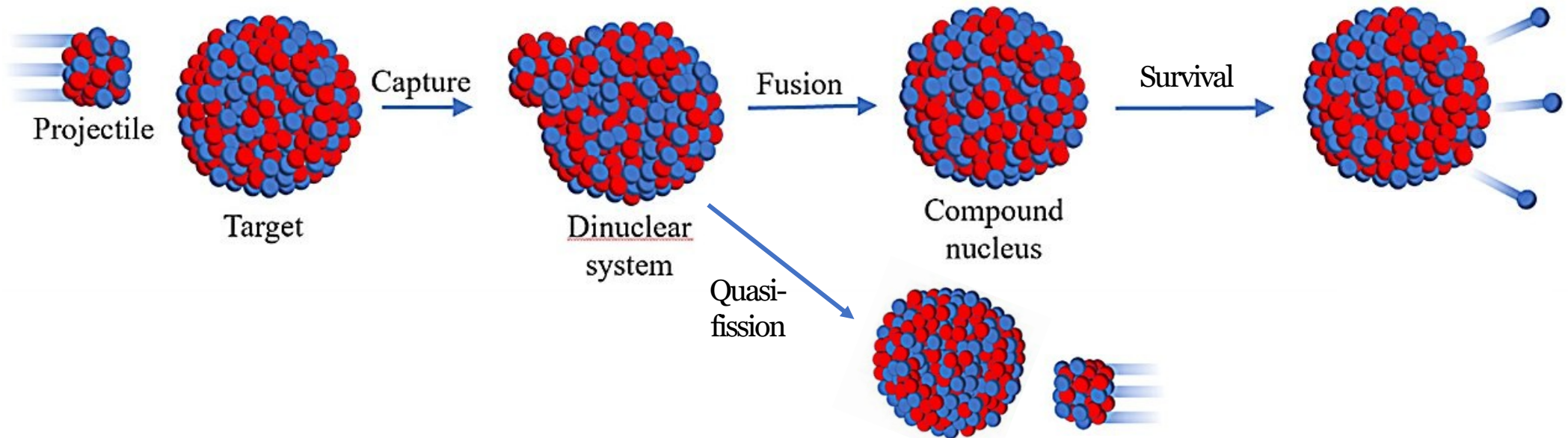


Description of the fusion mechanism in heavy ion collisions within full Langevin dissipative dynamics

Yannen Jaganathen (NCBJ, Poland),
Michał Kowal, Krzysztof Pomorski, Tomasz Cap (NCBJ, Poland)



The fusion process (Schematic view)



Micro-macroscopic description of fusion:

- ▶ Complexity/impossibility of tracking all internal degrees of freedom
- ▶ Identification of **slow collective degrees of freedom** immersed in a **bath** of faster dynamics
- ▶ Emergence of the mechanisms of **friction** and **random forces**
- ▶ Macroscopic model with some microscopic corrections.

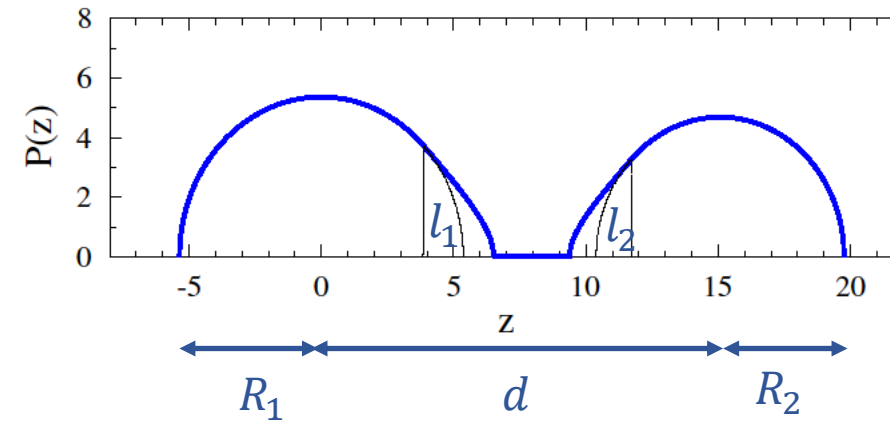
Collective variables adapted to fusion/fission – Shape variables

- ▶ Axially symmetric shapes
- ▶ **Spherical** cups connected by quadratic surfaces^[1]

▶ **Shape collective/slow variables:**

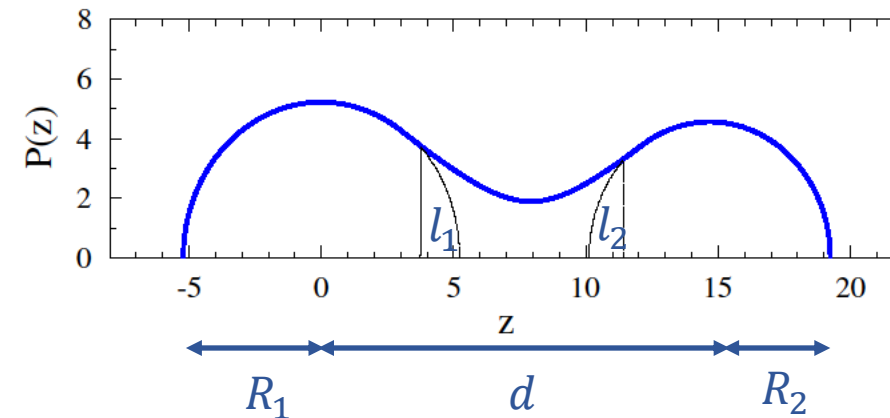
- ▶ **Distance/elongation :** $\rho = \frac{d}{R_1 + R_2}$
- ▶ **Neck/deformation :** $\lambda = \frac{l_1 + l_2}{R_1 + R_2}$
- ▶ **Asymmetry:** $\Delta = \frac{R_1 - R_2}{R_1 + R_2}$

$^{92}\text{Zr} + ^{64}\text{Ni}$



Bipartite

$$\begin{aligned}\rho &= 1.5 \\ \lambda &= 0.3 \\ \Delta &= \Delta_0\end{aligned}$$



Monopartite

$$\begin{aligned}\rho &= 1.5 \\ \lambda &= 0.4 \\ \Delta &= \Delta_0\end{aligned}$$

[1] J. Błocki, H. Feldmeier and W. J. Świątecki, Nucl. Phys. A 459 (1986) 145

Collective variables adapted to fusion/fission – Shape variables

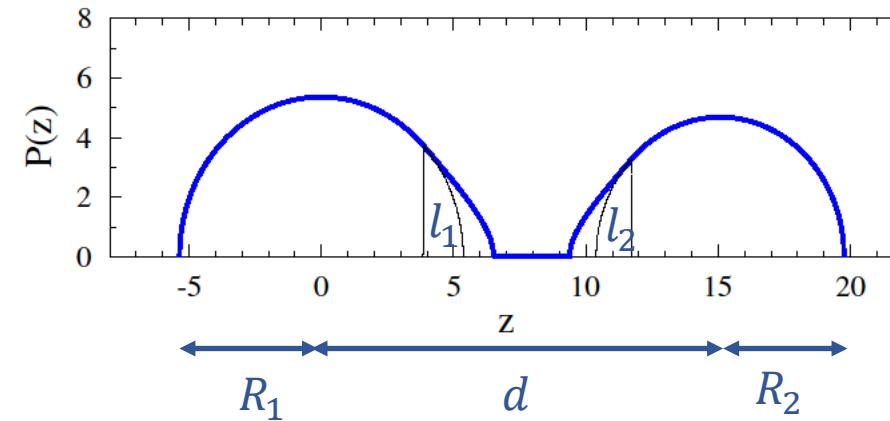
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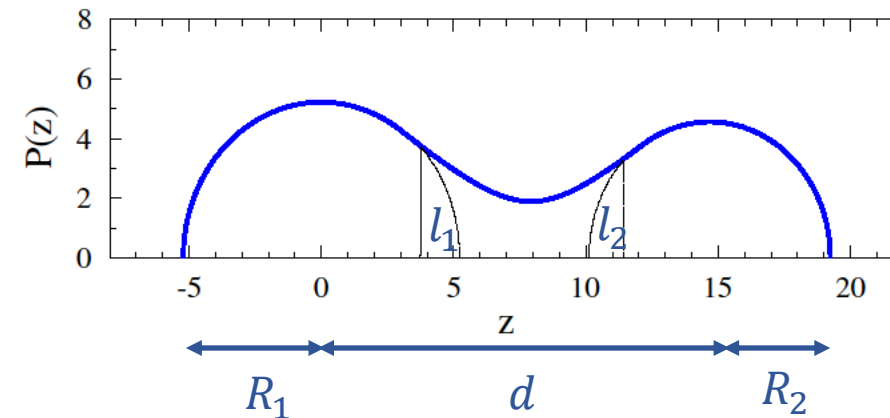
- ▶ **Scission is well-defined**: $\lambda_{\text{scission}} = 1 - \frac{1}{\rho_{\text{scission}}}$
 → Suited to describe fusion/fission
 (vs. multipole moments)

$^{92}\text{Zr} + ^{64}\text{Ni}$



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Collective variables adapted to fusion/fission – Angle variables

- ▶ **Collective angle variables**

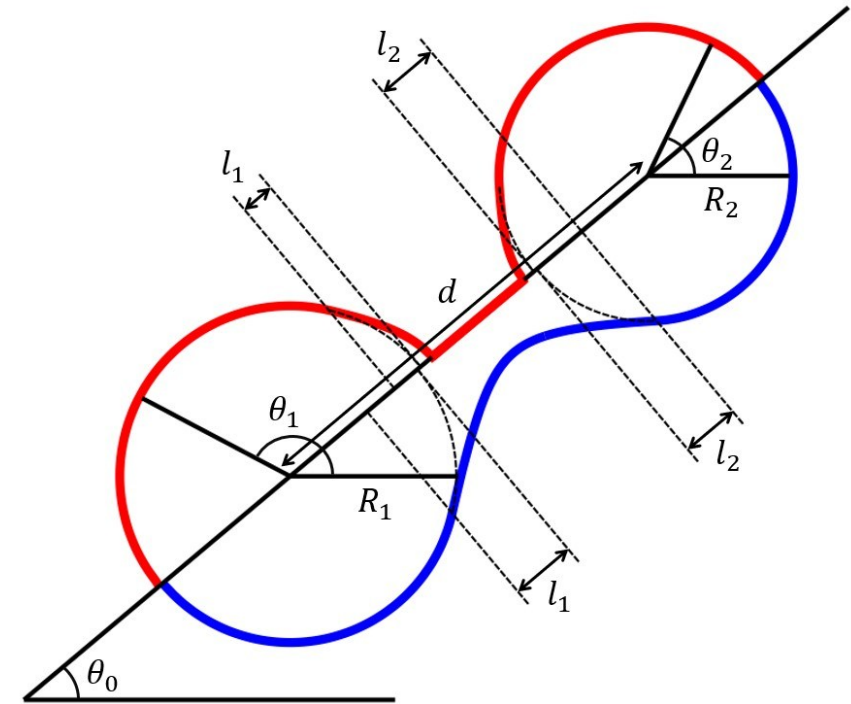
- ▶ Angle of the whole system θ_0
- ▶ Angle of the first sphere θ_1
- ▶ Angle of the second sphere θ_2

- ▶ Variations linked to angular momentum, in particular:

$$p_{\theta_0} + p_{\theta_1} + p_{\theta_2} = L_{init}$$

- ▶ **Exact treatment of angular momentum**

→ **Full Langevin 6-dimensional dissipative dynamics**



The Langevin system of equations

- Denoting **collective/slow variables** $q_i(t)$ and their **associated moments** $p_i(t)$, the **Langevin equations** read:

$$\begin{aligned}\dot{q}_i(t) &= \sum_k (\mathcal{M}^{-1})_{ik} p_k && \Leftrightarrow (P = MV) \\ \dot{p}_i(t) &= -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik} \dot{q}_k + \sum_k g_{ik} \xi_k(t) && \Leftrightarrow \left(\frac{dP}{dt} = \sum F \right)\end{aligned}$$

The Langevin system of equations

- Denoting **collective/slow variables** $q_i(t)$ and their **associated moments** $p_i(t)$, the **Langevin equations** read:

The diagram illustrates the Langevin system of equations. At the top left, a light blue box labeled "Mass tensor" has an arrow pointing to the first equation. Below it, the equation is $\dot{q}_i(t) = \sum_k (\mathcal{M}^{-1})_{ik} p_k$, followed by the equivalence $\Leftrightarrow (P = MV)$. Below this, the second equation is $\dot{p}_i(t) = -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik} \dot{q}_k + \sum_k g_{ik} \xi_k(t)$. Three yellow boxes at the bottom are connected to this equation: "Conservative forces ($H = T + V$)" points to the $-\frac{\partial H}{\partial q_i}$ term, "Friction forces" points to the $-\sum_k \gamma_{ik} \dot{q}_k$ term, and "Langevin/random forces" points to the $\sum_k g_{ik} \xi_k(t)$ term. To the right of the second equation is the equivalence $\Leftrightarrow \left(\frac{dP}{dt} = \sum E \right)$.

$$\dot{q}_i(t) = \sum_k (\mathcal{M}^{-1})_{ik} p_k \quad \Leftrightarrow (P = MV)$$

$$\dot{p}_i(t) = -\frac{\partial H}{\partial q_i} - \sum_k \gamma_{ik} \dot{q}_k + \sum_k g_{ik} \xi_k(t) \quad \Leftrightarrow \left(\frac{dP}{dt} = \sum E \right)$$

Mass tensor

Conservative forces ($H = T + V$)

