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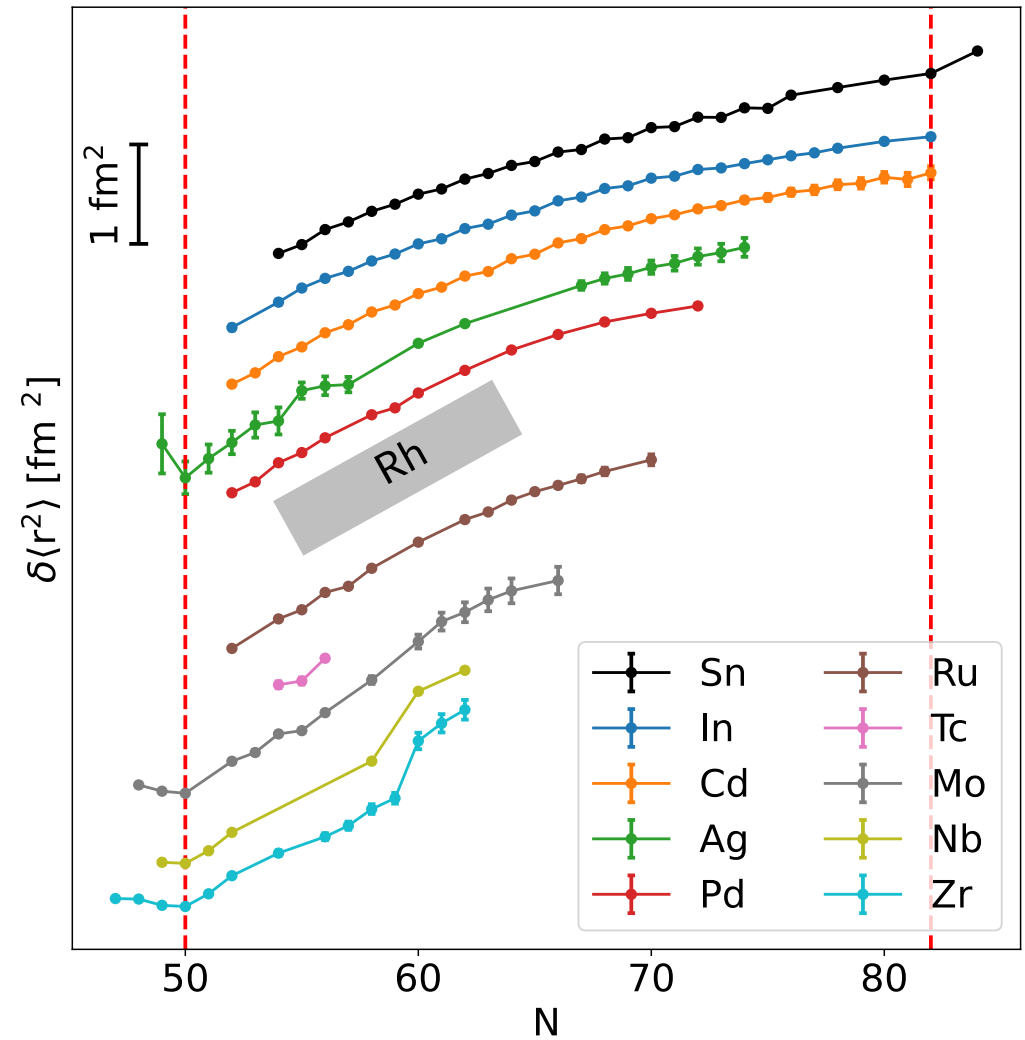
Nuclear moments and mean-squared charge radii of neutron deficient rhodium isotopes measured using collinear laser spectroscopy

S.Chinthakayala, A. Raggio, J. Warbinek, L. Caceres, P. Delahaye, I.D. Moore and IGISOL group



Motivation

- Understand structural evolution between two regimes
 - strongly deformed, collective nuclei near Zr ($Z \approx 40$, $N \approx 60$)
 - and the spherical ^{100}Sn ($Z=50$, $N=50$)
- Charge radius and nuclear moments as probes of nuclear structure

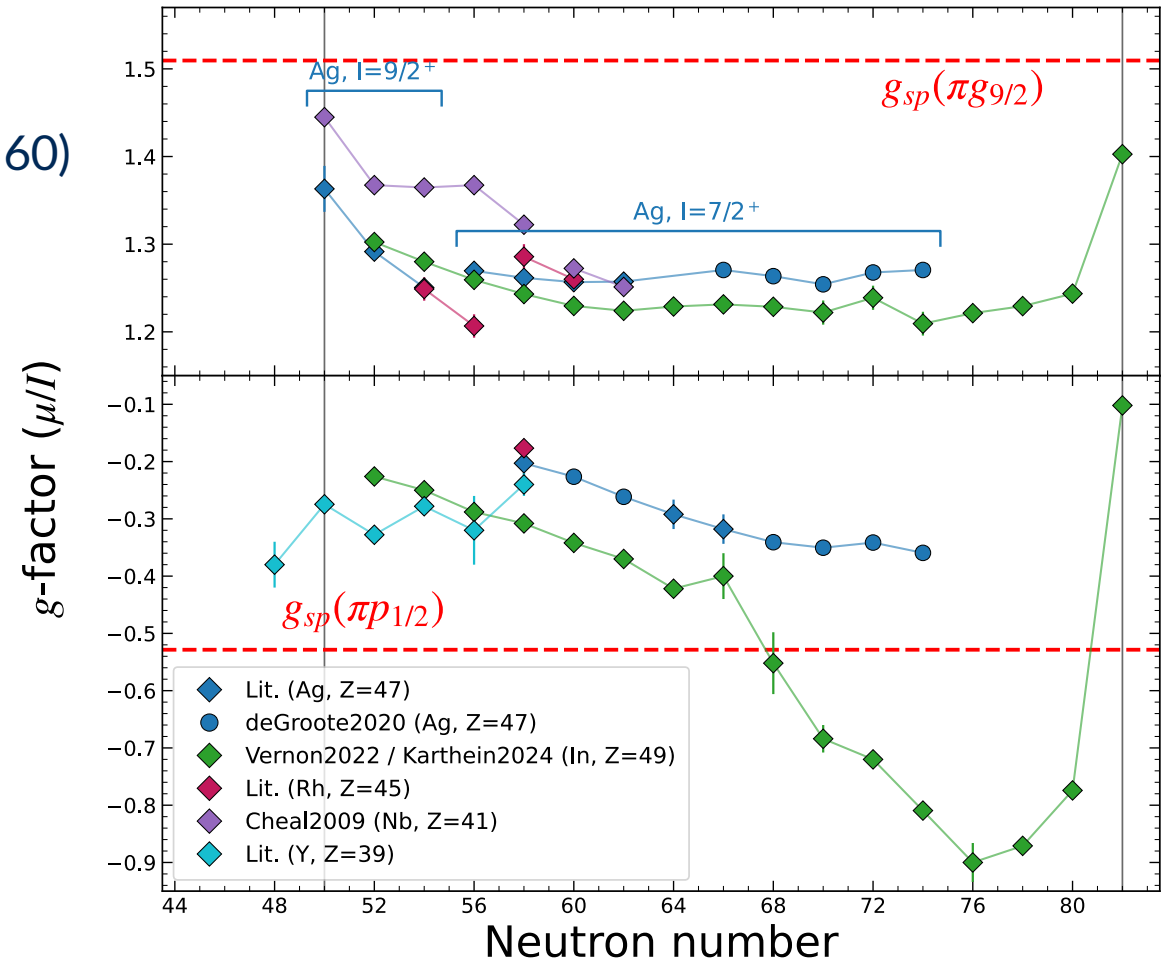




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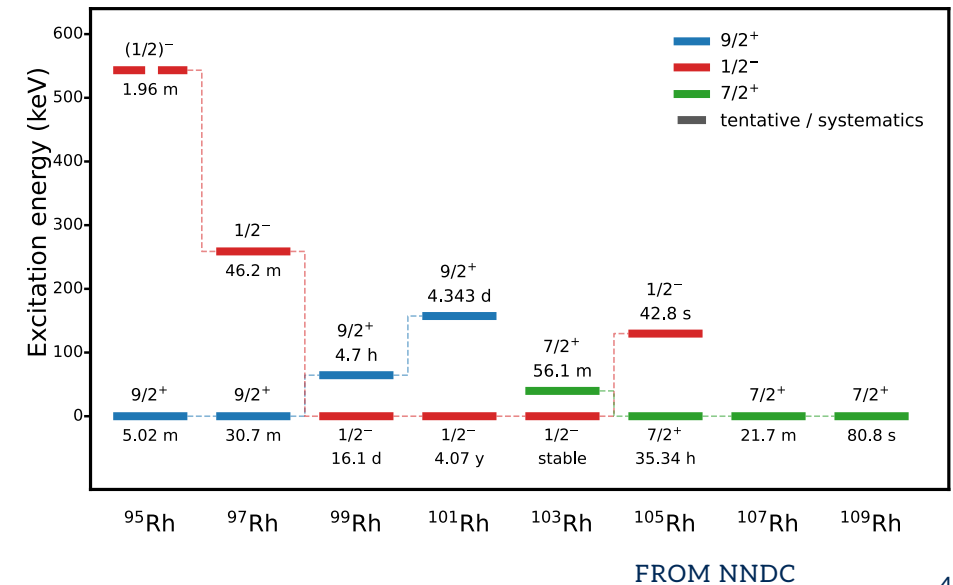
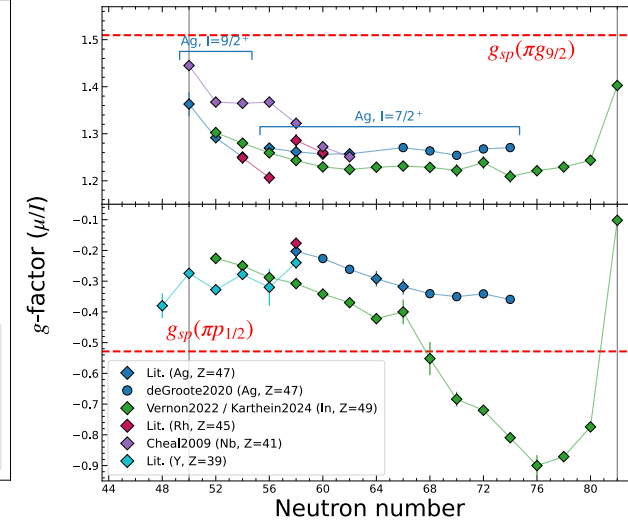
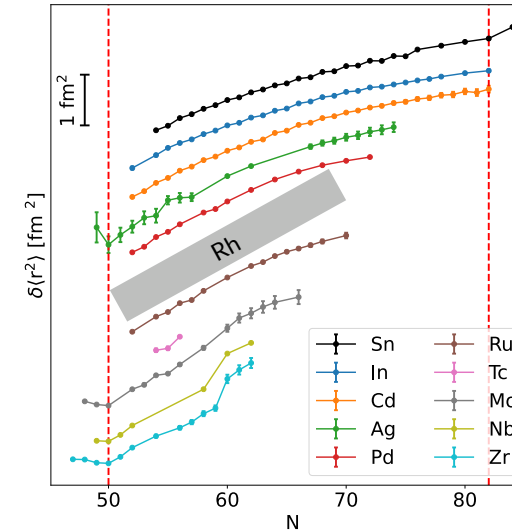
Odd Z - Odd A



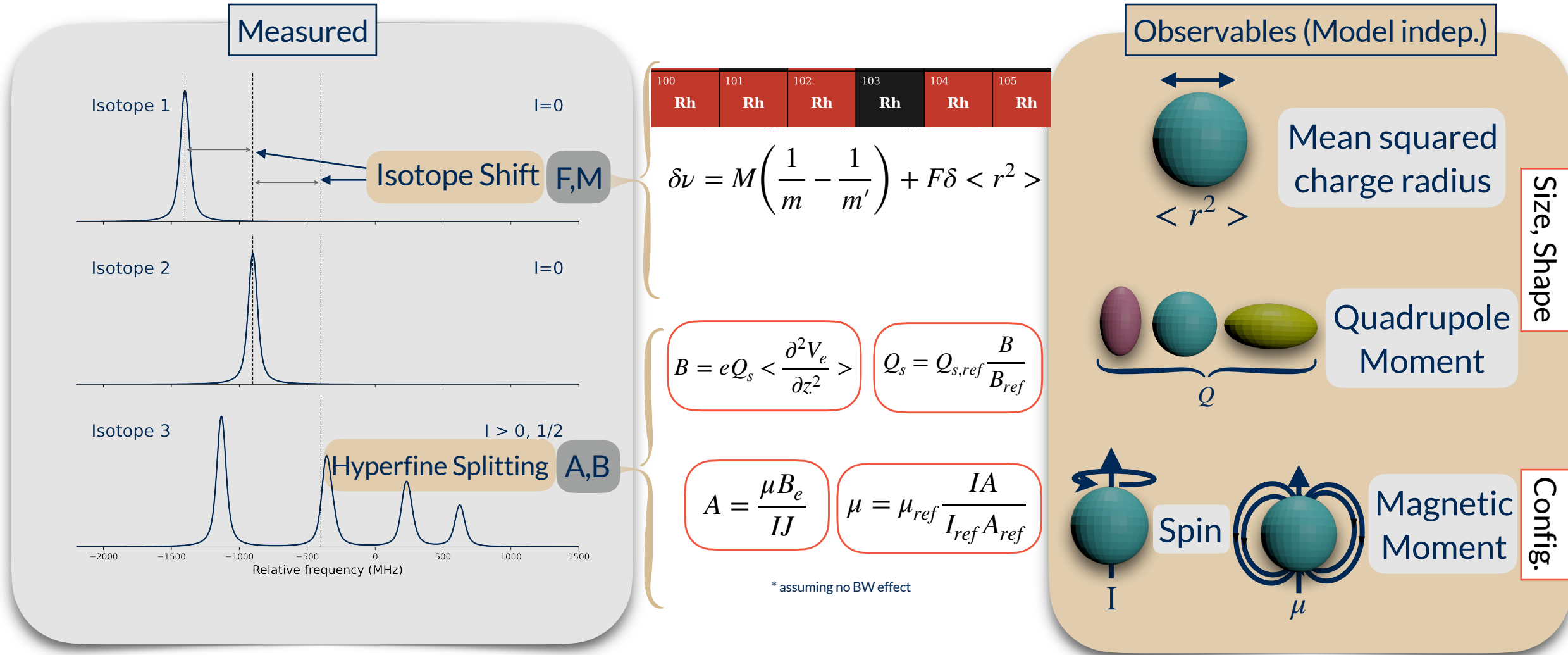


Motivation

- Understand structural evolution between two regimes
 - strongly deformed, collective nuclei near Zr (Z≈40, N≈60) and the spherical ^{100}Sn (Z=50, N=50)
- Charge radius and nuclear moments as probes of nuclear structure
- Rh (Z=45) - right at the mid shell, half filled $\pi g_{9/2}$
- Evolution of ground and isomer state spins, N=50 → mid shell
- Several n-deficient Rh isotopes lack measurements of charge radii, nuclear EM moments, and firm spin assignments



