

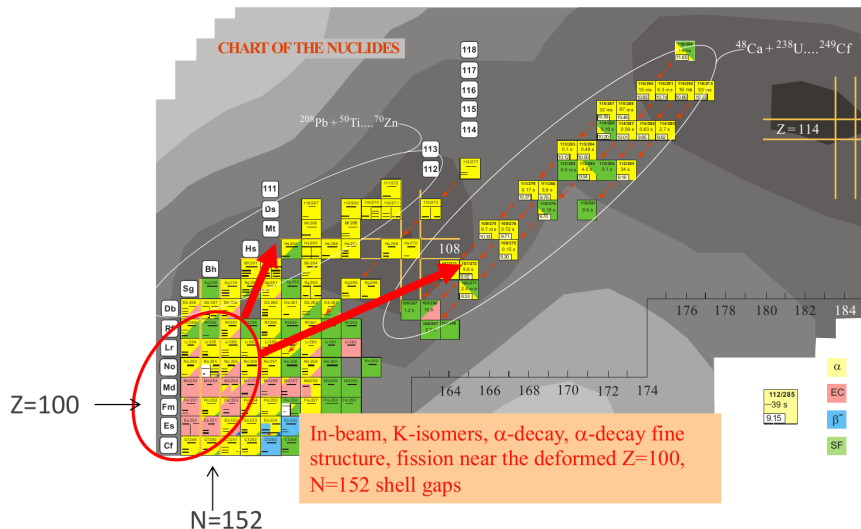
SHELL-MODEL : RECENT ADVANCES FROM RARE-EARTH TO SUPERHEAVY ELEMENTS

Duy-Duc Dao, F. Nowacki

24^{ème}
COLLOQUE GANIL
21 au 25 septembre 2026
Bussang, France



Landscape of superheavy nuclei



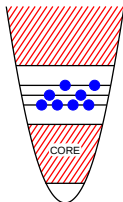
SM-CI approaches through the nuclear chart

◇ Nuclear Shell Model (SM) :

$$H_{\text{exact}}|\psi_{\text{exact}}\rangle = E|\psi_{\text{exact}}\rangle$$



$$H_{\text{eff}}|\psi_{\text{eff}}\rangle = E|\psi_{\text{eff}}\rangle$$



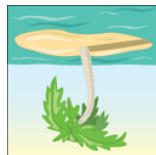
◇ Three Pillars of the Shell Model :

- A relevant valence space
- An associated effective interaction
- A “shell-model” code

(E. Caurier et al. Rev. Mod. Phys. 77, 427 (2005))

◇ Nuclear structure far from stability

- Deformation, Superdeformation
- Superfluidity, Dipole resonances
- Emergence of magic numbers
- Vanishing of shell closures

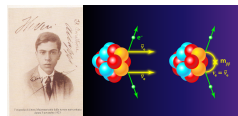


◇ Weak processes

- β decay
- double $\beta\beta$ decay

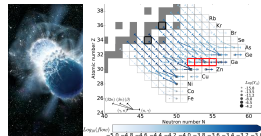
$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2$$

(nature of neutrinos)



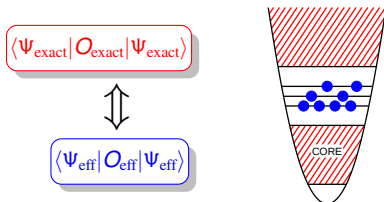
◇ Astrophysics and nucleosynthesis

- rp-process
- r-process



SM-CI approaches through the nuclear chart

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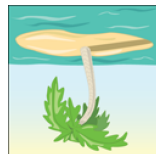
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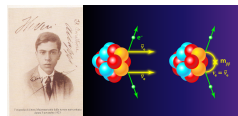


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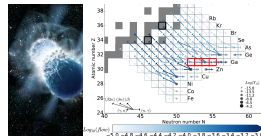
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- rp-process
- r-process



Diagonalization versus Variational Methods

Shell Model Exact Diagonalization

- Exponential growth of basis dimensions :

$$D \sim \begin{pmatrix} d_\pi \\ p \end{pmatrix} \cdot \begin{pmatrix} d_\nu \\ n \end{pmatrix}$$

In *pf* shell :

^{48}Cr 1,963,461

^{56}Ni 1,087,455,228

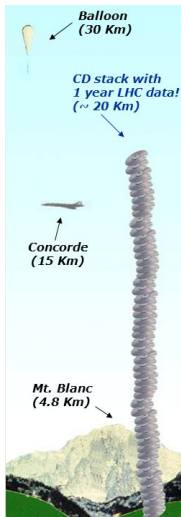
In *pf-sdg* space :

^{78}Ni 210,046,691,518

ANTOINE CODE (1989)

- Actual limits in giant diagonalizations :
 $\sim 10^{12}, \sim 10^{15} \neq 0$ matrix elements
- Some of the largest diagonalizations ever are performed in Strasbourg with relatively modest computational resources :
E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427

- m-scheme ANTOINE code
BIGSTICK
KHELL
- coupled scheme NATHAN code



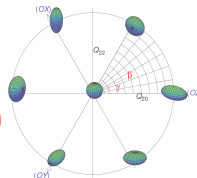
Variational Approximation

$$\mathcal{H}_{\text{eff}} |\Psi_\alpha^{JM}\rangle = E_\alpha^{(J)} |\Psi_\alpha^{JM}\rangle \rightarrow \delta \frac{\langle \Psi_\alpha^{JM} | \mathcal{H}_{\text{eff}} | \Psi_\alpha^{JM} \rangle}{\langle \Psi_\alpha^{JM} | \Psi_\alpha^{JM} \rangle} = 0$$

Mixing of shapes :

$$|\Psi_{\text{eff}}\rangle = \text{[shape 1]} + \text{[shape 2]} + \dots$$

Restoration of the rotational symmetry



Degree of freedom

- Multipole: $\beta, \gamma, Q_{30}, \dots$
- Cranking: J_x, J_z
- Pairing constraints
- Variation-After-Projection

Different implementations

- MCSM Tokyo group
- DNO-SM Strasbourg group
- PGCM Madrid group

Discrete Non-Orthogonal Shell Model (DNO-SM)

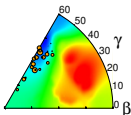
◇ Hill-Wheeler-Griffin Generator Coordinate Method :

- $\langle J_z \rangle$, (β, γ) (cranking, axial and triaxial)
- Selection of (β, γ) : Caurier technique
- Rotational symmetry restoration

$$|\Psi_{\text{eff}}\rangle = \text{[Diagram: Sum of three deformed spheres]} \dots$$

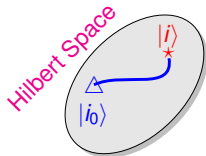
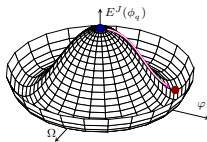
$$P_{\alpha}^{(J)}(q) = \sum_K |M_{\alpha}^{(J)}(q, K)|^2$$

