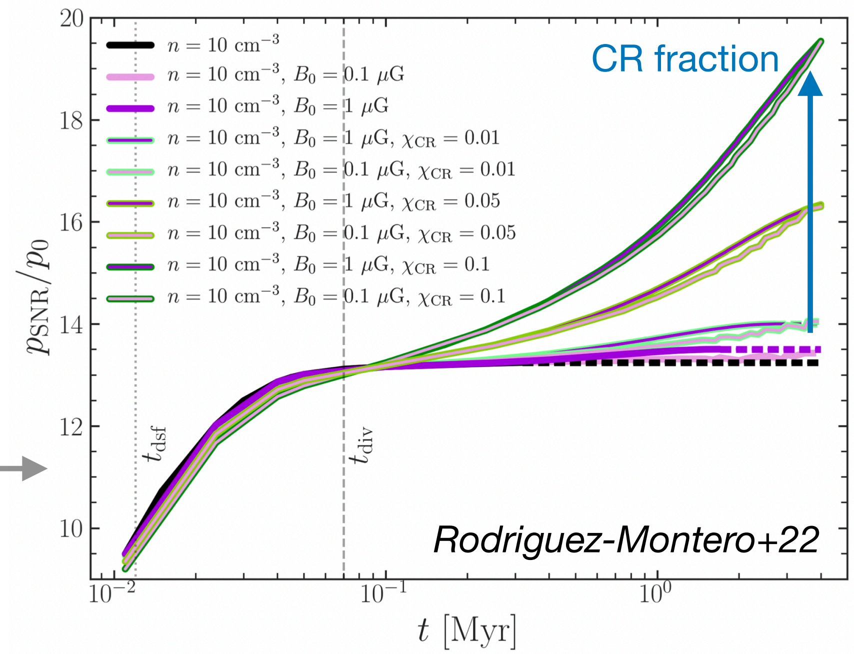


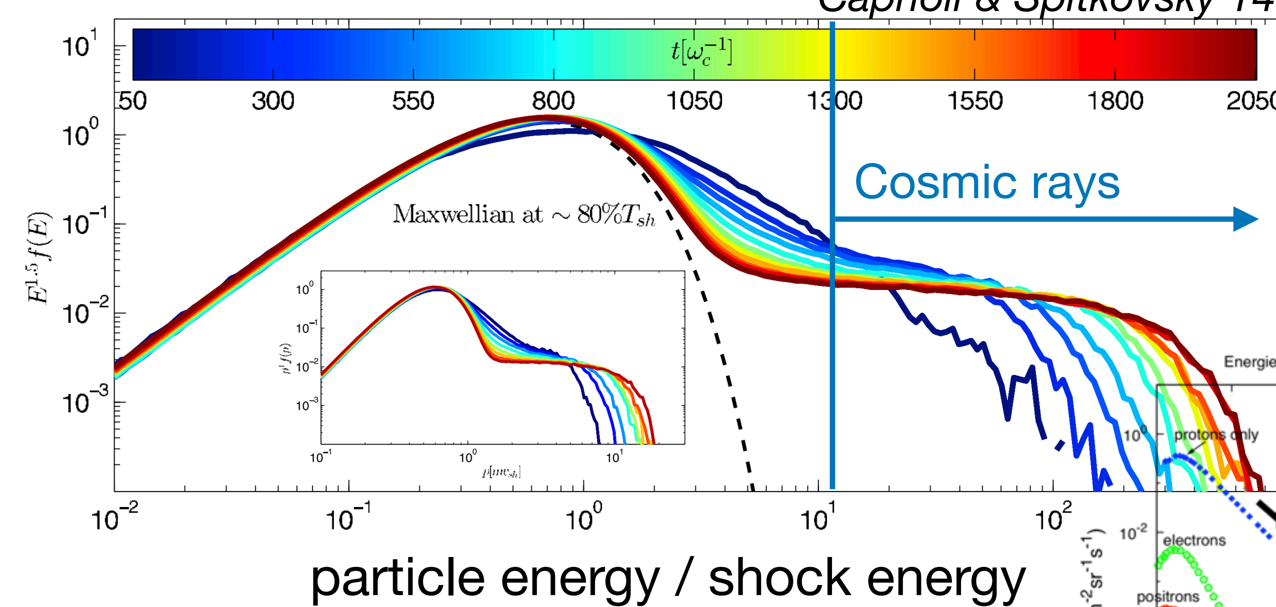
See also Ruszkowski+17; Ehlert+18; Su+20,21

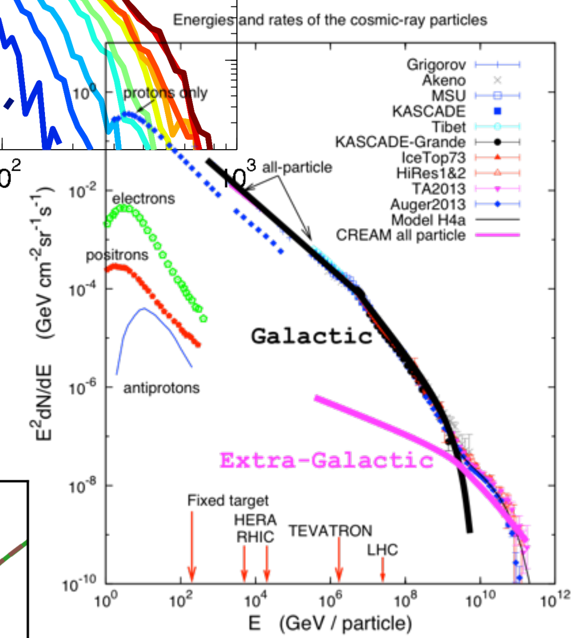
Galaxies: evolution and feedback



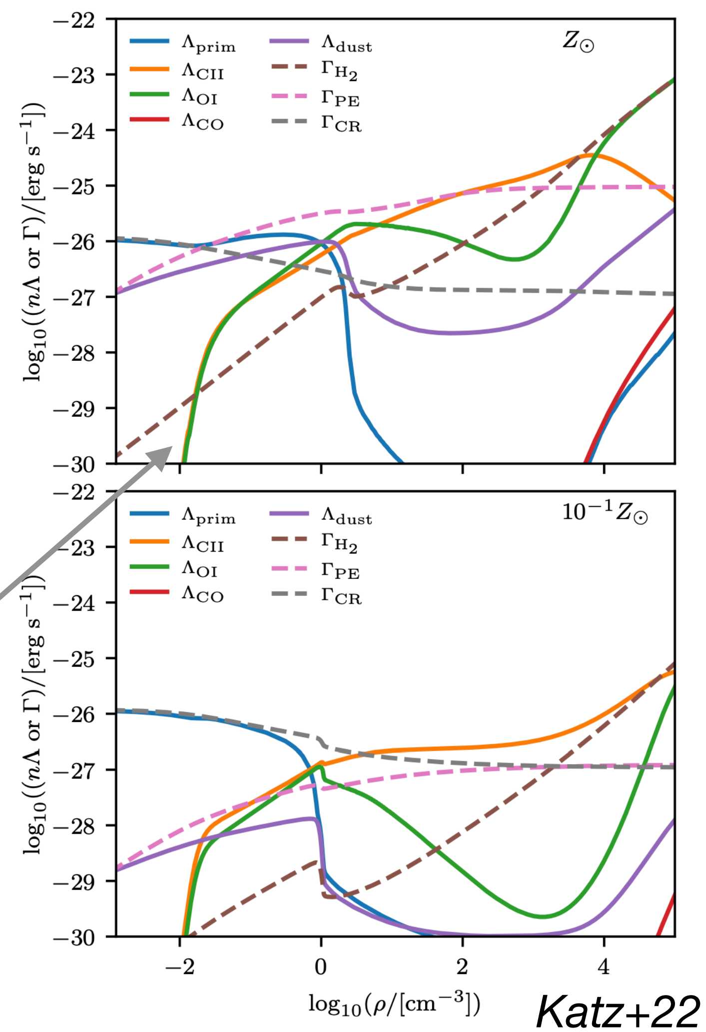
Why should we care about cosmic rays?







- **Equipartition of energies** (kinetic ~ thermal ~ magnetic ~ cosmic rays) in galaxy formation problems: intra-cluster medium, active galactic nuclei jets, galactic winds, interstellar medium
- As a relativistic population of particles **their adiabat and losses are different from that of the gas**
- **Diffusion** is a key aspect of cosmic ray transport
- Cosmic rays are **produced at shocks**: supernovae, jets, cosmic infall
- More momentum in SN explosions
- Important heating mechanism in the diffuse ISM
- Sets the electron fraction at high cloud densities due to CR ionisation losses



Cosmic ray magneto-hydrodynamics

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \, . \, (\rho \vec{u}) = 0$$ mass

$$\frac{\partial \rho \vec{u}}{\partial t} + \vec{\nabla} \, . \left(\rho \vec{u} \otimes \vec{u} + P - \frac{\vec{B} \otimes \vec{B}}{4\pi} \right) = - \vec{\nabla} P_{\rm cr}$$ momentum

$$\frac{\partial e}{\partial t} + \vec{\nabla} \, . \left((e + P)\vec{u} - \frac{(\vec{B} \, . \, \vec{u})\vec{B}}{4\pi} \right) = - \vec{u} \, . \, \vec{\nabla} P_{\rm cr} + \mathscr{H}_{\rm cr} + \Lambda_{\rm r}$$ total energy

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times (\vec{u} \times \vec{B}) = 0$$ magnetic field

CR energy

$$\frac{\partial e_{\rm cr}}{\partial t} + \vec{\nabla} \, . \left(\vec{u} e_{\rm cr} + \vec{u}_{\rm s} \gamma_{\rm cr} e_{\rm cr} \right) = - P_{\rm cr} \vec{\nabla} \, . \, \vec{u} - \vec{\nabla} \, . \, (-\kappa \vec{b}\vec{b} \, . \, \vec{\nabla} e_{\rm cr}) + \vec{u}_{\rm s} \, . \, \vec{\nabla} P_{\rm cr} + \mathscr{H}_{\rm acc} + \mathscr{L}_{\rm r}$$

Assume all CRs can be described by a single energy-momentum (e.g. 1GeV) bin

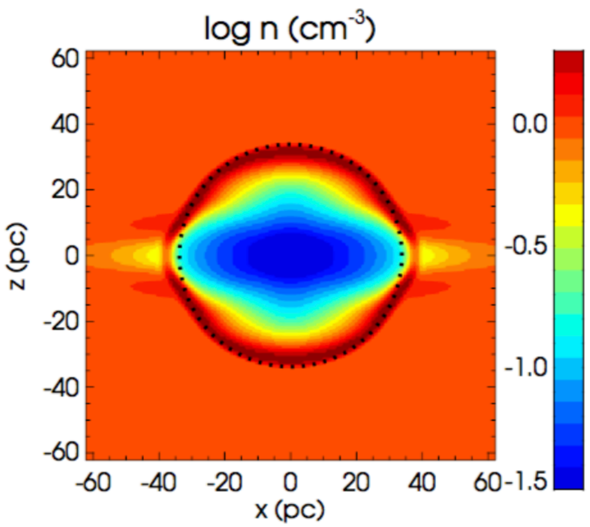
CR MHD

$$\frac{\partial e_{\rm cr}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} e_{\rm cr} + \vec{u}_{\rm s} \gamma_{\rm cr} e_{\rm cr} \right) = -P_{\rm cr} \vec{\nabla} \cdot \vec{u} - \vec{\nabla} \cdot \left(-\kappa \vec{b}\vec{b} \cdot \vec{\nabla} e_{\rm cr} \right) + \vec{u}_{\rm s} \cdot \vec{\nabla} P_{\rm cr} + \mathscr{H}_{\rm acc} + \mathscr{L}_{\rm r}$$

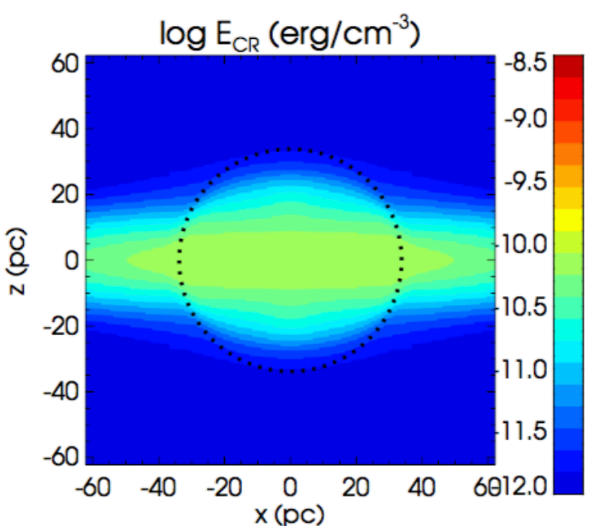
Advection — Streaming — Work — Diffusion — Streaming (heating of the plasma) — Shock acceleration/ injection — Radiative losses coulomb, ionisation, hadronic

SN explosion
with anisotropic diffusion

$$\vec{F}_{\rm CR} = -\kappa \vec{b}(\vec{b}.\nabla P_{\rm CR})$$



YD & Commerçon 16

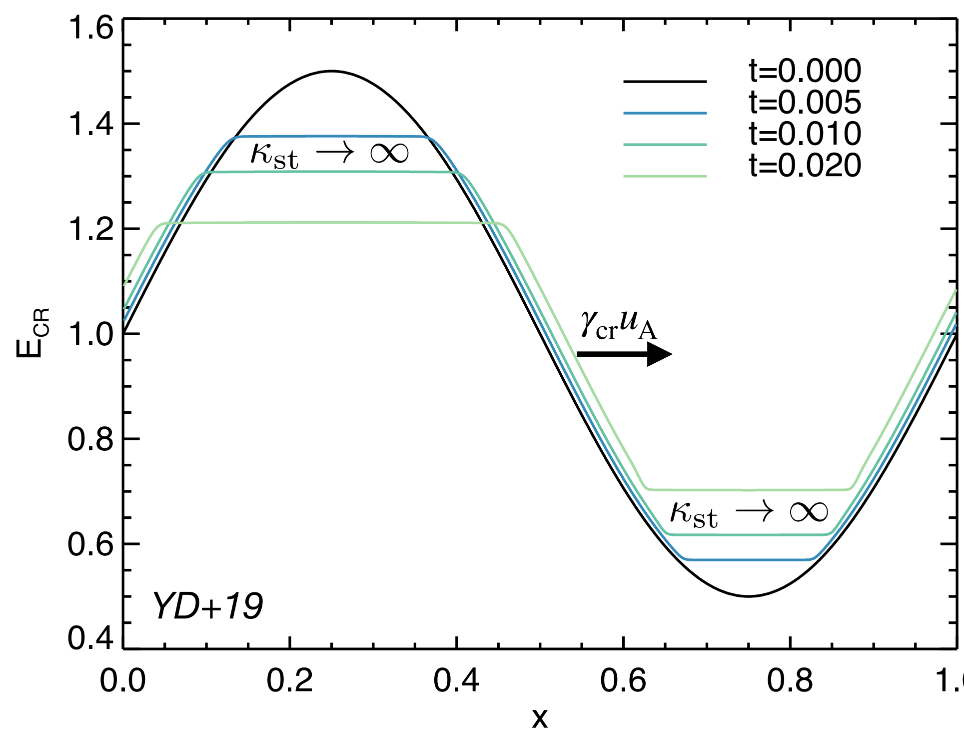


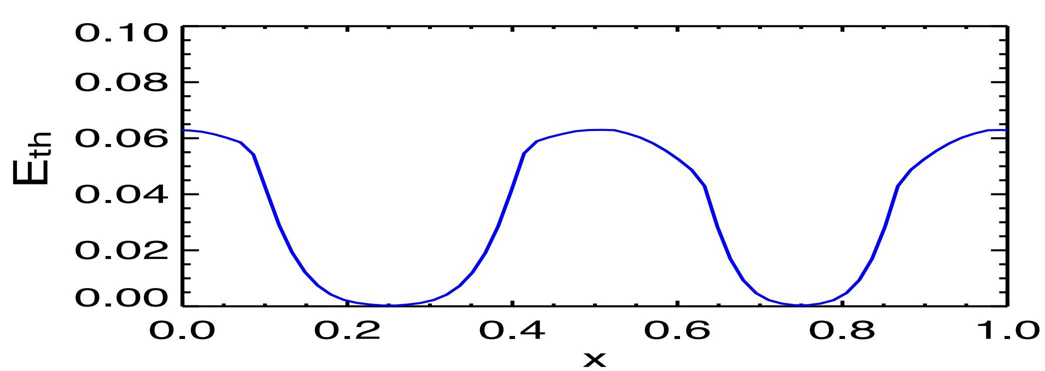
initial B-field

CR streaming

$$\nabla.\left((e_{\rm CR} + P_{\rm CR})\vec{u}_{\rm st}\right) = \nabla.\left(-\frac{(e_{\rm CR} + P_{\rm CR})|B|}{|\vec{b}.\nabla e_{\rm CR}|\sqrt{4\pi\rho}} \vec{b}(\vec{b}.\nabla e_{\rm CR}) \right)$$

$$= \nabla.(-\kappa_{\rm st}\vec{b}(\vec{b}.\nabla e_{\rm CR}))$$





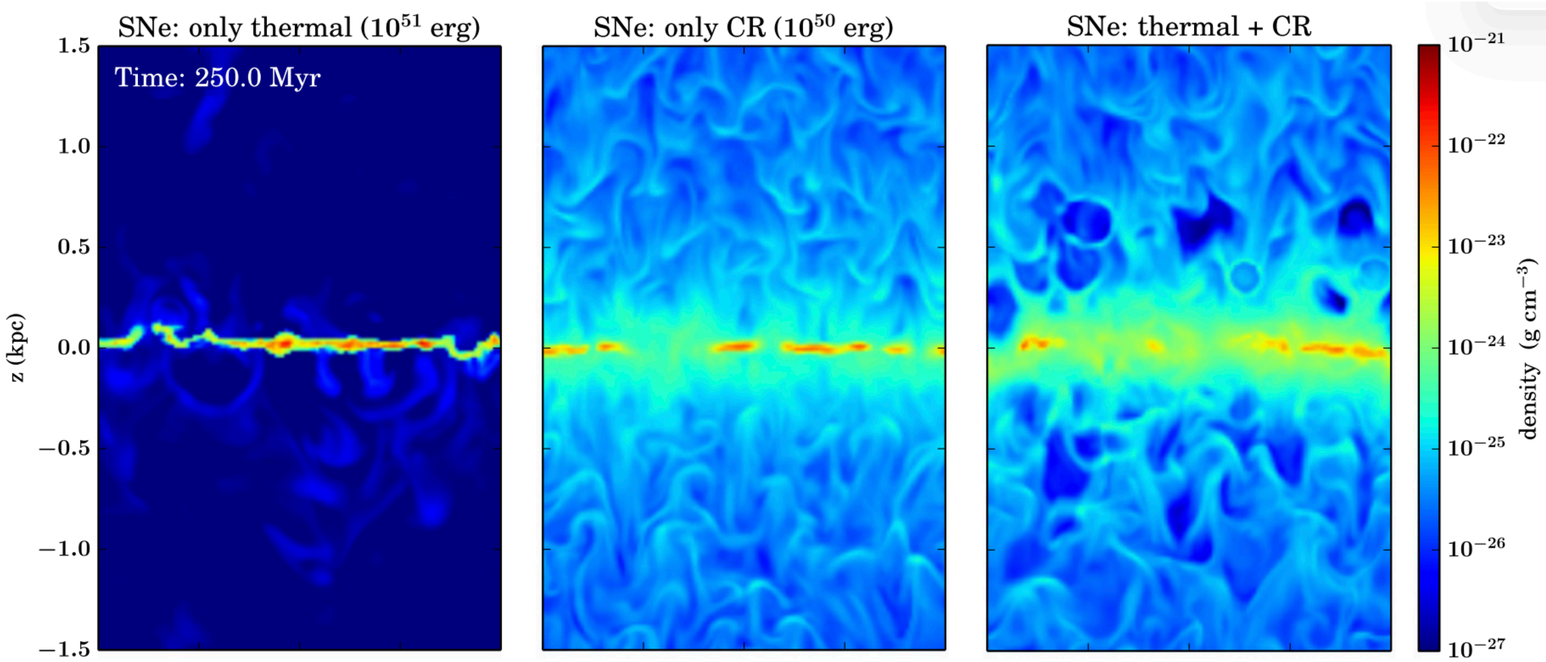
$$\vec{u}_{\rm st} = -\vec{u}_{\rm A}{\rm sign}(\vec{b}.\nabla e_{\rm CR})$$

$$\kappa_{\rm st} = \frac{(e_{\rm CR} + P_{\rm CR})|B|}{|\vec{b}.\nabla e_{\rm CR}|\sqrt{4\pi\rho}}$$

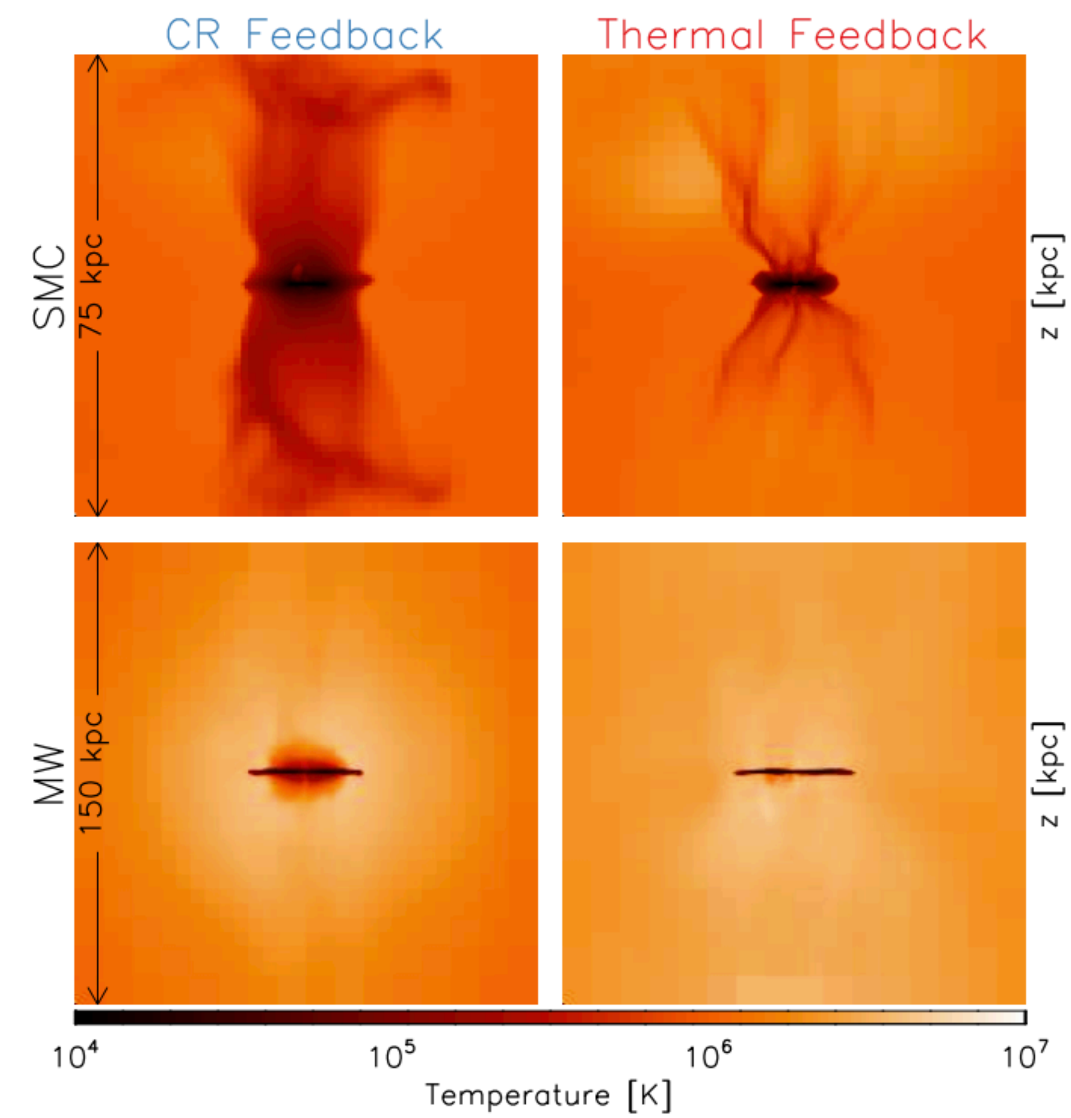
$$\mathcal{L}_{\rm st} = -\vec{u}_{\rm st}.\nabla P_{\rm CR} \leq 0$$

Implicit vs. explicite scheme
+
Transverse B-field flux limiters

CR feedback and winds

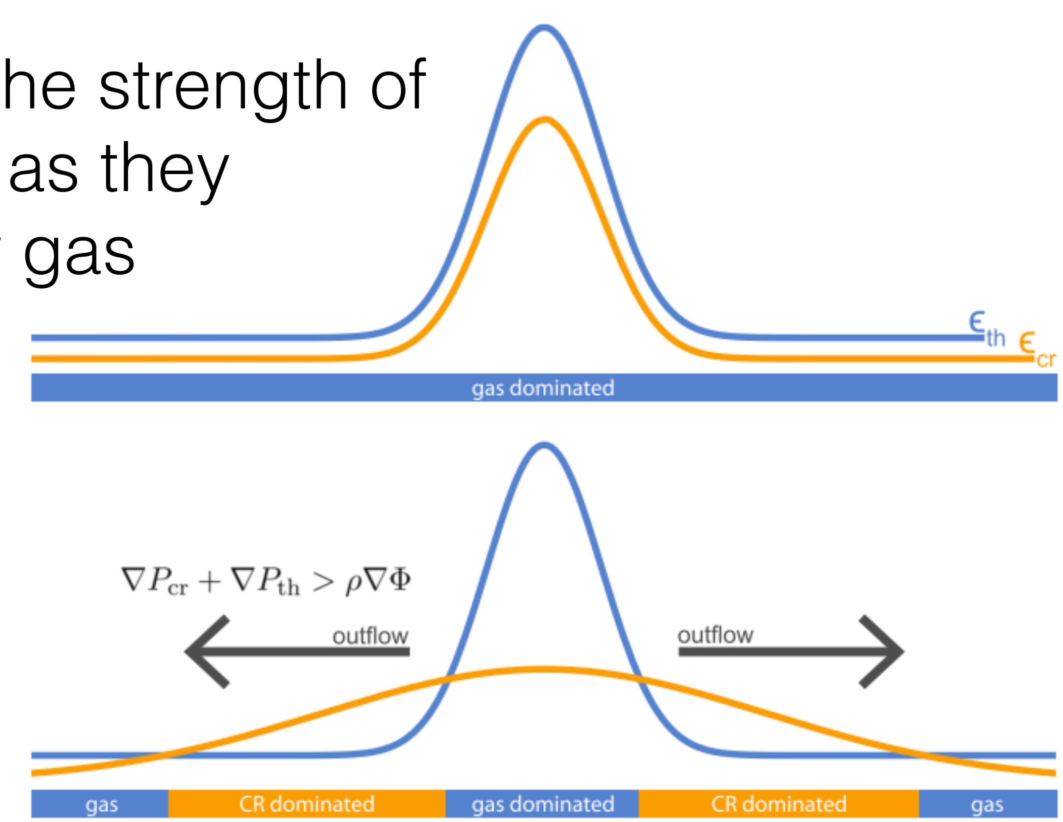


Girichidis+16



Booth+13

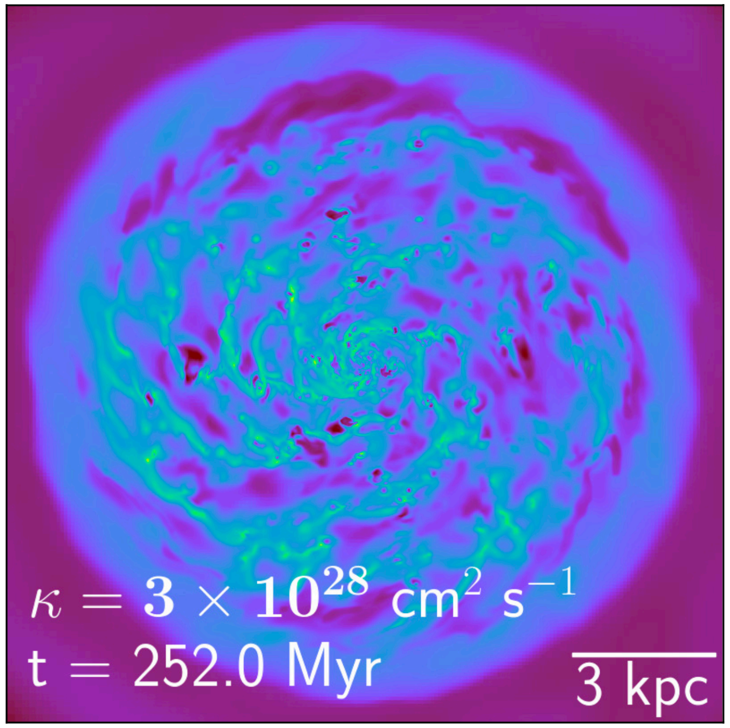
CRs reinforce the strength of galactic winds as they diffuse into low gas densities.



Salem & Bryan 14

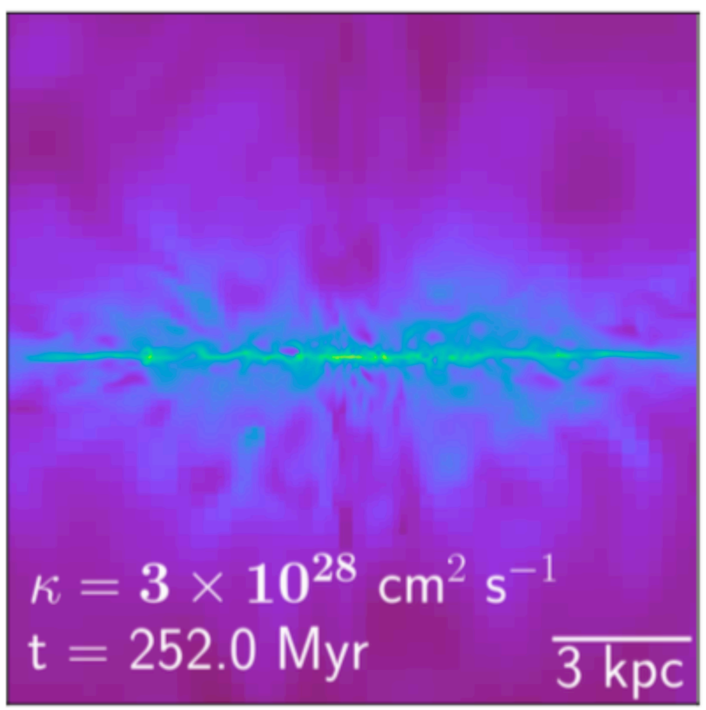
See: Jubelgas+08,
Uhlig+12,
Hanasz+13,
Booth+13,
etc.

Simulating galactic winds with CRMHD

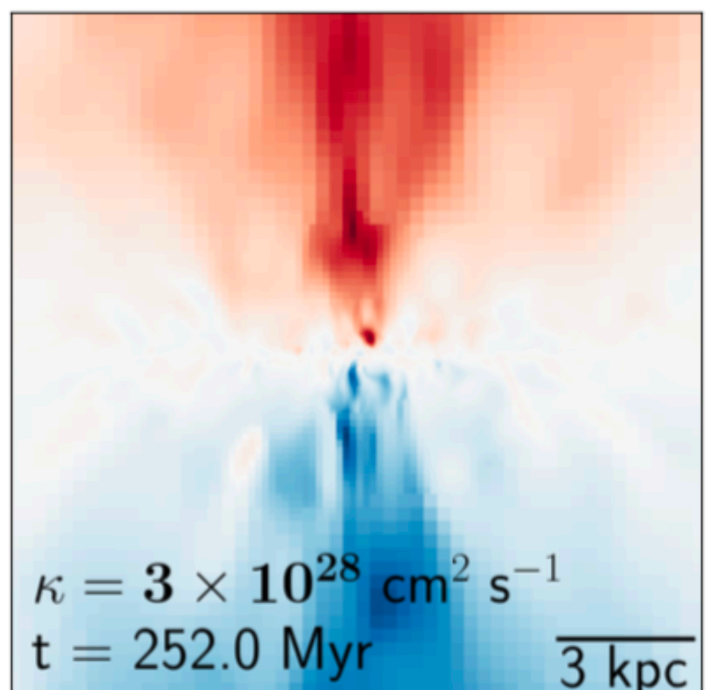


- Adaptive Mesh Refinement with the RAMSES code
- Isolated disc in an NFW DM halo: $M_{halo}=10^{10}$ and 10^{11} M_{sun}
- Stellar and gaseous disc (50% of gas) within a DM halo (f_{baryon}=3.5%)
- Initial toroïdal B field configuration (tried also poloidal B field) B~5-10 μG
- Gas cooling down to 10^3 K
- Star formation above gas density n_H=100 H.cm^{-3} with 2% efficiency
- SN feedback with $e_{SN}=10^{49}$ erg/M_{sun} and 10% in E_{CR}
- 10 pc spatial resolution and mass resolution of 10^3 M_{sun}

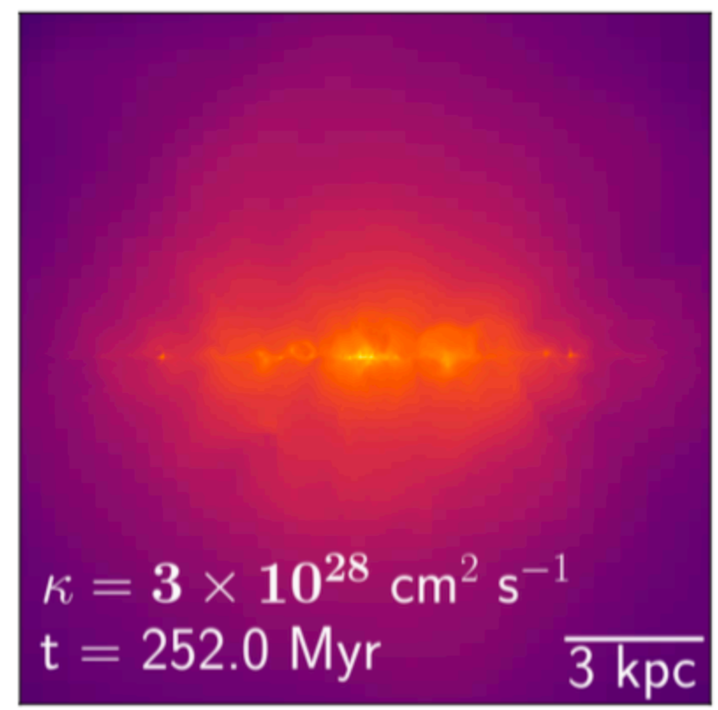
=> Play around with CR diffusion and streaming



density



velocity

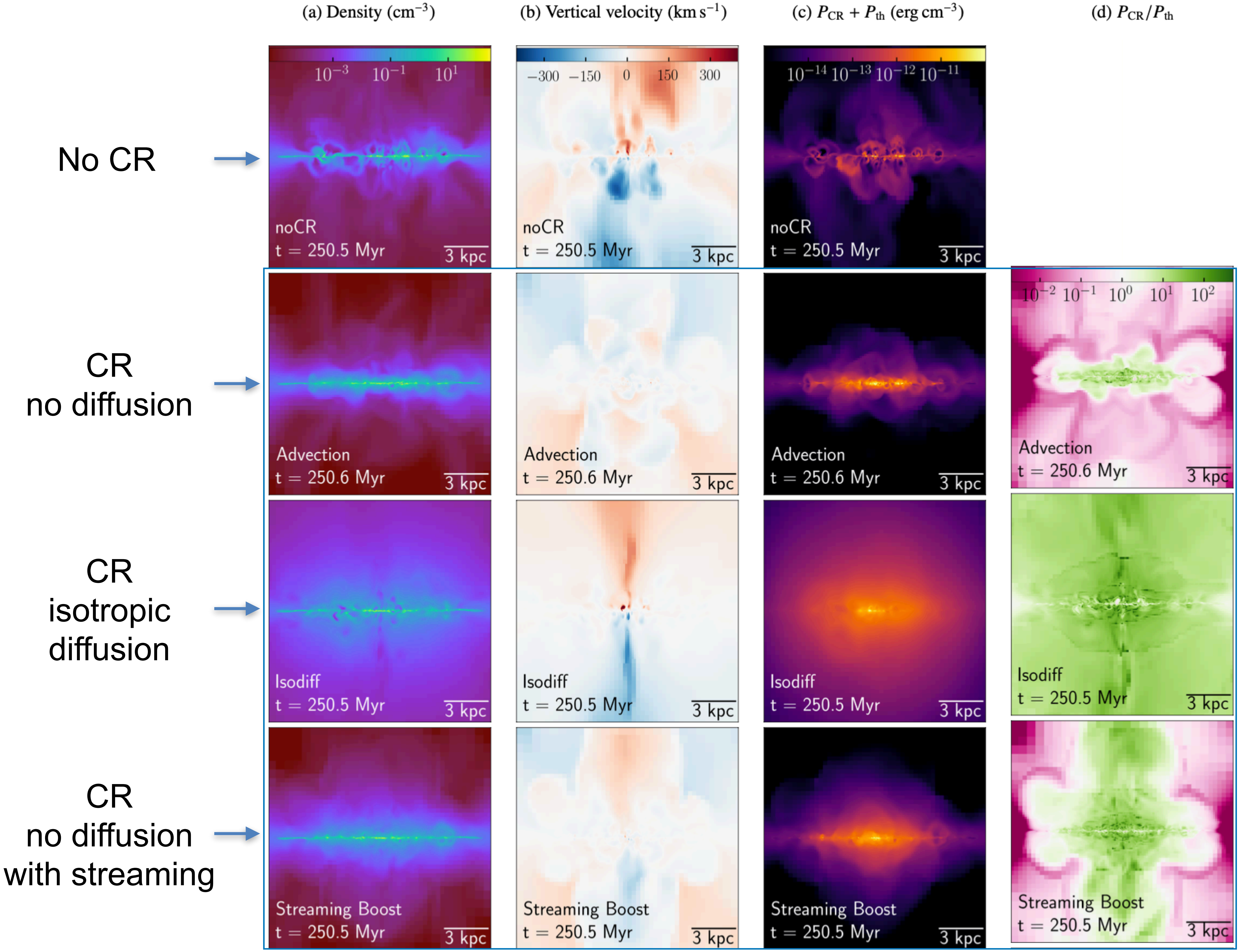


pressure

Dashyan & YD 20

SN-seeded CRs and large-scale galactic winds

$M_{\rm halo} = 10^{11}\,{\rm M}_\odot$



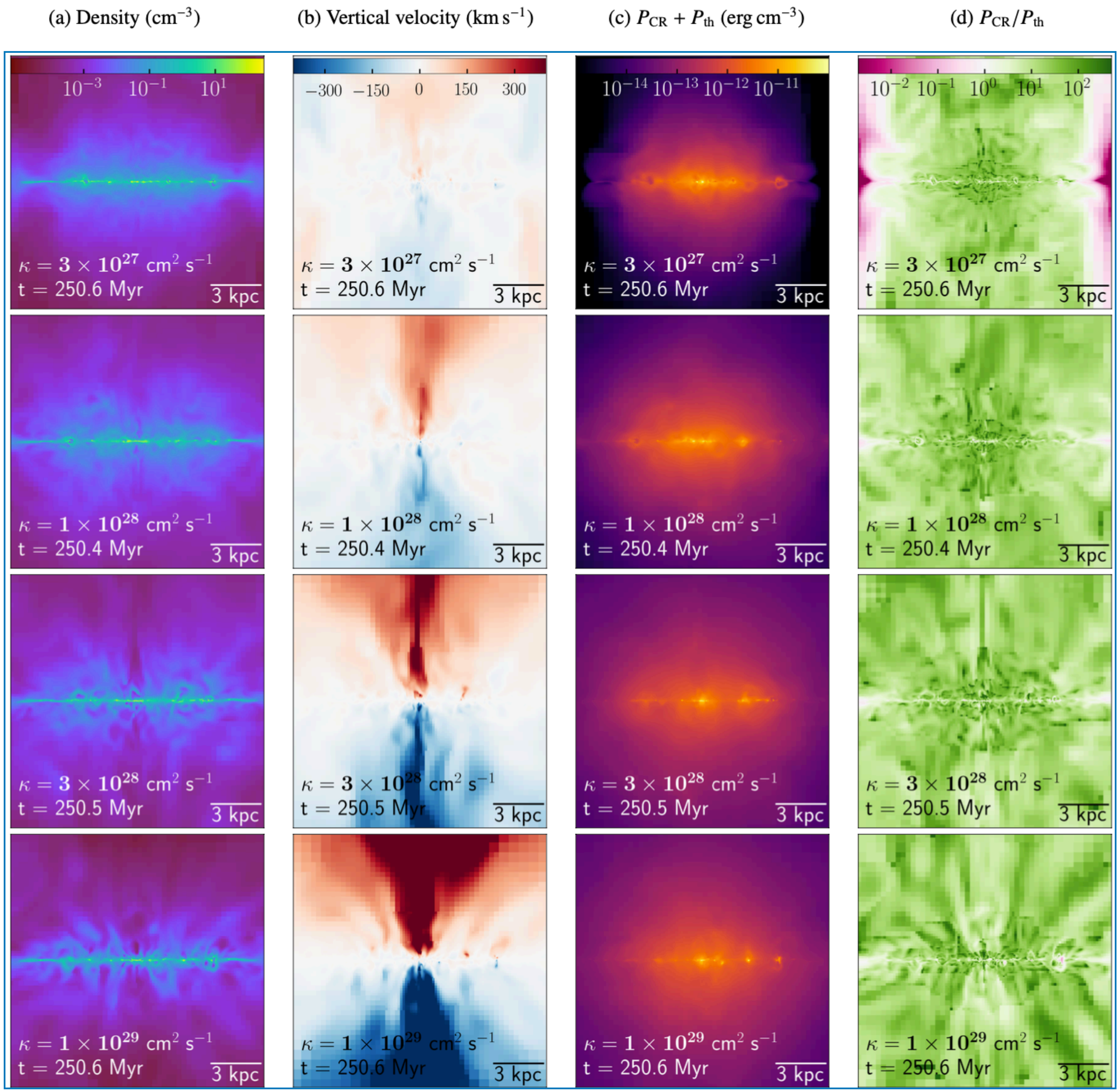
10% of $E_{SN}=10^{51}$ erg goes into E_{CR}

Dashyan & YD 20

SN-seeded CRs and large-scale galactic winds

CR **anisotropic** diffusion

More CR diffusion (κ)