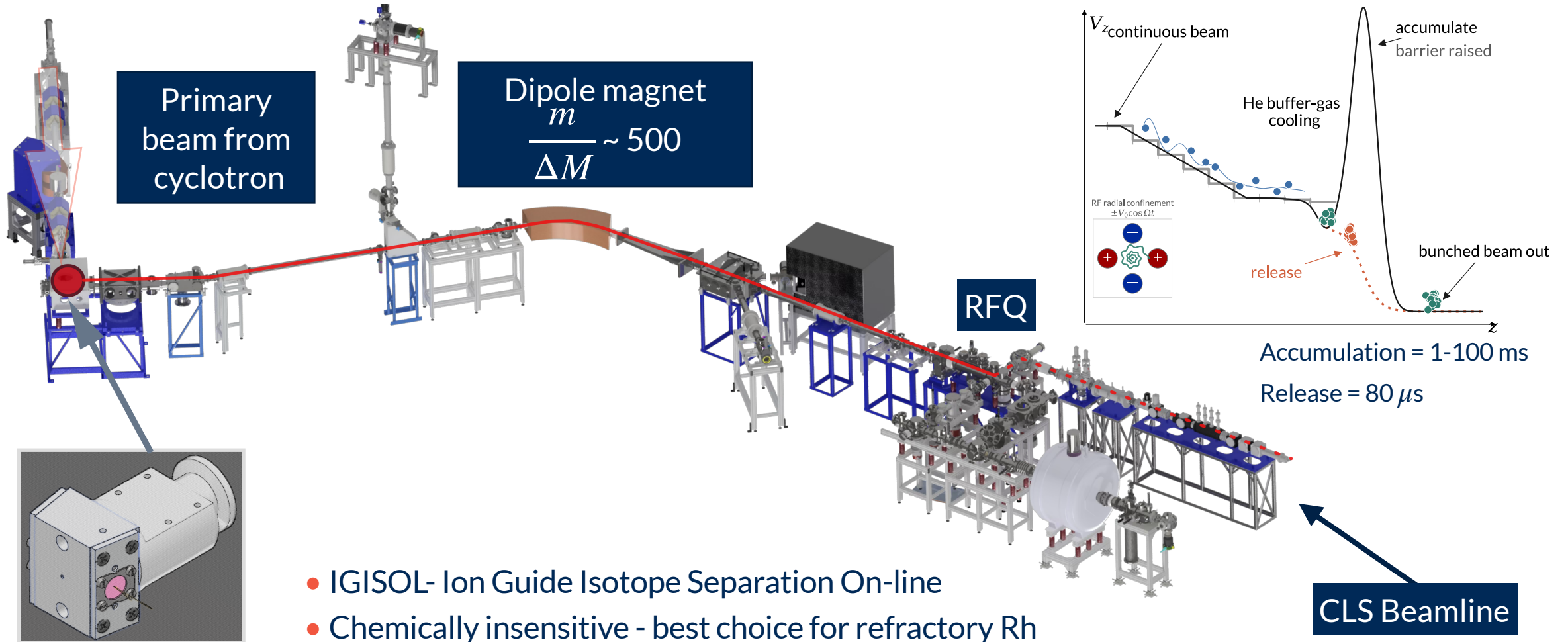
Laser Spectroscopy - Nuclear Structure



- Field shift Factor (F) , Mass shift factor (M) , V_{zz} from atomic theory - A. Borschevsky, E.H. Prinsen - University of Groningen



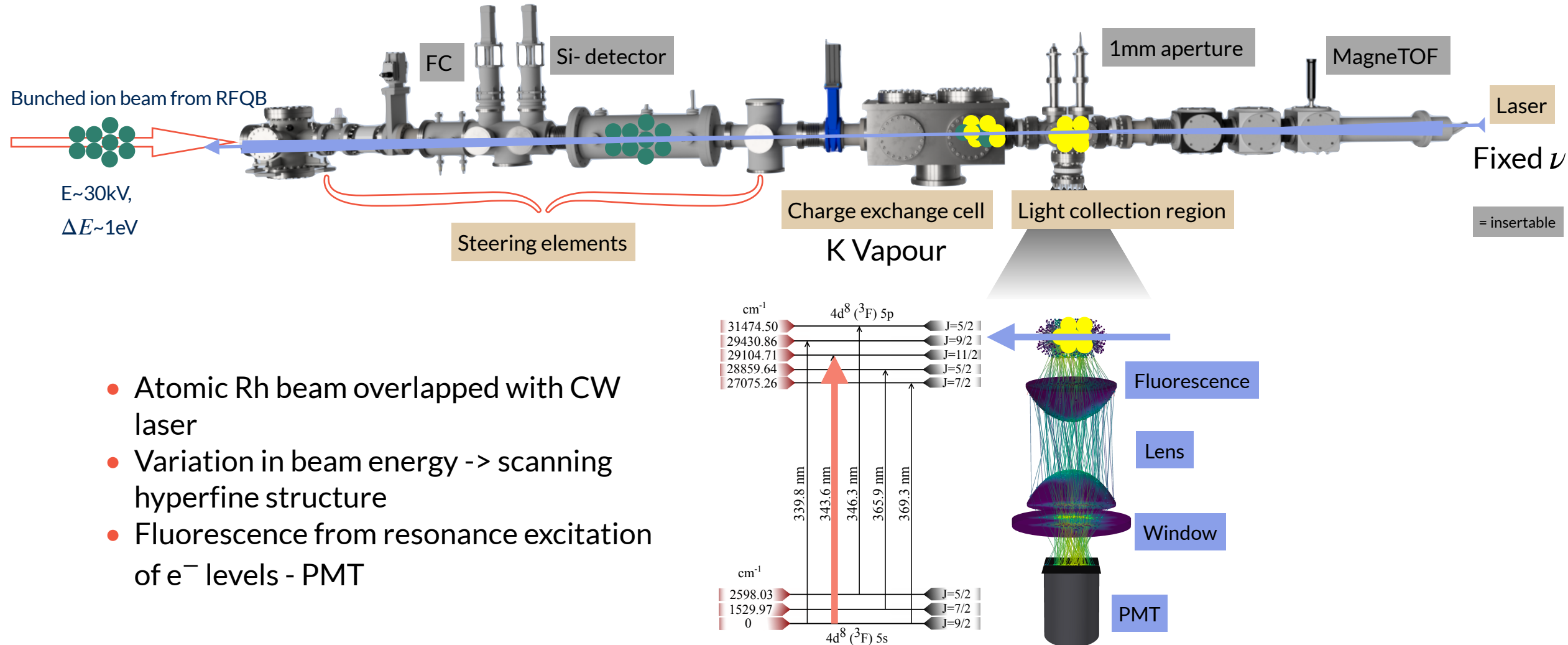
Radioactive beam production - IGISOL



Light ion guide

n-def isotopes close to stability

Collinear Laser Spectroscopy (CLS) at IGISOL



- Atomic Rh beam overlapped with CW laser
- Variation in beam energy -> scanning hyperfine structure
- Fluorescence from resonance excitation of e⁻ levels - PMT



Offline CLS studies - stable ^{103}Rh

$s \rightarrow p$ transition
Strong, low lying

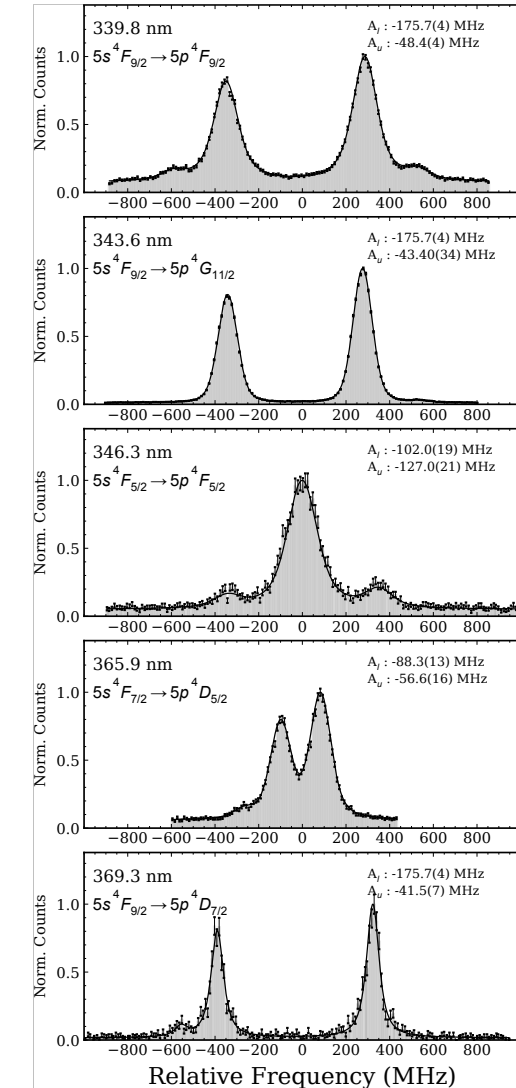
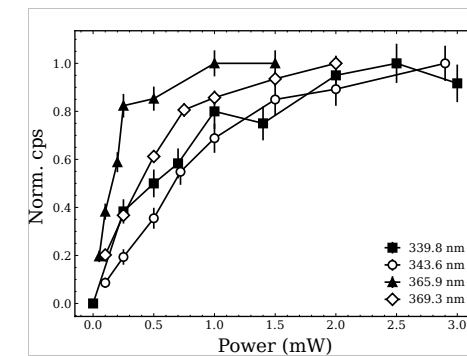
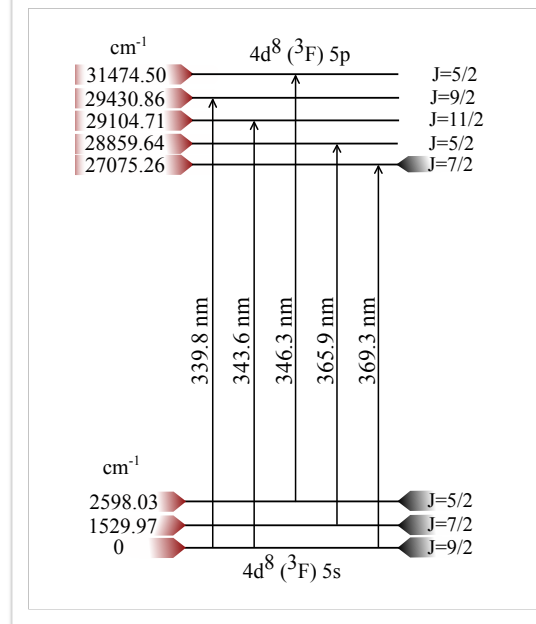
Photon yield $\geq 10^{-3}$
At least one per 1000 ions

Resolved Hyperfine peaks

Saturates ~ 1 mW
Low PMT background

Good e^- transition
online conditions

- Atomic $s \rightarrow p$ transitions, no suitable ion lines
- Photon yield spanned 1:300 to 1:100k
- **343.6 nm best online candidate**
- $^{103}\text{Rh}^+$ from spark discharge, neutralized in K vapour ($\sim 80\%$)
- CW light - SHG of OPO output was used