(D.D. Dao and F. Nowacki, PRC 105, 054314 (2022))



◇ Variation after Projection :

- $\frac{\delta E^J}{\delta \phi_q^*} = \frac{\delta}{\delta \phi_q^*} \sum_{KK'} f_q^{*J}(K) f_{q'}^J(K') \langle \phi_q | \hat{H} P_{KK'}^J | \phi_{q'} \rangle$
- $|\phi_q\rangle$: Slater or Bogoliubov wavefunctions
- $J_{\alpha}^{\pi} = 0_1^{+}, 2_1^{-}, \dots$
- Intrinsic states generated consistent with the symmetries : rotation, particle number



◇ Interplay of deformation/pairing correlations

- Missing correlations energies
- Rotational spectra
- Reduction of collectivity

LNPS effective interaction

^{62}Cr	(β, γ)	DNO-SM(VAP)	SM
$E(0_1^{+})$ (MeV)	-212.426	-214.138	-214.138
$E^{*}(2_1^{+})$ (MeV)	0.288	0.555	0.544
$B(E2, 2_1^{+} \rightarrow 0_1^{+})$ (e^2fm^4)	458	378	362

Discrete Non-Orthogonal Shell Model (DNO-SM)

◇ Hill-Wheeler-Griffin Generator Coordinate Method :

◇ Variation after Projection :

δE^J δ

PHYSICAL REVIEW C **105**, 054314 (2022)

Nuclear structure within a discrete nonorthogonal shell model approach: New frontiers

D. D. Dao  and F. Nowacki 

Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France



(Received 8 March 2022; accepted 6 May 2022; published 23 May 2022)

PHYSICAL REVIEW C **114**, 014327 (2026)

Editors' Suggestion

Exact solutions of the nuclear shell-model secular problem: Discrete nonorthogonal shell model within a variation-after-projection approach

Duy Duc Dao * and Frédéric Nowacki 

(e^{-1m})

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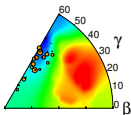
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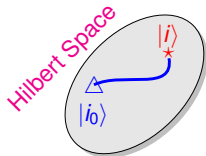
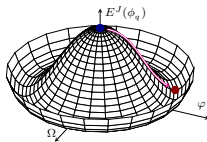
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(D.D. Dao and

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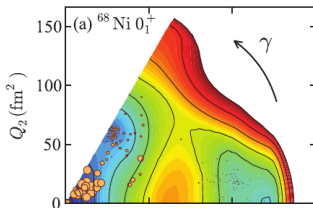
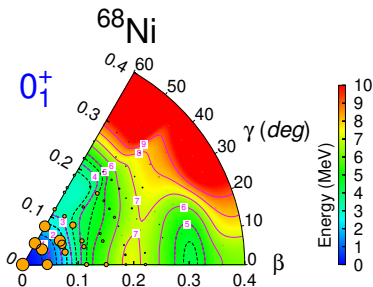
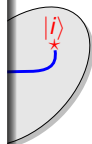
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functions

ent with

number



DNO-SM vs MCSM

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Effective Hamiltonian

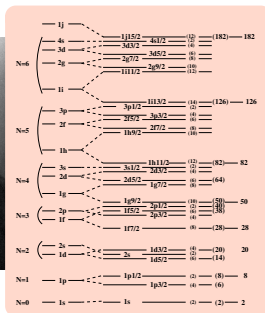
Monopole and multipole

Multipole expansion : $\mathcal{H} = \mathcal{H}_{monopole} + \mathcal{H}_{Multipole}$

- *Spherical mean-field*

$\mathcal{H}_{monopole}$: • *Evolution of the spherical single particle levels*

A. Poves and A. Zuker (Phys. Report 70, 235 (1981))



- *Correlations*
- *Energy gains*

$\mathcal{H}_{multipole}$:

- *Pairing (SU2)*
- *Quadrupole (SU3/pSU3/qSU3)*

semi-magic (n-n) (p-p)

p-n in H.O. or $\Delta j = 2$

M. Dufour and A. Zuker (PRC 54 1996 1641)

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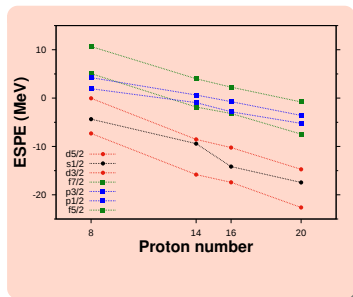
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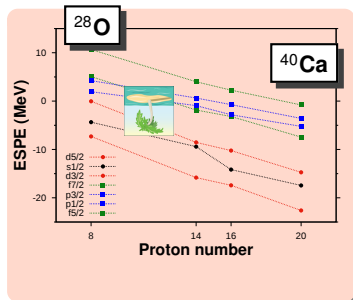
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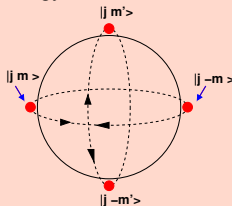
Effective Hamiltonian

Monopole and multipole

Multipole

- **Pairing regime : spherical nuclei**

ground state = pairs of like-particles coupled at $J=0$ (seniority $\nu=0$)
 2^+ state (break of pair ; $\nu=2$) at high energy

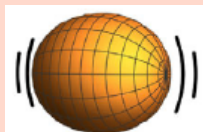


superfluid nucleus :

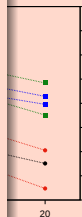
Typical example : **semi-magic Tin isotopes**

- **Quadrupole regime : deformed nuclei**

prolate nucleus :



