Evolution of the fission fragment angular momentum

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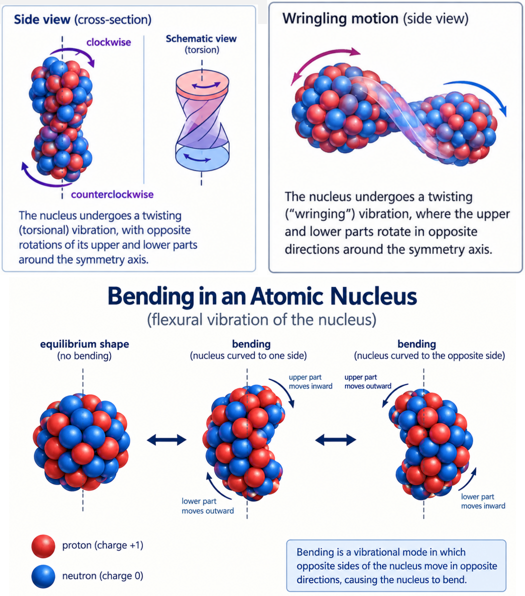
Outline

- **If normal modes can generate the angular momentum in fission fragments?**
- **Stochastic differential equations: Langevin (fission) Fokker-Planck (angular momentum)**
- **Results**
- **Plans**



Thanks to chatGPT

Stochastic approach

Dynamical effect

• path from equilibrium to scission slowed-down by the nuclear viscosity
• description of the time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a "heat bath".
• Monte Carlo method for choosing the shape, initial angular momentum, type and energy of emitted particles....

Coupling to the evaporation

Pre and post- scission emission of neutrons, protons, α and γ.

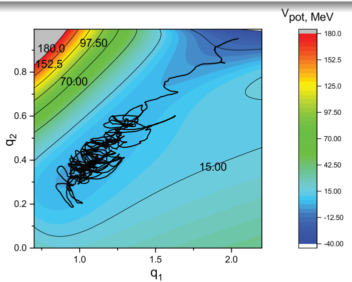
Ingredients

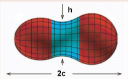
Inertia ($[M^{-1}(\vec{q})]_{ij}$); Friction ($\gamma_i(t)$) and fluctuation (g_{ik})
Macroscopic potential
$(V(\vec{q}, K) \rightarrow F(\vec{q}, K) = V(\vec{q}, K) - \mathbf{a(\tilde{q})}T^2)$

Langevin equations

$$\frac{dq_i}{dt} = \sum_j [M^{-1}(\vec{q})]_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{1}{2}\sum_{j,k}\frac{d[M^{-1}(\vec{q})]_{jk}}{dq_i} p_j p_k - \frac{dF(\vec{q}, K)}{dq_i} - \sum_{j,k}\gamma_{ij}(\vec{q})[M^{-1}(\vec{q})]_{jk} p_k + \sum_j g_{ij}(\vec{q})\Gamma_j(t)$$





- c - the elongation of the nucleus
- h - constriction coordinate
- α - mass-asymmetry parameter related to the ratio of the masses of nascent fragments

Model Ingredients

Energies

- Potential energy in deformation space e.g: FRLDM + Wigner or LSD + Congruence

$$\begin{aligned} \mathbf{E_{lsd}(q)} &= \mathbf{b_{vol}\{1-\kappa_{vol} I^2\}\,A + b_{surf}\{1-\kappa_{surf} I^2\}\,A^{2/3}B_{surf}(q)} \\ &+ \mathbf{b_{curv}\{1-\kappa_{curv} I^2\}\,A^{1/3}B_{curv}(q) + \frac{3}{5}e^2\frac{Z^2}{r_0^{ch}A^{1/3}}B_{Coul}(q)} \\ \mathbf{I} &= \mathbf{(N-Z)/A} \end{aligned}$$

- Rotational energy:

$E_{rot}(q,I,K) = \frac{\hbar^2 I(I+1)}{2J_\perp(q)} + \frac{\hbar^2 K^2}{2J_{eff}(q)}$ where $J_{eff}^{-1} = J_{\parallel}^{-1} - J_{\perp}^{-1}$ - the rigid-body moments of inertia.

- The friction parameter controlling coupling between K coordinate and the heat bath.

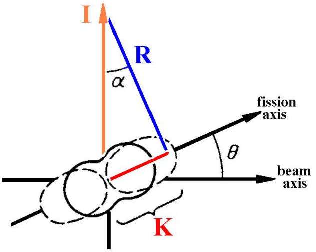
T. Dosing, J. Randrup, Nucl. Phys. A 433,215; J. Randrup, Nucl.Phys. A 383, 468

$$\gamma_K = \frac{1}{R_N R_{cm}\sqrt{2\pi^3 n_0}}\frac{J_{\parallel}|J_{eff}|J_R}{J_\perp^3}$$

where R_N - neck radius, R_{cm}- distance between the center of the nascent fragments, n_0=0.0263 MeV zs fm^{-4} - the bulk flux in standard nuclear mass and $J_R = M_0 R_{cm}^2/4$ for reflection symmetric shape.