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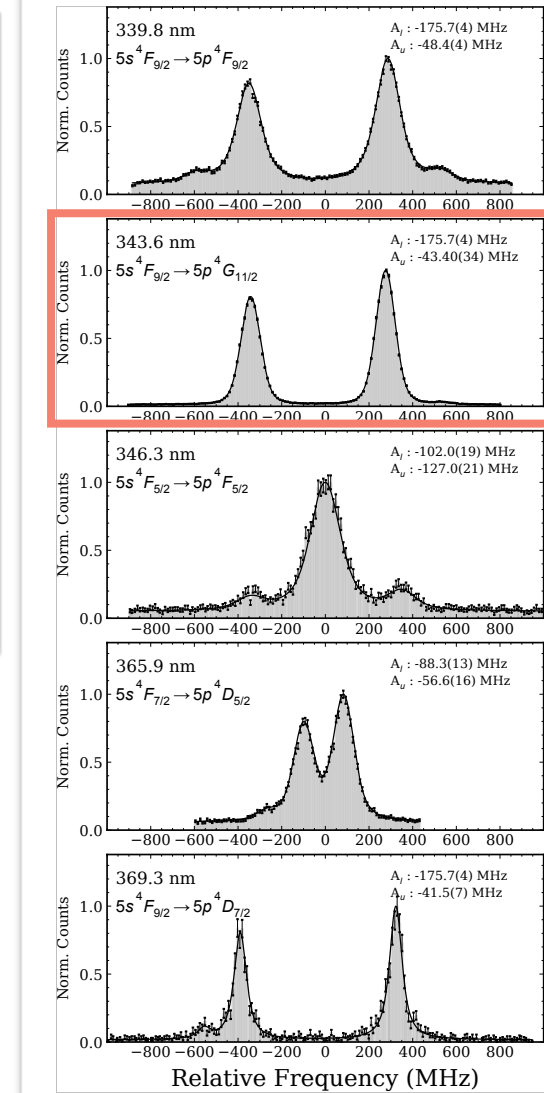
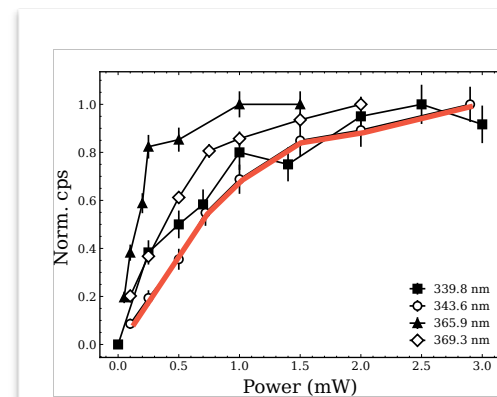
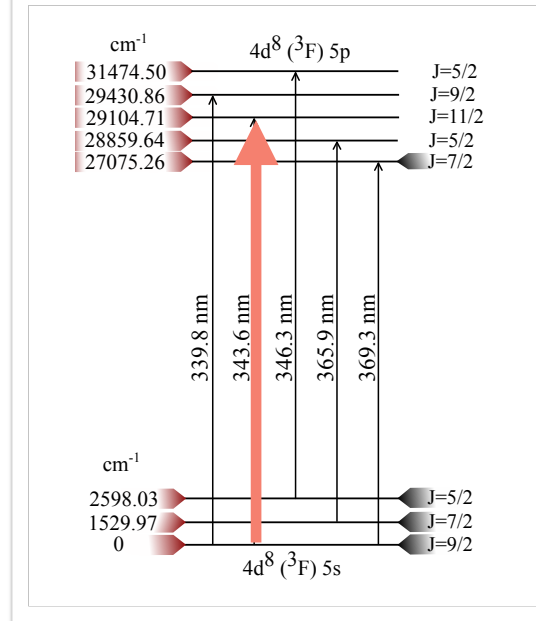
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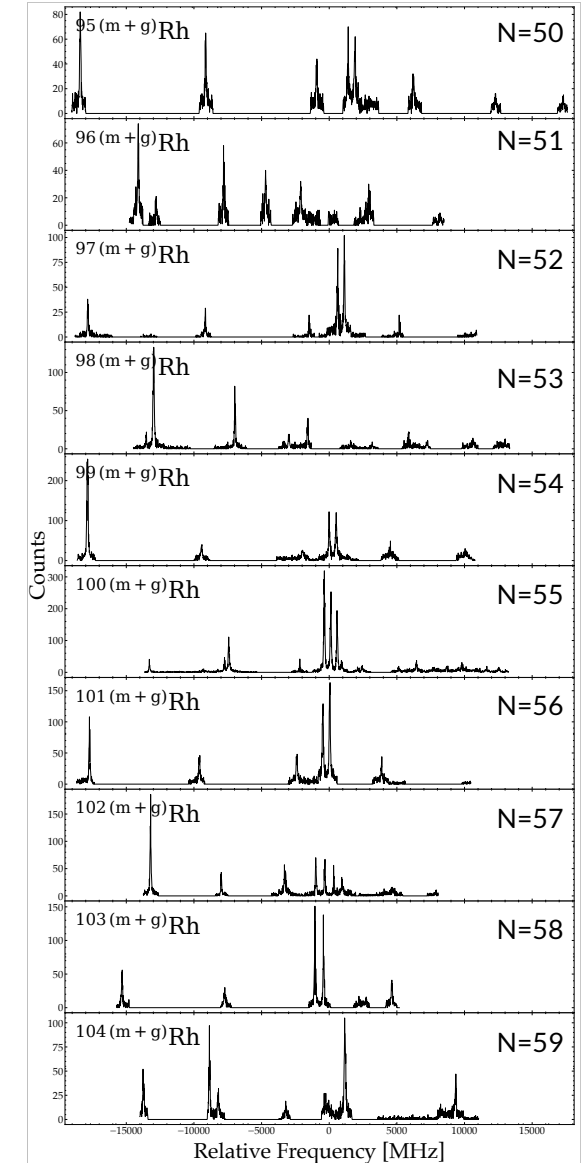
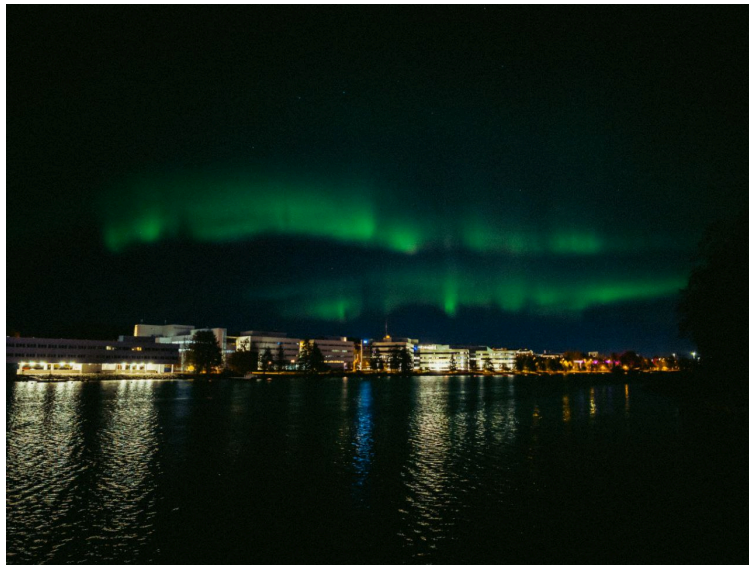
S. CHINTHAKAYALA ET.AL SUBMITTED TO NIM-B





Online studies n-def Rh isotopes

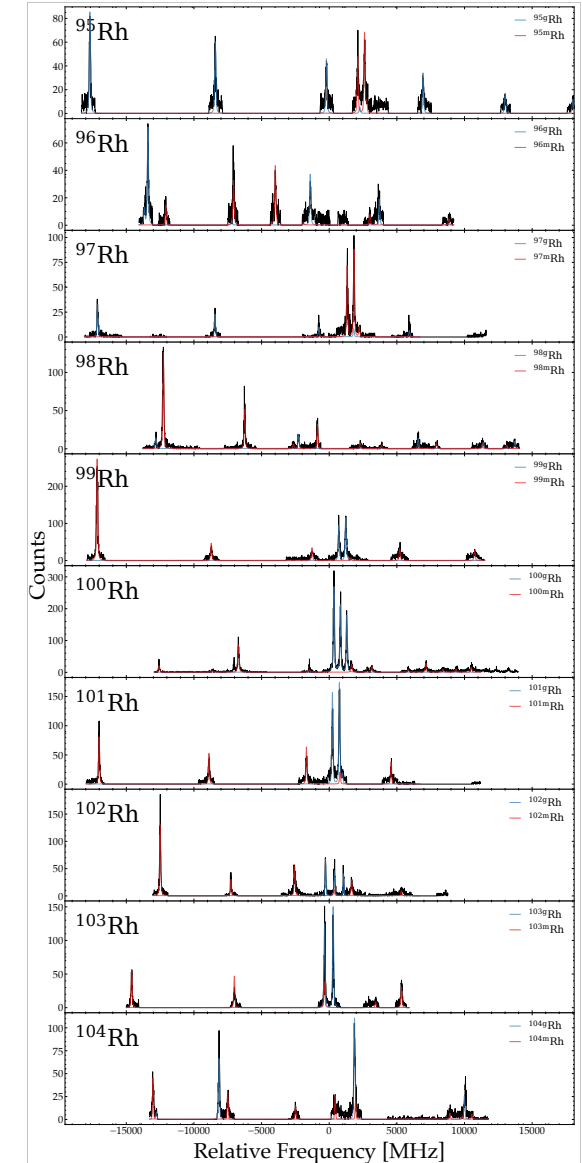
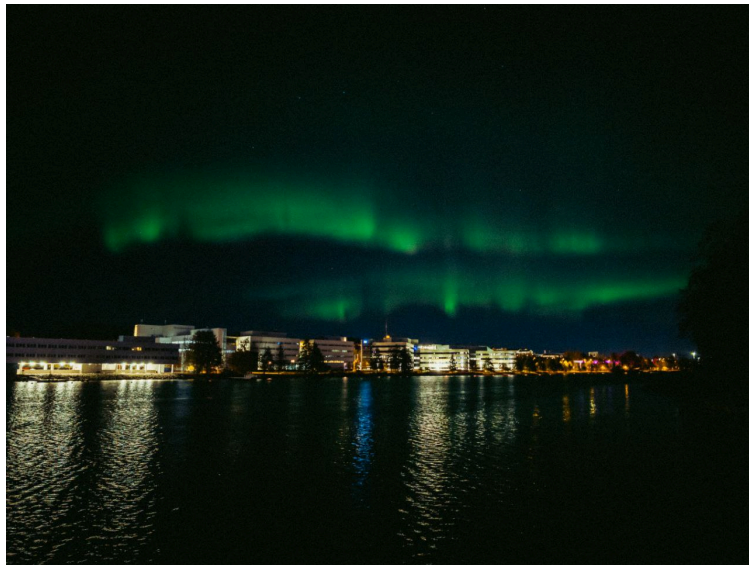
- $^{95-104}\text{Rh} \rightarrow 10$ isotopes, 20 states (g+m)
- Light ion guide - d beam at 15/30 MeV on ^{nat}Ru (2.1 mg/cm^2) and ^{96}Ru ($\sim \text{few mg/cm}^2$)