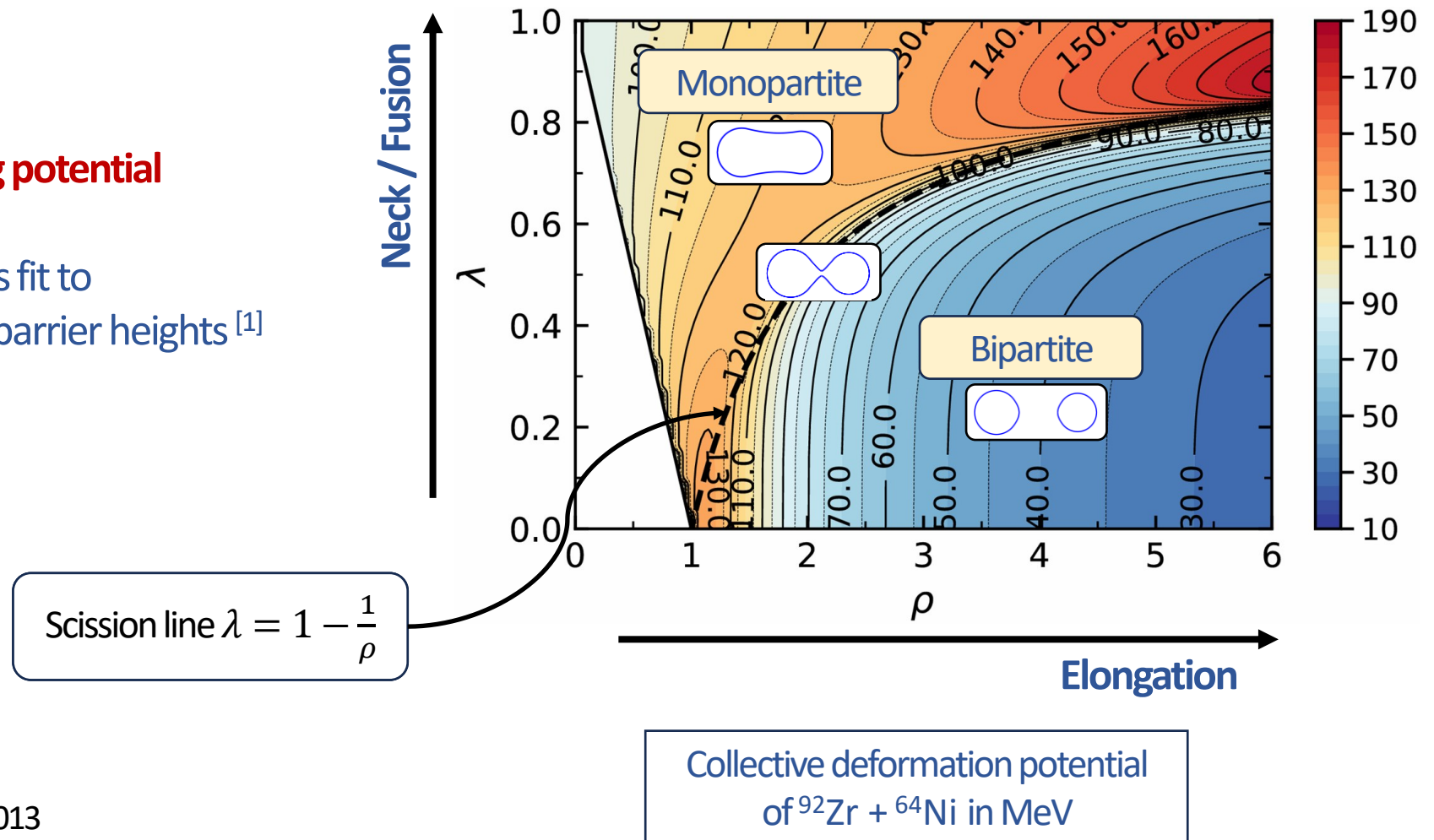
Friction forces

Langevin/random forces

→ A comprehensive understanding of the dynamical process.

Collective Potential Energy

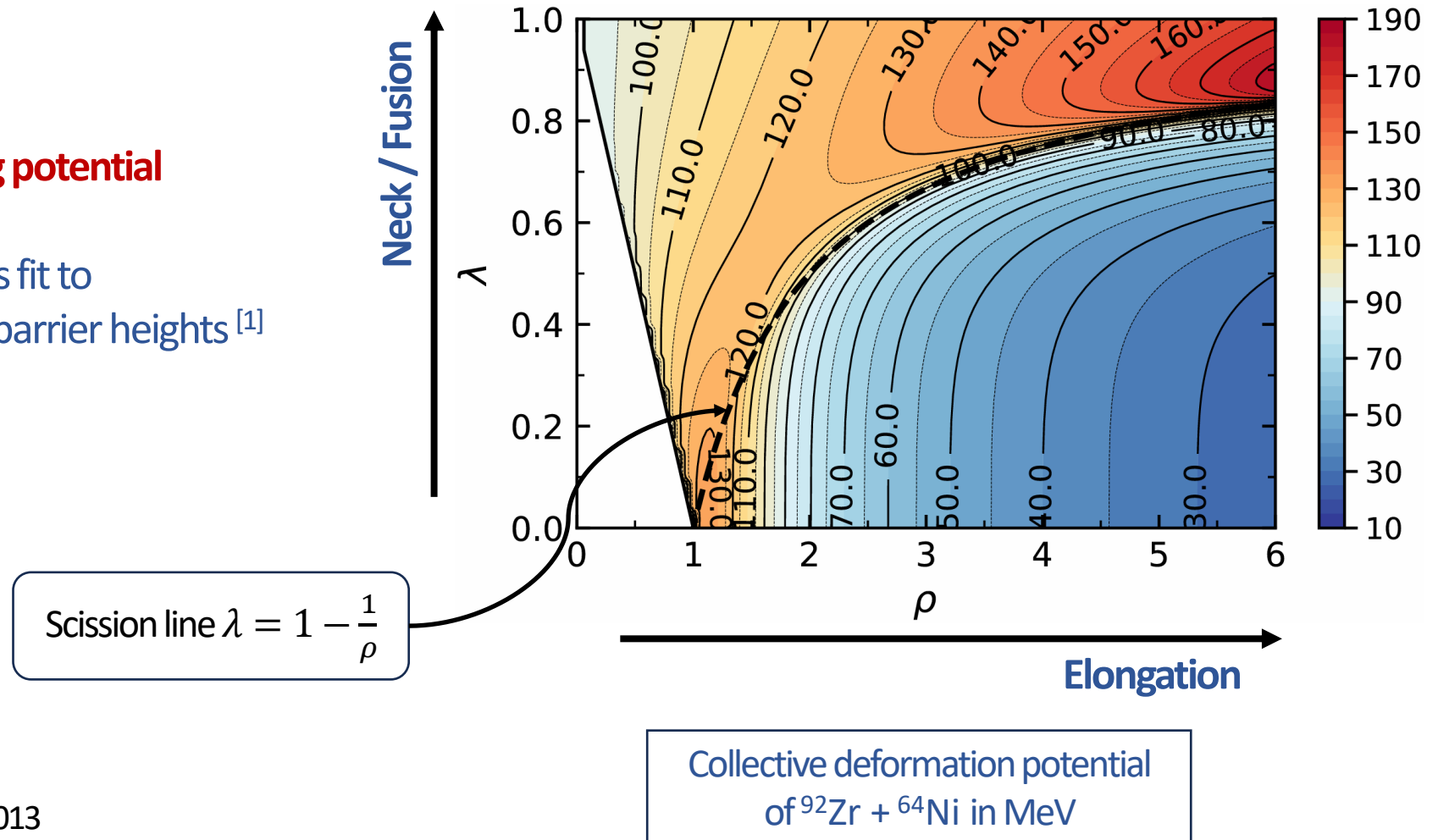
- ▶ **Yukawa-plus-exponential folding potential**
+ Coulomb potential
- ▶ Parameters taken from a previous fit to experimental masses and fusion barrier heights ^[1]
- ▶ **No shell effects at the moment.**



[1] H. J. Krappe *et al.*, Phys. Rev. C 20 (1979) 992–1013

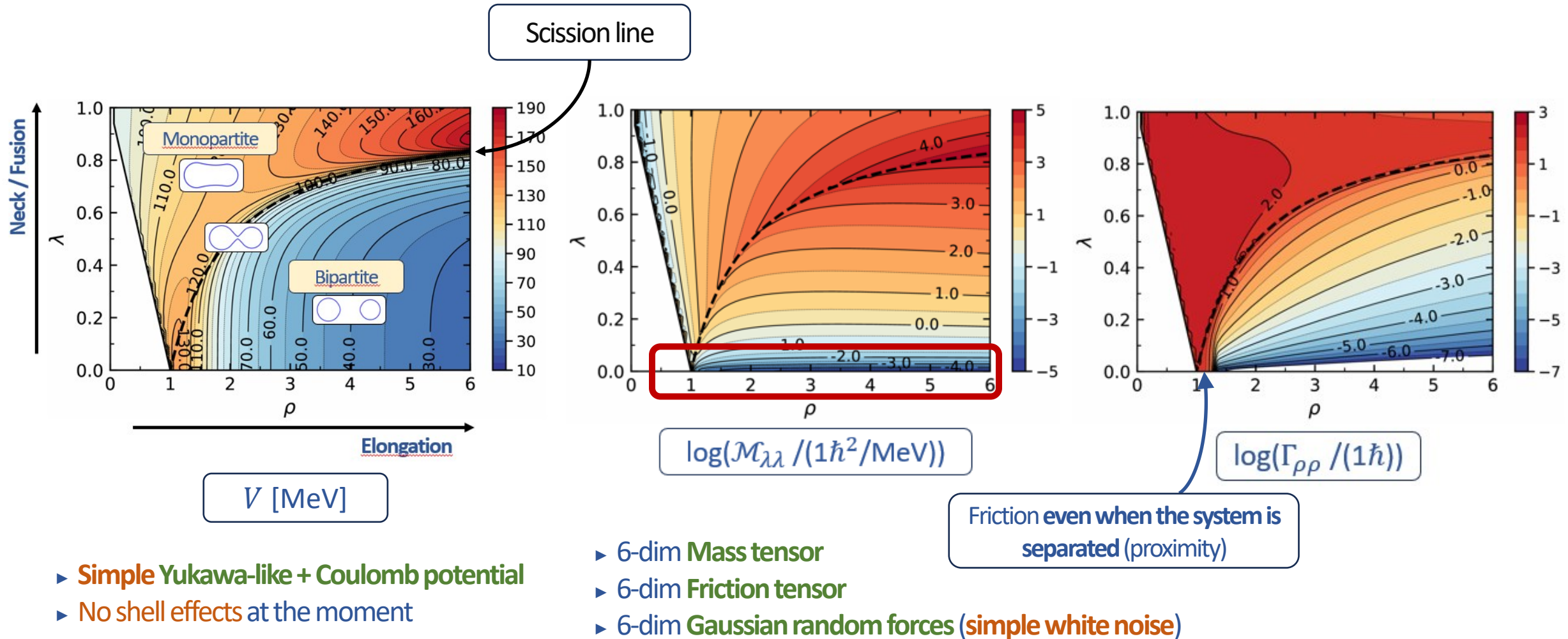
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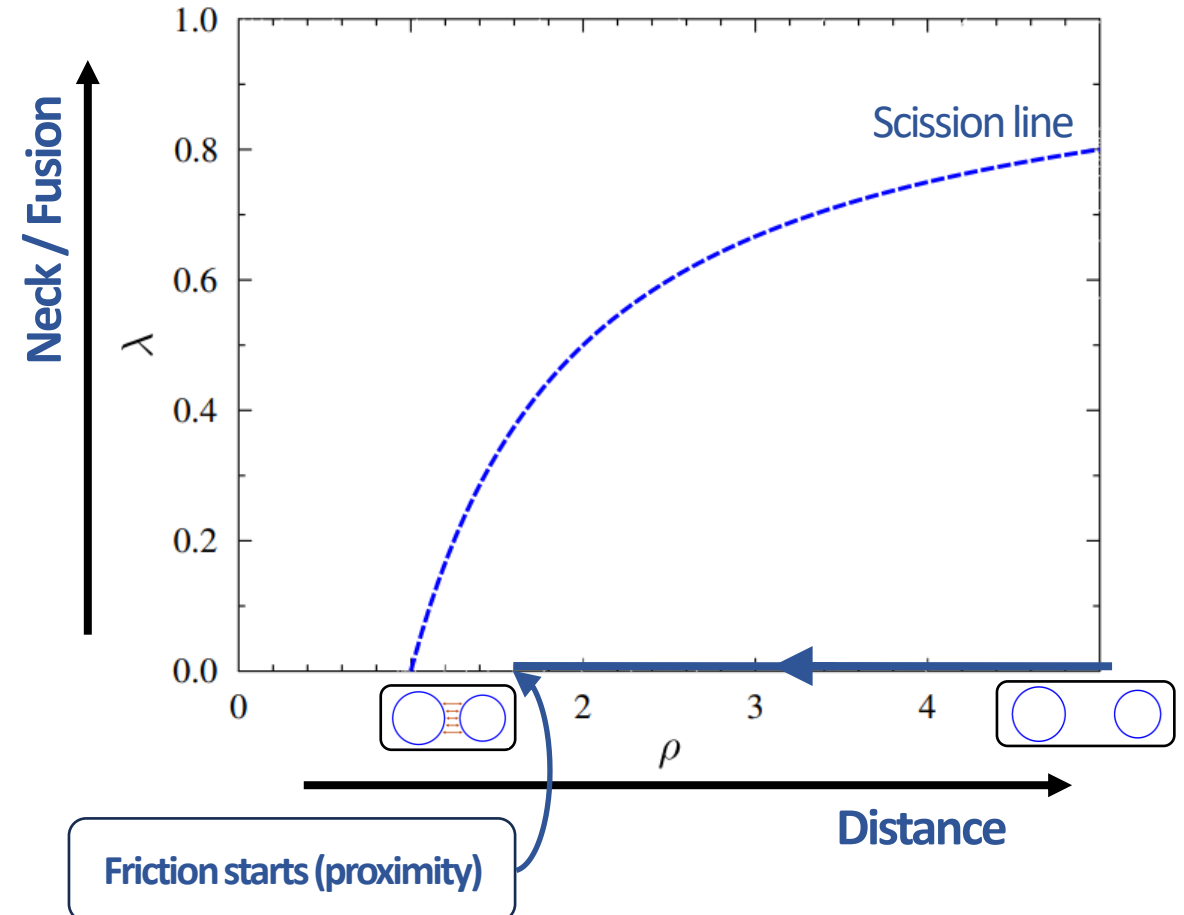
The Ingredients of the Langevin Formalism



