Description of the fusion mechanism in heavy ion collisions within full Langevin dissipative dynamics

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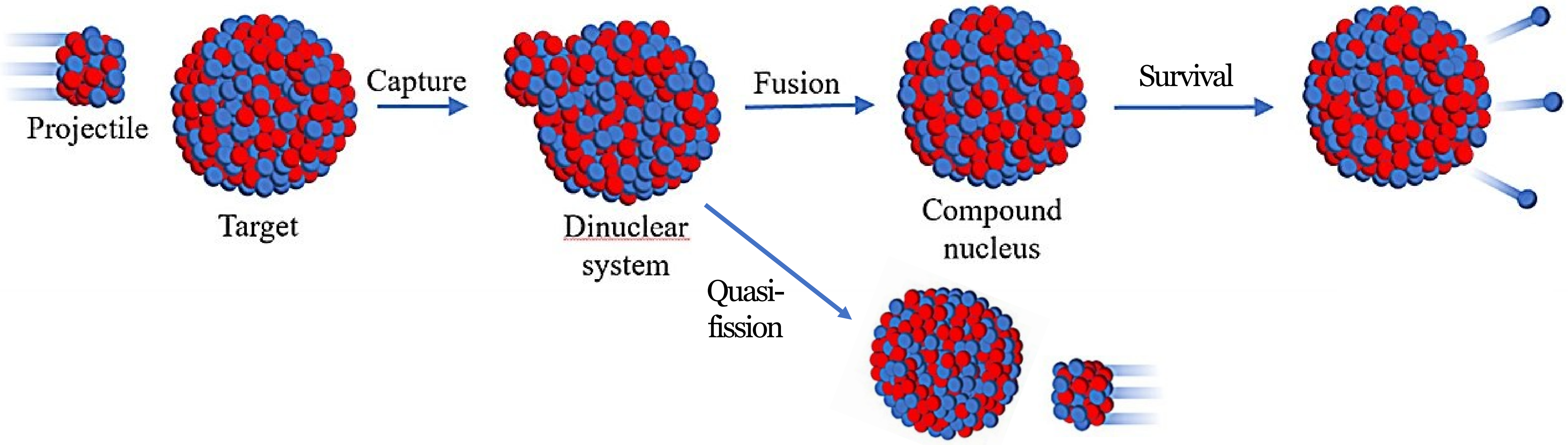
Bussang, France

NARODOWE CENTRUM BADAŃ JĄDROWYCH ŚWIERK

The fusion process (Schematic view)



Micro-macroscopic description of fusion:

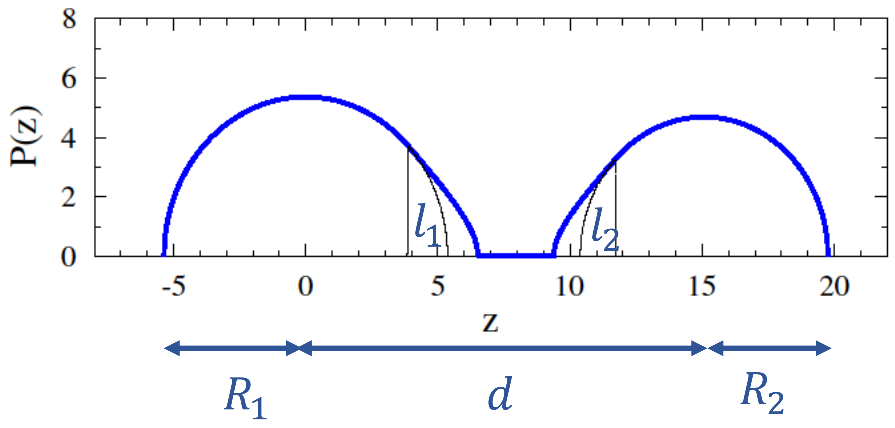
- Complexity/impossibility of tracking all internal degrees of freedom
- Identification of **slow collective degrees of freedom** immersed in a **bath** of faster dynamics
- Emergence of the mechanisms of **friction** and **random forces**
- Macroscopic model with some microscopic corrections.

Collective variables adapted to fusion/fission – Shape variables

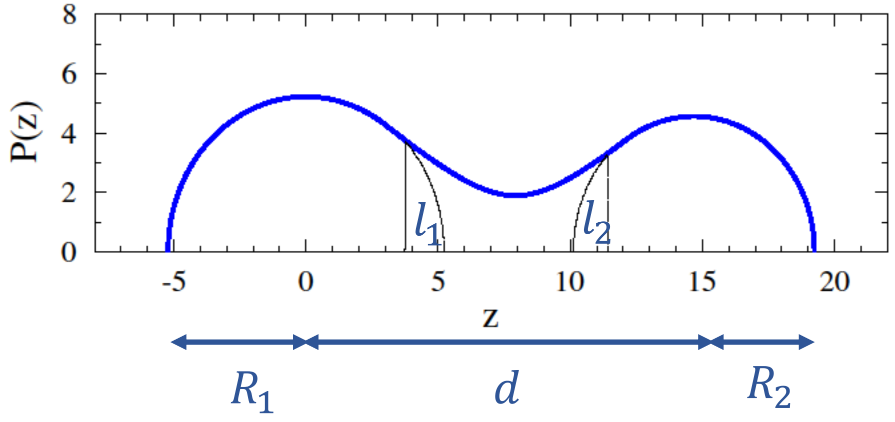
- Axially symmetric shapes
- **Spherical** cups connected by quadratic surfaces[1]

- **Shape collective/slow variables**:
  - **Distance/elongation** : $\rho = \frac{d}{R_1+R_2}$
  - **Neck/deformation** : $\lambda = \frac{l_1+l_2}{R_1+R_2}$
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$

$^{92}Zr + ^{64}Ni$



Bipartite
$\rho = 1.5$
$\lambda = 0.3$
$\Delta = \Delta_0$



Monopartite
$\rho = 1.5$
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[1] J. Błocki, H. Feldmeier and W. J. Świątecki, Nucl. Phys. A 459 (1986) 145

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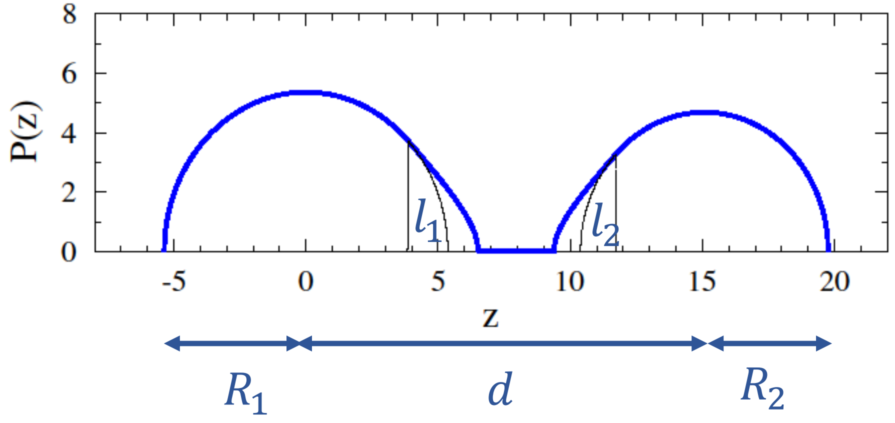
  - **Asymmetry**: $\Delta = \frac{R_1-R_2}{R_1+R_2}$

- **Scission is well-defined**: $\lambda_{\text{scission}} = 1 - \frac{1}{\rho_{\text{scission}}}$

  → **Suited to describe fusion/fission**
  (vs. multipole moments)

$^{92}Zr + {}^{64}Ni$



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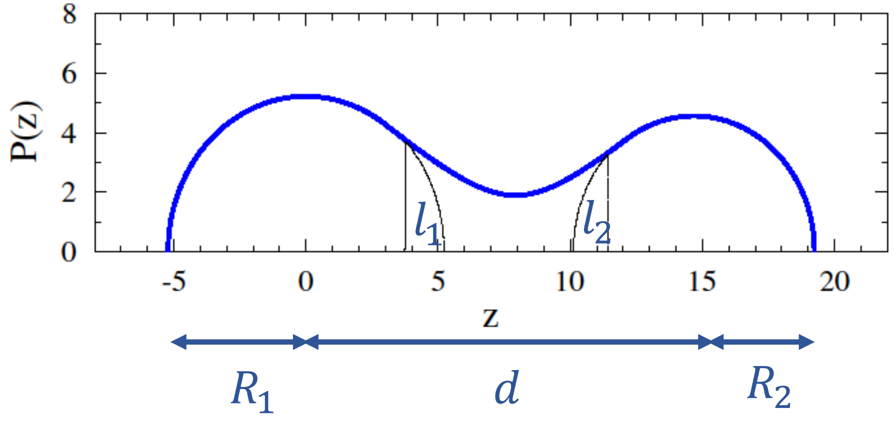

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Collective variables adapted to fusion/fission – Angle variables

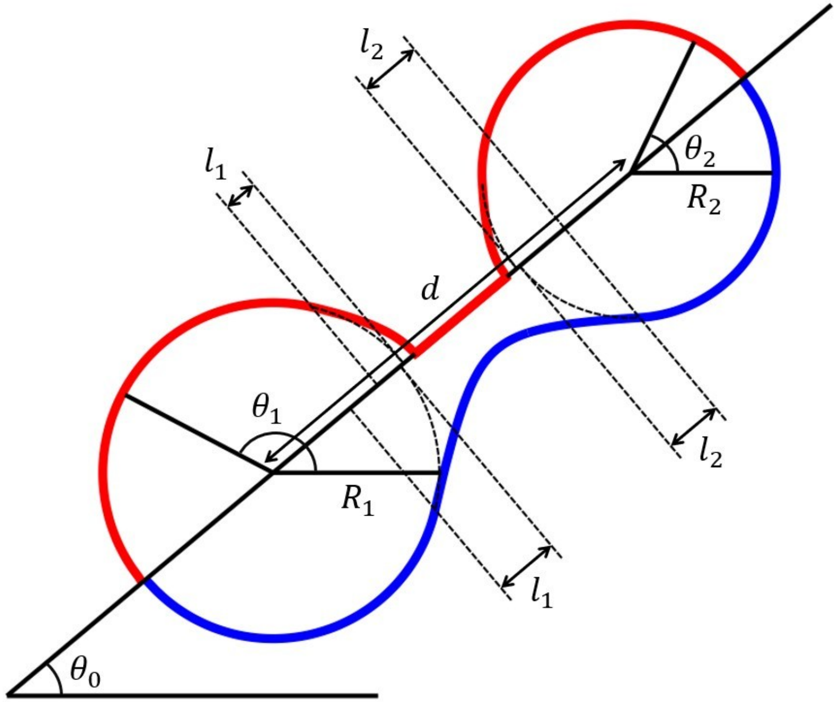
- **Collective angle variables**
  - Angle of the whole system θ_0
  - Angle of the first sphere θ_1
  - Angle of the second sphere θ_2

- Variations linked to angular momentum, in particular:

$$p_{\theta_0} + p_{\theta_1} + p_{\theta_2} = L_{init}$$

- **Exact treatment of angular momentum**

$\longrightarrow$ **Full Langevin 6-dimensional dissipative dynamics**



The Langevin system of equations

- Denoting **collective/slow variables** $q_i(t)$ and their **associated moments** $p_i(t)$, the **Langevin equations** read:

$$\dot{q}_i(t) = \sum_k (\mathcal{M}^{-1})_{ik}\, p_k \qquad \leftrightarrow (P = MV)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik}\, \dot{q}_k + \sum_k g_{ik}\, \xi_k(t) \qquad \leftrightarrow \left(\frac{dP}{dt} = \sum F\right)$$

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Conservative forces
$(\boldsymbol{H = T + V})$