Typical example : **open shell N=Z nuclei**



Effective Hamiltonian

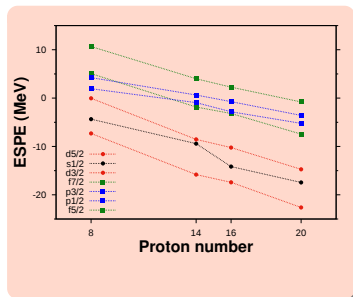
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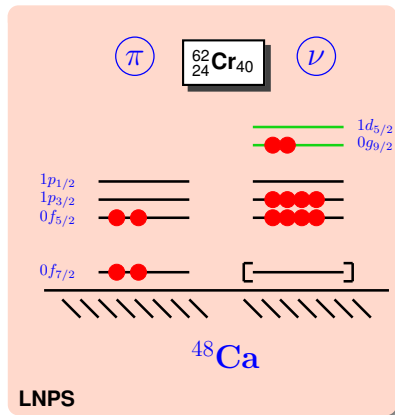
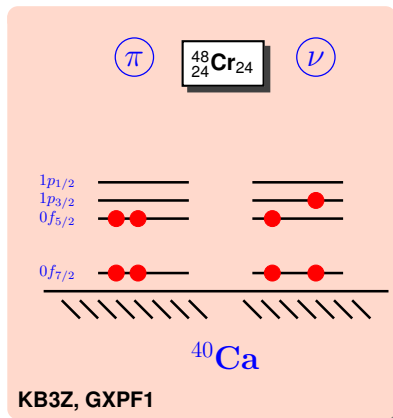
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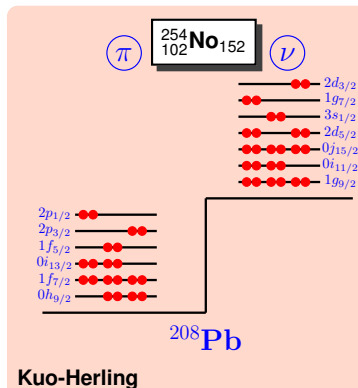
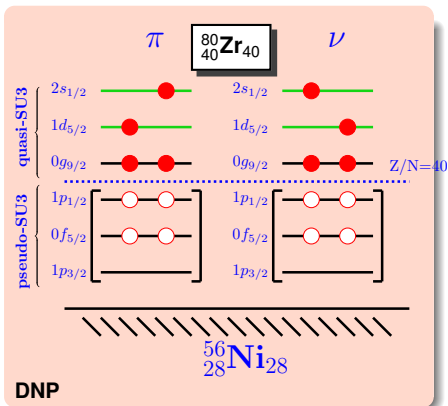
Choice of valence space

- Classical $0\hbar\omega$ model spaces like sd or pf -shell are successful for the description of a low lying states of nuclei, their transition rates and weak-decays.
- Neutron rich systems invoke asymmetric proton and neutron spaces

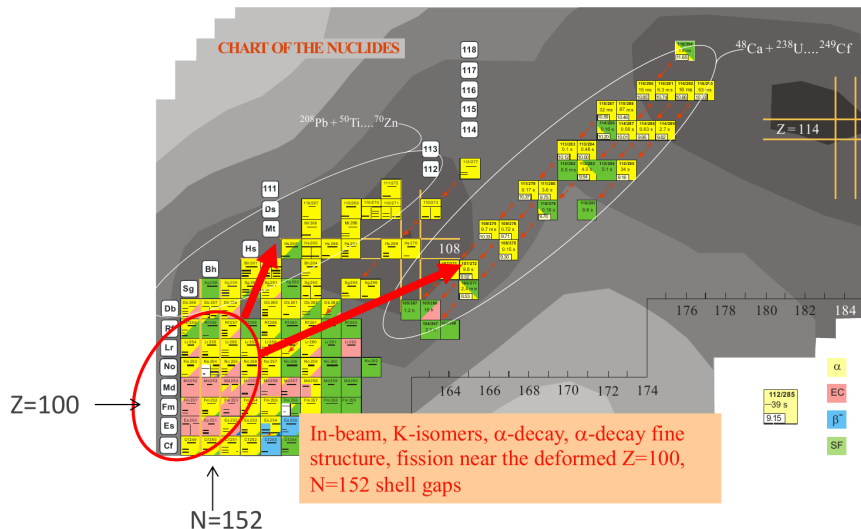


Choice of valence space

- Deformation/Superdeformation require enlarged valences spaces driven by underlying algebraic symmetries
(Duc's talk on Monday)
- Rare-earth region, Superheavy systems invoke also asymmetric proton and neutron and algebraic symmetries driven valence spaces
(Waely's talk on Tuesday)



Landscape of superheavy nuclei



In-beam, K-isomers, α -decay, α -decay fine structure, fission near the deformed $Z=100$, $N=152$ shell gaps

(From Seweryniak (SSNET 2024))

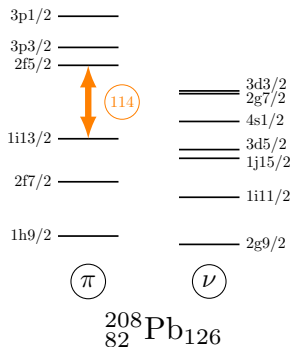
Applications to superheavy nuclei

Kuo-Herling interaction :

- $^{208}_{82}\text{Pb}_{126}$ core, realistic TBMEs
- $82 \leq Z \leq 126$ shells for proton and $126 \leq N \leq 184$ for neutrons
- monopole corrections (3N force)

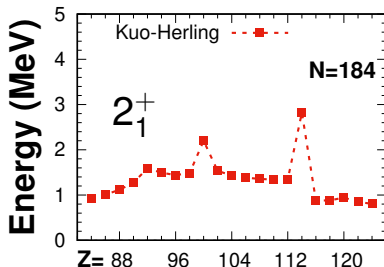
E. Caurier and F. Nowacki,

PRL 87 (2001),072511



Calculations : NATHAN & CARINA codes

- ◊ Diagonalization within the seniority scheme along the chains of $N = 126$ and $N = 184$
- ◊ Variation After Projection calculations : $^{253,254,255}\text{Es}$, $^{249,251,253}\text{Cf}$, $^{253,254}\text{No}$, ^{256}Fm
- ◊ Comparison of spectra and electromagnetic moments



Yrast systematics : odd-even $^{251}_{98}\text{Cf}_{153}$

PHYSICAL REVIEW C **114**, L021302 (2026)

Letter

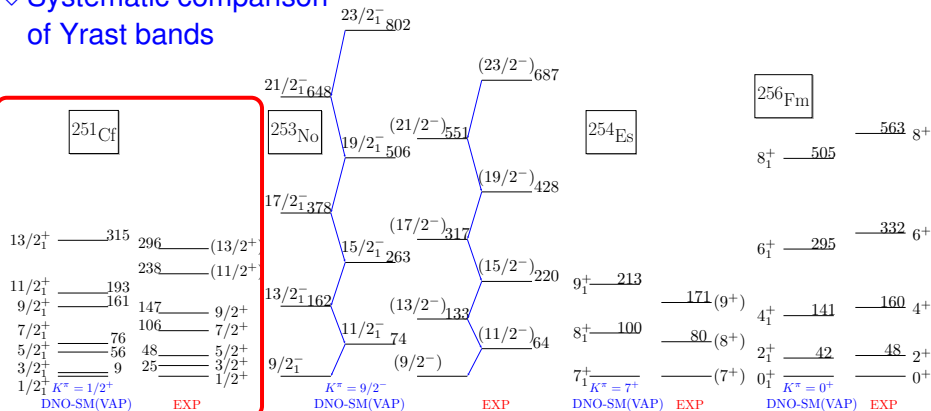
First complete shell-model description of low-lying spectroscopy in ^{254}No

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(Received 24 September 2024; revised 26 May 2026; accepted 15 July 2026; published 13 August 2026)