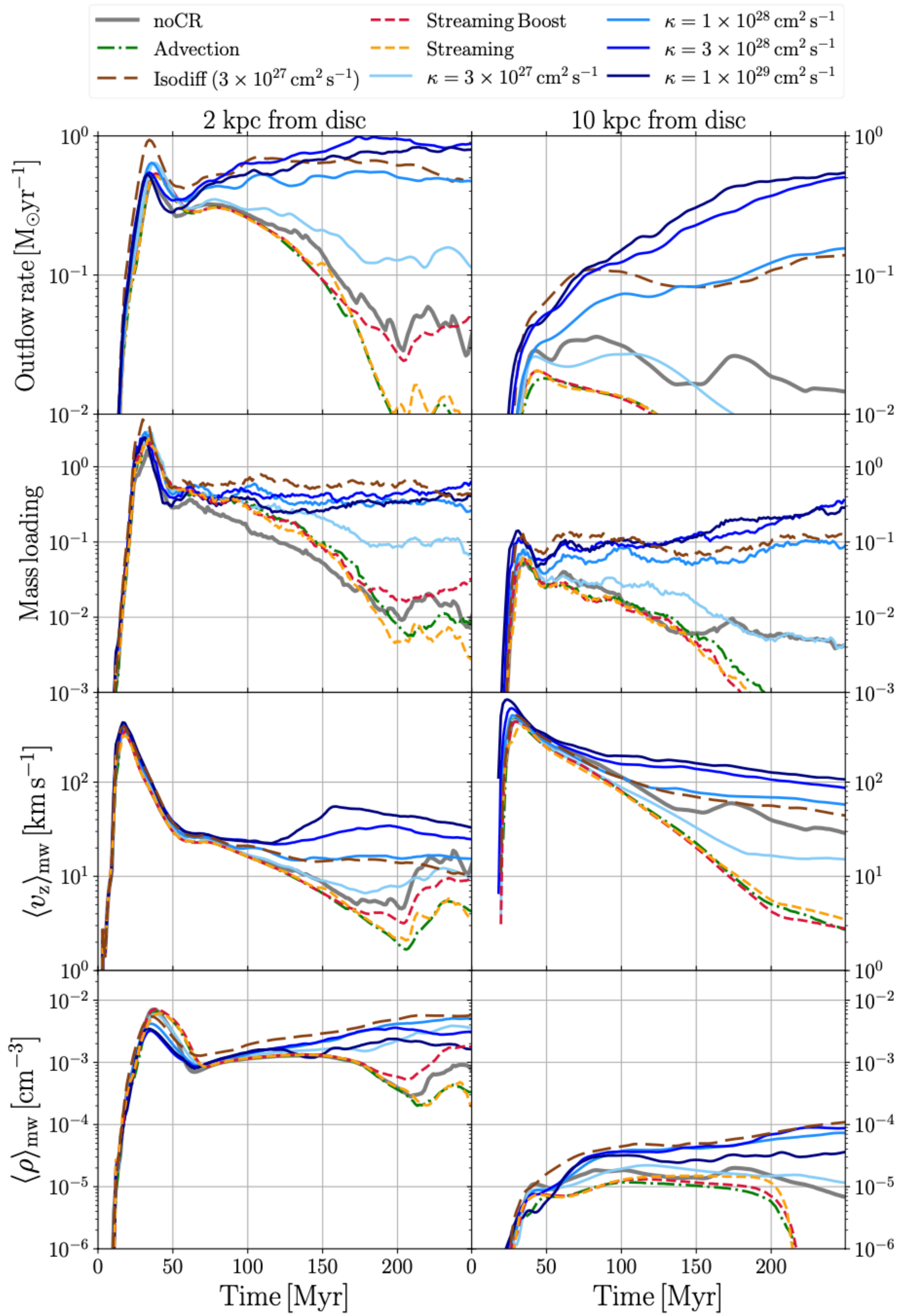


$M_{halo} = 10^{11}\,M_\odot$

10% of $E_{SN}=10^{51}$ erg goes into E_{CR}

Dashyan & YD 20

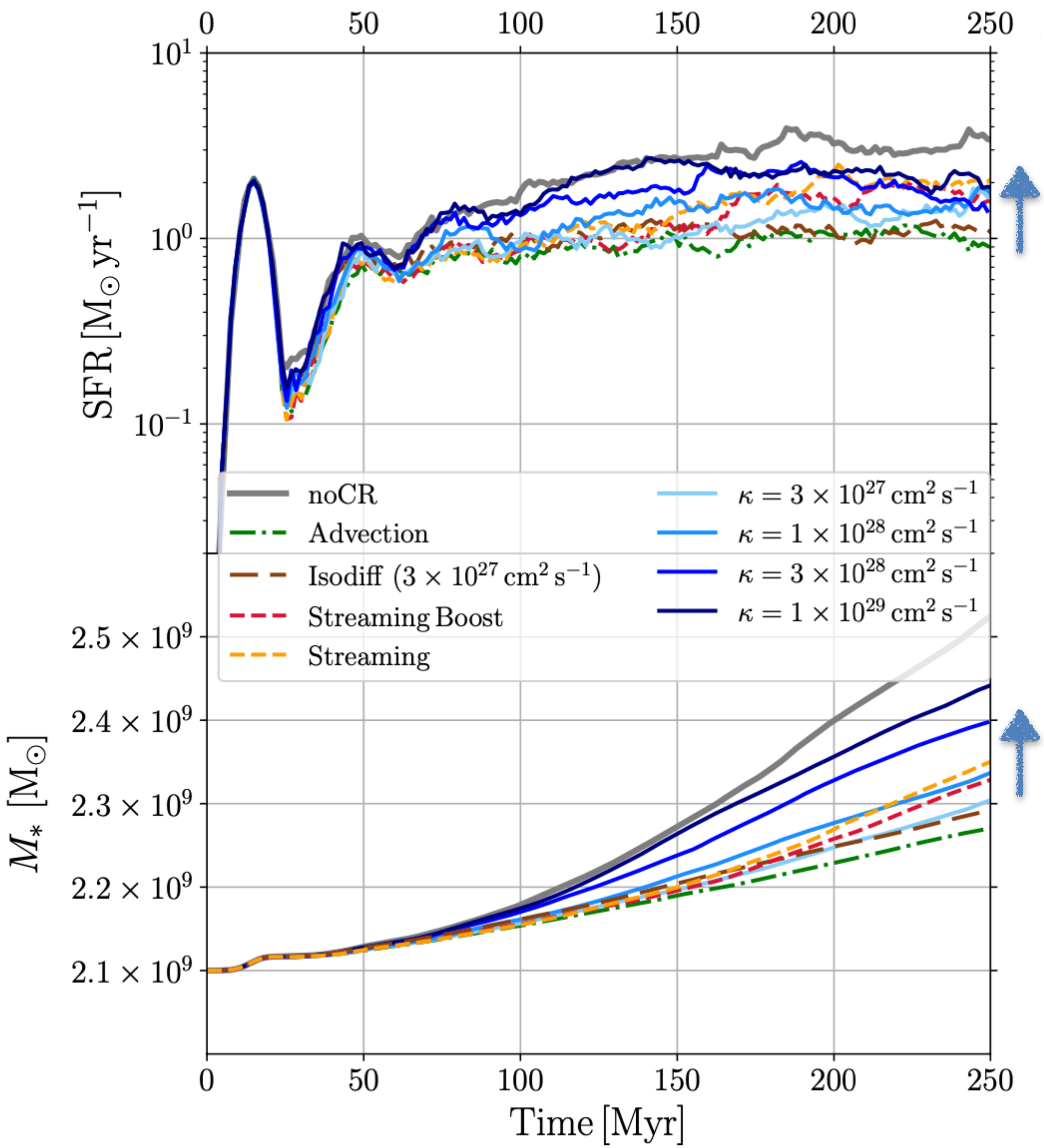
Large-scale winds much stronger with CRs



$M_{halo}=10^{11}$ M_{sun}

Dashyan & YD 20

Star formation rate reduced by CRs



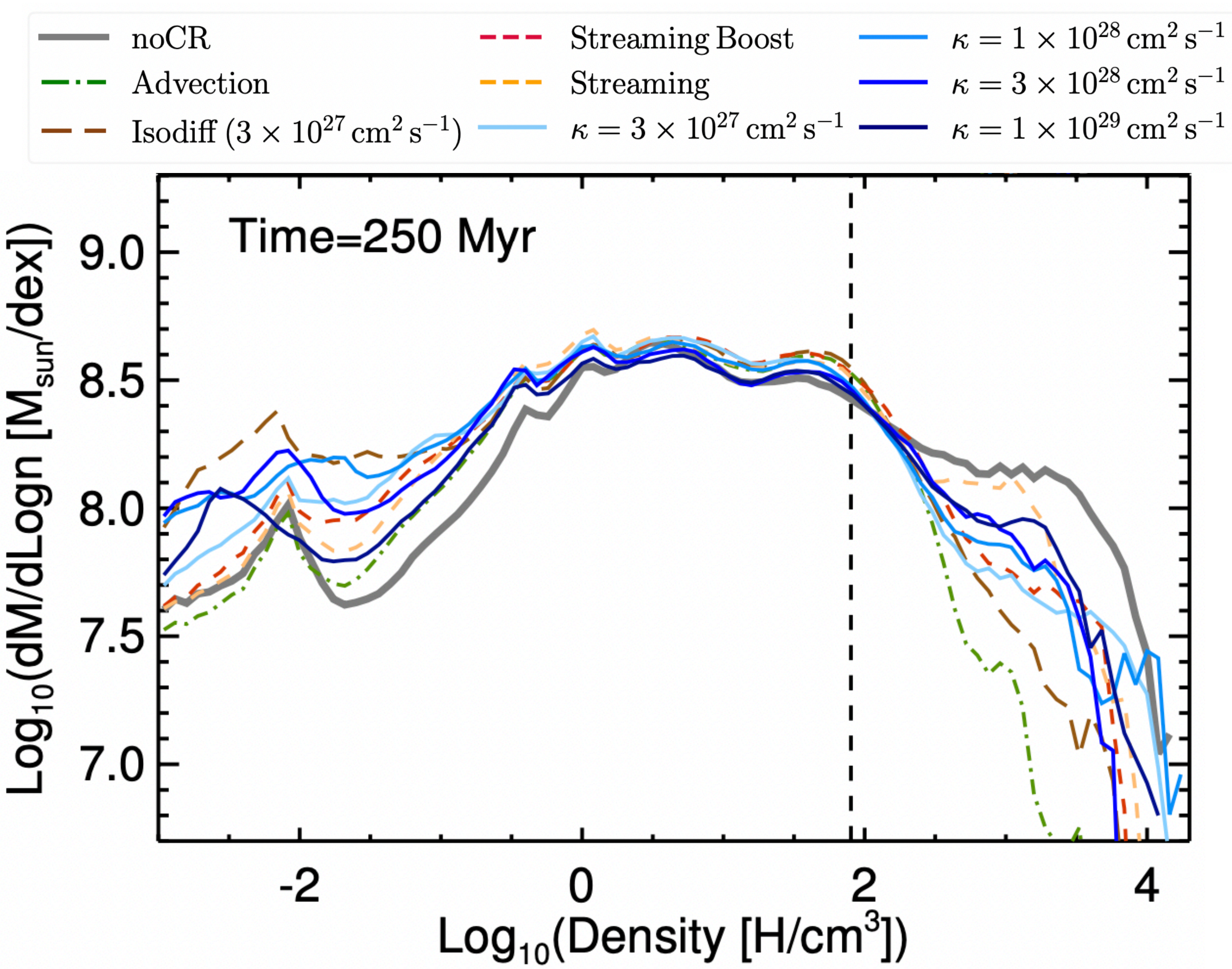
$M_{\mathrm{halo}} = 10^{11}\,\mathrm{M_\odot}$

More CR diffusion (κ)

More CR diffusion (κ)

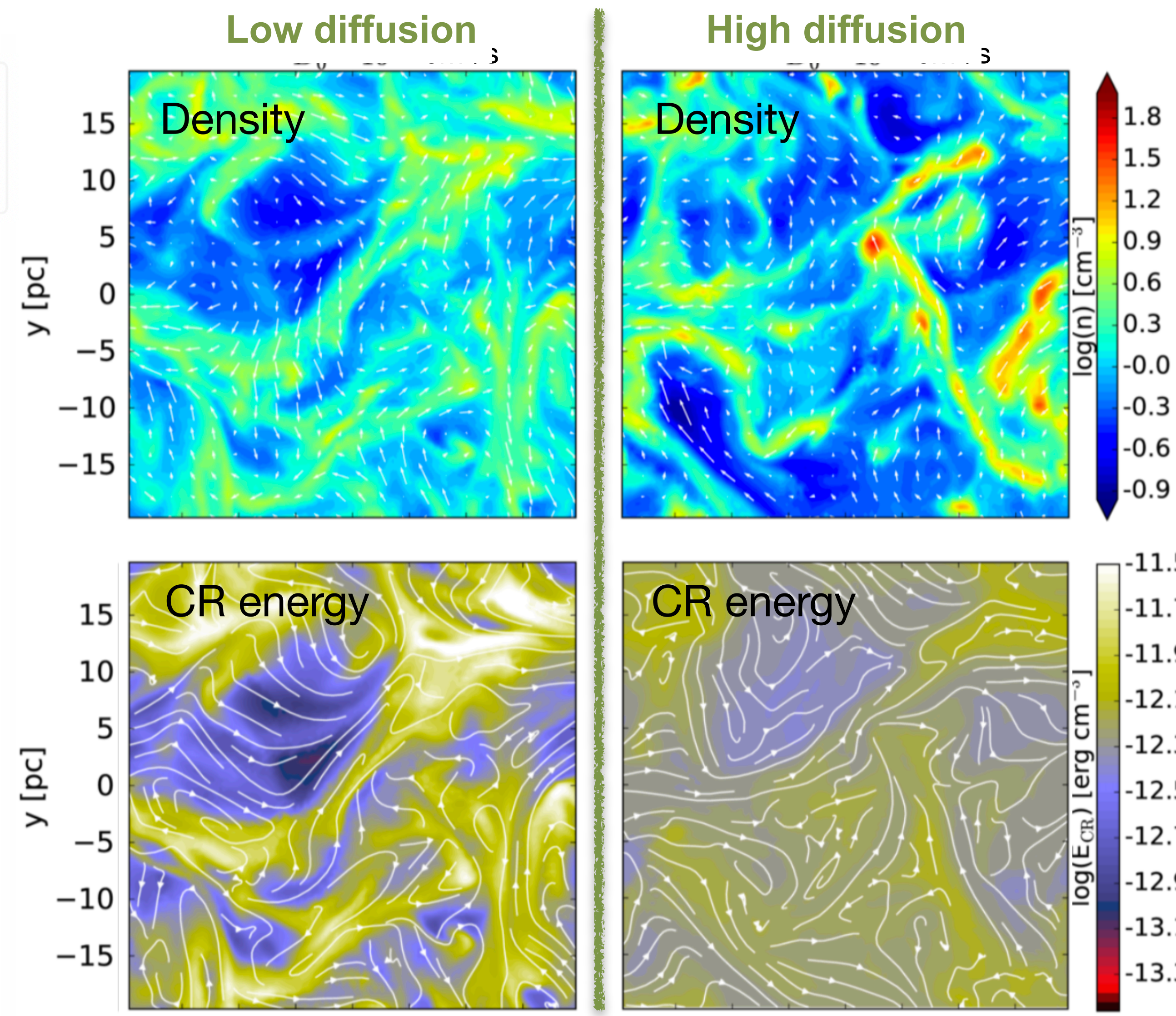
Dashyan & YD 20

CRs and ISM gas distribution



Dashyan & YD 20

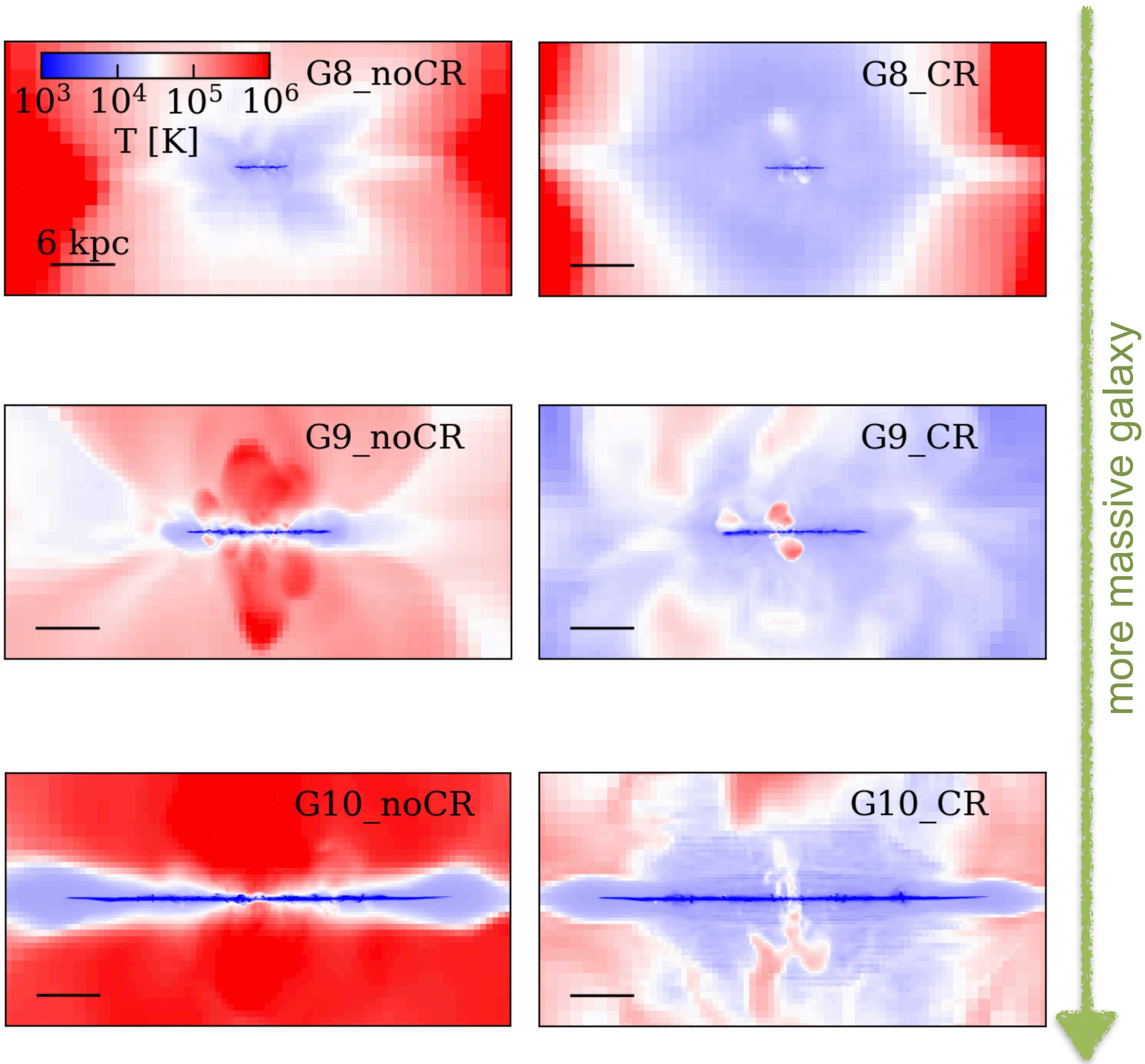
Idealised ISM box with forced turbulence

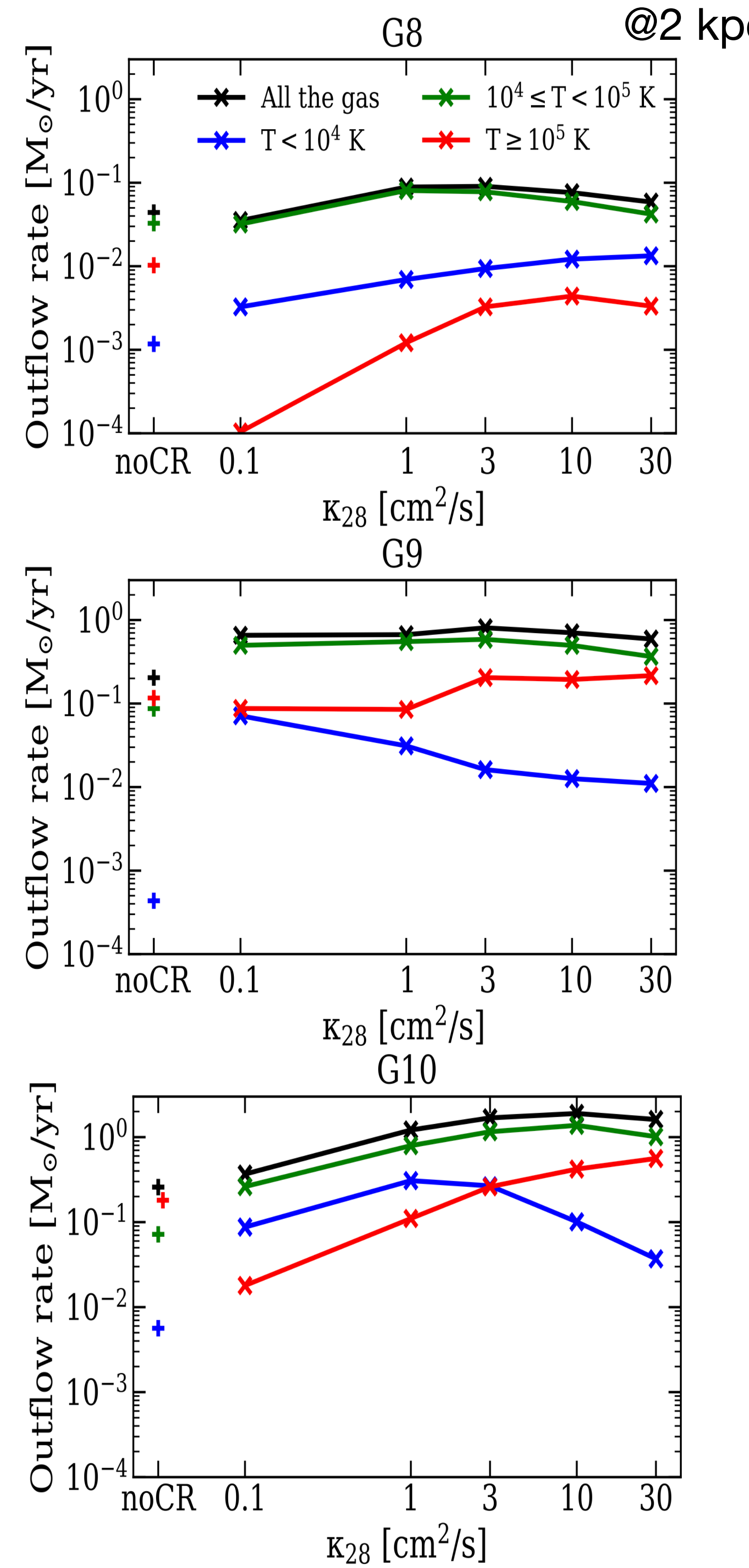


Commerçon, Marcowith & YD 19

See also *YD+19, Simpson+23, Sampson+26*

Wind thermodynamics depends on CRs

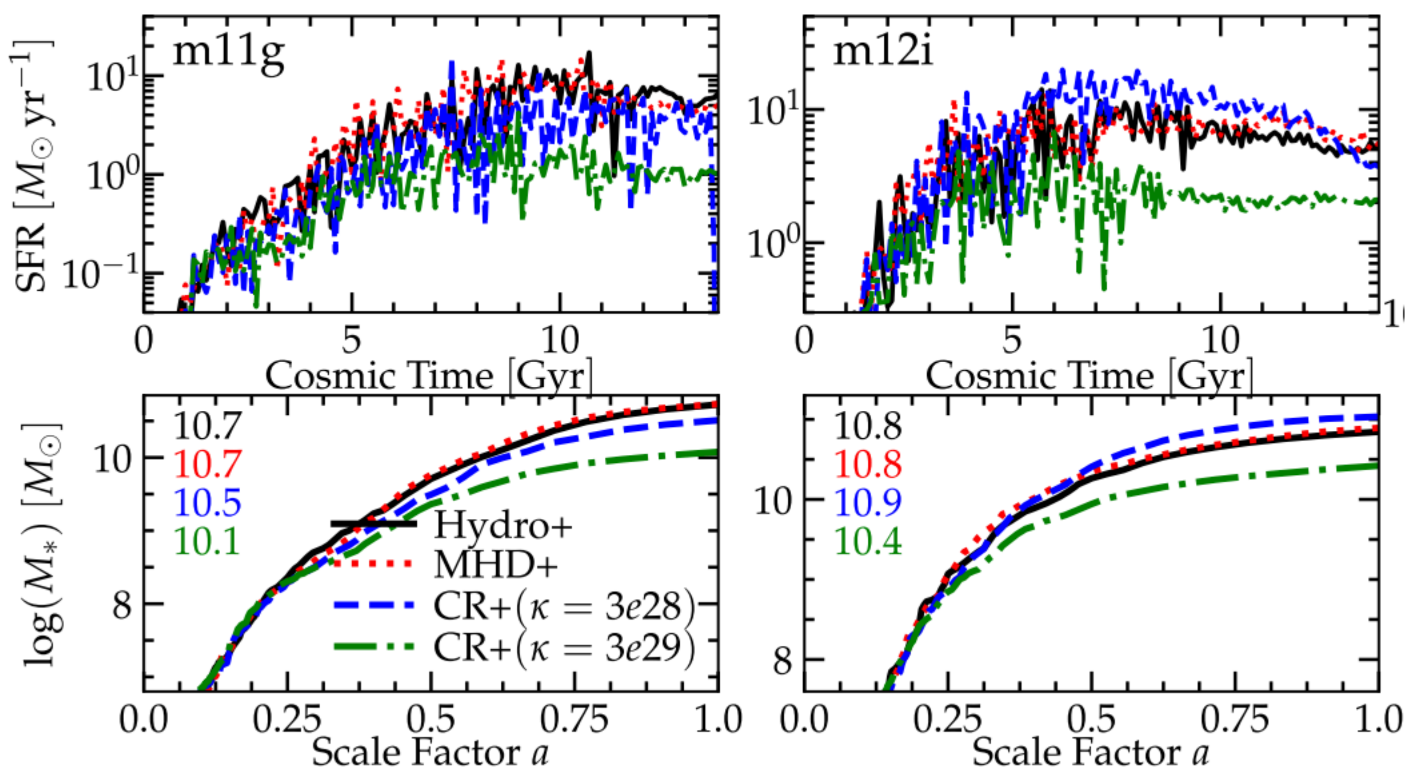




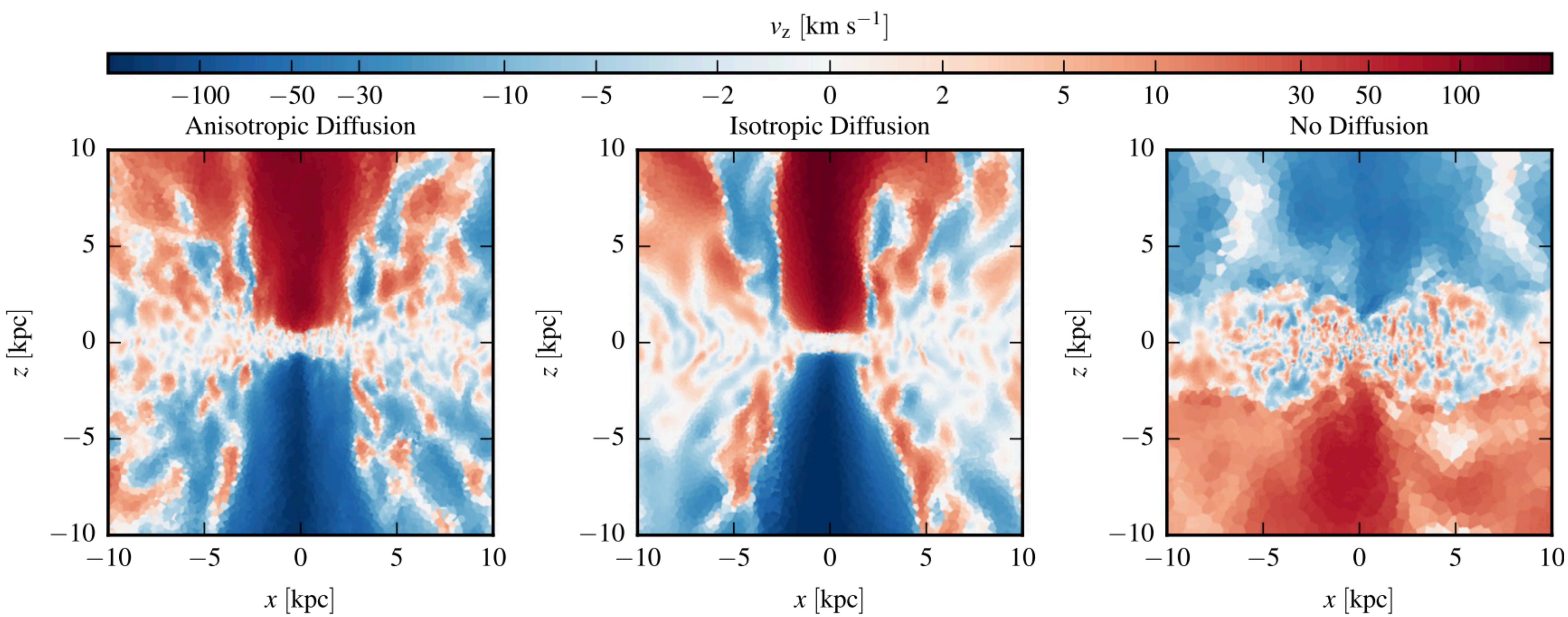
See also *Girichidis+18; Buck+20; Rodriguez Montero…YD+24; De Filippis+24; Thomas+24*

Farcy…YD+22

Diffusion plays a key role in CR ability to drive winds

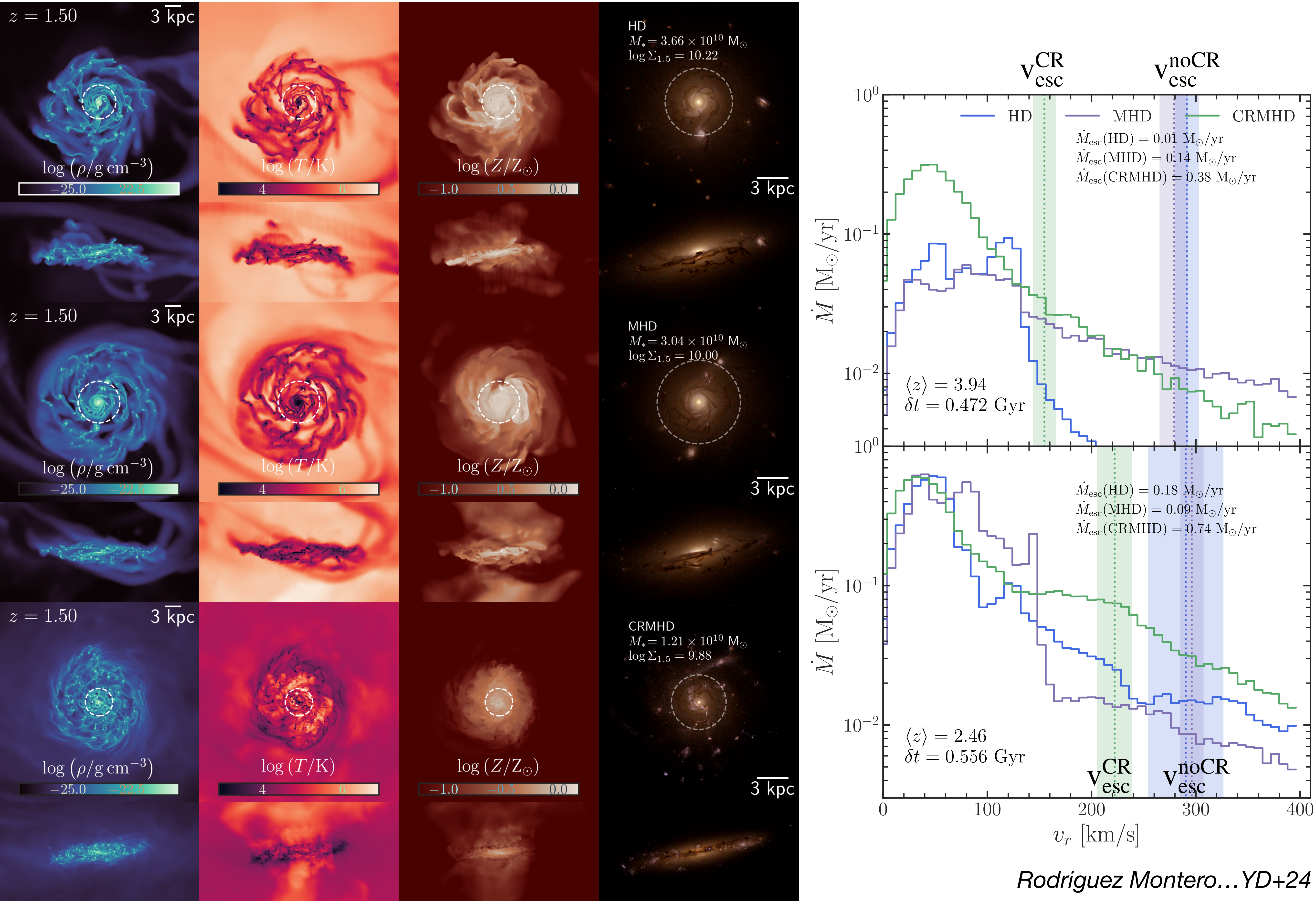


Hopkins+20



Pakmor+16

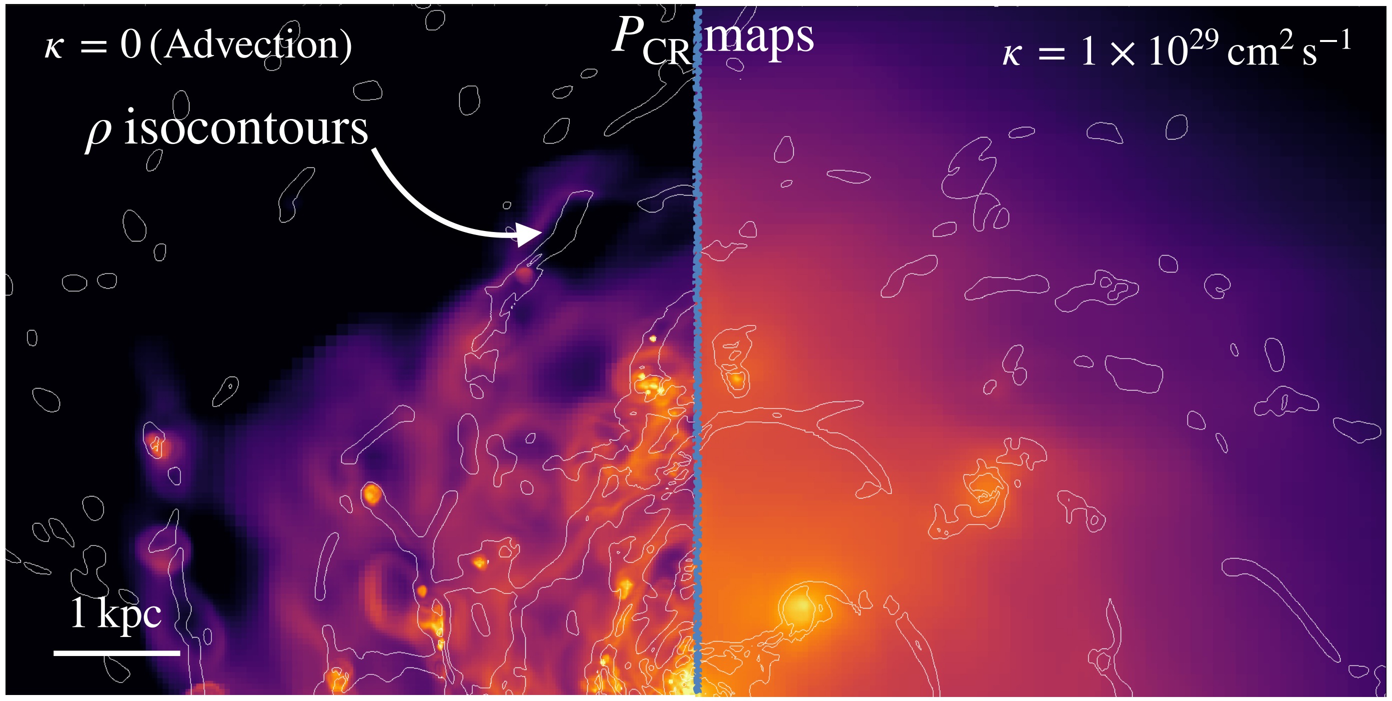
CRs in galaxies evolving in a cosmological context

