A shell model perspective on the Pygmy dipole resonance

Oscar Le Noan

Supervised by Kamila Sieja

I - Nuclear Electric Dipole Response: Definition And Motivation

Nuclear electric dipole response (E1): context

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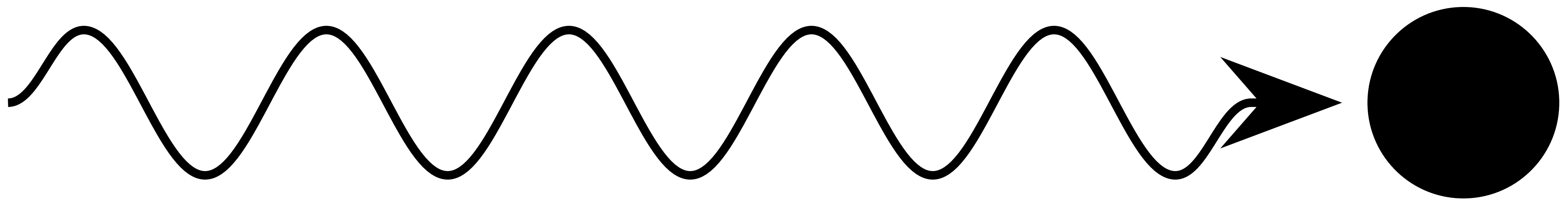

External perturbation

Nucleus

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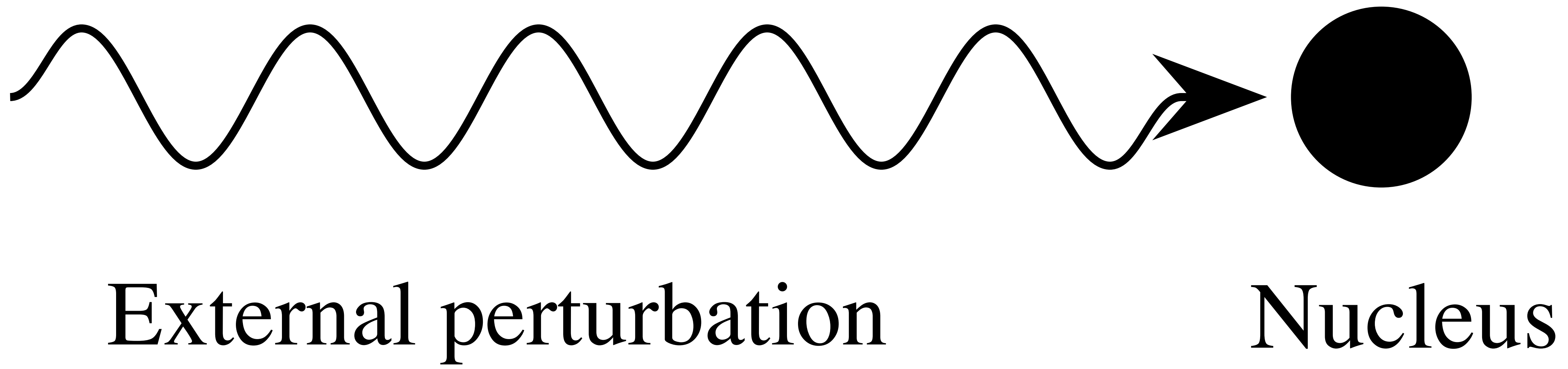

$$O^{\tau}_{X\,\lambda}$$

- $\tau \rightarrow$ IS, IV (Isoscalar, Isovector)
- $\lambda \rightarrow$ Multipolarity $(0, 1, 2, \cdots)$
- $X \rightarrow$ E, M (Electric, Magnetic)

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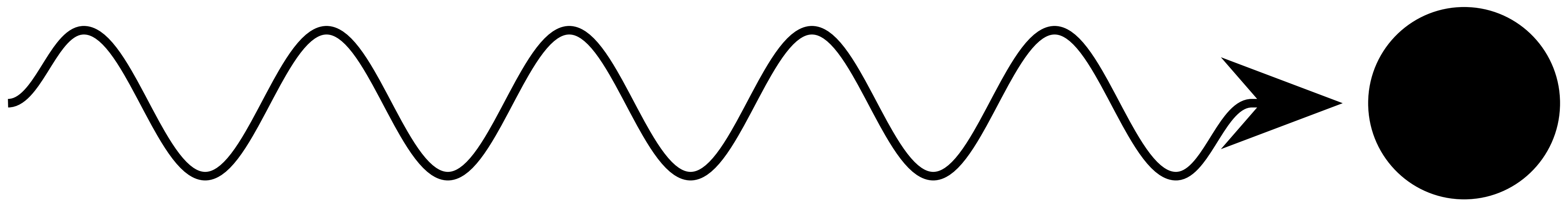

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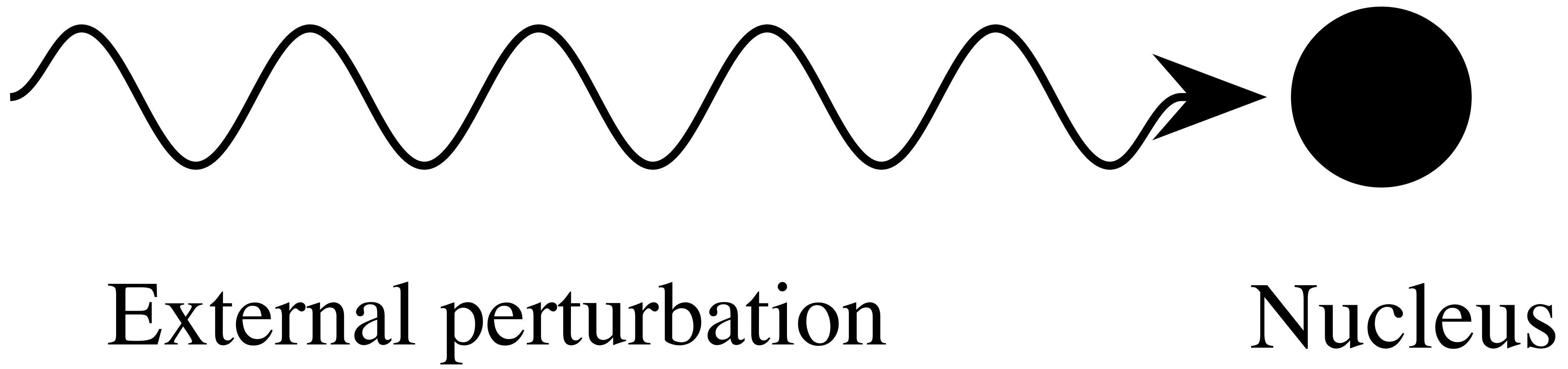

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Animation credit: Martin Bordeau

Nuclear electric dipole response (E1): definition

How to characterize the response to an external perturbation ?

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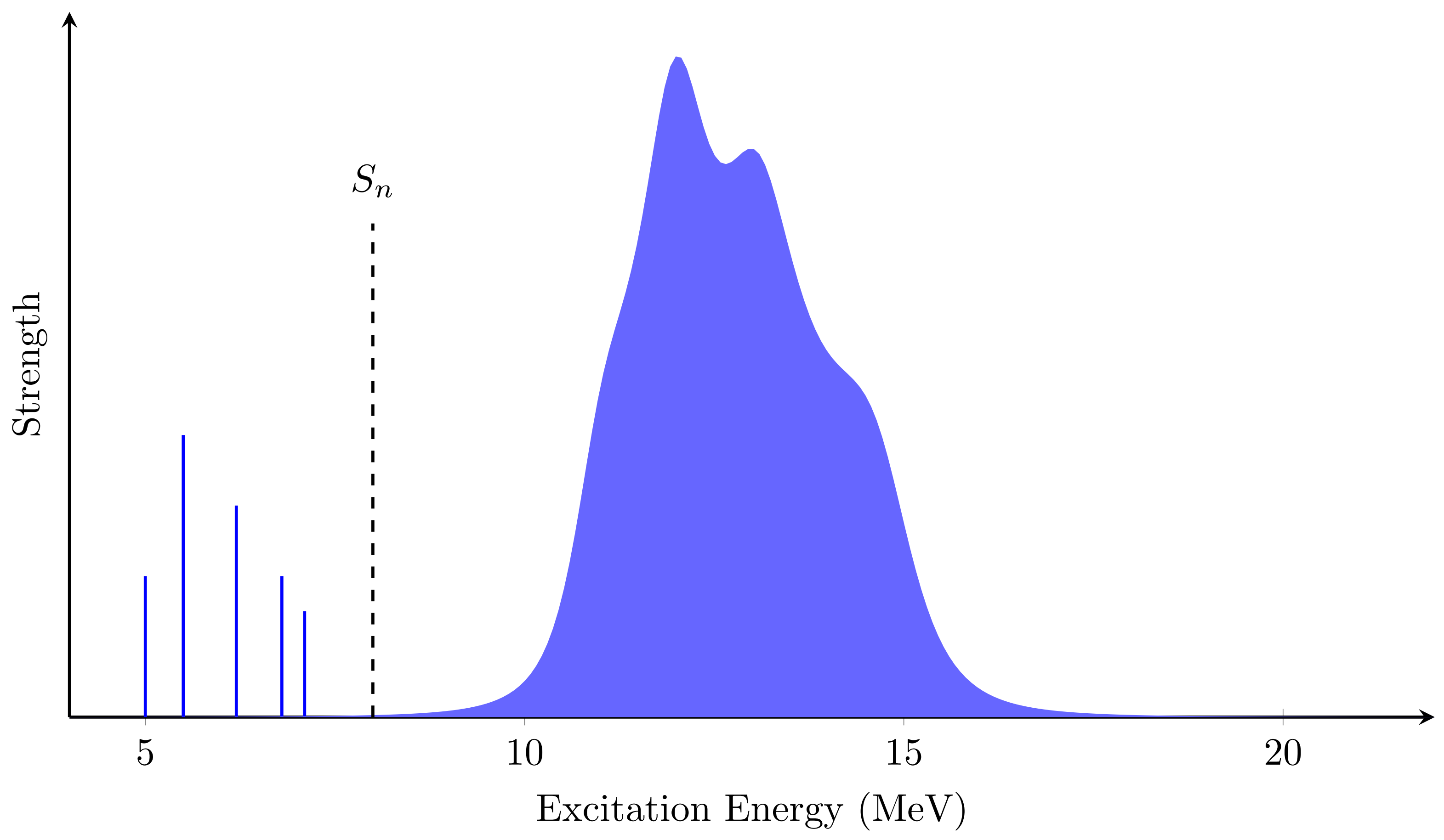
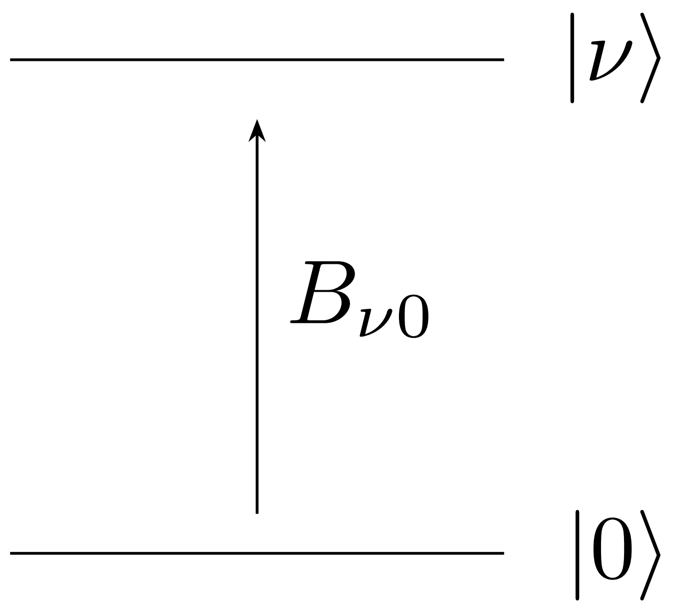
Strength function (response)



$$S(E) = \sum_{\nu} \underbrace{|\langle \nu | O | 0 \rangle|^2}\, \delta(E - E_\nu)$$