Systematic comparison of Yrast bands



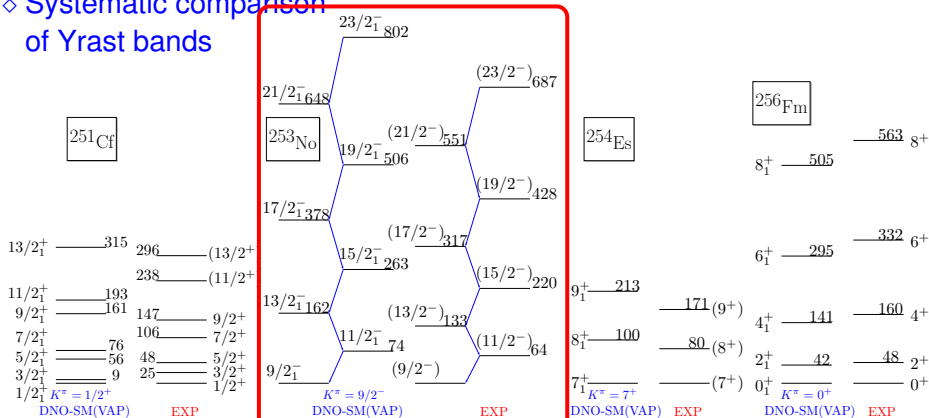
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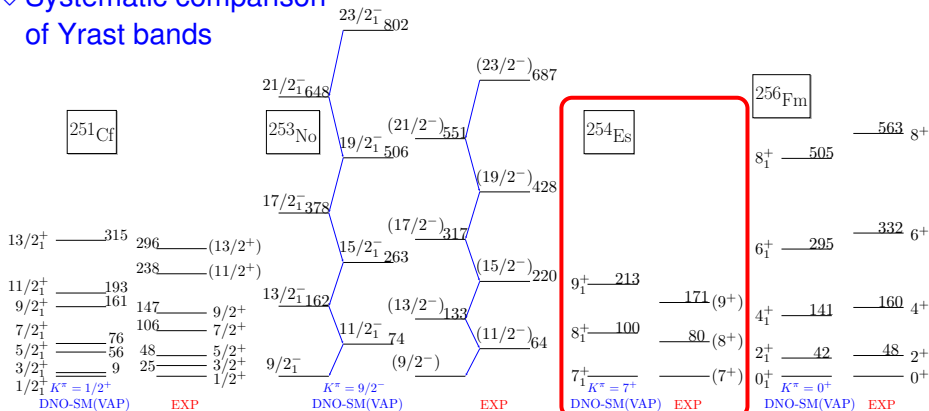
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Yrast systematics : even-even $^{256}_{100}\text{Fm}_{156}$

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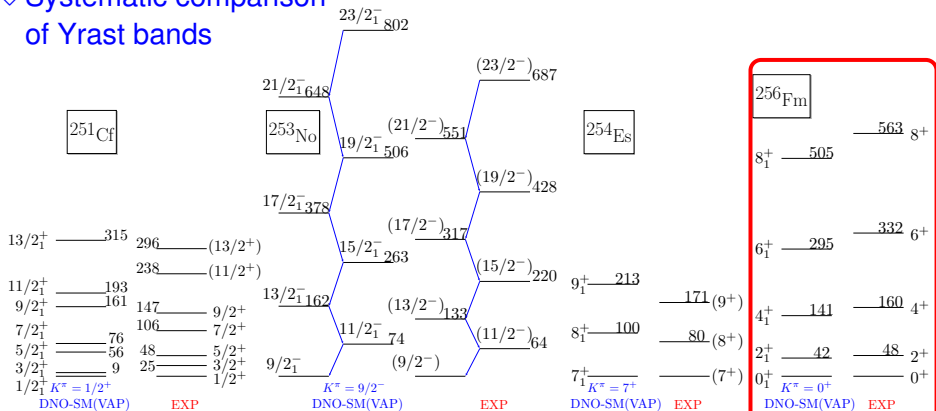
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Applications to superheavy nuclei : Electromagnetic moments

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◇ Comparison of magnetic and quadrupole moments

Effective charges : $e_p = 1.72$, $e_n = 0.75$

J^π_{gs}	E (MeV)			$\mu(\mu_N)$			Q_s (eb)		
		PHF	VAP	PHF	VAP	EXP	PHF	VAP	EXP
^{253}No 9/2 $^-$		-241.818	-242.816	+0.591	-0.493	-0.527(33)(75)	-3.5	+7.2	+5.9(1.4)(0.9)
^{253}Cf 7/2 $^+$		-253.402	-253.818	-0.677	-0.556	-0.731(35)	+5.77	+5.78	+5.53(51)
^{251}Cf 1/2 $^+$		-241.321	-241.724	-0.727	-0.610	-0.571(24)	-	-	-
^{249}Cf 9/2 $^-$		-229.021	-229.381	-0.480	-0.461	-0.395(17)	+6.62	+6.63	+6.27(33)
^{255}Es 7/2 $^+$		-263.512	-264.695	-1.10	+3.94	+4.14(10)	+6.0	+5.8	+5.1(1.7)
^{254}Es 7 $^+$		-257.492	-258.441	+0.778	+3.36	+3.42(7)	+1.8	+8.4	+9.6(1.2)
^{253}Es 7/2 $^+$		-251.837	-252.280	+3.63	+3.93	+4.10(7)	+5.87	+5.9	+6.7(8)
^{256}Fm 0 $^+$		-268.999	-269.717	+0.87	+0.89	-	-3.57	-3.60	-
^{254}No 0 $^+$		-249.568	-250.187	+0.87	+0.91	-	-3.78	-3.75	-

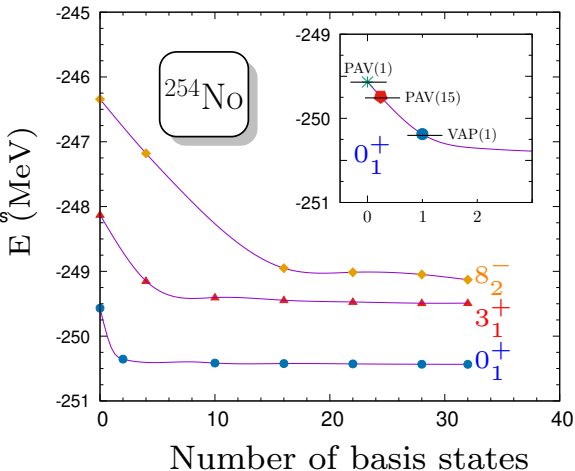
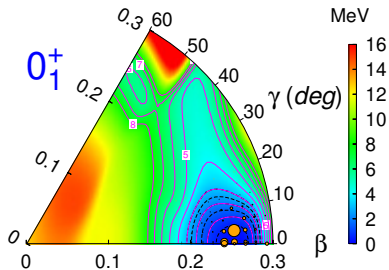
Applications to superheavy nuclei : $^{254}_{102}\text{No}$

Known spectroscopy :