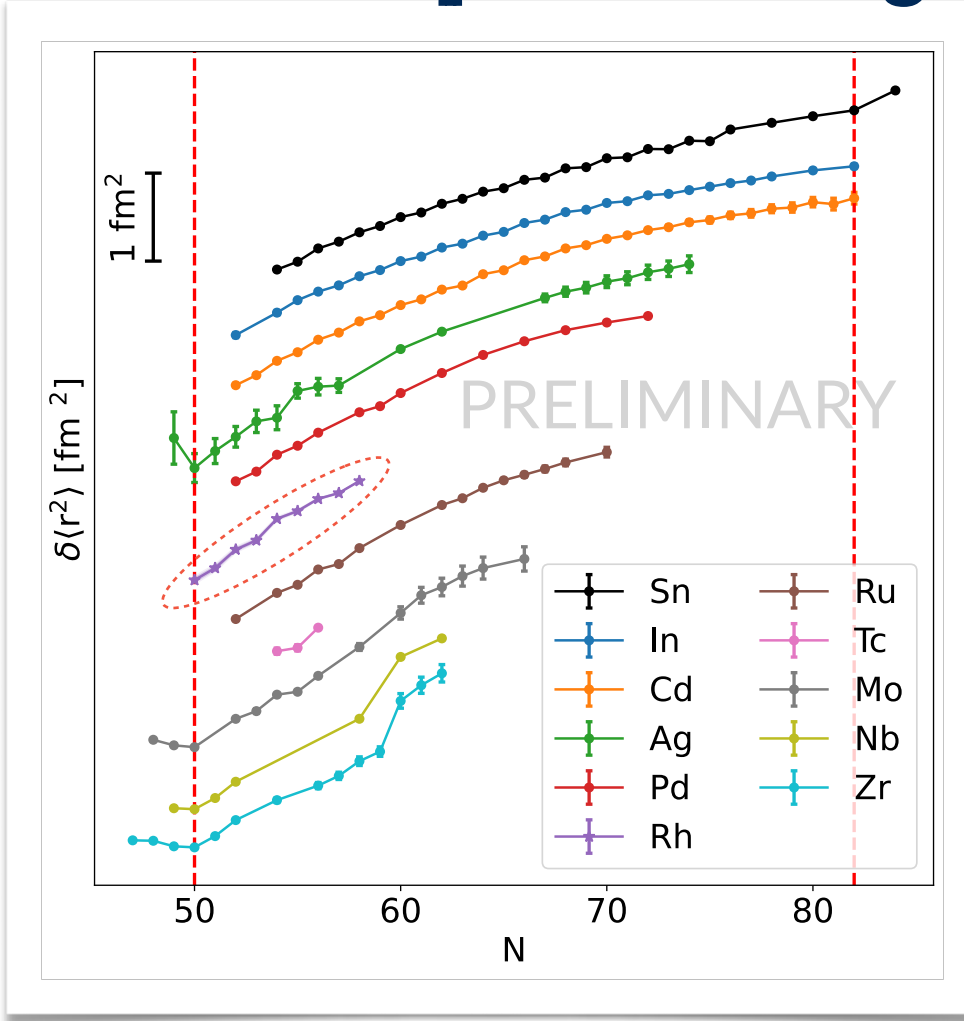
Online studies n-def Rh isotopes

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Mean Squared Charge Radius

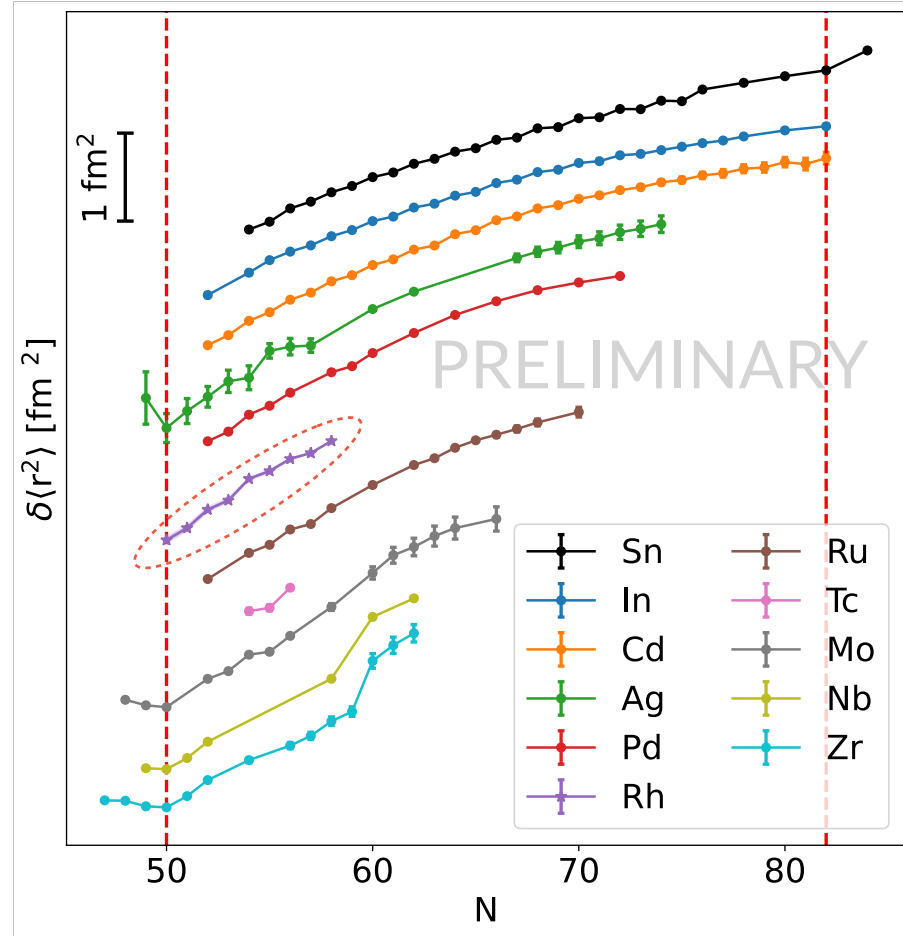


$$\delta\nu = M \left(\frac{1}{m} - \frac{1}{m'} \right) + F \delta \langle r^2 \rangle$$

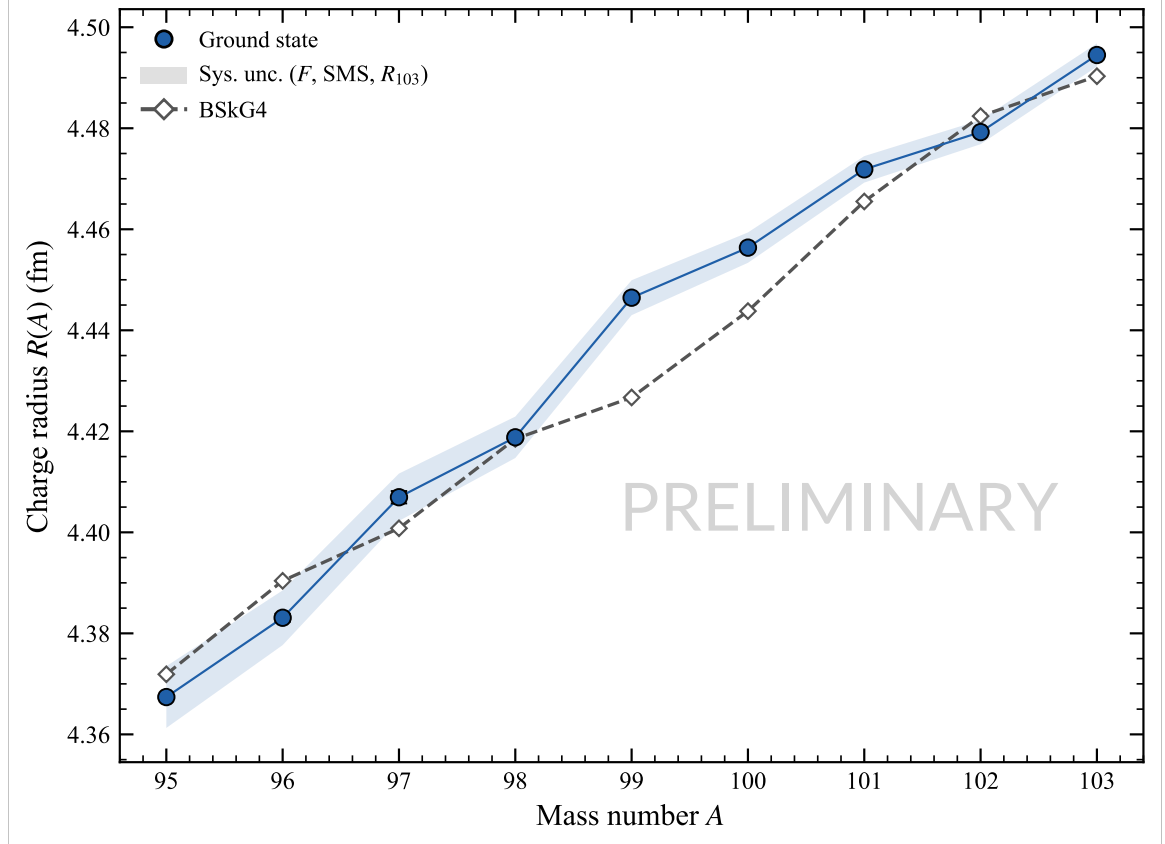
Measurement Atomic Params. Nuclear Observable

- No F from theory yet, estimated from neighboring chains

Mean Squared Charge Radius



GRAMS, G., SHCHECHILIN, N.N., SÁNCHEZ-FERNÁNDEZ, A. ET AL. EUR. PHYS. J. A 61, 35 (2025).



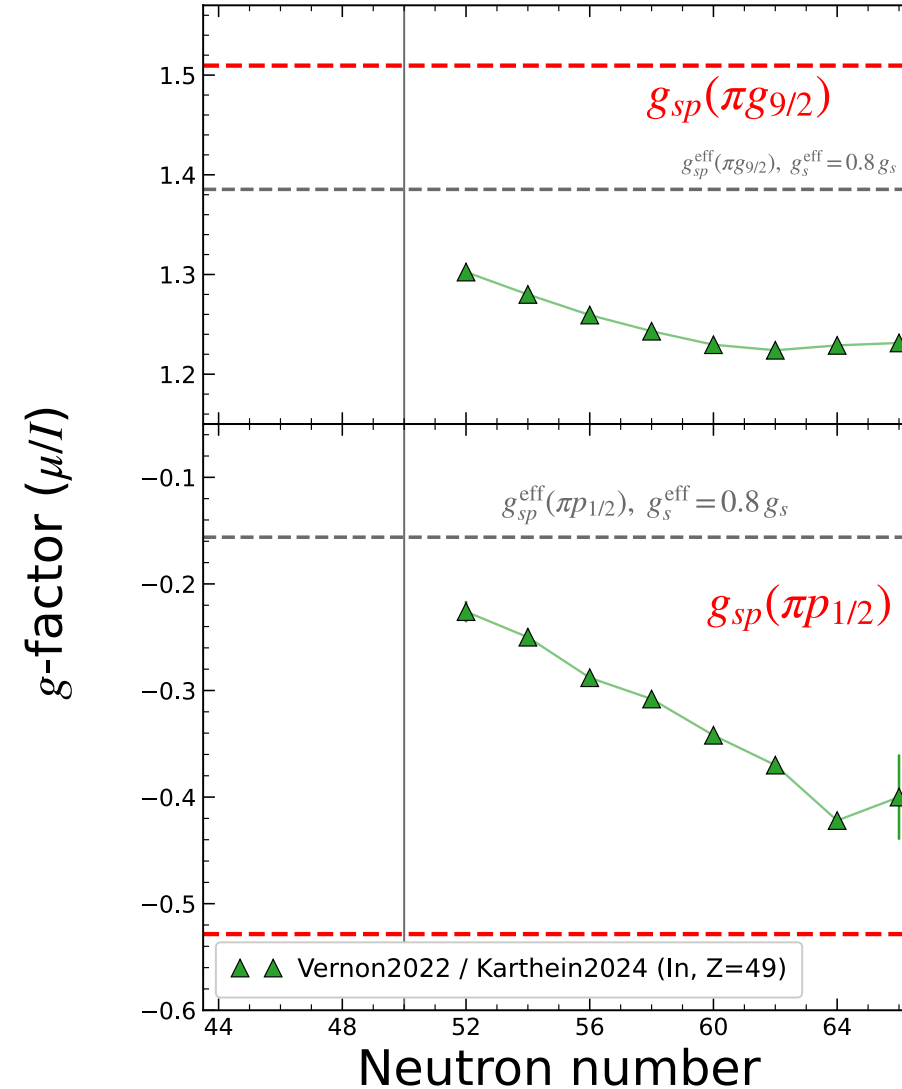
- No F from theory yet, estimated from neighboring chains
- Smooth increase of charge radii with N



Nuclear Magnetic Dipole moments - Odd A

- Near constant trend for $9/2^+$ states
- Trend of $1/2^-$ not constant - not simple particle - hole configuration

50	Sn
49	In
	Cd
47	Ag
	Pd
45	Rh
	Ru
43	Tc
	Mo
41	Nb
	Zr
39	Y

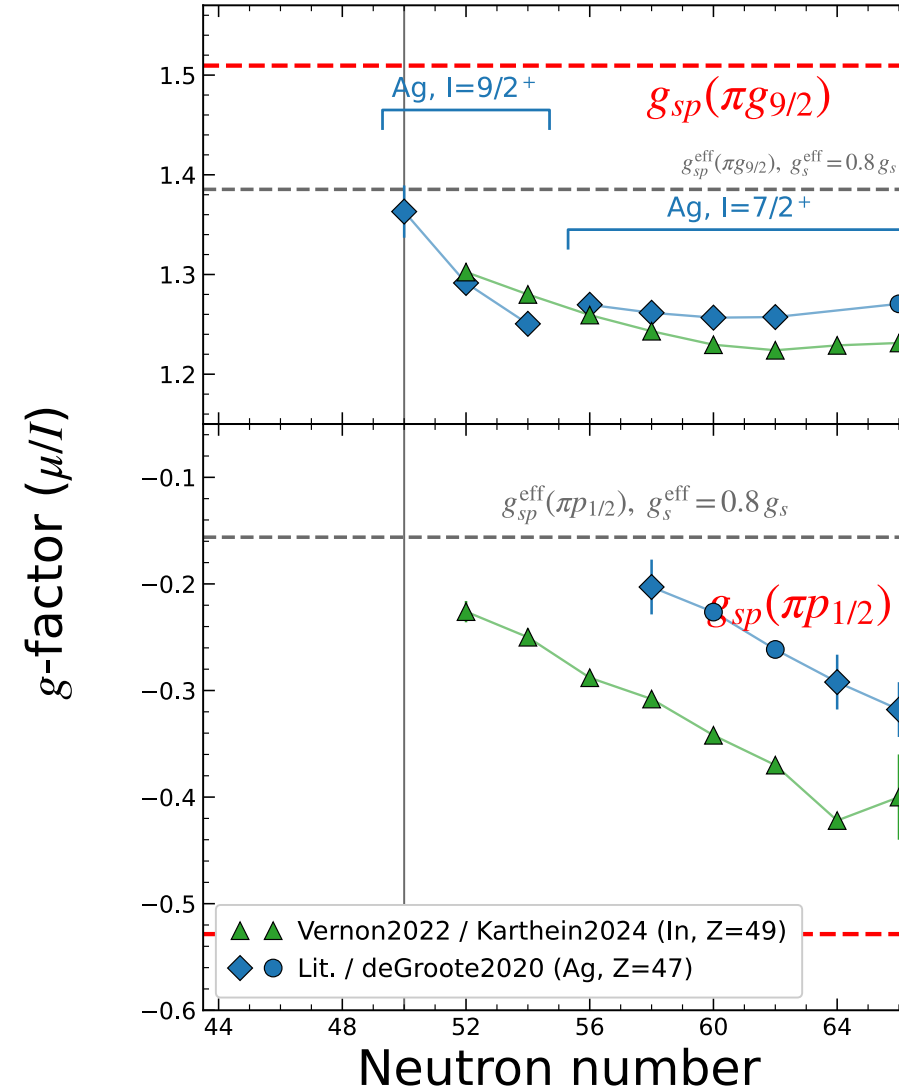




Nuclear Magnetic Dipole moments - Odd A

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- $7/2^+$ or $9/2^+$ states - constant + shell
effect towards N=50
- Trend for $1/2^-$ is not constant -
Similar trend seen in In (Z=49)

