On the correlation between the isospin-symmetry breaking correction and $E0$-transition strength in superallowed $0^+ \to 0^+$ nuclear β decay

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Contents

Motivation

- Lee-Yang model of β decay [*Phys. Rev. 104(1):254-258 (1956)*]

$$
\begin{aligned}
\mathcal{H}_{\mathrm{LY}} = \quad & +\bar{p}n\left(C_S\,\bar{e}\nu - C_S'\,\bar{e}\gamma_5\nu\right) && \text{scalar}\\
& +\bar{p}\gamma_\mu n\left(C_V\,\bar{e}\gamma^\mu\nu - C_V'\,\bar{e}\gamma^\mu\gamma_5\nu\right) && \text{vector}\\
& +\bar{p}\sigma_{\mu\nu}n\left(C_T\,\bar{e}\sigma^{\mu\nu}\nu - C_T'\,\bar{e}\sigma^{\mu\nu}\gamma_5\nu\right)/2 && \text{tensor}\\
& -\bar{p}\gamma_\mu\gamma_5 n\left(C_A\,\bar{e}\gamma^\mu\gamma_5\nu - C_A'\,\bar{e}\gamma^\mu\nu\right) && \text{axial-vector}\\
& +\bar{p}\gamma_5 n\left(C_P\,\bar{e}\gamma_5\nu - C_P'\,\bar{e}\nu\right) && \text{pseudoscalar}\\
& +\mathrm{H.c.}
\end{aligned}
$$

- Experiments are consistent with the dominant $V - A$ structure
- Are S and T exactly absent, or simply too small to be resolved?
- What mechanism prevents S and T?

Current experimental precision is $\mathcal{O}(10^{-4})$

Motivation

- Lee-Yang model of β decay [*Phys. Rev. 104(1):254-258 (1956)*]

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\mathcal{H}_{\mathrm{LY}} = \; & +\bar{p}n\left(C_S\,\bar{e}\nu - C'_S\,\bar{e}\gamma_5\nu\right) && \text{scalar} \\
& +\bar{p}\gamma_\mu n\left(C_V\,\bar{e}\gamma^\mu\nu - C'_V\,\bar{e}\gamma^\mu\gamma_5\nu\right) && \text{vector} \\
& +\bar{p}\sigma_{\mu\nu} n\left(C_T\,\bar{e}\sigma^{\mu\nu}\nu - C'_T\,\bar{e}\sigma^{\mu\nu}\gamma_5\nu\right)/2 && \text{tensor} \\
& -\bar{p}\gamma_\mu\gamma_5 n\left(C_A\,\bar{e}\gamma^\mu\gamma_5\nu - C'_A\,\bar{e}\gamma^\mu\nu\right) && \text{axial-vector} \\
& +\bar{p}\gamma_5 n\left(C_P\,\bar{e}\gamma_5\nu - C'_P\,\bar{e}\nu\right) && \text{pseudoscalar} \\
& +\mathrm{H.c.}
\end{aligned}
$$

- In the non-relativistic limit

$$
\begin{array}{lcll}
\text{vector + {\color{red}scalar}} & \propto & \tau^{\pm}, & \text{(Fermi)} \\
\text{axial-vector + {\color{red}tensor}} & \propto & \sigma\tau^{\pm}, & \text{(Gamow-Teller)} \\
\text{pseudoscalar} & \propto & \mathcal{O}(|q|/m) &
\end{array}
$$

Possible signatures of physics beyond the $V-A$ interaction

1) Scalar currents modify F transition strength (**CVC tests**)
2) Tensor currents modify GT transition strength (**mirror asymmetry**)
3) Small admixture of terms also modify the spectrum shape (**angular correlation measurements**).

Superallowed $0^+ \to 0^+$ nuclear β decay

- Quark flavor mixing

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Current data indicate $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1$ (~95 % C.L.)
with $|V_{ud}|$ from nuclear β decays,
$|V_{us}|$ from kaon decays,
$|V_{ub}| \sim 0$.