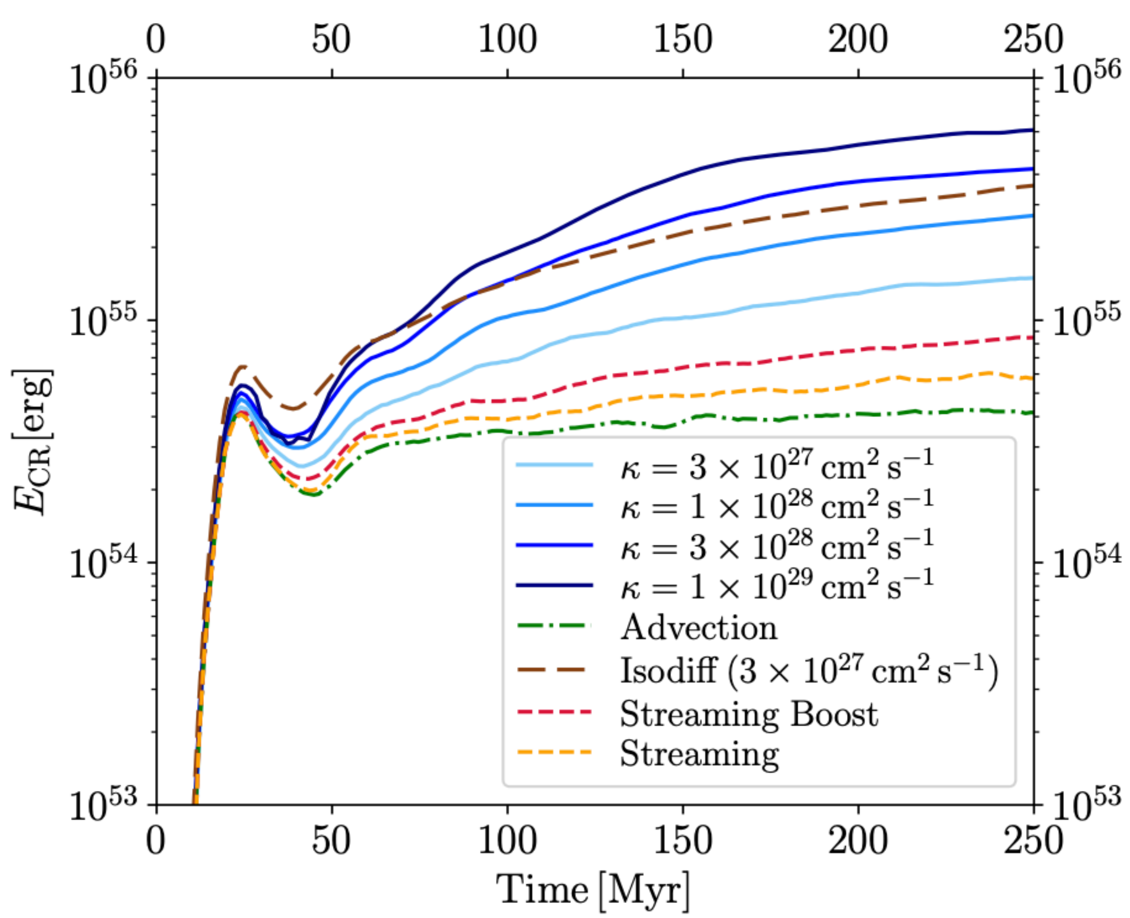


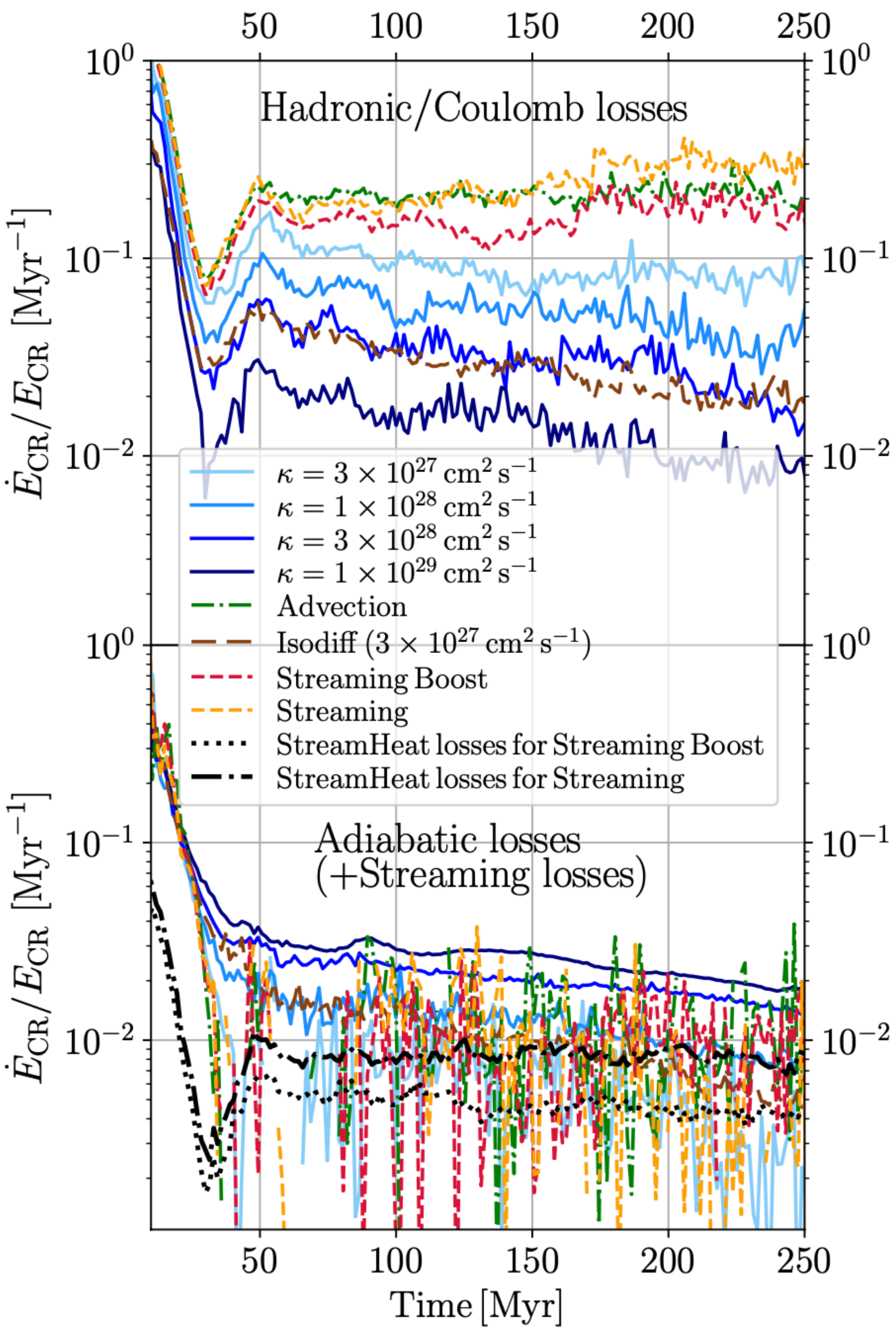
Rodriguez Montero…YD+24

Diffusion reduces the effective CR losses

$\dot{E}_{\mathrm{CR,loss}} \propto n$

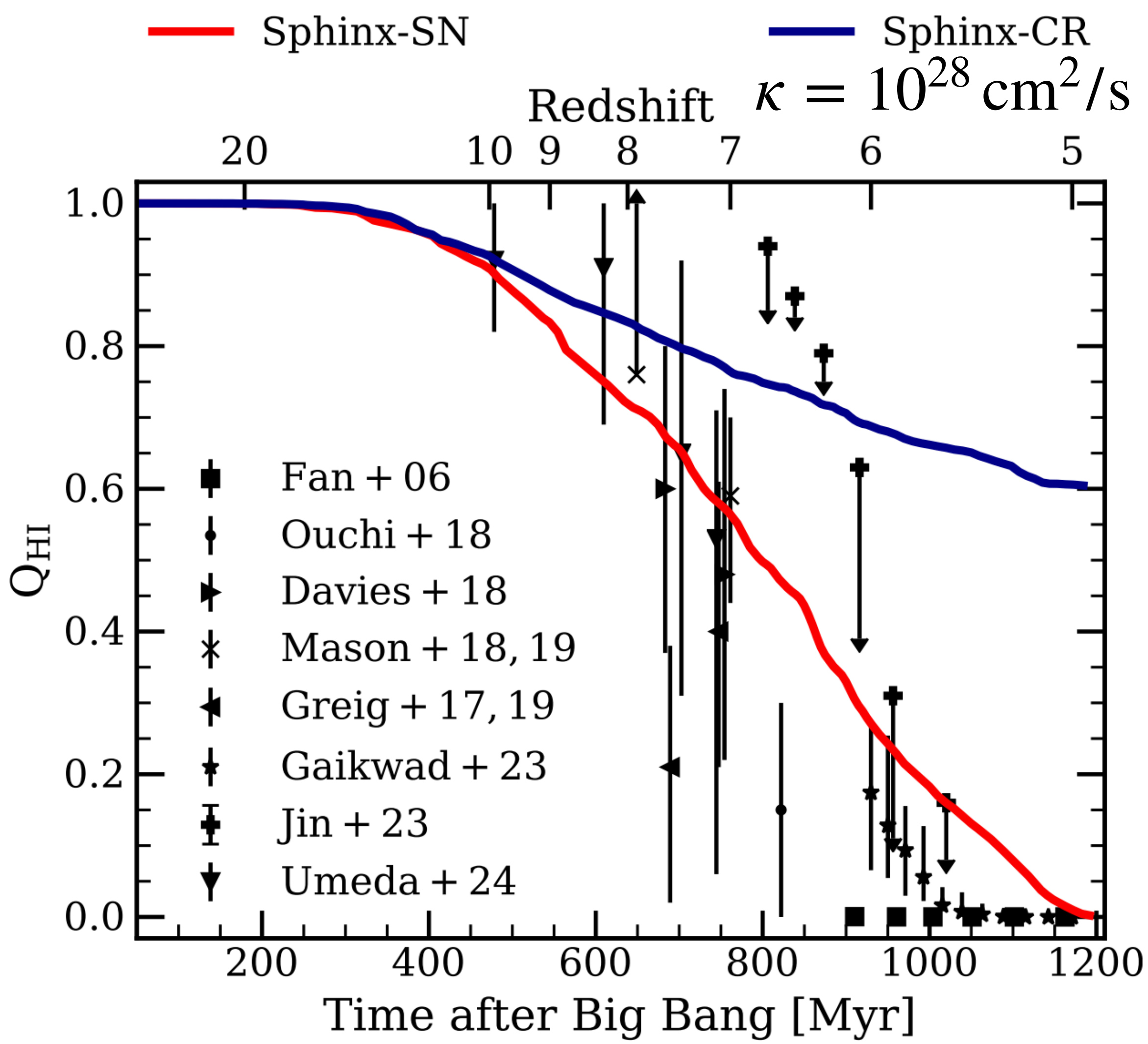


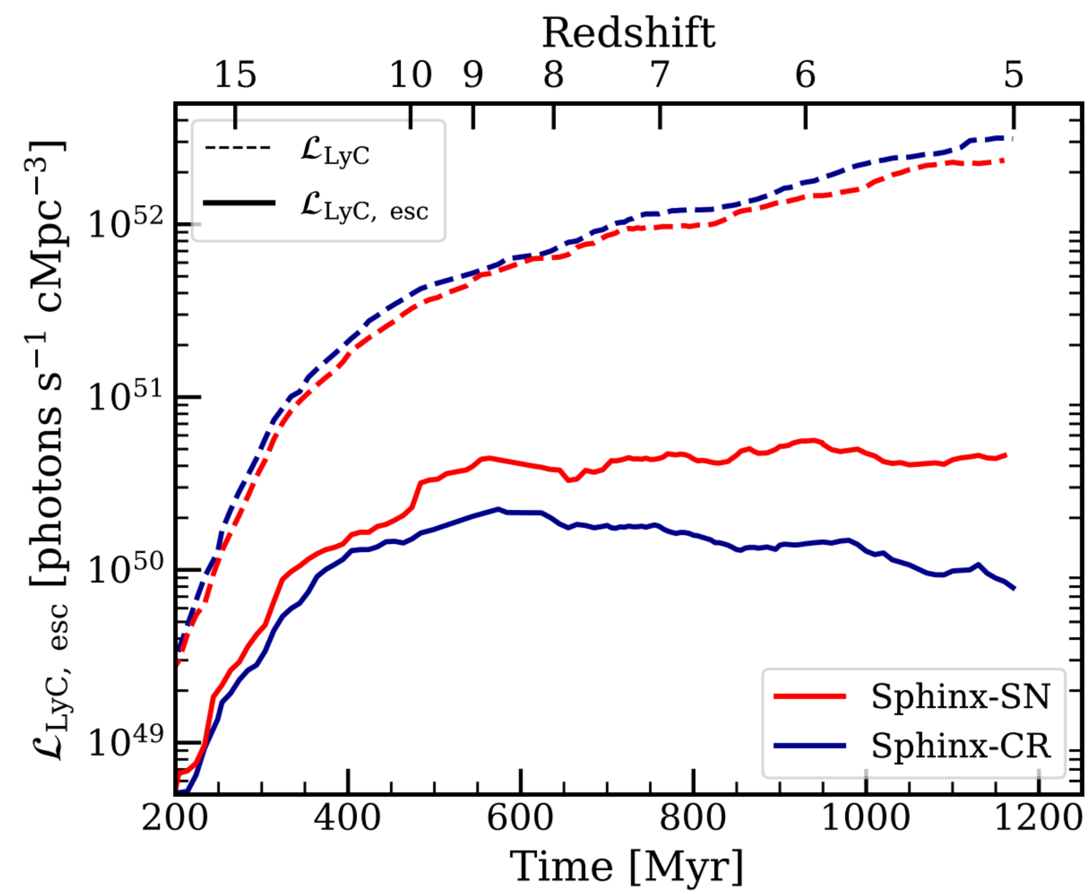


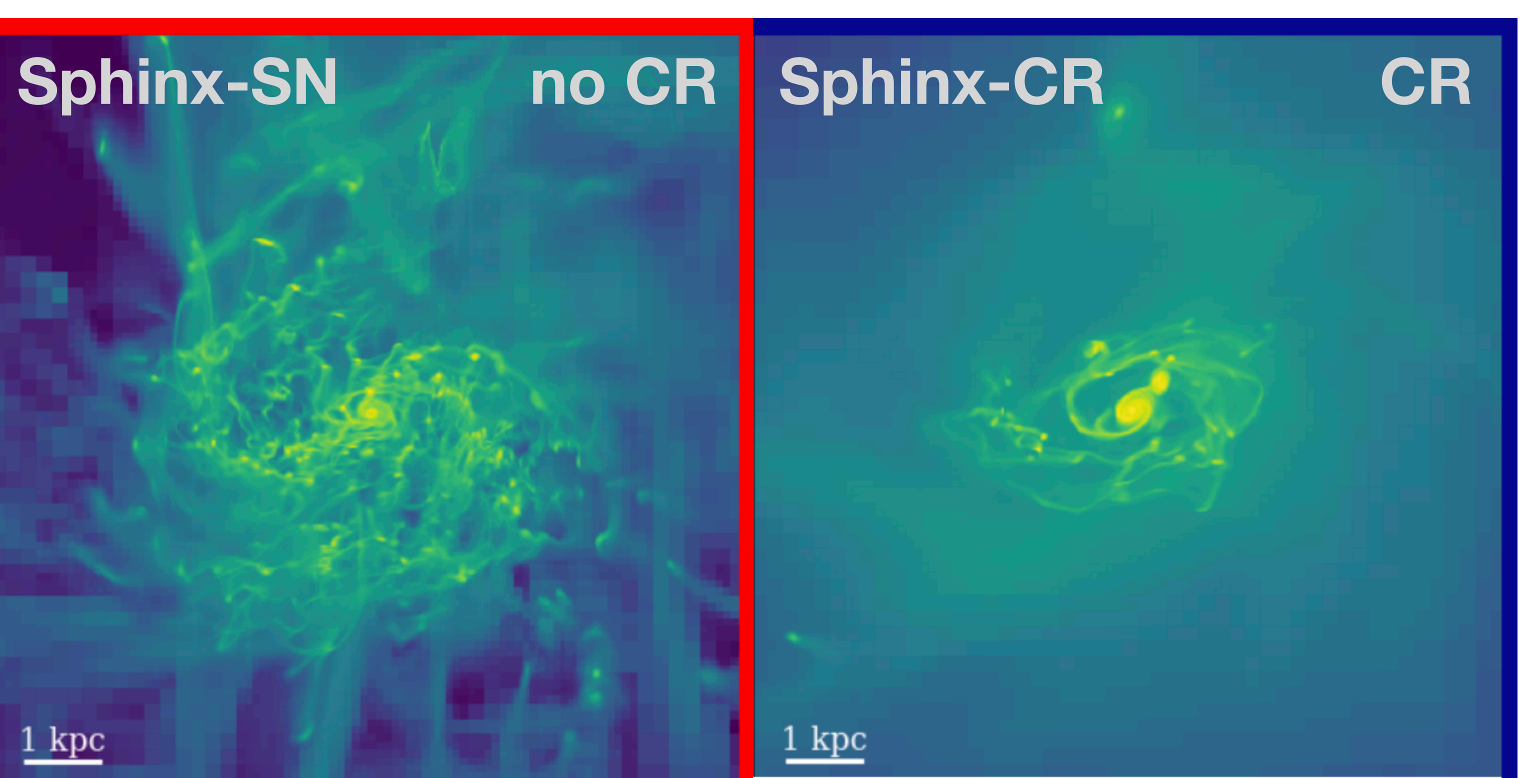


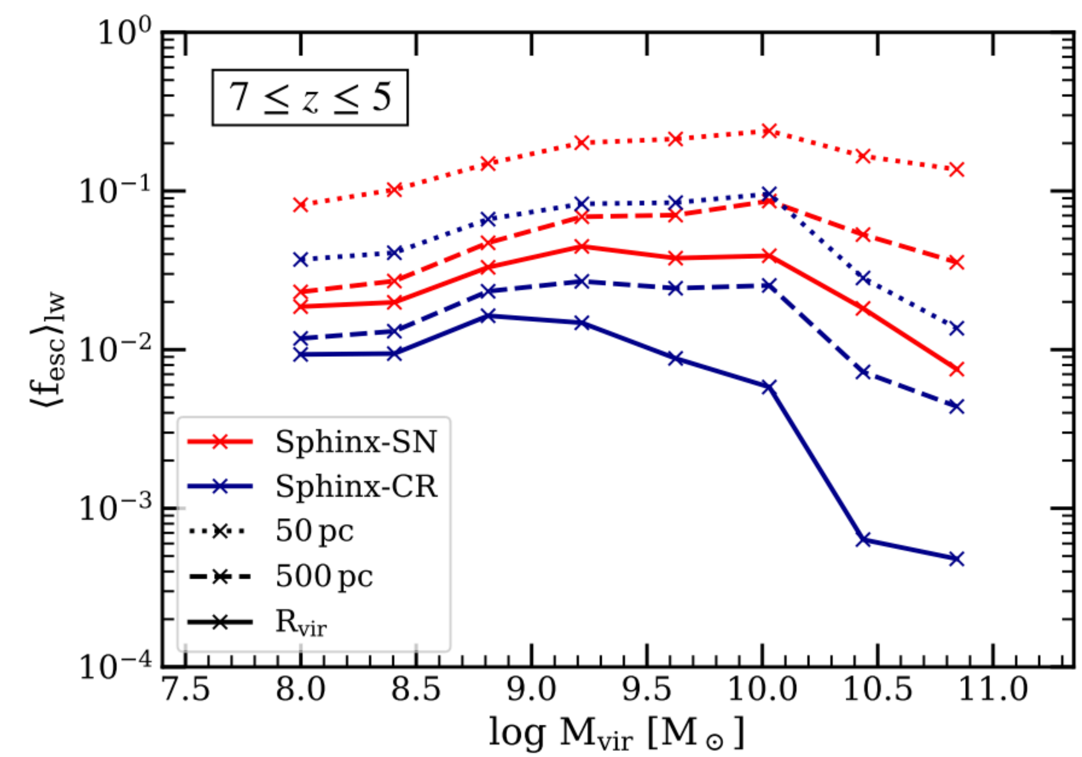
Dashyan & YD 20

CRs and the epoch of reionisation



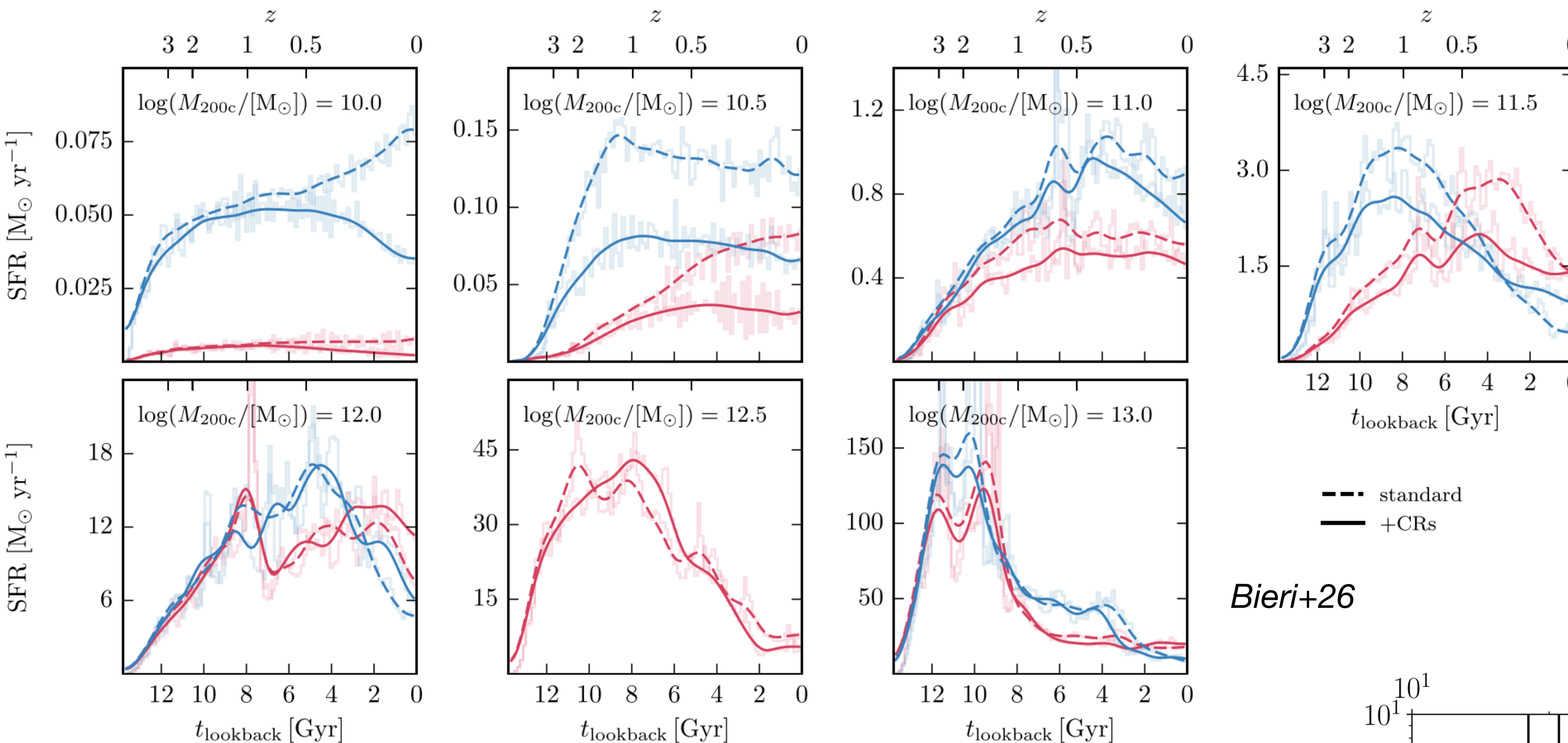






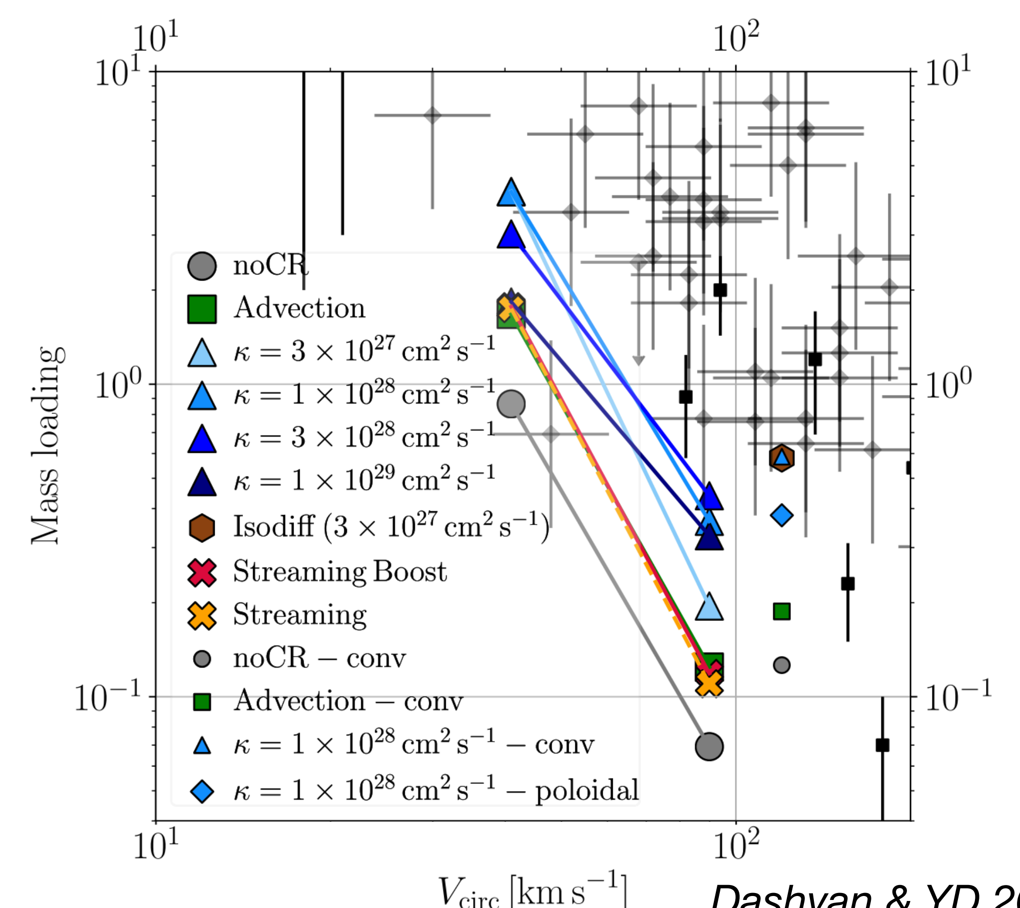
Farcy…YD+25

CR-driven large-scale galactic winds



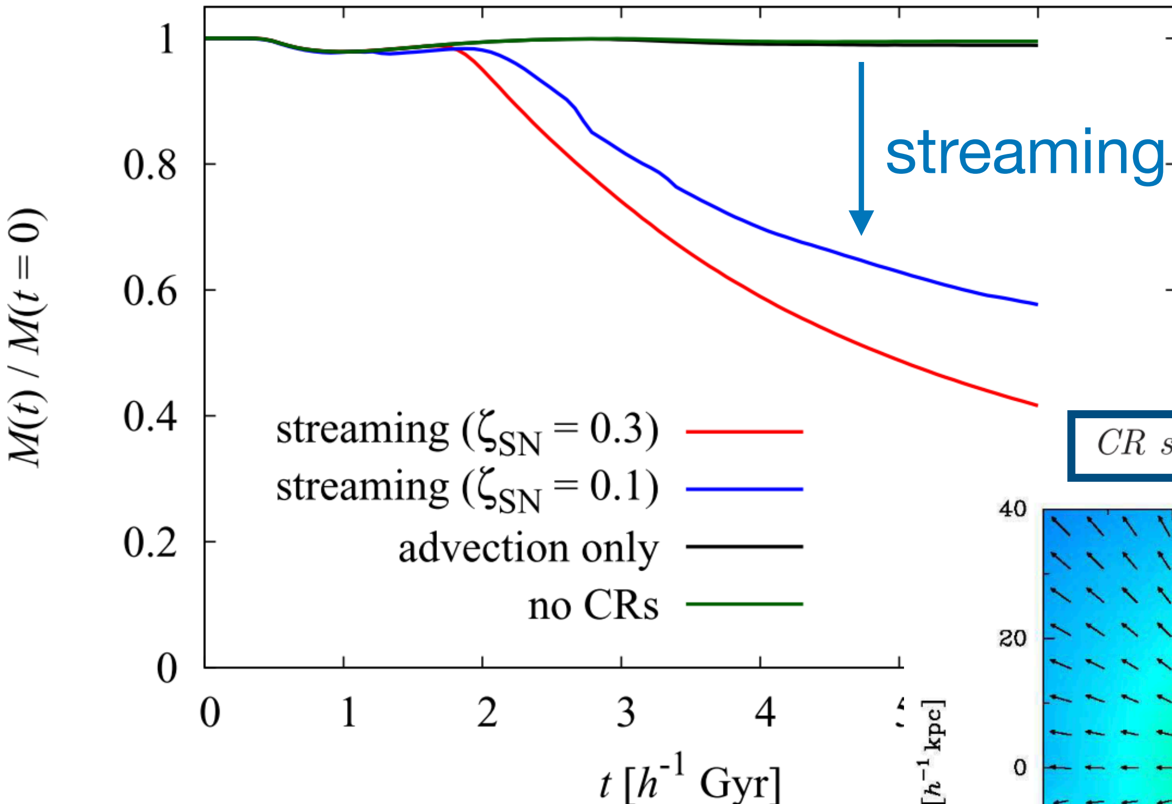
Bieri+26

- Better agreement with observations accounting for CRs and diffusion
- More mass removed by galactic winds
- Winds are CR pressure-dominated
- Winds are faster, denser, and colder
- CRs reduce the amount of very dense SF gas
- **Sensitive to the strength of CR diffusion κ**



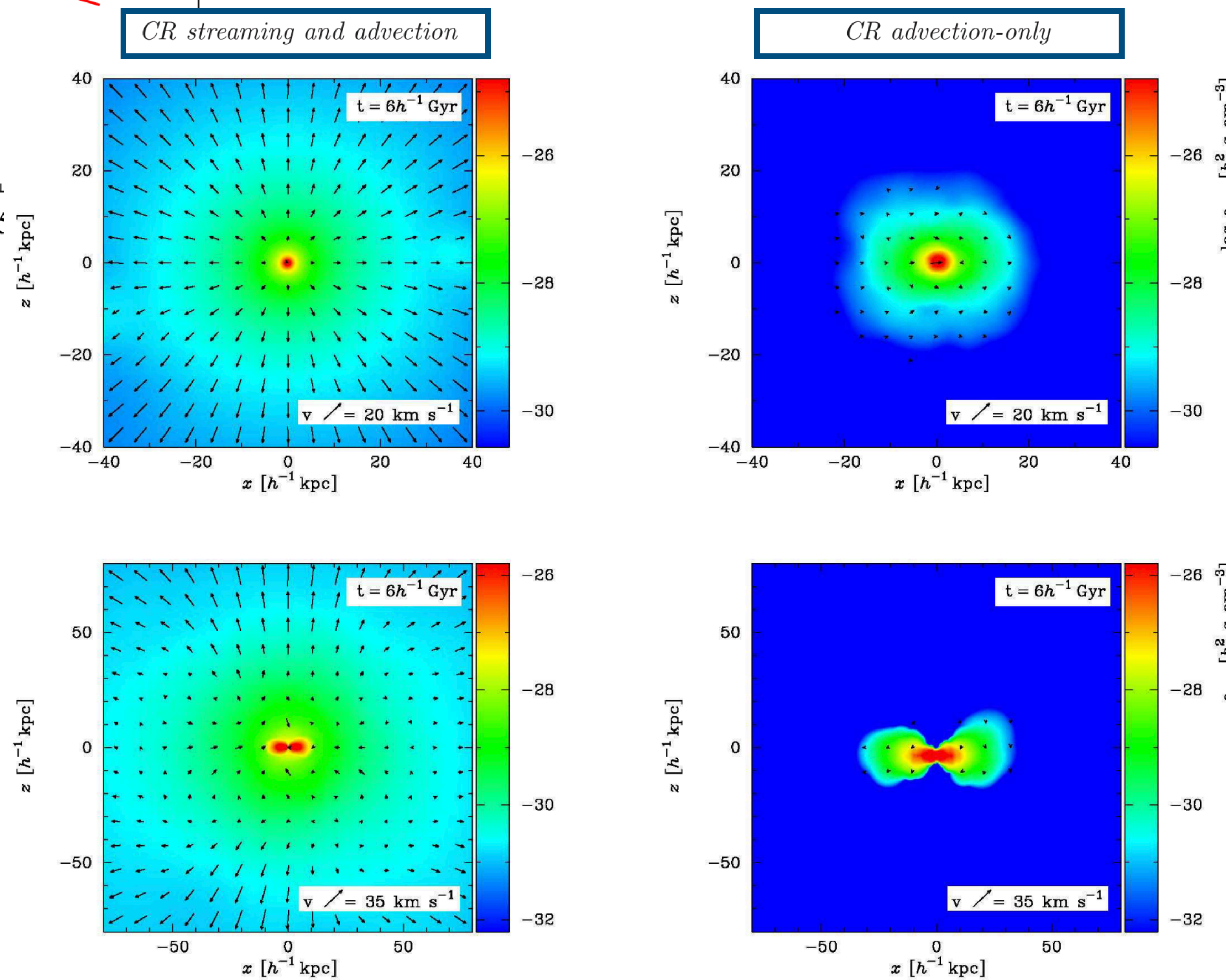
Dashyan & YD 20

What about CR streaming?



streaming reduces gas content

no diffusion, no MHD, streaming with $u_{\mathrm{st}} = c_{\mathrm{sound}}$



Streaming has a strong effect on wind launching

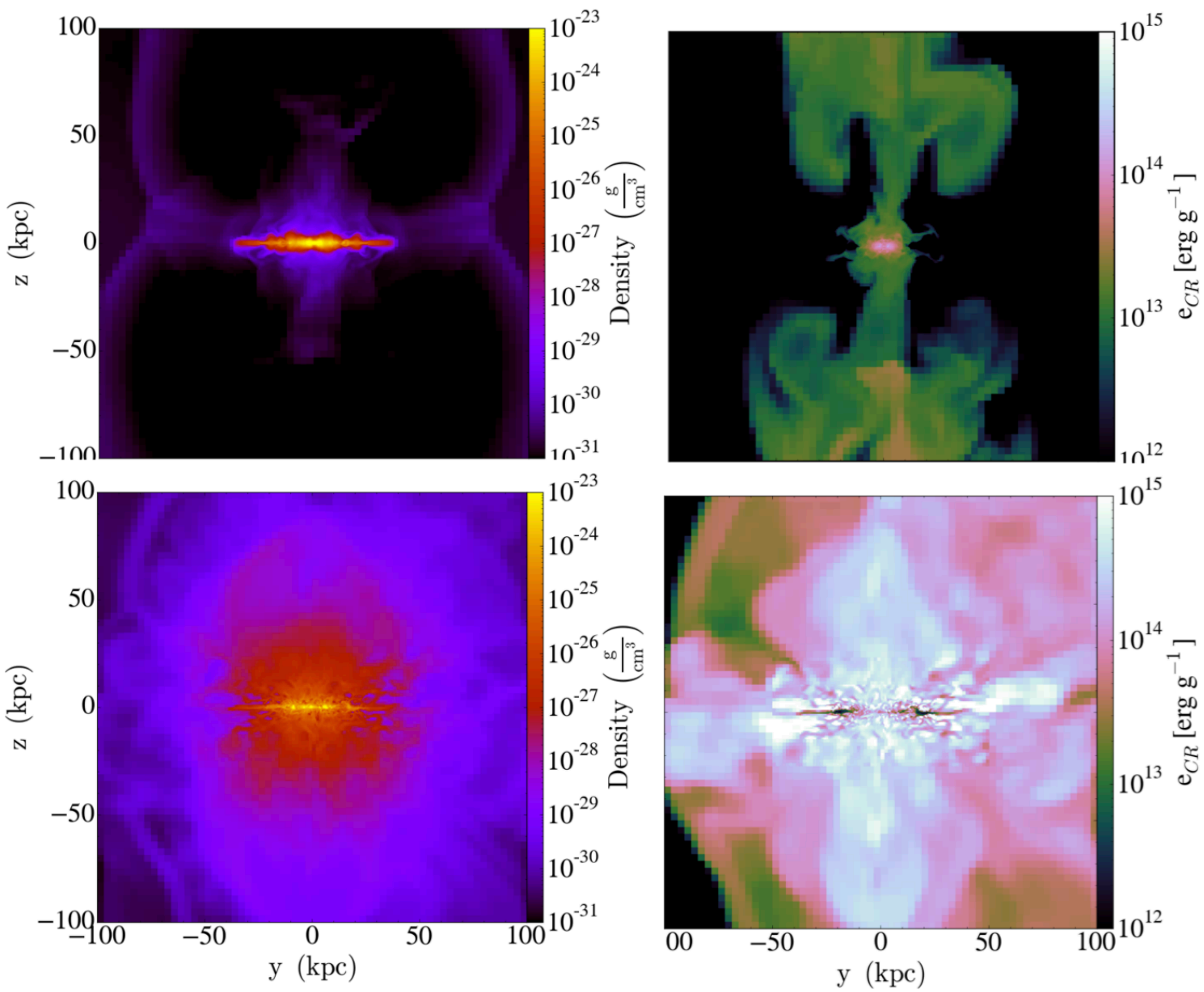
Uhlig+12

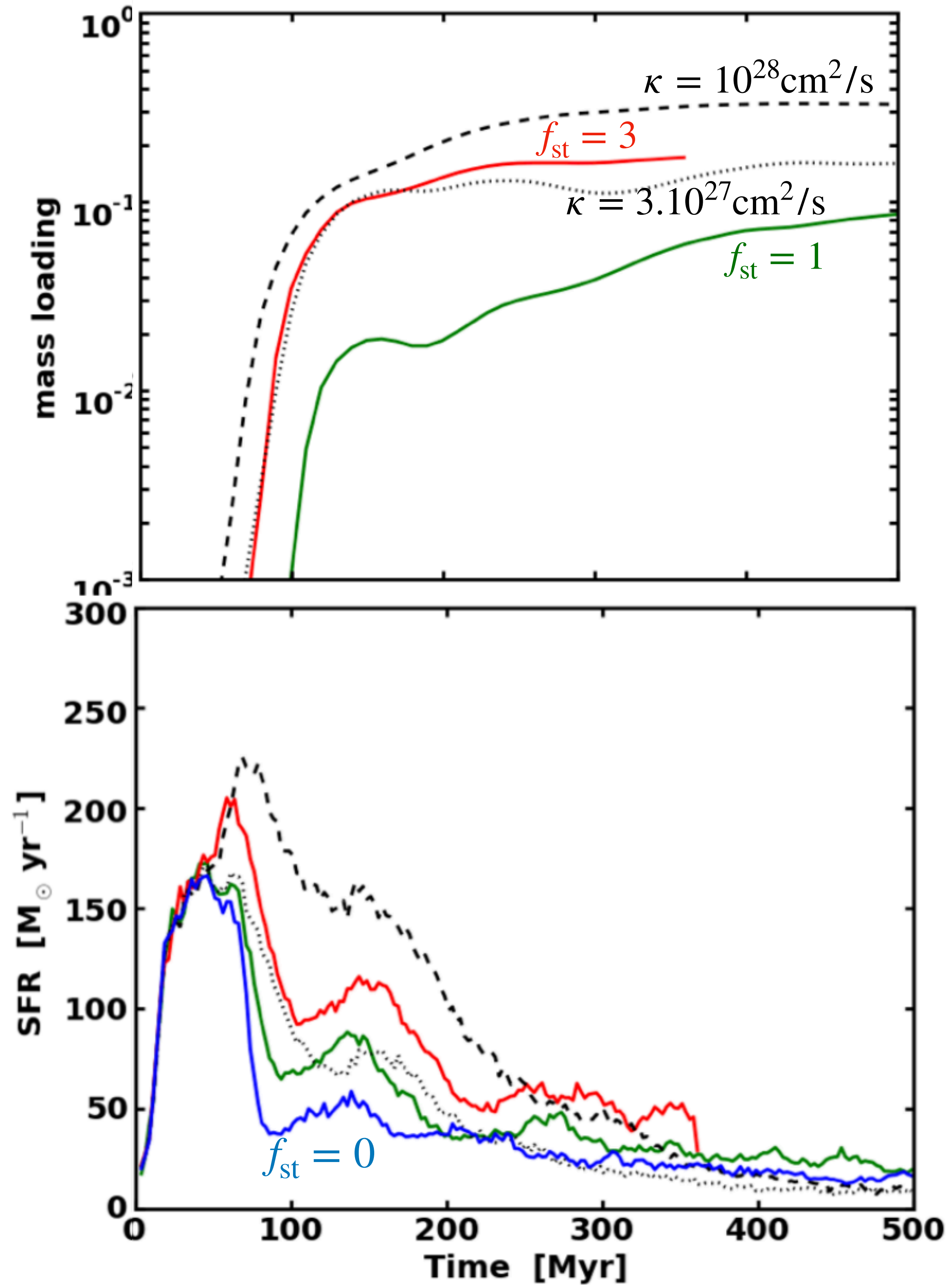
What about CR streaming?

MHD, streaming with $u_{st} = f_{st} u_A$
(either streaming **or** diffusion)

$f_{st} = 0$

$f_{st} = 4$



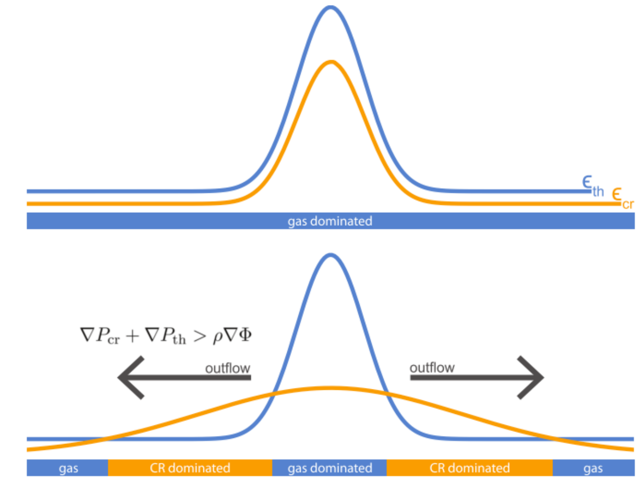


Streaming can play a significant role if value is boosted compared to Alfvén speed

Spatial resolution is 200 pc

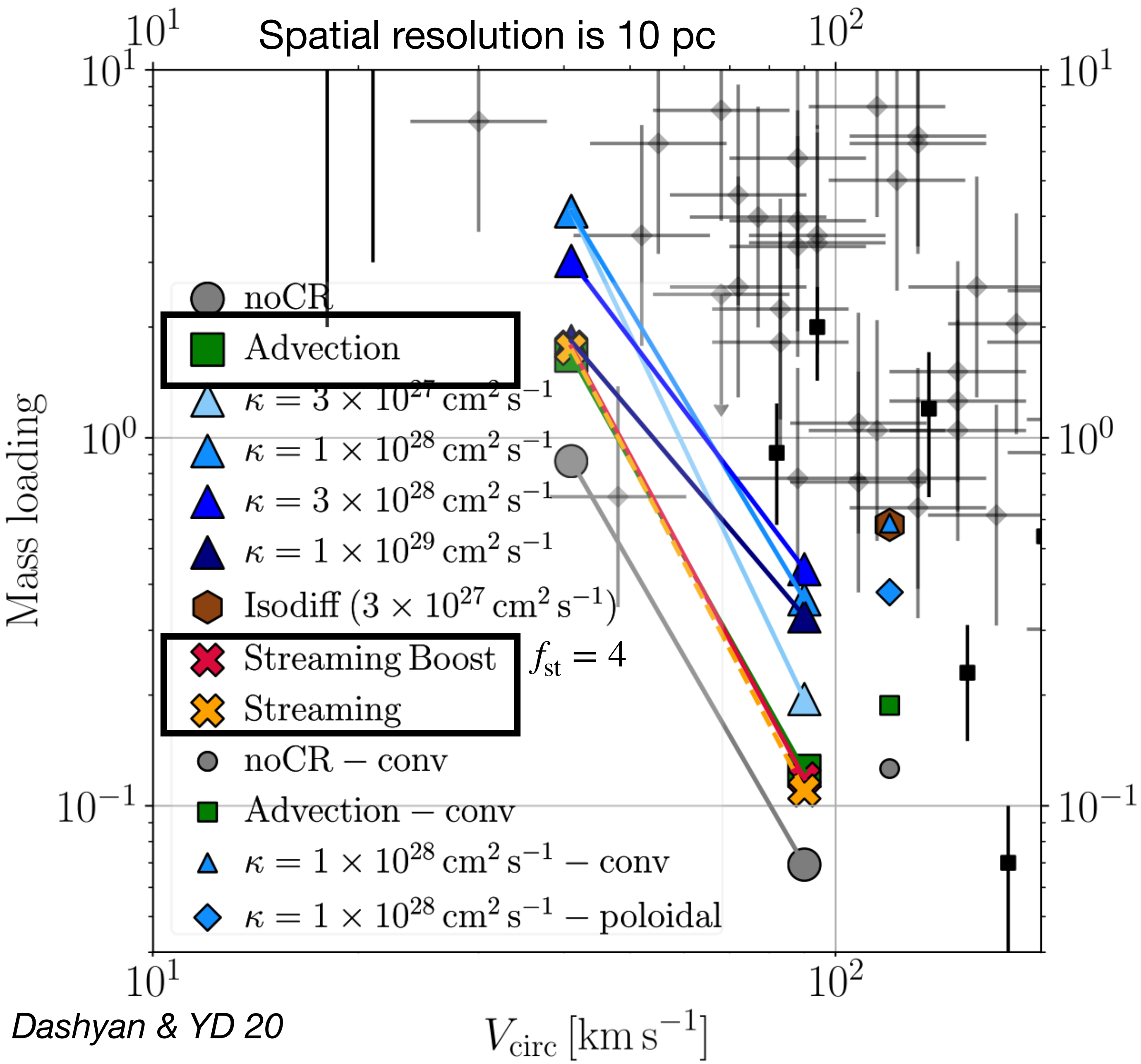
Ruszkowski+17

What about CR streaming?