Collective coordinates(4D)

- Description of the nuclear shape by elongation, neck and asymmetry – 3 parameters.
- **K** – spin about the fission (symmetry) axis



"Fission-fragment distributions within dynamical approach" K. M., P. N. Nadtochy, E. G. Ryabov, G. D. Adeev, Eur. Phys. J. A, 53 (2017) 79E

Energy Models study: **LSD or FRLDM, Congruence**

PHYSICAL REVIEW C **84**, 014610 (2011)

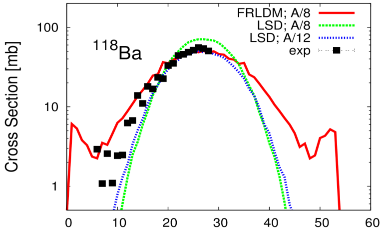
Critical insight into the influence of the potential energy surface on fission dynamics

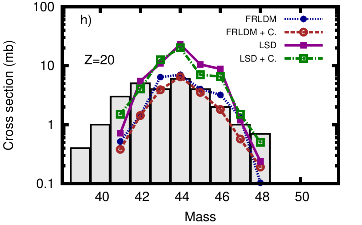
K. Mazurek,[1,2,*] C. Schmitt,[2] J. P. Wieleczko,[2] P. N. Nadtochy,[3] and G. Ademard[2]

PHYSICAL REVIEW C **88**, 054614 (2013)

Examining fine potential energy effects in high-energy fission dynamics

K. Mazurek,[1,*] C. Schmitt,[2] P. N. Nadtochy,[3] M. Kmiecik,[1] A. Maj,[1] P. Wasiak,[1] and J. P. Wieleczko[2]





EPJ Web of Conferences **17**, 16006 (2011)
DOI: 10.1051/epjconf/20111716006

Influence of the potential energy landscape on the fission dynamics

K. Mazurek[1,2,a], J.P. Wieleczko[1], C. Schmitt[1], and P. N. Nadtochy[3]

Vol. 44 (2013) *ACTA PHYSICA POLONICA B*

FISSION FRAGMENT MASS DISTRIBUTION AS A PROBE OF THE SHAPE-DEPENDENT CONGRUENCE ENERGY TERM IN THE MACROSCOPIC MODELS* **

K. Mazurek[a], C. Schmitt[b], P.N. Nadtochy[c], A. Maj[a]
P. Wasiak[a], M. Kmiecik[a], B. Wasilewska[a]

The change of the fission fragment mass distribution due to different PES taken to solve Langevin equations

Exp: Y. Futami, et al., Nucl. Phys. A607, 85 (1996)

Angular momentum of fission fragments (with J. Randrup and P. Nadtochy)

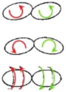
Useful framework for discussion of fission fragment angular momenta:

Normal modes:

wriggling

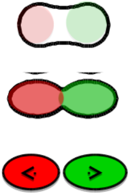
bending

twisting



Fission populates several distinct *rotational modes* whose presences reflect the physical *mechanisms* generating the angular momenta of the fragments

Generation of fission fragment angular momenta by nucleon exchange:



$$t_{\rm wrig} < t_{\rm fiss} \approx t_{\rm bend} < t_{\rm twst} << t_{\rm tilt}$$

Wriggling will reach equilibrium

Bending probably has some presence; it increases with the mass asymmetry

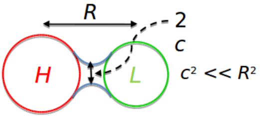
Twisting is unlikely to play a major role; it grows more prominent with excitation

Angular momentum of fission fragments

Relaxation times

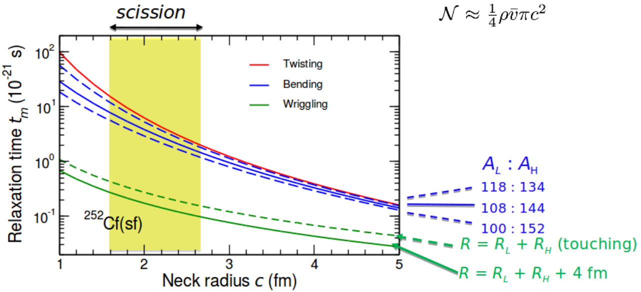
of dinuclear rotational modes caused by *nucleon exchange*

$$t_m = I_m / M_m \qquad \left\{ \begin{aligned} M_{\rm wrig} &= m\mathcal{N}R^2 \\ M_{\rm bend} &= m\mathcal{N}\left[\left(\frac{\mathcal{I}_H R_L - \mathcal{I}_L R_H}{\mathcal{I}_L + \mathcal{I}_H}\right)^2 + c_{\rm ave}^2\right] \\ M_{\rm twst} &= m\mathcal{N}c_{\rm ave}^2 \end{aligned} \right.$$