The fusion process

The three main stages of the collision:

1. **A first violent deceleration** during which:
 - The **system loses most of its kinetic energy**
 - There is **almost no deformation of the nuclei**
$$\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda$$
$$= -\infty + \infty$$
 - **Unstable balance close to the lambda border**



The fusion process

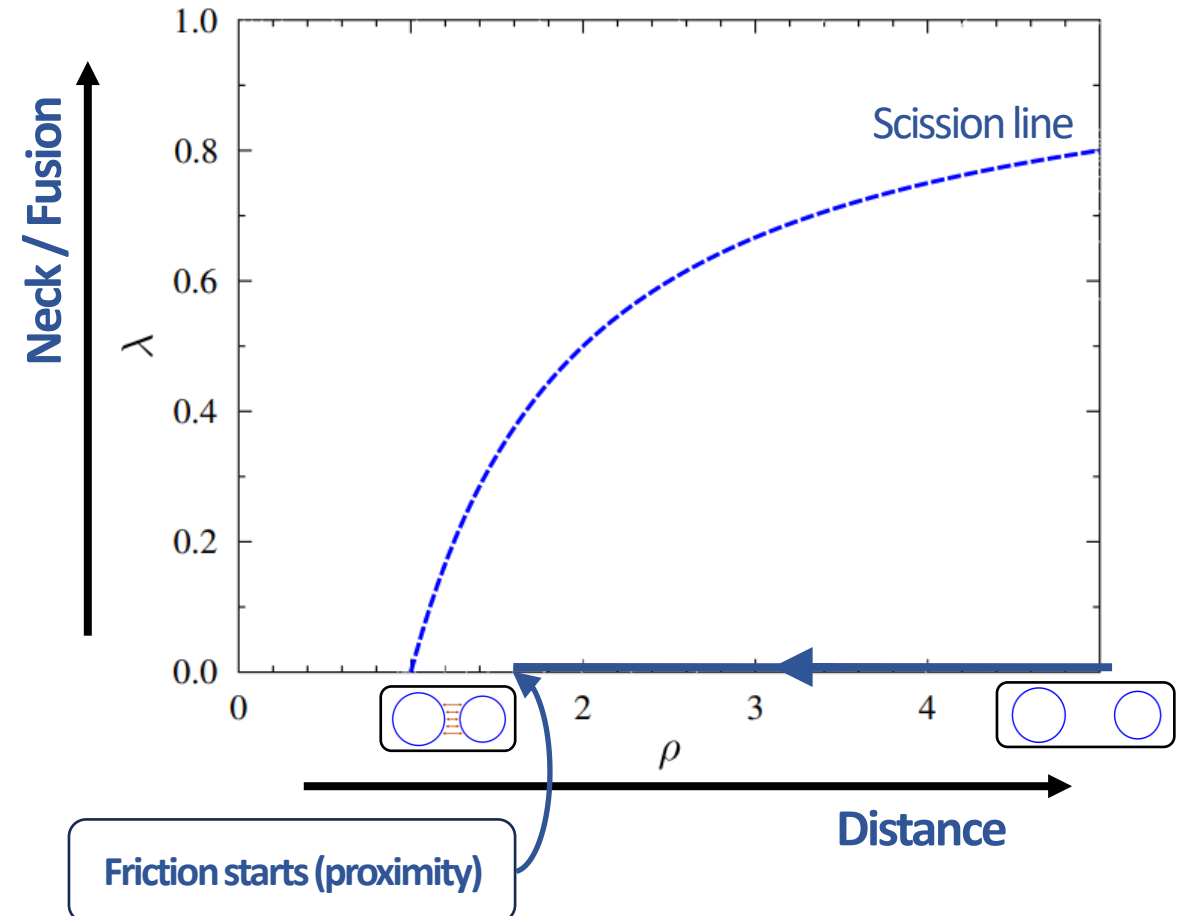
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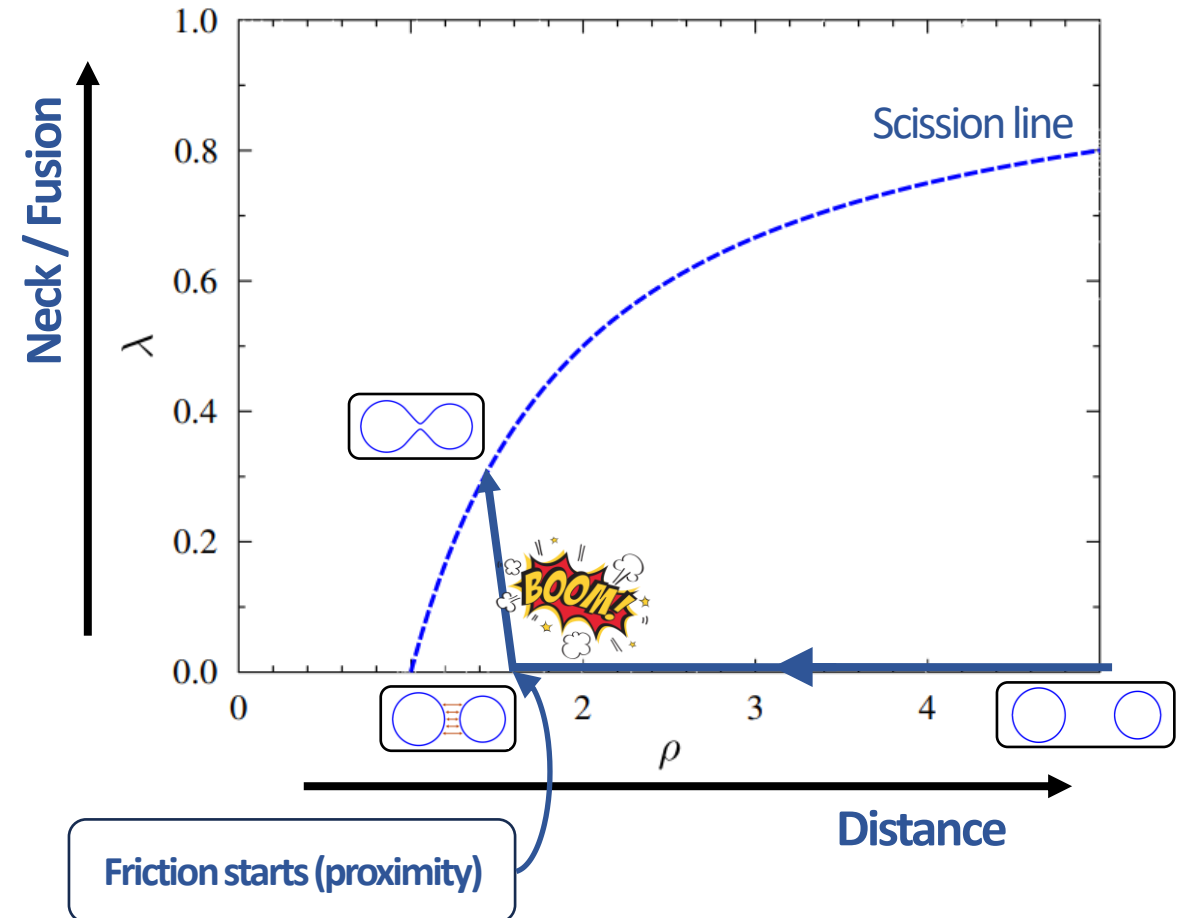
- **Unstable balance close to the lambda border**
- **Physical interpretation: (Extra)-deformation can only occur when the fragments interact with each other**
- **Treated exactly** (conservative forces).



The fusion process

2. The **"Kiss of death"** when friction starts

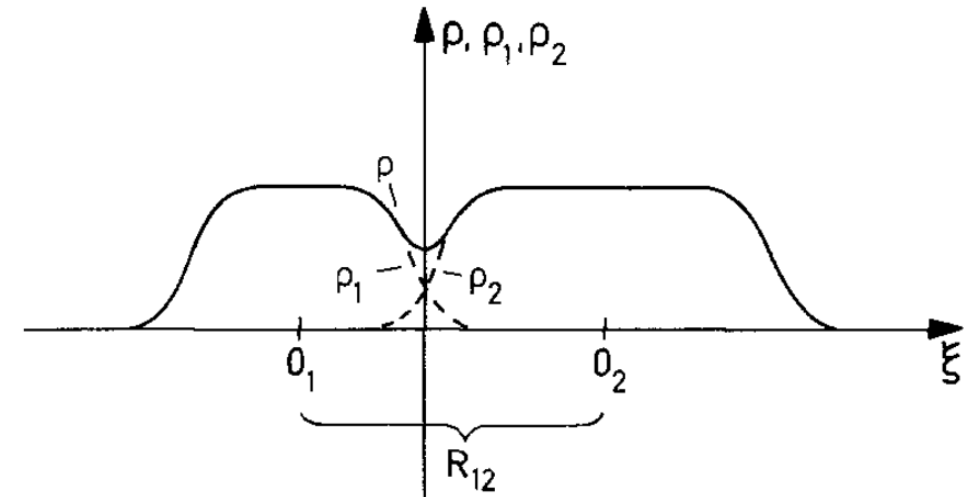
- A little sudden change in $p_\rho \rightarrow$ infinite push to the scission line ($\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda$
 $= -\infty + \infty$)
- Deformation starts and remains.



The fusion process

2. The **"Kiss of death"** when friction starts

- A little sudden change in $p_\rho \rightarrow$ infinite push to the scission line ($\dot{\lambda} = (\mathcal{M}_{\rho\lambda})^{-1} p_\rho + (\mathcal{M}_{\lambda\lambda})^{-1} p_\lambda = -\infty + \infty$)
- Deformation starts and remains.
- **Inherent instability of non-saturated nucleonic densities**
(Sudden approximation in HF when the nuclear tails touch)
- **This step is NOT treated numerically:**
We start the calculation from the touching point



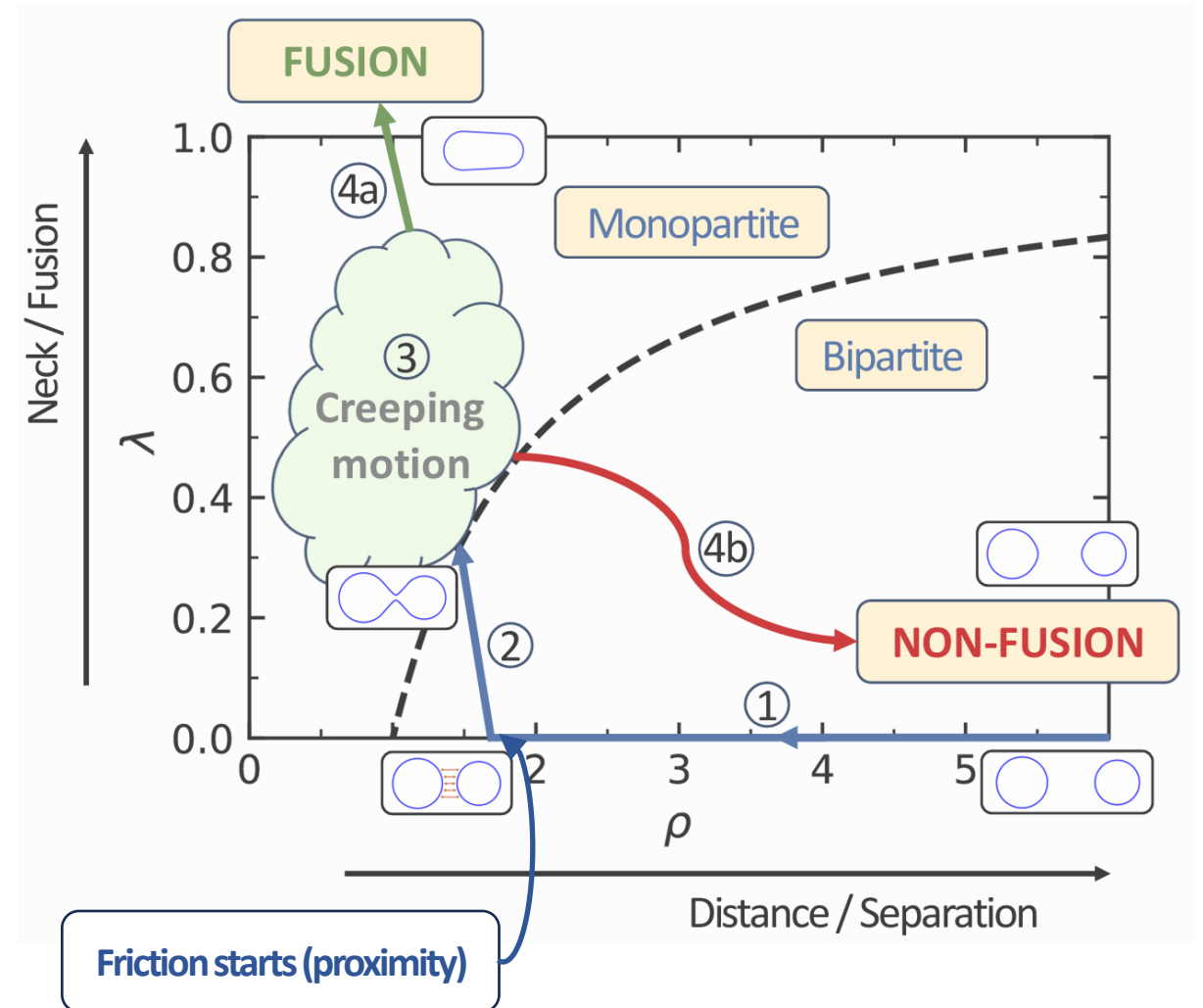
"Sudden approximation" – K. Pomorski, K. Dietrich
Z. Phys. A, 295 (1980) 335