- Nuclear extractions of $|V_{ud}|$

1) Superallowed $0^+ \to 0^+$ nuclear β decay (pure Fermi process)

$$\begin{aligned} \mathcal{F}t &= ft(1+\delta'_R)(1-\delta_C+\delta_{NS}) \\ &= \frac{K}{2G_F^2|V_{ud}|^2(1+\Delta_R^V)} \end{aligned}$$

- ft values measured with $\sim 0.1\,\%$ precision for 15 cases.
- Radiative corrections: Δ_R^V (nucleus-independent), δ'_R (dependent on Z and Q_β) and δ_{NS} (structure-dependent)
- δ_C is the isospin-symmetry breaking correction

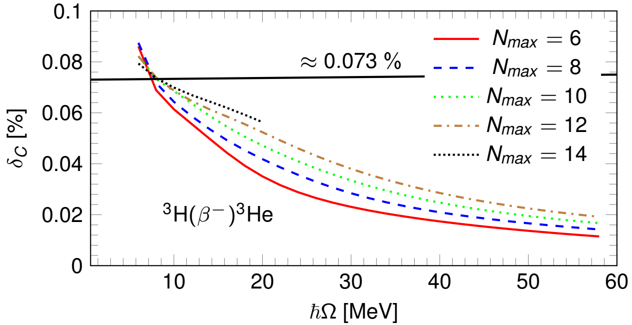
$$|M_F|^2 = |\mathcal{M}_F|^2(1-\delta_C), \quad |\mathcal{M}_F|^2 = 2 \text{ for } T=1$$

Towner and Hardy, PRC102, 045501 (2020)
Xayavong et al. (2016-present)

2) Complementary data from mirror β transitions (admixture of Fermi and Gamow-Teller)

Severijns et al., PRC107, 015502 (2023)

- Convergence problem in first-principles treatments of δ_C



NCSM w/o 3-body forces!

TBMEs: N3LO-EMN500, SRG evolved with λ =2 fm^{-1}.

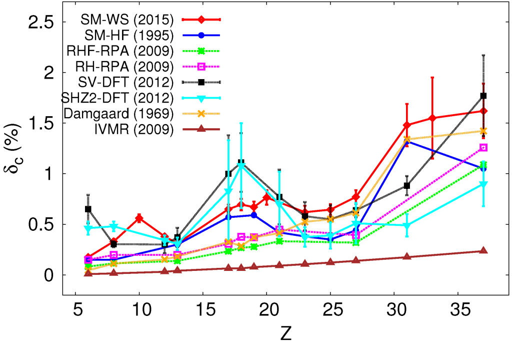
Xayavong et al., PRC113, 034317 (2026)
Miyagi, EPJA59, 150 (2023)
Johnson et al., arXiv:1801.08432

Because it is driven by a long-range interaction, Coulomb mixing can extend across multiple shells.
Stetcu et al., PRC71, 044325 (2005)

A similar difficulty is also encountered in IMSRG calculations.
Stroberg, Particles 4, 521–535 (2021)

Isospin-symmetry breaking correction

- Phenomenological calculations of δ_C are available



Towner and Hardy, PRC102, 045501 (2020);

Ormand and Brown, PRC52, 2455 (1995);
Liang et al., PRC79, 064316 (2009);

Satula et al., PRC86, 054316 (2012);

Damgaard, NPA130, 233 (1969);

Auerbach, PRC79, 035502 (2009);

Xayavong, Smirnova, Lim et al., PRC112, 055503 (2025); PRC113, 034317 (2026); PRC109, 014317 (2024); PRC105, 044308 (2022); PRC97, 024324 (2018)

- The predictions diverge and the accuracy of individual approaches is difficult to assess
- Experimental tests are therefore needed.

- Electric monopole operator

$$\hat{T}(E0) = \sum_{q\in p,n} e_q \sum_{i=1}^{\mathcal{N}_q} r_i^2, \qquad \mathcal{N}_p = Z, \quad \mathcal{N}_n = N$$

$$\underbrace{e^{(0)} \sum_{i=1}^{A} r_i^2}_{T^{(0)}(E0)} + \underbrace{e^{(1)} \sum_{i=1}^{A} \tau_{zi} r_i^2}_{T^{(1)}(E0)}$$

where

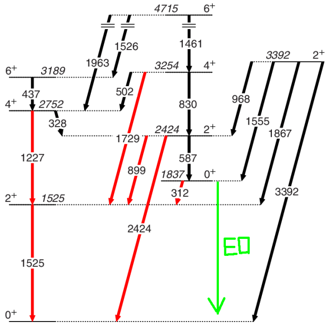
$$e^{(0)} = \frac{e_p + e_n}{2}, \quad e^{(1)} = \frac{e_n - e_p}{2}$$