Friction forces

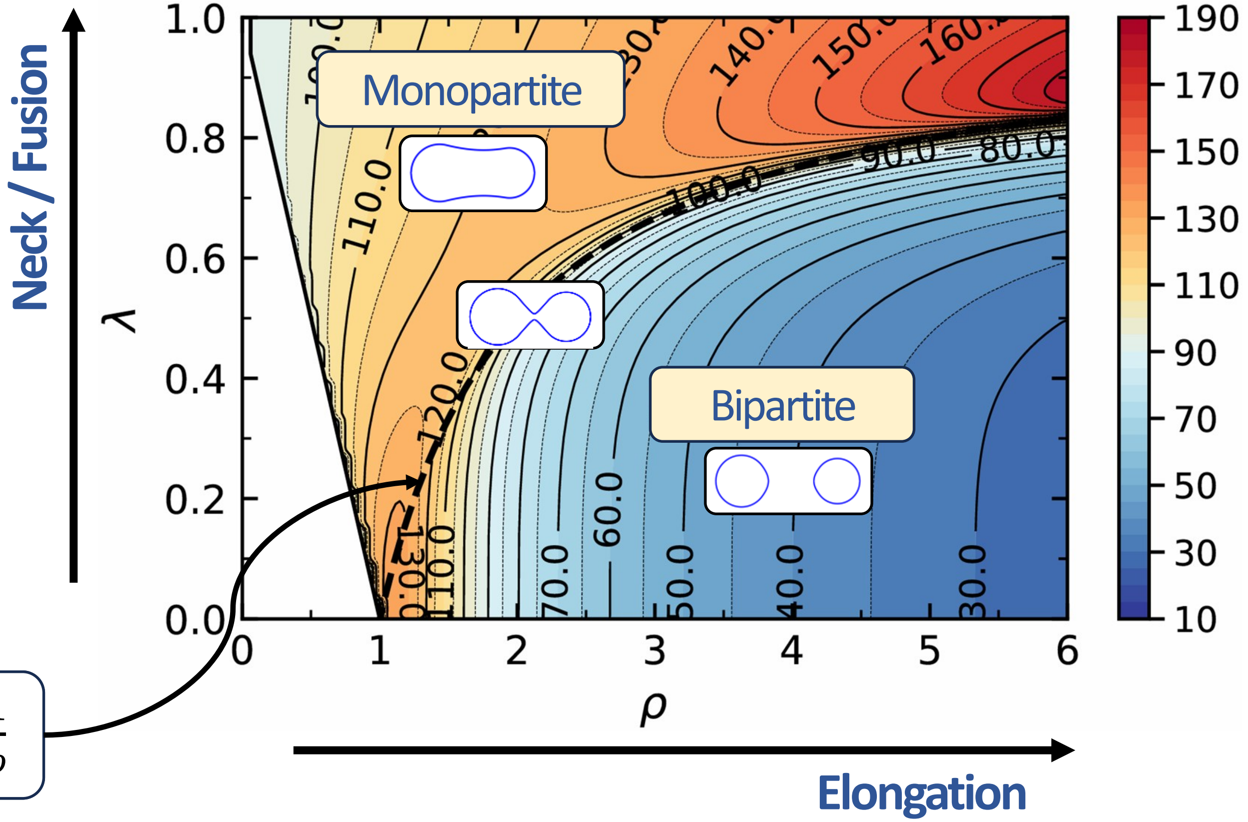
Langevin/random forces

$\longrightarrow$ A **comprehensive understanding of the dynamical process.**

Collective Potential Energy

- **Yukawa-plus-exponential folding potential**
  + Coulomb potential
- Parameters taken from a previous fit to experimental masses and fusion barrier heights [1]

- **No shell effects at the moment.**



Scission line $\lambda = 1 - \frac{1}{\rho}$

Collective deformation potential of $^{92}Zr + ^{64}Ni$ in MeV

[1] H. J. Krappe *et al.*, Phys. Rev. C 20 (1979) 992–1013

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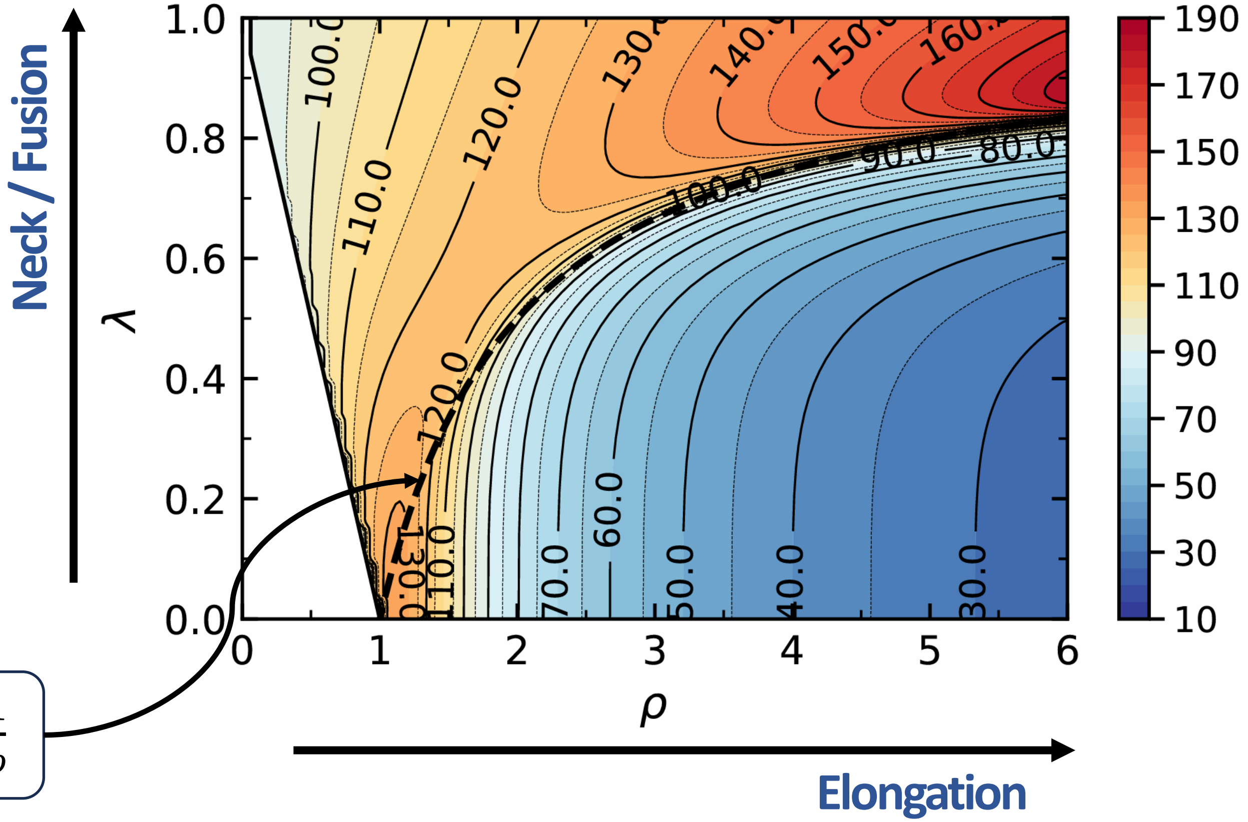

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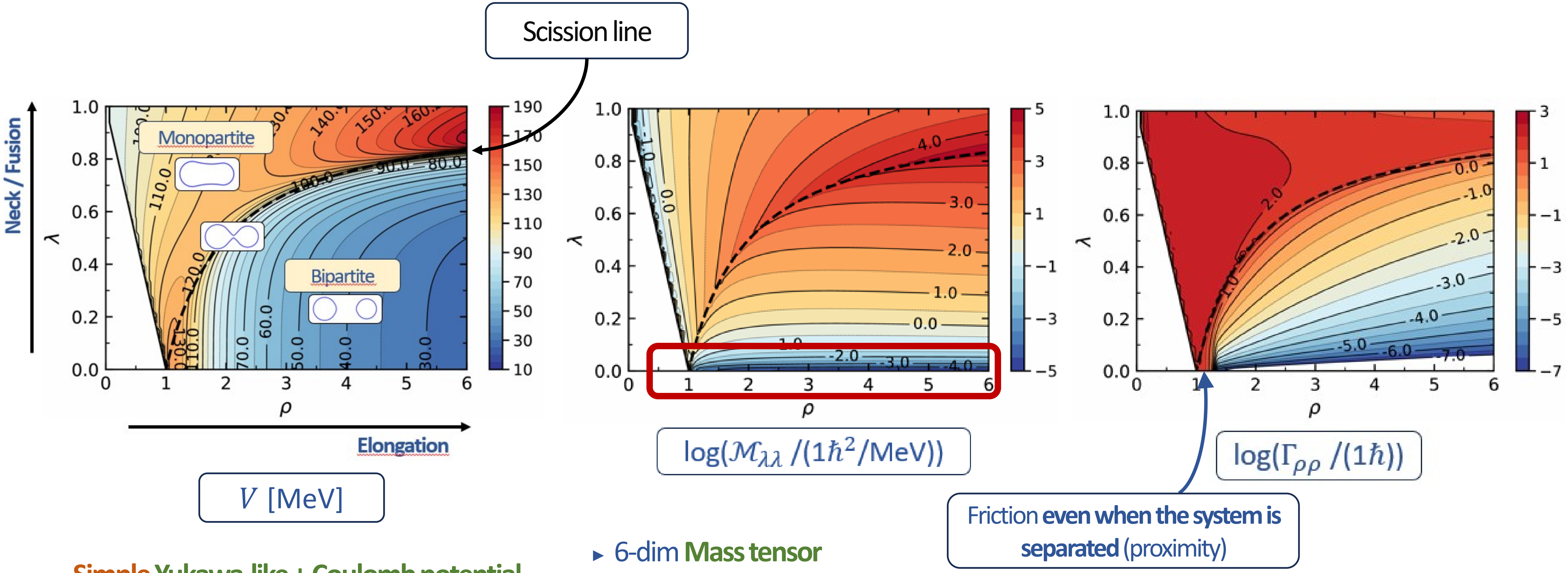
The Ingredients of the Langevin Formalism



- **Simple Yukawa-like + Coulomb potential**
- No shell effects at the moment

- 6-dim **Mass tensor**
- 6-dim **Friction tensor**
- 6-dim **Gaussian random forces** (**simple white noise**)

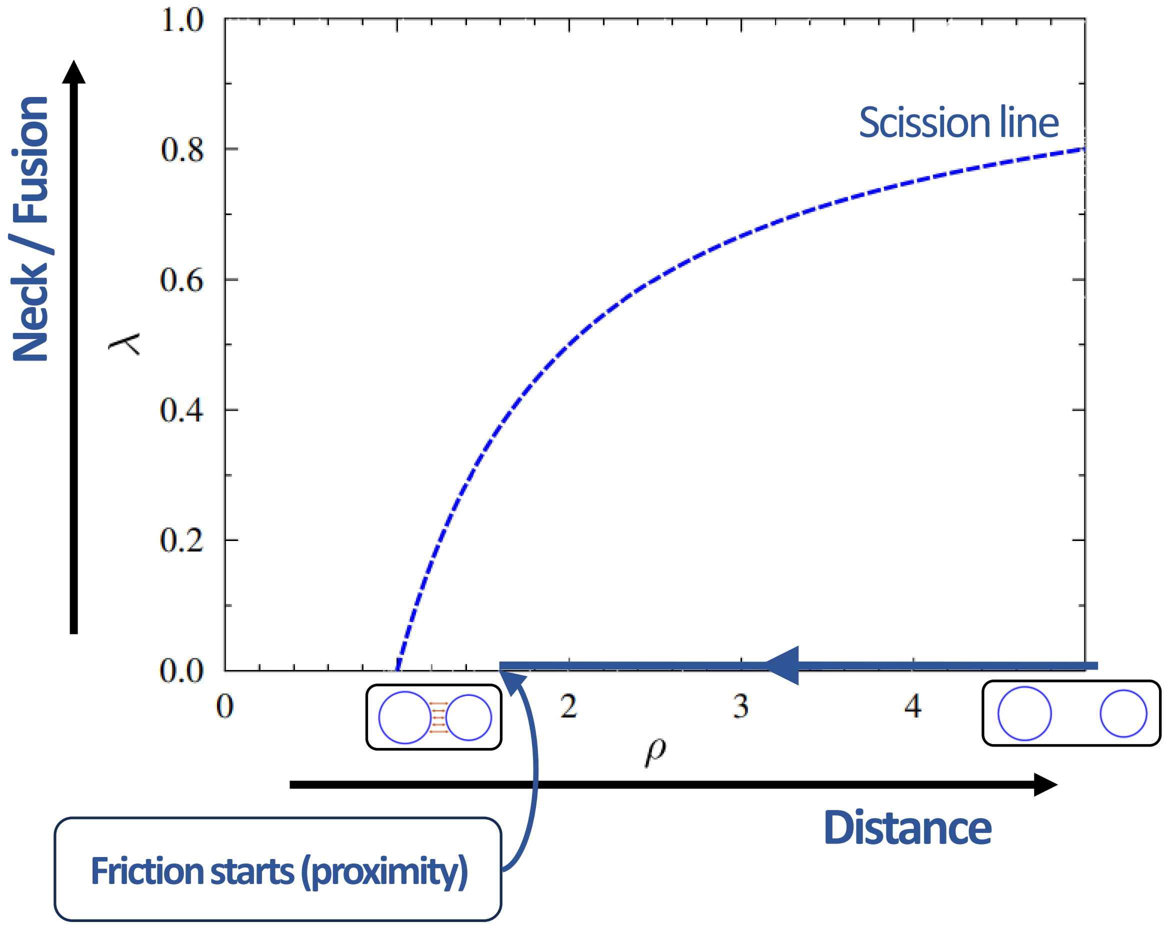
The fusion process

The three main stages of the collision:

1. **A first violent deceleration** during which:
   - The **system loses most of its kinetic energy**
   - There is **almost no deformation of the nuclei**

   $(\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda$
   $= -\infty + \infty)$

   - **Unstable balance close to the lambda border**



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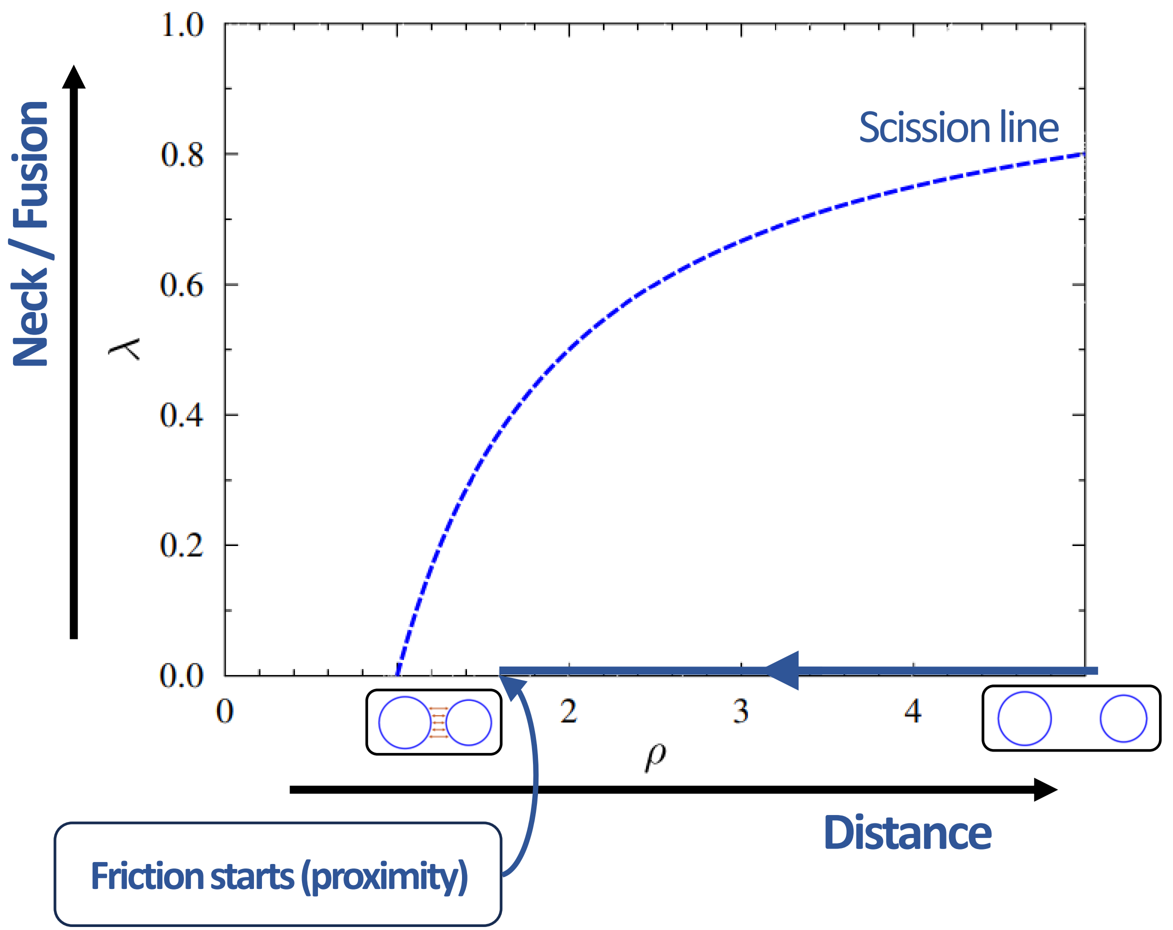
   - **Unstable balance close to the lambda border**
   - **Physical interpretation: (Extra)-deformation can only occur when the fragments interact with each other**
   - **Treated exactly** (conservative forces).

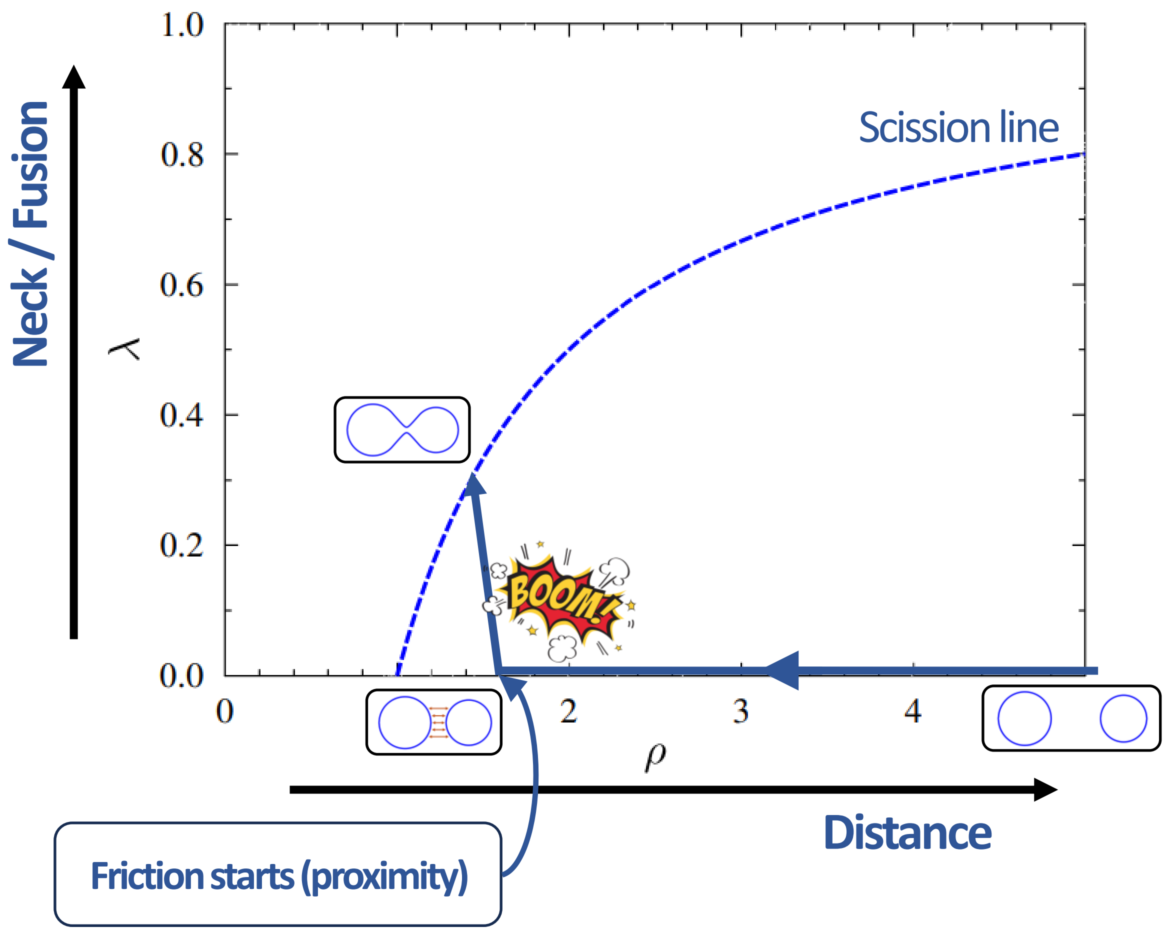


The fusion process

2. The **"Kiss of death"** when friction starts

- **A little sudden change in** p_ρ **→ infinite push to the scission line** $(\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda = -\infty + \infty)$

- **Deformation starts and remains.**



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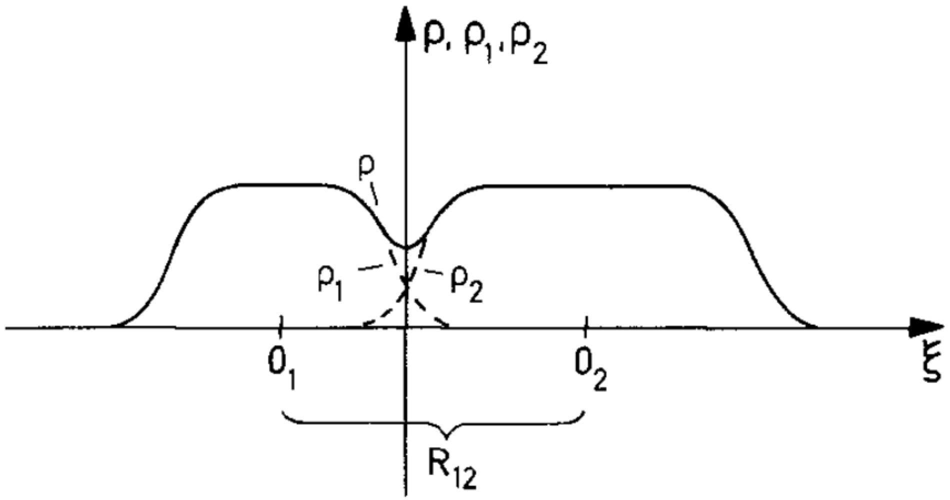
$= -\infty + \infty)$

- **Deformation starts and remains.**

- **Inherent instability of non-saturated nucleonic densities**
(**Sudden approximation** in HF when the nuclear tails touch)

- **This step is <u>NOT</u> treated numerically**:
We start the calculation from the touching point

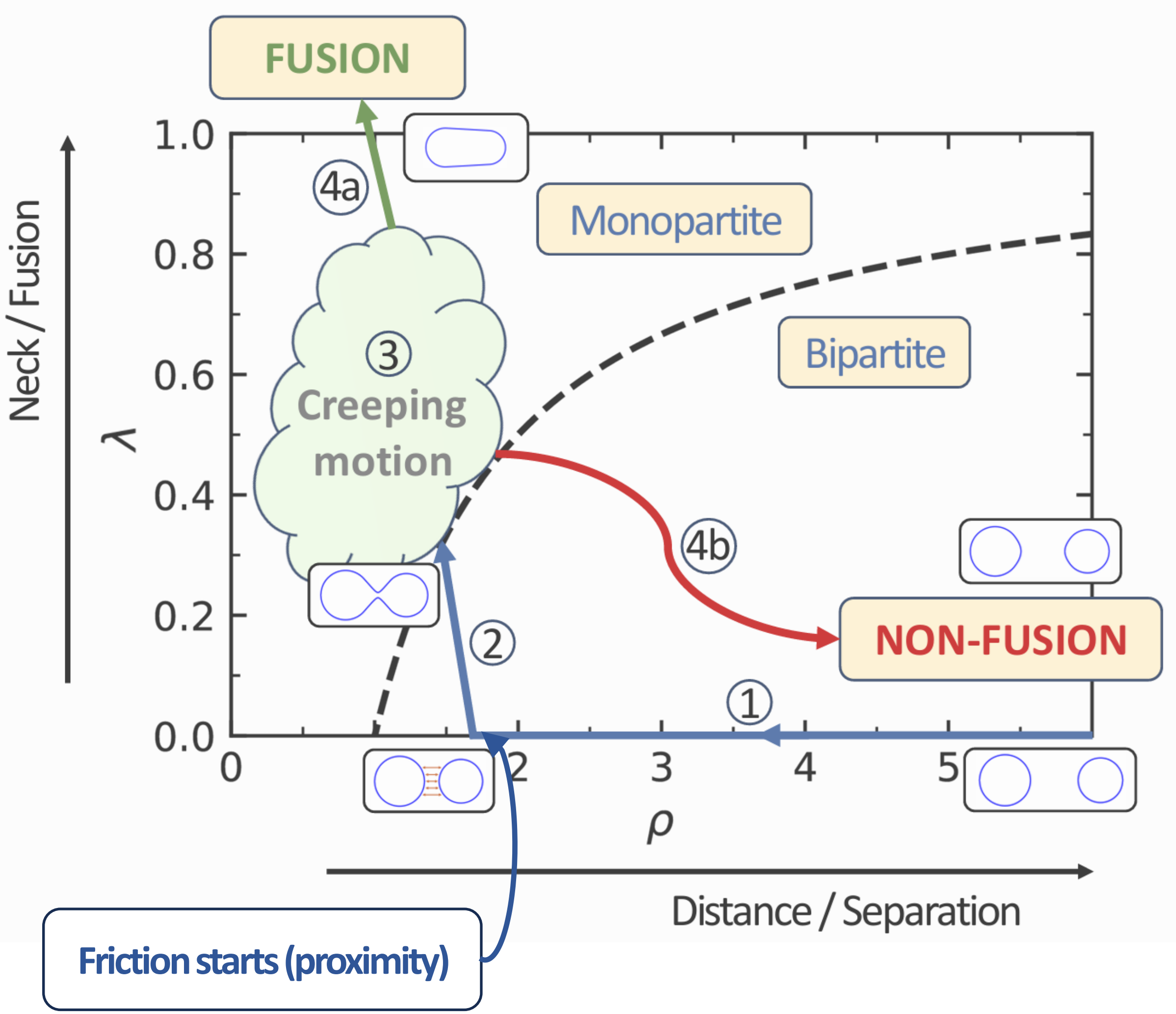


"Sudden approximation" – K. Pomorski, K. Dietrich
Z. Phys. A, 295 (1980) 335

The fusion process

The three main stages of the collision:

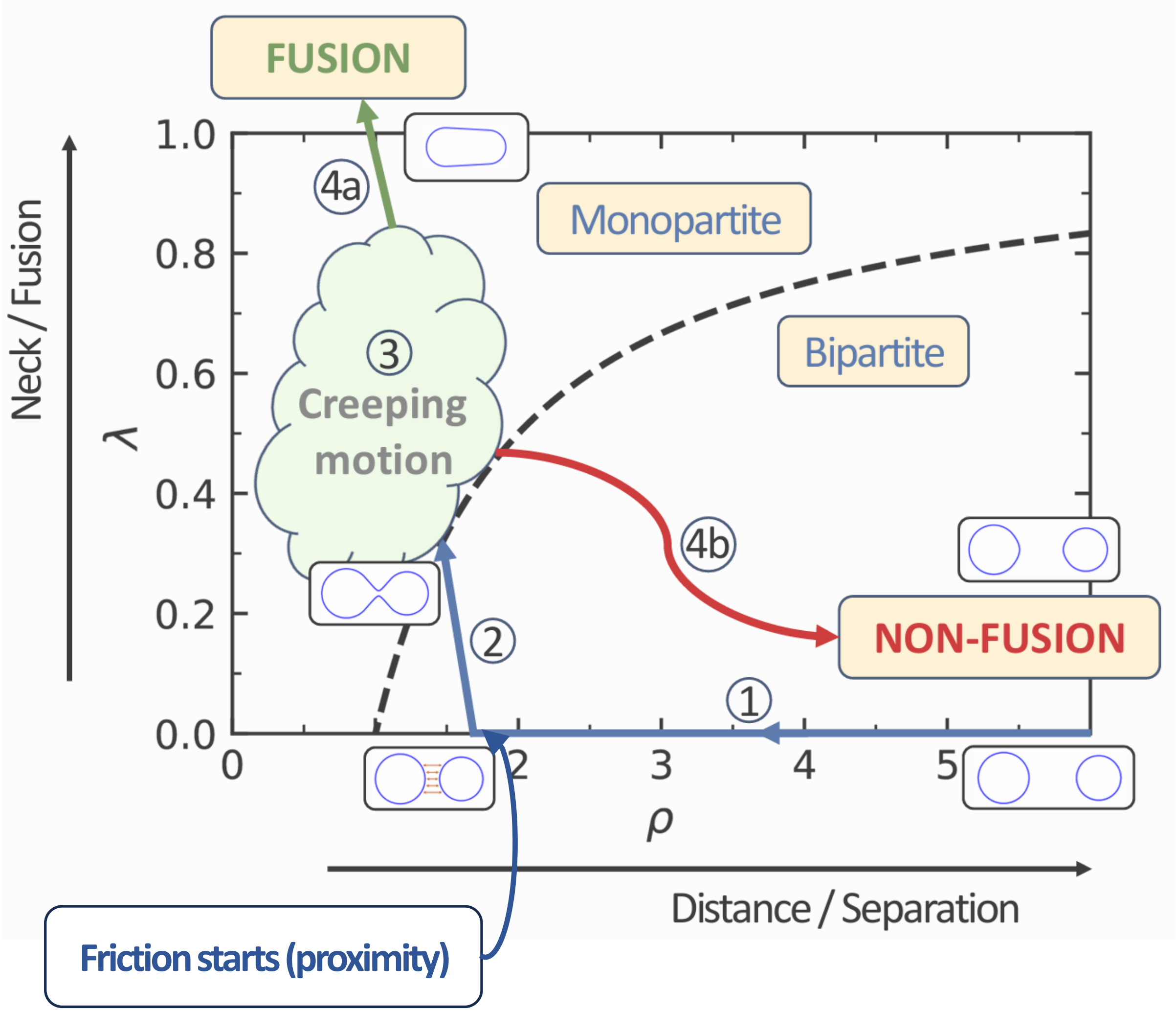
3. A **long creeping motion**
   - which leads to fusion or separation
   - **Solved numerically**



Physically-guided definition of fusing and non-fusing events

- **Fusing conditions**:
  - $\lambda = 1$ (half of the spheres are mixed)
  - $\rho(1-\lambda) = \Delta^2$ (window angle fully open)
  - $\rho = 0.5$

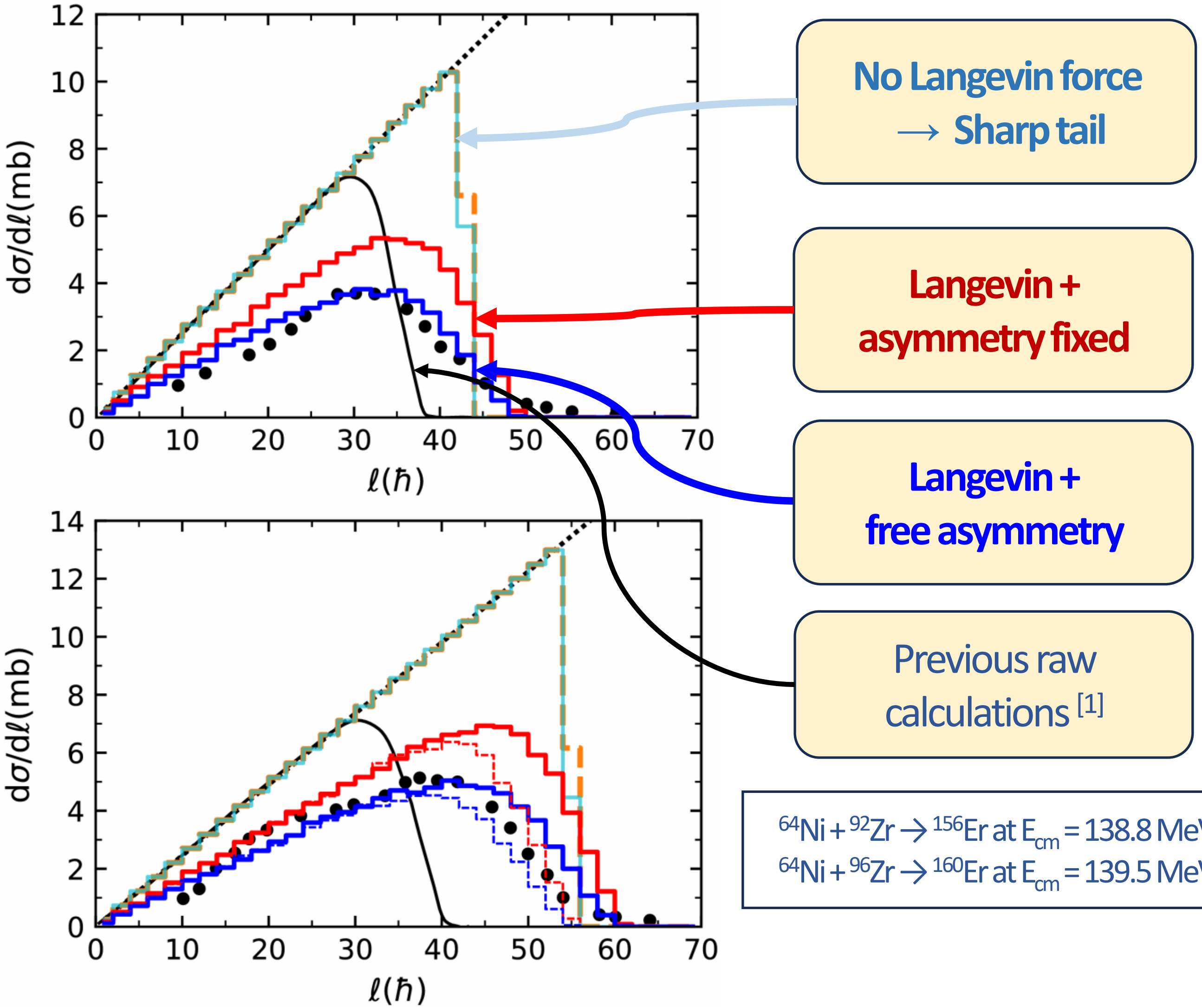
- **Non-fusing conditions**:
  - $\lambda \rightarrow \lambda_{min} = 10^{-2}$
  - $\rho \rightarrow \rho_{max} = 3$
  - No fusion after $N_{max} = 500{,}000$ steps.



Pilot paper – “Symmetric” system

Y. Jaganathen, M. Kowal, K. Pomorski, Phys. Lett. B 862 (2025) 139302

- **Spin distributions are more natural with the Langevin force.**
- They **drop at the correct angular momentum**
- **Great agreement to the experimental data[2,3] .**



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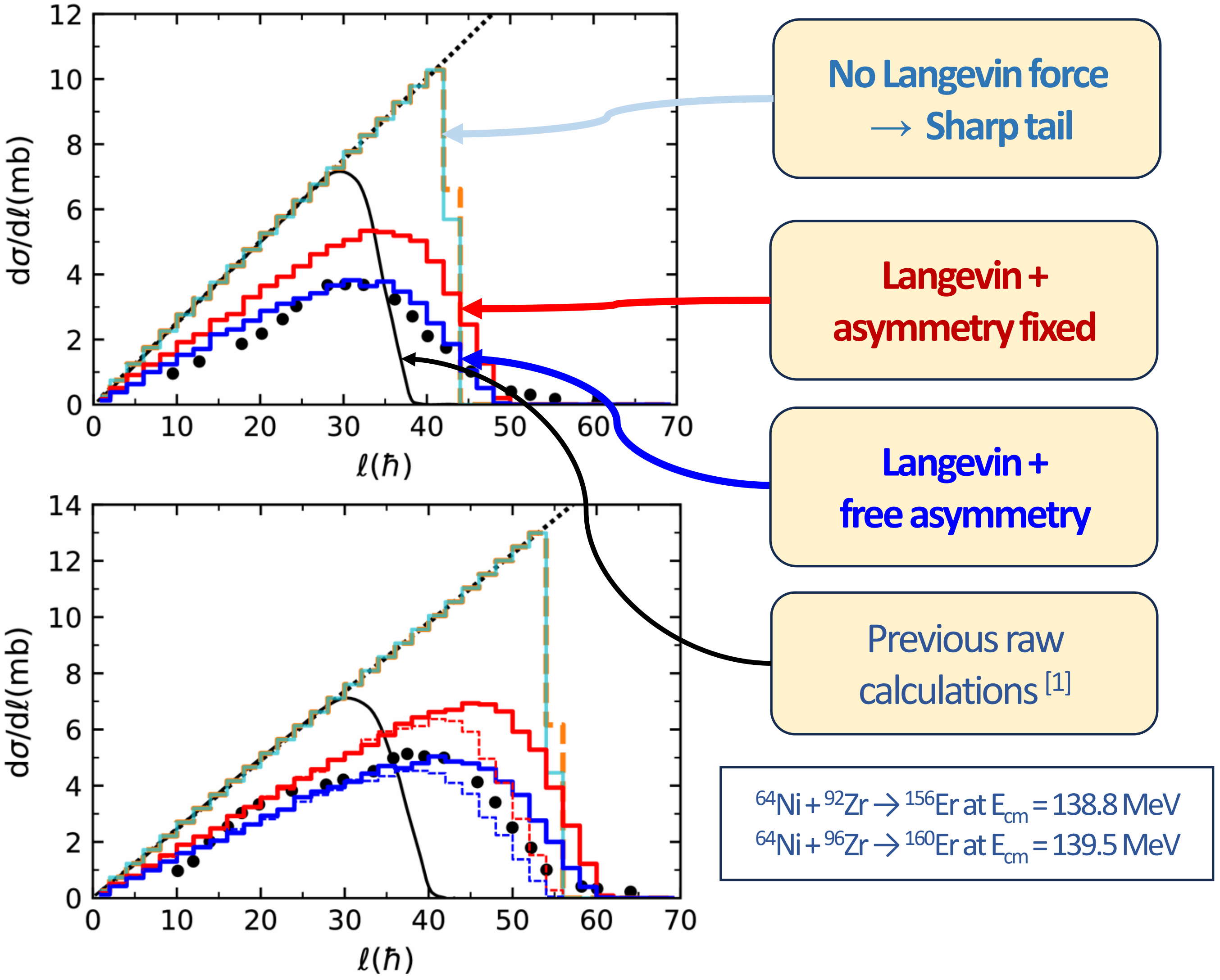
- Open question: **Why do the cross sections go down when asymmetry is a free variable?**