MHD, streaming with $u_{st} = f_{st} u_A$

(either streaming **<u>or</u>** diffusion)



Dashyan & YD 20

Even if boosted streaming is inefficient at wind launching compared to diffusion

Streaming velocity

$$u_{st} = f_{st} u_A$$

$$u_A \simeq 10\,\mathrm{km/s}$$

Diffusion velocity

$$u_D = \frac{\kappa}{L} \simeq 200 \frac{\kappa}{3.10^{27}\mathrm{cm^2/s}} \left(\frac{L}{50\mathrm{pc}}\right)^{-1} \mathrm{km/s}$$

Low resolution (200pc)

$$u_D \simeq 50 \frac{\kappa}{3.10^{27}\mathrm{cm^2/s}} \mathrm{km/s}$$

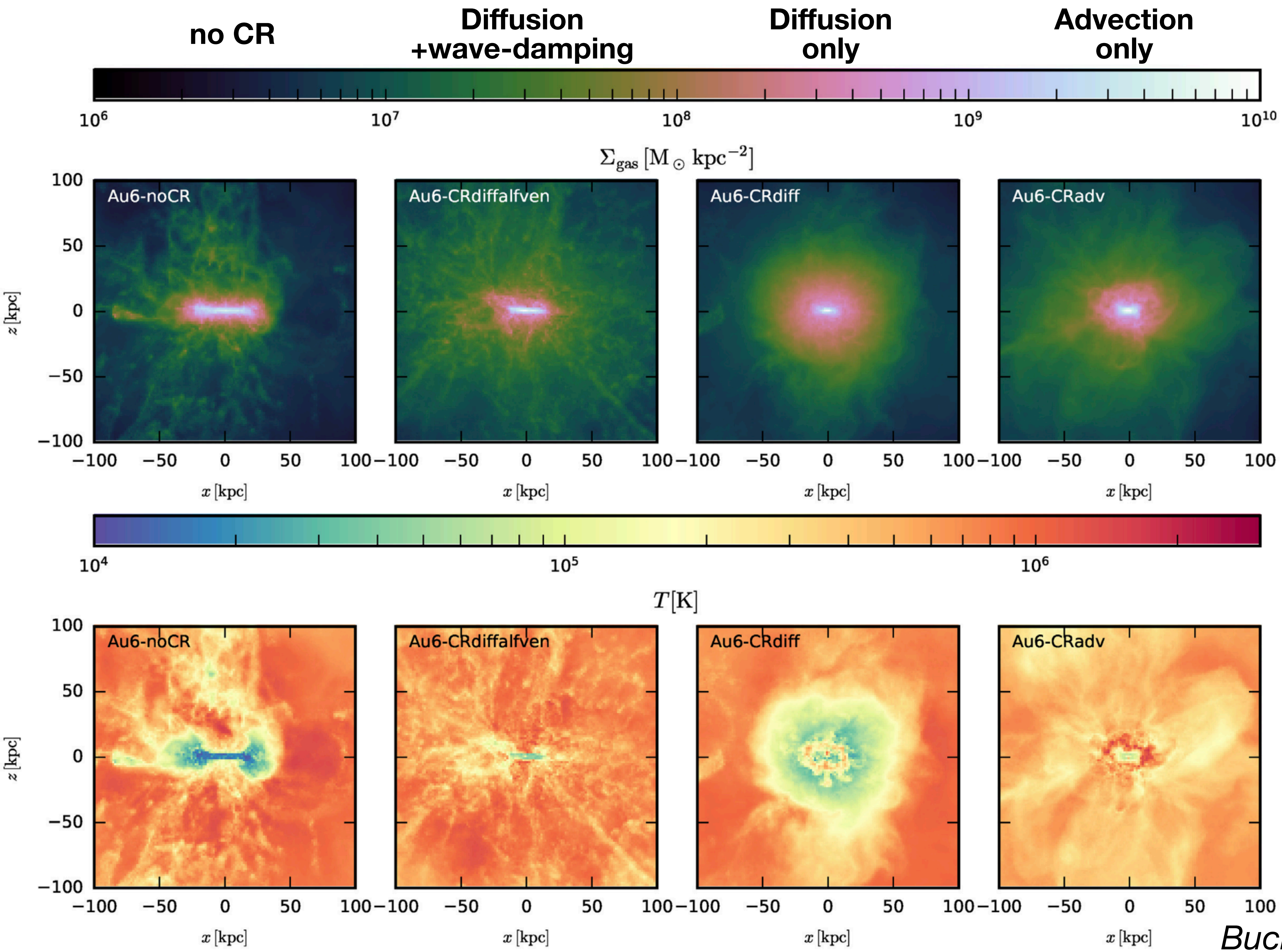
This is likely why low-res sim. find important effect of streaming

Caveat: <u>ion</u> Alfvén speed

$$u_{A,ion} = \frac{B}{\sqrt{4\pi\rho_{ion}}} \gg u_A$$

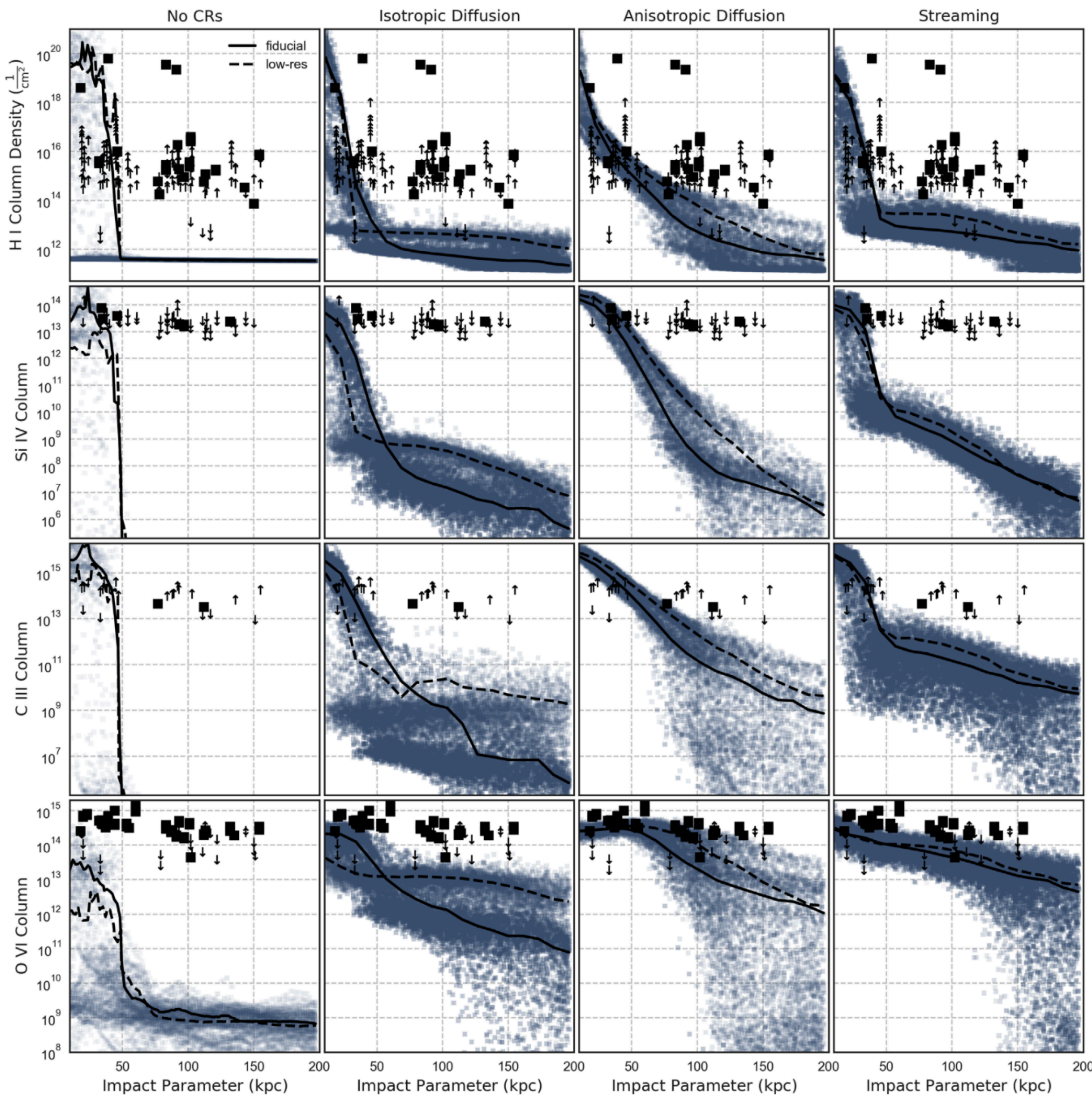
Requires to know accurately the ion fraction in the CNM

Alfvén-wave heating important for CGM re-heating

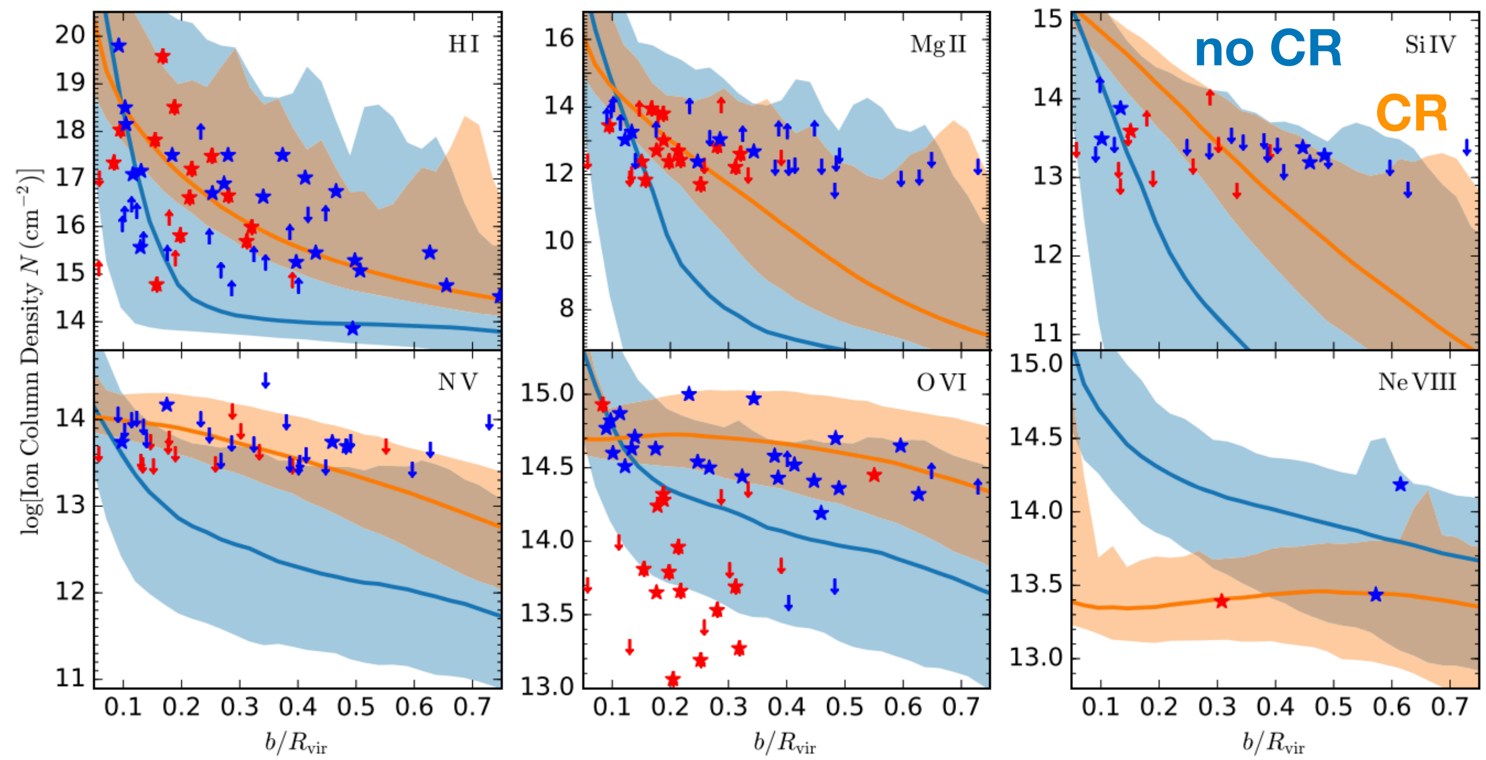


Buck+20

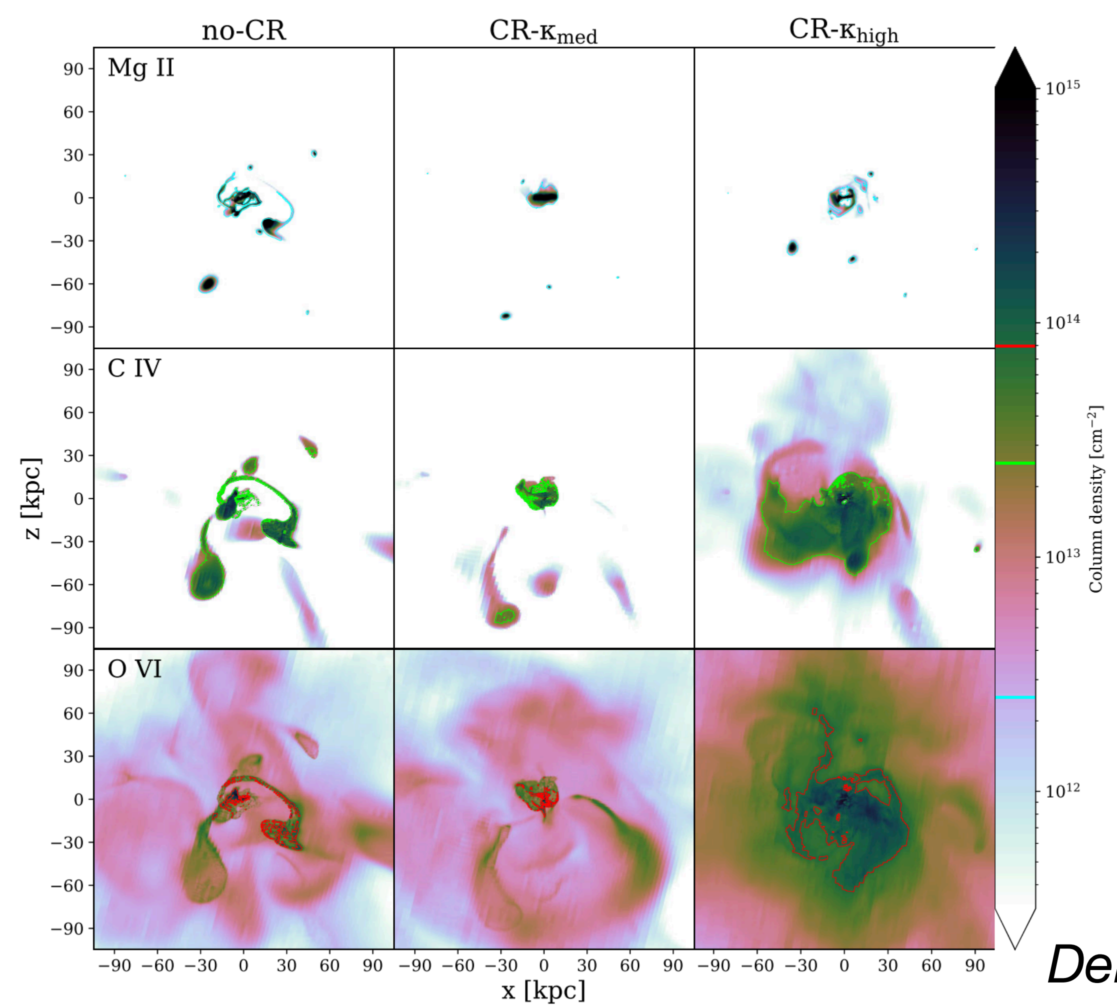
And, hence, for various CGM tracers



Butsky&Quinn18



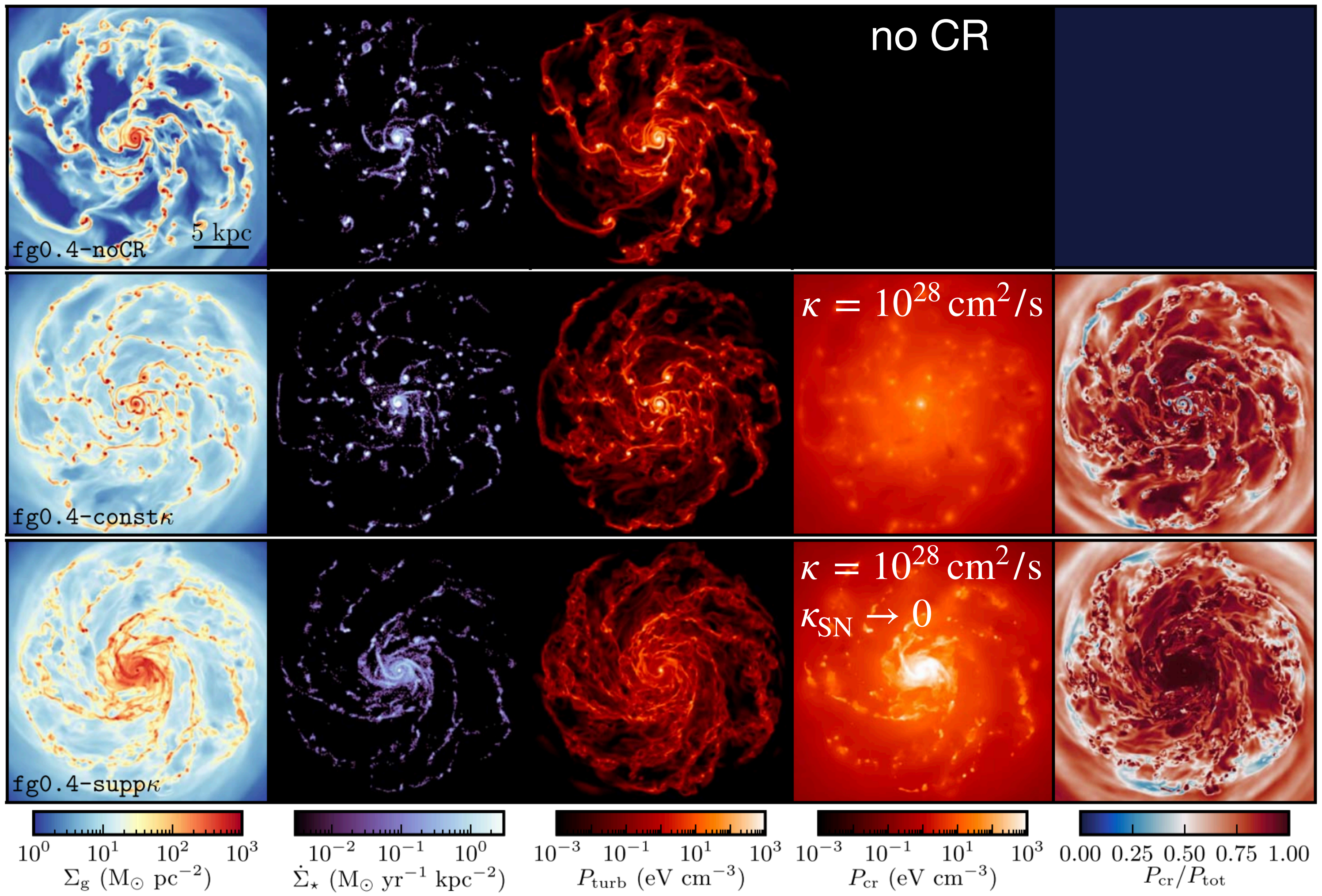
Ji+20

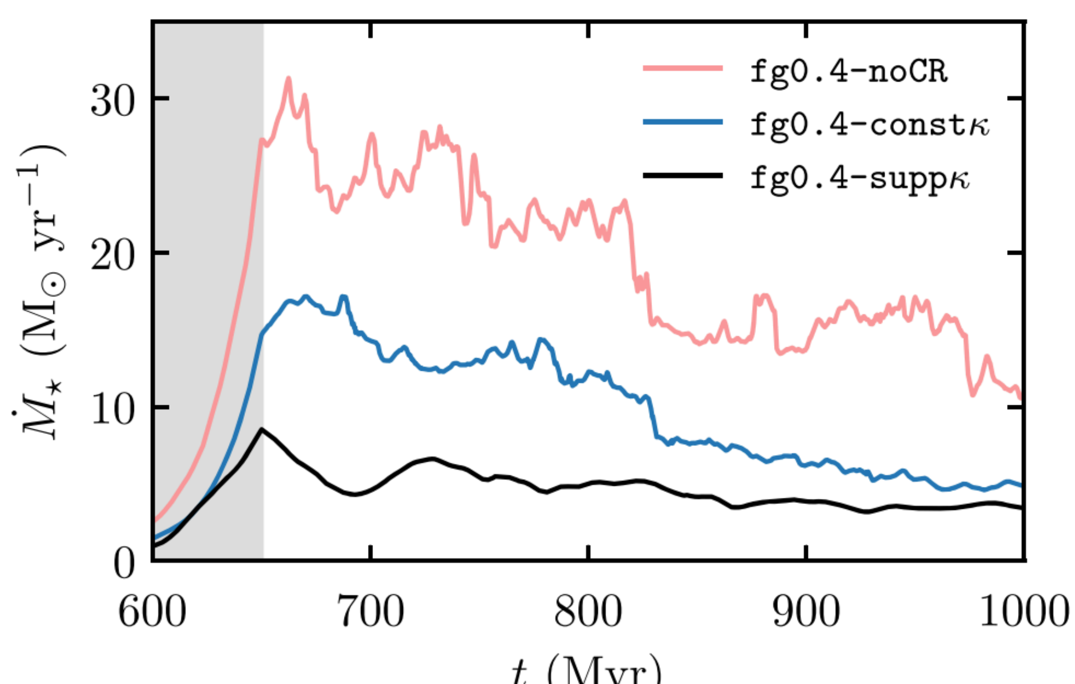


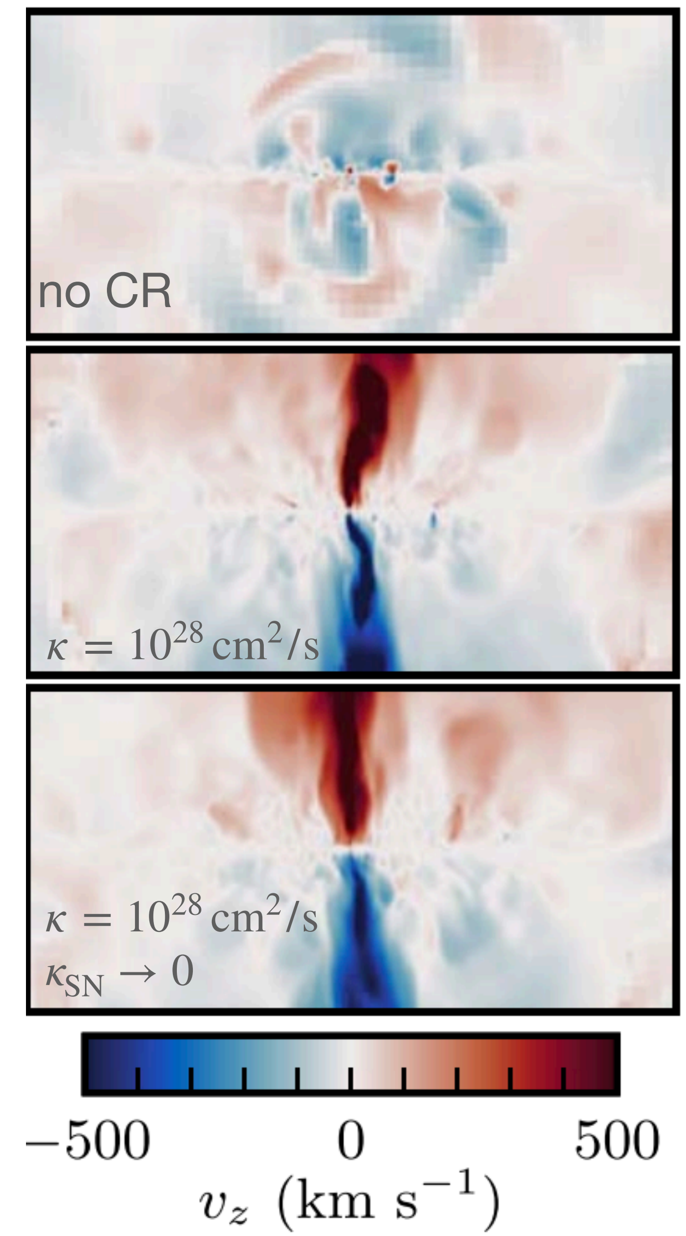
DeFilippis+24

Beyond assuming constant diffusion coefficient

Suppression of diffusion around CR sources (SNe)





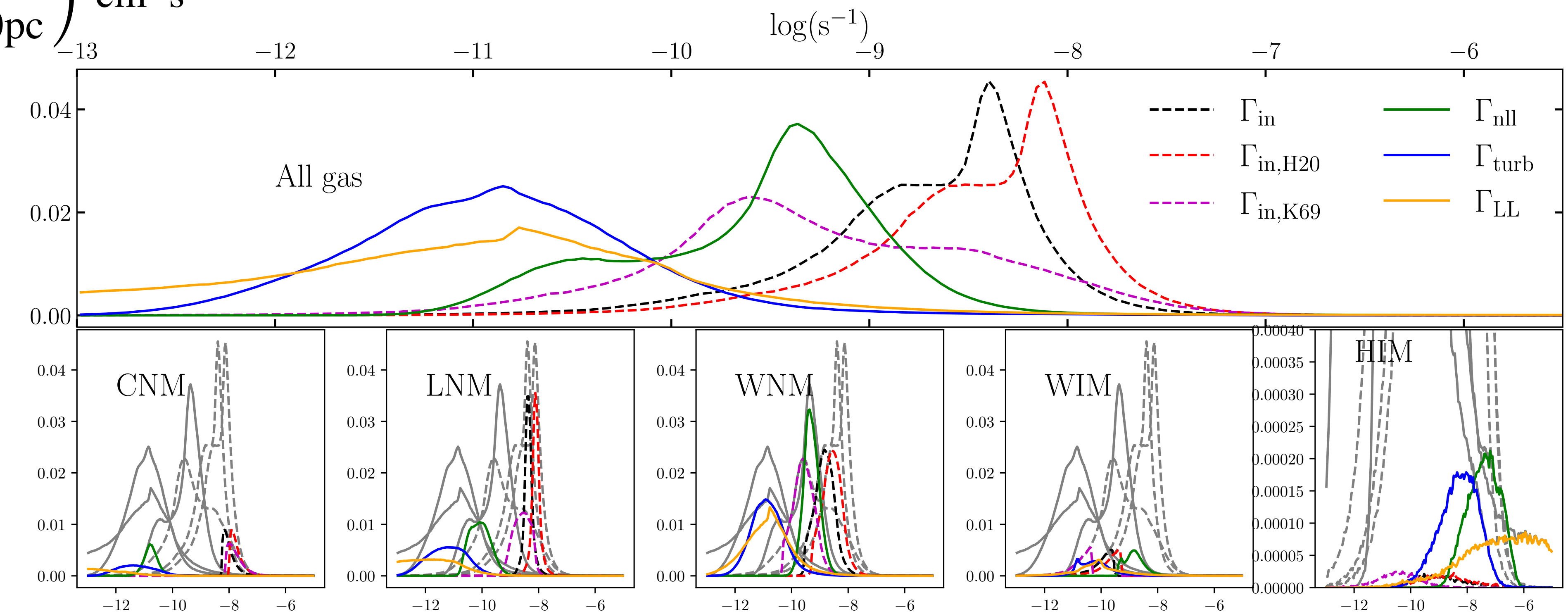


Semenov, Kravtsov & Caprioli 21

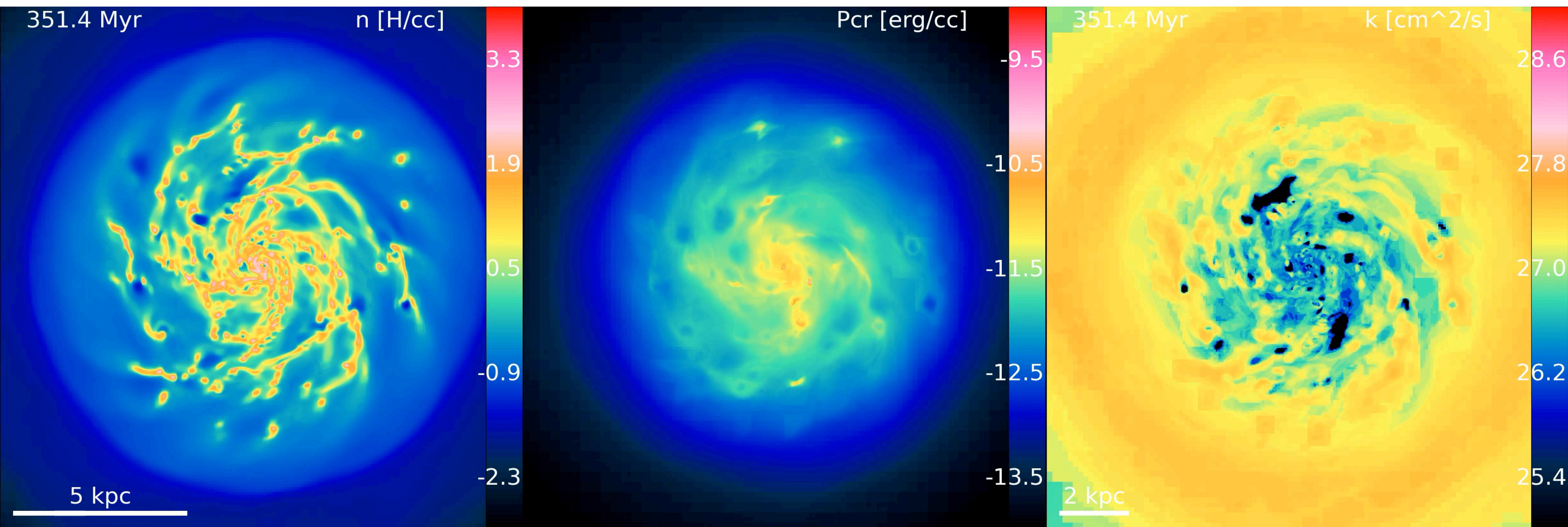
Beyond assuming constant diffusion coefficient

Self-confinement scenario with various damping processes

$$\kappa_{\rm SC} \simeq 2.2 \times 10^{27} \left(\frac{\Gamma_{\rm da}}{10^{-10}{\rm s}^{-1}}\right) \left(\frac{\ell_{\rm c}}{100{\rm pc}}\right) {\rm cm}^2\,{\rm s}^{-1}$$



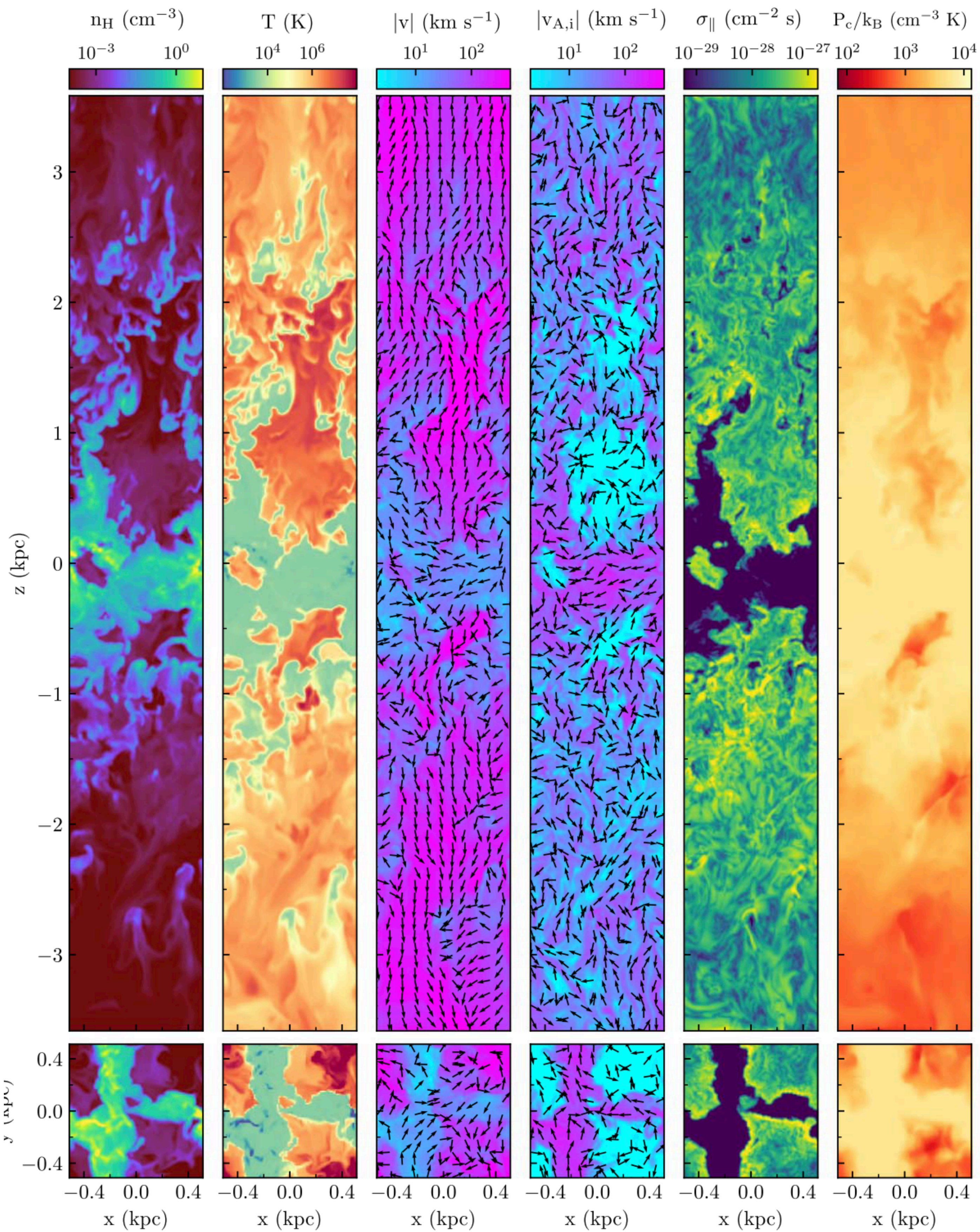
Núñez-Castiñeyra+ in prep.

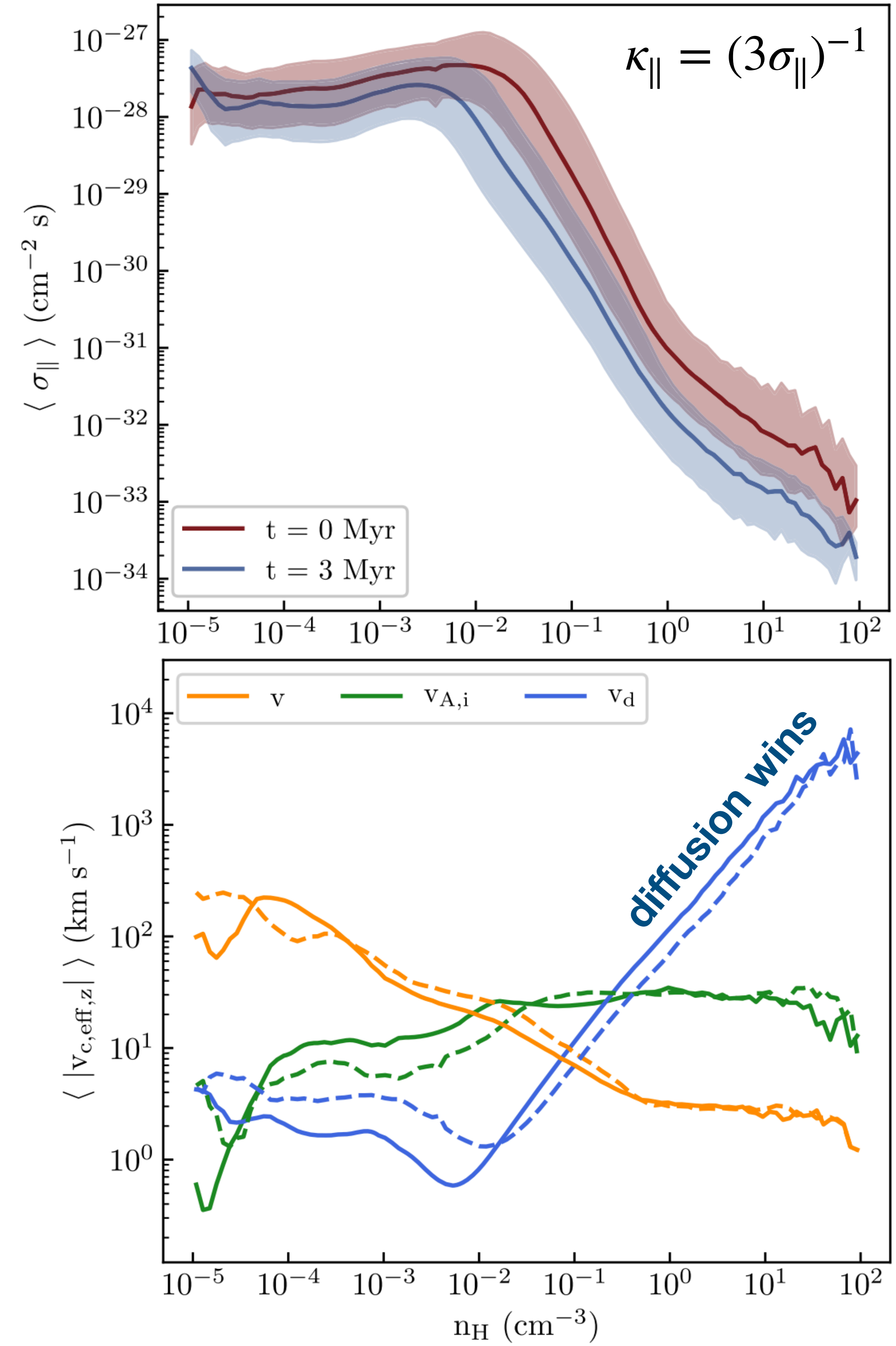


Beyond assuming constant diffusion coefficient

Self-confinement scenario with various damping processes

Armillotta+24

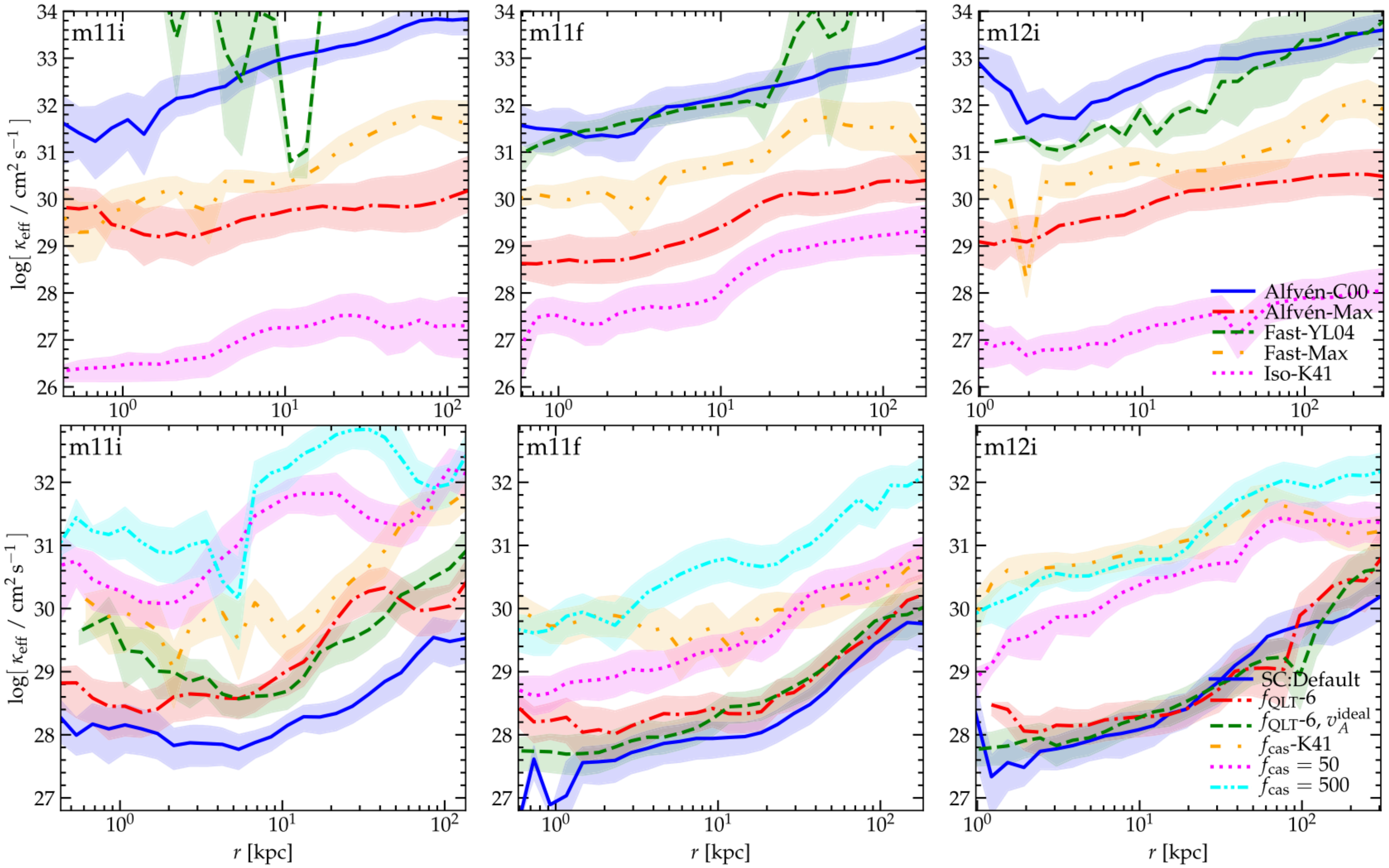




Beyond assuming constant diffusion coefficient

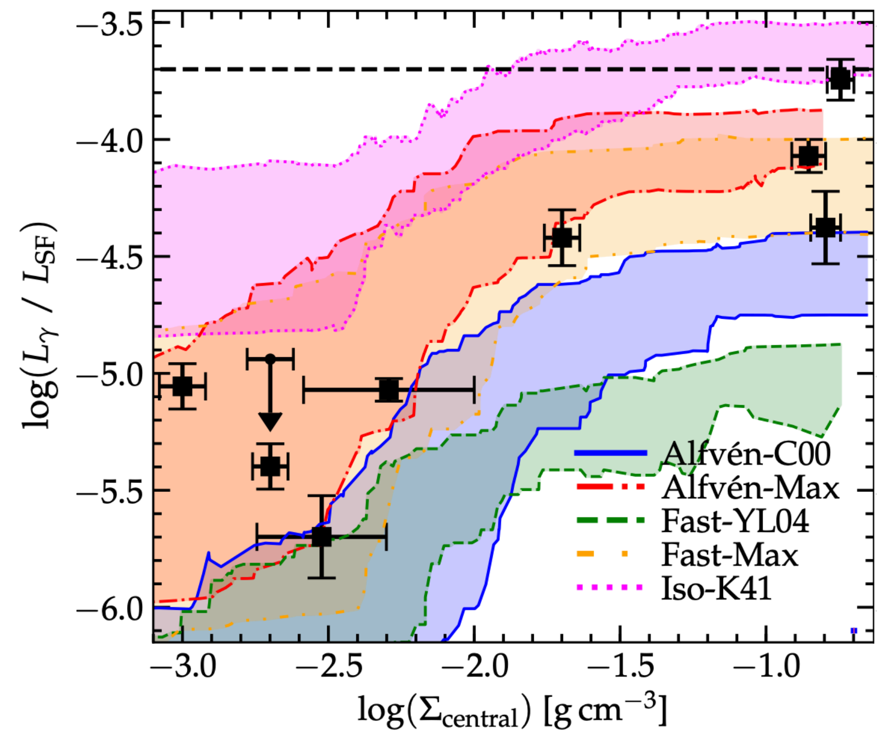
Testing a vast range of models

standard ET produces too high effective diffusion

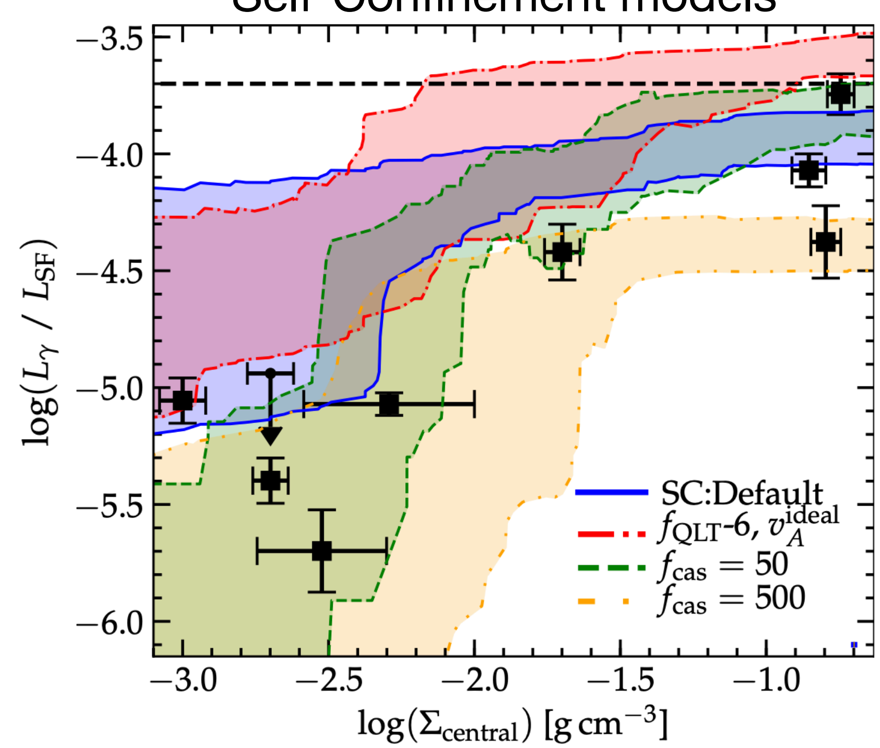


standard SC produces too low effective diffusion
(controlled by diffusion in ionised phase)