50	Sn
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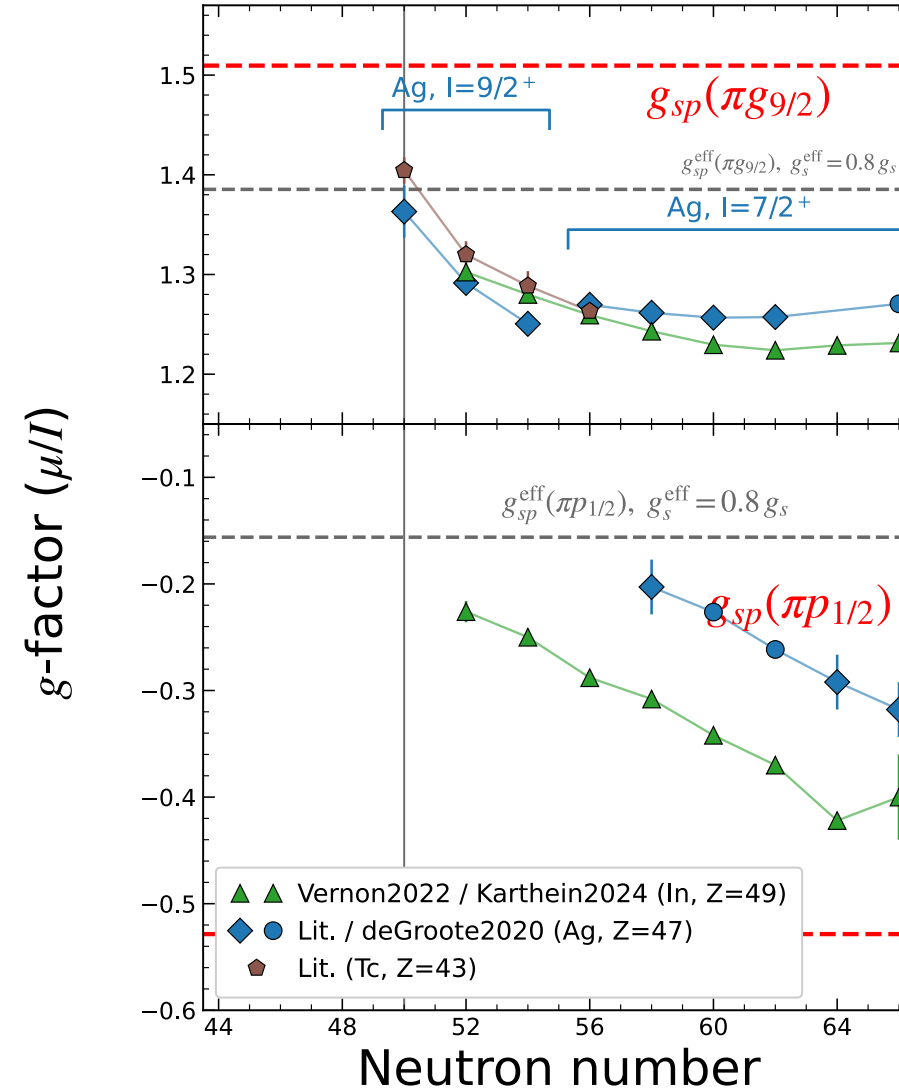




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- $9/2^+$ states - shell effect towards $N=50$
- Limited available data

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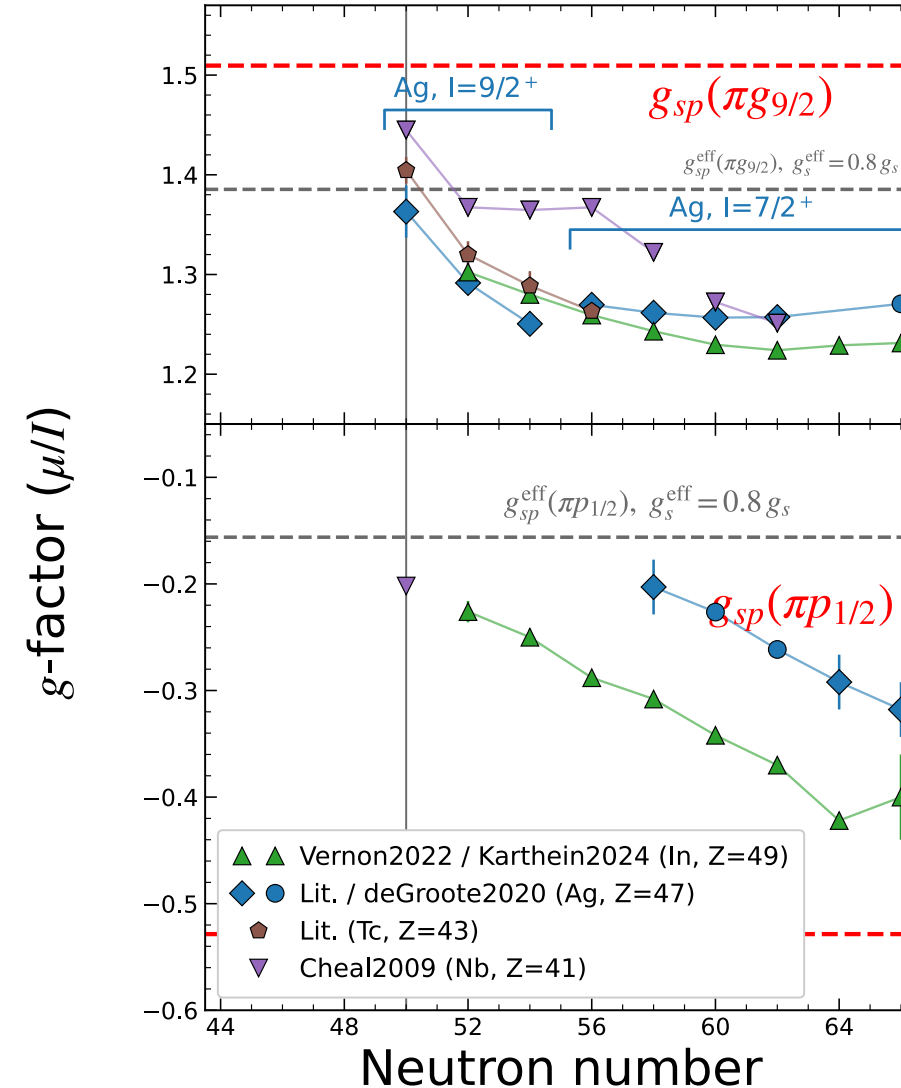




Nuclear Magnetic Dipole moments - Odd A

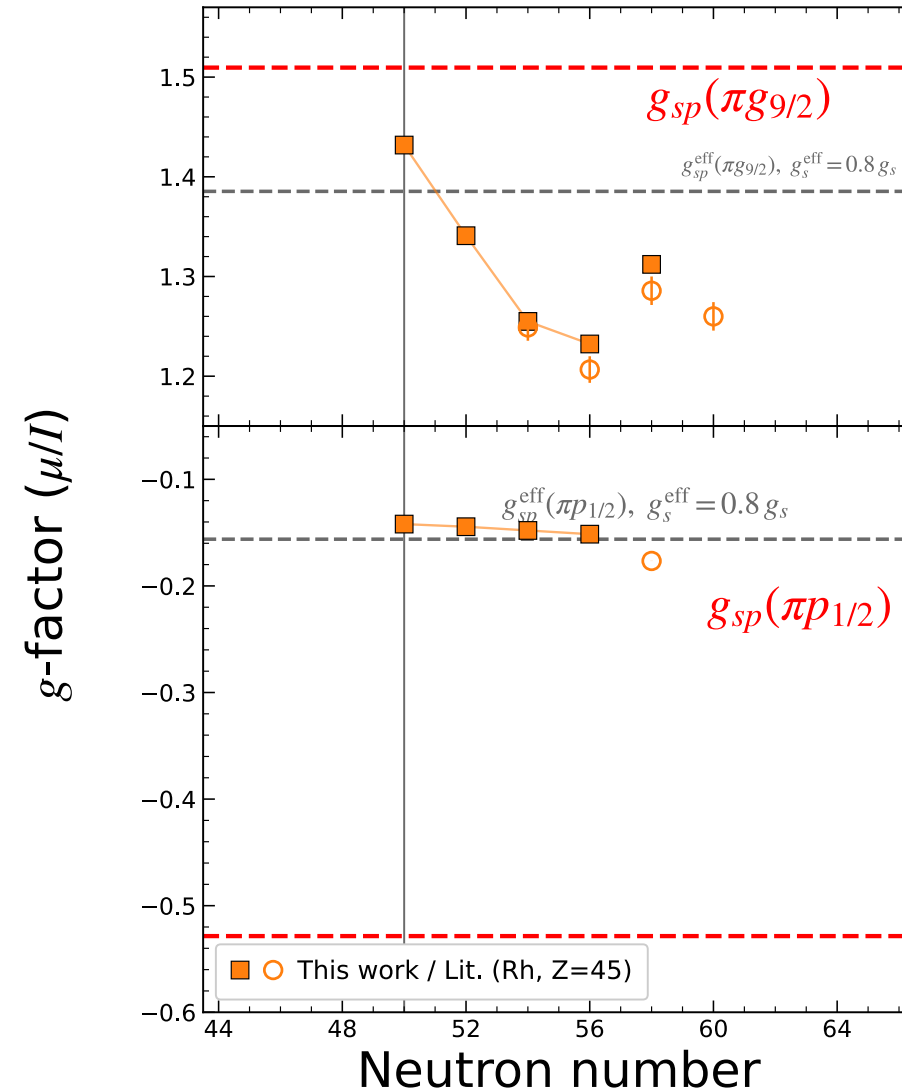
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- $9/2^+$ states - shell effect towards $N=50$
- Limited available data
- $9/2^+$ states shows nearly constant trend for $N>50$