- $E0$-related observables

$$\langle \hat{T}(E0)\rangle_{\text{diag}} = \text{squared radii}$$

$$\boldsymbol{\langle \hat{T}(E0)\rangle_{\text{off-diag}} = \rho(E0) \sim \textbf{shape mixing}}$$

A notable example, ^{42}Ca



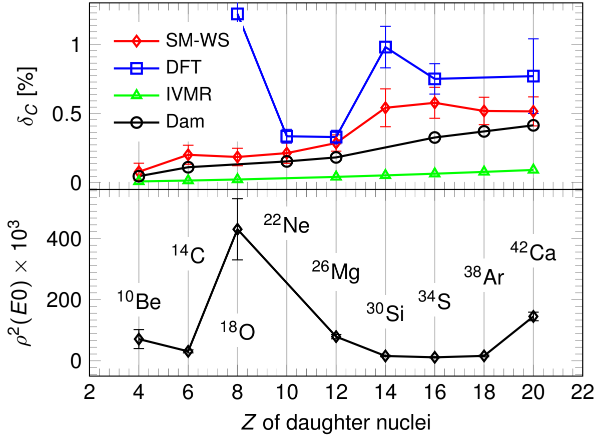
Nearly spherical in 0_1^+ and strong prolate deformed in 0_2^+ => large $\rho^2(E0)$

Sensitive to cross-shell configurations

See Prog. Part. Nucl. Phys. 123 (2022) 103930.

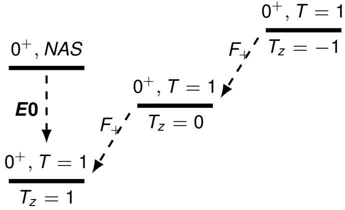
Could $E0$ strength be a new probe for δ_C?

- An anti-correlation between δ_C and E0 transition strengths



Exp data from Prog. Part. Nucl. Phys. 123 (2022) 103930.

- Transition diagram



- $E0$ strengths are suppressed in mid shells, and could be substantial in cross-shell regions
- Exp $\rho^2(E0)$ values are available for neutron-rich members of isotriplets up to ^{42}Ca

Could $E0$ strength be a new probe for δ_C?

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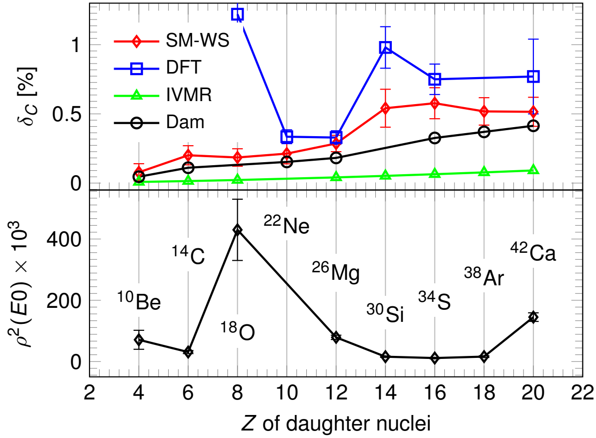

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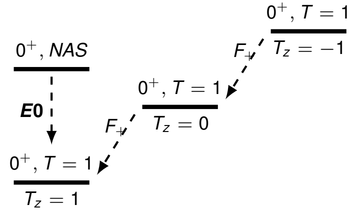


Not a direct experimental extraction of δ_C, but a way to benchmark theoretical predictions

Theoretical approaches should consistently reproduce the exp $\rho^2(E0)$ values

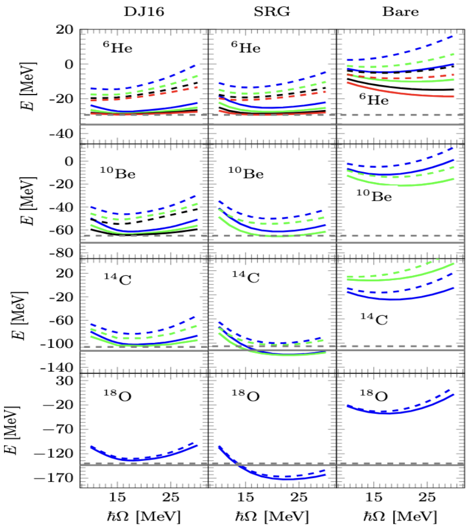
Is this correlation physical, or merely accidental?