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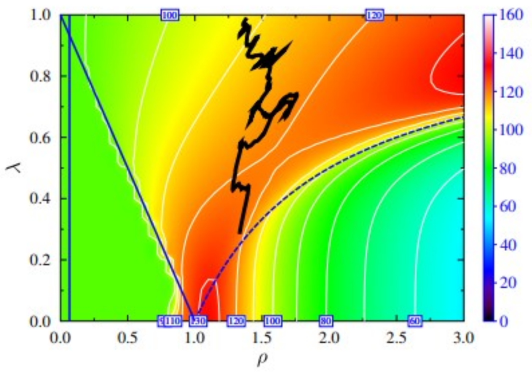
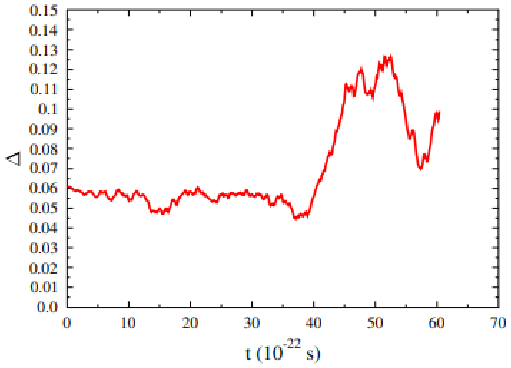
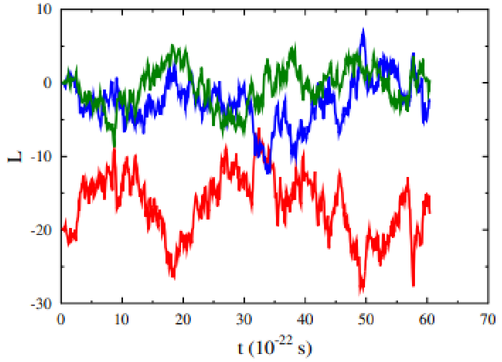
A deep understanding of the fusion process (Preliminary examples)

Paths | Asymmetry vs. time | Angular momenta

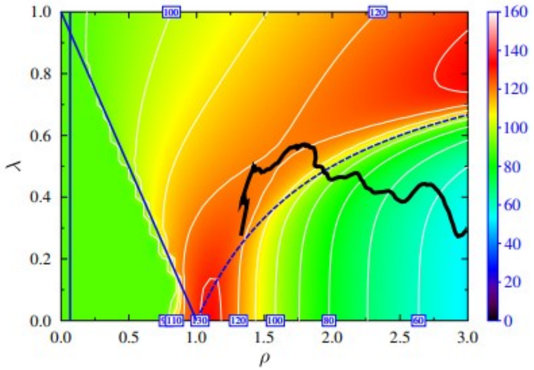
- $\ell = 20\hbar$,
  Fusion case:

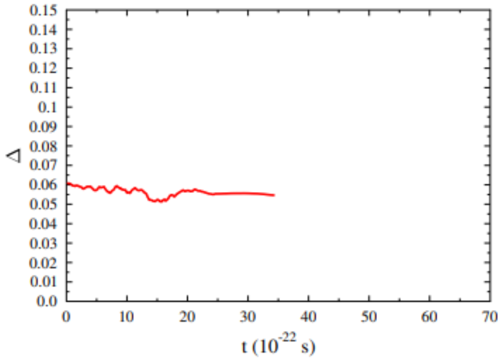


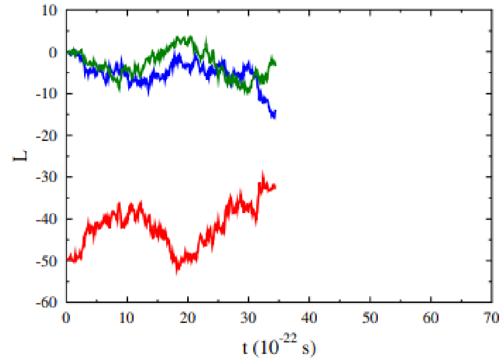




- $\ell = 50\hbar$,
  No-fusion case:







- And various other observables: **diffusion tensor**, the **number of rotations the system undergoes before fusion** etc.

Mass asymmetry effects – ^{220}Th (Preliminary)

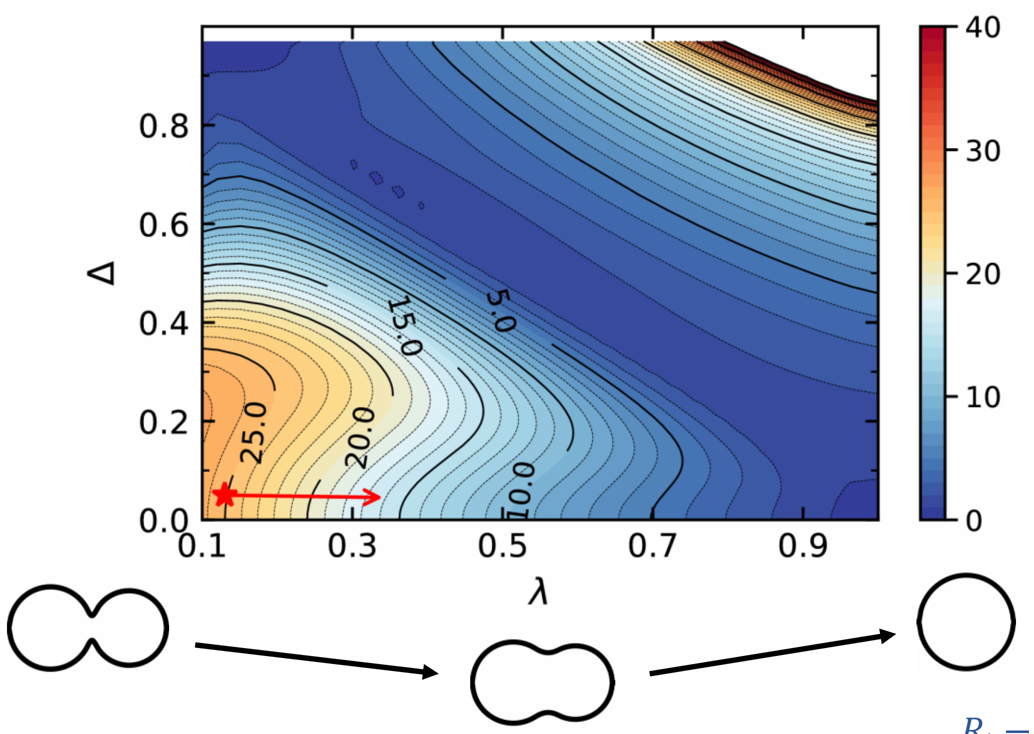
- **Several well-studied reactions leading to the same compound nucleus** ^{220}Th:
  - ^{16}O + ^{204}Pb $\longrightarrow$ ^{220}Th **[Experimental transmission = 100 %]**
  - ^{40}Ar + ^{180}Hf $\longrightarrow$ ^{220}Th
  - ^{48}Ca + ^{172}Yb $\longrightarrow$ ^{220}Th
  - ^{96}Zr + ^{124}Sn $\longrightarrow$ ^{220}Th

- Same excitation energy of the compound nucleus
- $\boldsymbol{E^*_{CN}}$ **= 50 MeV** (no pre-fusion neutron emission)

Two different fusion mechanisms

- Symmetric to quite asymmetric systems:
  $\longrightarrow$ **Symmetrization of the shape/slow equilibration**
  ($R_1 = R_2 = R_0$ + large neck)

$V - V_{comp}$ [MeV]
(ρ adapted)



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Two different fusion mechanisms

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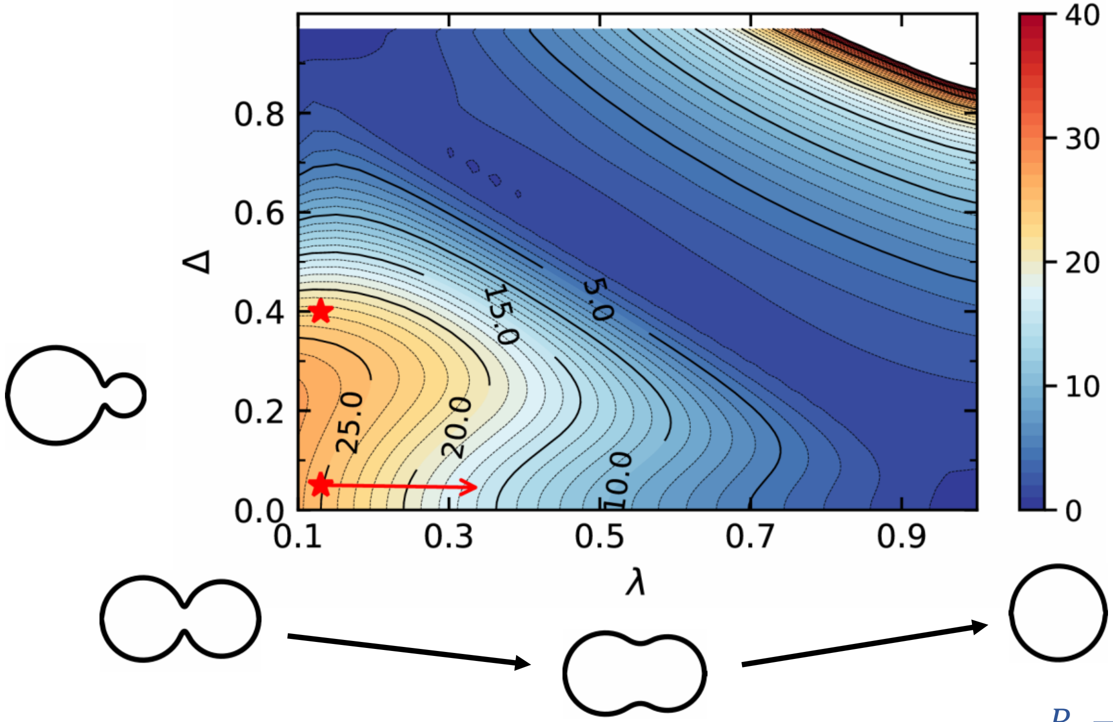