Extrinsic Turbulence models



Self-Confinement models

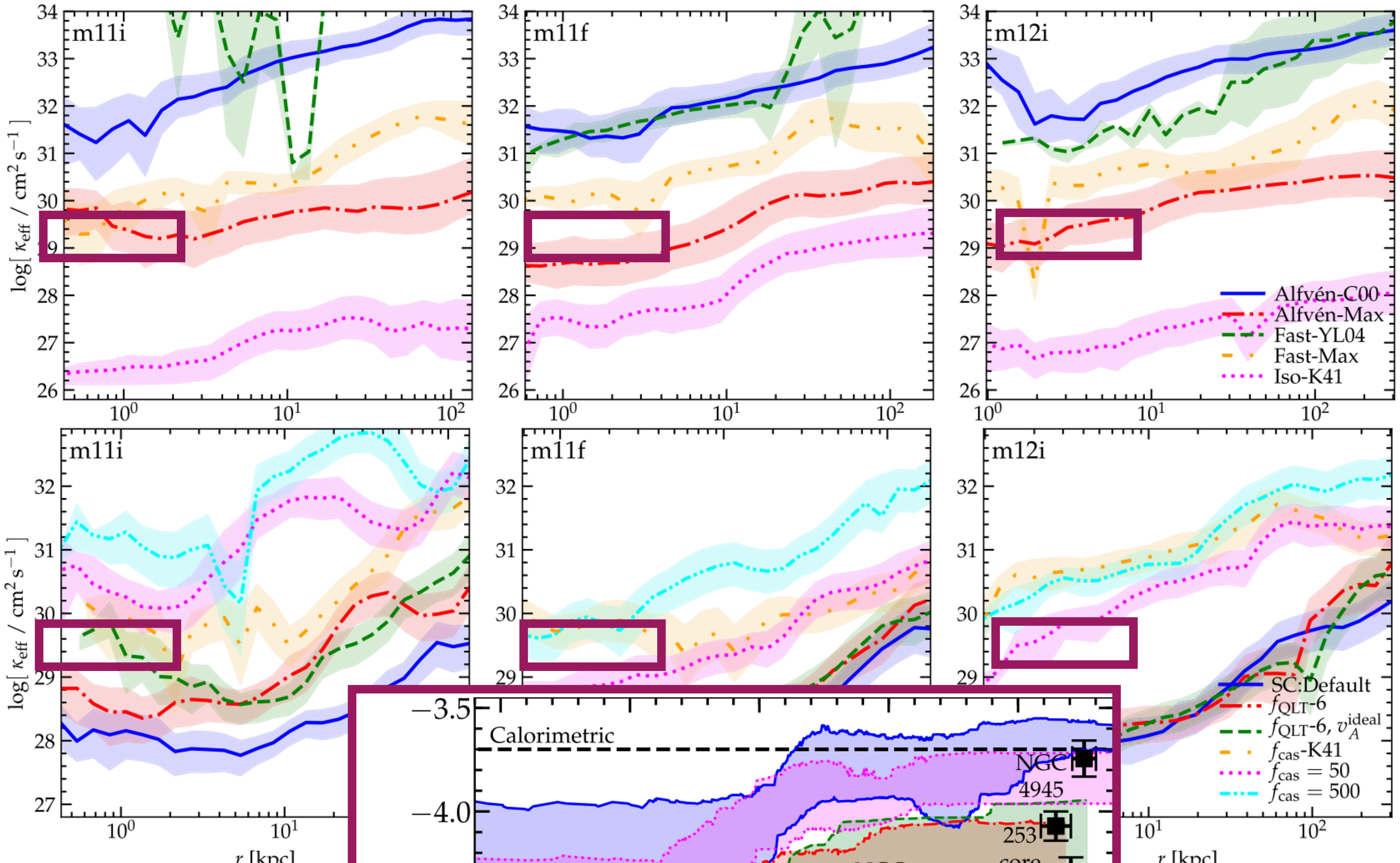


Hopkins+21

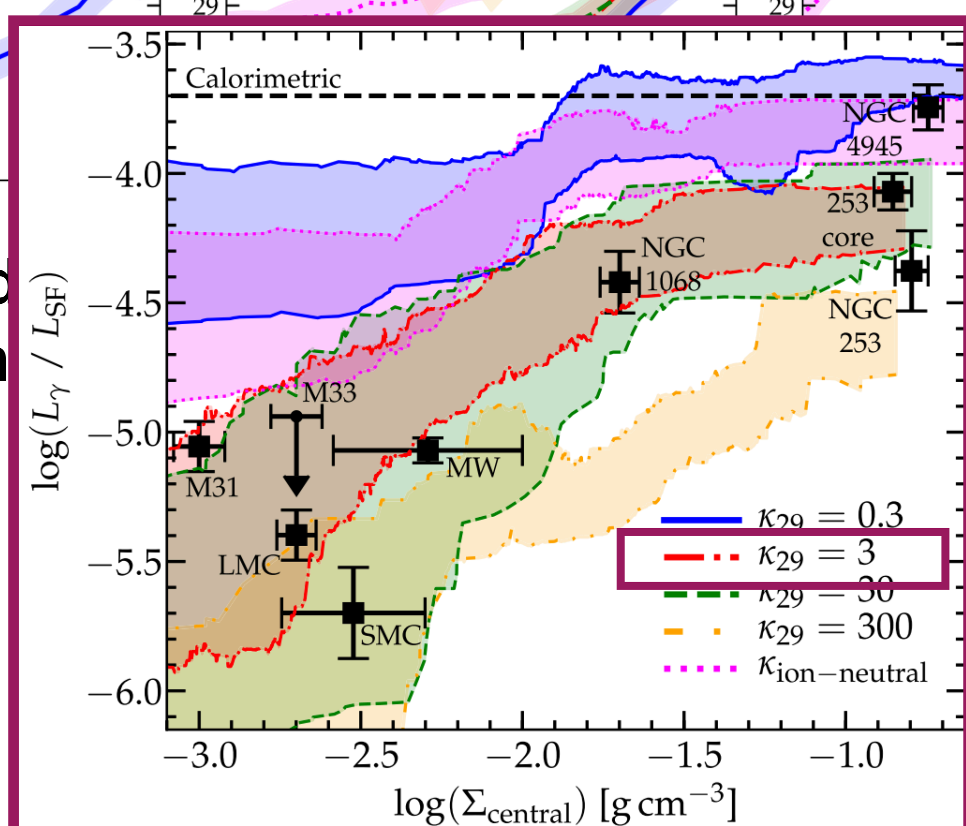
Beyond assuming constant diffusion coefficient

Testing a vast range of models

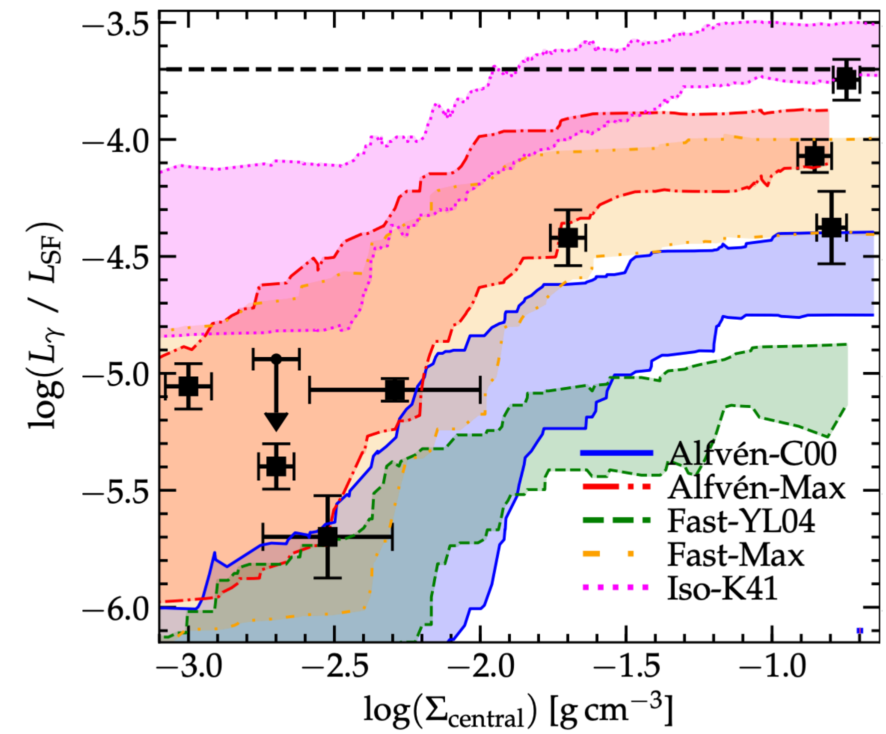
standard ET produces too high effective diffusion



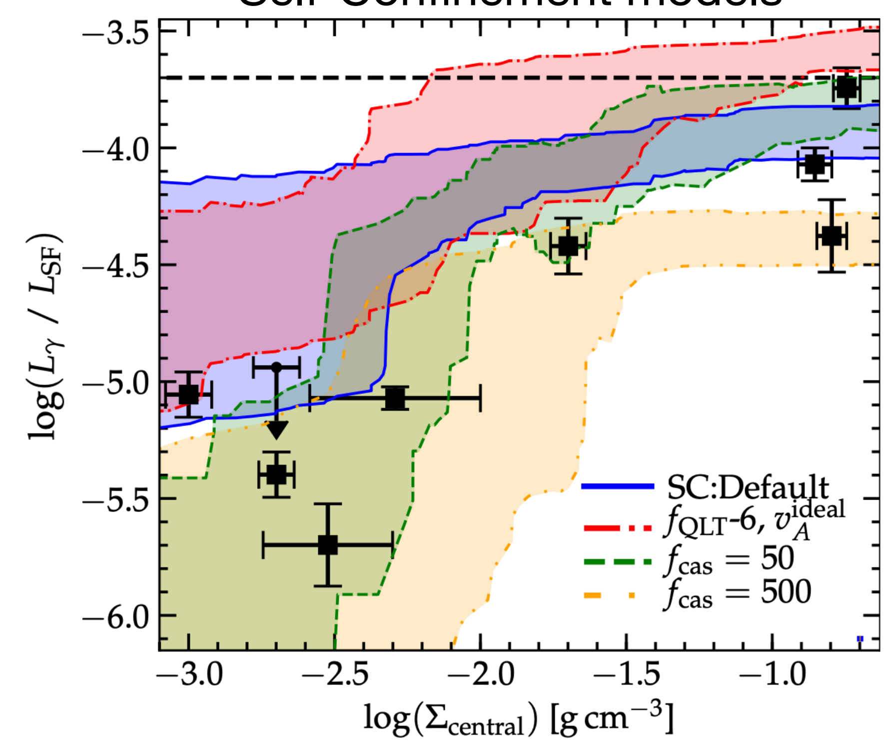
standard … diffusion
(con … ase)



Extrinsic Turbulence models



Self-Confinement models

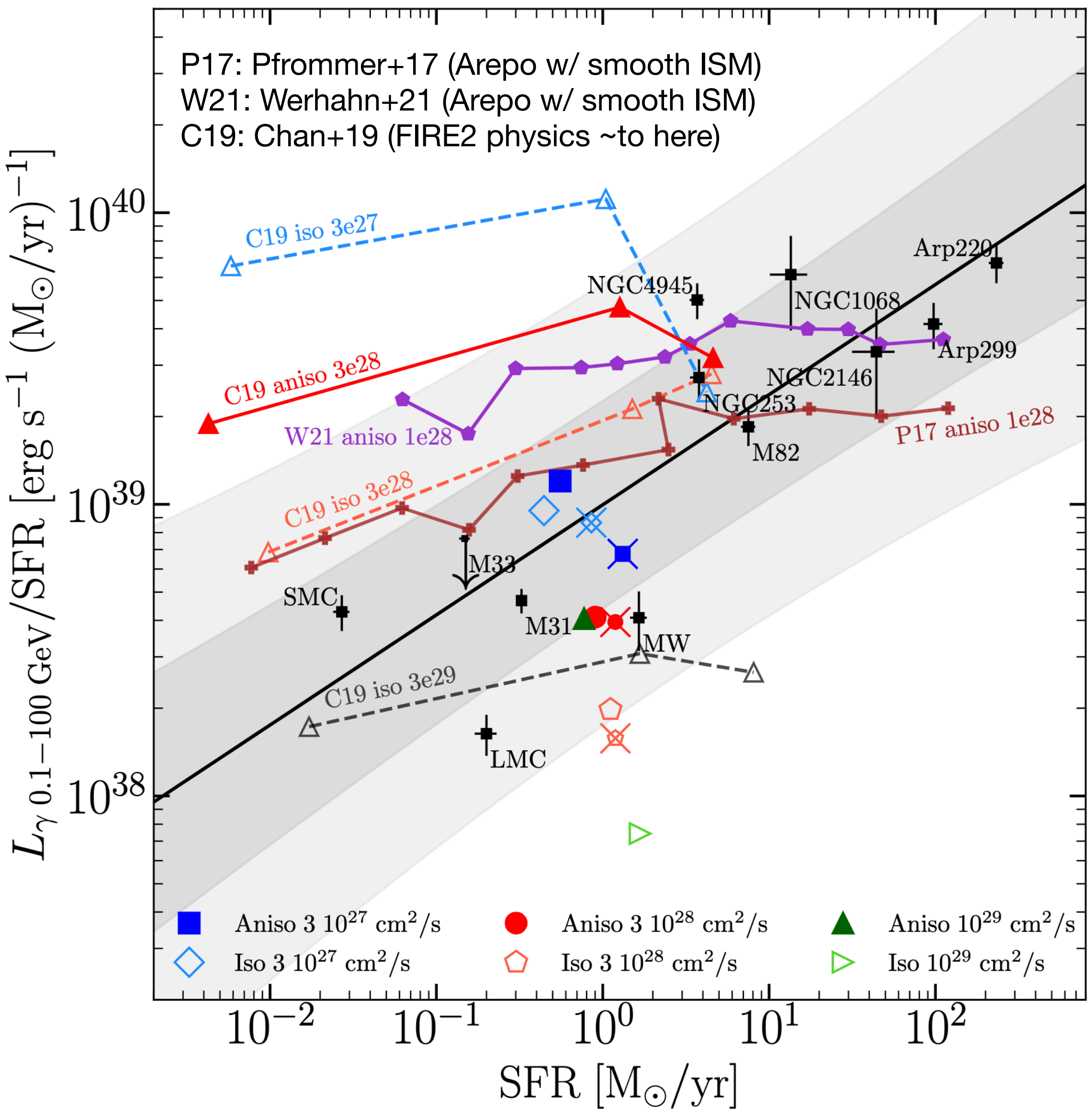


Hopkins+21

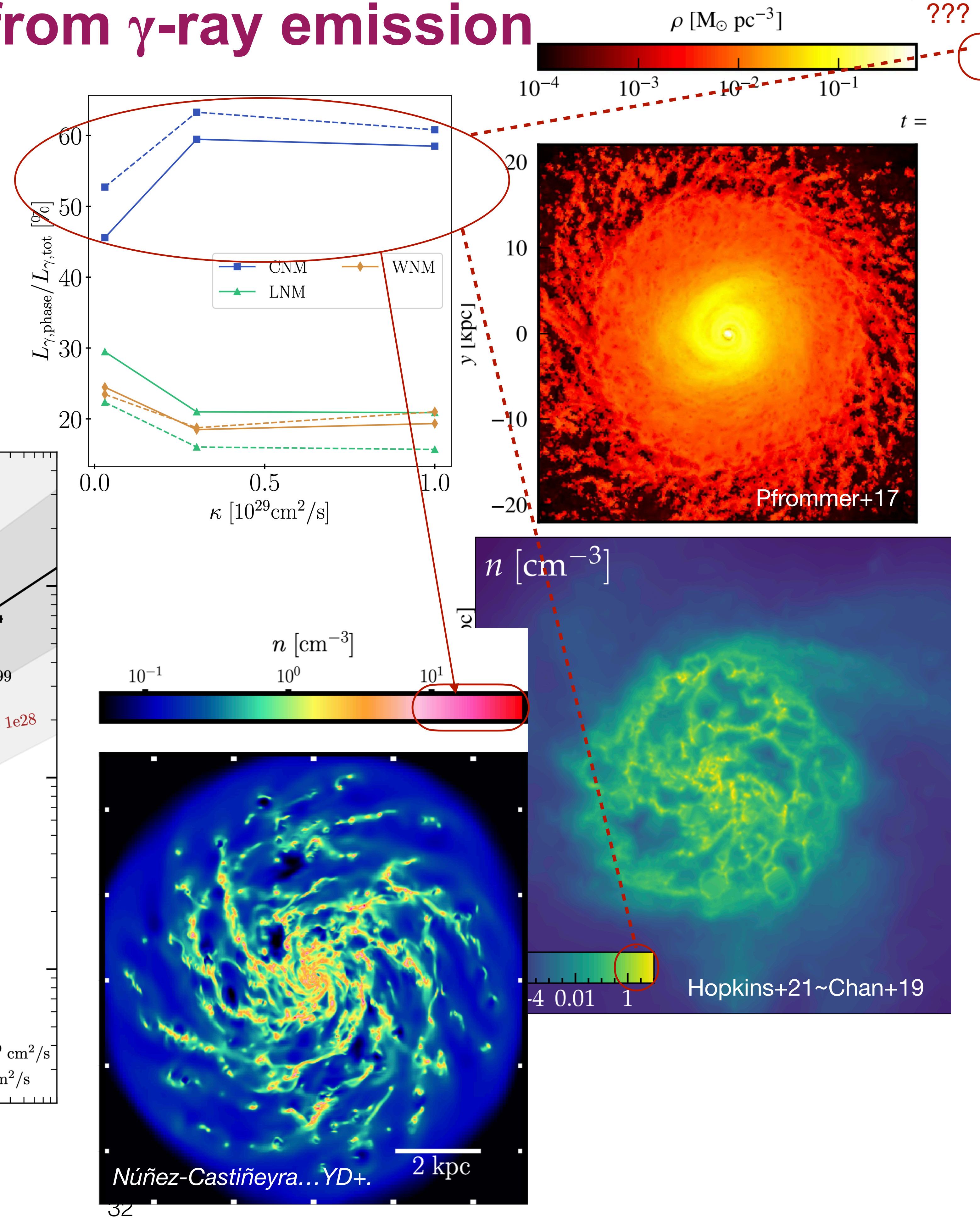
Constraints from γ-ray emission

The least resolved phase (CNM) dominates the γ-ray signal.

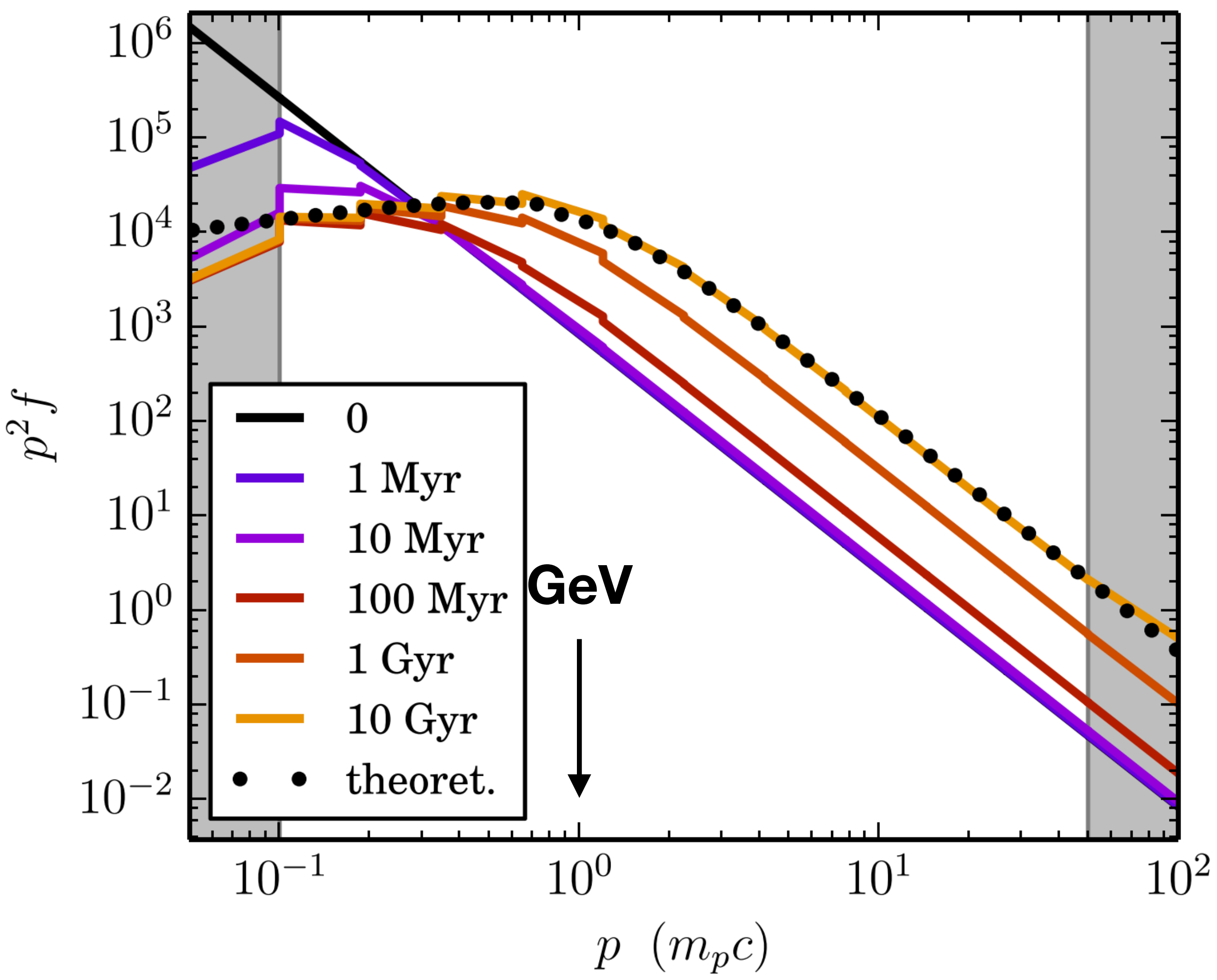
Hence the significant differences amongst simulations?



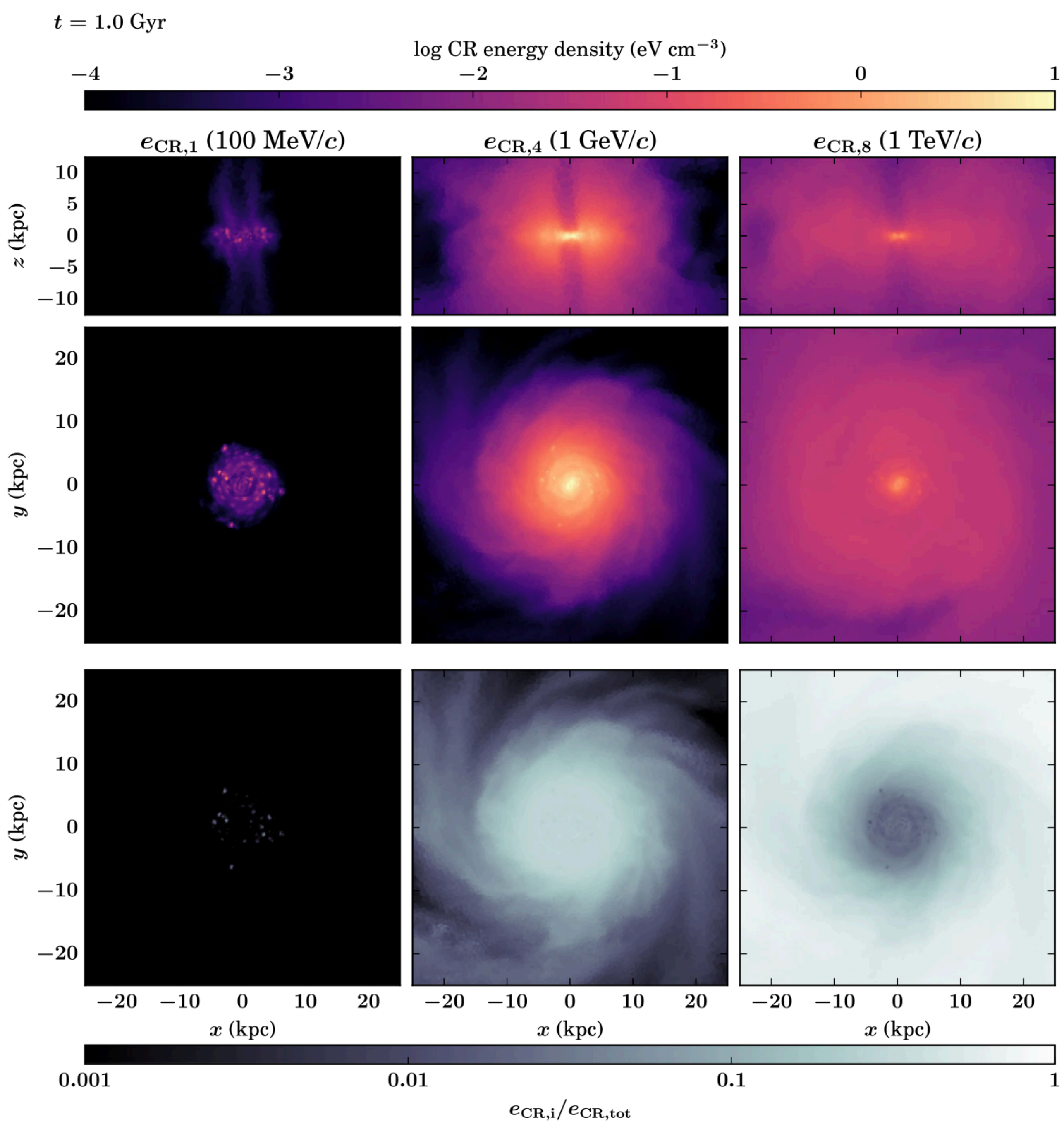
Nuñez-Castiñeyra…YD+



Beyond assuming 1GeV-only particles



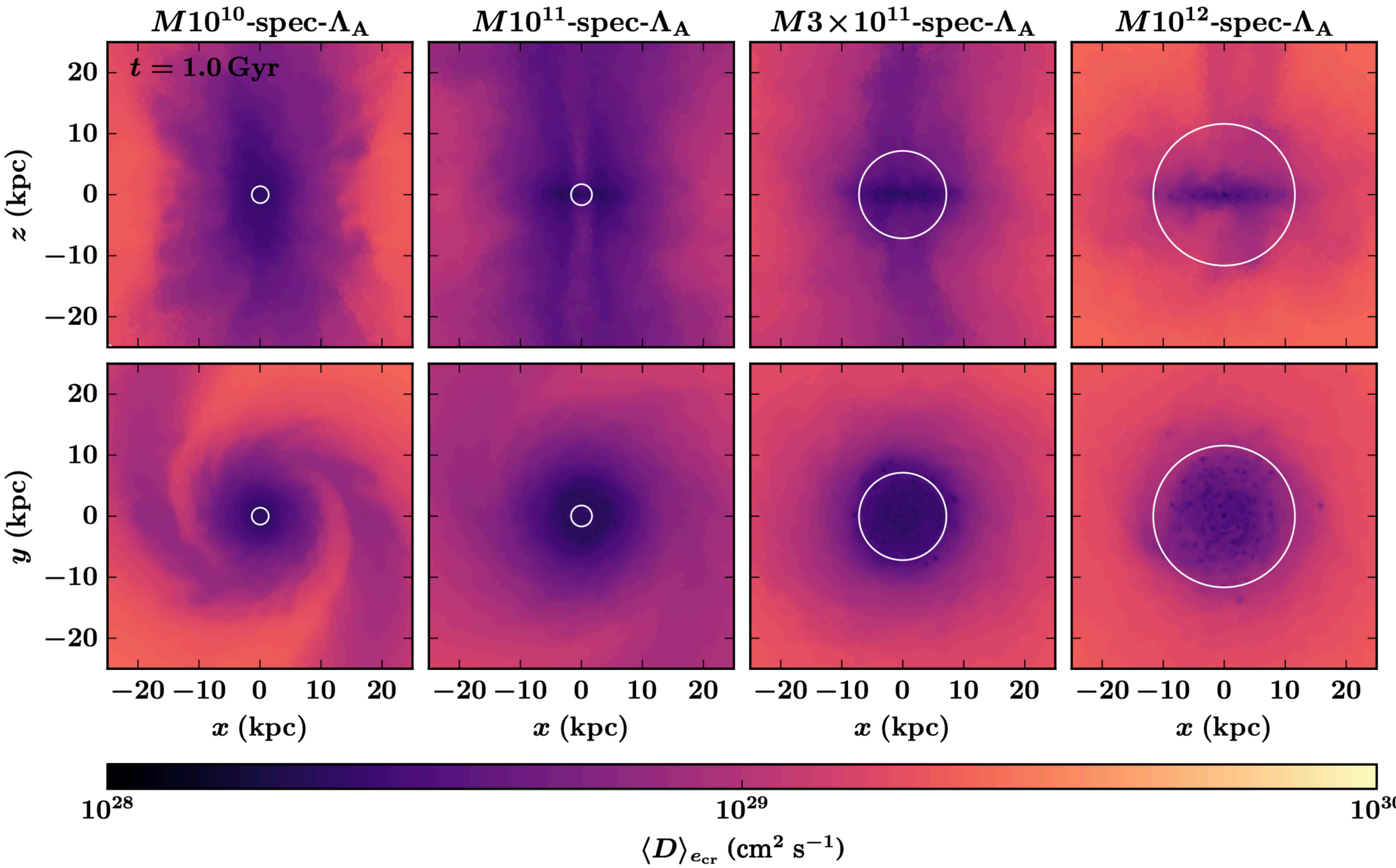
Girichidis+20

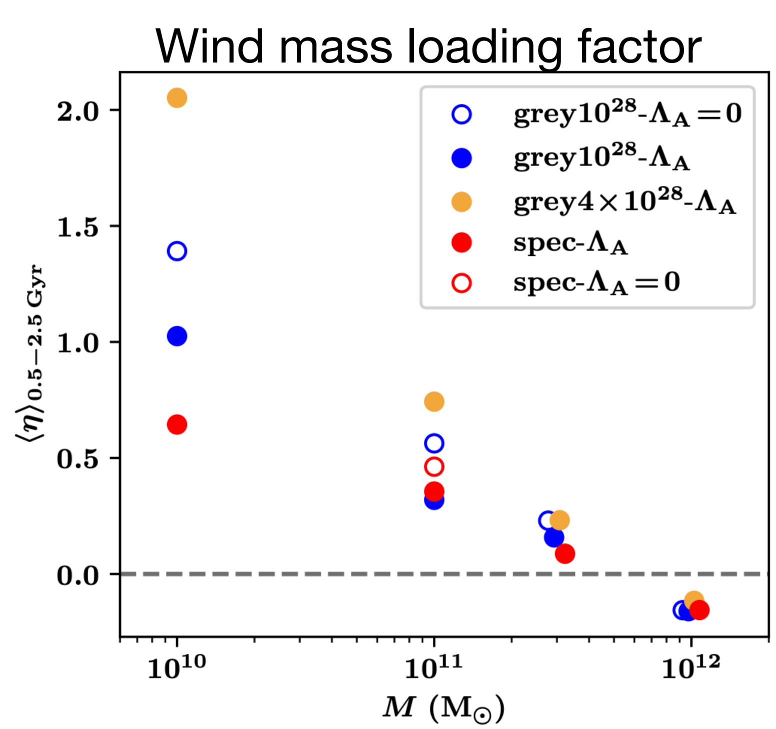


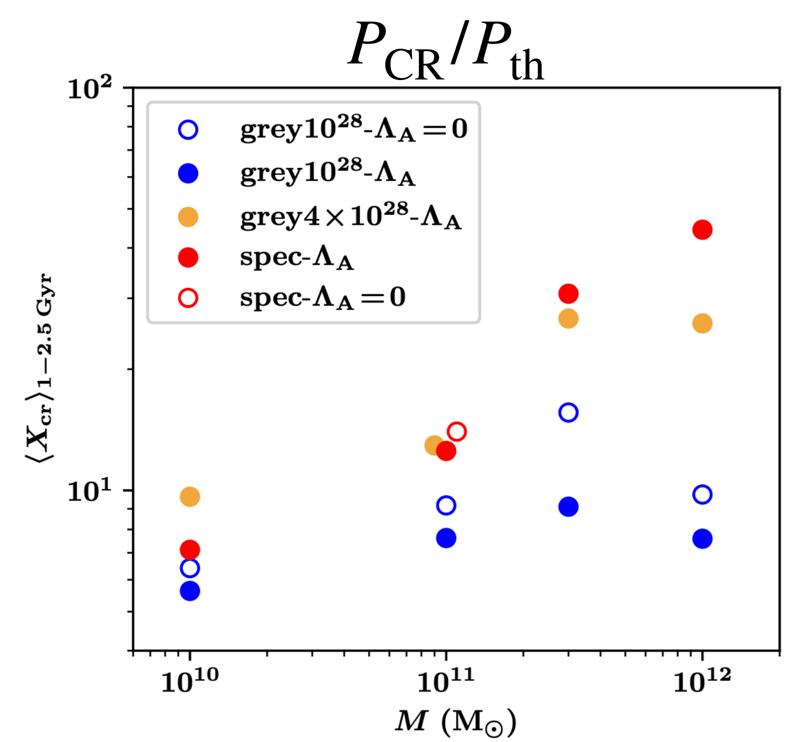
$$D(p) = 10^{28} \left(\frac{p}{1\,\mathrm{GeV}/c} \right)^{0.3-0.5} \mathrm{cm}^2/\mathrm{s}$$

Girichidis+23

Beyond assuming 1GeV-only particles







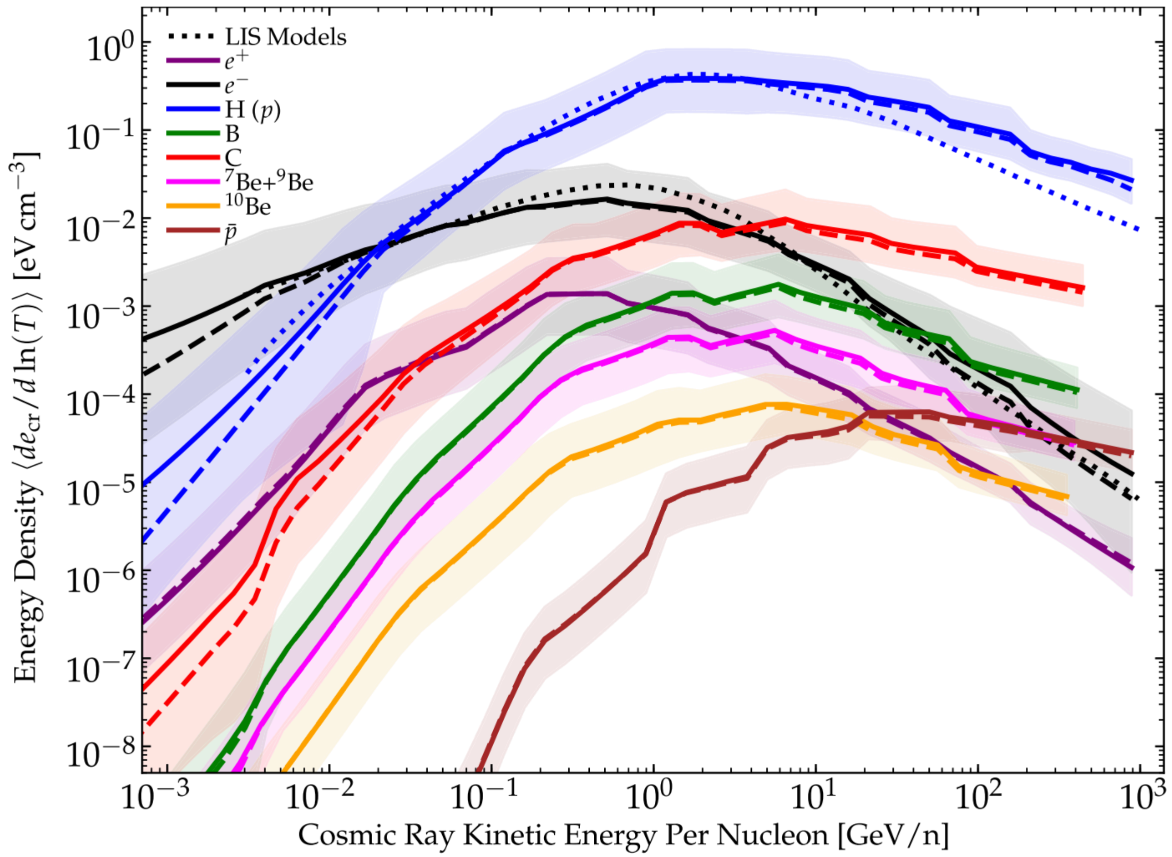
Girichidis+23

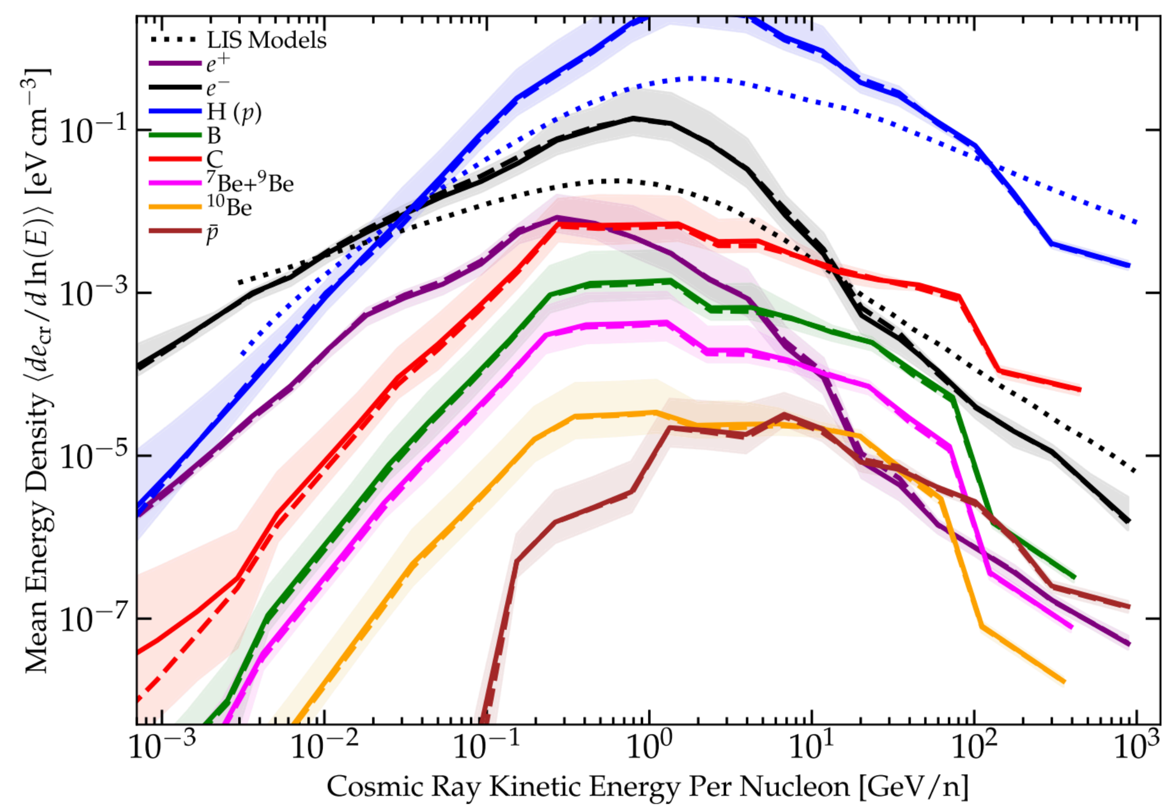
Beyond assuming 1GeV-only particles

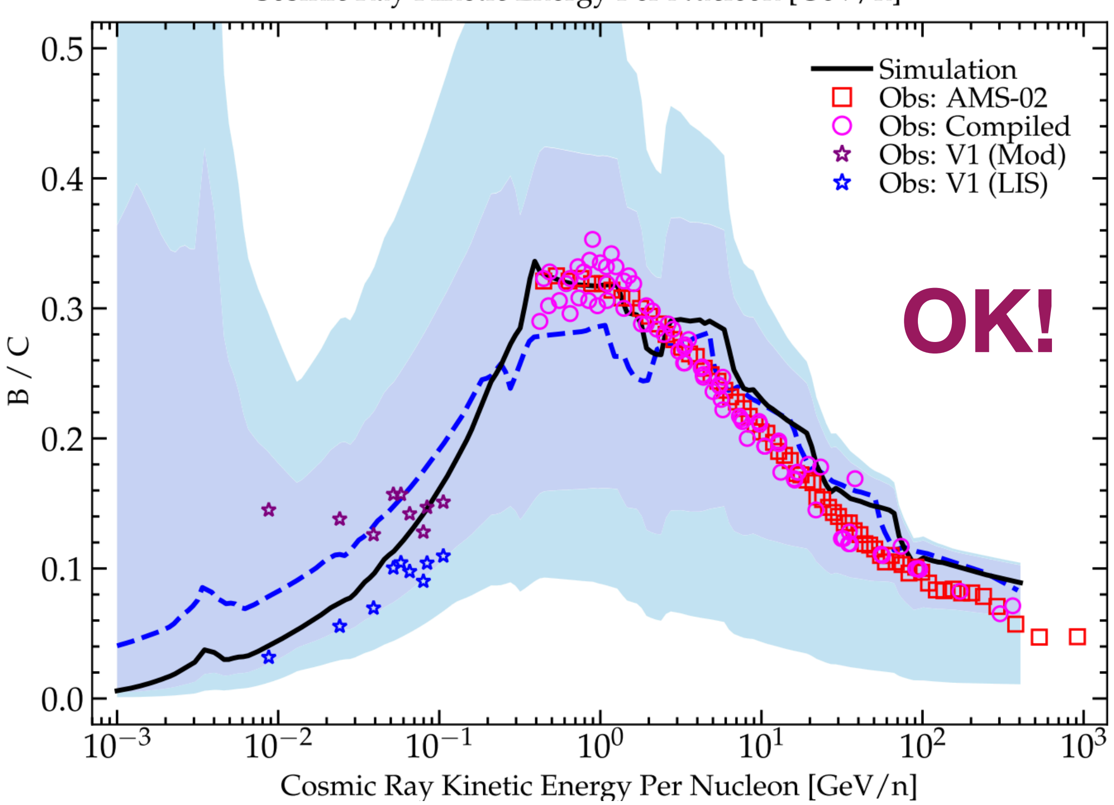
$$D(p) = 10^{29}\left(\frac{p}{1\,\mathrm{GeV}/c}\right)^{\delta}\ \mathrm{cm^2/s},\ \text{with}\ \ \delta = 0.5$$