Transition probability $B_{\nu 0} \propto |\langle \nu | O | 0 \rangle|^2$



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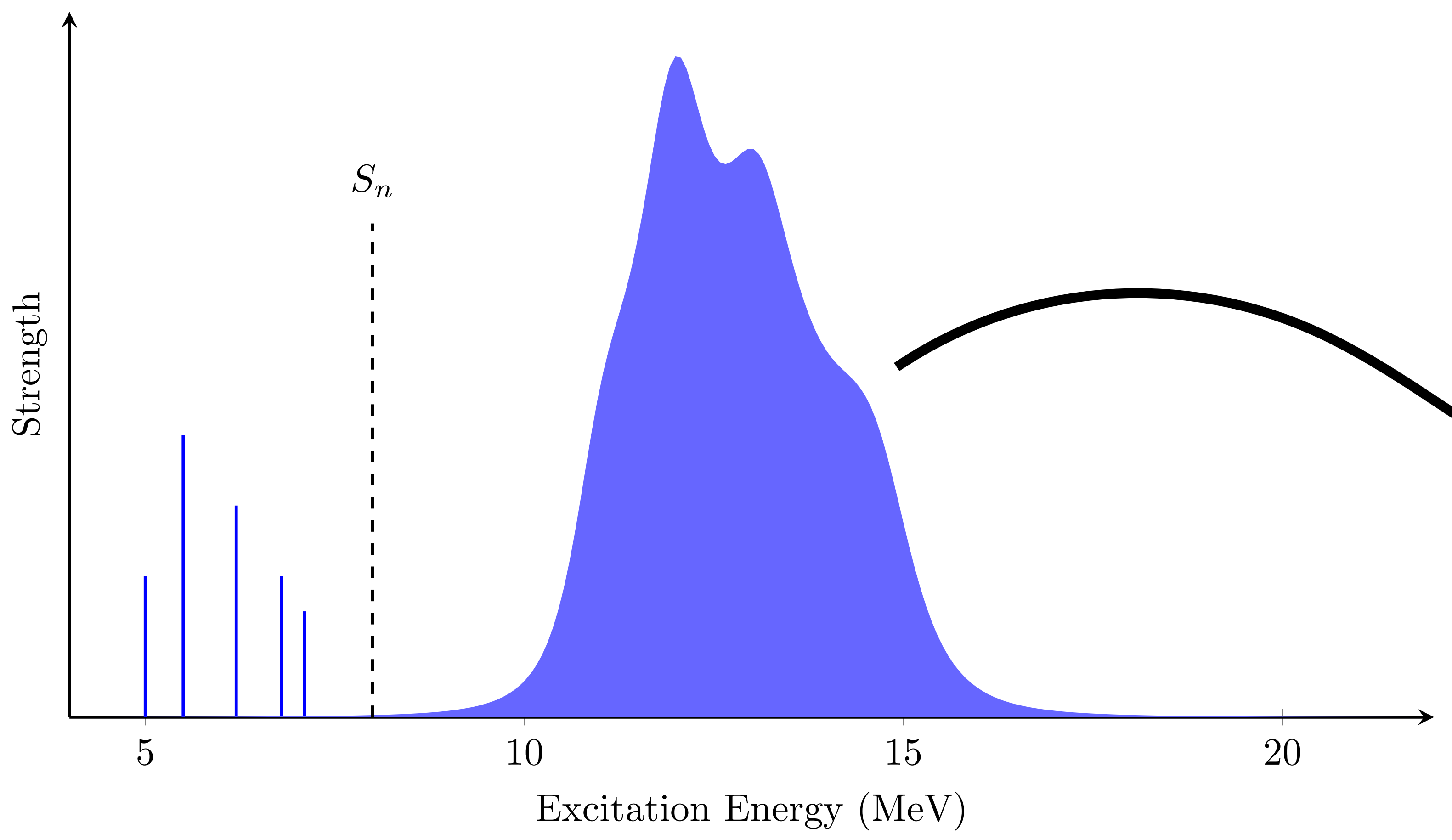

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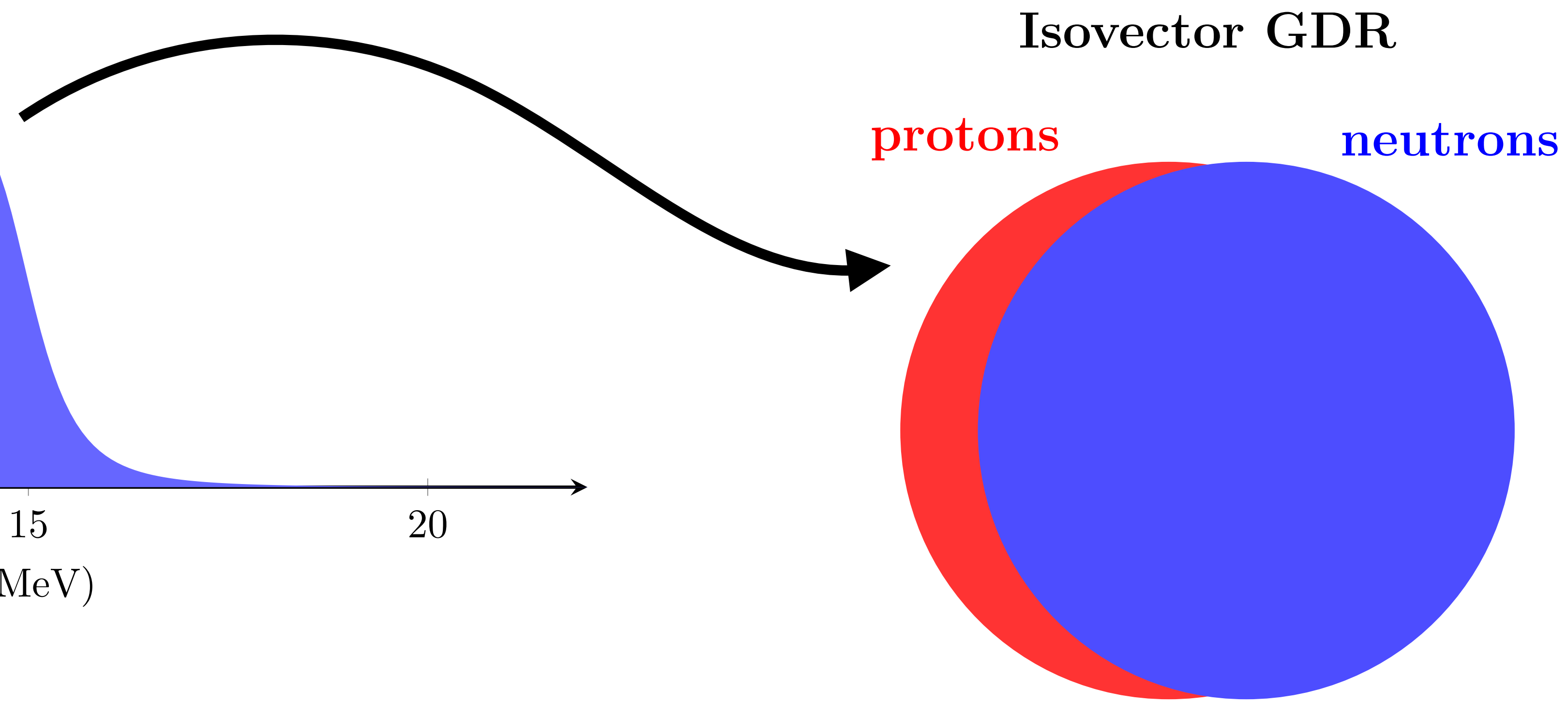

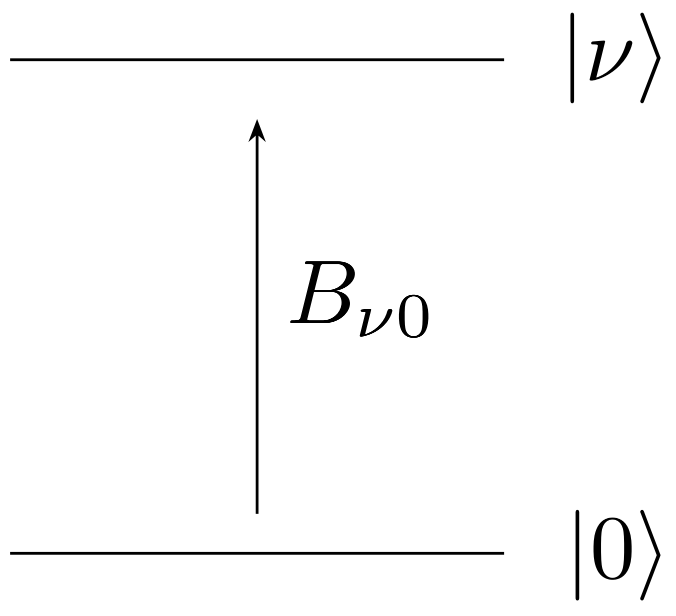

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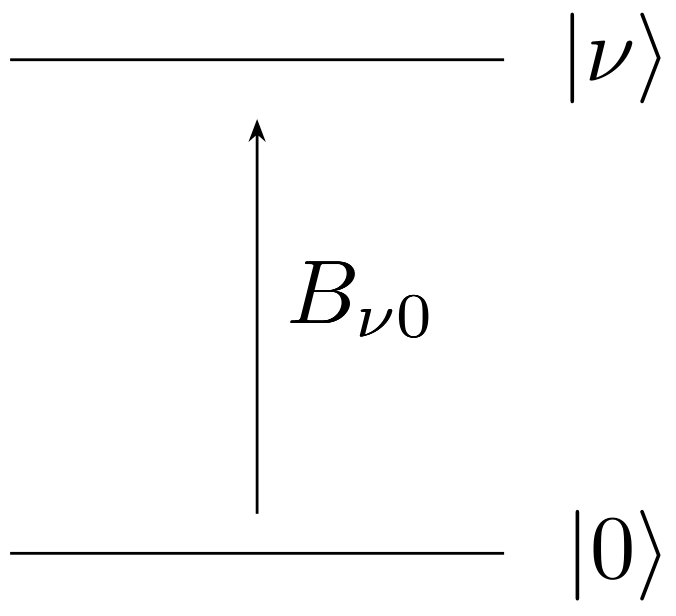