- NCSM yields reasonable correlations between some pairs of observables, e.g.,

$$\boldsymbol{Q_2} \ vs \ \boldsymbol{B(E2)}$$

- Can NCSM reveal a physical correlation between δ_C and $\rho^2(E0)$?
- Four isotriplets: ^{6}He, ^{10}Be, ^{14}C, and ^{18}O
- DJ16, bare N3LO (EMN500), and SRG-evolved N3LO (EMN500)

Solid line :	0_1^+ *state*
Dashed line :	0_2^+ *state*
Gray lines :	*exp values*
Blue lines :	$N_{\text{max}} = 4$
Green lines :	$N_{\text{max}} = 6$
Black lines :	$N_{\text{max}} = 8$
Red lines :	$N_{\text{max}} = 10$



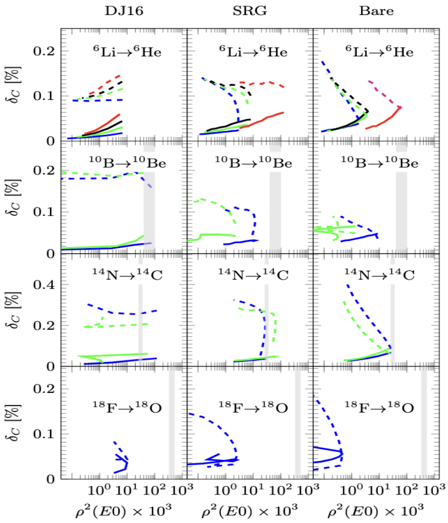
NCSM analysis

- **NCSM results show no anti-correlation for any of the isotriplets.** **This may not reflect the correct behavior, but could be a consequence of energy error**
- Perturbation theory gives

$$\delta_C \propto 1/(\Delta E)^2$$

When scaling with exp $(\Delta E)^2$, the trend reverses for $A = 6$ (**except for DJ16!**)

Solid line :	*exact*
Dashed line :	*scaled*
Gray bands :	*exp values*
Blue lines :	$N_{\max} = 4$
Green lines :	$N_{\max} = 6$
Black lines :	$N_{\max} = 8$
Red lines :	$N_{\max} = 10$



- An anti-correlation emerges when $\hbar\Omega$ is selected at the energy minimum
- These points follow the convergence path from the variational principle

Blue lines :	$N_{\text{max}} = 4$
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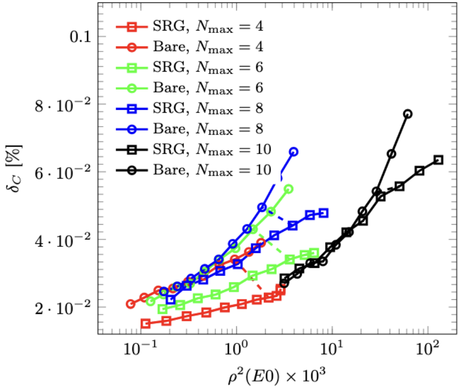


Strong model-space dependence remains

- Our starting point is

$$[F_{\mp}, [\hat{T}(E0), F_{\pm}]] = -2\hat{T}^{(1)}(E0).$$

The expectation value is written as

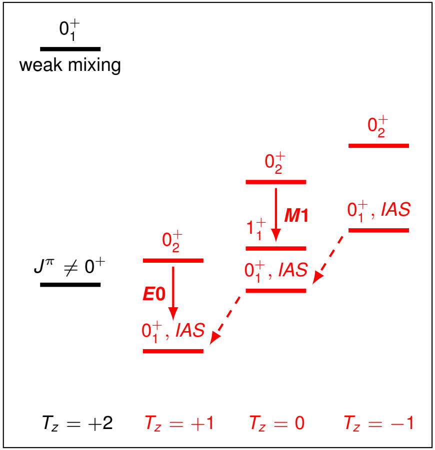
$$\begin{aligned}
-2\langle a|\hat{T}^{(1)}(E0)|a\rangle &= \sum_{bb'} \langle a|F_{\mp}|b\rangle \langle b|\hat{T}(E0)|b'\rangle \langle b'|F_{\pm}|a\rangle \\
&- \sum_{ba'} \langle a|F_{\mp}|b\rangle \langle b|F_{\pm}|a'\rangle \langle a'|\hat{T}(E0)|a\rangle \\
&- \sum_{ca'} \langle a|\hat{T}(E0)|a'\rangle \langle a'|F_{\pm}|c\rangle \langle c|F_{\mp}|a\rangle \\
&+ \sum_{cc'} \langle a|F_{\pm}|c'\rangle \langle c'|\hat{T}(E0)|c\rangle \langle c|F_{\mp}|a\rangle ,
\end{aligned}$$

where $T_{zb} = T_{za} \pm 1$ and $T_{zc} = T_{za} \mp 1$.