50	Sn
49	In
	Cd
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	Pd
45	Rh
	Ru
43	Tc
	Mo
41	Nb
	Zr
39	Y



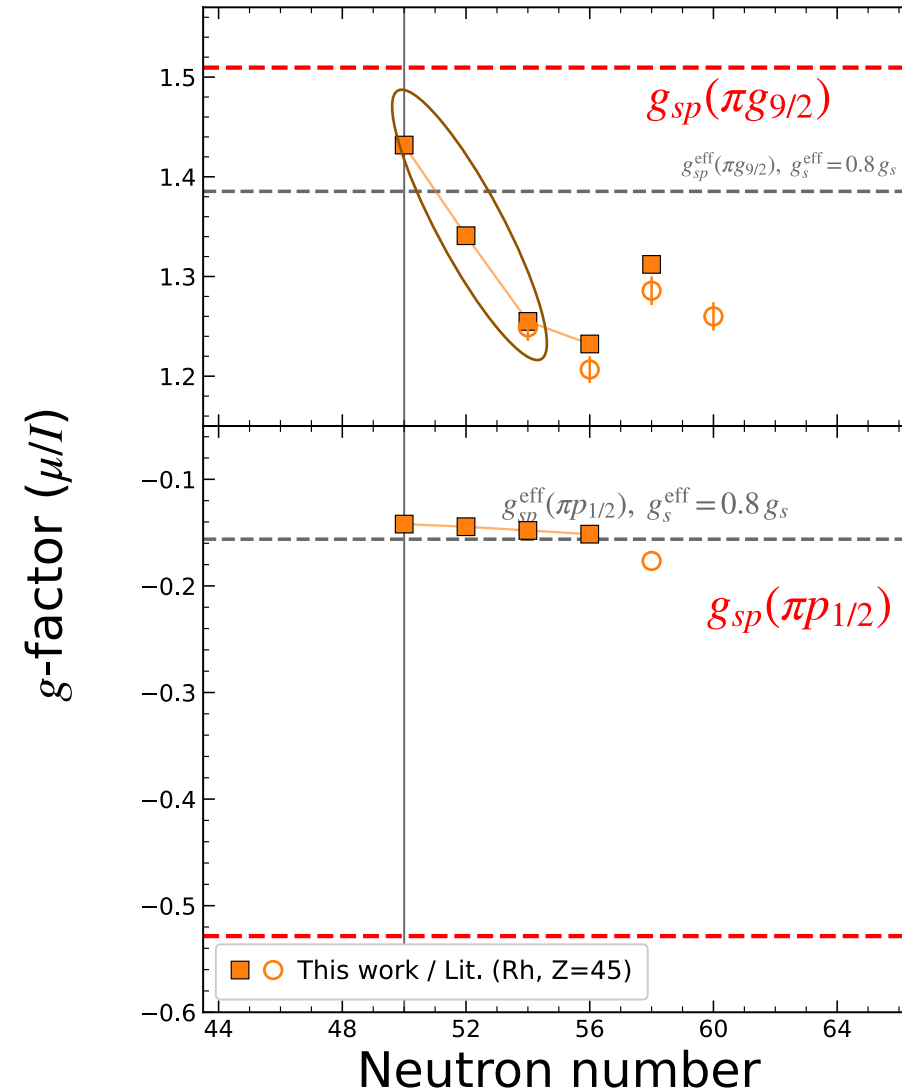


Nuclear Magnetic Dipole moments - Odd Rh





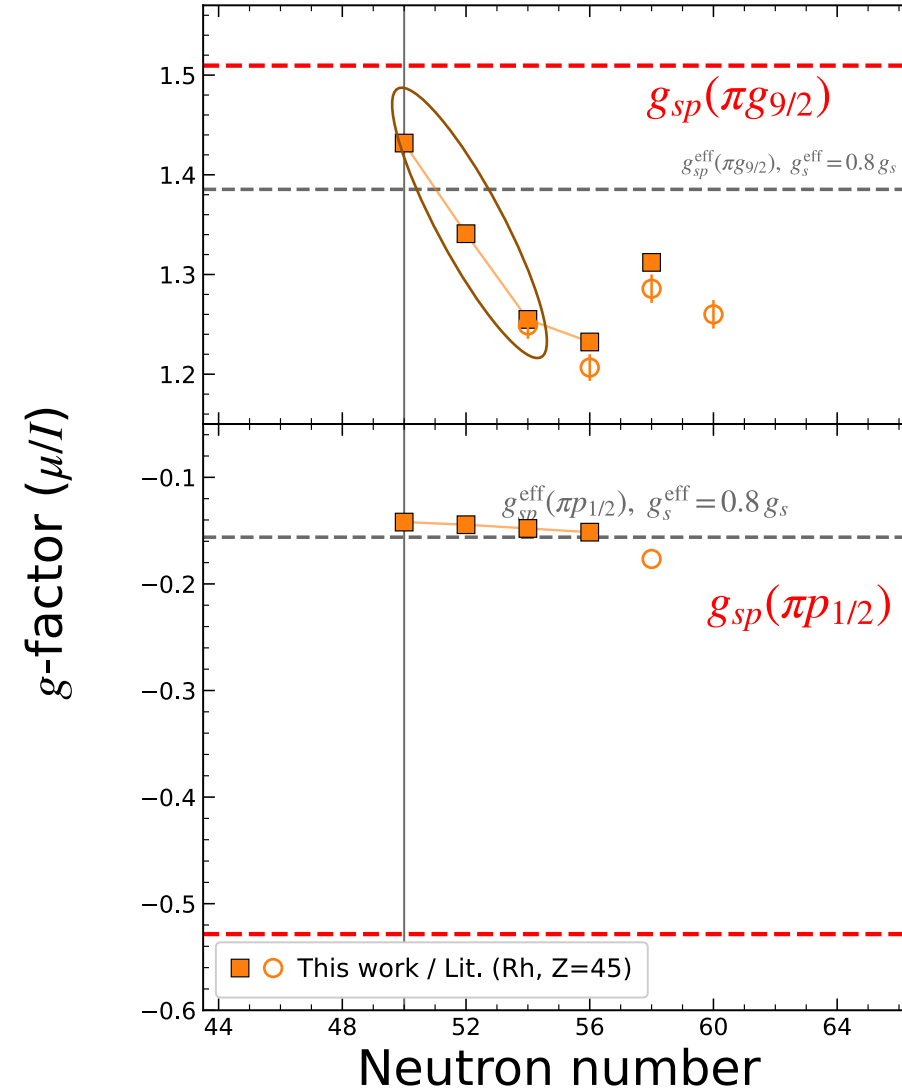
Nuclear Magnetic Dipole moments - Odd Rh





Nuclear Magnetic Dipole moments - Odd Rh

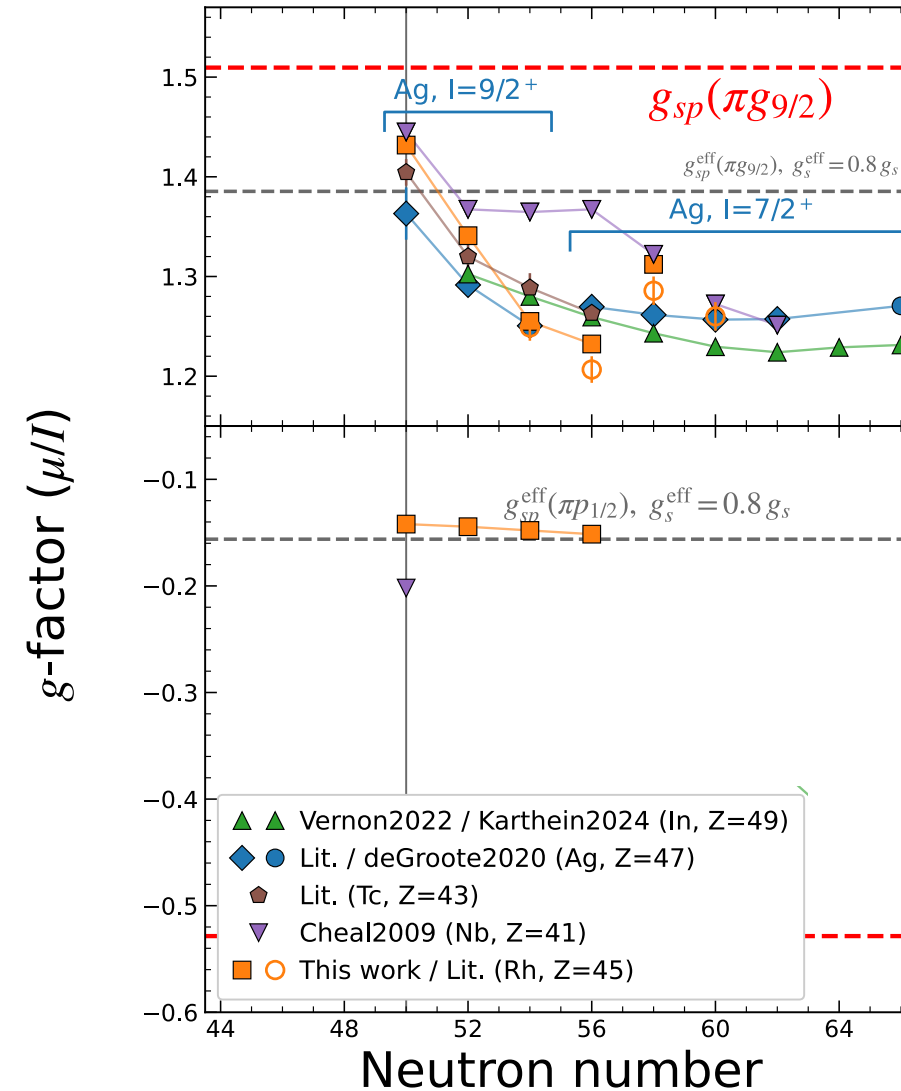
- $9/2^+$ shows increasing trend approaching N=50, similar to In, Ag, Tc - evidence of shell closure





Nuclear Magnetic Dipole moments - Odd Rh

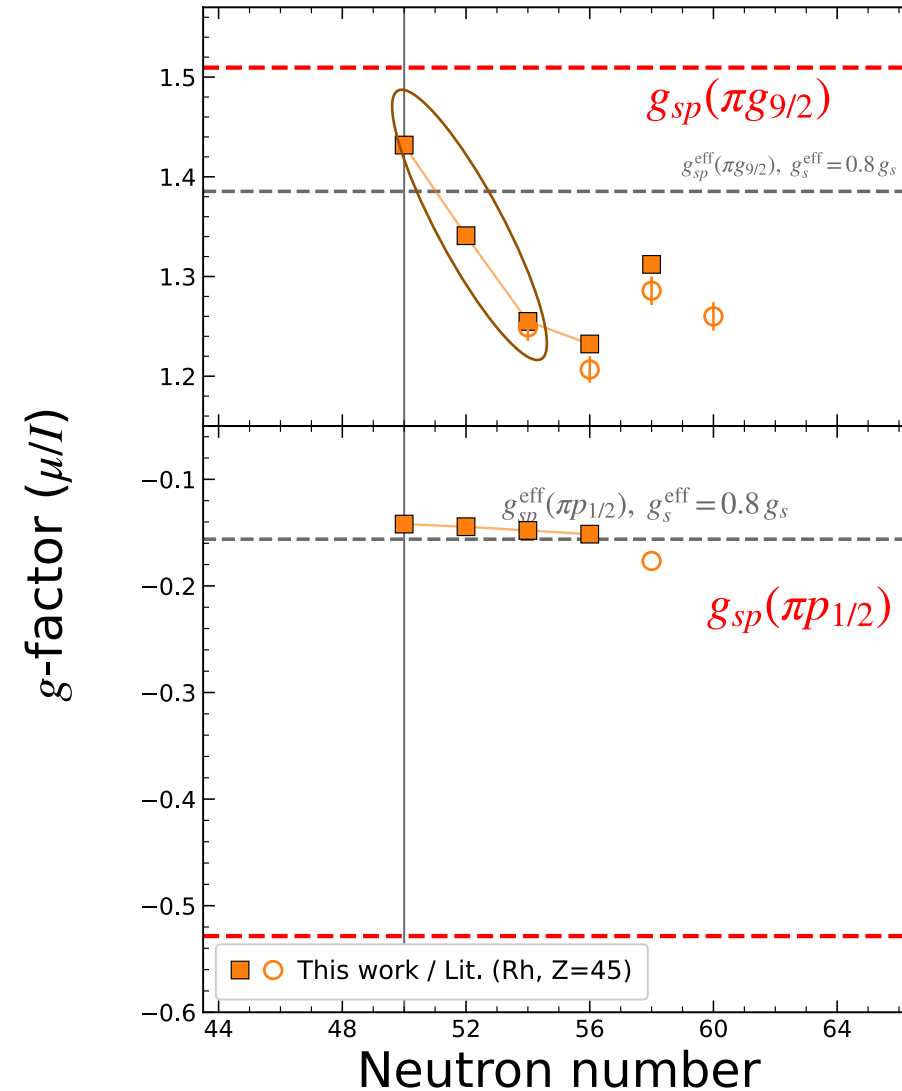
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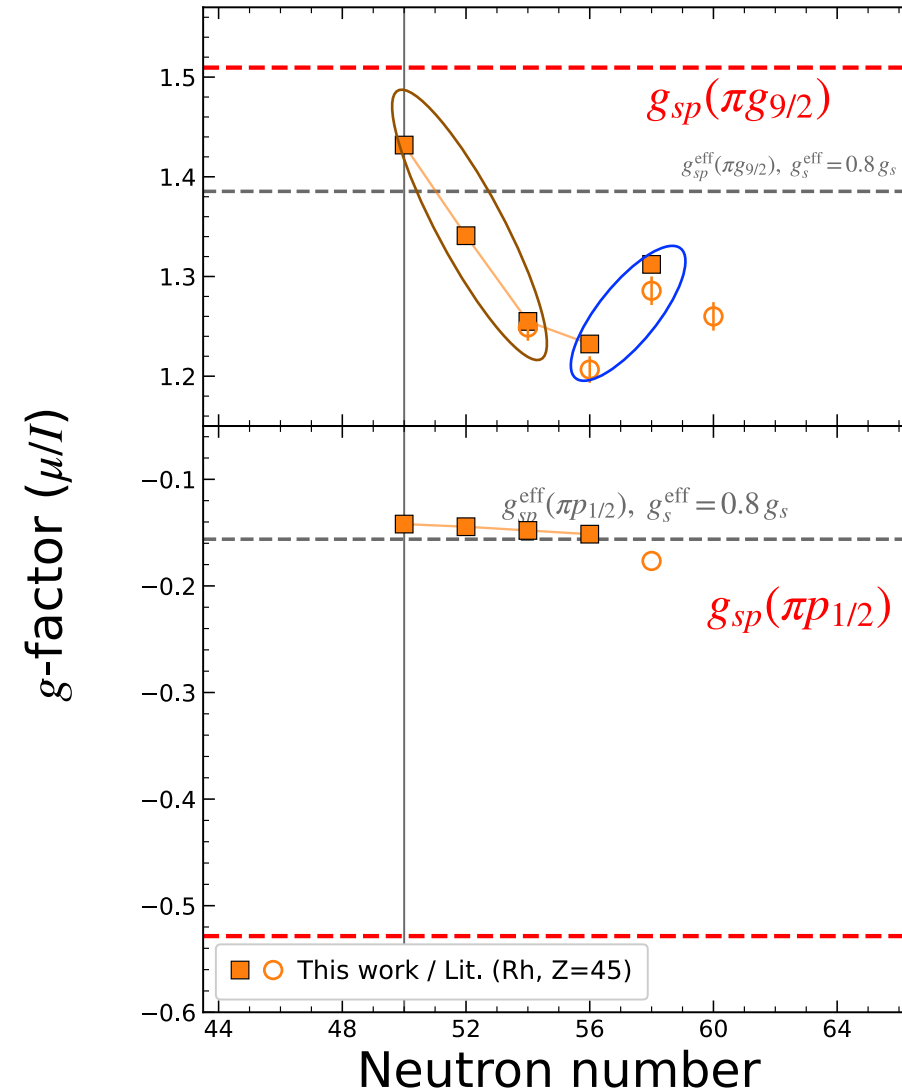
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- Difference in g-factor for $9/2^+$ and $7/2^+$ states