The fusion process

The three main stages of the collision:

3. A **long creeping motion**

- which leads to fusion or separation
- **Solved numerically**



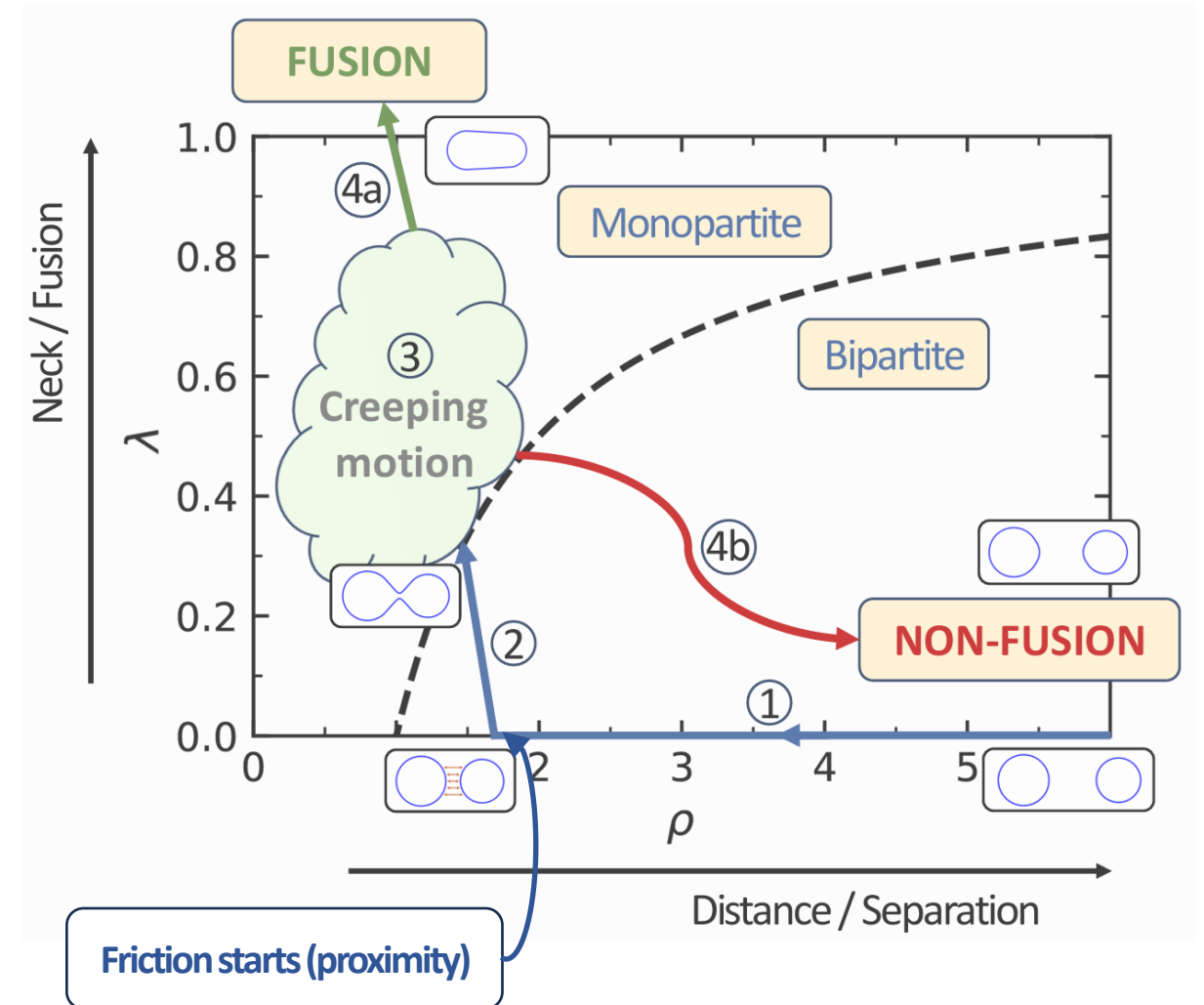
Physically-guided definition of fusing and non-fusing events

► Fusing conditions:

- $\lambda = 1$ (half of the spheres are mixed)
- $\rho(1 - \lambda) = \Delta^2$ (window angle fully open)
- $\rho = 0.5$

► Non-fusing conditions:

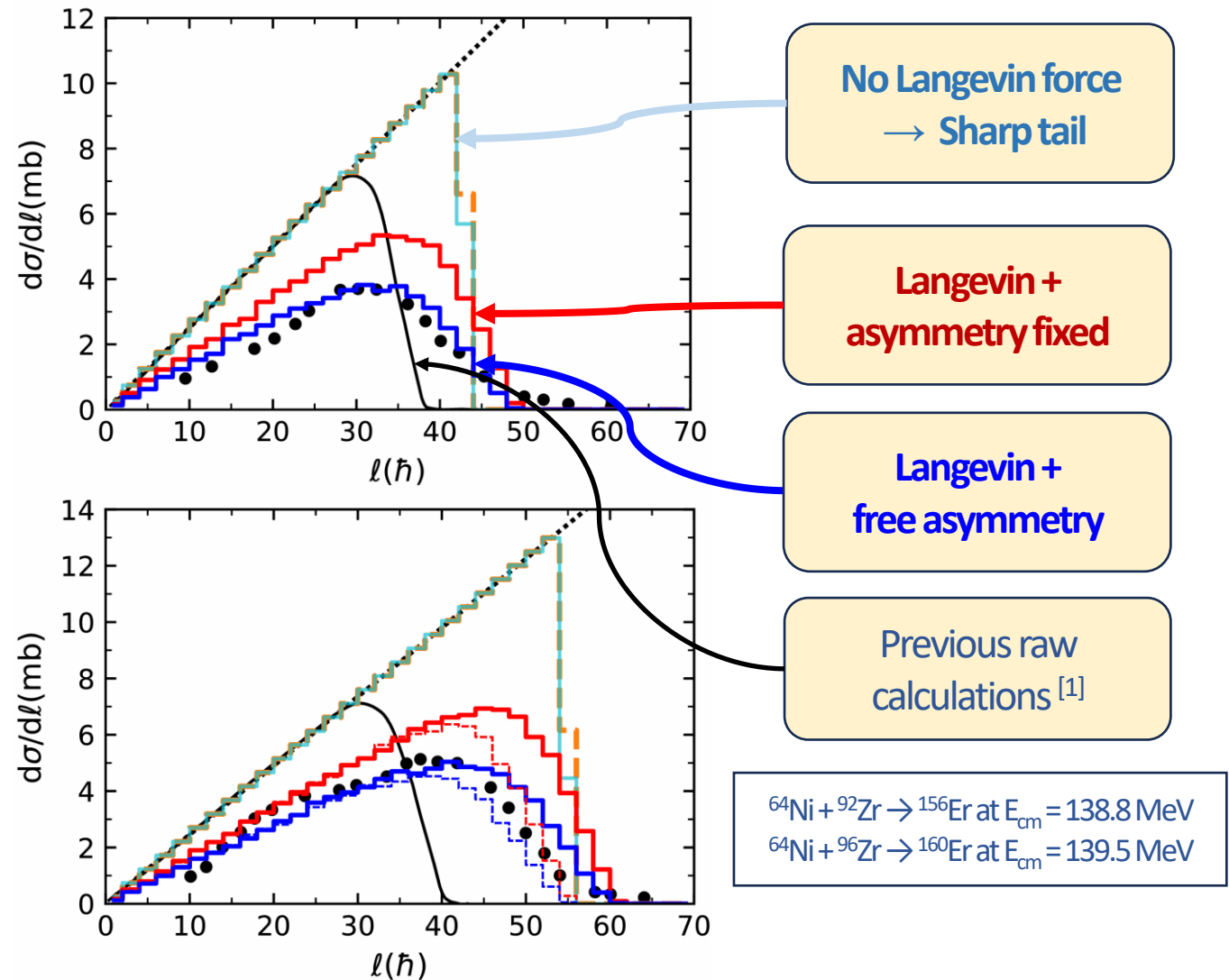
- $\lambda \rightarrow \lambda_{min} = 10^{-2}$
- $\rho \rightarrow \rho_{max} = 3$
- No fusion after $N_{max} = 500,000$ steps.



Pilot paper – “Symmetric” system

Y. Jaganathen, M. Kowal, K. Pomorski, Phys. Lett. B 862 (2025) 139302

- ▶ Spin distributions are more natural with the Langevin force.
- ▶ They drop at the correct angular momentum
- ▶ Great agreement to the experimental data^[2,3].



[1] W. Przystupa, K. Pomorski, Nud. Phys. A 572(1) (1994) 153

[2] W. Kuhn *et al.*, Phy. Rev. Lett. 62 (1989) 1103

[3] A. M. Stefanini *et al.*, Phys. Lett. B 252 (1990) 43

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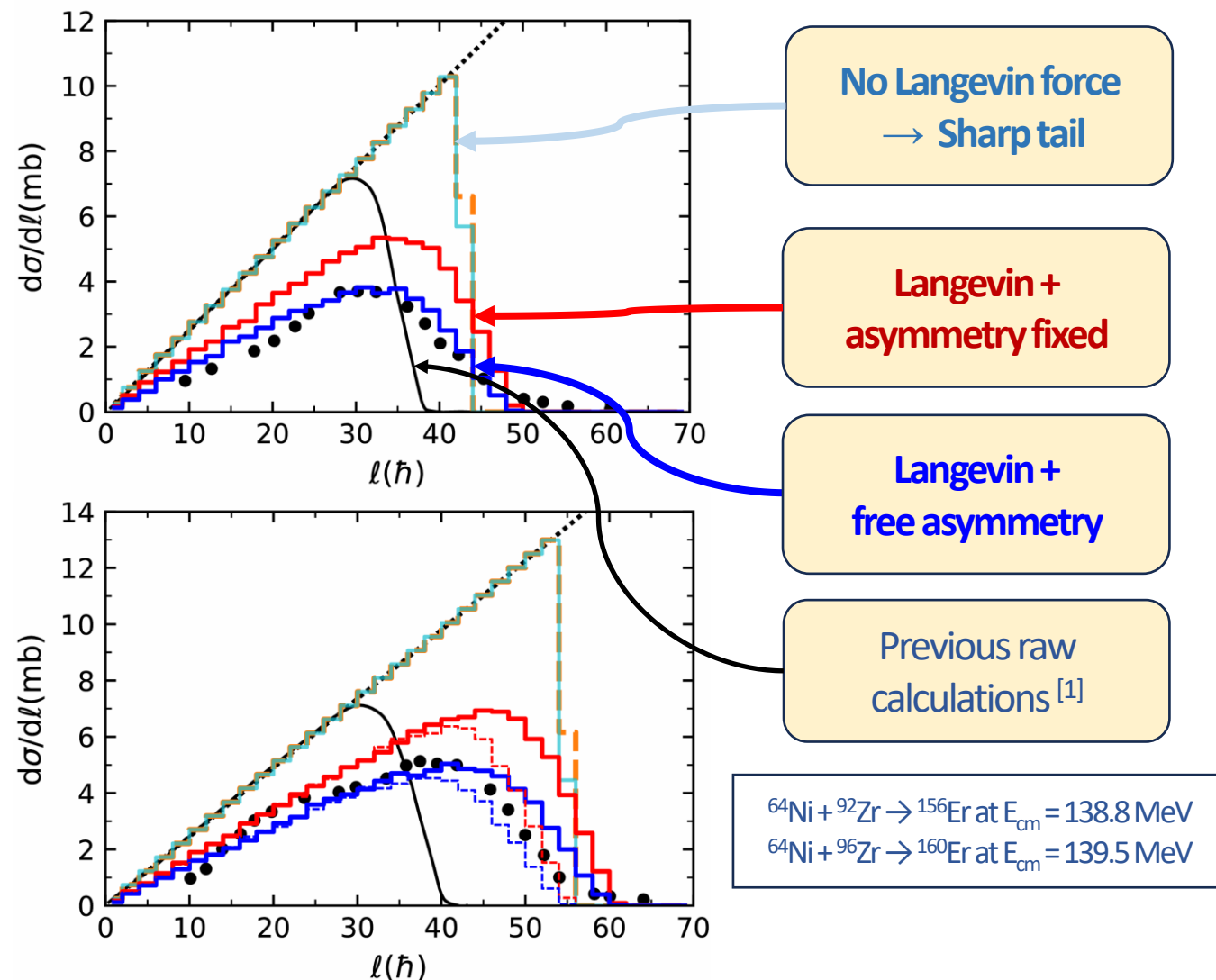
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- ▶ They drop at the correct angular momentum
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- ▶ Open question: **Why do the cross sections go down when asymmetry is a free variable?**