At the limit of exact isospin, the expression is model-independent

Simplification could be made though specific choices of { a,b,c }

Typical picture



A model-independent analysis

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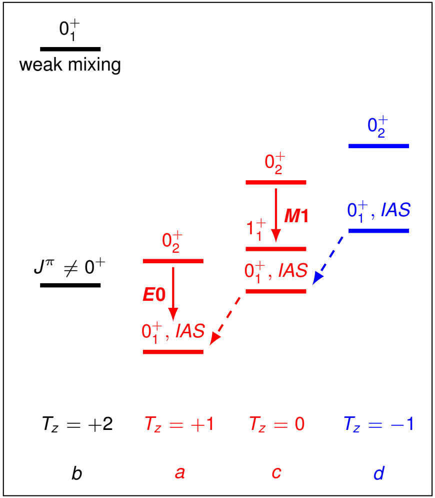
&+ \sum_{cc'} \langle a|F_{\pm}|c'\rangle \langle c'|\hat{T}(E0)|c\rangle \langle c|F_{\mp}|a\rangle\,,
\end{aligned}$$

where $T_{zb} = T_{za} \pm 1$ and $T_{zc} = T_{za} \mp 1$.

- The proton-rich $T_z = -1$ nucleus (d) is selectively excluded (no $E0$ data available)
- In c, the $E0$ transition is completely suppressed due to $M1$ transitions
- Since b is not an isotriplet member, its contribution is negligible (**its 0_1^+ stateis largely separate**)

Typical picture



A model-independent analysis

- At the limit of two-level mixing, we obtain

$$\delta_{\boldsymbol{C}} \approx \left| \frac{-\Delta T_{11} \boldsymbol{T^a_{21}} \pm \sqrt{(\Delta T_{11} \boldsymbol{T^a_{21}})^2 - \left[(\Delta T_{11})^2 - (V^a_{11})^2\right]\left[(\boldsymbol{T^a_{21}})^2 - 4(\Delta T_{11})^2 + 2\Delta T_{11} S^c_{11}\right]}}{\left[(\boldsymbol{T^a_{21}})^2 - 2(\Delta T_{11})^2 + \Delta T_{11} S^c_{11}\right]} \right|^2$$

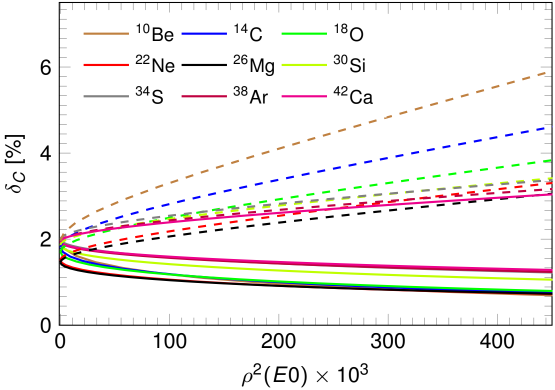
Here, $\Delta T_{11} = (S^c_{11} - S^a_{11}) + (V^c_{11} - V^a_{11})$, where

$$\begin{aligned} T^a_{21} &= \rho(E0) \\ S^c_{11} &= \langle 0^+_1, c|\hat{T}^{(0)}(E0)|0^+_1, c\rangle = e^{(0)} A \langle R^2_{m,c}\rangle \\ V^a_{11} &= \langle 0^+_1, a|\hat{T}(E0)^{(1)}|0^+_1, a\rangle = e^{(1)} \left[(N-Z)\langle R^2_{p,a}\rangle + 2N\Delta r_{np,a}\langle R^2_{p,a}\rangle^{\frac{1}{2}} + N(\Delta r_{np,a})^2\right]. \end{aligned}$$

- This analysis indeed suggests an anti-correlation.
- The other independent-observables include only the matter radii and neutron-skin thicknesses of the *a* and *c* nuclei

A model-independent analysis

- With Skyrme-Hartree-Fock calculations of $\langle R^2_{m,c} \rangle$, $\langle R^2_{p,a} \rangle$ and $\Delta r_{np,a}$, we obtain



The result clearly suggests an anti-correlation.
(The dashed curves are unphysical as they violate the conservation of total Fermi strength)

The correlation seems stronger at small $\rho^2(E0)$.