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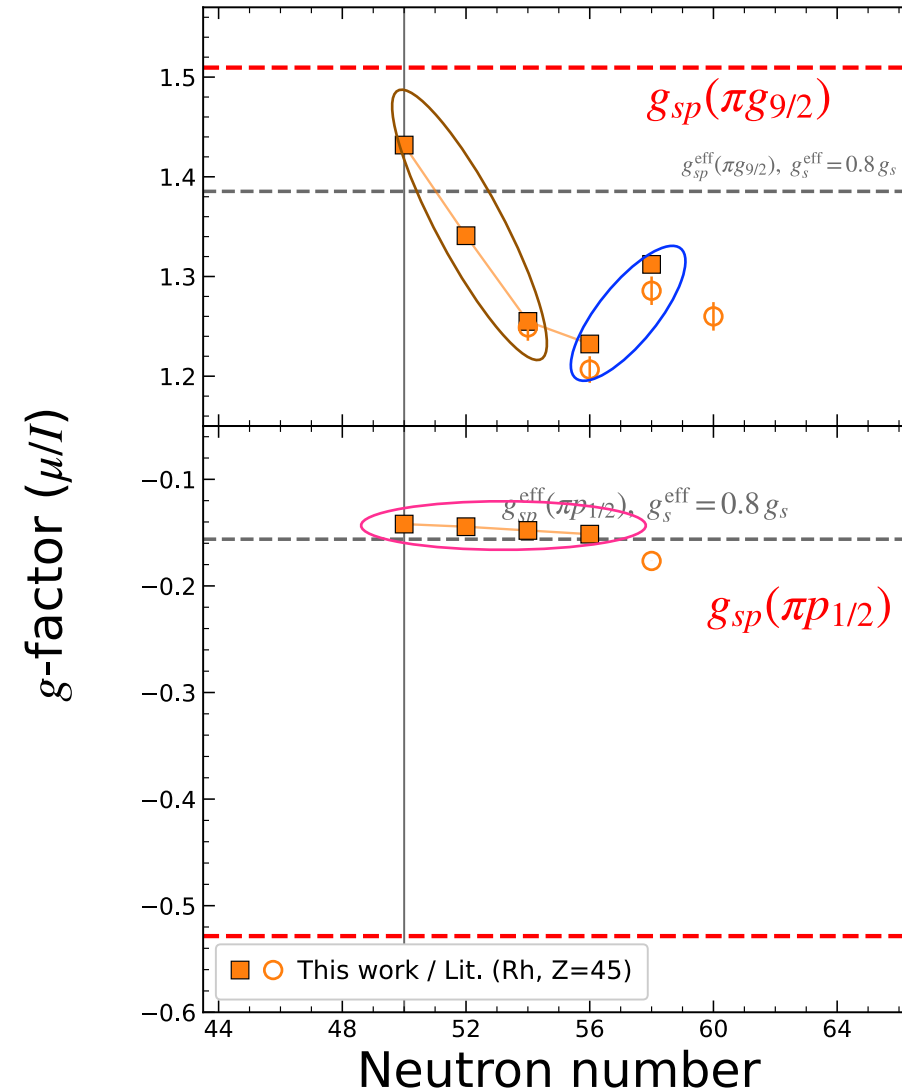
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Nuclear Magnetic Dipole moments - Odd Rh

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- Difference in g-factor for $9/2^+$ and $7/2^+$ states
- $1/2^-$ shows quasi-constant trend approaching N=50, close to quenched Schmidt line - particle hole configuration $\pi p_{1/2}$

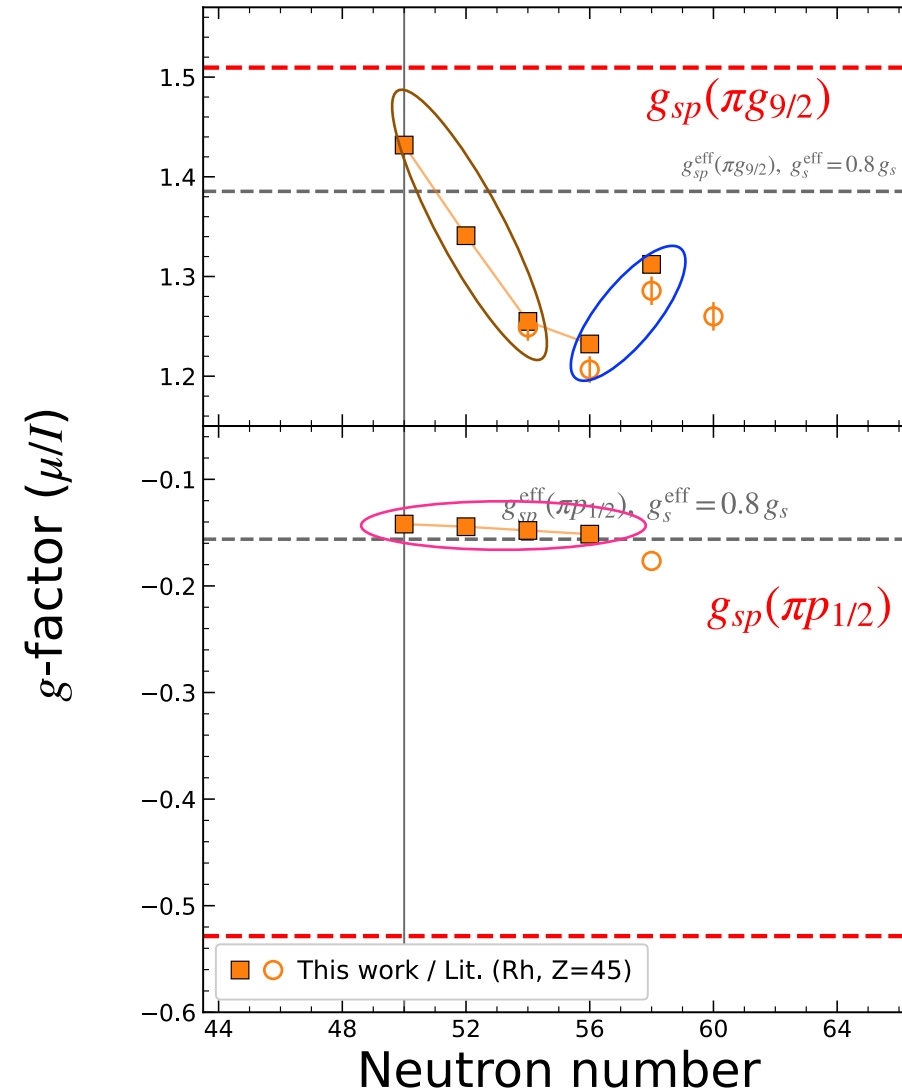




Nuclear Magnetic Dipole moments - Odd Rh

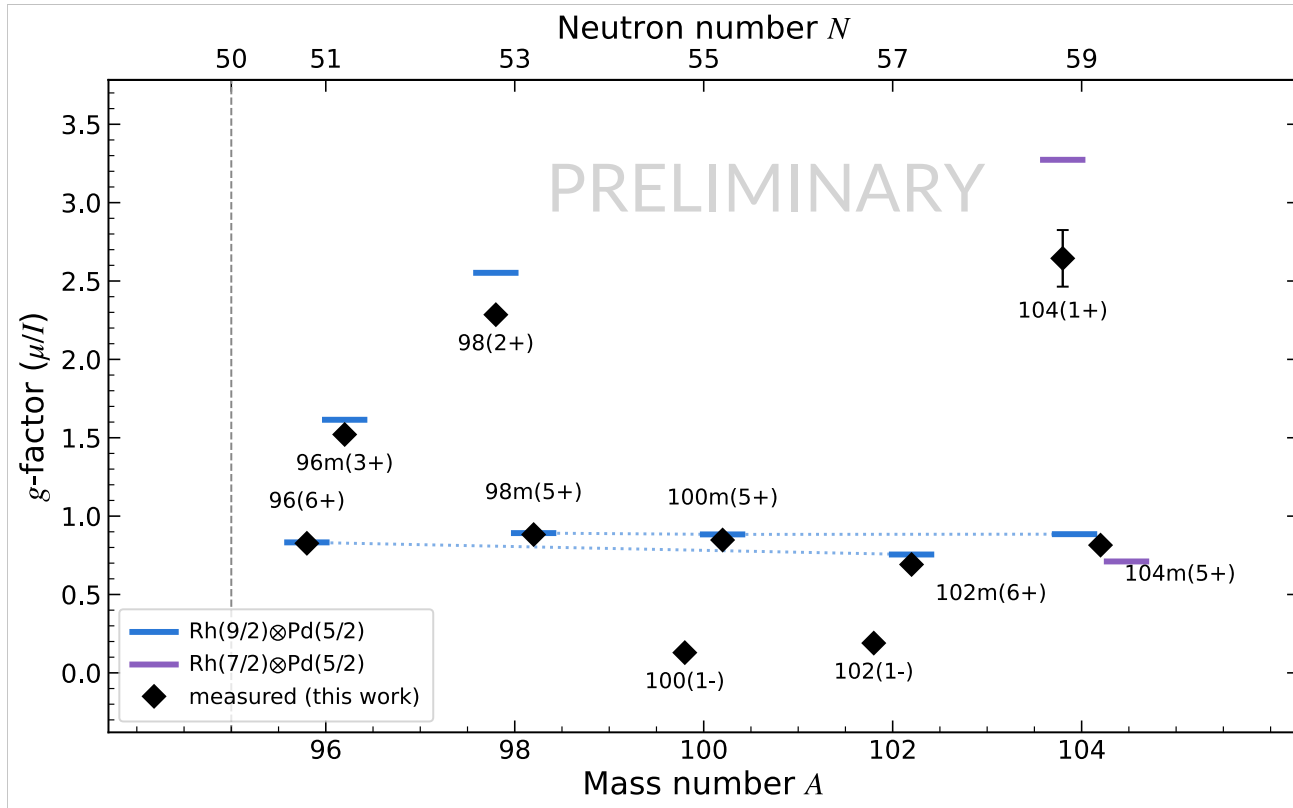
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Need of exp. Data on n-rich Rh isotopes - proposal submitted JYFL- PAC





Nuclear Magnetic Dipole moments - Even Rh



Additivity rule

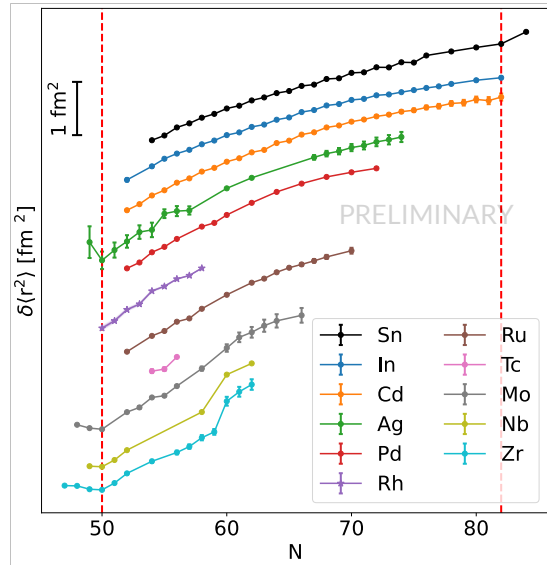
$$\mu(J) = \frac{J}{2} \left[\frac{\mu_{\pi}}{J_{\pi}} + \frac{\mu_{\nu}}{J_{\nu}} + \left(\frac{\mu_{\pi}}{J_{\pi}} - \frac{\mu_{\nu}}{J_{\nu}} \right) \frac{J_{\pi}(J_{\pi} + 1) - J_{\nu}(J_{\nu} + 1)}{J(J + 1)} \right]$$

^{97}Pd	^{98}Pd	^{99}Pd $5/2^+$	^{100}Pd
^{96}Rh	^{97}Rh	^{98}Rh	^{99}Rh $m=9/2^+$
^{95}Ru	^{96}Ru	^{97}Ru	^{98}Ru

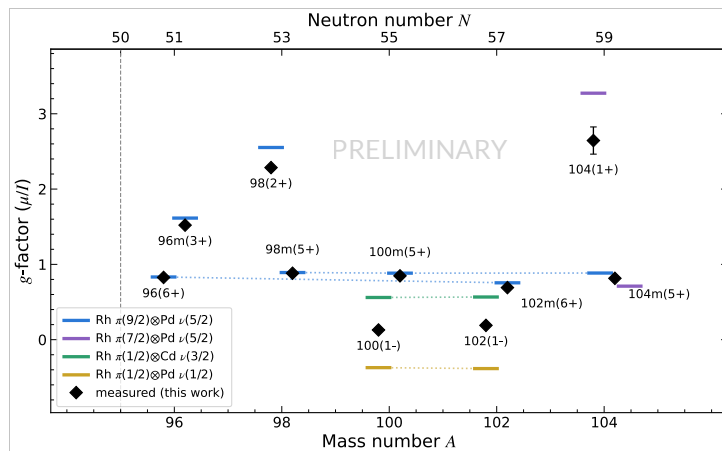
- Good agreement of experimental g -factors with empirical estimates
- No μ data for $3/2^+$ or $1/2^+$ states around $A=100, 102$



Summary and Outlook



- First laser spectroscopy measurements of n-def Rh isotopes
- Charge radius show smooth trend - reached N=50 - no abrupt changes in shape
- g-factor - $9/2^+$ shows increasing trend approaching N=50 - evidencing shell closure N=50
- g-factor - $1/2^-$ nearly-constant trend, following simple p-h config.
- General good agreement of g-factor trends of even Rh isotopes compared to g-factors calculated using additivity rule
- Data analysis to obtain electric quadrupole moments -> deformation is ongoing



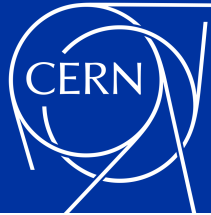
- Proposal to study n-rich Rh isotopes submitted to JYFL PAC, experiment in 2027
- Attempts to reach Rh isotopes N<50 - RIS studies at S3-LEB or LASAGN?