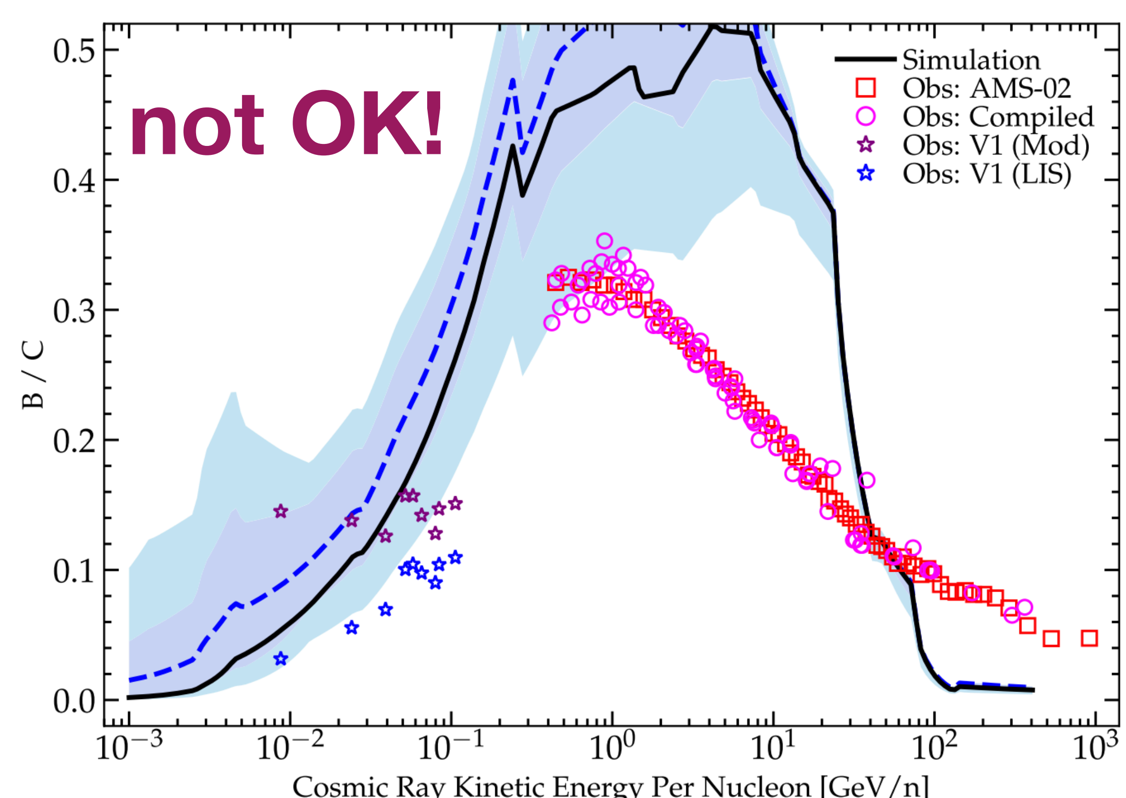
self-confinement model

$-1 \lesssim \delta_{<\mathrm{GeV}} \lesssim 0.5$ $\qquad$ $0.8 \lesssim \delta_{>\mathrm{GeV}} \lesssim 1.7$







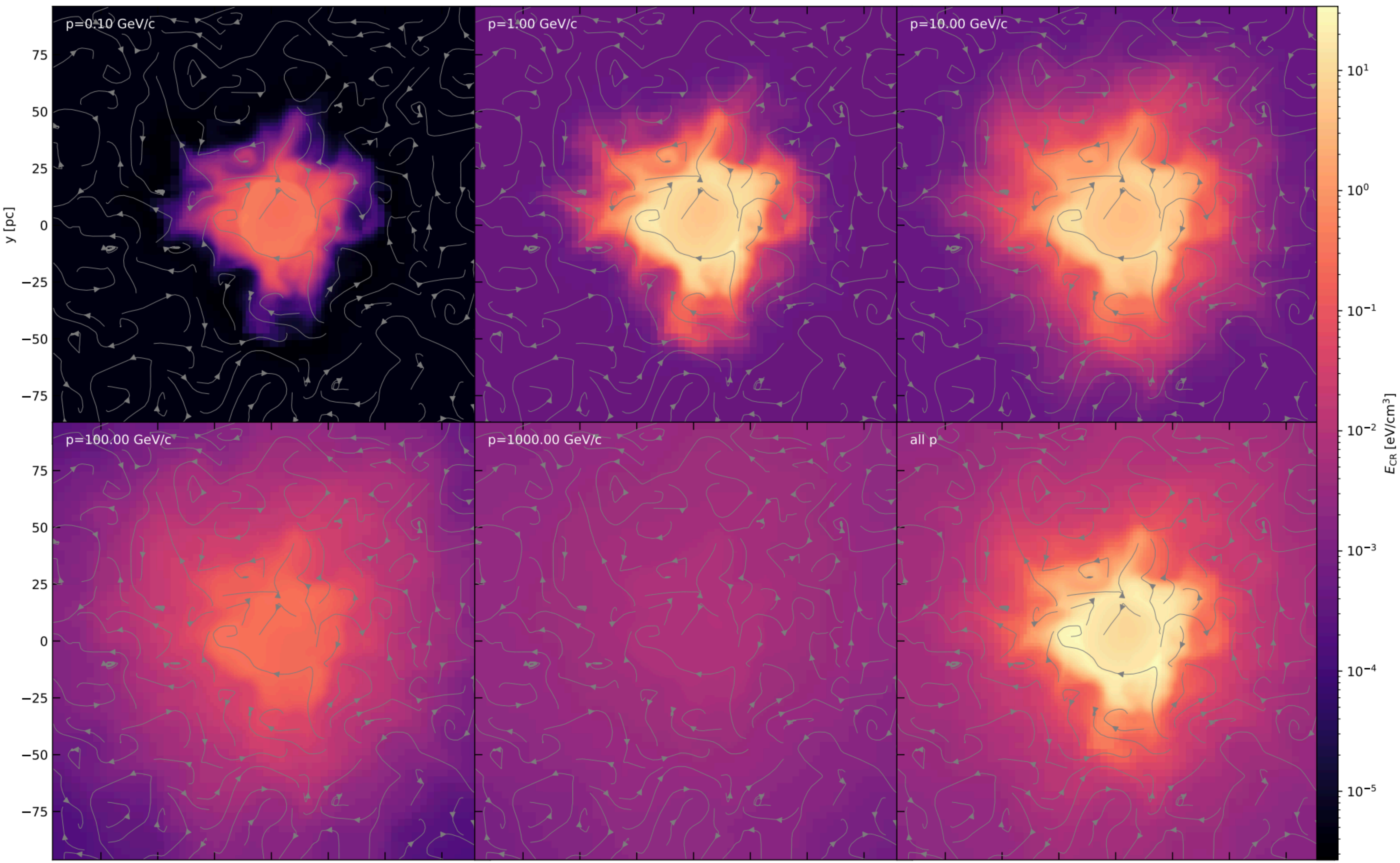


Hopkins+23

The MCR method

Developed by **Nimatou Seydi Diallo (final year PhD student IAP)** in RAMSES

Currently looking for a PostDoc position!

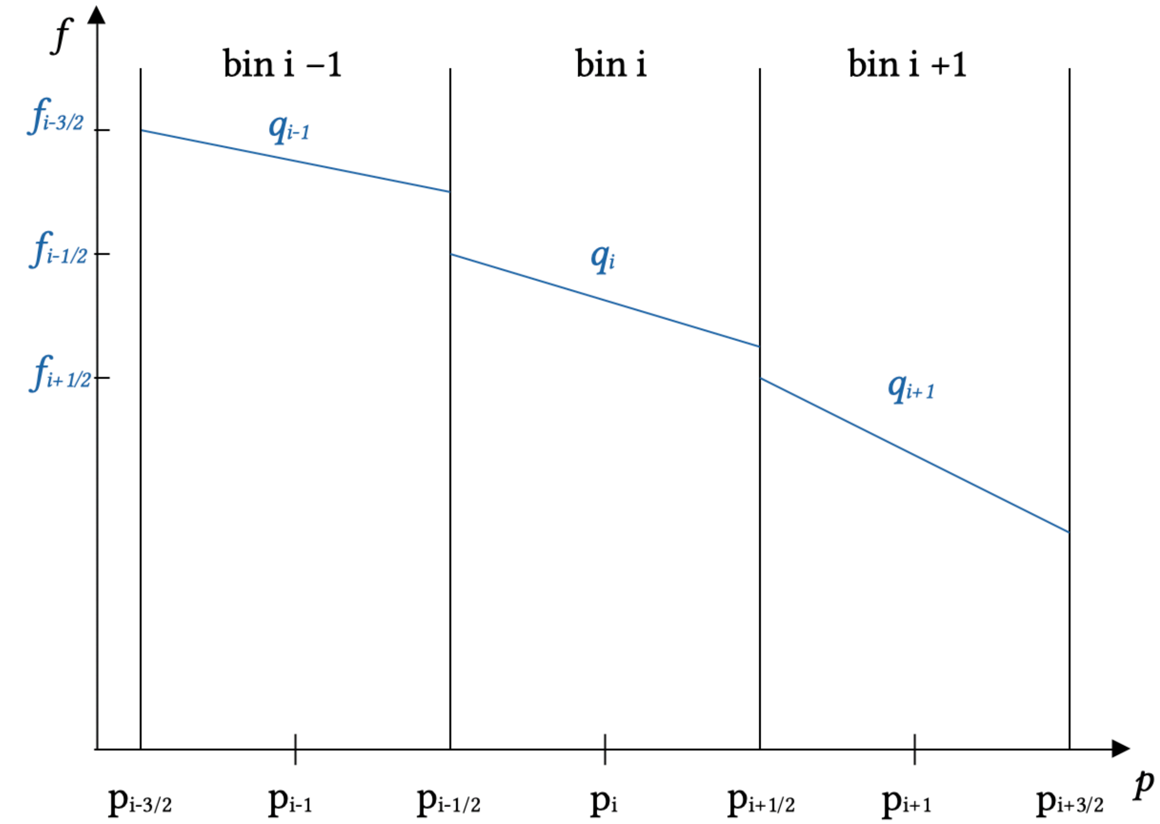


MCR method: discretisation in momentum

- Assume piece-wise power law CR distribution function

$$f(p) = f_{i-1/2} \left(\frac{p}{p_{i-1/2}} \right)^{-q_i}$$

- Decompose in equally log-spaced bins of p



See Jones+99; Miniati 01; Girichidis+20; Hopkins+23

MCR method: discretisation in momentum

- Assume piece-wise power law CR distribution function

$$f(p) = f_{i-1/2}\left(\frac{p}{p_{i-1/2}}\right)^{-q_i}$$

- Decompose in equally log-spaced bins of p
- CR number density:

$$n_i = 4\pi\int_{p_{i-1/2}}^{p_{i+1/2}} p^2\, f(p)dp$$

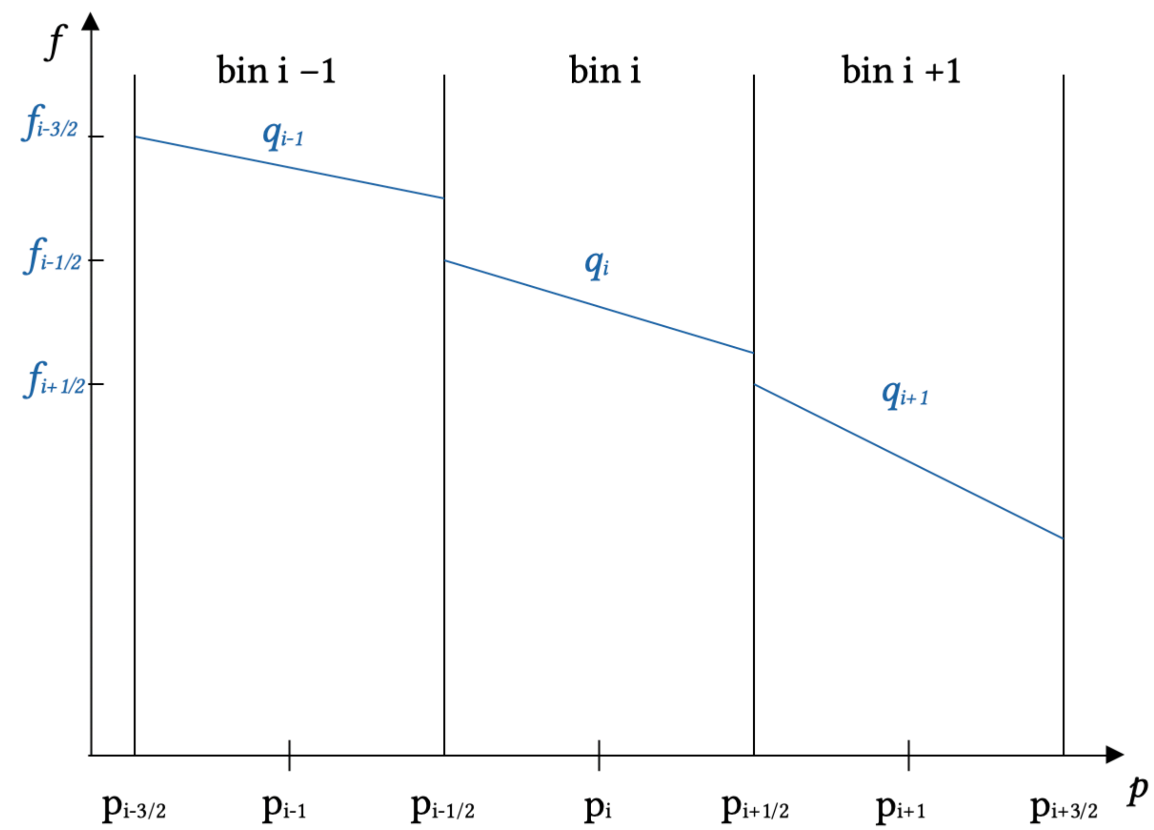
- CR energy density:

$$e_i = 4\pi\int_{p_{i-1/2}}^{p_{i+1/2}} p^2\, T(p)f(p)dp$$

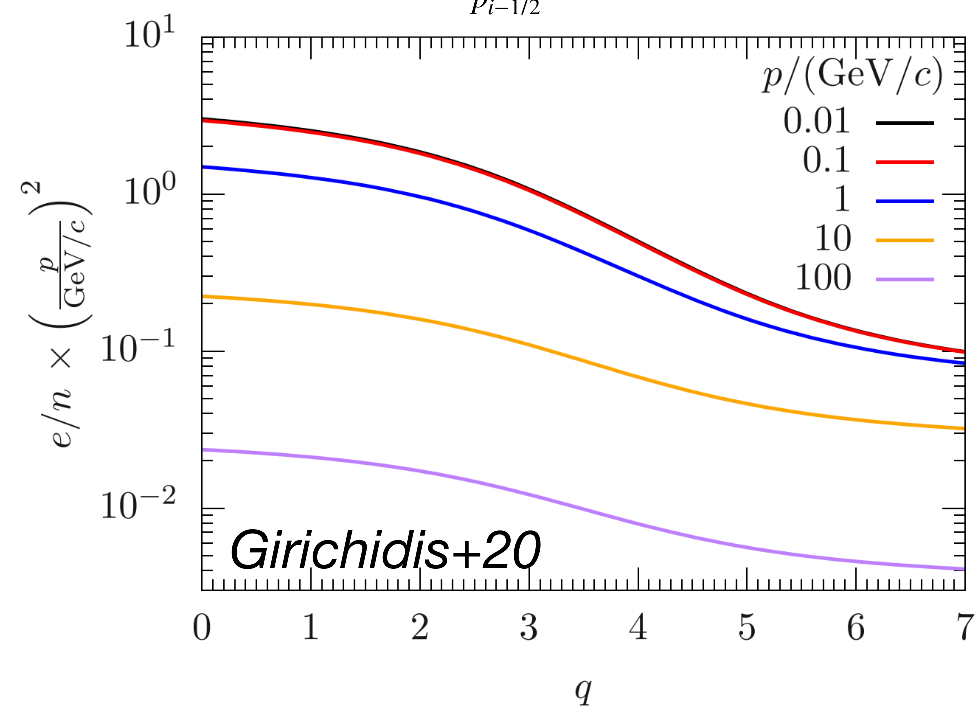
Evolution of $(f_{i-1/2}, q_i)$ **<=>** **Evolution of** (e_i, n_i)

$$T(p) = \sqrt{p^2c^2 - m^2c^4} - mc^2$$

$m \simeq 1\,\mathrm{GeV}$ for protons

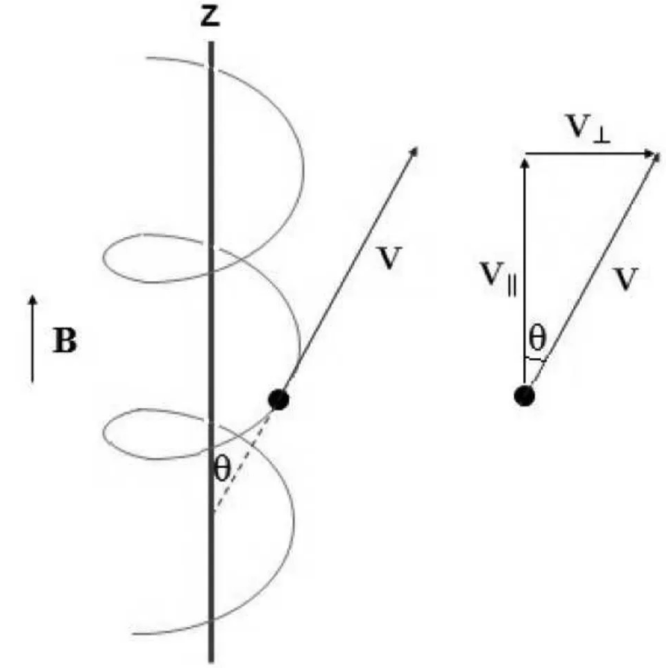


$$\frac{e_i}{n_i} = \frac{\int_{p_{i-1/2}}^{p_{i+1/2}} p^{2-q_i}\mathrm{d}p}{\int_{p_{i-1/2}}^{p_{i+1/2}} T(p)p^{2-q_i}\mathrm{d}p}$$



See Jones+99; Miniati 01; Girichidis+20; Hopkins+23

MCR method: basic equations on $f(x, p, \mu, t)$



General focused CR transport equation (*Skilling 71*)

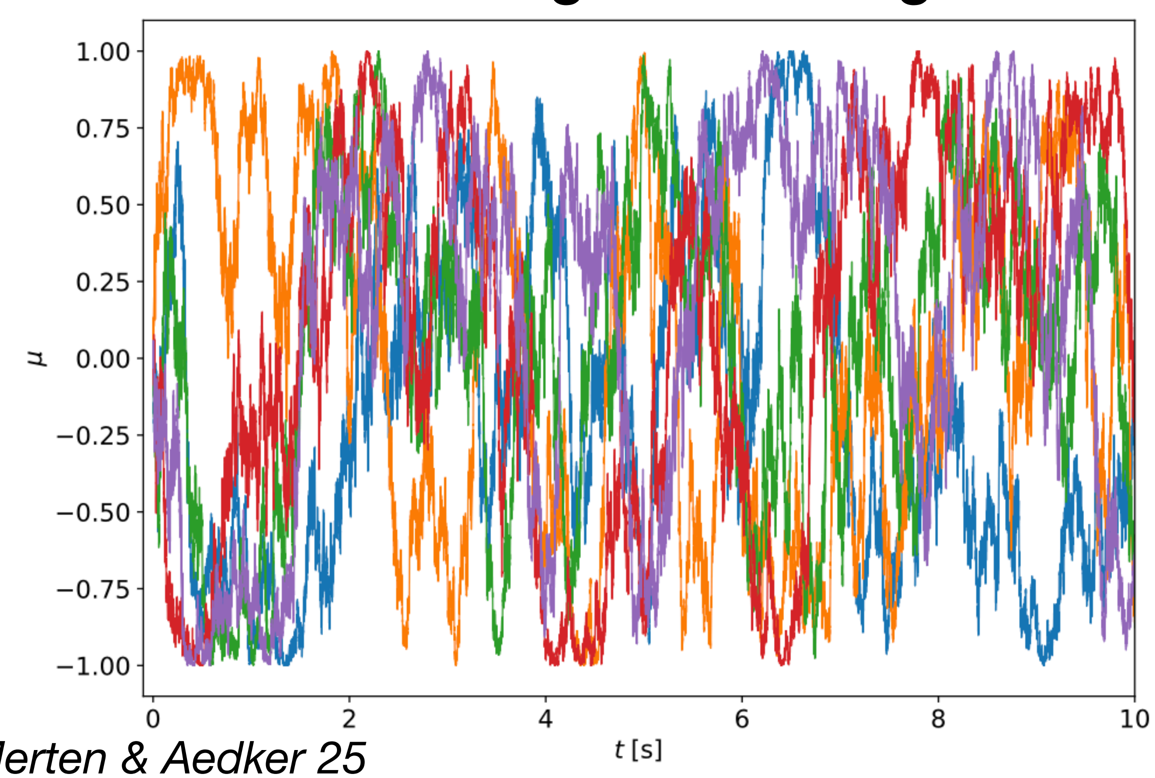
$$
\begin{aligned}
&\frac{1}{c}D_t f + \mu\beta\hat{\mathbf{b}}\cdot\nabla f - f\nabla\cdot\boldsymbol{\beta}_u \\
&+ \left[\frac{1-3\mu^2}{2}(\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\boldsymbol{\beta}_u) - \frac{1-\mu^2}{2}\nabla\cdot\boldsymbol{\beta}_u - \frac{\mu\hat{\mathbf{b}}\cdot\mathbf{a}}{\beta c^2}\right] p\frac{\partial f}{\partial p} \\
&+ \left[\beta\nabla\cdot\hat{\mathbf{b}} + \mu\nabla\cdot\boldsymbol{\beta}_u - 3\mu(\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\boldsymbol{\beta}_u) - \frac{2\hat{\mathbf{b}}\cdot\mathbf{a}}{\beta c^2}\right]\frac{1-\mu^2}{2}\frac{\partial f}{\partial \mu} \\
&= \frac{1}{c}\frac{\partial f}{\partial t}\Big|_{\rm coll}.
\end{aligned}
$$

$$\mu = \cos\theta = \vec{p}\,.\,\vec{b}$$

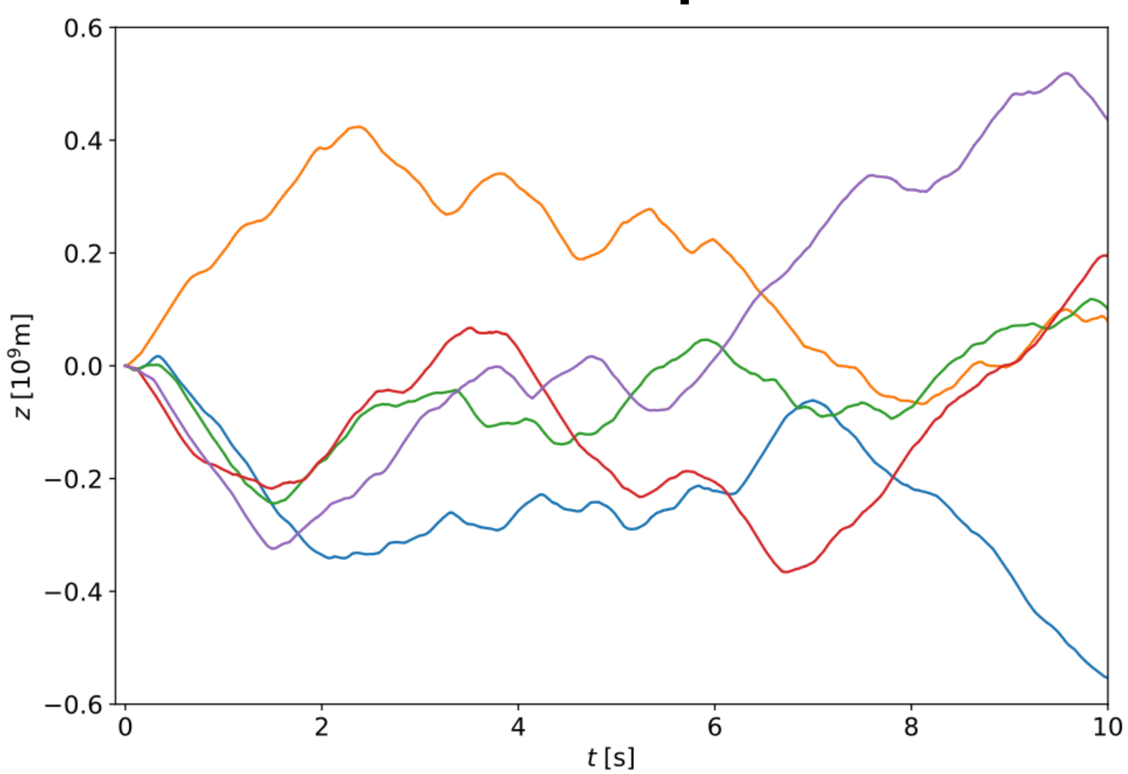
is the CR particle pitch angle

random walk along B field →

Pitch-angle scattering



Diffusion in space



Merten & Aedker 25

MCR method: basic equations on $f(x, p, \mu, t)$

$\mu = \vec{p}\,.\,\vec{b}$ is the CR particle pitch angle

General focused CR transport equation (*Skilling 71*)

$$\frac{1}{c} D_t f + \mu \beta \hat{\mathbf{b}} \cdot \nabla f - f \nabla \cdot \boldsymbol{\beta}_u + \left[\frac{1 - 3\mu^2}{2} (\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \boldsymbol{\beta}_u) - \frac{1 - \mu^2}{2} \nabla \cdot \boldsymbol{\beta}_u - \frac{\mu \hat{\mathbf{b}} \cdot \mathbf{a}}{\beta c^2} \right] p \frac{\partial f}{\partial p} + \left[\beta \nabla \cdot \hat{\mathbf{b}} + \mu \nabla \cdot \boldsymbol{\beta}_u - 3\mu (\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \boldsymbol{\beta}_u) - \frac{2\hat{\mathbf{b}} \cdot \mathbf{a}}{\beta c^2} \right] \frac{1 - \mu^2}{2} \frac{\partial f}{\partial \mu} = \frac{1}{c} \frac{\partial f}{\partial t}\bigg|_{\text{coll}}.$$

Quasi-linear theory (*Schlickeiser 89*)

$$\frac{\partial f}{\partial t}\bigg|_{\text{sc}} = \frac{\partial}{\partial \mu} \left(D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_{\mu p} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 \left(D_{\mu p} \frac{\partial f}{\partial \mu} + D_{pp} \frac{\partial f}{\partial p} \right) \right]$$

We don't like μ, so let's take the μ-moments of the general equation

$$\int_{-1}^{+1} [\ldots] \mathrm{d}\mu$$

$$\int_{-1}^{+1} [\ldots] \mu \mathrm{d}\mu$$

$$f(x, p, \mu, t) = f_0(x, p, t) + 3\mu f_1(x, p, t)$$

$$\frac{\partial f_0}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_0) + \vec{\nabla}\,.\,(\mathrm{v} \vec{b} f_1) = \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 L(p) f_0 \right] + j_0$$

$$\frac{\partial f_1}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_1) + \mathrm{v}\, \tilde{\nabla}(f_0) = - \left[\bar{D}_{\mu\mu} f_1 + \bar{D}_{\mu p} \frac{\partial f_0}{\partial p} \right] + j_1$$

MCR method: basic equations on $f(x, p, \mu, t)$

$\mu = \vec{p}\,.\,\vec{b}$ is the CR particle pitch angle

gas velocity

CR part. velocity

CR injection

Equation evolving the isotropic part of f:

$$\frac{\partial f_0}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_0) + \vec{\nabla}\,.\,(\mathrm{v}\vec{b} f_1) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 L(p) f_0\right] + j_0$$

Equation evolving the anisotropic part of f:

$$\frac{\partial f_1}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_1) + \mathrm{v}\,\tilde{\nabla}(f_0) = -\left[D_{\mu\mu} f_1 + D_{\mu p}\frac{\partial f_0}{\partial p}\right] + j_1$$

radiative losses

streaming losses

$$L(p) = L_{\mathrm{r}} + p\mathbb{D} : \nabla u + D_{p\mu}\frac{f_1}{f_0} + \frac{D_{pp}}{f_0}\frac{\partial f_0}{\partial p}$$

adiabatic change

Fermi II acceleration

Alfvén speed

diffusion

streaming losses

$$D_{\mu\mu} = \bar{\nu},\ D_{\mu p} = \chi\frac{p u_a}{\mathrm{v}}\bar{\nu},\ D_{p\mu} = \frac{p u_a}{\mathrm{v}}\bar{\nu},\ D_{pp} = \chi\frac{p^2 u_a^2}{\mathrm{v}^2}\bar{\nu}$$

streaming transport

Fermi II acceleration

Scattering rate

MCR method: basic equations on $f(x, p, \mu, t) \rightarrow$ Turning them into (n_i, F_i^n)

$\mu = \vec{p}\,.\,\vec{b}$ is the CR particle pitch angle

$$\int_{p_{i-1/2}}^{p_{i+1/2}} [\ldots] \times p^2 \mathrm{d}p$$

$$\frac{\partial f_0}{\partial t} + \overrightarrow{\nabla}\,.\,(\vec{u} f_0) + \overrightarrow{\nabla}\,.\,(\mathrm{v}\vec{b} f_1) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 L(p) f_0\right] + j_0 \longrightarrow \text{evol. for } n_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 f_0(p) dp$$

$$\frac{\partial f_1}{\partial t} + \overrightarrow{\nabla}\,.\,(\vec{u} f_1) + \mathrm{v}\,\tilde{\nabla}(f_0) = -\left[D_{\mu\mu} f_1 + D_{\mu p}\frac{\partial f_0}{\partial p}\right] + j_1 \longrightarrow \text{evol. for } F_i^n = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} f_1(p) dp$$

$$\int_{p_{i-1/2}}^{p_{i+1/2}} [\ldots] \times p^2 \mathrm{v}\mathrm{d}p$$

MCR method: basic equations on $f(x, p, \mu, t) \rightarrow$ Turning them into (e_i, F_i^e)

$\mu = \vec{p} \,.\, \vec{b}$ is the CR particle pitch angle

$$\int_{p_{i-1/2}}^{p_{i+1/2}} [\ldots] \times T(p) p^2 \mathrm{d}p$$

$$\frac{\partial f_0}{\partial t} + \vec{\nabla} \,.\, (\vec{u} f_0) + \vec{\nabla} \,.\, (\mathrm{v} \vec{b} f_1) = \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^2 L(p) f_0 \right] + j_0 \longrightarrow \text{evol. for } e_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} T(p) p^2 f_0(p) dp$$

$$\frac{\partial f_1}{\partial t} + \vec{\nabla} \,.\, (\vec{u} f_1) + \mathrm{v}\, \tilde{\nabla}(f_0) = - \left[D_{\mu\mu} f_1 + D_{\mu p} \frac{\partial f_0}{\partial p} \right] + j_1 \longrightarrow \text{evol. for } F_i^e = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} T(p) p^2 \mathrm{v} f_1(p) dp$$

$$\int_{p_{i-1/2}}^{p_{i+1/2}} [\ldots] \times T(p) p^2 \mathrm{v} \mathrm{d}p$$

$$T(p) = \sqrt{p^2 c^2 - m^2 c^4} - mc^2$$

Spectral method: basic equations on n_i, F_i^n, e_i, F_i^e

What we really track in the code

After a bit of algebra + assumptions*

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\,.\,(\vec{u} n_i) + \vec{\nabla}\,.\,\vec{F}_i^n = \left[4\pi p^2 L(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} + j_{0,i}^n$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b}\,.\,\vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\left[\vec{F}_i^n - \frac{q_i}{3}\bar{u}_\mathrm{A} n_i\right]$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla}\,.\,(\vec{u} e_i) + \vec{\nabla}\,.\,\vec{F}_i^e = \left[4\pi p^2 L(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} L(p) f_0 \mathrm{d}p + j_{0,i}^e$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b}\,.\,\vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{q_i}{3}\bar{u}_\mathrm{A} e_i\right]\,.$$

$$\kappa_i = \frac{\mathrm{v}^2}{3\bar{\nu}_i}$$

$$\bar{u}_A = \vec{u}_A \mathrm{sign}(\vec{b}\,.\,\vec{\nabla}\,e_i)$$

*Drop the smaller terms (v^{-2}), assume P1 $\langle \mu \rangle \simeq 0$, use Dirac delta function $\delta(p - p_i)$ to evaluate $\bar{\nu}_i$ and $\int \mathrm{v}\,\tilde{\nabla} f_0[\,\ldots\,]\mathrm{d}p$

Link with the grey method: basic equations on e_i, F_i^e

$$[\dots]_{p_{i-1/2}}^{p_{i+1/2}} = 0 \qquad \gamma_i = 4/3 \qquad q_i = 4$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} . (\vec{u} n_i) + \vec{\nabla} . \vec{F}_i^n = j_{0,i}^n$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b} . \vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\left[\vec{F}_i^n - \frac{4}{3}\bar{u}_\mathrm{A} n_i\right]$$

no need to follow n

$$-4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} L(p) f_0 \mathrm{d}p$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla} . (\vec{u} e_i) + \vec{\nabla} . \vec{F}_i^e = - P_i \vec{\nabla} . \vec{u} - \frac{\bar{u}_\mathrm{A}}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{4}{3}\bar{u}_\mathrm{A} e_i\right] + \mathscr{L}_\mathrm{rad} + j_{0,i}^e$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b} . \vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{4}{3}\bar{u}_\mathrm{A} e_i\right]$$

This is the two-moment CR equation (Rosdahl,YD+25)

Link with the grey method: basic equations on e_i, F_i^e

$$[\ldots]_{p_{i-1/2}}^{p_{i+1/2}} = 0 \qquad \gamma_i = 4/3 \qquad q_i = 4$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\,.\,(\vec{u}n_i) + \vec{\nabla}\,.\,\vec{F}_i^n = j_{0,i}^n$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b}\,.\,\vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\left[\vec{F}_i^n - \frac{4}{3}\bar{u}_\mathrm{A} n_i\right]$$

no need to follow n

$$-4\pi\int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} L(p) f_0 \mathrm{d}p$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla}\,.\,(\vec{u}e_i) + \vec{\nabla}\,.\,\vec{F}_i^e = -P_i\vec{\nabla}\,.\,\vec{u} - \frac{\bar{u}_\mathrm{A}}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{4}{3}\bar{u}_\mathrm{A} e_i\right] + \mathscr{L}_\mathrm{rad} + j_{0,i}^e$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b}\,.\,\vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{4}{3}\bar{u}_\mathrm{A} e_i\right]$$

This is the two-moment CR equation (Rosdahl,YD+25)

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} \rightarrow 0$$ (flux-limited diffusion)

$$\frac{\partial e_i}{\partial t} + \vec{\nabla}\,.\left(\left[\vec{u} + \frac{4}{3}\bar{u}_\mathrm{A}\right]e_i\right) = -\vec{\nabla}\,.\left(-\kappa_i^e\vec{b}\vec{b}\,\vec{\nabla}e_i\right) - P_i\vec{\nabla}\,.\,\vec{u} + \vec{u}_\mathrm{s}\,.\,\vec{\nabla}P_i + \mathscr{L}_\mathrm{rad} + j_0^e$$

advection+streaming; anisotropic diffusion; pressure work; streaming loss; radiative loss; injection

This is Dubois+16,19

MCR method: basic equations on $f(x, p, \mu, t)$

It's the CR two-moment method repeated for $i = 1,\ldots,N$ bins of p

+

transfer between spectral bins — **the core of the MCR method**

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} . (\vec{u} n_i) + \vec{\nabla} . \vec{F}_i^n = \left[4\pi p^2 L(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} + j_{0,i}^n$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b} . \vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\left[\vec{F}_i^n - \frac{q_i}{3}\bar{u}_{\mathrm{A}} n_i\right]$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla} . (\vec{u} e_i) + \vec{\nabla} . \vec{F}_i^e = \left[4\pi p^2 L(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} L(p) f_0 \mathrm{d}p + j_{0,i}^e$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b} . \vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{q_i}{3}\bar{u}_{\mathrm{A}} e_i\right] .$$

The spectral step

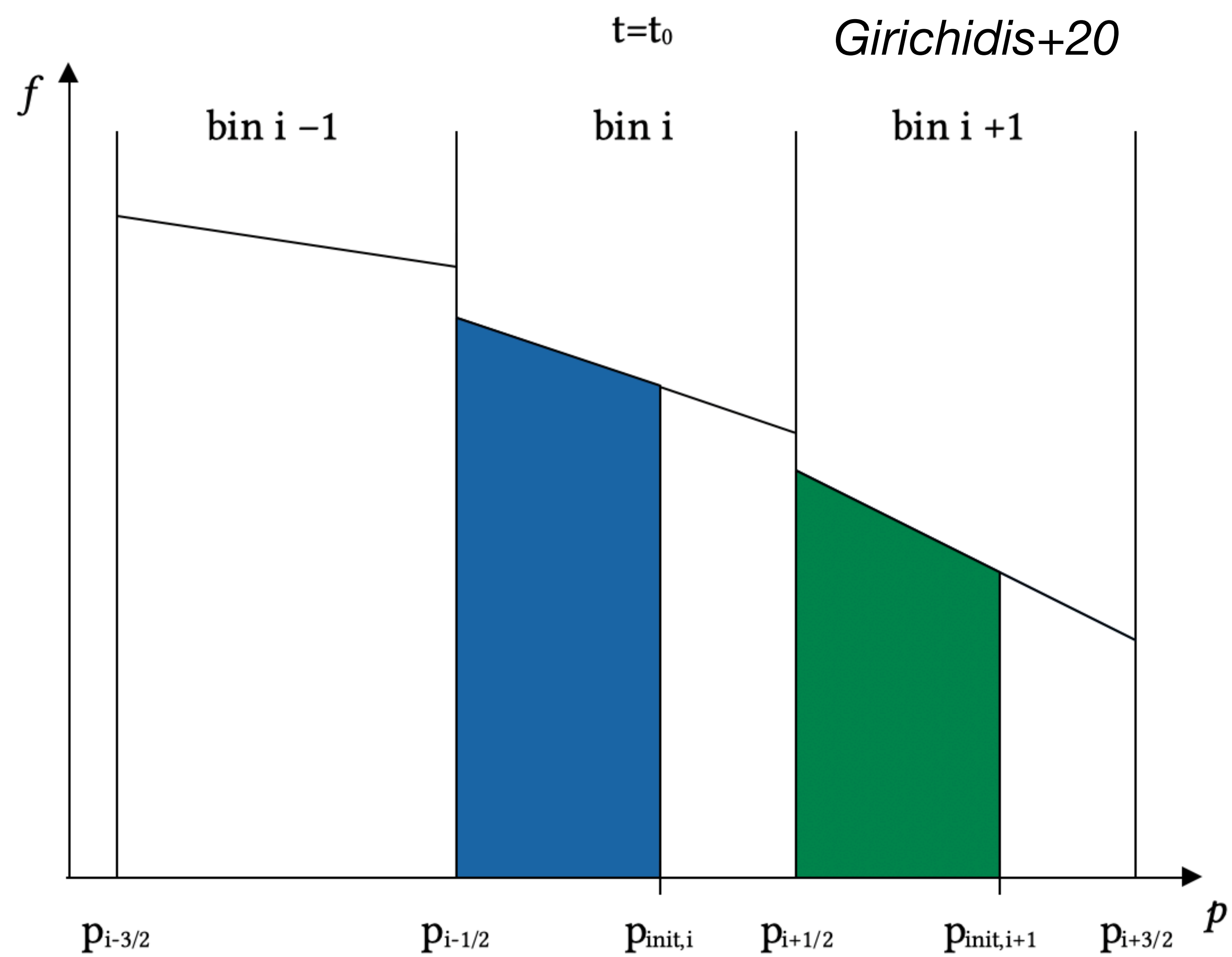
$$\frac{\partial n_i}{\partial t} = \left[4\pi p^2 L(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}}$$

$$\to n_i(t+\Delta t) = n_i(t) + \Delta n_{i+1/2} - \Delta n_{i-1/2}$$

$$\Delta n_{i-1/2} = \int_t^{t+\Delta t} 4\pi p_{i-1/2}^2 L(p_{i-1/2}) f(p_{i-1/2}) \mathrm{d}t$$

$$-\int_{p_{\mathrm{ini}}}^{p_{i-1/2}} \frac{\mathrm{d}p}{L(p)} = \int_t^{t+\Delta t} \mathrm{d}t = \Delta t$$

$$\to \Delta n_{i-1/2} = \int_{p_{i-1/2}}^{p_{\mathrm{ini}}} 4\pi p^2 f(p) \mathrm{d}p$$



- Manifestly conservative for n_i
- Chose Δt wisely (Courant-like condition)
- Boundary conditions at the edge of the p-domain…
- Same story for $\Delta e_{i-1/2}$?