Transfer rate:

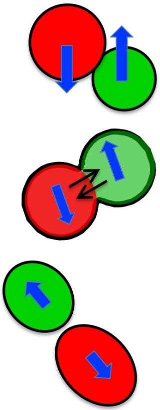
$$\mathcal{N} \approx \tfrac{1}{4}\rho\bar{v}\pi c^2$$

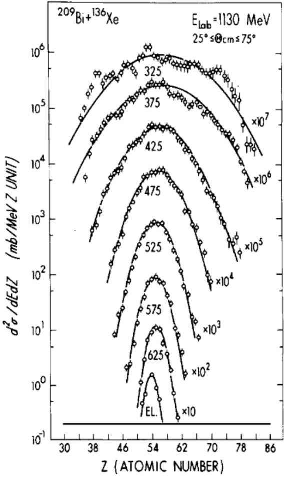


J. Randrup, T. Døssing, R. Vogt, Phy.s Rev. C **106,** 014609 (2022)

Multiple nucleon exchange was found to be a dominant mechanism in *damped nuclear reactions*

W.U. Schröder and J.R. Huizenga, *Damped Nuclear Reactions*, Treatise on Heavy-Ion Science II, p. 113 (1984) Plenum Press, Ed: D.A. Bromley



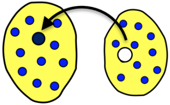


W.U. Schröder *et al.*, PRL **36**, 514 (1975)

The participating nucleons are *near the Fermi surface*

=> The dissipation is *strong!*

W.U. Schröder *et al.*, PRL **44**, 308 (1980)



window formula

Multiple nucleon transfers produce a dissipative force affecting the linear and angular momenta of the binary partners:

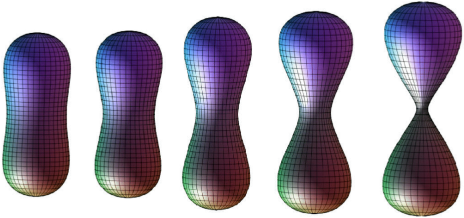
$Z_H, N_H, \boldsymbol{P}_H, \boldsymbol{S}_H, T_H, \; Z_L, N_L, \boldsymbol{P}_L, \boldsymbol{S}_L, T_L$

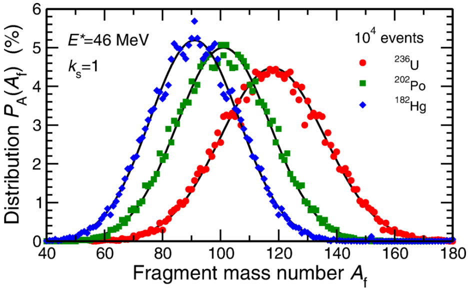
Nucleon Exchange Transport Theory

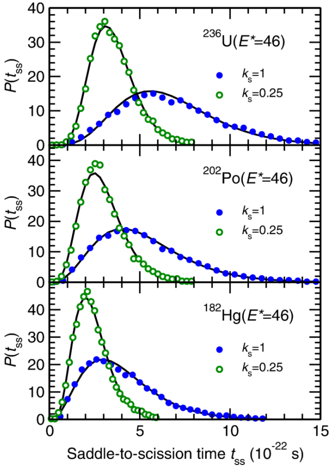
J. Randrup, Nucl. Phys. A **327**, 490 (1979)
J. Randrup, Nucl. Phys. A **383**, 468 (1983)
T. Døssing & J. Randrup, NPA **433**, 215 (1985)

Angular momentum of fission fragments

From saddle to scission







J. Randrup, P. Nadtochy, C. Schmitt, and K. Mazurek, Phys. Rev. C 113, 044605

Angular momentum of fission fragments

For each shape evolution, we wish to calculate the evolution of the distribution of the correlated fragment spins, $P(S_A, S_B)$

Rotational energy:

$$E_{\rm rot} = \underbrace{\frac{J_\perp^2}{2\mathcal{I}_\perp} + \frac{J_\parallel^2}{2\mathcal{I}_\parallel}}_{\textit{overall rotation}} + \underbrace{\frac{s_{\rm wrig}^2}{2\mathcal{I}_{\rm wrig}} + \frac{s_{\rm bend}^2}{2\mathcal{I}_{\rm bend}} + \frac{s_{\rm twst}^2}{2\mathcal{I}_{\rm twst}}}_{\textit{internal rotations}}$$