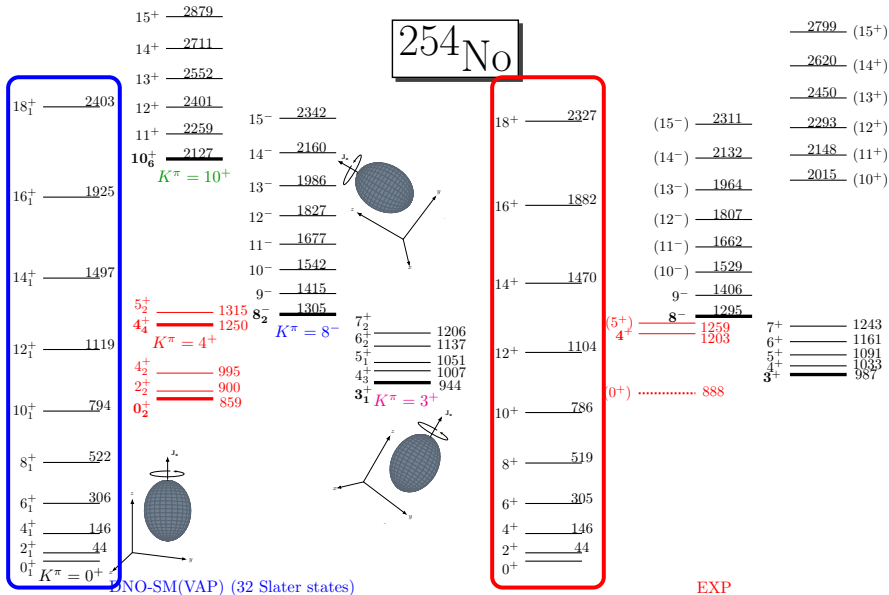
- Axially well-deformed nucleus
- Ground-state rotational band
- Several isomeric states (8^- , 3^+ , 10^+)

Calculations :

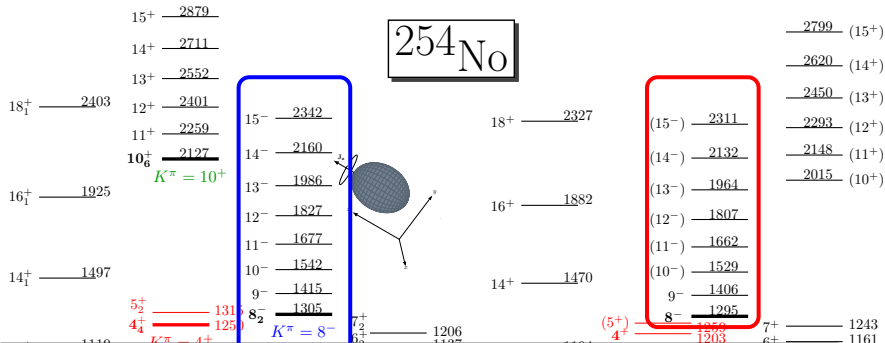
- Deformed Axial Minimum at $\beta = 0.25$
- Mixing of triaxial configurations (β, γ) (PAV(15))
- VAP calculations for band-head structures



Reproduction of spectra : Yrast rotational band



Isomeric $8^- (K = 8)$ band



8_2^-	Calc.	EXP.
E^* (keV)	1305	1295 ^a

(^a M. Forge *et al.*, Phys. Rev. C114, 0243024 (2026))

Single-particle configurations :

~ Two-neutron excitations

$$\nu j15/2 \otimes \nu g9/2$$

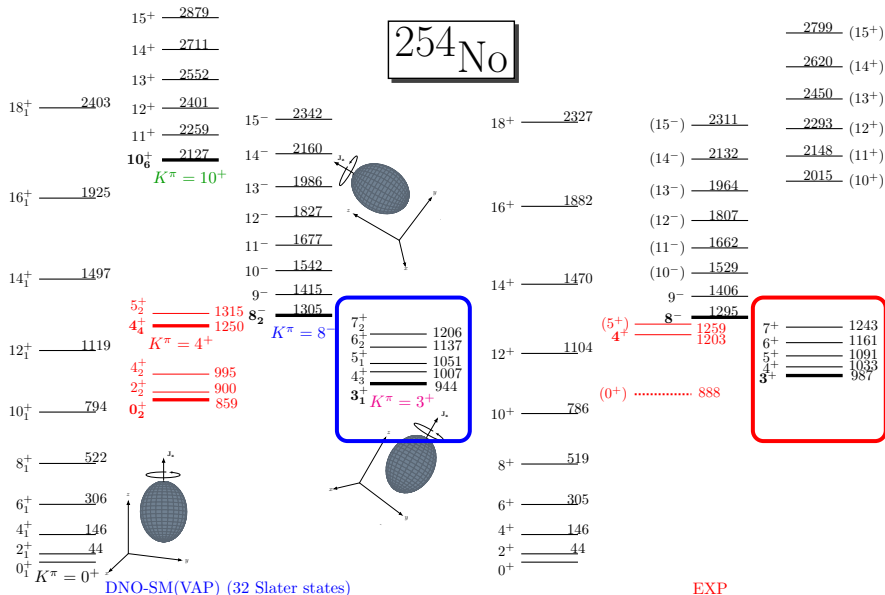
2₁⁺ — 44
0₁⁺ $K^\pi = 0^+$

DNO-SM(VAP) (32 Slater states)

2⁺ — 44
0⁺ —

EXP

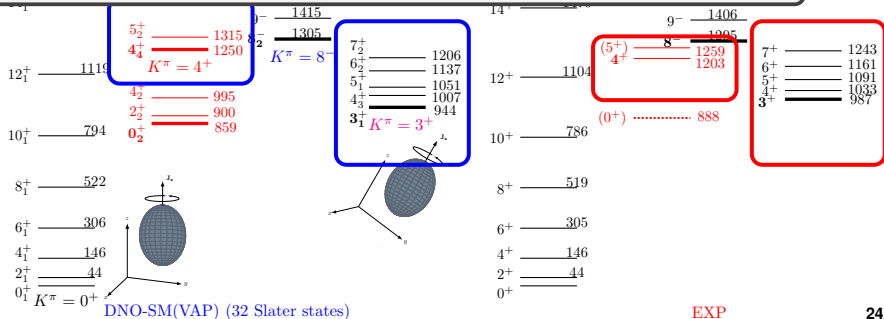
Isomeric $3^+(K=3)$ band



Isomeric $3^+(K=3)$ band : Gallagher-Moszkowski partner $4^+(K=4)$

^{254}No	Calc.	EXP.	$K = \Omega_n \pm \Omega_p $
$E^*(3^+)$ (keV)	944	987 ^a	$K=3$ (spin-aligned) lies lower than
$E^*(4^+)$ (keV)	1250	1203 ^a	$K=4$ (spin-anti-aligned)
$\Delta E_{GM} = E_{\uparrow\downarrow} - E_{\uparrow\uparrow}$	306	216 ^a	

(^a M. Forge *et al.*, Phys. Rev. C114, 024324 (2026))