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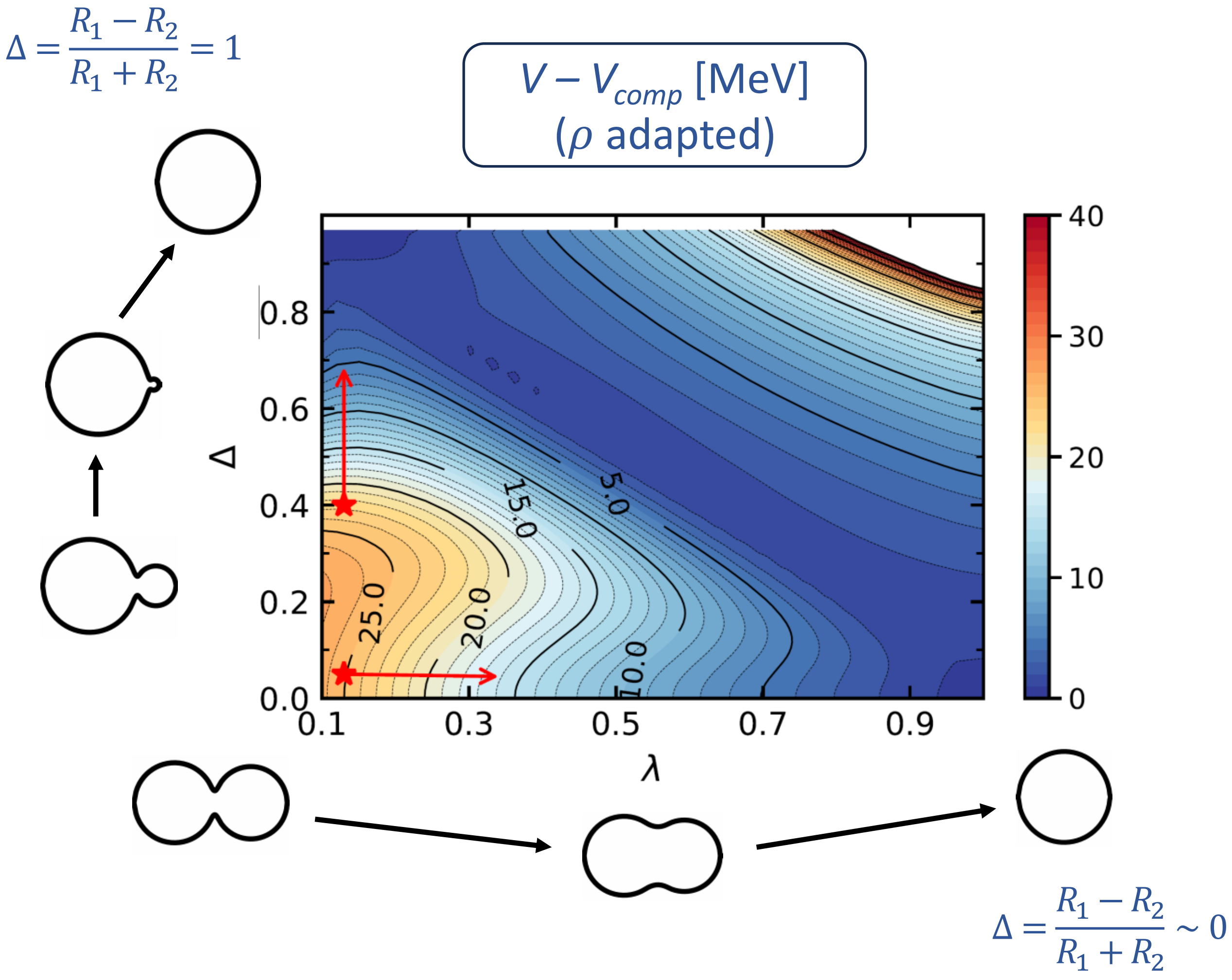
Two different fusion mechanisms

- <u>Symmetric to quite asymmetric systems</u>:
  $\longrightarrow$ **Symmetrization of the shape/slow equilibration**
  ($R_1 = R_2 = R_0$ + large neck)

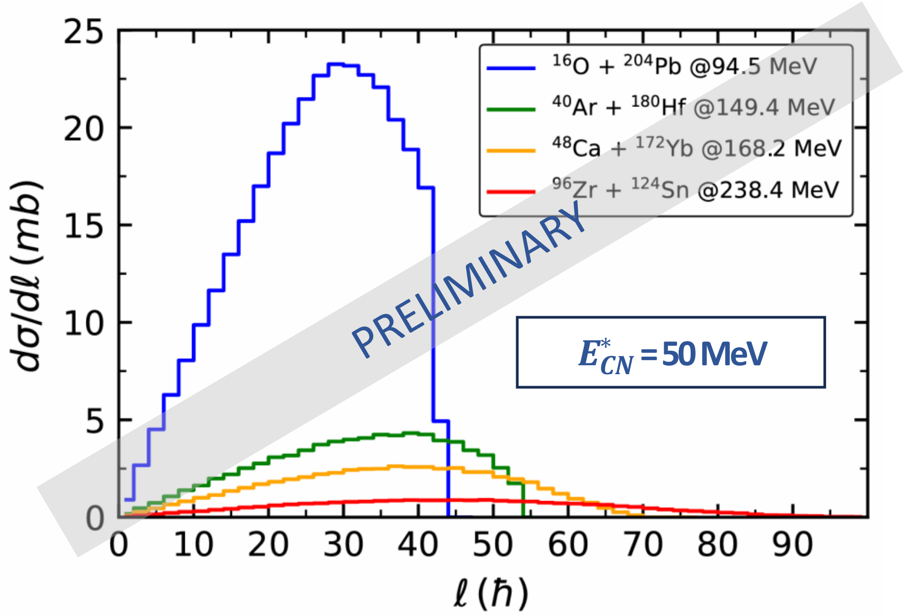
- <u>Extremely asymmetric systems</u>:
  $\longrightarrow$ **Fast absorption** of the smaller fragment
  ($R_1 = R_0$, $R_2 = 0$ + no neck = **dinuclear**)



Mass asymmetry effects – ^{220}Th (Preliminary)

- **Two different mechanisms: modification of the fusion conditions.**

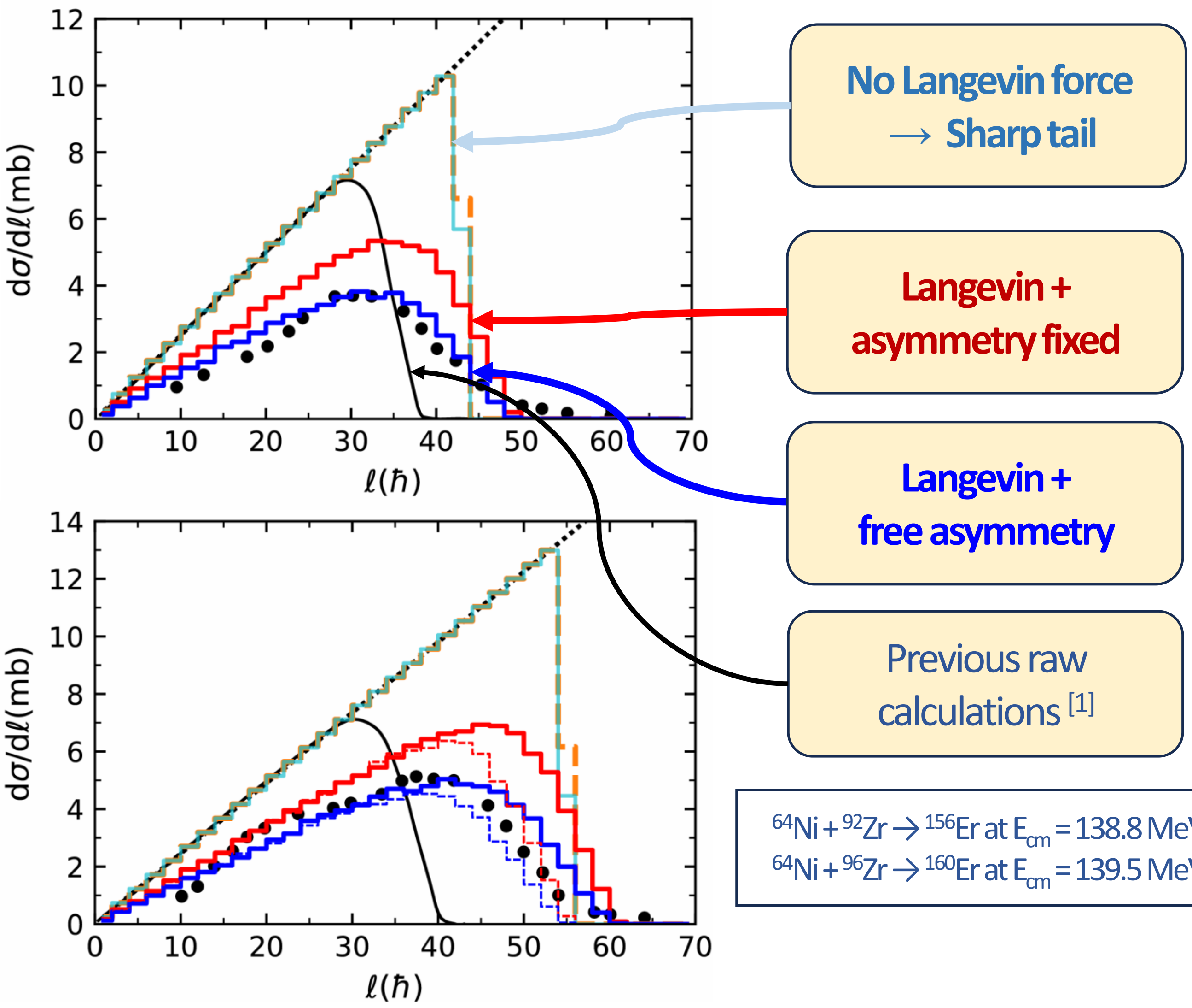
- **Cross sections decrease with Z_1Z_2**
- More symmetric system $\longrightarrow$ broader spin distributions (higher energies)



Insights into earlier results

Y. Jaganathen, M. Kowal, K. Pomorski, Phys. Lett. B 862 (2025) 139302

Open question: **Why do the cross sections go down when asymmetry is a free variable?**



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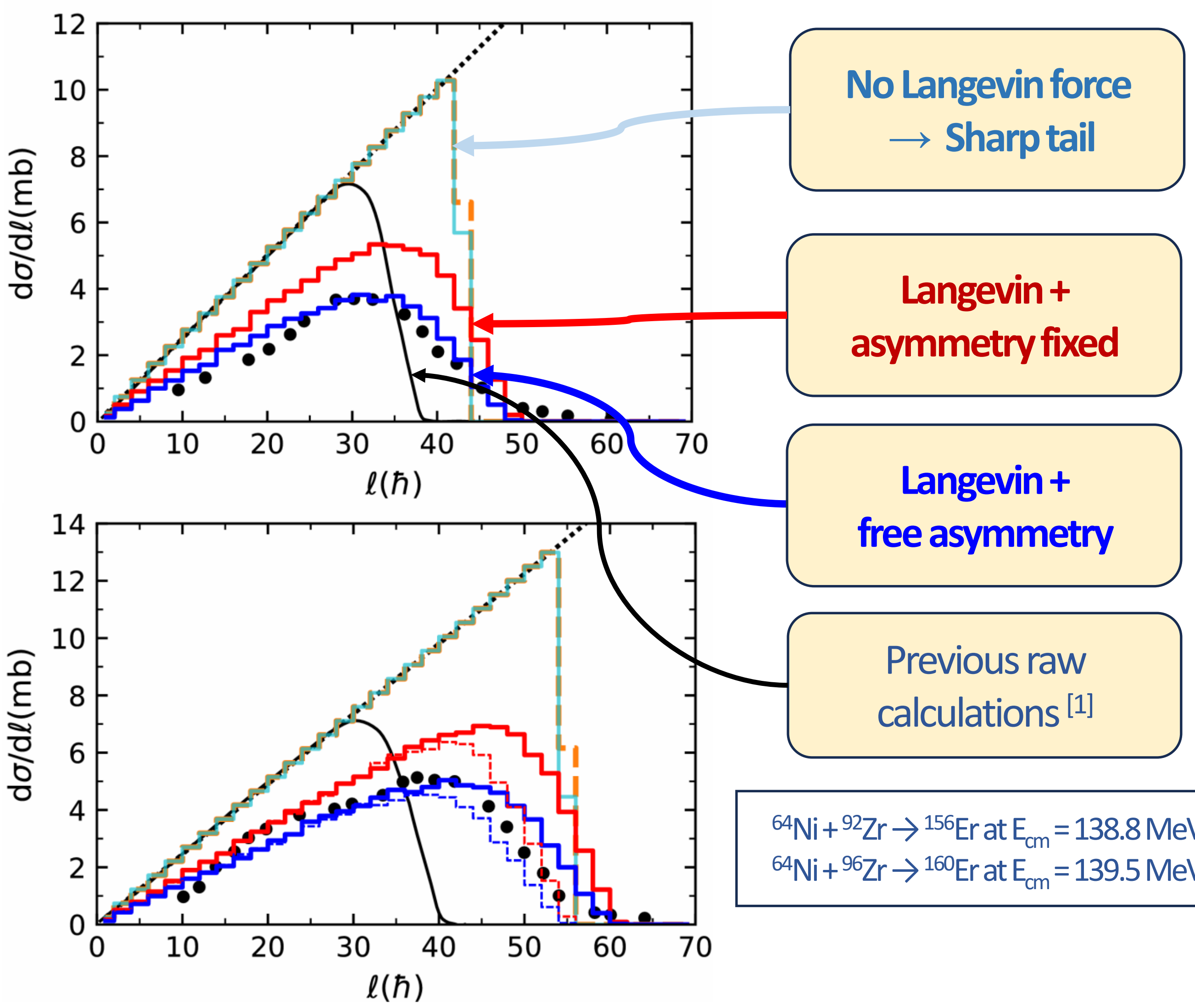
- The **equilibration by symmetrization** is a **slow process**
  → **Favors quasi-fission.**



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Summary and Perspectives

- We have derived a fully **6-dimensional dissipative dynamics Langevin**-based formalism to describe the **unrestricted motion of the systems** in terms of **elongation**, **neck** and **asymmetry** variables.