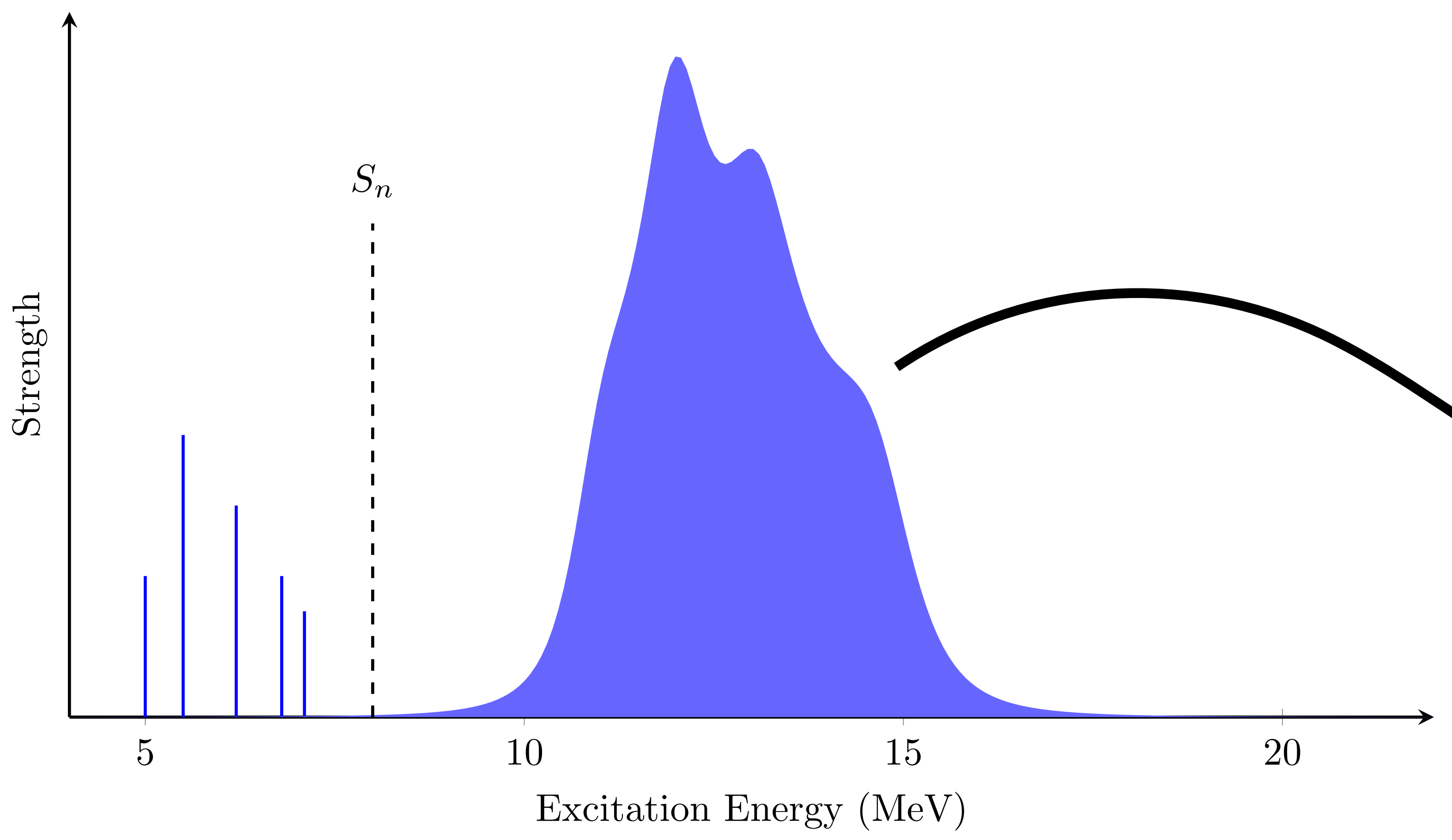

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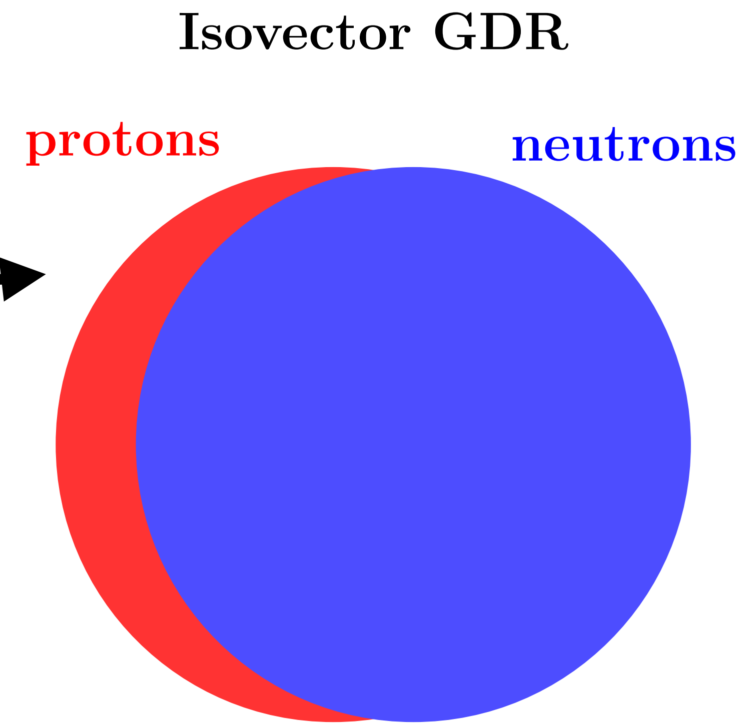

Focus on the isoscalar (IS) and isovector (IV) $E1$ response

$$\boxed{O^{\mathrm{IV}}_{E1\mu} = -e\frac{Z}{A}\sum_{i=1}^{N} r_i Y_{1\mu}(\hat{r}_i) + e\frac{N}{A}\sum_{i=1}^{Z} r_i Y_{1\mu}(\hat{r}_i)}$$

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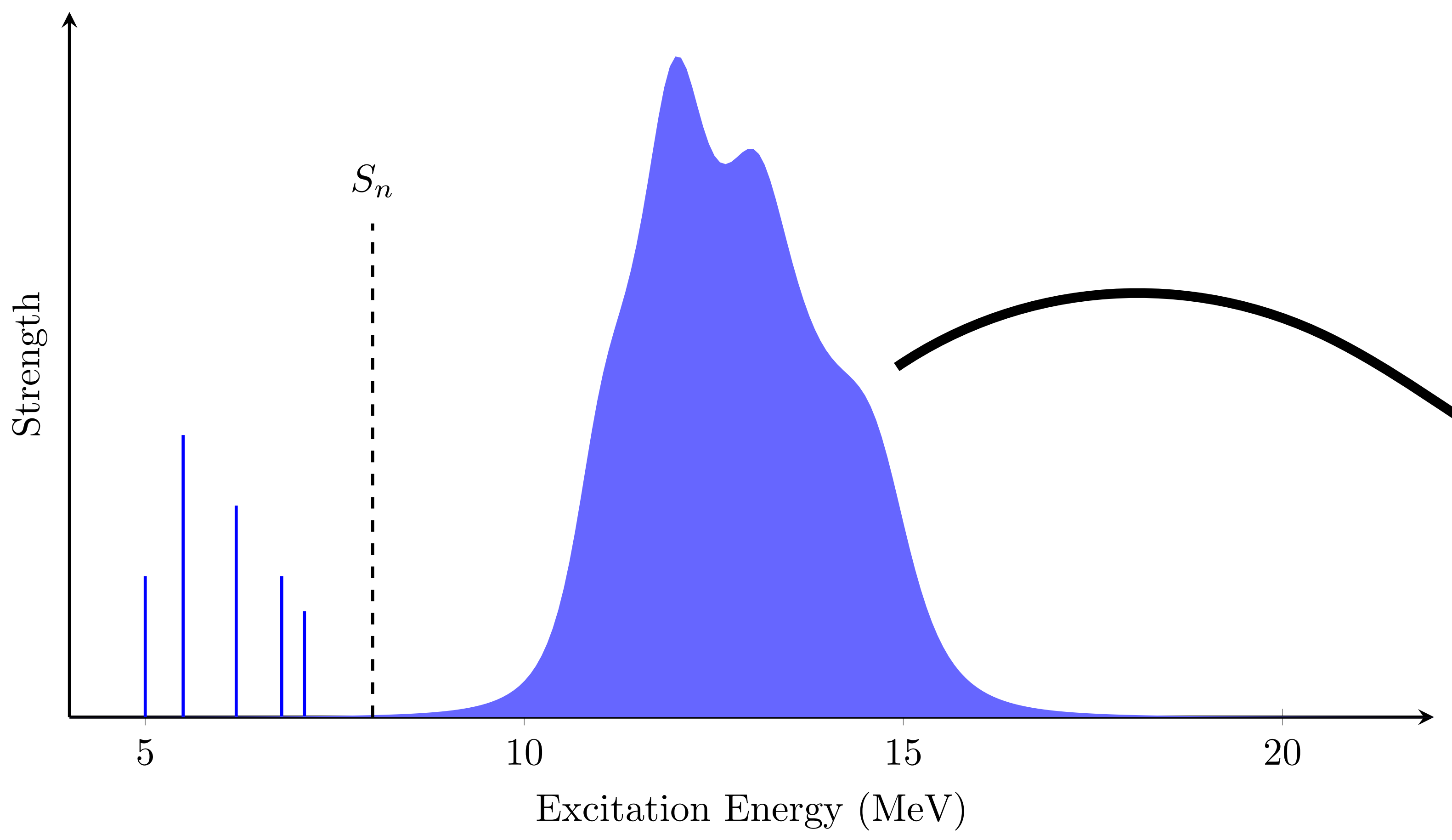