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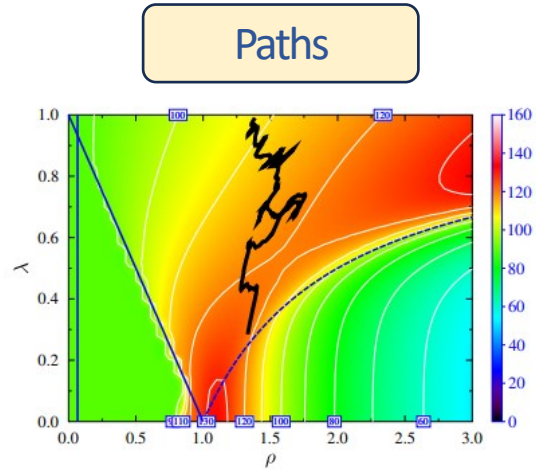
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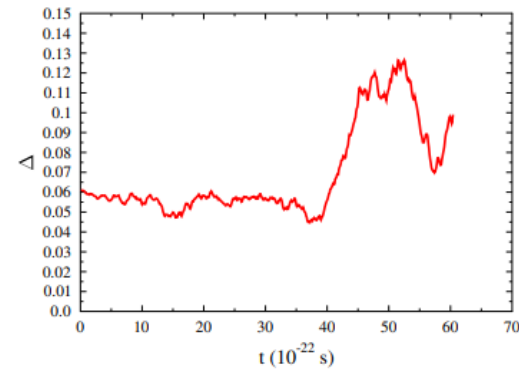


A deep understanding of the fusion process (Preliminary examples)

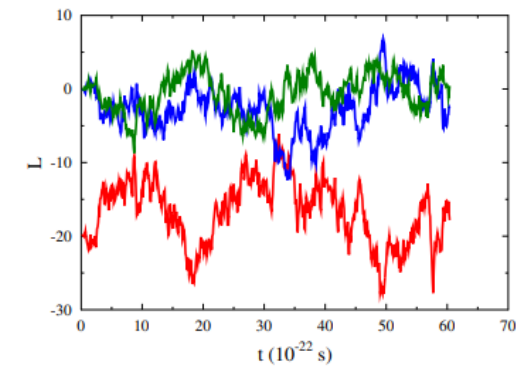
► $\ell = 20\hbar$,
Fusion case:



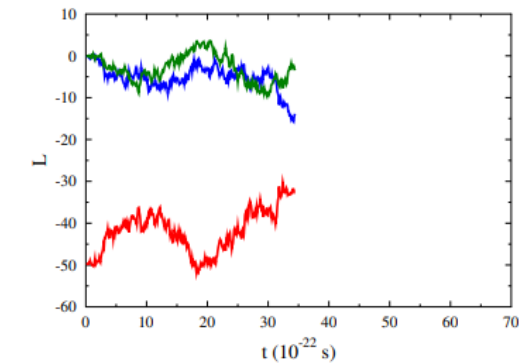
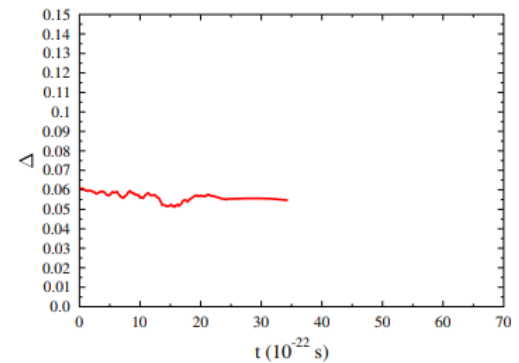
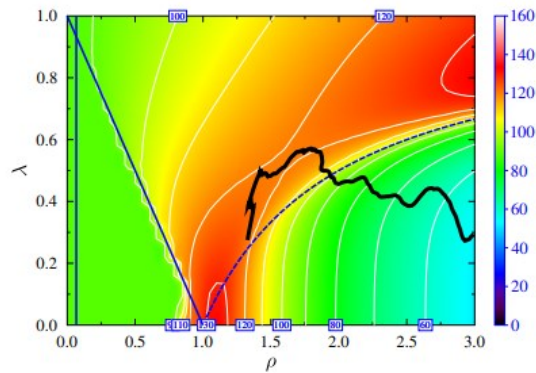
Asymmetry vs. time



Angular momenta



► $\ell = 50\hbar$,
No-fusion case:



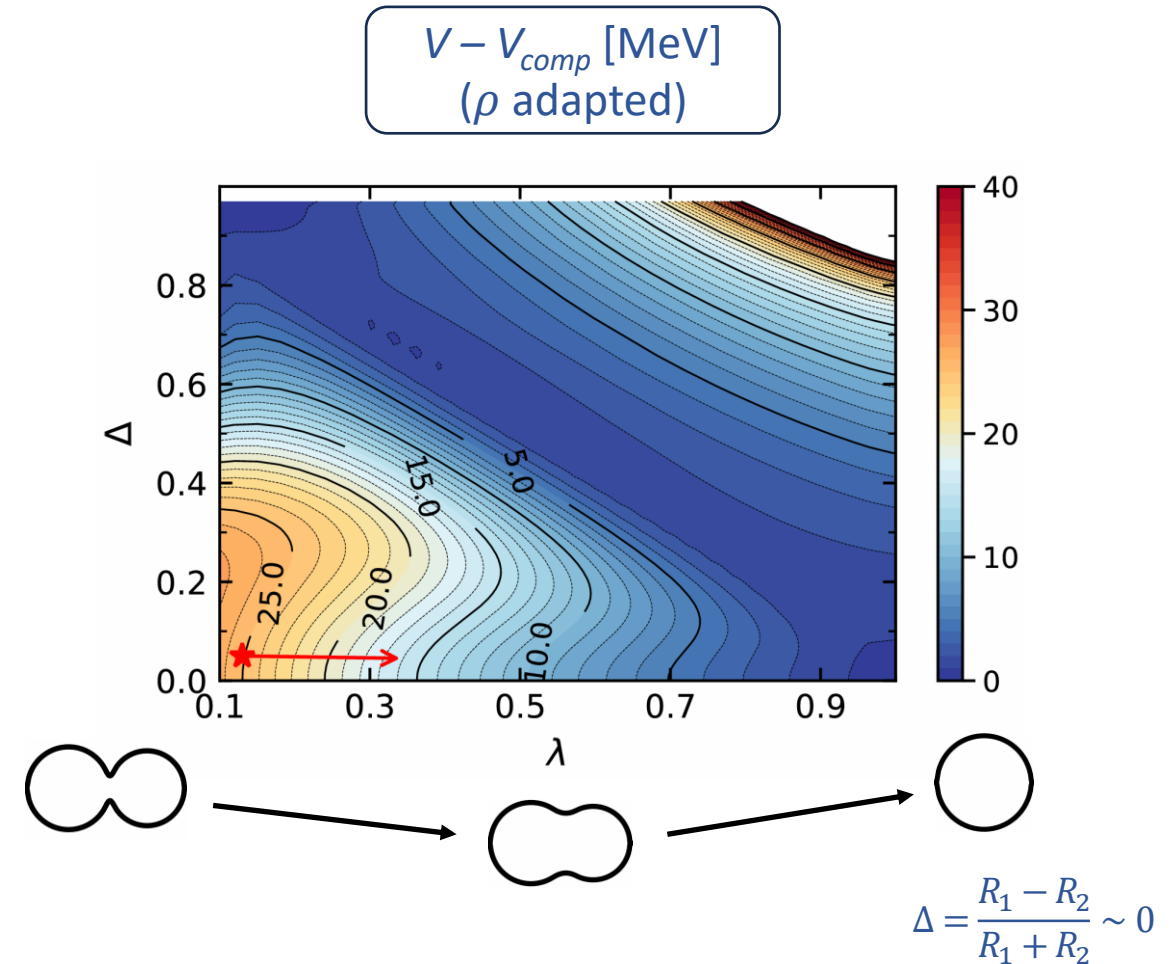
► And various other observables: **diffusion tensor**, the **number of rotations the system undergoes before fusion** etc.

Mass asymmetry effects – ^{220}Th (Preliminary)

- ▶ Several well-studied reactions leading to the same compound nucleus ^{220}Th :
 - ▶ $^{16}\text{O} + ^{204}\text{Pb} \rightarrow ^{220}\text{Th}$ [Experimental transmission = 100 %]
 - ▶ $^{40}\text{Ar} + ^{180}\text{Hf} \rightarrow ^{220}\text{Th}$
 - ▶ $^{48}\text{Ca} + ^{172}\text{Yb} \rightarrow ^{220}\text{Th}$
 - ▶ $^{96}\text{Zr} + ^{124}\text{Sn} \rightarrow ^{220}\text{Th}$
- ▶ Same excitation energy of the compound nucleus
- ▶ $E_{CN}^* = 50 \text{ MeV}$ (no pre-fusion neutron emission)

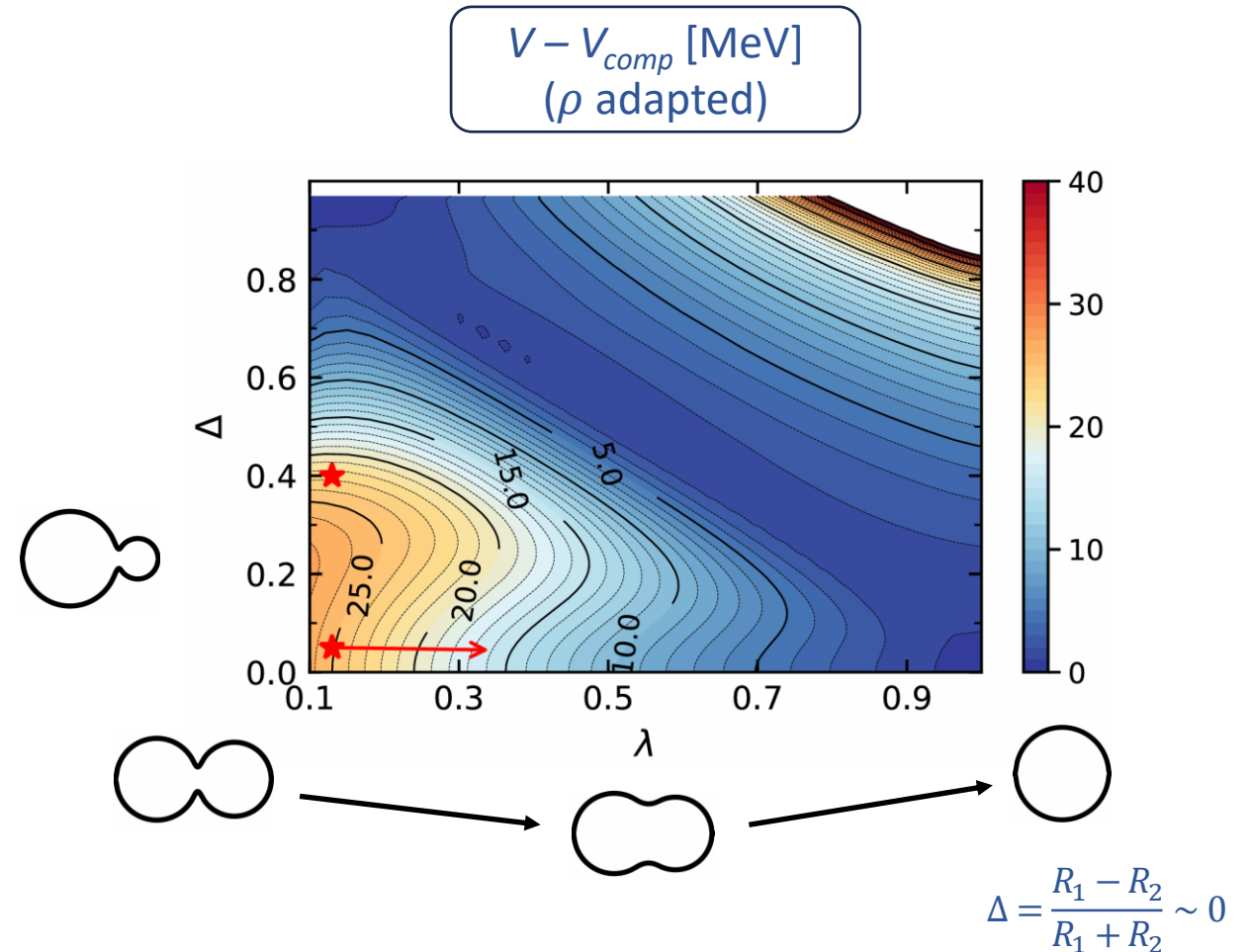
Two different fusion mechanisms

- Symmetric to quite asymmetric systems:
→ **Symmetrization of the shape/slow equilibration**
($R_1 = R_2 = R_0$ + large neck)