The spectral step

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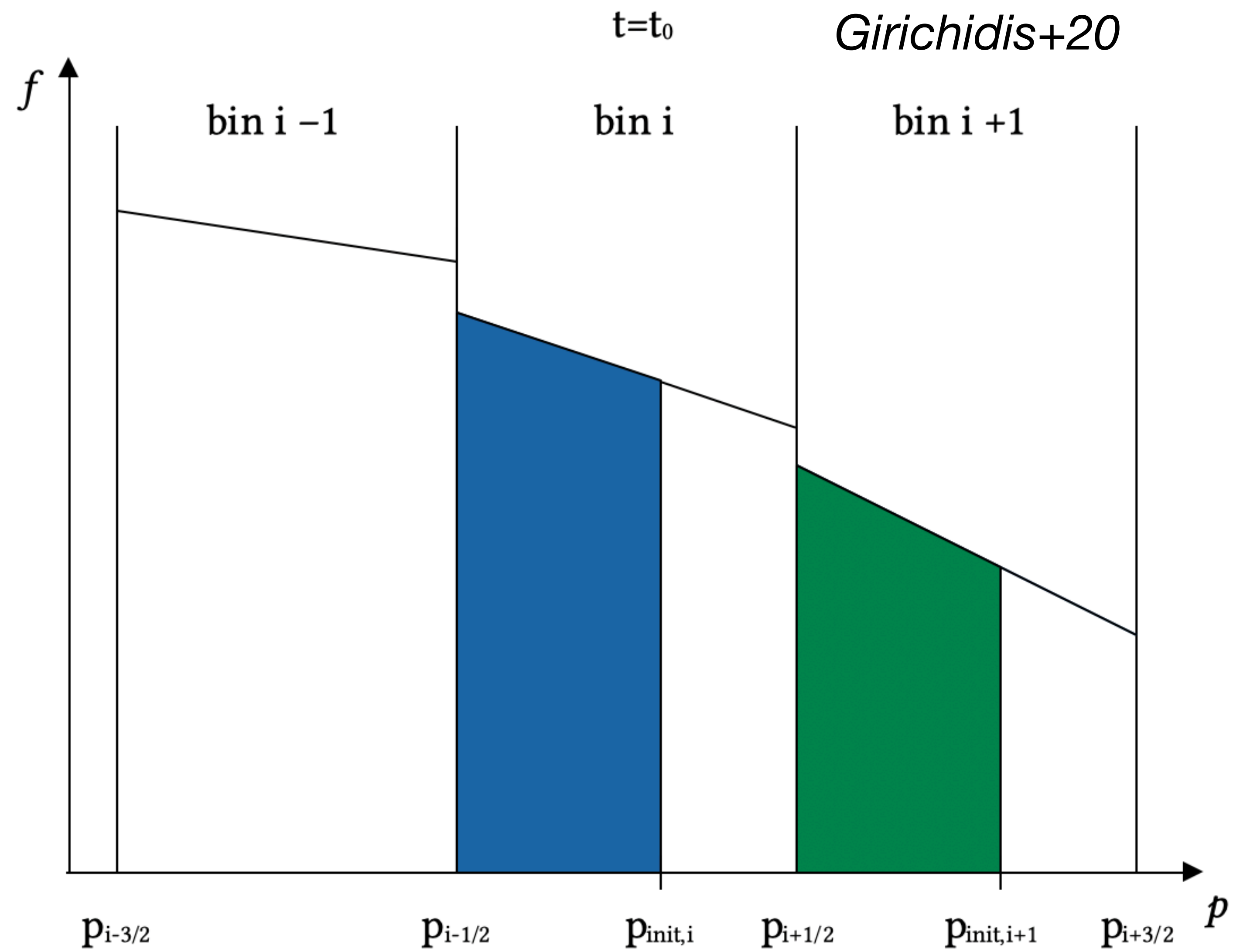


- Manifestly conservative for n_i
- Chose Δt wisely (Courant-like condition)
- Boundary conditions at the edge of the p-domain…
- Same story for $\Delta e_{i-1/2}$?

$$\mathcal{R} = -\frac{\int_{p_{i-1/2}}^{p_{i+1/2}} [\ldots] \mathrm{d}p}{e_i(t)}$$

$$\frac{\partial e_i}{\partial t} = \left[4\pi p^2 L(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - \int_{p_{i-1/2}}^{p_{i+1/2}} 4\pi p^2 \mathrm{v} L(p) f_0 \mathrm{d}p \quad \rightarrow \quad e_i(t+\Delta t) = \frac{(1 + 0.5\mathcal{R}\Delta t) e_i(t) + \Delta e_{i+1/2} - \Delta e_{i-1/2}}{1 - 0.5\mathcal{R}\Delta t}$$

new non-conservative term!

Test: free cooling

- Coulomb/ionisation losses dominant below 1 GeV

  $L_{\mathrm{r,Coul}} \propto p^{-2}$

→ should converge toward $f(p) \propto \mathrm{constant}$
→ all CR losses are returned to plasma

- Hadronic losses dominant above 1 GeV

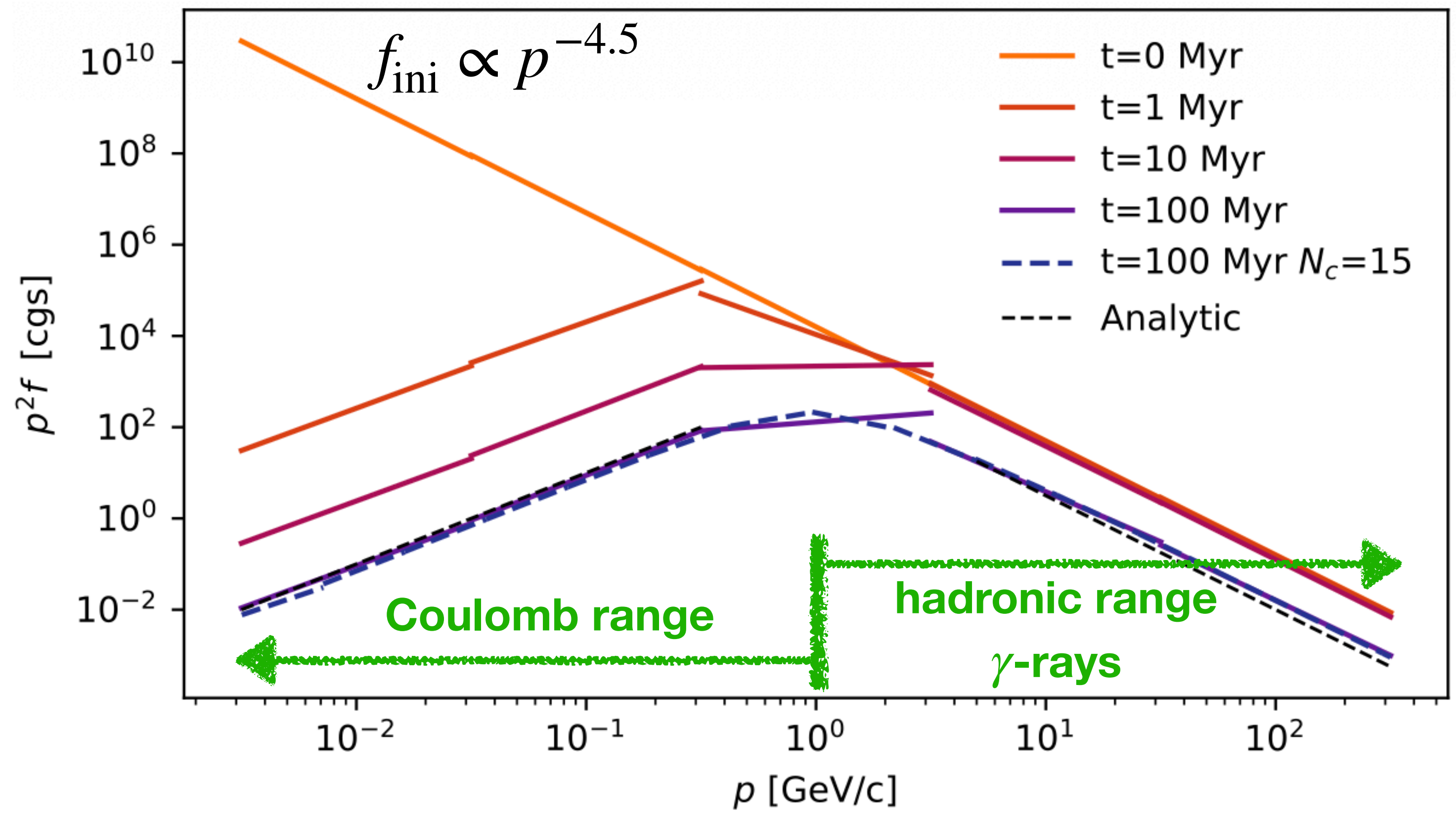
  $L_{\mathrm{r,hadr}} \propto p$

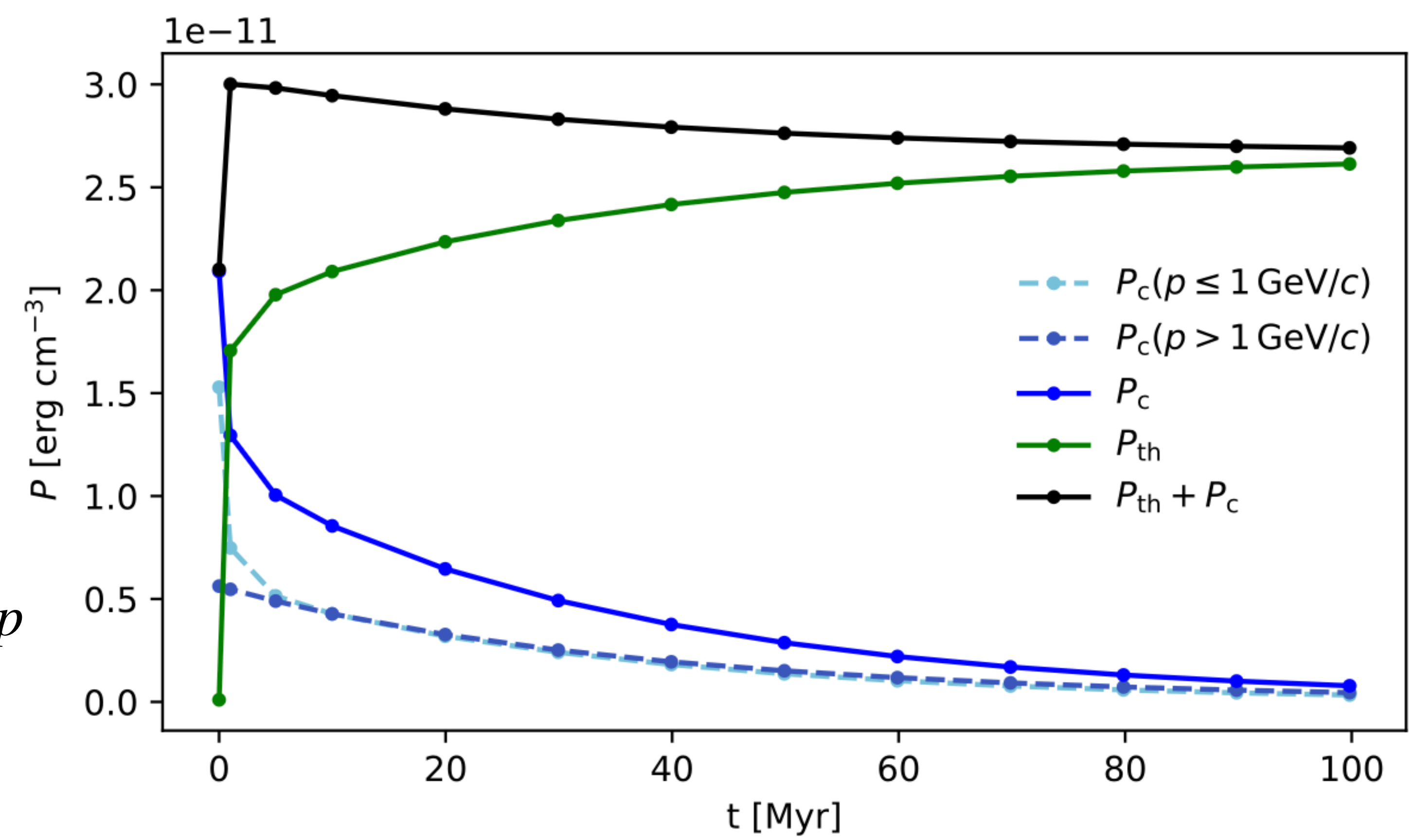
→ should conserve the initial slope of $f(p)$
→ 1/6th CR losses are returned to plasma

$$\frac{\partial n_i}{\partial t} = \left[4\pi p^2 L_{\mathrm{r}}(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}}$$

$$\frac{\partial e_i}{\partial t} = \left[4\pi p^2 L_{\mathrm{r}}(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} L_{\mathrm{r}}(p) f_0 \mathrm{d}p$$

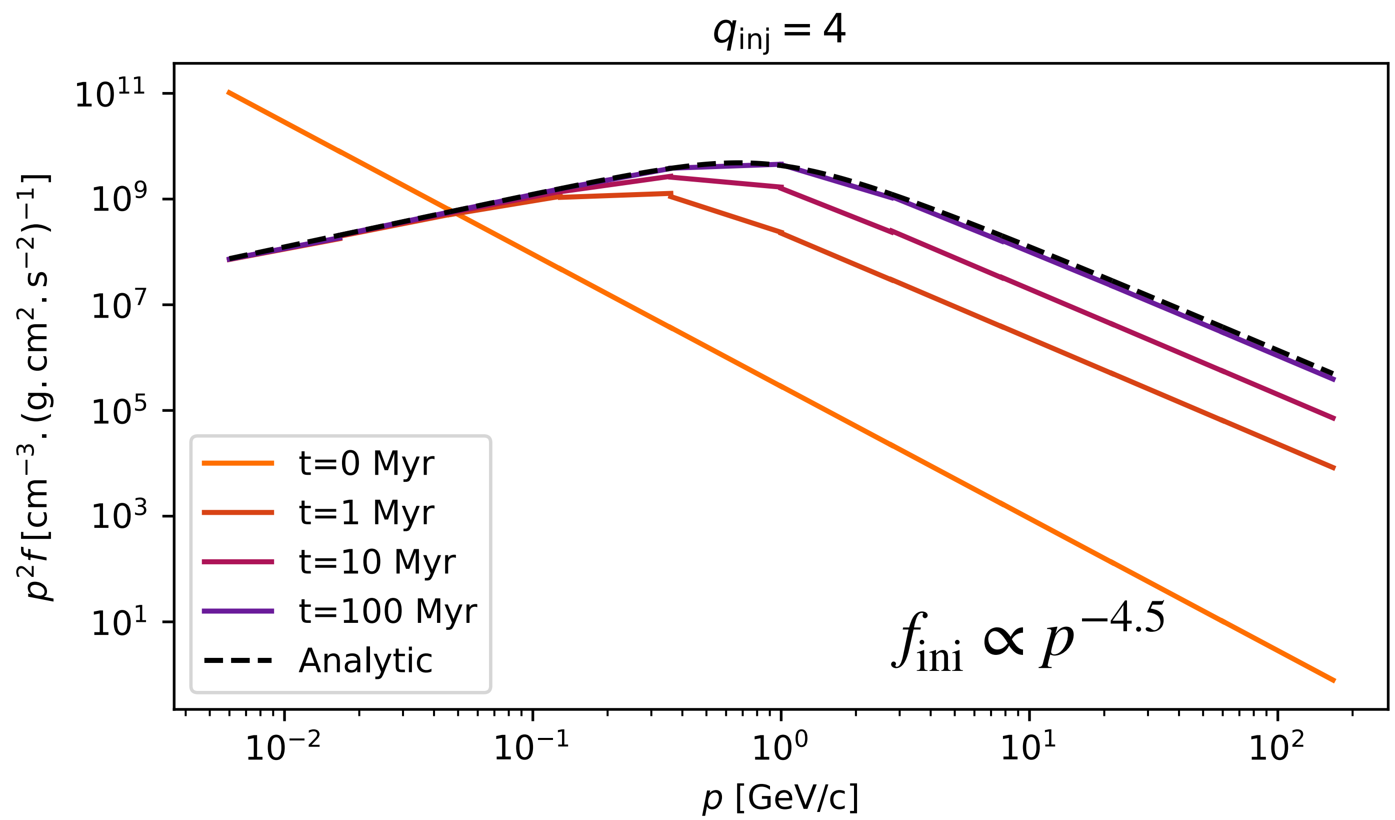
$$L_{\mathrm{r}} = L_{\mathrm{r,Coul}} + L_{\mathrm{r,hadr}}$$





Test: steady-state with cooling + injection

$$f_{\rm eq}(p) = \frac{Ap^{1-q_{\rm inj}}}{(q_{\rm inj}-3)L_{\rm r}(p)}$$

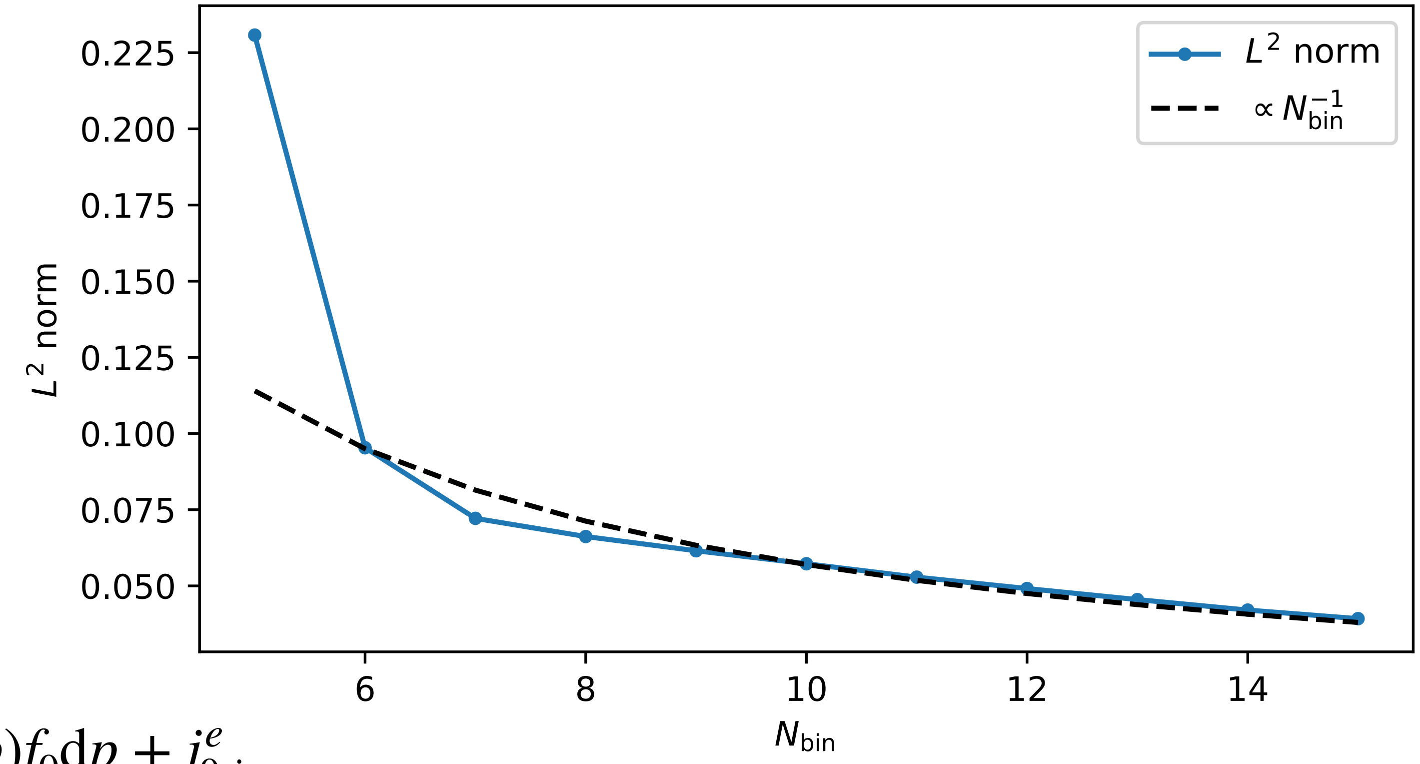


$L_{\rm r} = L_{\rm r,Coul} + L_{\rm r,hadr}$

$L_{\rm r,Coul} \propto p^{-2}$

$L_{\rm r,hadr} \propto p$

$j_0 = Ap^{-q_{\rm inj}}$



$$\frac{\partial n_i}{\partial t} = \left[4\pi p^2 L_{\rm r}(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} + j_{0,i}^n$$

$$\frac{\partial e_i}{\partial t} = \left[4\pi p^2 L_{\rm r}(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 {\rm v} L_{\rm r}(p) f_0 {\rm d}p + j_{0,i}^e$$

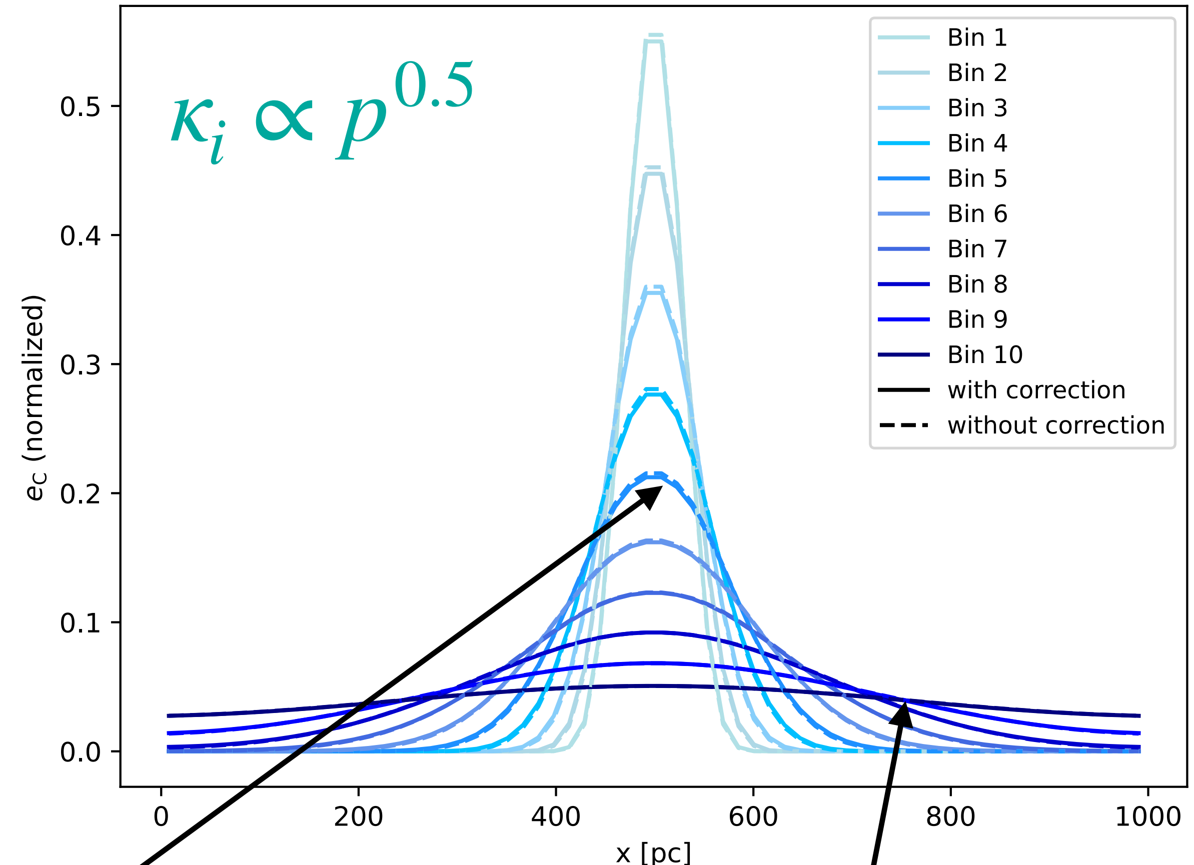
Test: 1D diffusion

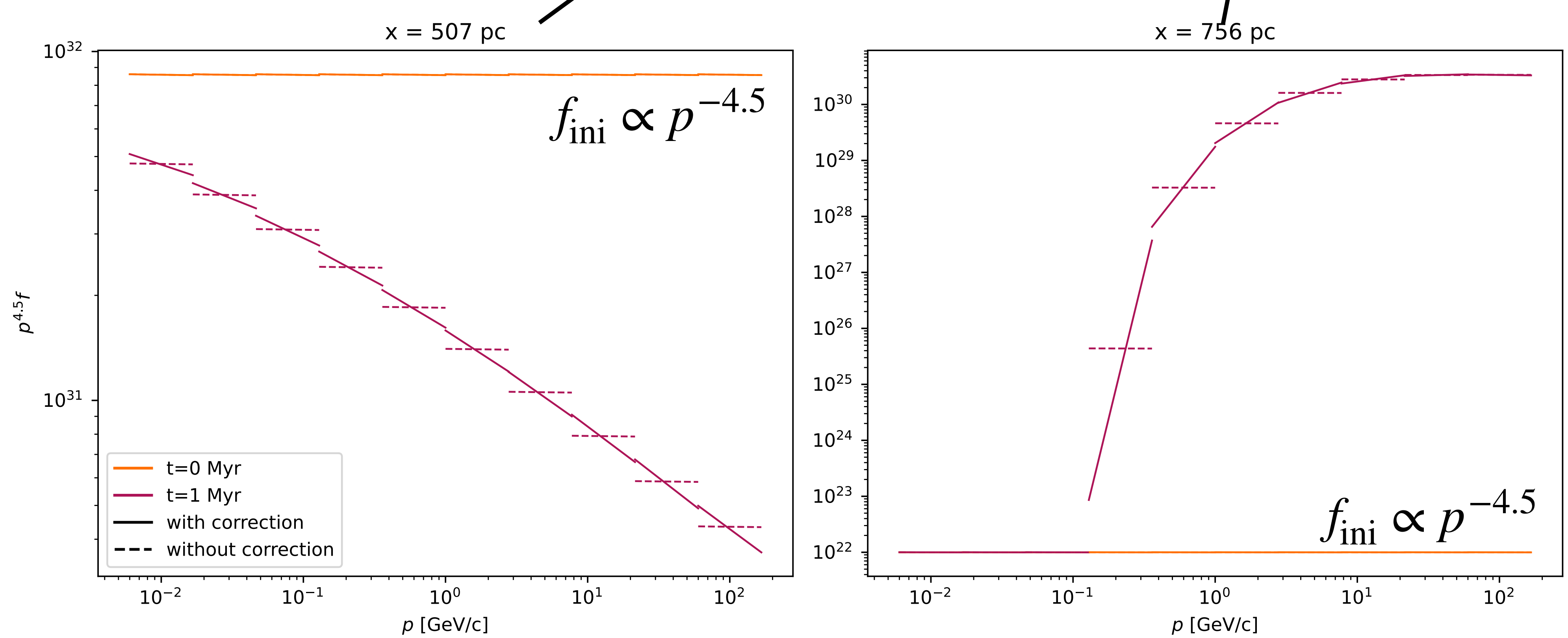
$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \,.\, \vec{F}_i^n = 0$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b} \,.\, \vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\vec{F}_i^n$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla} \,.\, \vec{F}_i^e = 0$$

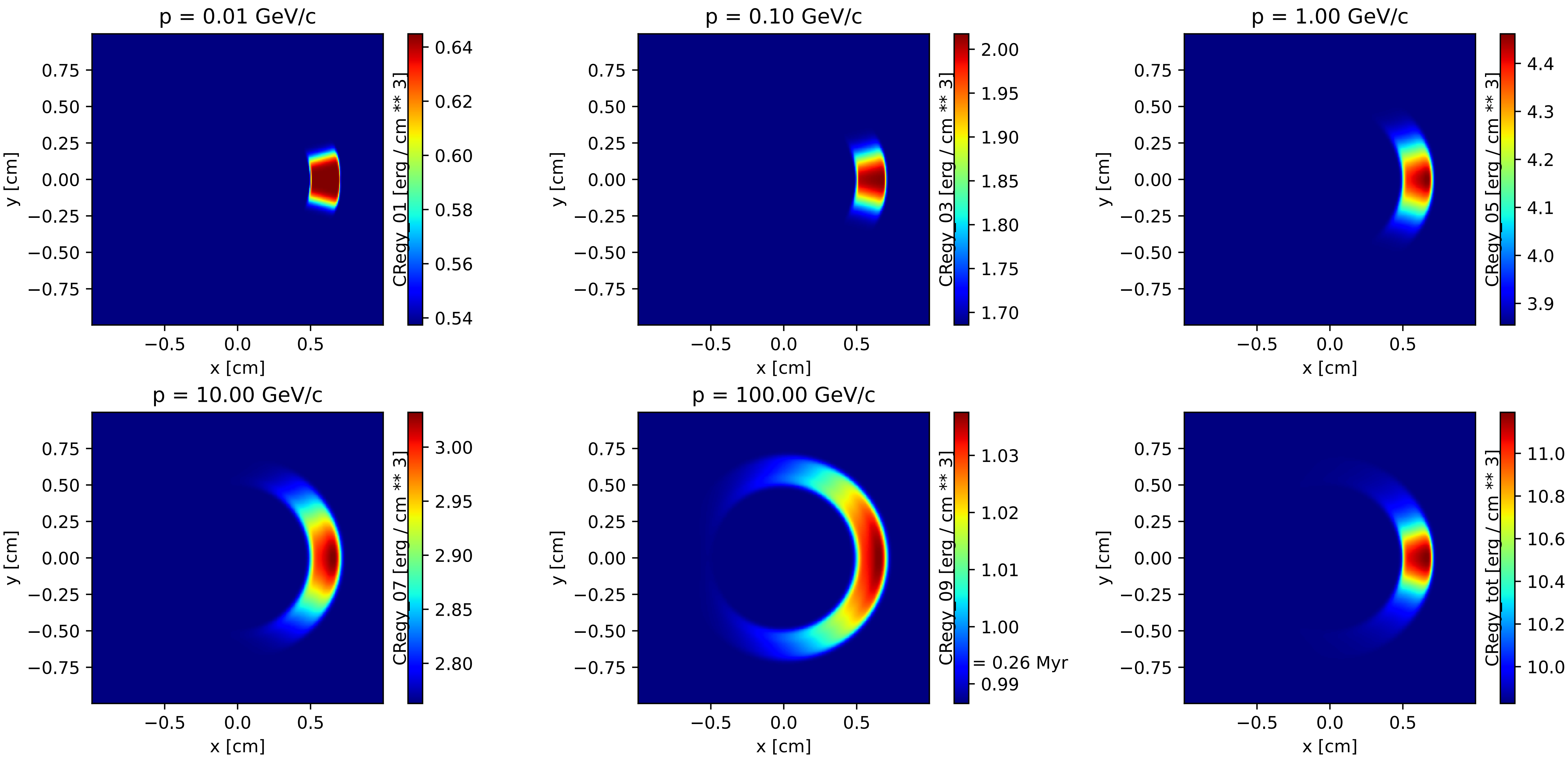
$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b} \,.\, \vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\vec{F}_i^e$$





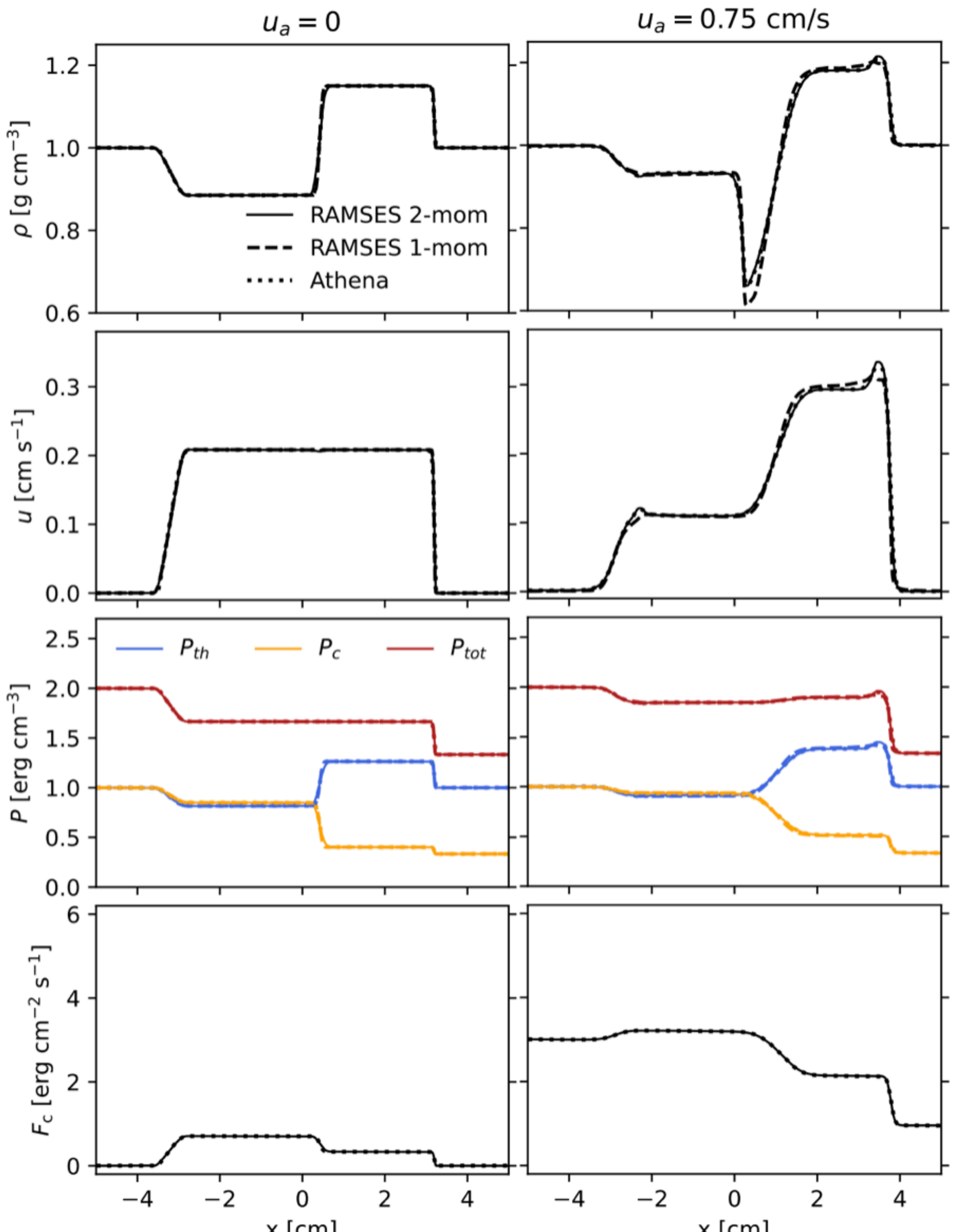
Test: 2D anisotropic diffusion

$$\kappa_i \propto p^{0.5}$$



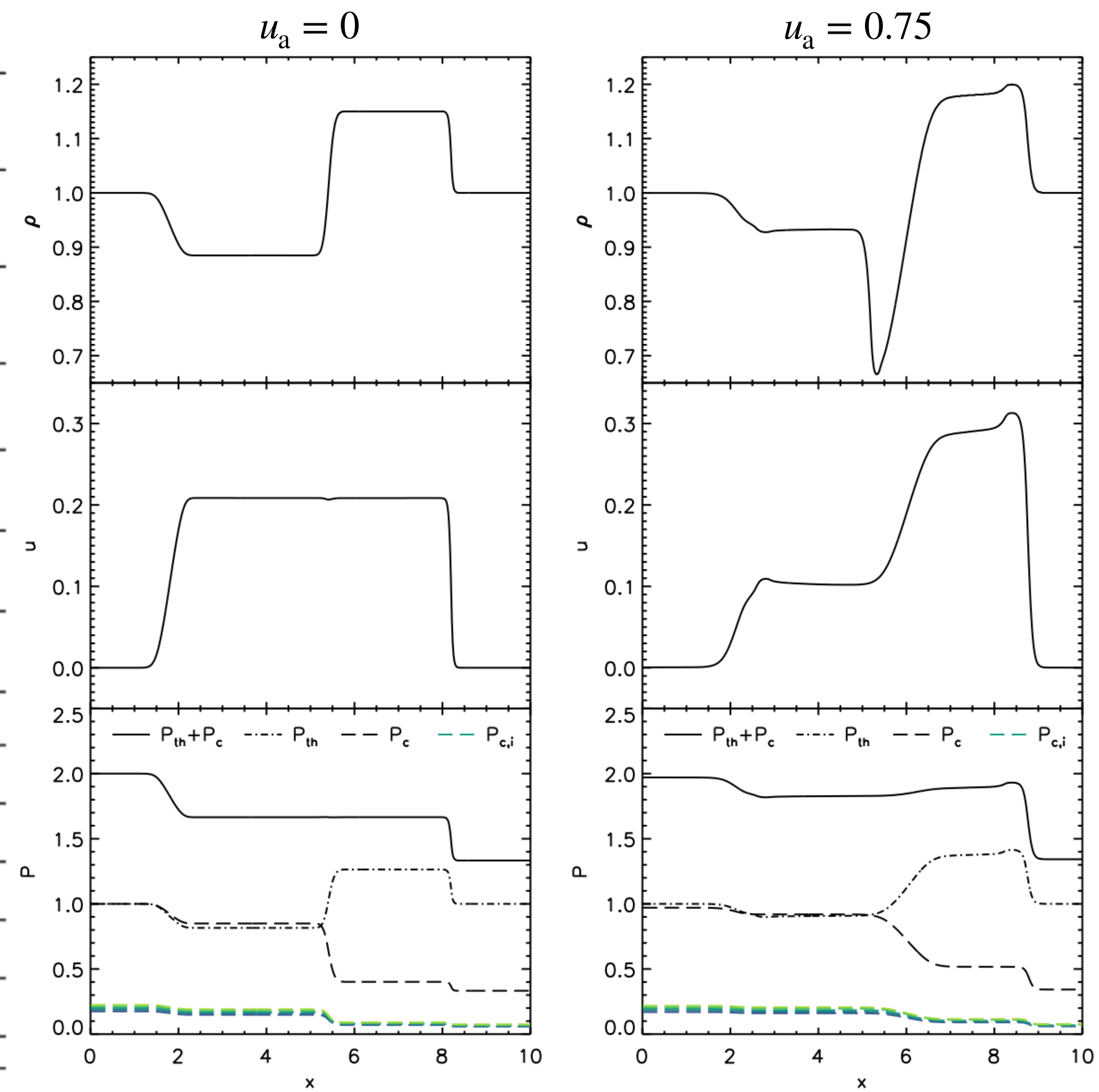
Test: Shock tube with streaming

RAMSES mono-group



Rosdahl+25

RAMSES multi-group

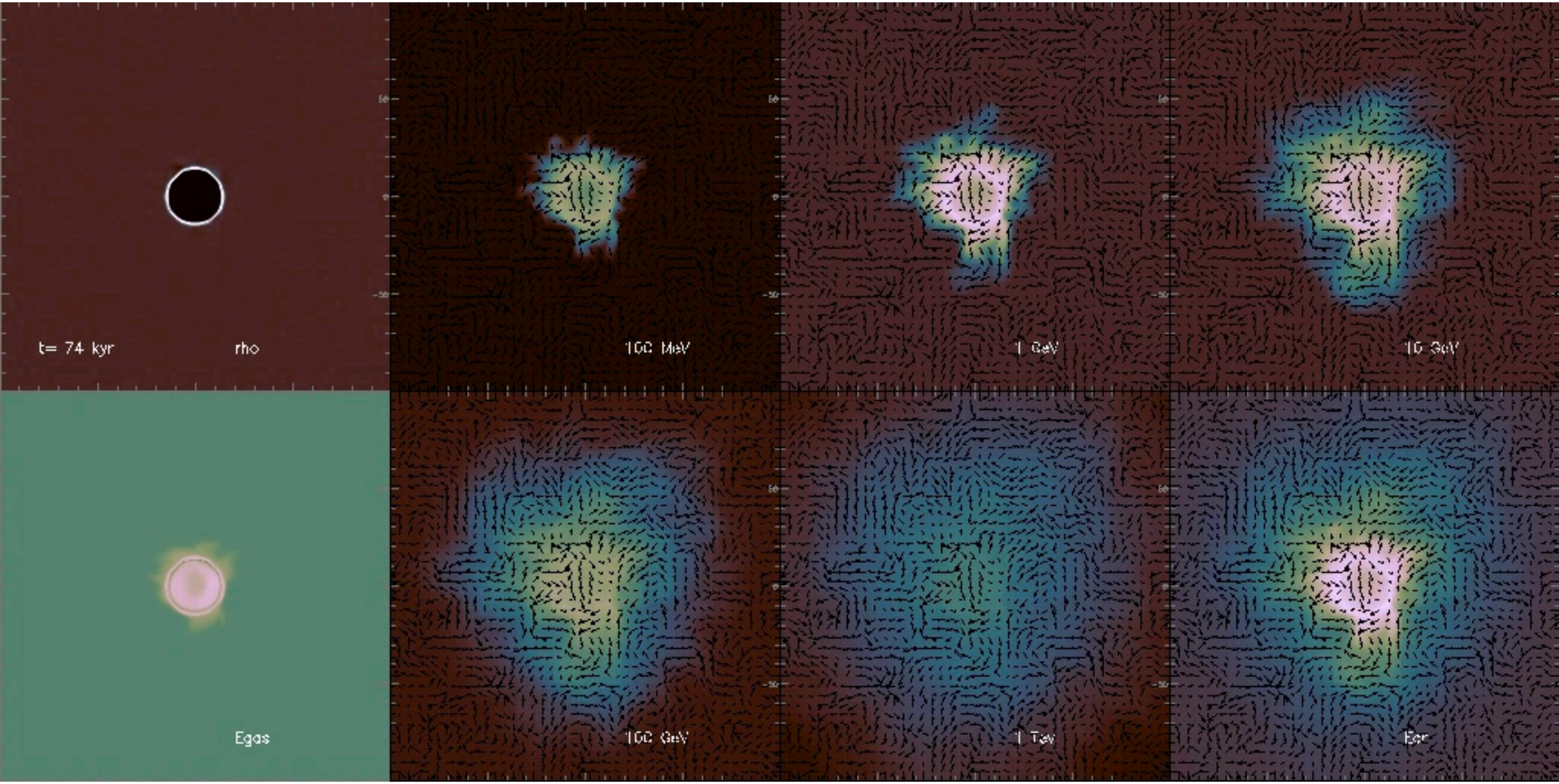


Diallo+26

Supernova Remnant with RAMSES-MCR

$E_{sn} = 10^{51}\,\mathrm{erg}$
$E_{sn,gas} = 9 \times 10^{50}\,\mathrm{erg}$
$E_{sn,cr} = 10^{50}\,\mathrm{erg}$