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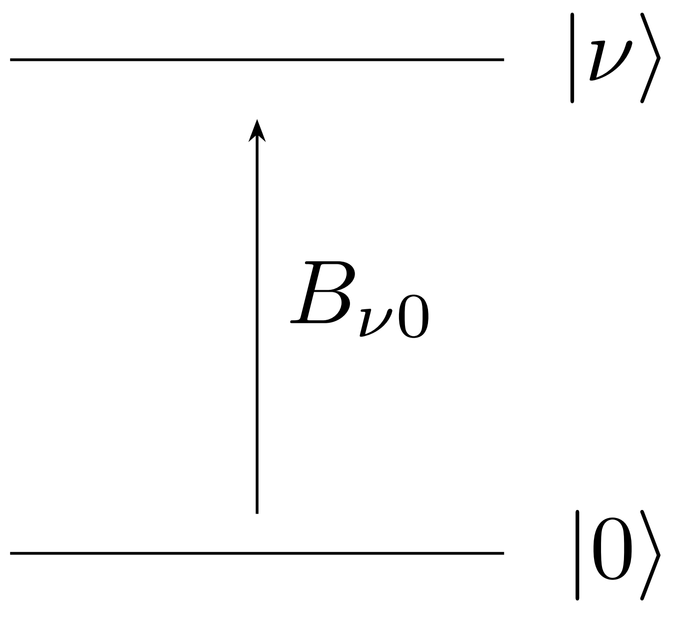
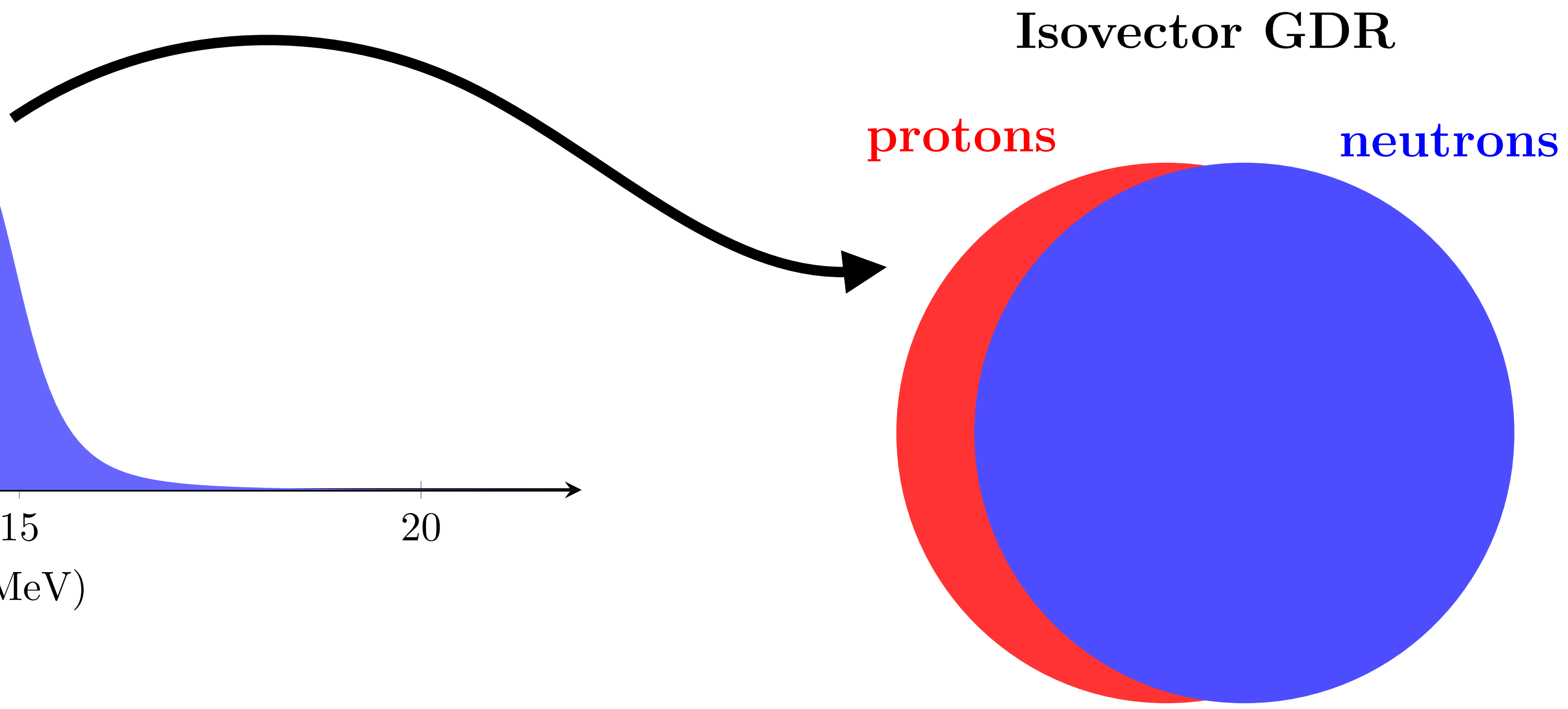
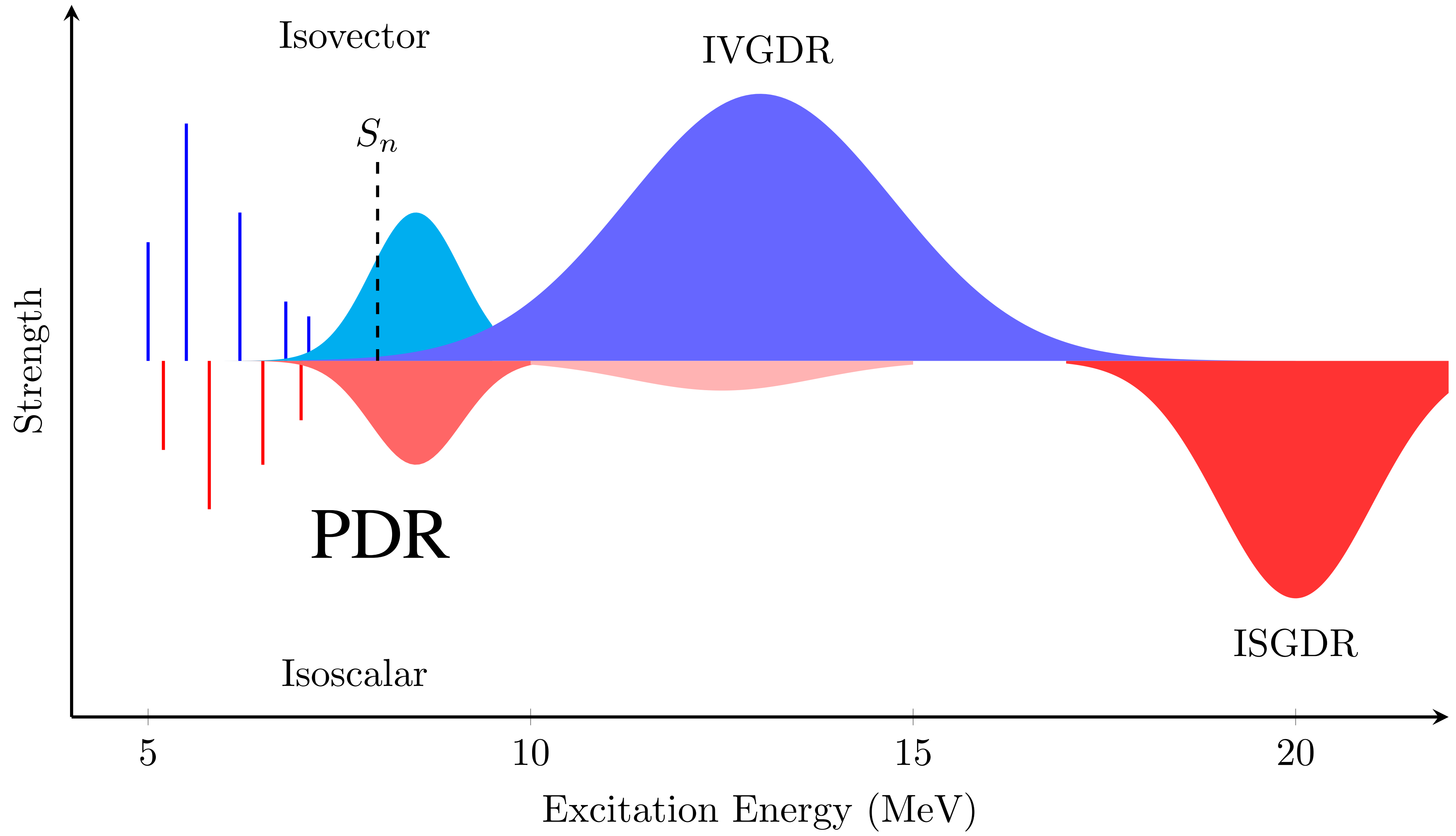




Neutron-rich nucleus: Pygmy Dipole Resonance (PDR)



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- Collectiveness ?
- Classical interpretation ?
- Isospin splitting ?
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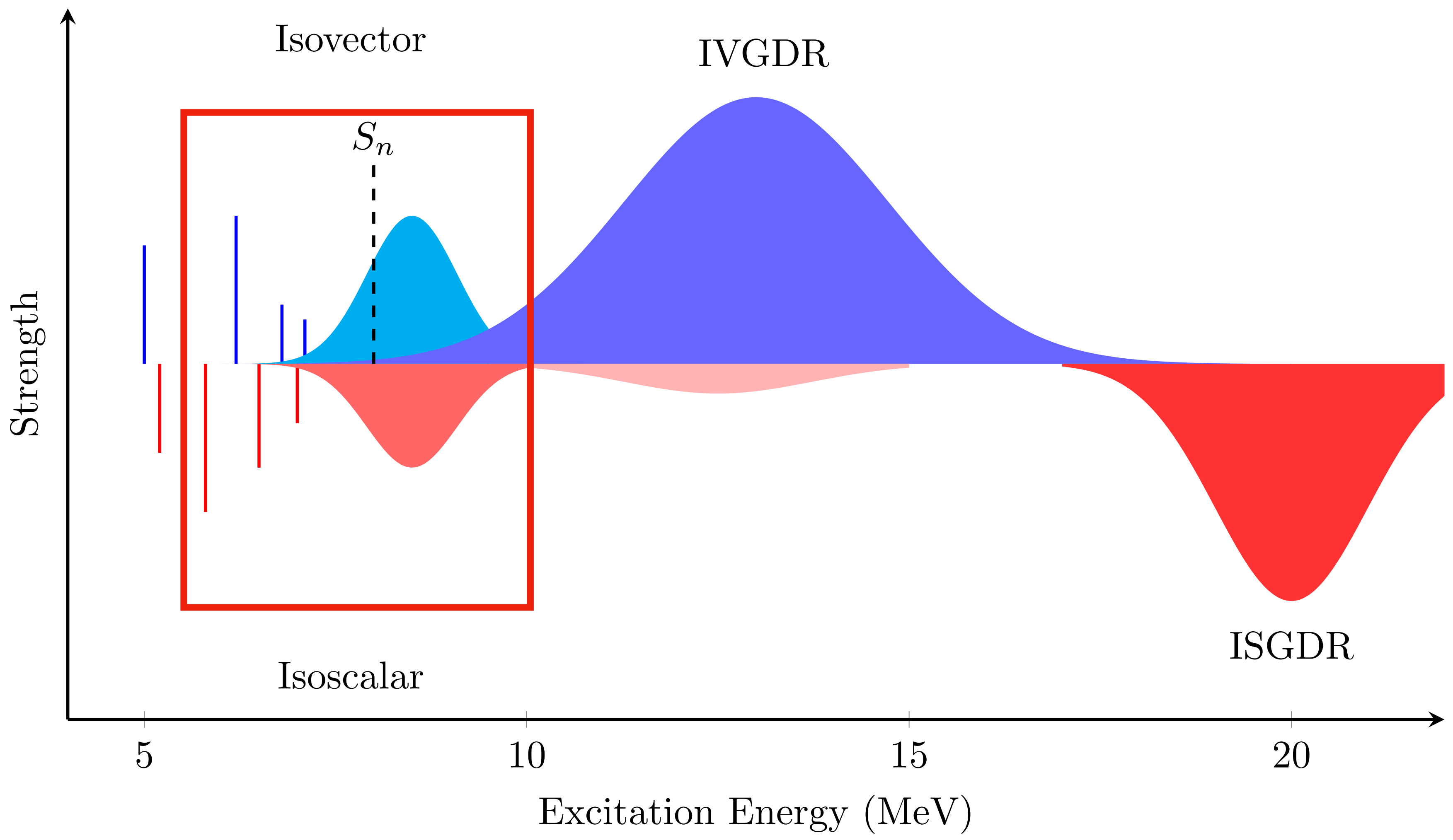

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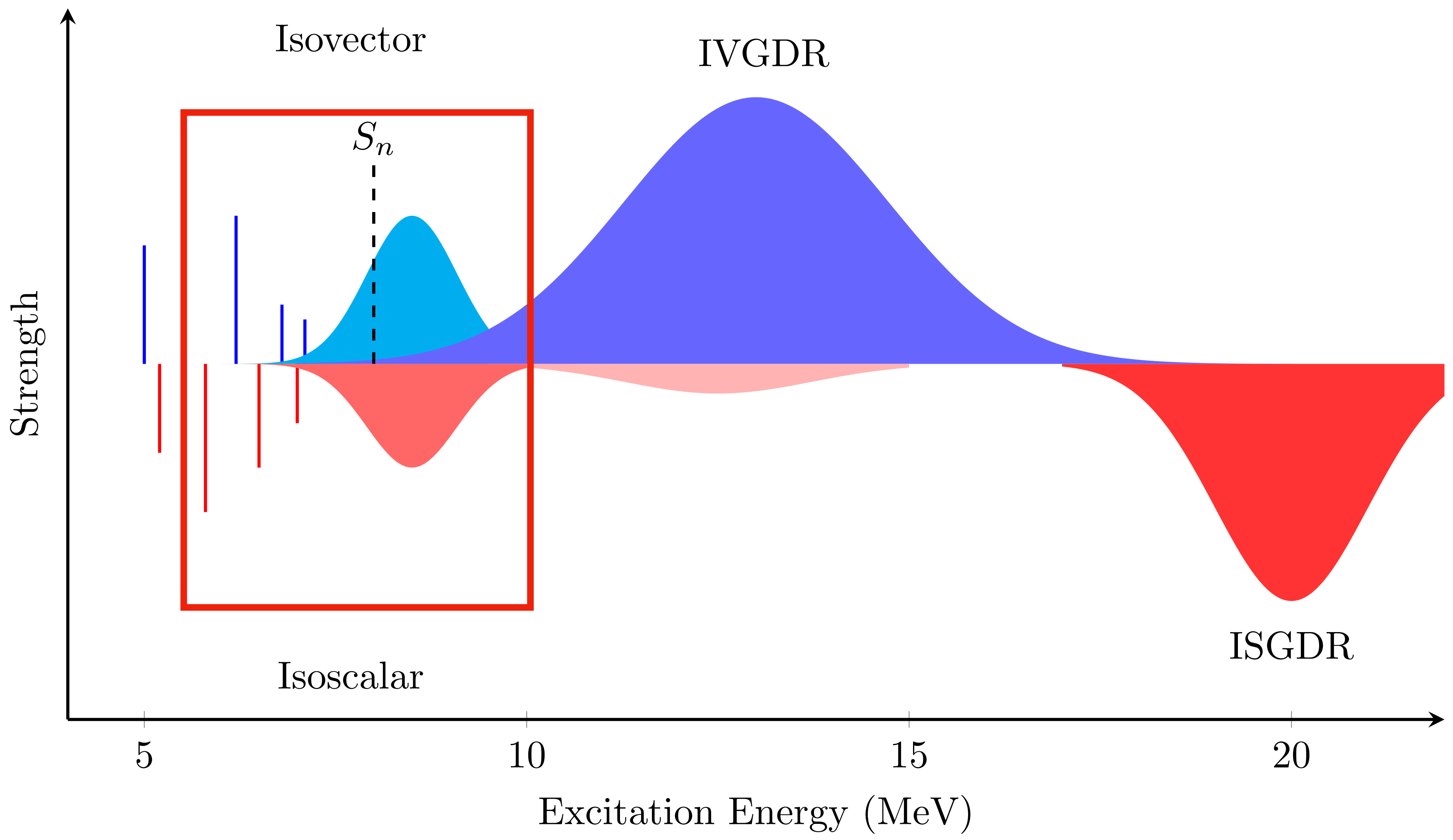