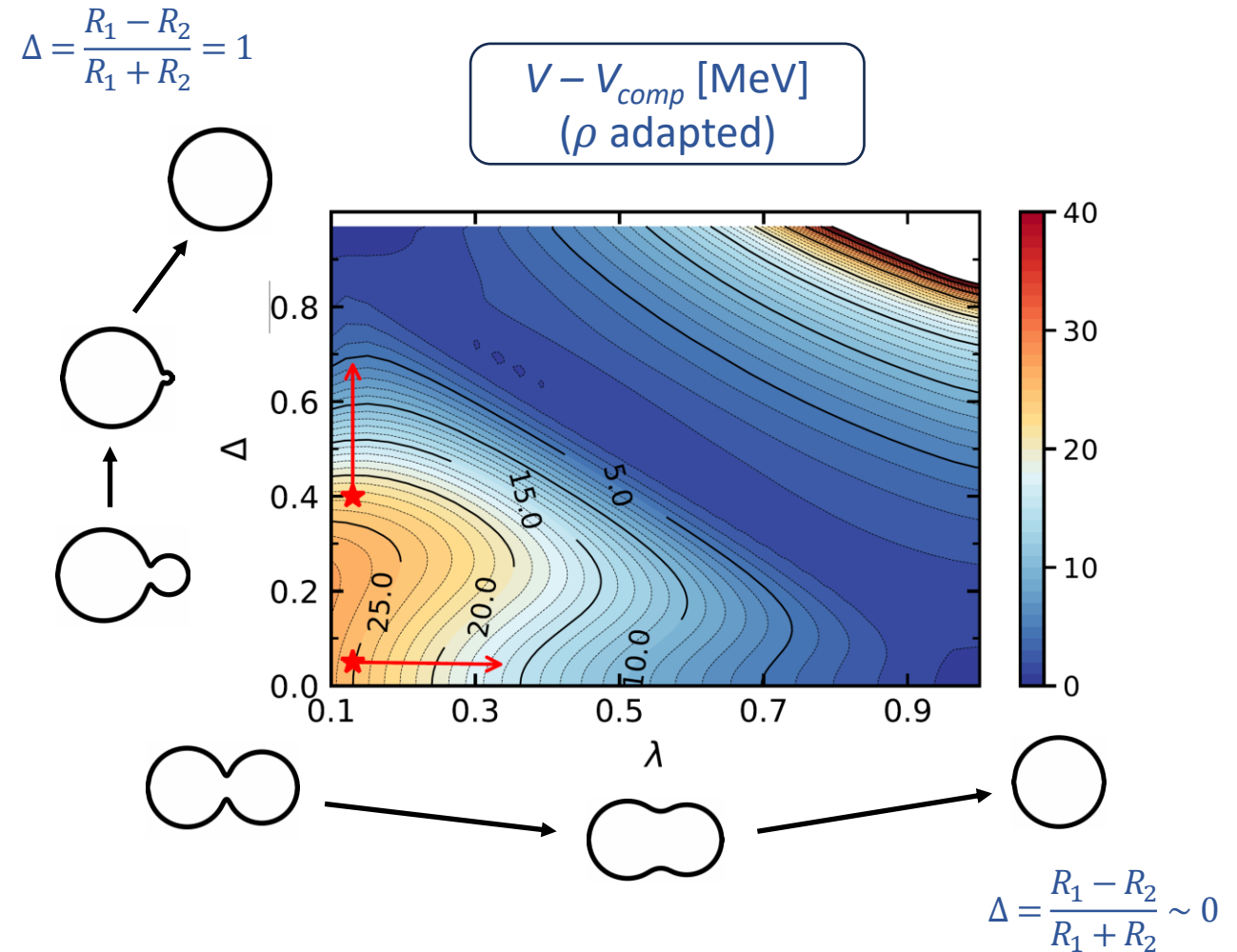
Two different fusion mechanisms

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- ▶ Extremely asymmetric systems:



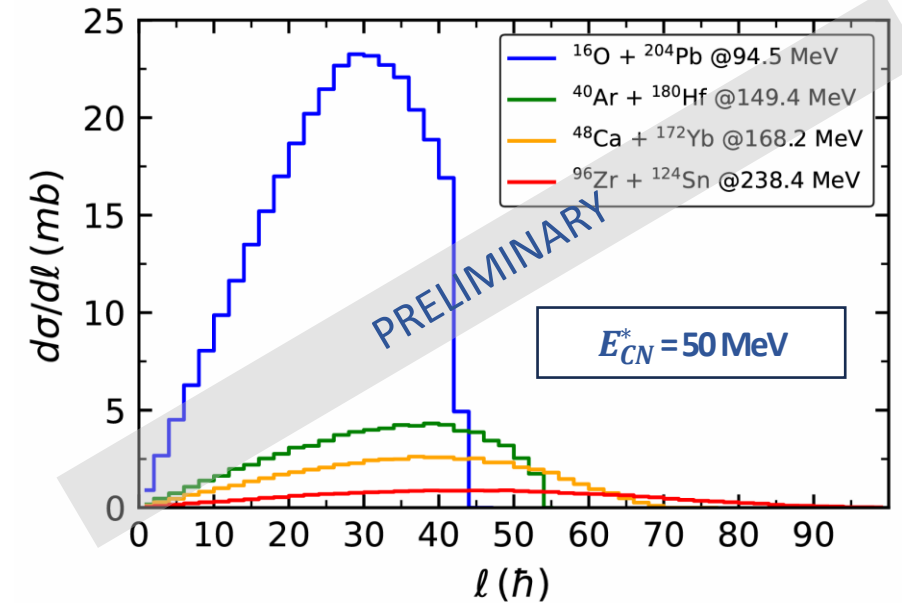
Two different fusion mechanisms

- Symmetric to quite asymmetric systems:
→ **Symmetrization of the shape/slow equilibration**
($R_1 = R_2 = R_0$ + large neck)
- Extremely asymmetric systems:
→ **Fast absorption** of the smaller fragment
($R_1 = R_0, R_2 = 0$ + no neck = **dinuclear**)



Mass asymmetry effects – ^{220}Th (Preliminary)

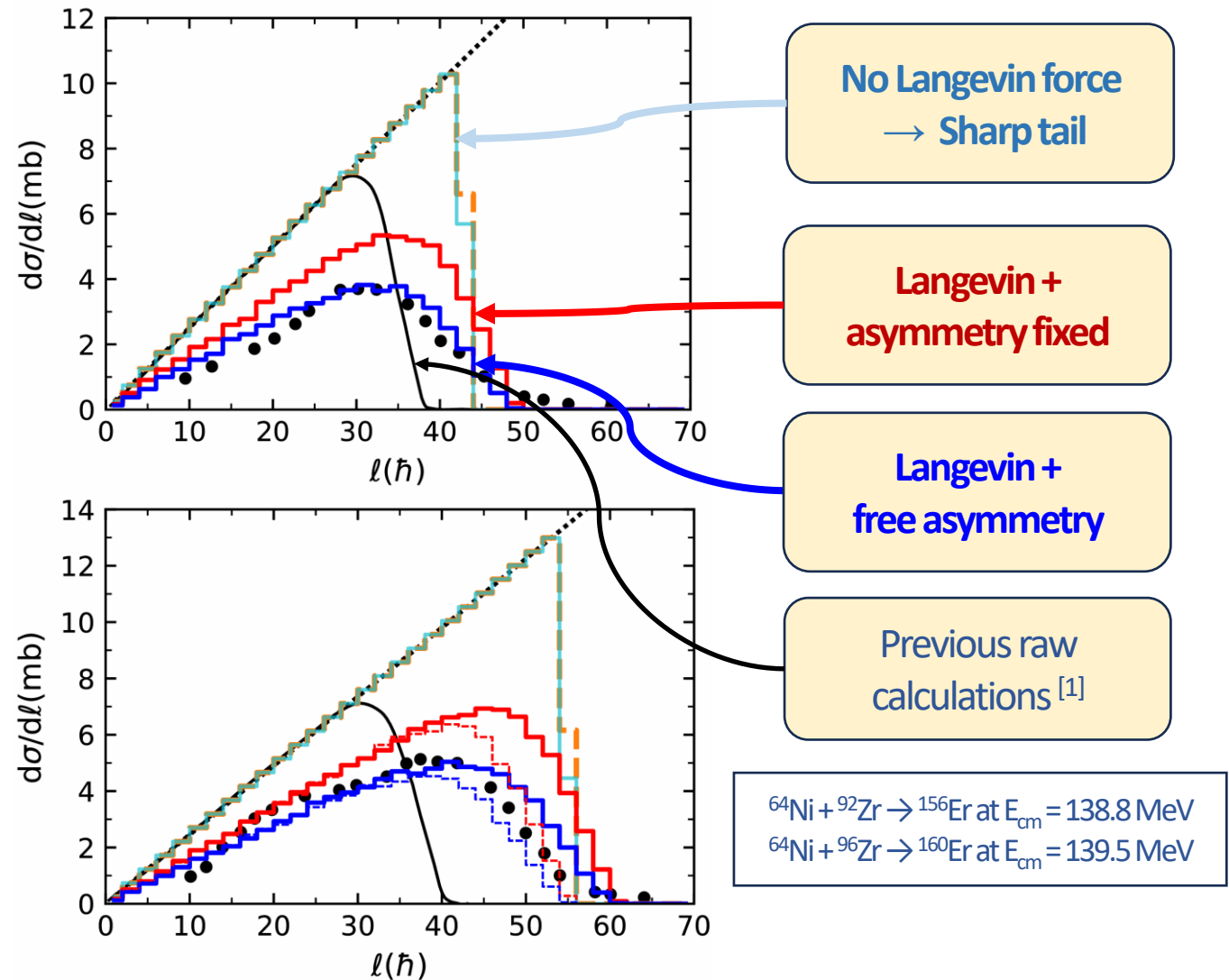
- ▶ Two different mechanisms: **modification of the fusion conditions.**
- ▶ Cross sections decrease with $Z_1 Z_2$
- ▶ More symmetric system $\rightarrow$ broader spin distributions (higher energies)



Insights into earlier results

Y. Jaganathen, M. Kowal, K. Pomorski, Phys. Lett. B 862 (2025) 139302

Open question: **Why do the cross sections go down when asymmetry is a free variable?**



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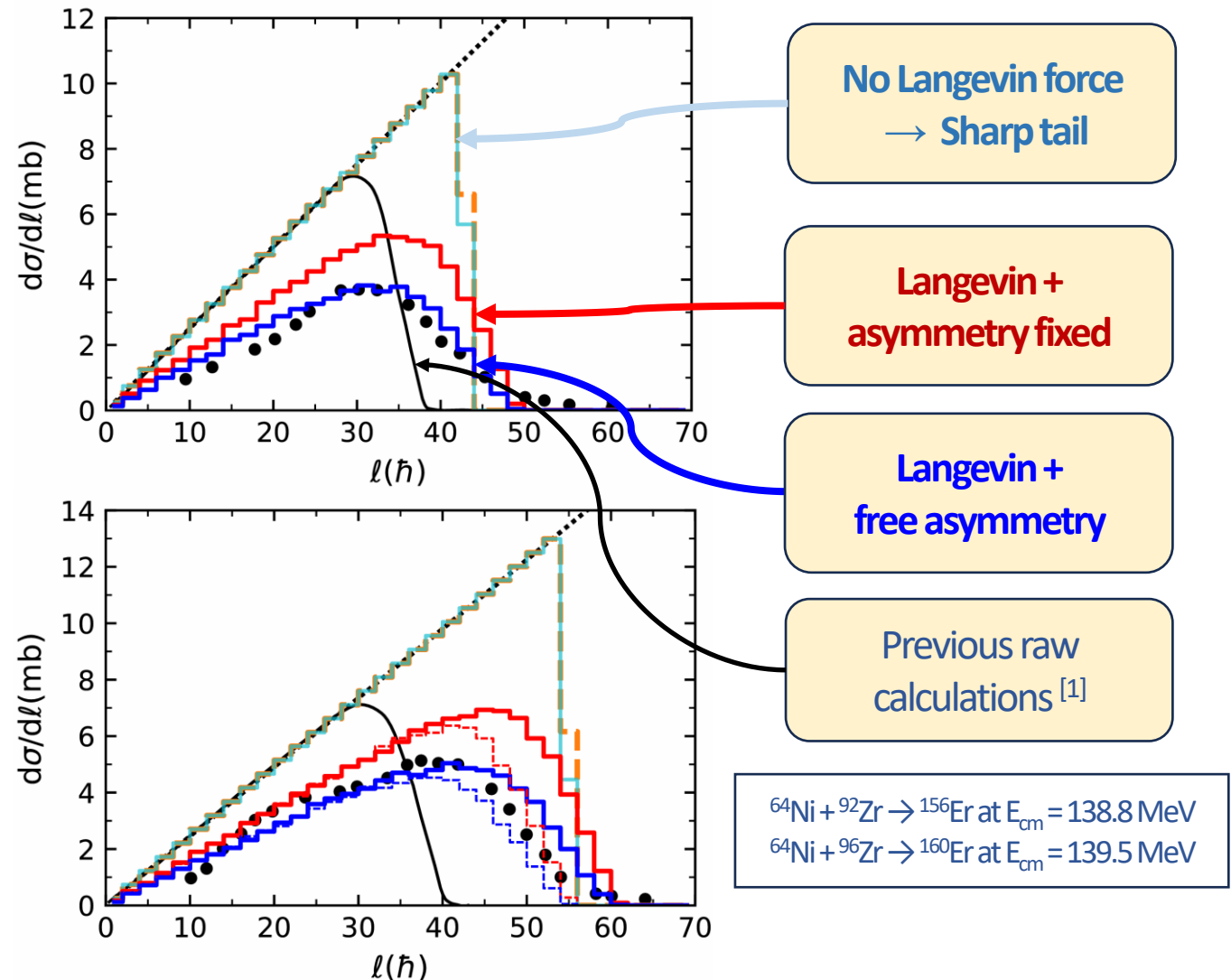
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- The **equilibration by symmetrization** is a slow process
→ **Favors quasi-fission.**

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Summary and Perspectives

- ▶ We have derived a fully **6-dimensional dissipative dynamics Langevin**-based formalism to describe the **unrestricted motion of the systems** in terms of **elongation**, **neck** and **asymmetry** variables.
- ▶ Thanks to a **correct treatment of the different stages of fusion**, the spin distributions for the Ni + Zr reactions are **in great agreement with experimental data**.
- ▶ We are currently studying the **entrance channel asymmetry effects** in many reactions leading to ^{220}Th .
- ▶ The Langevin formalism allows for a deep understanding of the fusion process: in particular, depending on the asymmetry of the systems, **two different modes of fusion (symmetrization and absorption) emerge**.
- ▶ We are planning to make the following improvements of the formalism:
 - ▶ The addition of **shell effects and deformations/orientations** for a **fully microscopic-macroscopic picture**
 - ▶ The testing of different forms of stochastic noises (**color noises**),
 - ▶ The incorporation of **neutron emission** throughout the process.
- ▶ Ultimately, we aim to tackle the **hindrance problem** that prevents the formation of superheavy elements.