The theoretical approaches to δ_C should accurately reproduce $\rho^2(E0)$.

This model is too simplified to produce a qualitative result. It nicely captures the essential behaviors of the observable/correction of interest

Shell-model benchmark

- Can the shell-model approach can consistently reproduce $\rho^2(E0)$?

  - Single-shell model spaces

Nuclei	Model Spaces	Interactions	Effective Charges		$\rho^2(E0) \times 10^3$	
			Protons	Neutrons	Calculations	Experiments
^{10}Be	p $(0\hbar\Omega)$	CKPOT/CKPOTcd	1.3	0.5	0	71(31)
^{14}C	p $(0\hbar\Omega)$	CKPOT/CKPOTcd	1.3	0.5	0	31(5)
^{26}Mg	sd $(0\hbar\Omega)$	USD/USDA/USDB and cd	1.36	0.45	0	79(7)
^{30}Si	sd $(0\hbar\Omega)$	USD/USDA/USDB and cd	1.36	0.45	0	15.8(23)
^{34}S	sd $(0\hbar\Omega)$	USD/USDA/USDB and cd	1.36	0.45	0	11.4(21)

By construction, $\boldsymbol{\rho^2(E0)}$ vanishes in single-shell model spaces. Extended valence spaces are needed for these isotriplets

Because of the anti-correlation, this implies an overestimate of $\boldsymbol{\delta_C}$
The corresponding $|\boldsymbol{V_{ud}}|$) value is therefore too large

Improving this deficiency will further increase tension in the CKM unitarity

Shell-model benchmark

- Can the shell-model approach can consistently reproduce $\rho^2(E0)$?

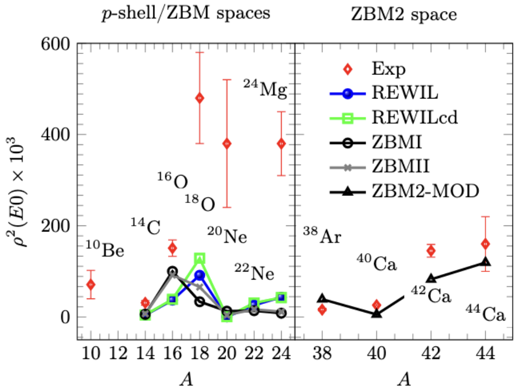
  - Mixed-shell model spaces

Nuclei	Model Spaces	Interactions	Effective Charges		$\rho^2(E0) \times 10^3$	
			Protons	Neutrons	Calculations	Experiments
^{14}C	ZBM ($N\hbar\Omega$)	REWIL	1.5	0.5	4.209	31(5)
		REWILcd	1.5	0.5	4.278	
		ZBMI	1.5	0.5	5.978	
		ZBMII	1.5	0.5	6.922	
^{18}O	ZBM ($N\hbar\Omega$)	REWIL	1.5	0.5	91.568	430(100)
		REWILcd	1.5	0.5	128.689	
		ZBMI	1.5	0.5	33.466	
		ZBMII	1.5	0.5	65.592	
^{22}Ne	ZBM ($N\hbar\Omega$)	REWIL	1.5	0.5	25.745	N.A
		REWILcd	1.5	0.5	30.603	
		ZBMI	1.5	0.5	13.730	
		ZBMII	1.5	0.5	16.660	
^{38}Ar	ZBM2 ($N\hbar\Omega$)	ZBM2-mod	1.5	0.5	38.830	16.1(28)
^{42}Ca	ZBM2 ($N\hbar\Omega$)	ZBM2-mod	1.5	0.5	82.556	145(14)

Similar to the single-shell cases, exp data are typically underestimated.

Shell-model benchmark

- Within the phenomenological shell model, there is an observable-related issue



The evaluation of $\rho^2(E0)$ requires effective charges, which are generally known. The values extracted from $B(E2)$ systematics may not be appropriate for $E0$ transitions

Shell-model predictions of δ_C may be okay. Further investigation of effective charges could be helpful.

Summary

- Using NCSM calculations and model-independent analysis, we demonstrate the existence of anti-correlation between δ_C and $\rho^2(E0)$
- Therefore, $\rho^2(E0)$ may provide a sensitive probe for δ_C
- The δ_C values obtained from shell-model calculations show a notable consistency
- However, the shell model does not quantitatively reproduce the exp $\rho^2(E0)$ values
- The problem likely lies in the effective charges