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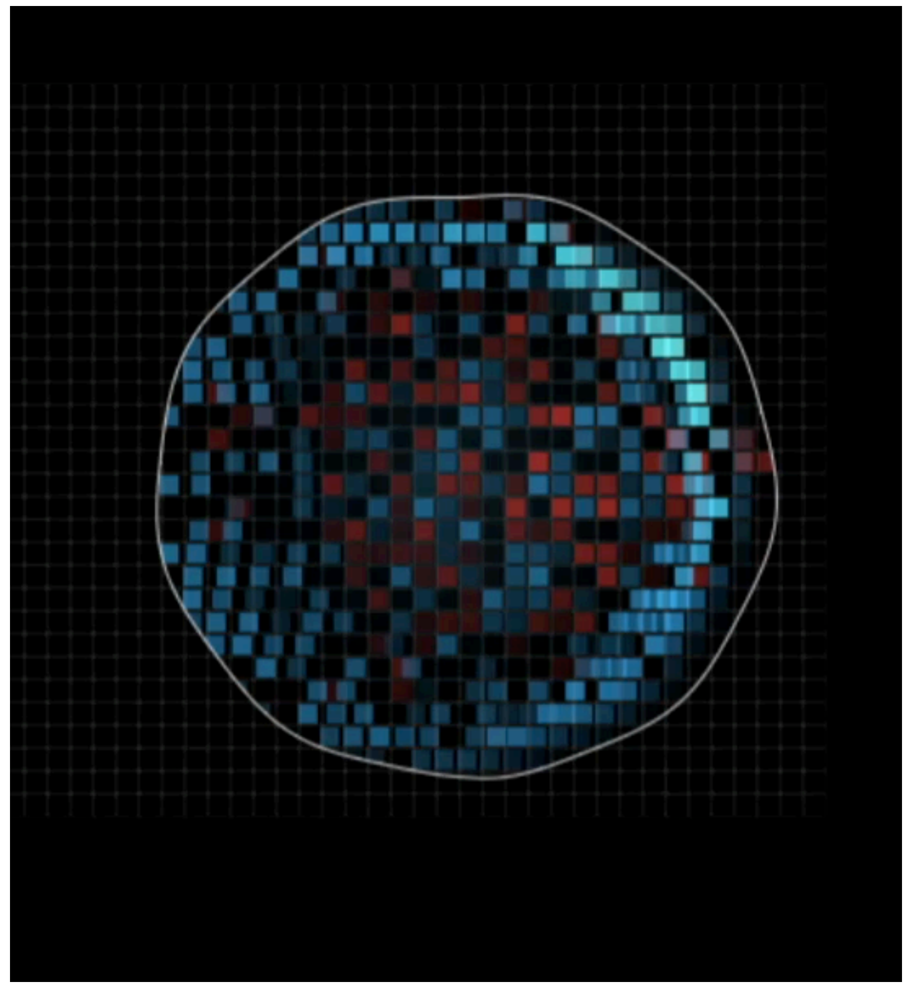

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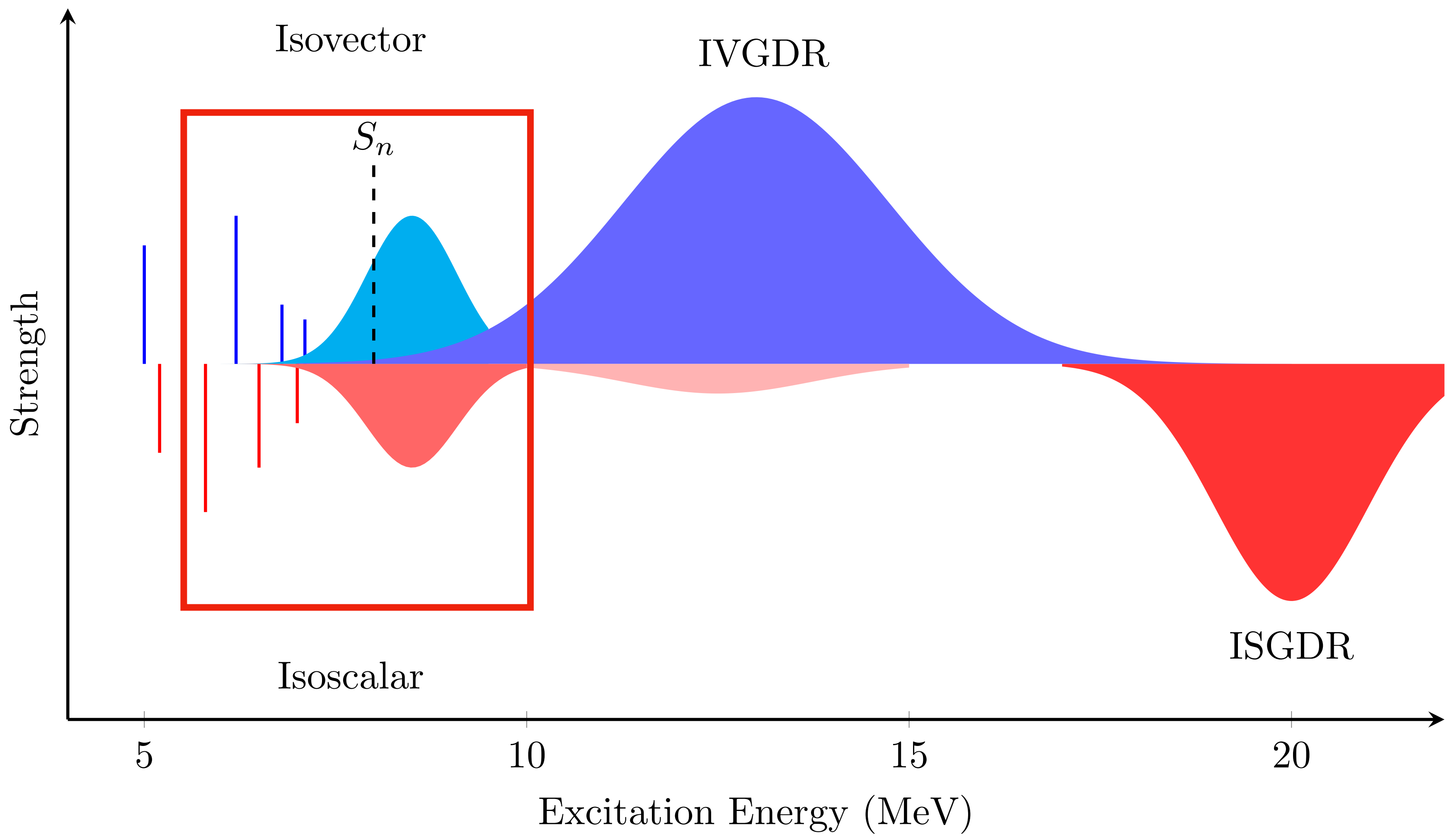

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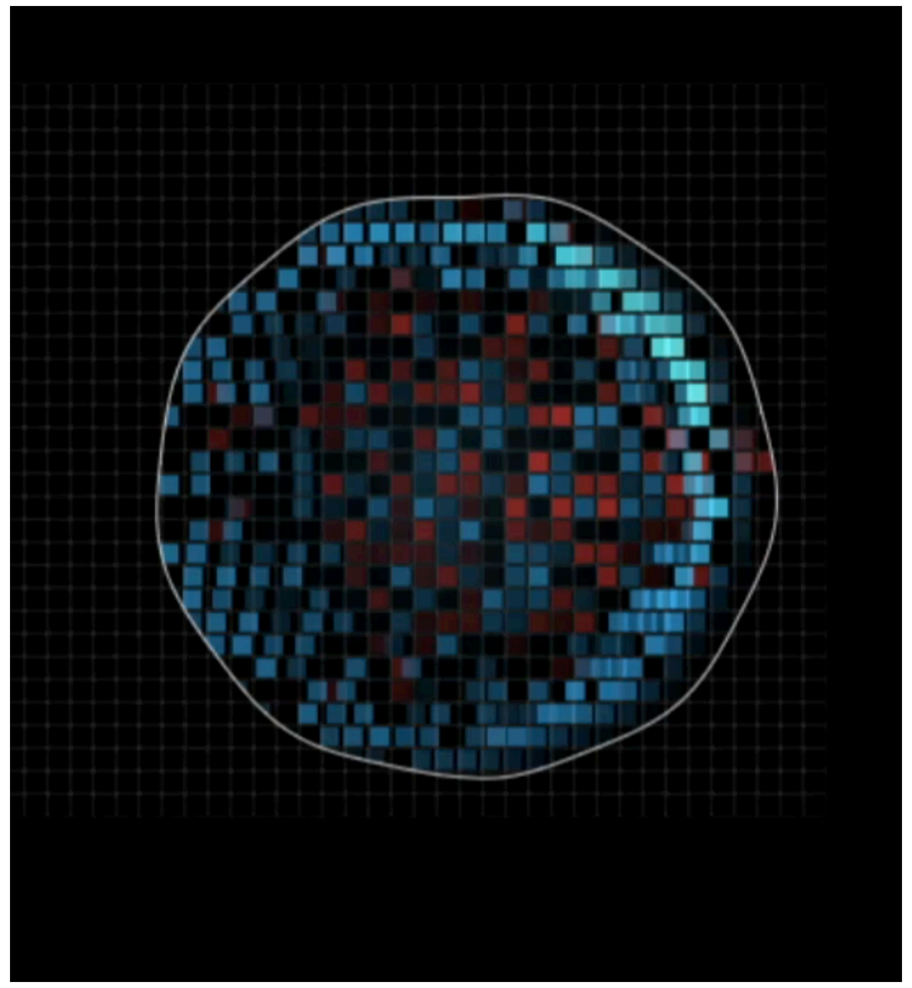