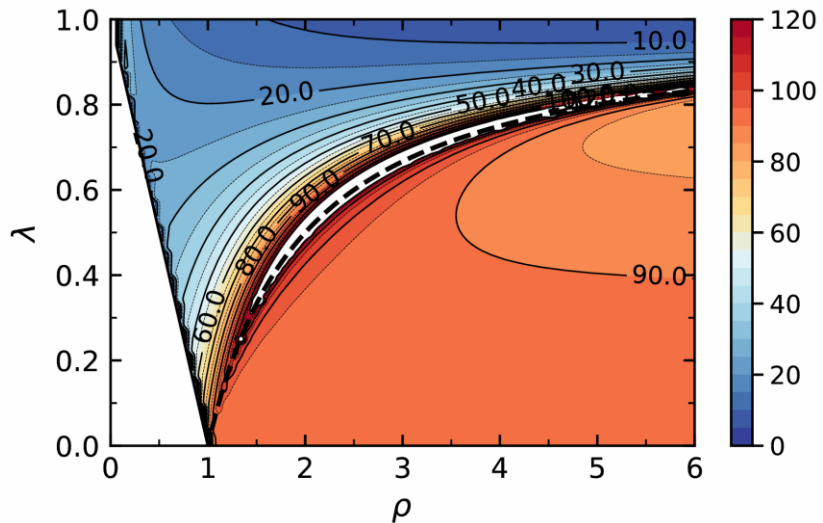
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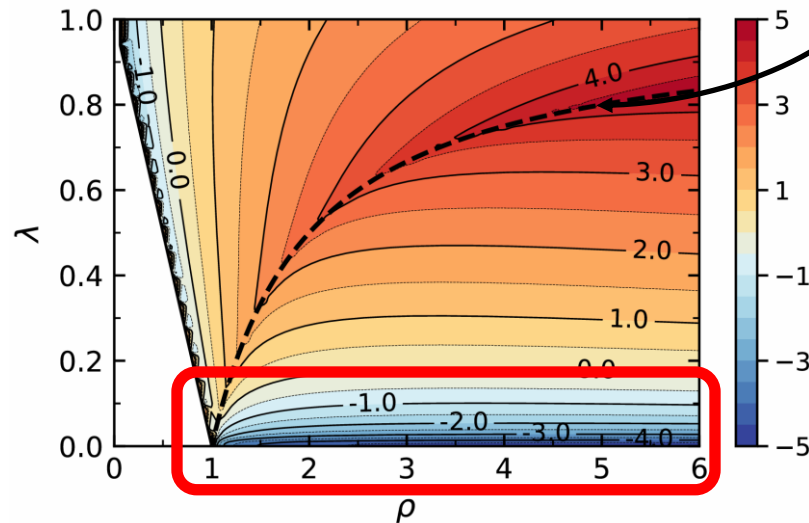
**Thank you for
your attention!**

Mass tensor / Kinetic Energy

- ▶ **Werner-Wheeler flow approximation:**
 - ▶ **Incompressibility** (matter density is uniformly distributed)
 - ▶ The flow is **irrotational** (the moving planes remain plane)