Single-particle Occupations : $3^+(K=3)$, $4^+(K=4)$

Ground- and excited-states occupations

Proton orbits	$0h_{9/2}$	$0i_{13/2}$	$1f_{7/2}$	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$	
0_1^+	6.03	7.75	3.43	1.49	0.77	0.52	
0_2^+	7.08	7.91	3.23	0.89	0.69	0.21	
$3_1^+(K=3)$	6.47	7.98	3.34	1.15	0.72	0.34	
$4_4^+(K=4)$	6.50	7.83	3.41	1.18	0.72	0.36	
$8_2^-(K=8)$	6.48	7.90	3.36	1.19	0.70	0.37	
$10_6^+(K=10)$	6.55	7.03	3.48	1.56	0.79	0.58	
Neutron orbits	$0i_{11/2}$	$0j_{15/2}$	$1g_{9/2}$	$1g_{7/2}$	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$
0_1^+	7.30	9.91	5.43	1.00	1.09	0.84	0.43
0_2^+	7.36	9.95	5.45	0.96	1.05	0.80	0.42
$3_1^+(K=3)$	7.32	9.94	5.46	0.97	1.07	0.81	0.42
$4_4^+(K=4)$	7.34	9.79	5.48	1.03	1.11	0.81	0.44
$8_2^-(K=8)$	7.42	9.00	6.30	0.98	1.06	0.81	0.43
$10_6^+(K=10)$	7.23	8.89	5.72	1.40	1.34	0.97	0.45

- $3_1^+, 4_4^+$: similar structure
involving a $\pi h9/2$
recoupled with others orbitals

Branching Ratio : $3^+(K = 3)$ band

15+ — 2879

14+ — 2711

13+ — 2552

^{254}No

— 2799 (15+)

— 2620 (14+)

— 2450 (13+)

— 2293 (12+)

— 2148 (11+)

— 2015 (10+)

$$\Delta E_{GM} = E_{\uparrow\downarrow} - E_{\uparrow\uparrow}$$

Calc.

EXP.

$$\Delta E_{GM} \text{ (keV)}$$

306

216^a

$$K = |\Omega_n \pm \Omega_p|$$

$K = 3$ (spin-aligned)

lies lower

$K = 4$ (spin-anti-aligned)

(^a M. Forge *et al.*, Phys. Rev. C114, 024324 (2026))

14+ — 1497

10- — 1342

1470

(10-) — 1529

^{254}No

$$B(E2, 7_2^+ \rightarrow 5_1^+)$$

$$23971 \text{ e}^2\text{fm}^4$$

$$B(M1, 7_2^+ \rightarrow 6_2^+)$$

$$0.228 \mu_N^2$$

Branching Ratio

Calc.

EXP.^a

0.94

1.1(4)

(^a S. K. Tandel *et al.*, Phys. Rev. Lett. **97**, 082502 (2006))

1243
1161
1091
1033
987

DNO-SM(VAP) (32 Slater states)

EXP

Low-lying excited 0_2^+ state

15+ — 2879

14+ — 2711

13+ — 2552

^{254}No

— 2799 (15+)

— 2620 (14+)

(13+)

(12+)

(11+)

(10+)

Structure :

Involving essentially proton excitations

0_2^+

Calc.

EXP.

E^* (keV)

859

888^a

(^a M. Forge *et al.*, Phys. Rev. C114, 024324 (2026))

14₁⁺ — 1491

12₁⁺ — 1119

10₁⁺ — 794

8₁⁺ — 522

6₁⁺ — 306

4₁⁺ — 146

2₁⁺ — 44

0₁⁺ — 0

$K^\pi = 0^+$

DNO-SM(VAP) (32 Slater states)

5₂⁺ — 1315

4₄⁺ — 1250

$K^\pi = 4^+$

4₂⁺ — 995

2₂⁺ — 900

0₂⁺ — 859

9⁻ — 1415

8₂⁻ — 1305

$K^\pi = 8^-$

7₂⁺ — 1206

6₂⁺ — 1137

5₁⁺ — 1051

4₃⁺ — 1007

3₁⁺ — 944

$K^\pi = 3^+$

14₊ — 1470

12₊ — 1104

10₊ — 786

8₊ — 519

6₊ — 305

4₊ — 146

2₊ — 44

0₊ — 0

9⁻ — 1406

8⁻ — 1295

(5⁺) — 1259

4⁺ — 1203

(0⁺) 888

7₊ — 1243

6₊ — 1161

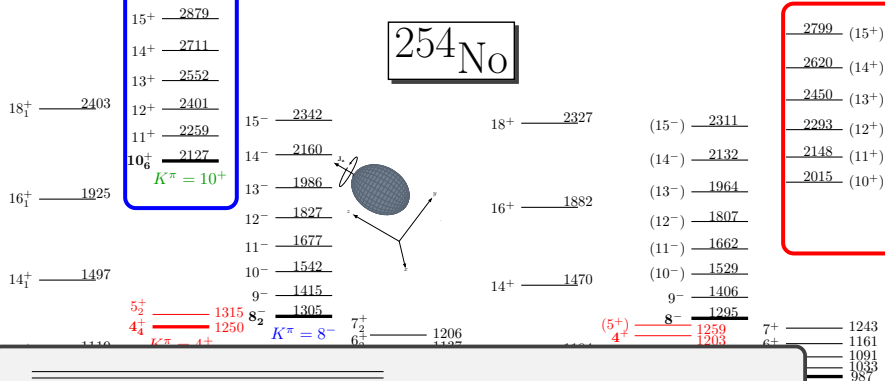
5₊ — 1091

4₊ — 1033

3₊ — 987

EXP

Isomeric $10^+(K=10)$ band



10^+

Calc.

EXP.

^a S. G. Wahid *et al.*,

E^* (keV)

2127

2015^a

Phys. Rev. C 111, 034320 (2025)

$0_1^+ \text{---} 0$
 $4_1^+ \text{---} 146$
 $2_1^+ \text{---} 44$
 $0_1^+ \text{---} 0$ ($K^\pi = 0^+$)

DNO-SM(VAP) (32 Slater states)

$0^+ \text{---} 0$
 $4^+ \text{---} 146$
 $2^+ \text{---} 44$
 $0^+ \text{---} 0$

EXP

Isomeric $10^+(K=10)$ band

15+ — 2879

2711

254

2799 (15+)

2620 (14+)

2450 (13+)

2293 (12+)

2148 (11+)

2015 (10+)

18+

PHYSICAL REVIEW C **114**, L021302 (2026)

Letter

16+

First complete shell-model description of low-lying spectroscopy in ^{254}No

Duy Duc Dao[✉] and Frédéric Nowacki[✉]

Université de Strasbourg, CNRS, IPHC UMR7178, 23 Rue du Loess, F-67000 Strasbourg, France

14+



(Received 24 September 2024; revised 26 May 2026; accepted 15 July 2026; published 13 August 2026)