- Thanks to a **correct treatment of the different stages of fusion**, the spin distributions for the Ni + Zr reactions are **in great agreement with experimental data**.

- We are currently studying the **entrance channel asymmetry effects** in many reactions leading to ^{220}Th.
- The Langevin formalism allows for a deep understanding of the fusion process: in particular, depending on the asymmetry of the systems, **two different modes of fusion (symmetrization and absorption) emerge**.

- We are planning to make the following improvements of the formalism:
  - The addition of **shell effects and deformations/orientations** for a **fully microscopic-macroscopic picture**
  - The testing of different forms of stochastic noises (**color noises**),
  - The incorporation of **neutron emission** throughout the process.

- Ultimately, we aim to tackle the **hindrance problem** that prevents the formation of superheavy elements.

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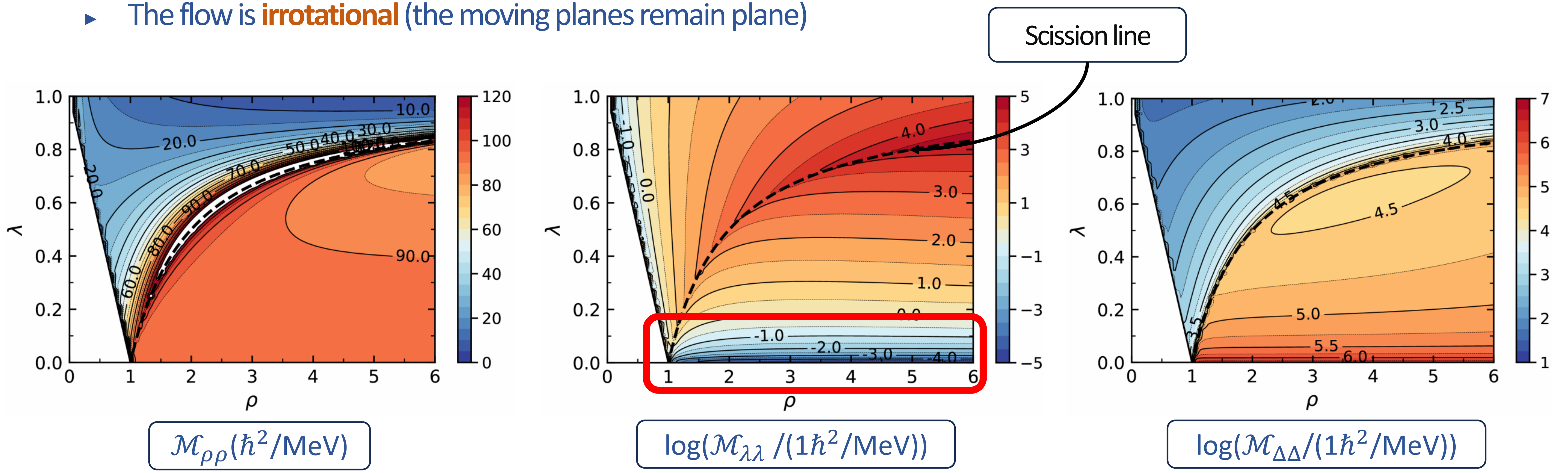
Thank you for your attention!

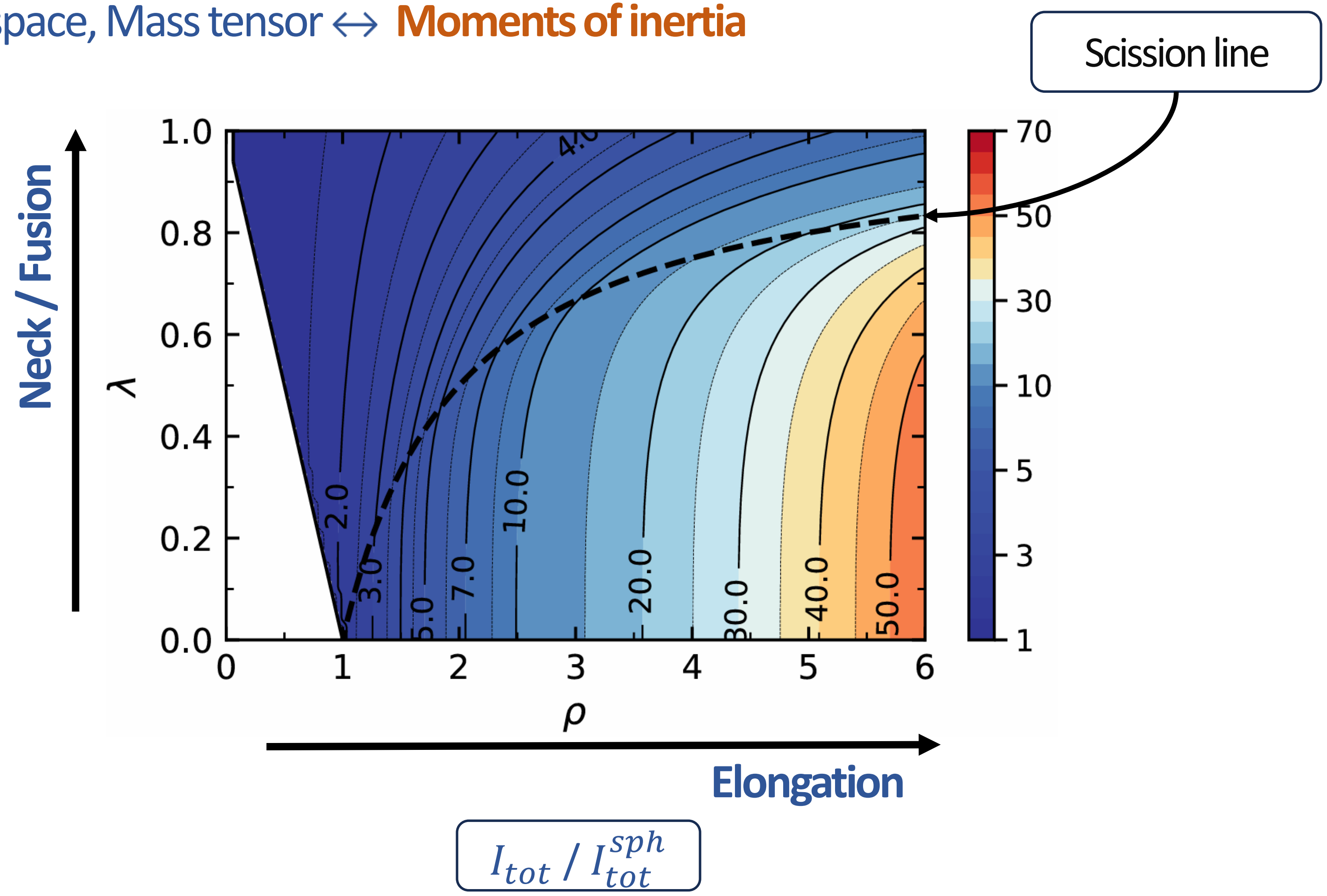
Mass tensor / Kinetic Energy

- **Werner-Wheeler flow approximation**:
  - **Incompressibility** (matter density is uniformly distributed)
  - The flow is **irrotational** (the moving planes remain plane)



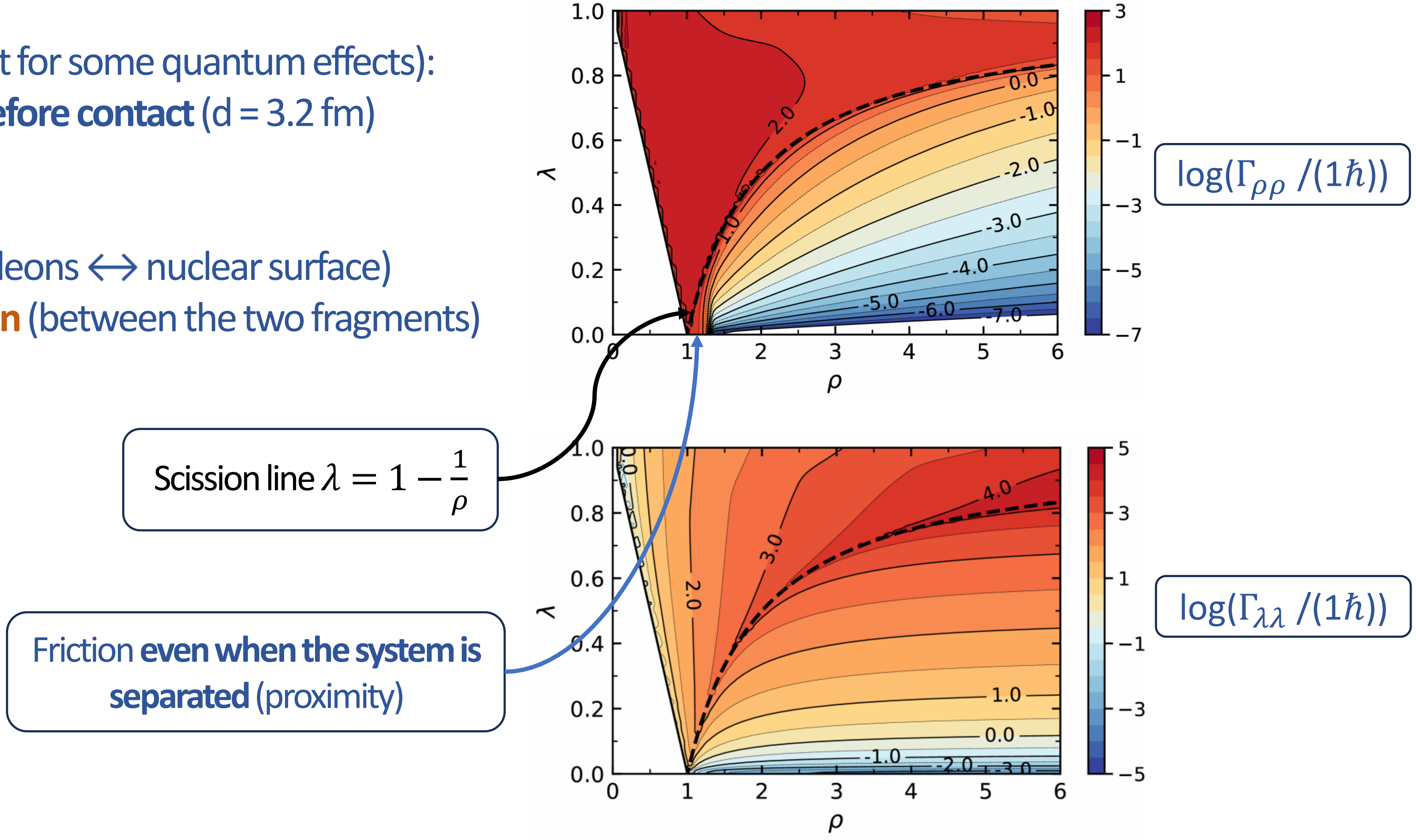
Mass tensor / Kinetic Energy

- In the rotational space, Mass tensor ↔ **Moments of inertia**



Friction forces

- **Proximity formalism** (to account for some quantum effects): **Possible matter flow/friction before contact** (d = 3.2 fm)
- **Shape friction**:
  - **Wall friction** (collisions nucleons ↔ nuclear surface)
  - + **Wall-plus-window friction** (between the two fragments)