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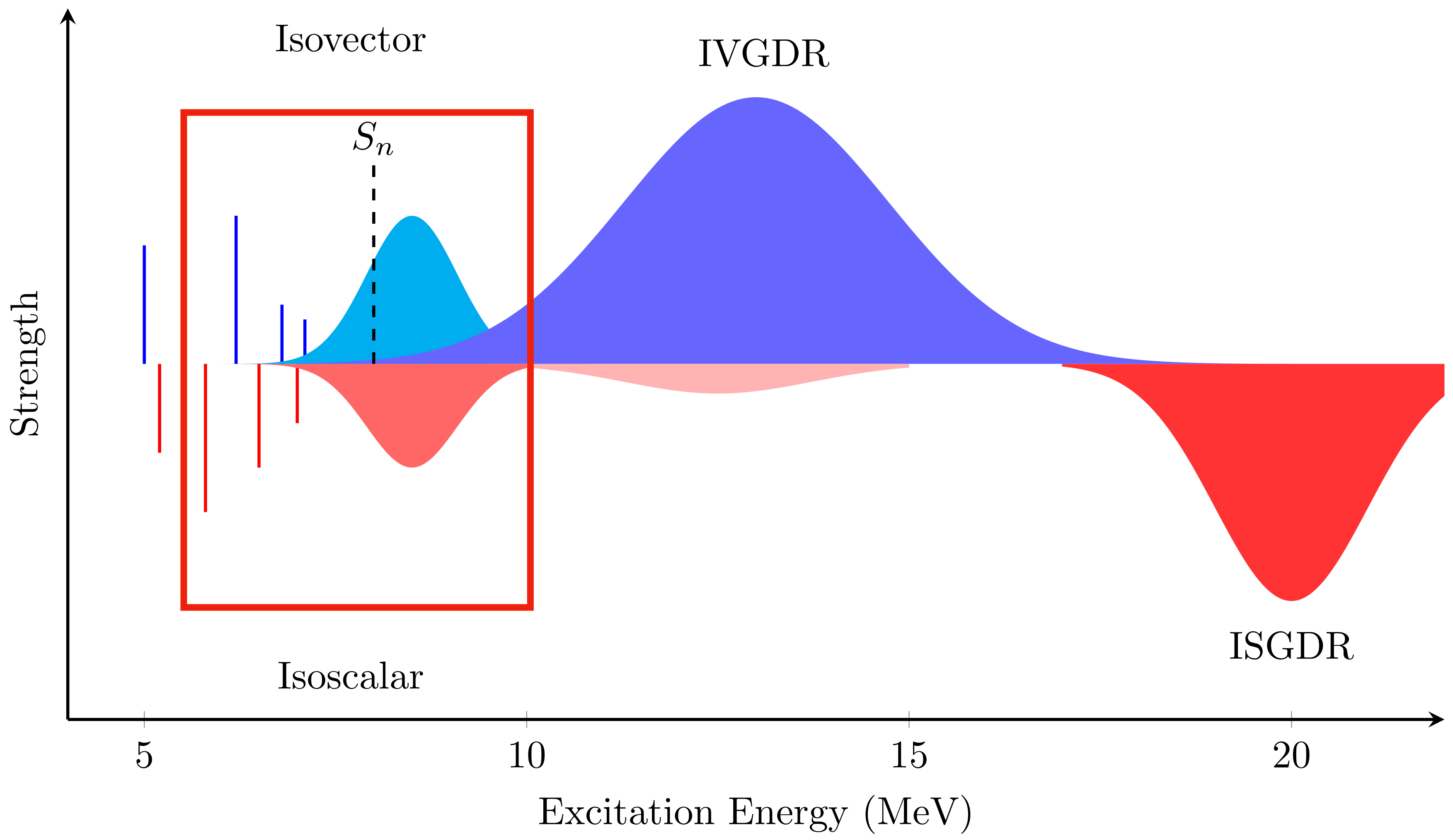

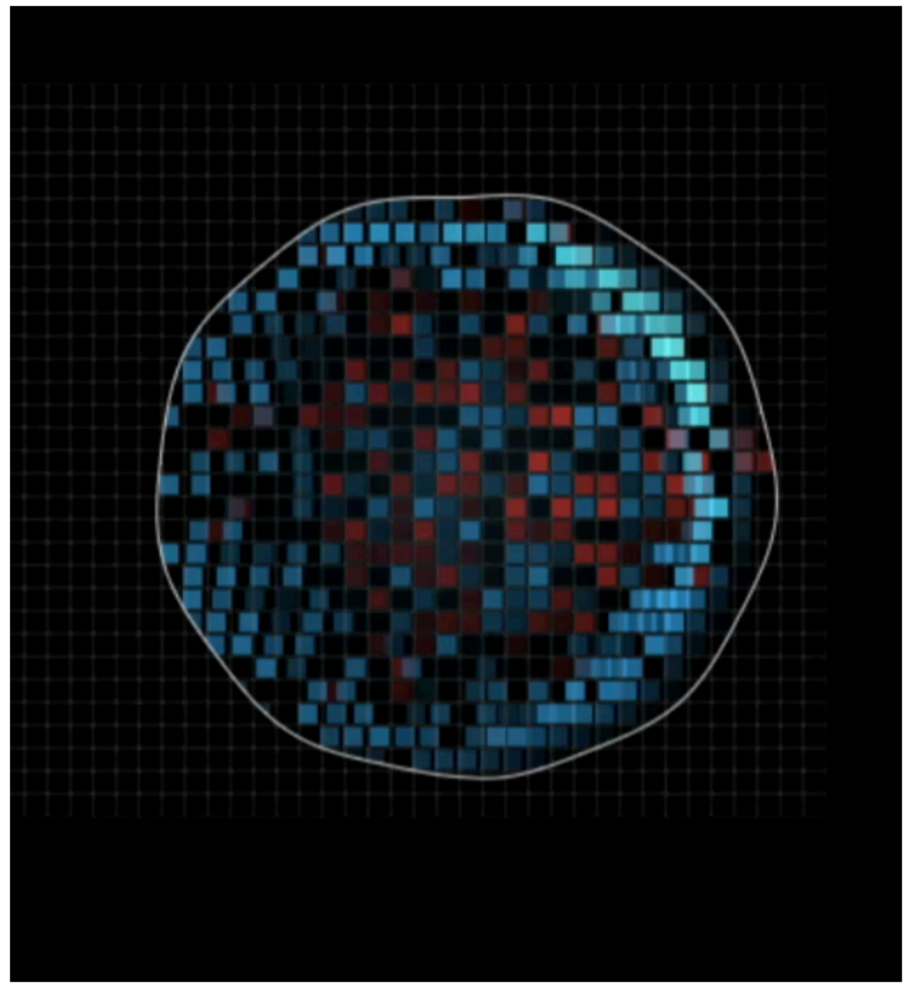

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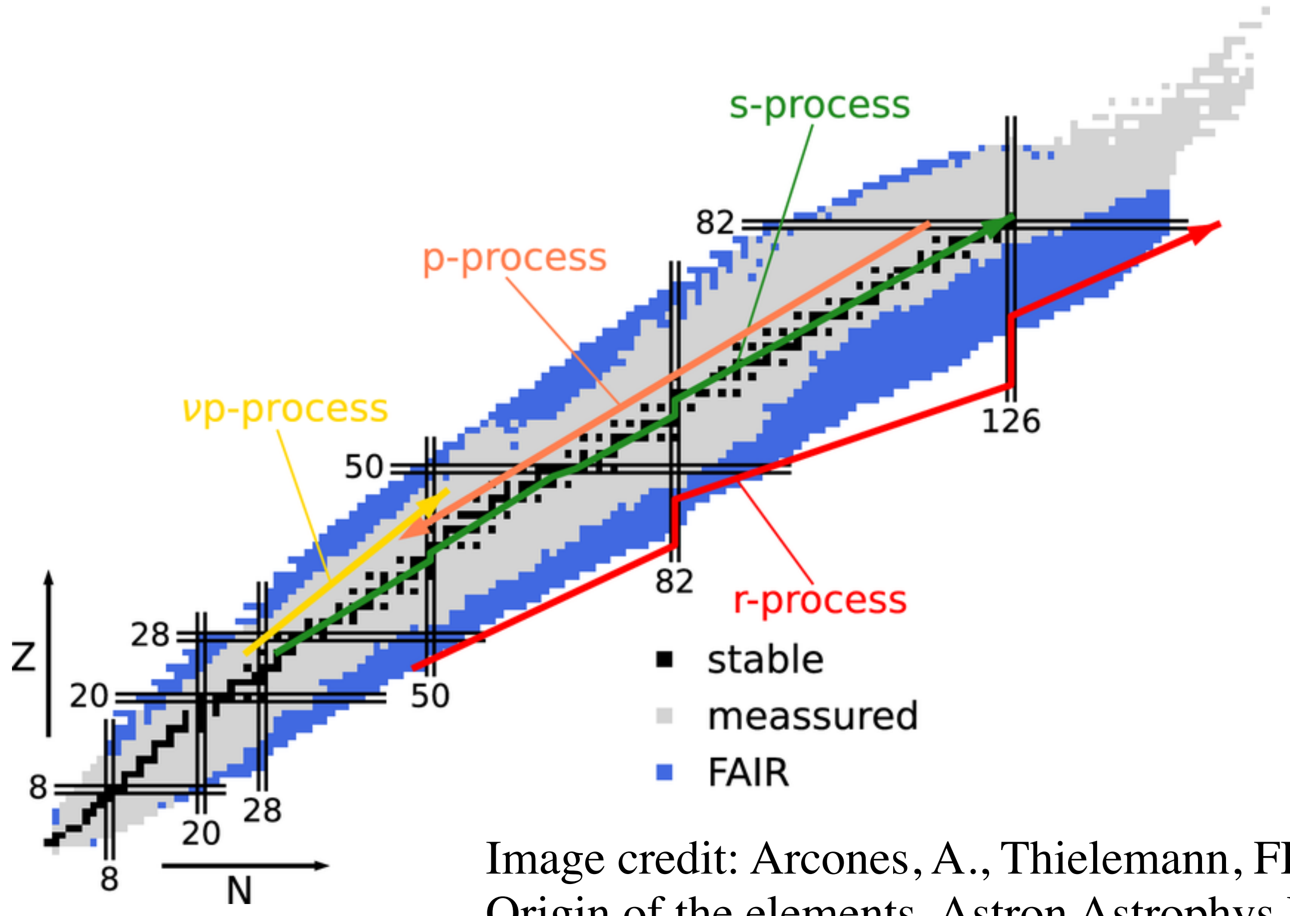

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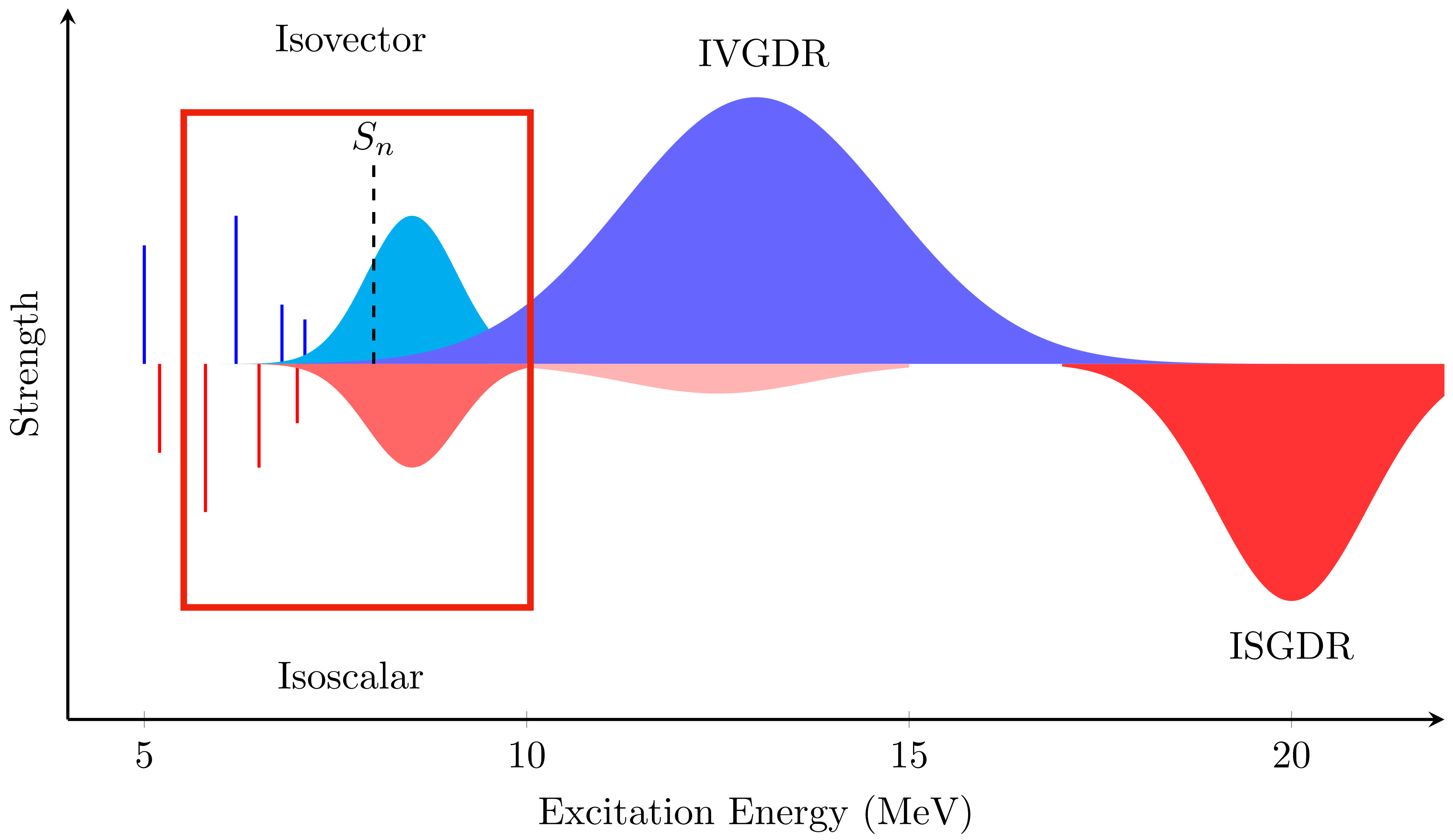