PHYSICAL REVIEW C **114**, 024324 (2026)

First experimental measurement of spin splitting and evidence for a second 0^+ state in $^{254}_{102}\text{No}_{152}$

M. Forge^{✉,1} O. Dorvaux^{✉,1} A. Lopez-Martens^{✉,2} K. Kessaci^{✉,1} B. J. P. Gall^{✉,1} K. Hauschild^{✉,2} D. D. Dao^{✉,1}
F. Nowacki^{✉,1} S. Péru^{✉,3} Z. Asfari^{✉,1} R. Chakma^{2,*} A. V. Yeremin^{4,5} M. L. Chelnokov^{✉,4} V. I. Chepigin⁴ A. V. Isaev^{✉,4,5}
I. N. Izosimov^{✉,4} D. Katrasev⁴ A. A. Kuznetsova⁴ O. N. Malyshev^{4,5} R. Mukhin⁴ A. G. Popeko^{4,5} Yu. A. Popov⁴
B. Saylaubekov^{✉,4} A. I. Svirikhin^{✉,4,5} E. A. Sokol^{✉,4} M. S. Tezekbayeva⁴ N. I. Zamyatin⁶ and J. Piot^{✉,7}

1243

1161

1091

1033

987

0+

4+

2+

0+

136

44

$K^\pi = 0^+$

DNO-SM(VAP) (32 Slater states)

4+

2+

0+

136

44

EXP

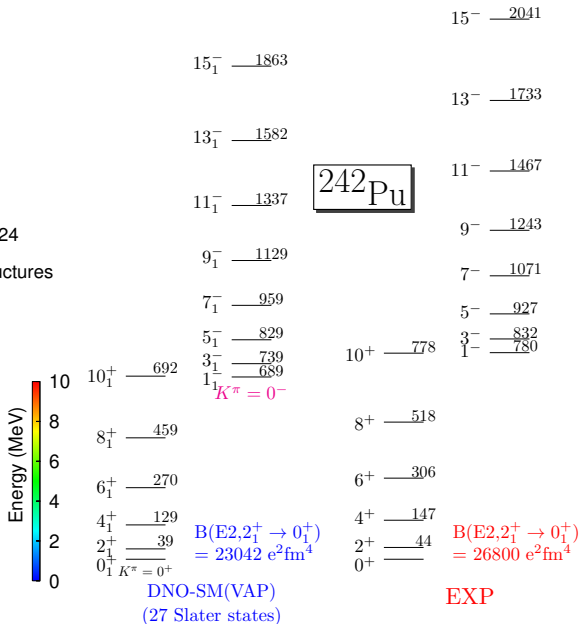
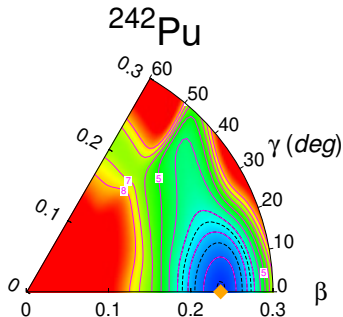
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

Known spectroscopy :

- Axially well-deformed nucleus
- Ground-state rotational band
- Several side bands :
 $1^-(K=0), 3^-(K=3), 7^-$

Calculations :

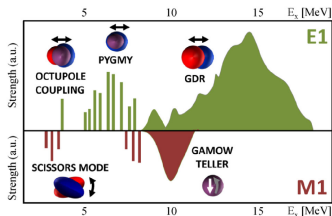
- Deformed Axial Minimum at $\beta = 0.24$
- VAP calculations for band-head structures



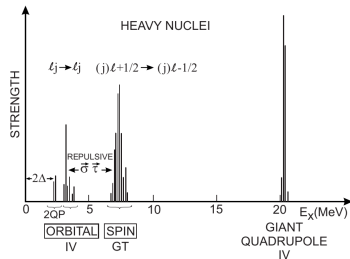
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

◇ Strength function description of $M1$, $E1$:

- Scissors mode in deformed nuclei
- DNO-SM(VAP) : capture correlations



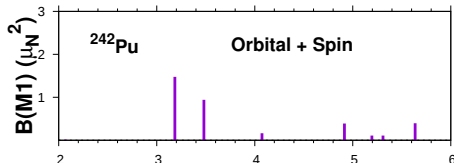
D. Budker *et al.*, Ann. Phys. **2022**, 534, 2100284.



K. Heyde *et al.*, Rev. Mod. Phys. **104**, 045801 (2021).

◇ Open questions :

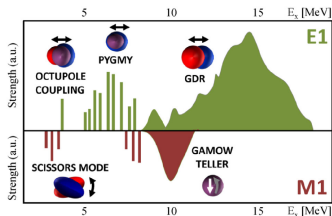
- Generate easily structure function for many final states
- E1 transitions forbidden in the natural valence space
→ add QSU3 higher orbitals



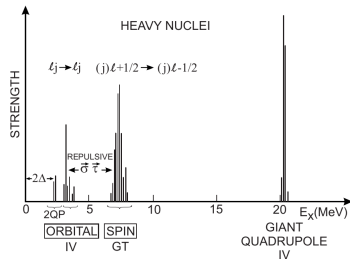
Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

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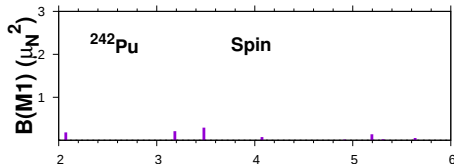
D. Budker *et al.*, Ann. Phys. **2022**, 534, 2100284.



K. Heyde *et al.*, Rev. Mod. Phys. **104**, 045801 (2021).

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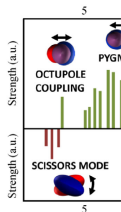
- Generate easily structure function for many final states
- E1 transitions forbidden in the natural valence space
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Dipole excitations in $^{242}_{94}\text{Pu}_{148}$

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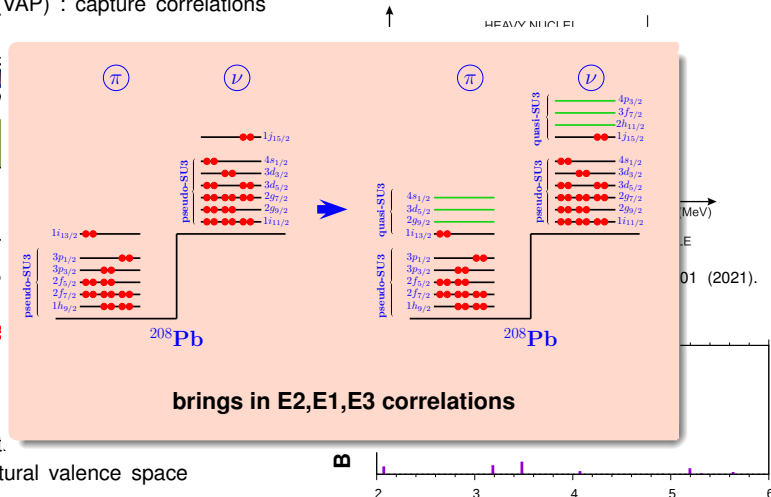
- Scissors mode in deformed nuclei
- DNO-SM(VAP) : capture correlations