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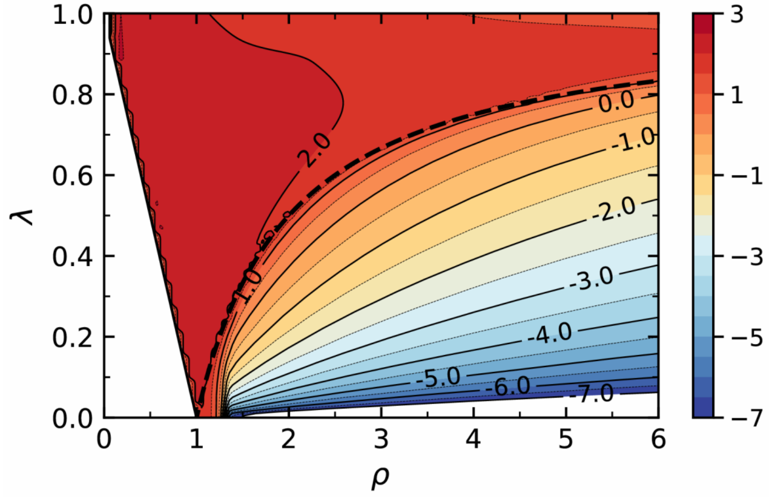
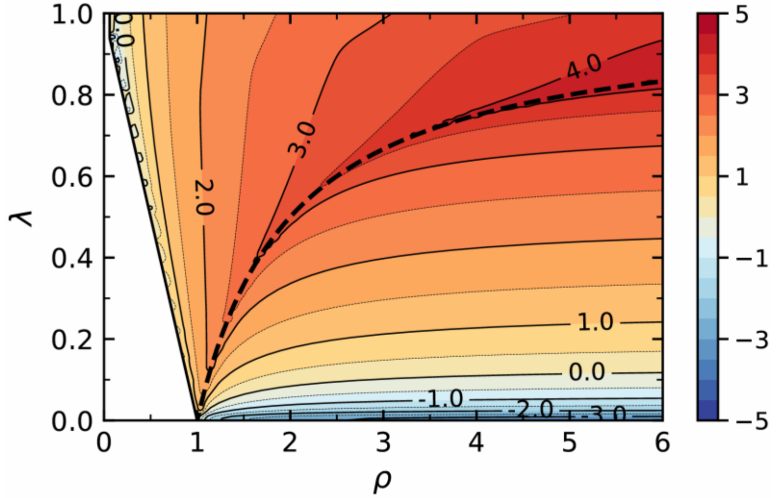
- **Shape friction**:
  - **Wall friction** (collisions nucleons ⟷ nuclear surface)
  - + **Wall-plus-window friction** (between the two fragments)
- **Strong friction at the scission line**:
  - **The system slows down at the scission line**
  - **Re-separation is irreversible**
- **Angular friction**:
  - **Sliding friction**
  - **No rolling friction**



$\log(\Gamma_{\rho\rho}\,/(1\hbar))$



$\log(\Gamma_{\lambda\lambda}\,/(1\hbar))$

Langevin/random forces

- We assume a simple memoryless Langevin force (white noise):

$$F_i = \sum_k g_{ik}\xi_k(t)$$

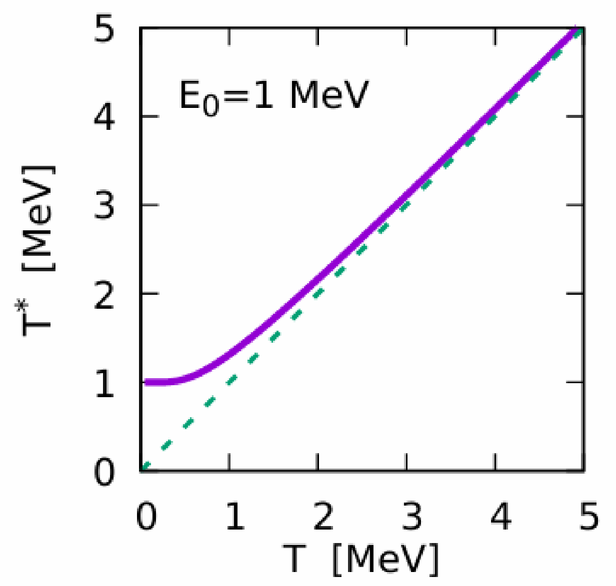
$\xi_k(t)$ are **time-dependent Gaussian random variables:**

$$\langle \xi_k(t)\rangle = 0$$
$$\langle \xi_k(t), \xi_{k'}(t')\rangle = 2\delta_{kk'}\delta(t-t')$$

- The diffusion tensor is given by the **Einstein relation**:

$$\sum_k g_{ik}g_{kj} = D_{ij} = k_B T^* \gamma_{ij}\,, \qquad T^* = \frac{E_0}{\tanh(E_0/T)}, \qquad T = \sqrt{E^*/a}$$

$E_0 = 2$ MeV is the zero-point collective energy of the heat bath oscillators
E^* is the **dissipated energy**, $a = A/8$ MeV is the **level density parameter**.



Quantum-corrected temperature
(Courtesy of K. Pomorski)

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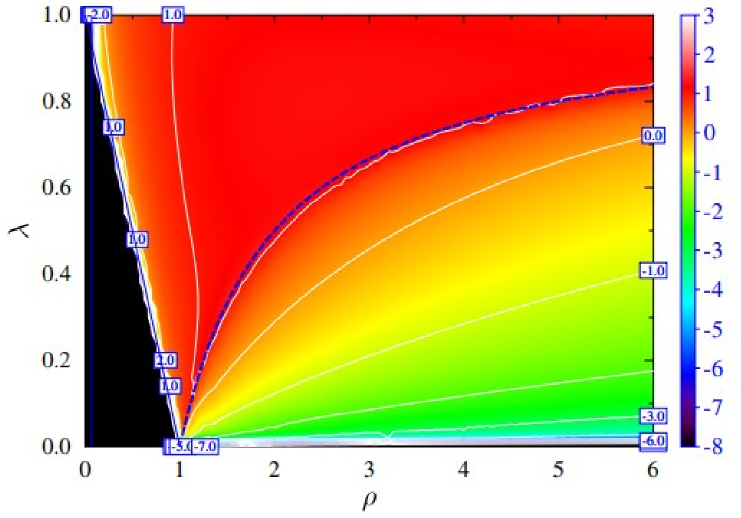
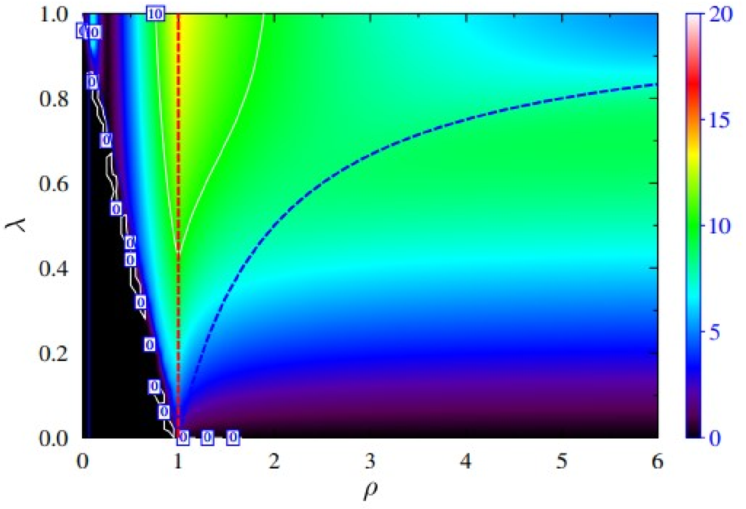


$\log(\hat{g}_{\rho\lambda})$

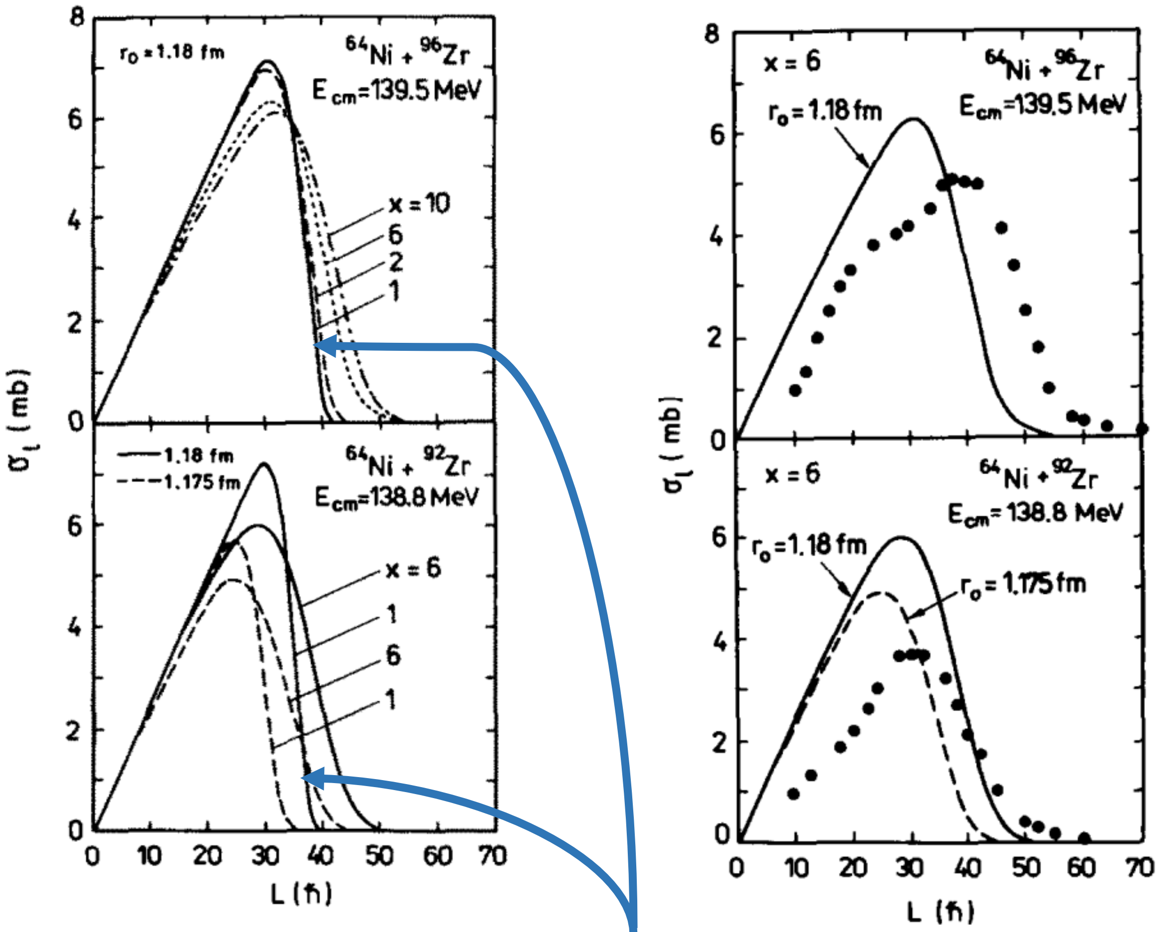


$g_{\theta_2\theta_2}$

A 30 year-old project

W. Przystupa, K. Pomorski, Nucl. Phys. A 572(1) (1994) 153

- Systems: $^{64}Ni + ^{92,96}Zr \rightarrow ^{156}Er$
- **Minimal shells effects** at the incident energies (50 MeV).
- Calculations with the **asymmetry variable frozen**.
- **Correction of the diffusion tensor** by a factor 6 to reproduce the tails of the spin distributions
- **Possible improvement**:
  - **Full calculation** with asymmetry needed



Raw results

Physical vs. practical collective variables

- Unlike the multipole moments, the (ρ, λ, σ) variables are **extremely irregular**:
  - **Many borders,**
  - **Small proximity region,**
  - **Regime change.**

- **A correct treatment of numerical precision is needed.**

- **Extrapolation** is needed for the calculations of the physical quantities and their derivatives.
  The regions of extrapolation should **avoid the borders.**

- The process of fusion itself is not fully tractable numerically.