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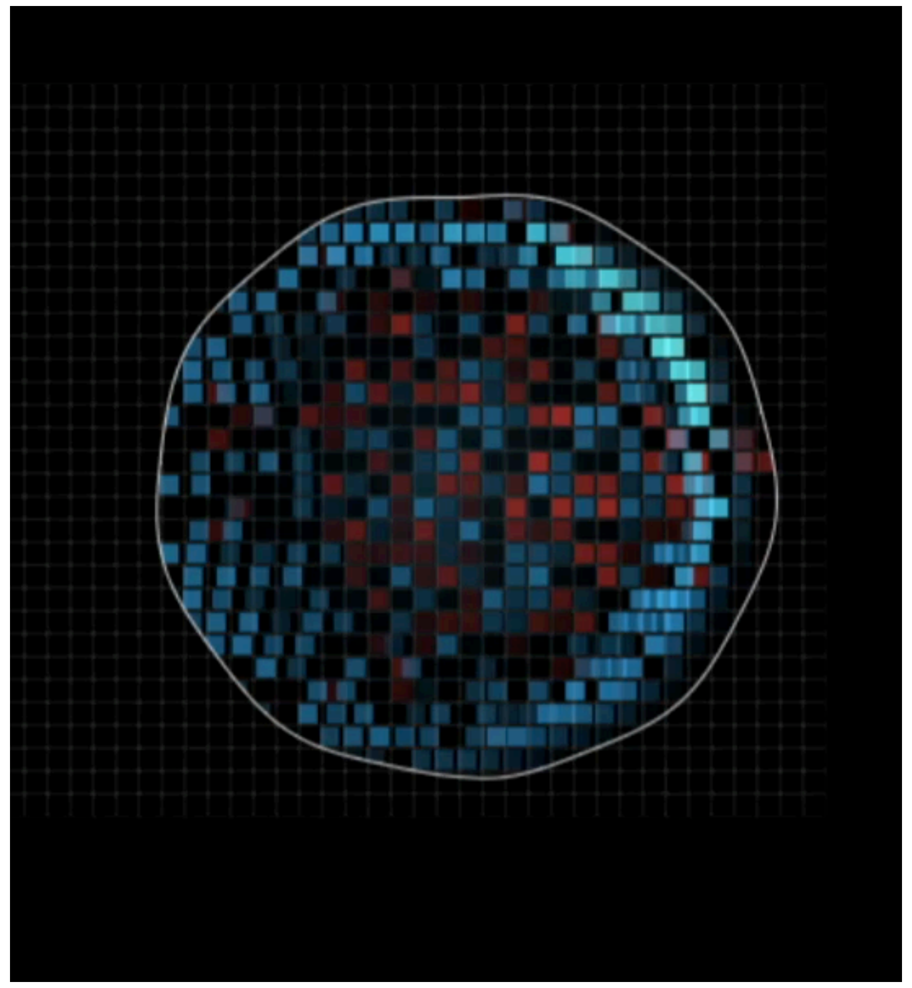

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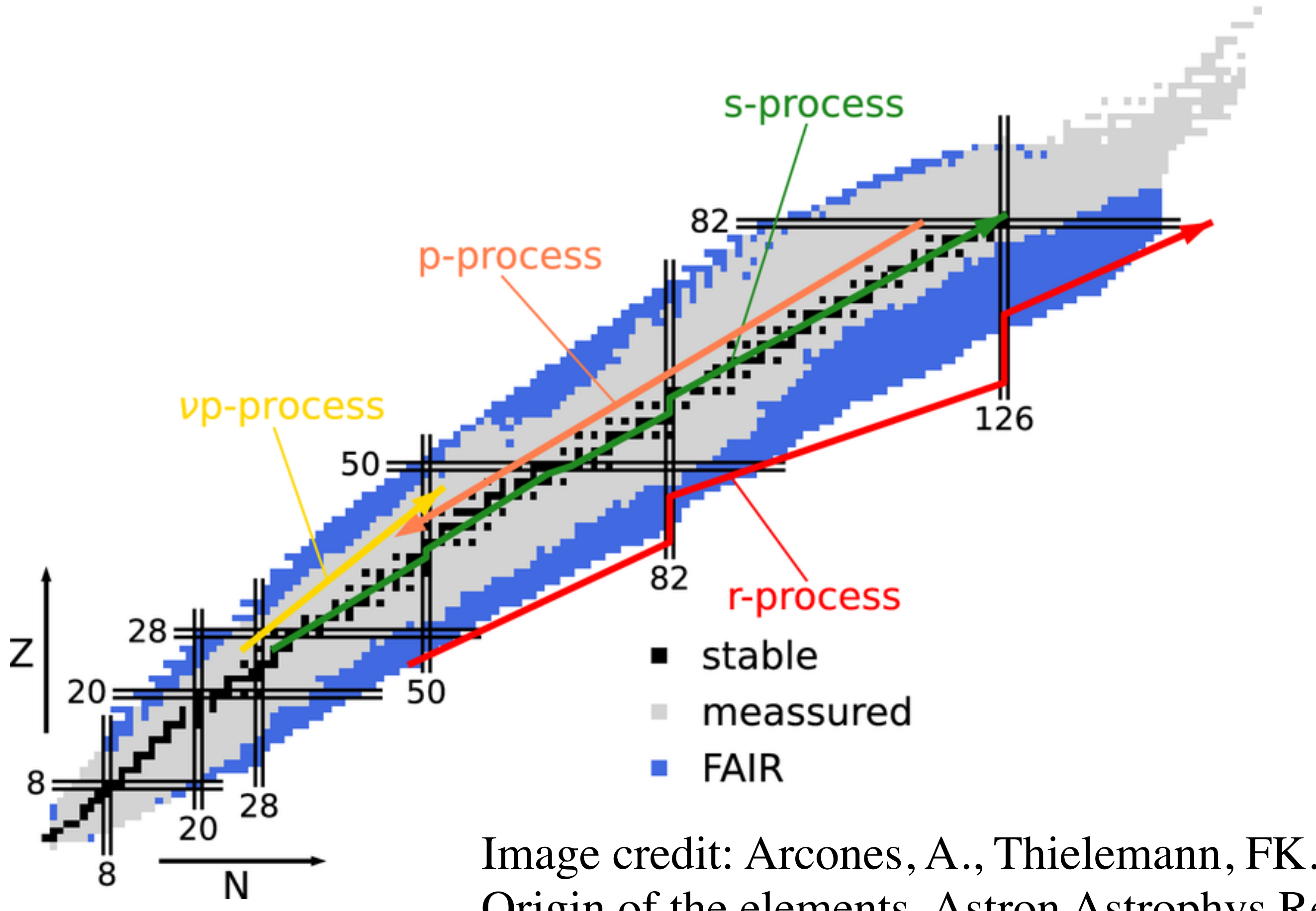

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Neutron matter equation of state

$$\mathcal{E}(\rho,\alpha) = \mathcal{E}_{\mathrm{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$\rho = (\rho_n + \rho_p)$ $\alpha = (\rho_n - \rho_p)/\rho$

where the density-dependent symmetry energy is:

$$\mathcal{S}(\rho) = J + L\frac{(\rho-\rho_0)}{3\rho_0} + \ldots.$$

symmetry energy at saturation density

slope parameter, related to pressure of pure neutron matter at saturation density

5 Image credit: Francesca Bonaiti, JGU Mainz Rising Researchers Seminar Series (2023).

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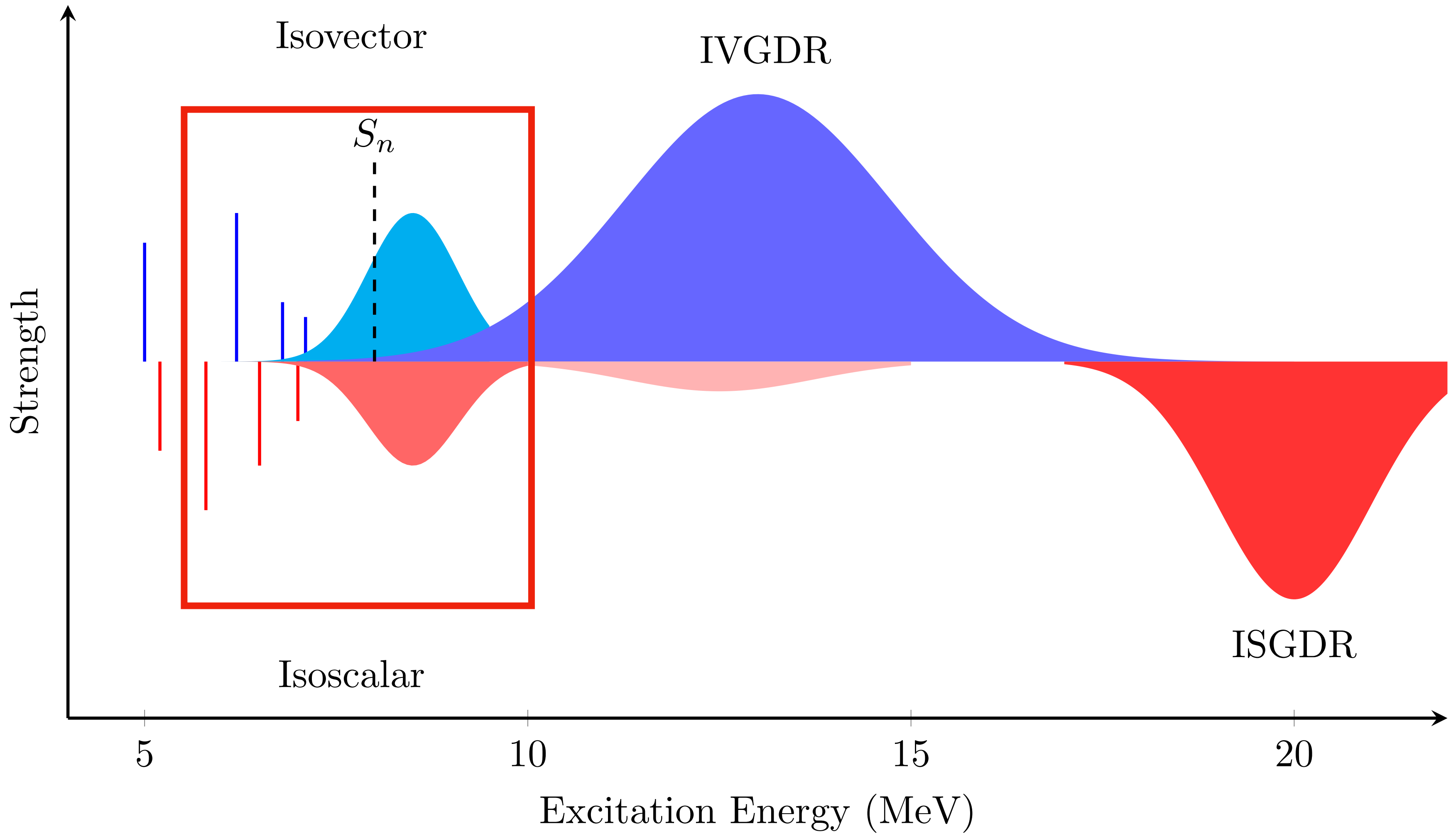