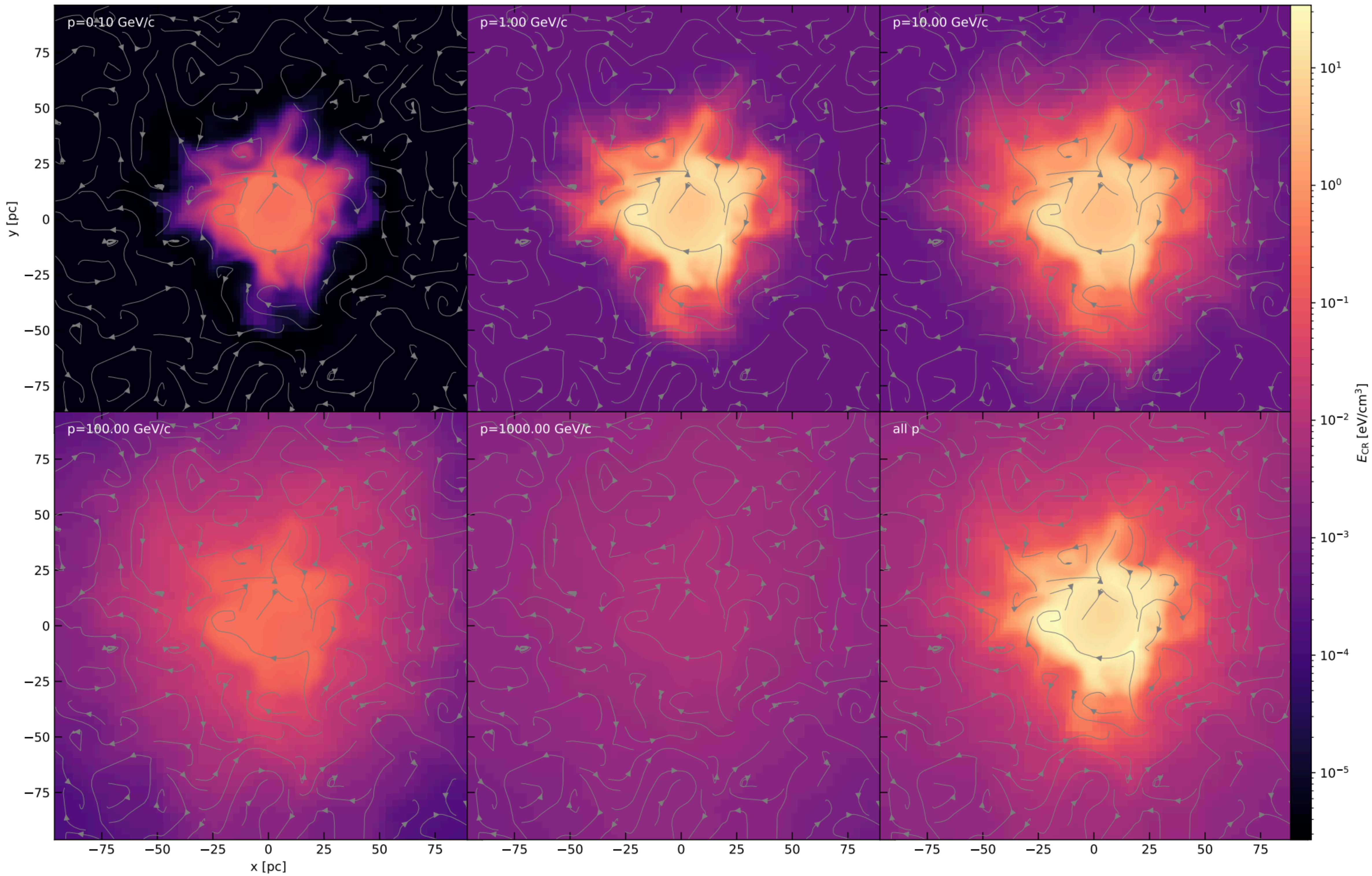
$f_{ini}(p) \propto p^{-4.5}$



Random B-field, gas and CR cooling, anisotropic diffusion

$$\kappa = 3 \times 10^{26} \left(\frac{p}{1\mathrm{GeV}} \right)^{0.5} \mathrm{cm}^2\,\mathrm{s}^{-1}$$

Supernova Remnant with RAMSES-MCR



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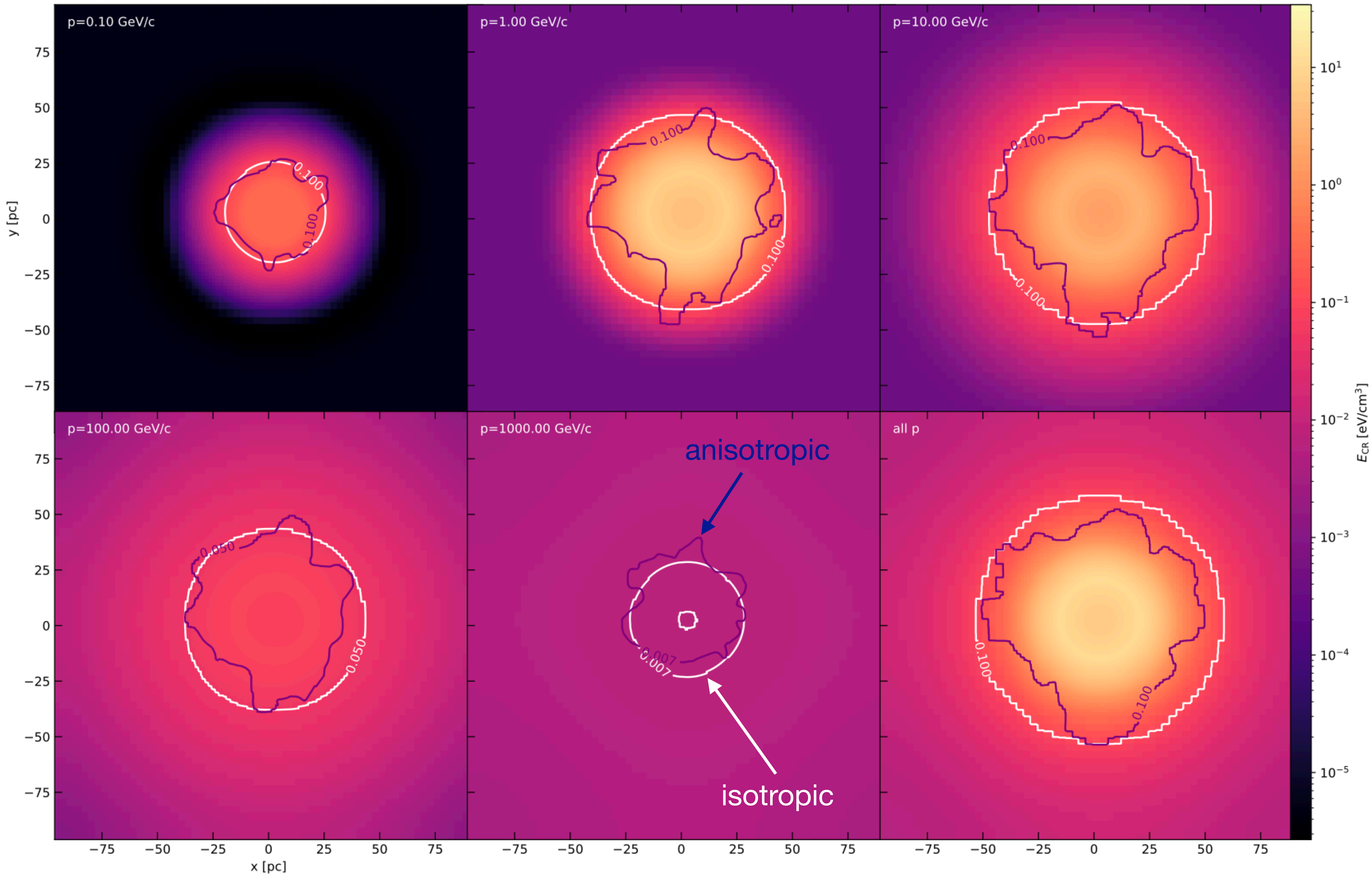
Supernova Remnant with RAMSES-MCR

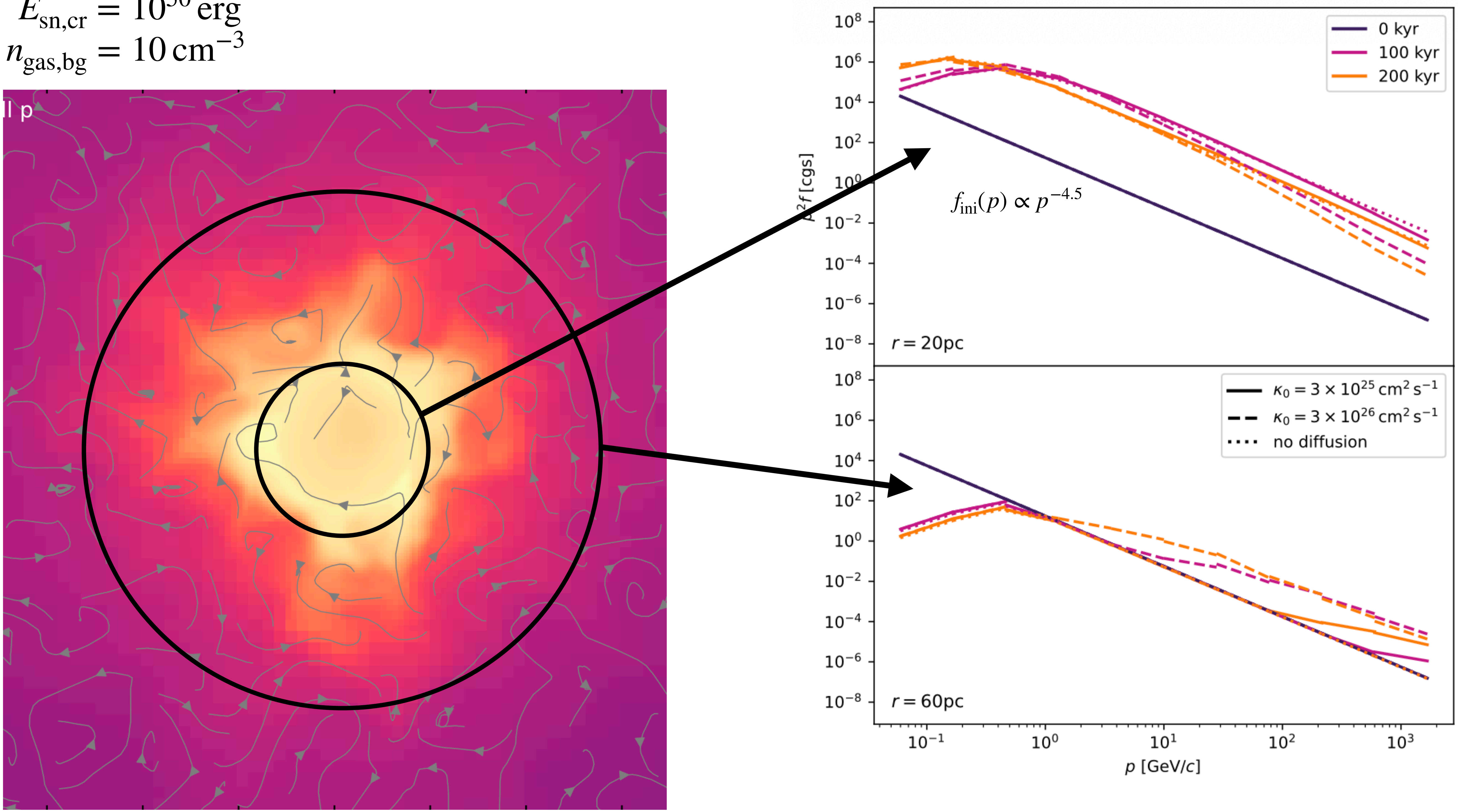


Random B-field, gas and CR cooling, ~~anisotropic~~ diffusion **isotropic!**

$$\kappa = 10^{26}\left(\frac{p}{1\mathrm{GeV}}\right)^{0.5}\ \mathrm{cm^2\,s^{-1}}$$

Supernova Remnant with RAMSES-MCR

$$E_{sn} = 10^{51}\,\mathrm{erg}$$
$$E_{sn,gas} = 9 \times 10^{50}\,\mathrm{erg}$$
$$E_{sn,cr} = 10^{50}\,\mathrm{erg}$$
$$n_{gas,bg} = 10\,\mathrm{cm}^{-3}$$



Random B-field, gas and CR cooling, anisotropic diffusion

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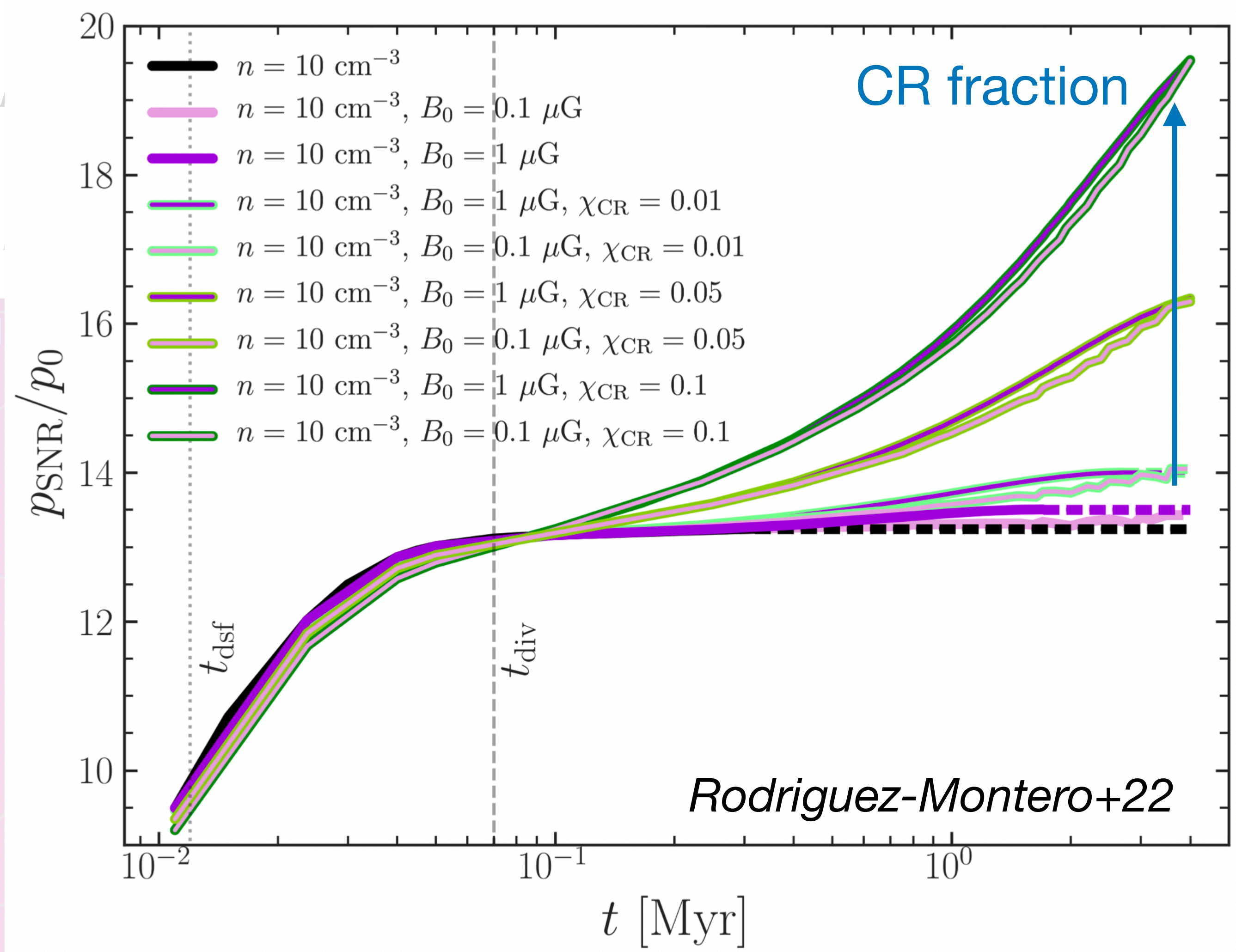
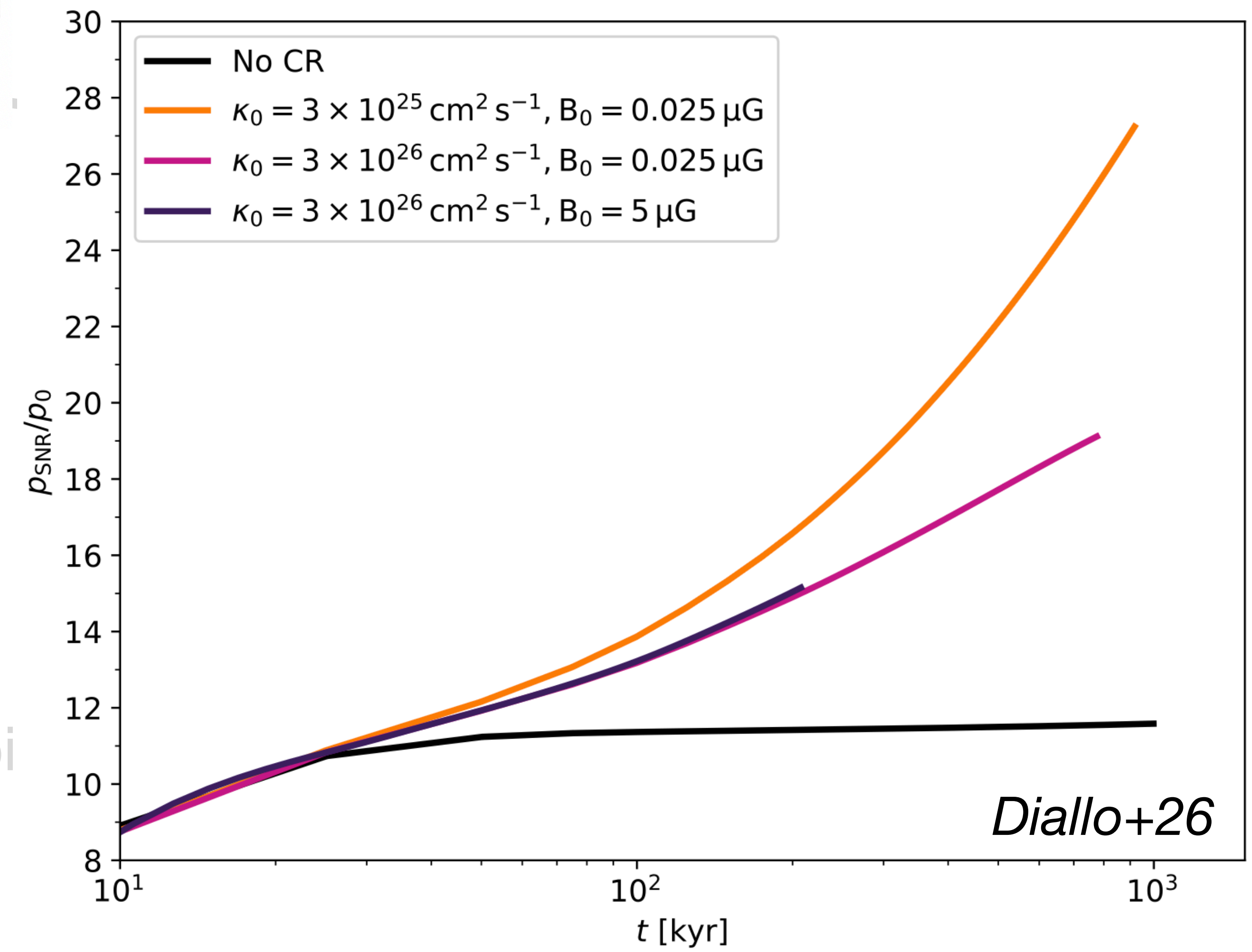
Supernova Remnant with RAMSES-MCR

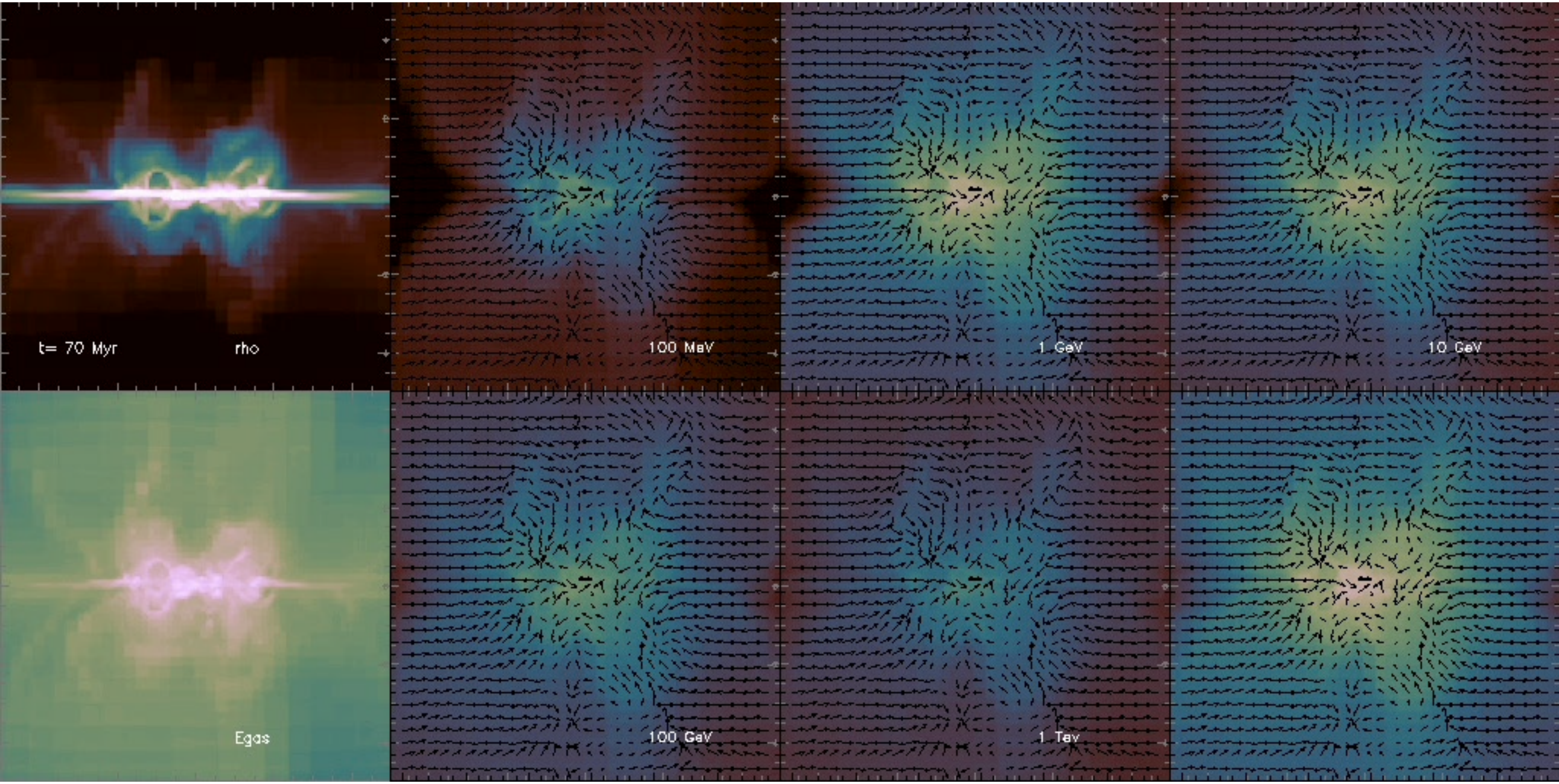




Random B-field, gas and CR cooling, anisotropi

$$\kappa = 3 \times 10^{26} \left(\frac{p}{1\mathrm{GeV}}\right)^{0.5} \mathrm{cm^2\,s^{-1}}$$

G8 with RAMSES-MCR



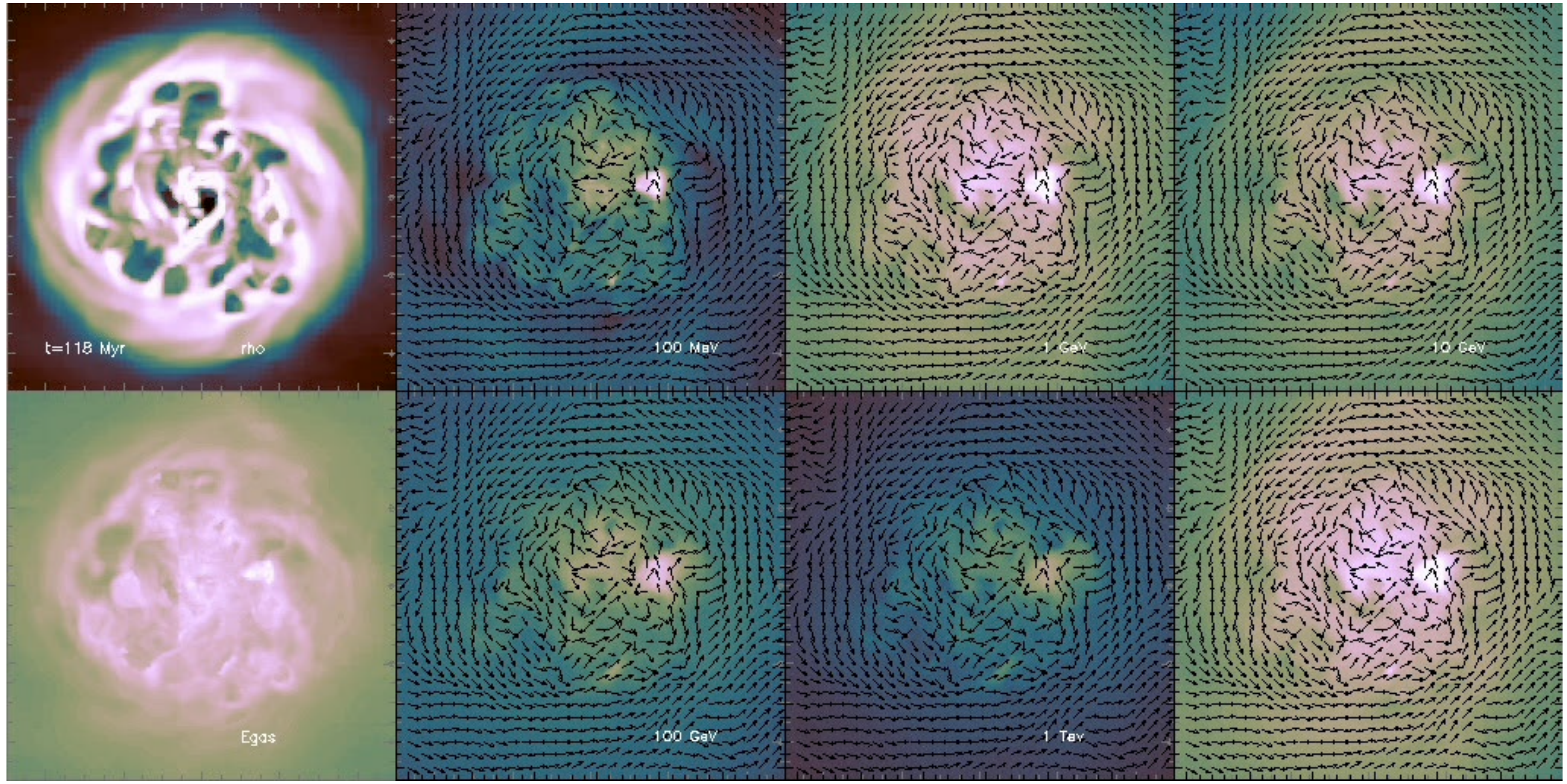
$M_h = 10^{10}\,M_\odot$ $\quad f_{gas} = 50\,\%$

40pc resolution, 10% SNII energy in CR

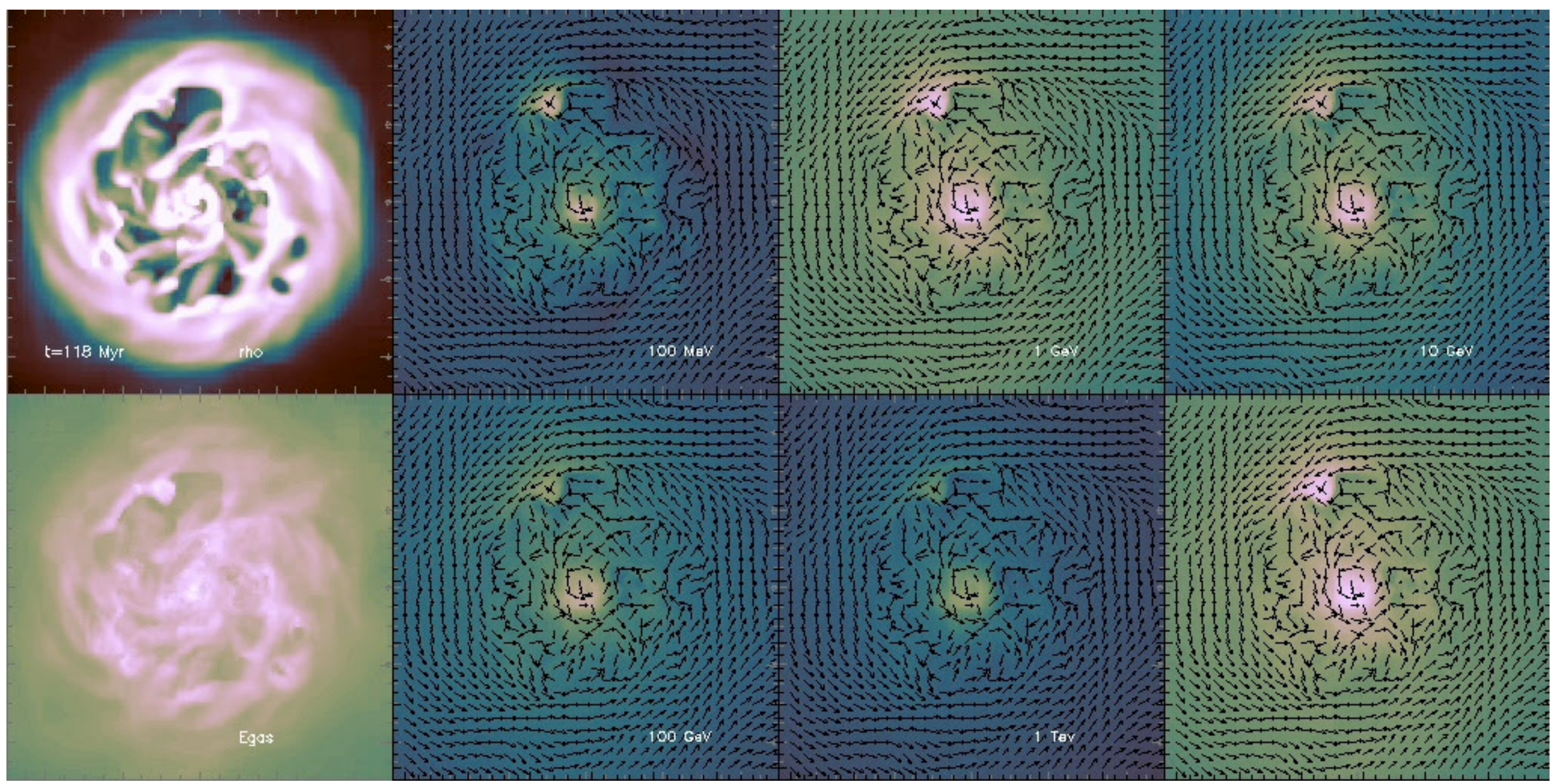
$$\kappa = 3 \times 10^{28} \left(\frac{p}{1\text{GeV}}\right)^{0.5}\ \text{cm}^2\,\text{s}^{-1}$$

10% of SN 10^{51} erg in e_c, q_{inj}=4.5, anisotropic diffusion, no streaming, Coulomb+ionisation+hadronic losses

$\kappa_{1\mathrm{GeV}} = 3 \times 10^{27}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$

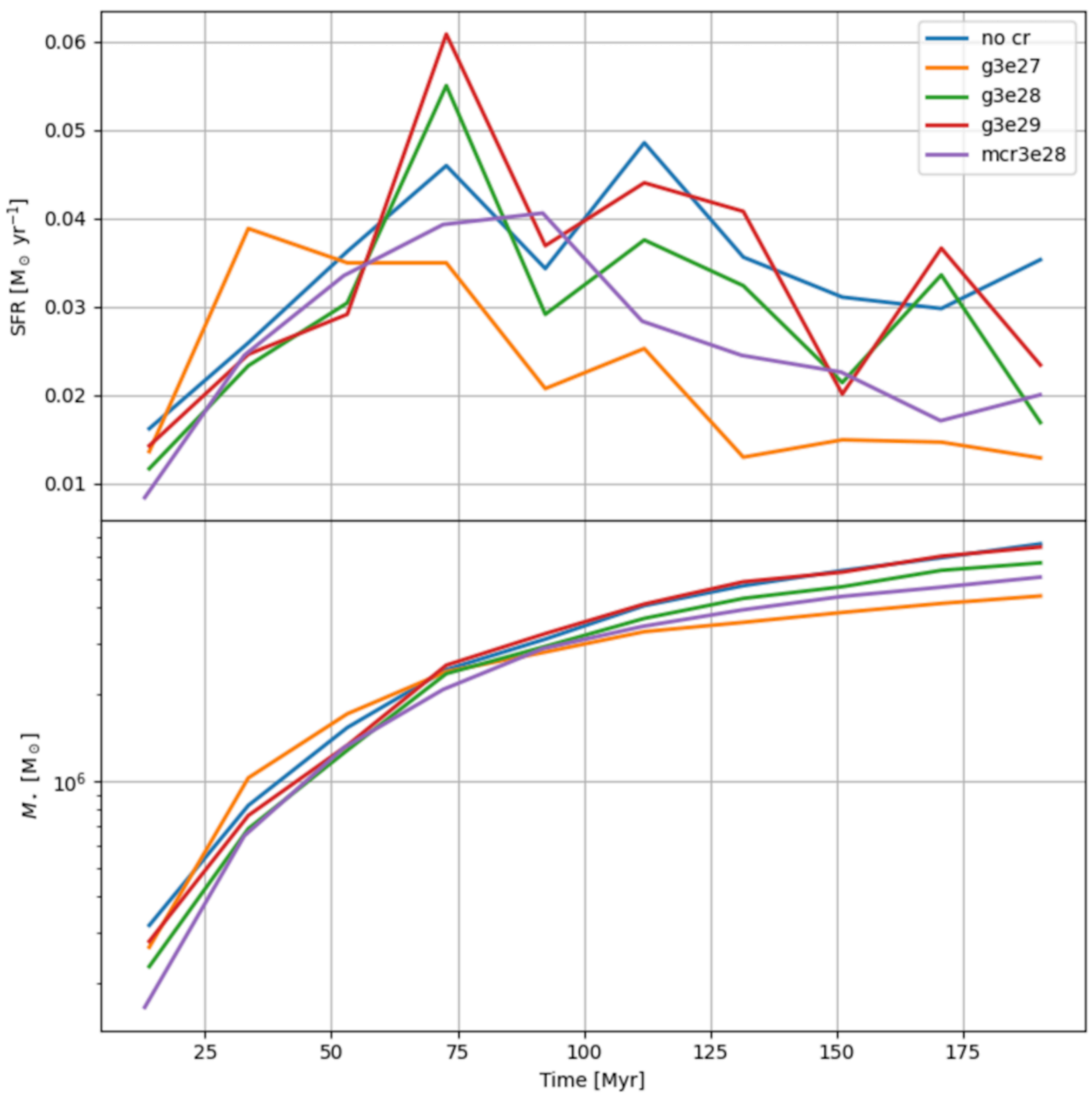


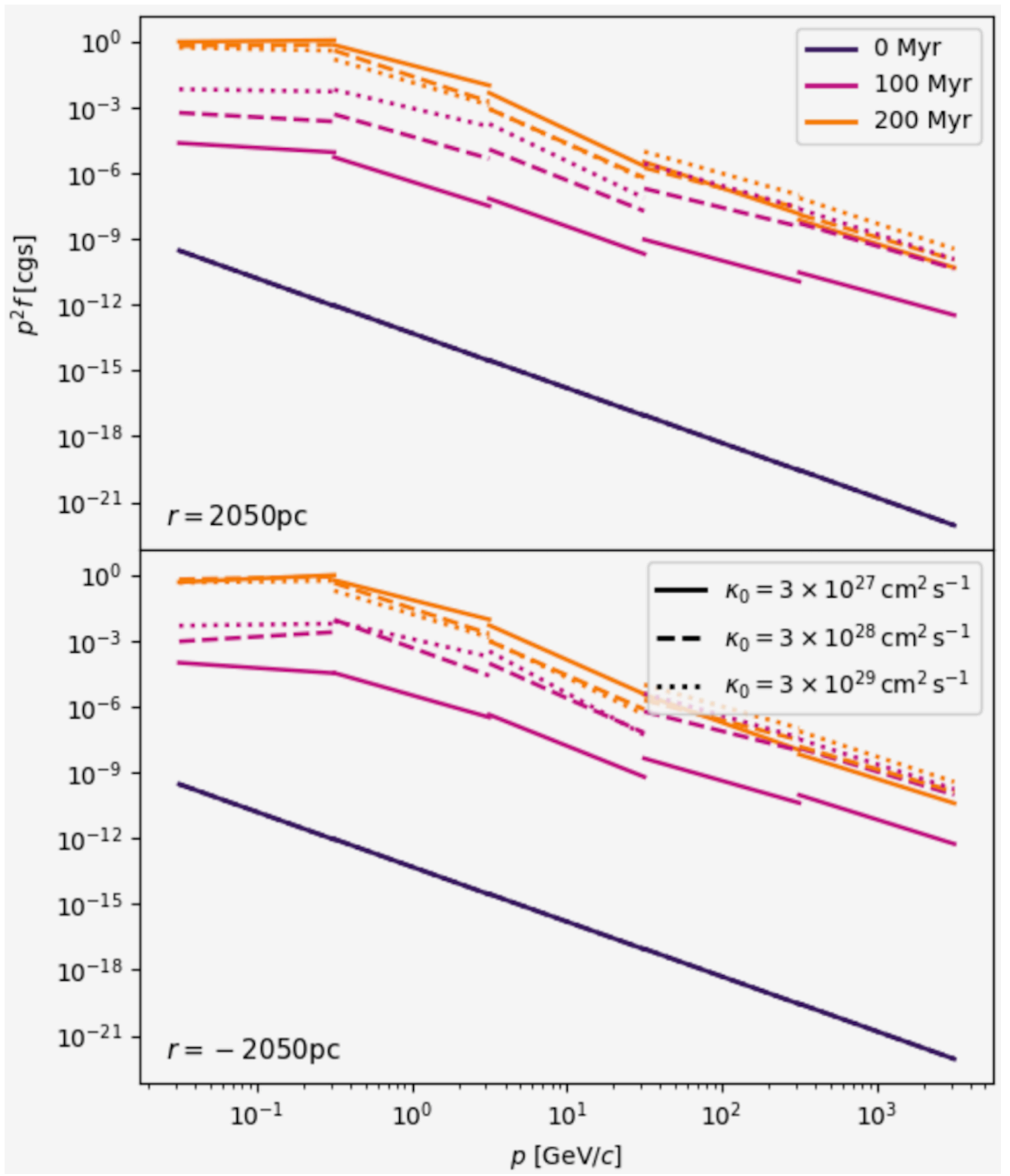
$\kappa_{1\mathrm{GeV}} = 3 \times 10^{28}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$



G8 with RAMSES-MCR

Preliminary results



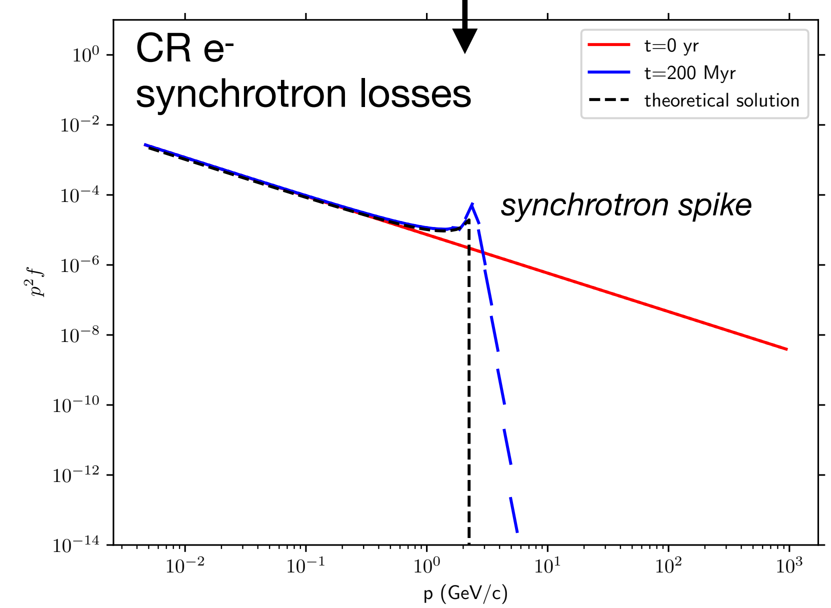


Conclusions/Perspectives

- CRMHD sims: CRs are important for galaxy feedback. **They agree at a qualitative level.** At a quantitive level: jury is still out → Depends on diffusion.

- CR transport is **multi-scale**: resolve ISM phases → better than 100pc res.
- CR transport is **energy-dependent**: multi-group methods → RAMSES-MCR

- RAMSES-MCR can be extended to multiple CR species (heavier nuclei, electrons, positrons)
  ➡Just need to change the mass of the particle and loss rates
  ➡We tested synchrotron losses for CR electrons



- **Galactic winds**
  ➡How does it affect large-scale wind properties?
  ➡Test various scaling of κ: SC, ET…

- Supernova remnants
  ➡shell dynamics modified?
  ➡x-ray, γ-ray emission?
- Interstellar medium
  ➡Diffuse re-acceleration by supersonic turbulence
  ➡MeV CR-driven ionisation losses
  ➡MeV production in stellar winds, jets, accretion shocks