Normal spins: $s_{\rm wrig}$, $s_{\rm bend}$, $s_{\rm twst}$

Fokker-Planck equation for $P(\boldsymbol{s}_A, \boldsymbol{s}_B; t)$

$$\boxed{\frac{\partial}{\partial t}P = -\sum_i \frac{\partial}{\partial s_i} V_i P + \sum_{ij} \frac{\partial^2}{\partial s_i \partial s_j} D_{ij} P}$$

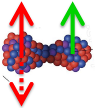
Drift coefficient $V_i(\chi) = \Sigma_j M_{ij}(\chi) f_j$ determines the *mean* evolution of $s_i(t)$: $\boxed{\frac{d\bar{s}_i}{dt} = V_i(\bar{\chi})}$

Mobility coefficients

Diffusion coefficient $D_{ij}(\chi) = M_{ij}(\chi)\, T$ generates *correlated spin fluctuations* $\sigma_{ij}(t)$:

$$\sigma_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle : \quad \boxed{\frac{d}{dt}\sigma_{ij} = 2D_{ij}(\bar{\chi}) - \sum_k [\nu_{ik}\sigma_{kj} + \nu_{jk}\sigma_{ki}]}$$

Angular momentum of fission fragments



Results for a single shape evolution:

Spin components *perpendicular* to the fission axis

Wriggling & bending

$$\mathcal{I}_{\rm wrig} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp)\,\mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R} \qquad \tilde{\sigma}^2_{\rm wrig} = \mathcal{I}_{\rm wrig} T$$

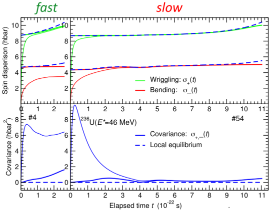
$$\tilde{\sigma}_{\rm wrig,bend} = 0$$

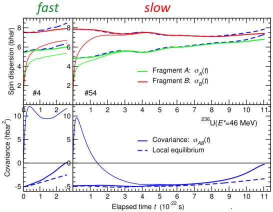
$$\mathcal{I}_{\rm bend} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \qquad \tilde{\sigma}^2_{\rm bend} = \mathcal{I}_{\rm bend} T$$

Individual fragments:

$$\boldsymbol{s}_A = \frac{\mathcal{I}_A^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}\,\boldsymbol{s}_{\rm wrig} + \boldsymbol{s}_{\rm bend}$$

$$\boldsymbol{s}_B = \frac{\mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}\,\boldsymbol{s}_{\rm wrig} - \boldsymbol{s}_{\rm bend}$$





$$\boldsymbol{M}_\pm = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c^2_{\rm ave} \end{pmatrix} \qquad \boldsymbol{M}^\perp_{AB} = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} a^2 + c^2_{\rm ave} & ab - c^2_{\rm ave} \\ ab - c^2_{\rm ave} & b^2 + c^2_{\rm ave} \end{pmatrix}$$

$\delta \;=\; (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B)$ small

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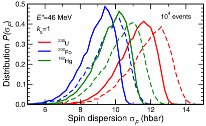
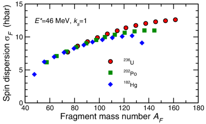
Angular momentum of fission fragments

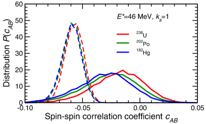


The fragment spins relax to the local equilibrium values throughout most of the evolution from saddle towards scission.

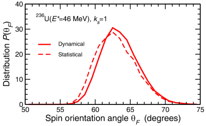


But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decrease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, resulting in smaller magnitudes.

Gating on fragment mass may be informative.



The spin-spin correlation is quite small in equilibrium and it is even smaller when generated dynamically The fragment spins are practically uncorrelated (J. Wilson et al., Nature 590, 2021)



The spin orientation angle θ_F is about 70°, both in equilibrium and dynamically
Can be measured !!!

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Summary and plans

- The fission description with Langevin type transport equation solving gives good estimation of various observables: mass, charge, TKE, angular distributions, emitted particles multiplicities and spectra and so on.
- The study of the correlation between mass and angular distributions allows to distinguish between fission and quasifission reactions.
- **The angular momentum of the fission fragments is estimated, including the effects of exotic rotation on the path from the saddle to the scission point.**
- **The spin dispersion and space orientation have been tested, for 4 reactions assuming two viscosity parameters: k_s=0.25 and 1. and 0 initial angular momentum.**
- **The spin of fragments are uncorrelated.**

Plans

Follow the study of angular momentum of fission fragments by adding particle emission.