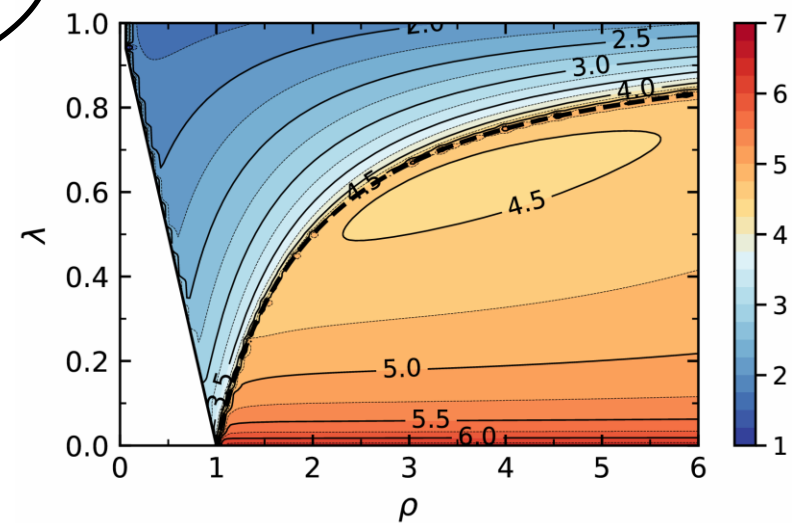


$\mathcal{M}_{\rho\rho}(\hbar^2/\text{MeV})$



$\log(\mathcal{M}_{\lambda\lambda}/(1\hbar^2/\text{MeV}))$

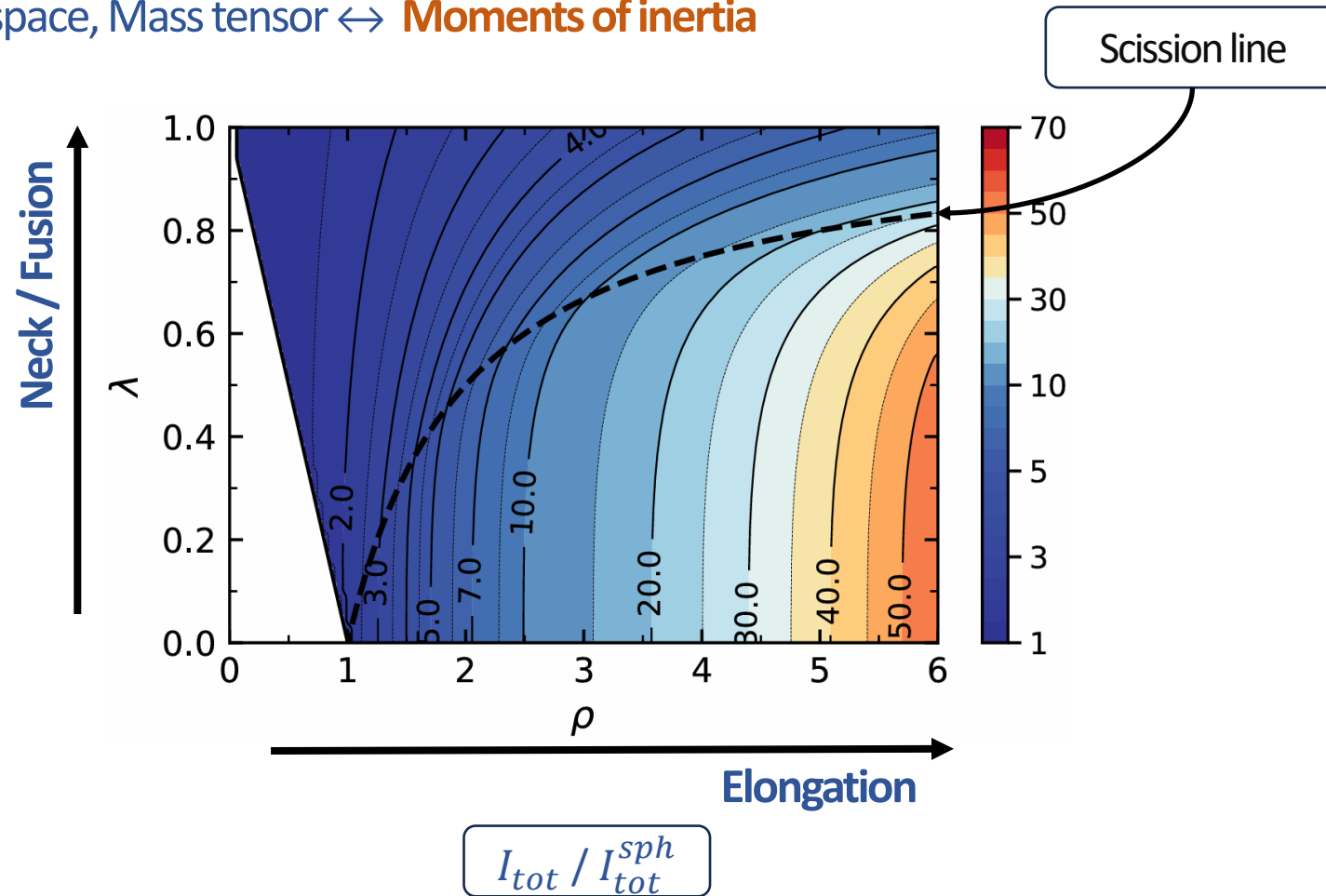
Scission line



$\log(\mathcal{M}_{\Delta\Delta}/(1\hbar^2/\text{MeV}))$

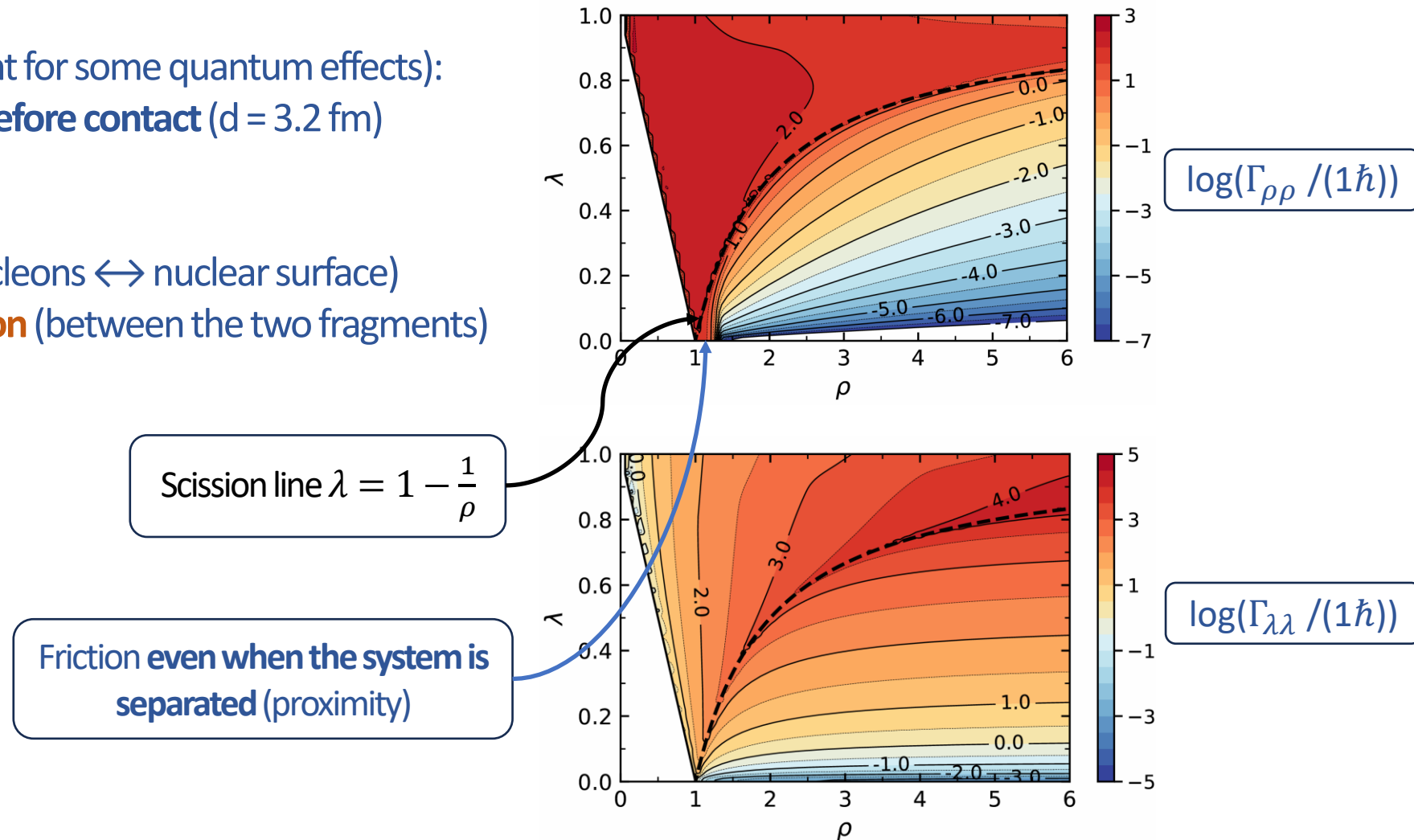
Mass tensor / Kinetic Energy

- In the rotational space, Mass tensor $\leftrightarrow$ **Moments of inertia**



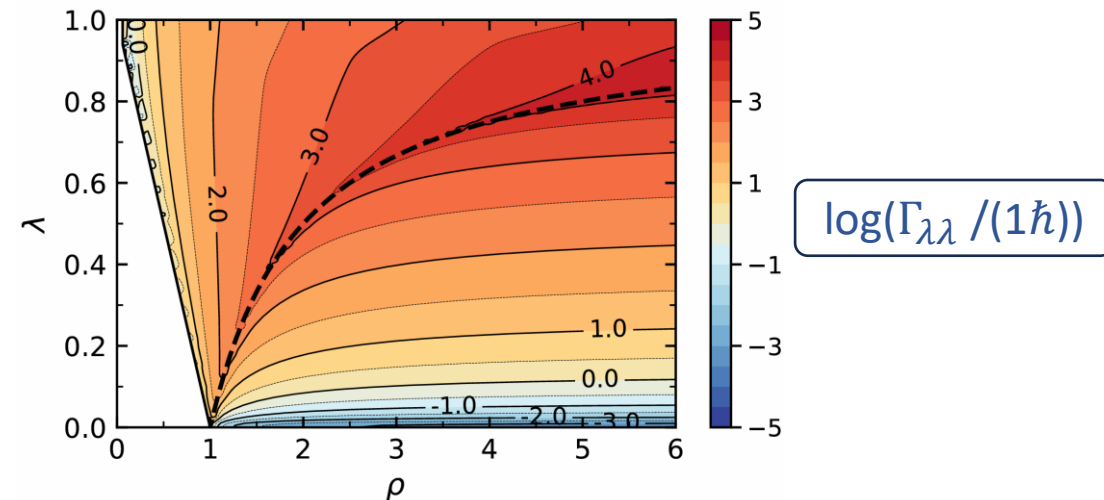
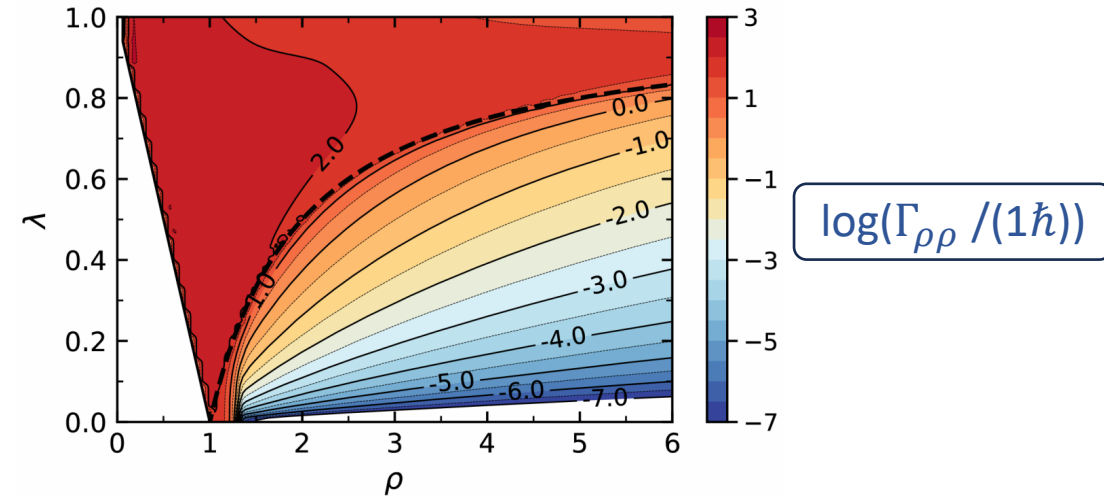
Friction forces

- ▶ **Proximity formalism** (to account for some quantum effects):
Possible matter flow/friction before contact ($d = 3.2$ fm)
- ▶ **Shape friction**:
 - ▶ **Wall friction** (collisions nucleons $\leftrightarrow$ nuclear surface)
 - ▶ + **Wall-plus-window friction** (between the two fragments)



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Possible matter flow/friction before contact ($d = 3.2$ fm)
- ▶ **Shape friction:**
 - ▶ **Wall friction** (collisions nucleons $\leftrightarrow$ nuclear surface)
 - ▶ + **Wall-plus-window friction** (between the two fragments)
- ▶ **Strong friction at the scission line:**
 - ▶ The system slows down at the scission line
 - ▶ Re-separation is irreversible
- ▶ **Angular friction:**
 - ▶ **Sliding friction**
 - ▶ **No rolling friction**



Langevin/random forces

- We assume a simple memoryless Langevin force (white noise):

$$F_i = \sum_k g_{ik} \xi_k(t)$$

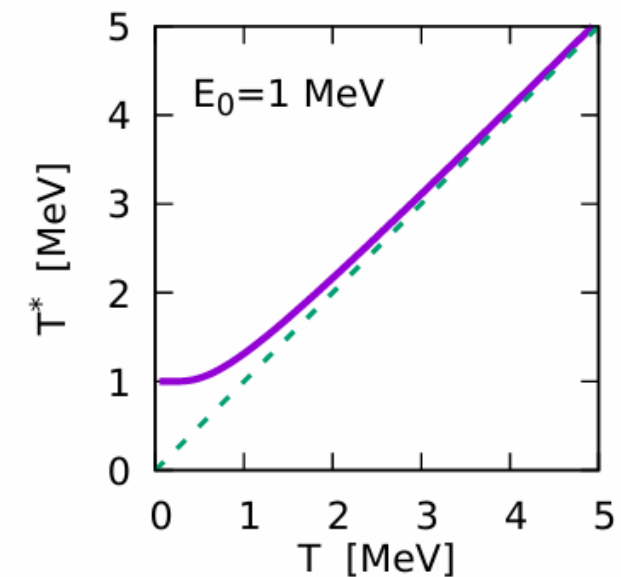
$\xi_k(t)$ are **time-dependent Gaussian random variables**:

$$\begin{aligned}\langle \xi_k(t) \rangle &= 0 \\ \langle \xi_k(t), \xi_{k'}(t') \rangle &= 2\delta_{kk'}\delta(t - t')\end{aligned}$$

- The diffusion tensor is given by the **Einstein relation**:

$$\sum_k g_{ik} g_{kj} = D_{ij} = k_B T^* \gamma_{ij}, \quad T^* = \frac{E_0}{\tanh(E_0/T)}, \quad T = \sqrt{E^*/a}$$

$E_0 = 2$ MeV is the zero-point collective energy of the heat bath oscillators
 E^* is the **dissipated energy**, $a = A/8$ MeV is the **level density parameter**.



Quantum-corrected temperature
(Courtesy of K. Pomorski)

Langevin/random forces

- We assume a simple memoryless Langevin force (white noise):

$$F_i = \sum_k g_{ik} \xi_k(t)$$

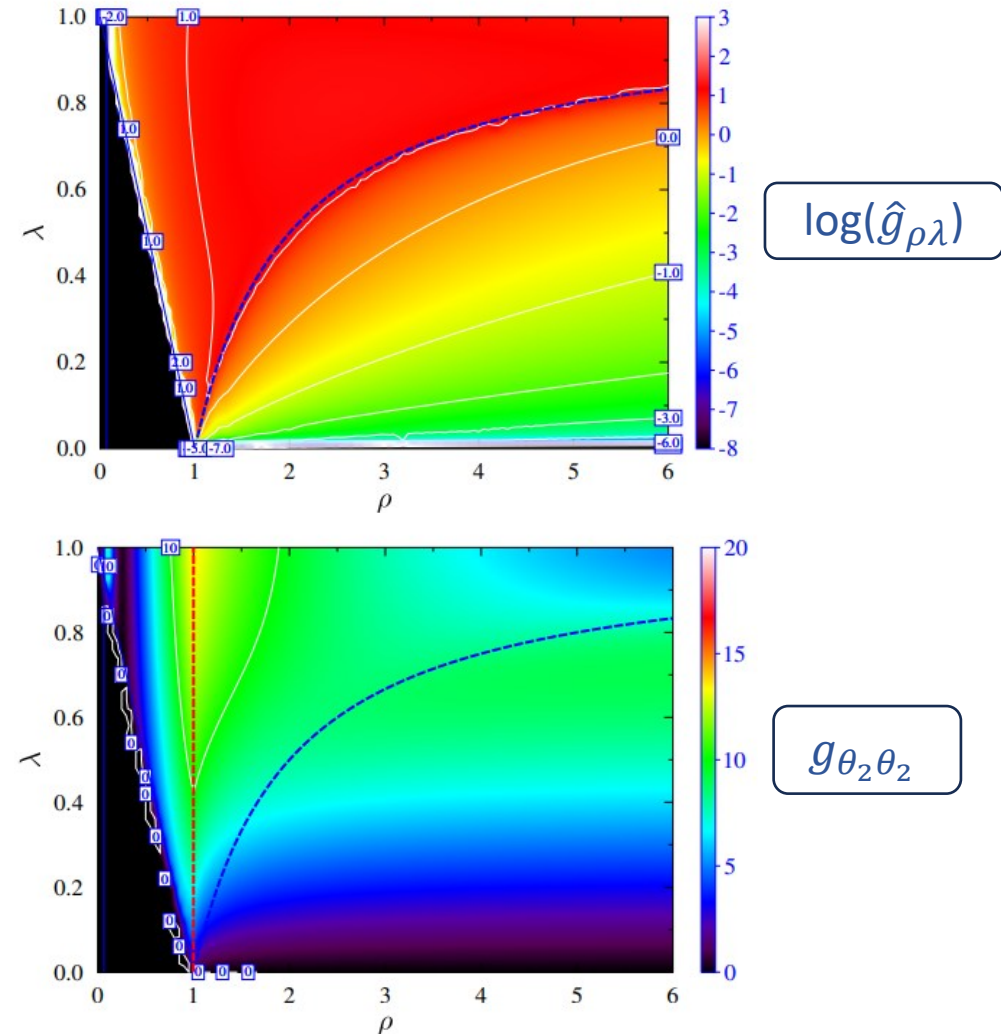
$\xi_k(t)$ are **time-dependent Gaussian random variables**:

$$\begin{aligned} \langle \xi_k(t) \rangle &= 0 \\ \langle \xi_k(t), \xi_{k'}(t') \rangle &= 2\delta_{kk'}\delta(t - t') \end{aligned}$$

- The diffusion tensor is given by the **Einstein relation**:

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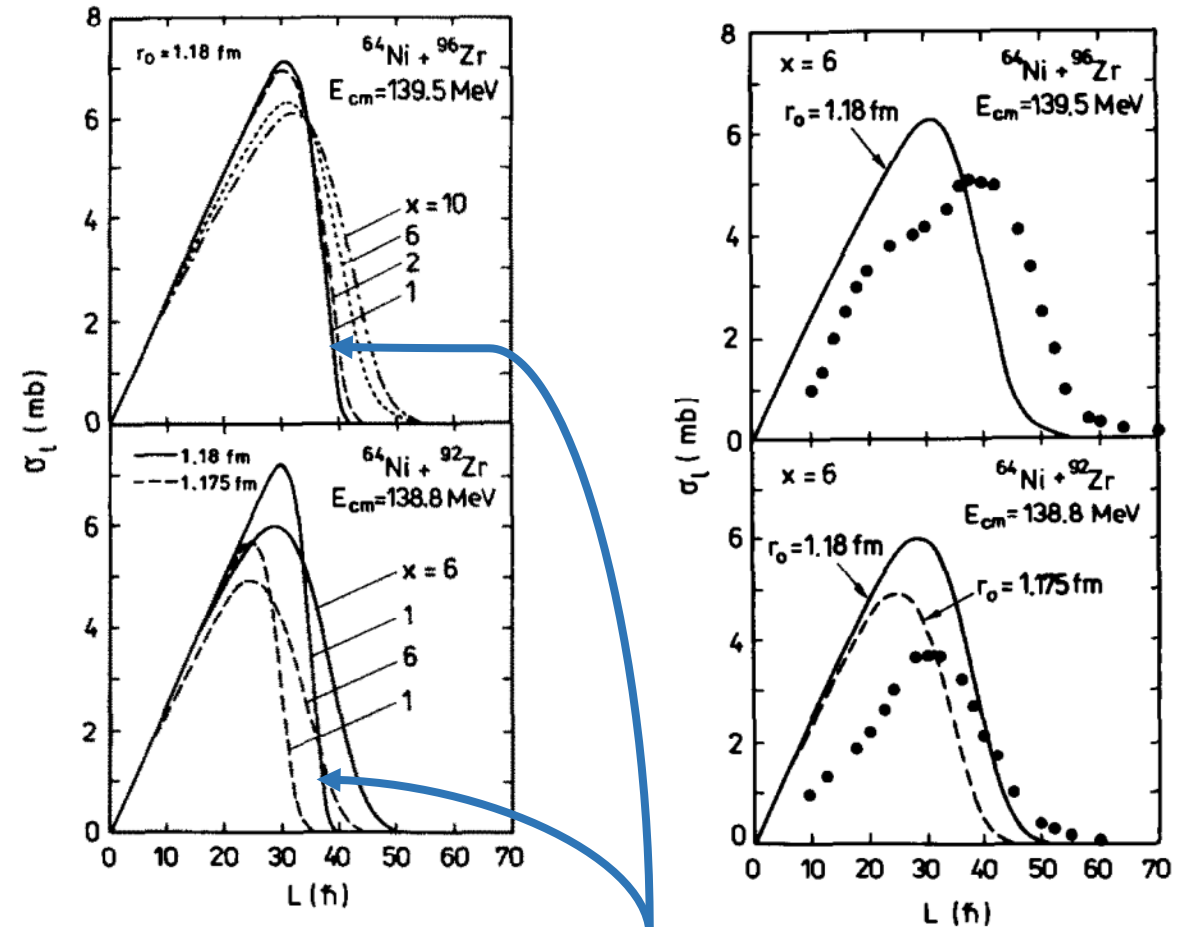
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A 30 year-old project

W. Przystupa, K. Pomorski, Nucl. Phys. A 572(1) (1994) 153

- ▶ Systems: $^{64}\text{Ni} + ^{92,96}\text{Zr} \rightarrow ^{156}\text{Er}$
- ▶ Minimal shells effects at the incident energies (50 MeV).
- ▶ Calculations with the **asymmetry variable frozen**.
- ▶ **Correction of the diffusion tensor** by a factor 6 to reproduce the tails of the spin distributions
- ▶ Possible improvement:
 - ▶ **Full calculation** with asymmetry needed



Raw results

Physical vs. practical collective variables

- ▶ Unlike the multipole moments, the (ρ, λ, σ) variables are **extremely irregular**:
 - ▶ Many borders,
 - ▶ Small proximity region,
 - ▶ Regime change.
- ▶ A correct treatment of numerical precision is needed.
- ▶ **Extrapolation** is needed for the calculations of the physical quantities and their derivatives.
The regions of extrapolation should **avoid the borders**.
- ▶ The process of fusion itself is not fully tractable numerically.