D. Budker *et al.*,

◇ Open que

- Generate for many
- E1 transit. in the natural valence space
→ add QSU3 higher orbitals



Spectroscopy in rare-earth region

Few spectroscopy :

- Well-deformed nucleus
- Ground-state rotational band
- few negative parity states bands : 3^- , 4^-

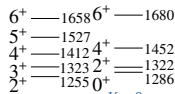
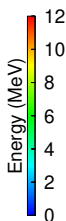
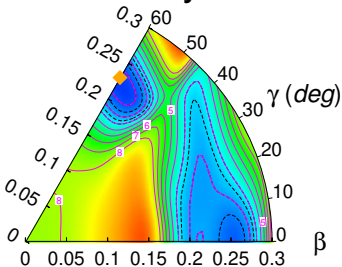
Calculations :

- realistic N3LO interaction for ^{132}Sn core and

$$\begin{array}{ccccccc} \pi & g_{7/2} & d_{5/2} & d_{3/2} & s_{1/2} & h_{11/2} & \\ \nu & h_{9/2} & f_{7/2} & f_{5/2} & p_{3/2} & p_{1/2} & i_{13/2} \\ & 2 & 2 & 2 & 2 & 2 & 2 \end{array} \quad \text{orbitals}$$

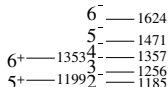
- Deformed Oblate minimum at $\beta = 0.23$
- VAP calculations for band-head structures

^{168}Dy



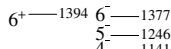
γ -band

$K=0$
(β)-band



$K=2$
Octupole band

γ -band

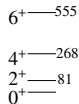


$K=2$
Octupole band

γ -band



SCCM



DNO-SM



EXPERIMENT

Level scheme for the ground state band, the γ band, the $K^\pi = 0^+$ band and the $K^\pi = 2^-$ band in ^{168}Dy computed with the Symmetry-Conserving Configuration Mixing Model and DNO-SM, compared to experimental values

E. L. Larsson *et al.*, "Revealing the shapes of ^{168}Dy by first-time projectile-fragmentation of ^{170}Er ", to be submitted

Octupole excitations in $^{168}_{66}\text{Dy}_{102}$

Few spectroscopy :

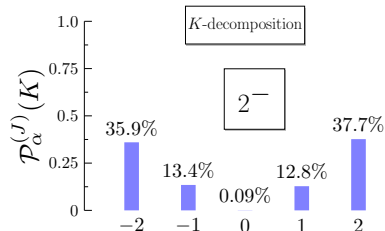
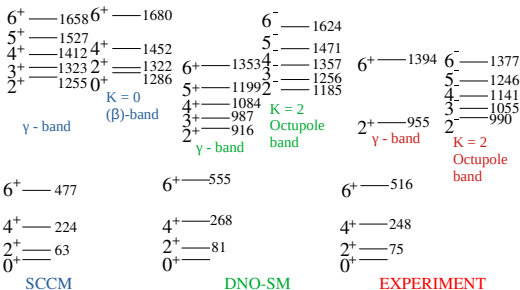
- Well-deformed nucleus
- Ground-state rotational band
- few negative parity states bands : 3^- , 4^-

Calculations :

- realistic N3LO interaction for ^{132}Sn core and

$$\begin{array}{c} \pi \quad g_{7/2}^2 \quad d_{5/2}^2 \quad d_{3/2}^2 \quad s_{1/2}^2 \quad h_{11/2}^2 \\ \nu \quad h_{9/2}^2 \quad f_{7/2}^2 \quad f_{5/2}^2 \quad p_{3/2}^2 \quad p_{1/2}^2 \quad i_{13/2}^2 \end{array} \quad \text{orbitals}$$

- Deformed Oblate minimum at $\beta = 0.23$
- VAP calculations for band-head structures



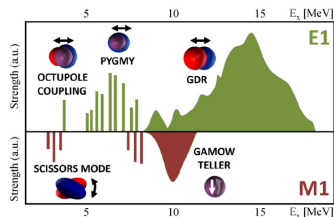
Level scheme for the ground state band, the γ band, the $K^{\pi} = 0^+$ band and the $K^{\pi} = 2^-$ band in ^{168}Dy computed with the Symmetry-Conserving Configuration Mixing Model and DNO-SM, compared to experimental values

E. L. Larsson *et al.*, "Revealing the shapes of ^{168}Dy by first-time projectile-fragmentation of ^{170}Er ", to be submitted

Dipole excitations in Rare-Earth region

◇ Strength function description of $M1$, $E1$:

- Scissors mode in deformed nuclei
- DNO-SM(VAP) : capture correlations



D. Budker *et al.*, Ann. Phys. **2022**, 534, 2100284.

◇ Open questions :

- Generate easily structure function for many final states
- E1 transitions forbidden in the natural valence space
→ add QSU3 higher orbitals

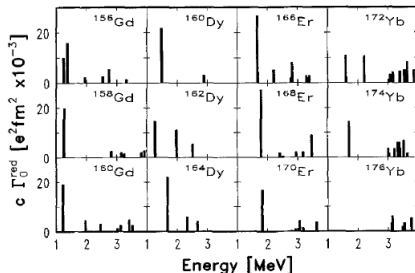


Fig. 1. Distribution of $\Delta K=0$ dipole strengths in the rare earth nuclei $^{156, 158, 160}\text{Gd}$, $^{160, 162, 164}\text{Dy}$, $^{166, 168, 170}\text{Er}$ and $^{172, 174, 176}\text{Yb}$. In case of negative parity the ordinate value can be identified with the $10^{-3} e^2 \text{fm}^2$, i.e. $c \cdot \Gamma_0^{\text{red}}(1^- \rightarrow 0^+) = B(E1, 866 \cdot 10^{-3} e^2 \text{fm}^2 (\text{MeV})^{-1})$ and Γ_0^{red} is the in MeV defined in (1)

A. Zilges *et al.*, Z. Phys. A **340**, 155 (1991).

Summary

- Non-Orthogonal Multi-Slater Determinants give an exact representation of shell-model wave functions
- Pairing correlations are fully captured by the variation after projection where the rotational symmetry is consistently restored
- Isospin symmetric Island of Inversion around $A \sim 80$
- First shell-model description of superheavy nuclei : $^{253,254,255}\text{Es}$, $^{249,251,253}\text{Cf}$, $^{253,254}\text{No}$, ^{256}Fm
- Complete reproduction of known spectroscopy of ^{254}No : yrast and excited isomeric structures
- Ongoing study of dipole excitations in ^{242}Pu
- Ongoing study of dipole/octupole excitations in ^{168}Dy