Thank you

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O. Ahmad, D. Bettaney , A. Borschevsky, L. Caceres, A. Candiello, P. Campbell, B. Cheal, T. Christen , P. Delahaye, R.P de Groote, J. Hughes, A. Karadimas, A. Kayaalp , A. Koszorus I. D. Moore, A. Raggio, M. Reponen, M. Stryczyk, J. Warbinek, and IGISOL group



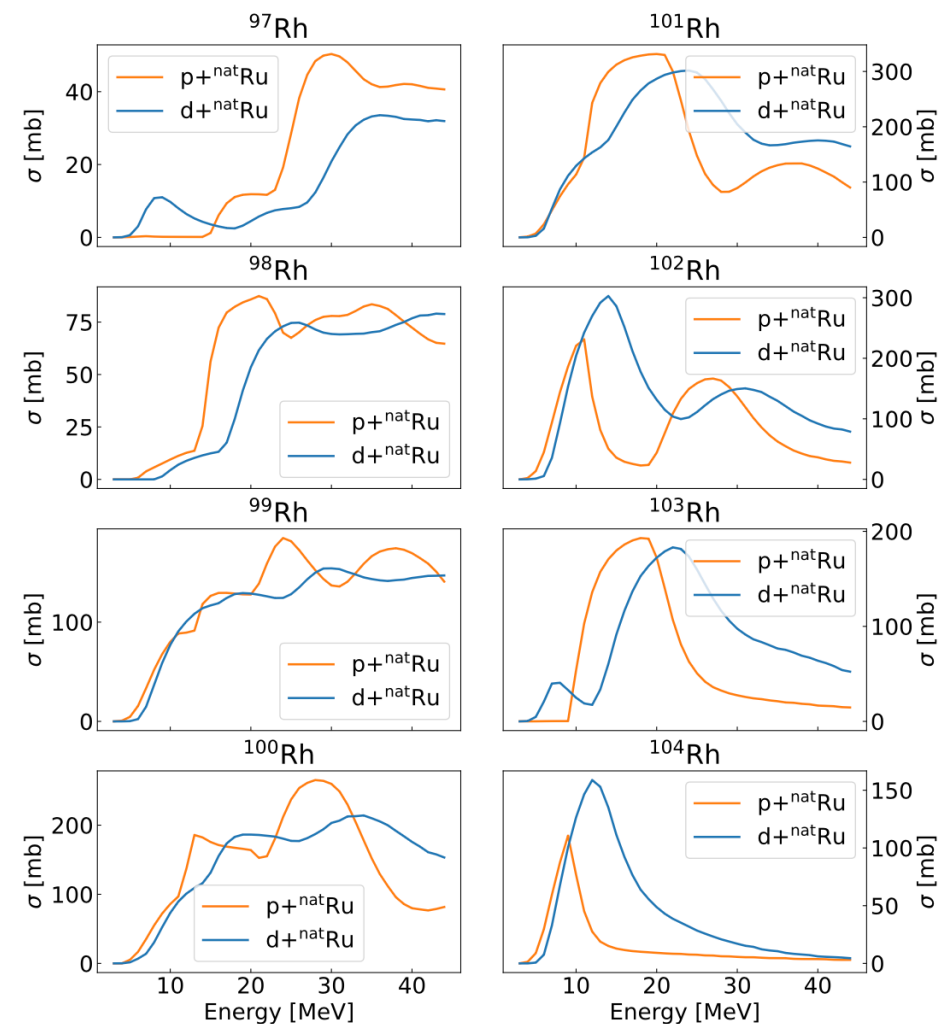
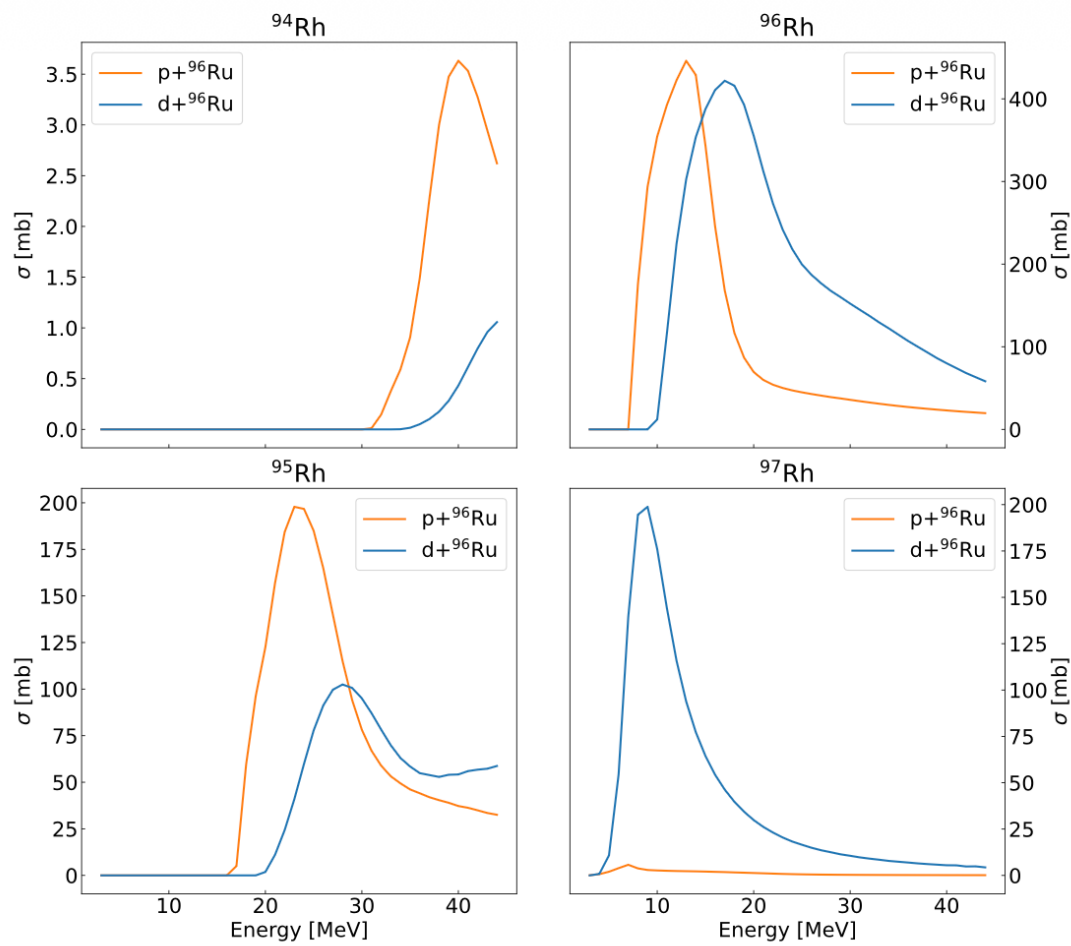
Extra Slides

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Yields





Estimated yields at SWSi, considering 15cps per mbarn per uA of d beam

Isotope	Target	Energy	Approx. d-beam σ	Expt. Est. Yield
95Rh	96Ru	30 MeV	~80–100 mb	~18–23 kcps
96Rh	96Ru	30 MeV	~180–200 mb	~40–45 kcps
97Rh	natRu	15 MeV	~3–5 mb	~0.7–1.1 kcps
98Rh	natRu	30 MeV	~70–75 mb	~16–17 kcps
99Rh	natRu	15 MeV	~110–120 mb	~25–27 kcps
100Rh	natRu	30 MeV	~200–210 mb	~45–47 kcps
101Rh	natRu	15 MeV	~220–250 mb	~50–56 kcps
102Rh	natRu	30 MeV	~130–150 mb	~29–34 kcps
103Rh (g+m)	natRu	15 MeV	~20–30 mb	~4–7 kcps
103Rh (g+m)	natRu	30 MeV	~80–100 mb	~18–23 kcps
104Rh	natRu	15 MeV	~140–160 mb	~31–36 kcps



Derivation - additivity rule

1. Setup and Definitions

- Total Angular Momentum:** The total spin J is the vector sum of the proton and neutron spins:

$$J = J_\pi + J_\nu$$

- Total Magnetic Moment:** The total magnetic moment operator μ is the sum of the individual moments:

$$\mu = \mu_\pi + \mu_\nu$$

- Gyromagnetic Ratios (g-factors):** By definition, $\mu = gJ$. For the individual nucleons, $g_\pi = \frac{\mu_\pi}{J_\pi}$ and $g_\nu = \frac{\mu_\nu}{J_\nu}$. Substituting these in gives:

$$\mu = g_\pi J_\pi + g_\nu J_\nu$$

2. The Projection Theorem

Because J is a constant of motion, the magnetic moment vector μ rapidly precesses around J . The only observable component is the part of μ that lies parallel to J .

We project μ onto J by taking the dot product. The effective total g-factor (g) is:

$$g = \frac{\langle \mu \cdot J \rangle}{\langle J^2 \rangle} = \frac{\langle \mu \cdot J \rangle}{J(J+1)}$$

Substitute the definition of μ into the dot product:

$$\mu \cdot J = (g_\pi J_\pi + g_\nu J_\nu) \cdot J = g_\pi (J_\pi \cdot J) + g_\nu (J_\nu \cdot J)$$

3. Solving the Dot Products

To find $J_\pi \cdot J$ and $J_\nu \cdot J$, we use a vector trick based on the law of cosines.

Starting from $J_\nu = J - J_\pi$, we square both sides:

$$J_\nu^2 = (J - J_\pi)^2 = J^2 + J_\pi^2 - 2(J_\pi \cdot J)$$

Rearranging to solve for the dot product and replacing the squared operators with their quantum mechanical eigenvalues (where $J^2 \rightarrow J(J+1)$):

$$J_\pi \cdot J = \frac{1}{2}[J(J+1) + J_\pi(J_\pi+1) - J_\nu(J_\nu+1)]$$

By symmetry, doing the same starting from $J_\pi = J - J_\nu$ gives:

$$J_\nu \cdot J = \frac{1}{2}[J(J+1) + J_\nu(J_\nu+1) - J_\pi(J_\pi+1)]$$

4. Assembling the Final Equation

Plug these dot products back into the total g-factor equation $g = \frac{\langle \mu \cdot J \rangle}{J(J+1)}$:

$$g = \frac{g_\pi}{2} \left[\frac{J(J+1) + J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right] + \frac{g_\nu}{2} \left[\frac{J(J+1) + J_\nu(J_\nu+1) - J_\pi(J_\pi+1)}{J(J+1)} \right]$$

Distribute the $J(J+1)$ denominator to separate the terms:

$$g = \frac{g_\pi}{2} \left[1 + \frac{J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right] + \frac{g_\nu}{2} \left[1 - \frac{J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right]$$

Group the parts multiplied by 1 and the parts multiplied by the fraction together:

$$g = \frac{1}{2}(g_\pi + g_\nu) + \frac{1}{2}(g_\pi - g_\nu) \left[\frac{J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right]$$

Remember from our setup that $g = \frac{\mu(J)}{J}$, $g_\pi = \frac{\mu_\pi}{J_\pi}$, and $g_\nu = \frac{\mu_\nu}{J_\nu}$. Substitute these back in:

$$\frac{\mu(J)}{J} = \frac{1}{2} \left(\frac{\mu_\pi}{J_\pi} + \frac{\mu_\nu}{J_\nu} \right) + \frac{1}{2} \left(\frac{\mu_\pi}{J_\pi} - \frac{\mu_\nu}{J_\nu} \right) \left[\frac{J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right]$$

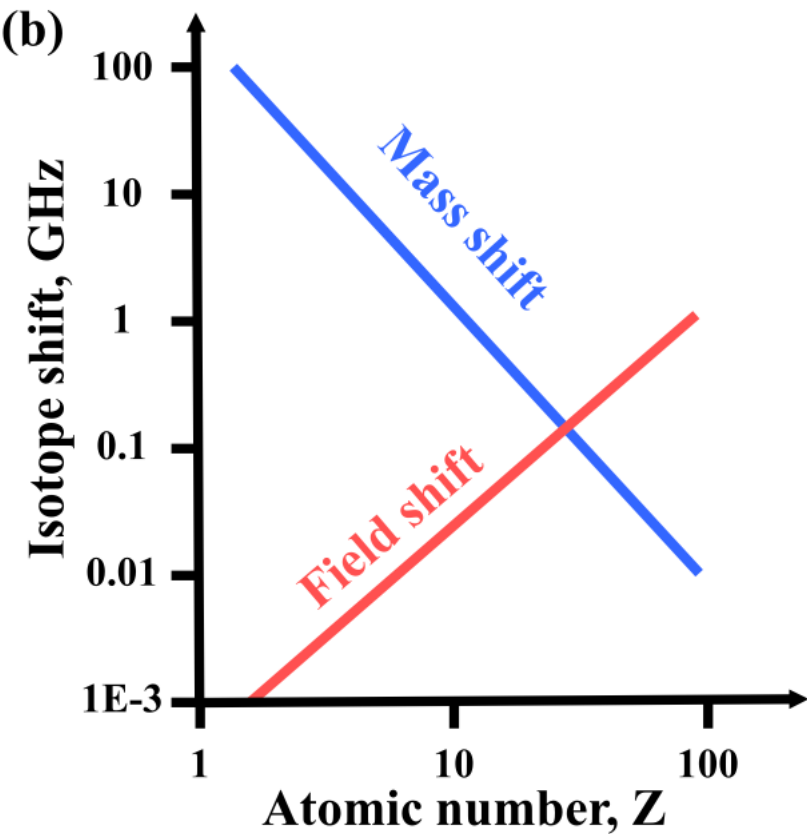
Multiply both sides by J to isolate $\mu(J)$, which brings you exactly to the Additivity Rule formula:

$$\mu(J) = \frac{J}{2} \left[\frac{\mu_\pi}{J_\pi} + \frac{\mu_\nu}{J_\nu} + \left(\frac{\mu_\pi}{J_\pi} - \frac{\mu_\nu}{J_\nu} \right) \frac{J_\pi(J_\pi+1) - J_\nu(J_\nu+1)}{J(J+1)} \right]$$



Isotope shift contributions

Different atomic transitions in Pd



	Field shift	Mass shift	Total IS
Transition (nm)	$\delta\nu_{\text{FS}}$ (MHz)	$\delta\nu_{\text{MS}}$ (MHz)	Total $\delta\nu^{\{106,108\}}$ (MHz)
363.6	644 (133)	-148 (117)	496 (178)
361.1	733 (133)	-236 (135)	497 (190)
357.2	644 (133)	-142 (120)	501 (179)
369.1	888 (444)	-428 (487)	460 (659)