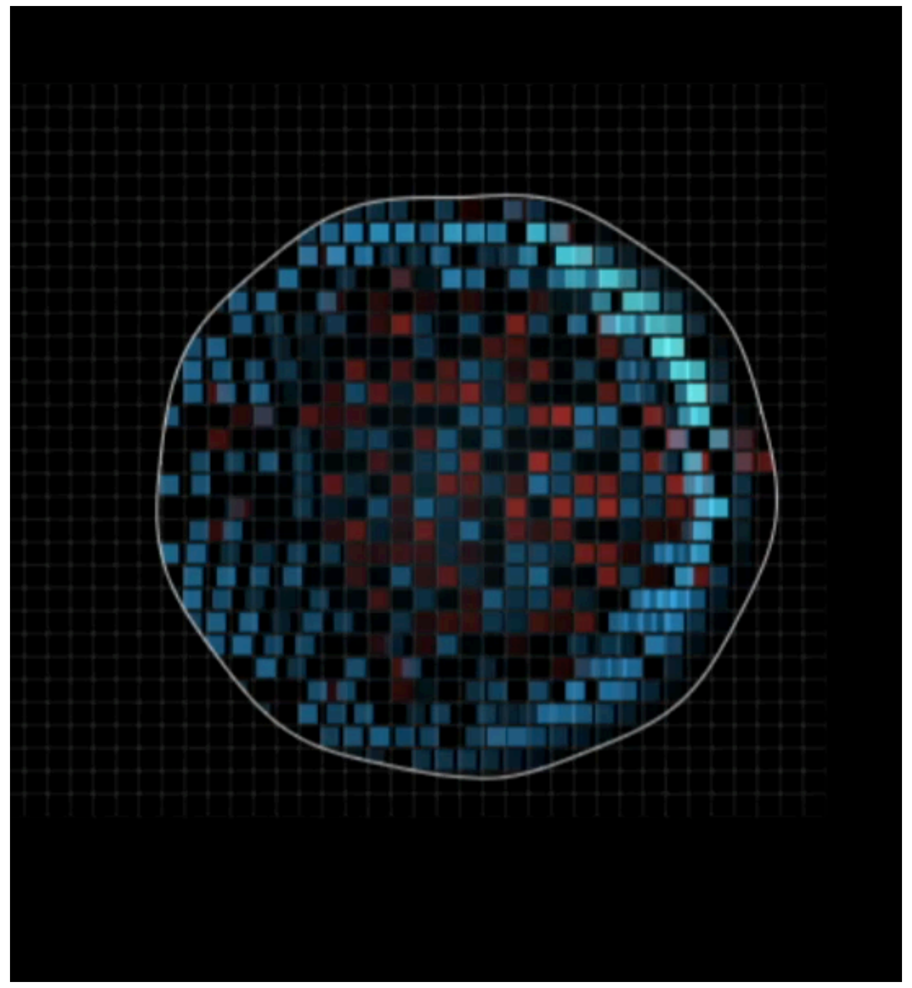

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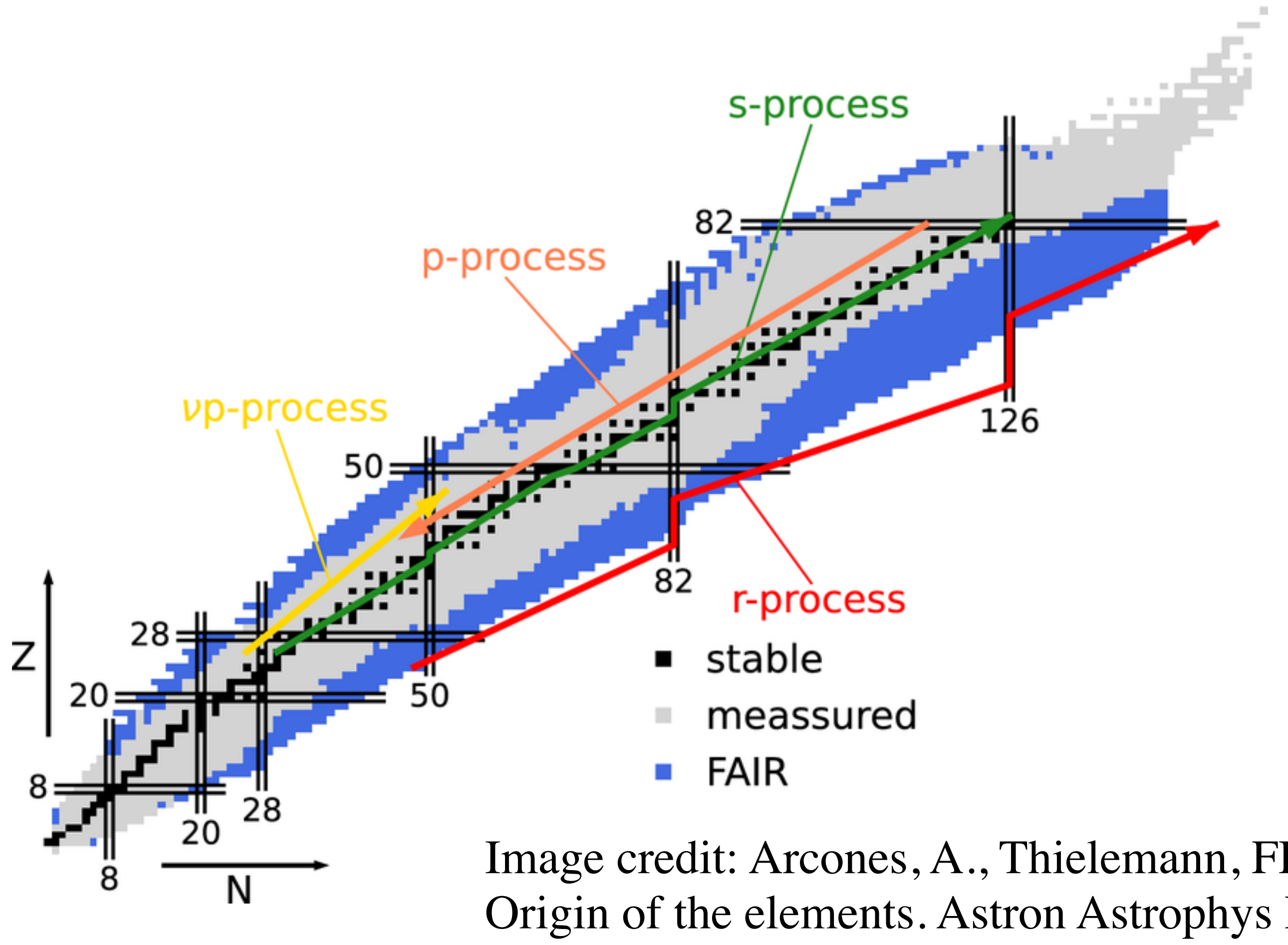

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Ultra High Energy Cosmic Rays

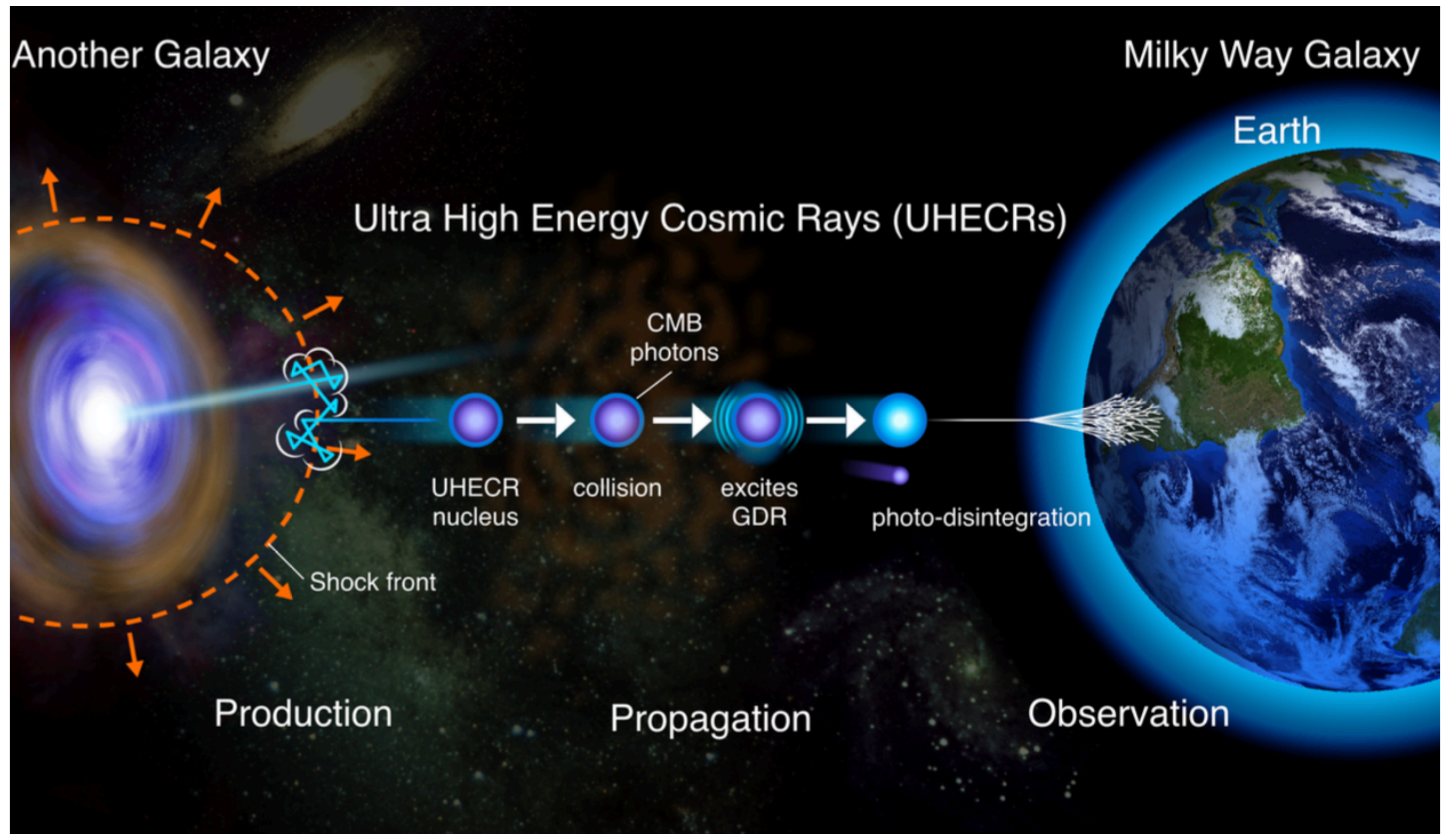

Image credit: Atsushi Tamii, 9th Workshop on Nuclear Level Density and Gamma Strength, 2024, Oslo.

II - Theoretical Framework: Configuration-Interaction Shell Model

Theoretical framework: CI-SM

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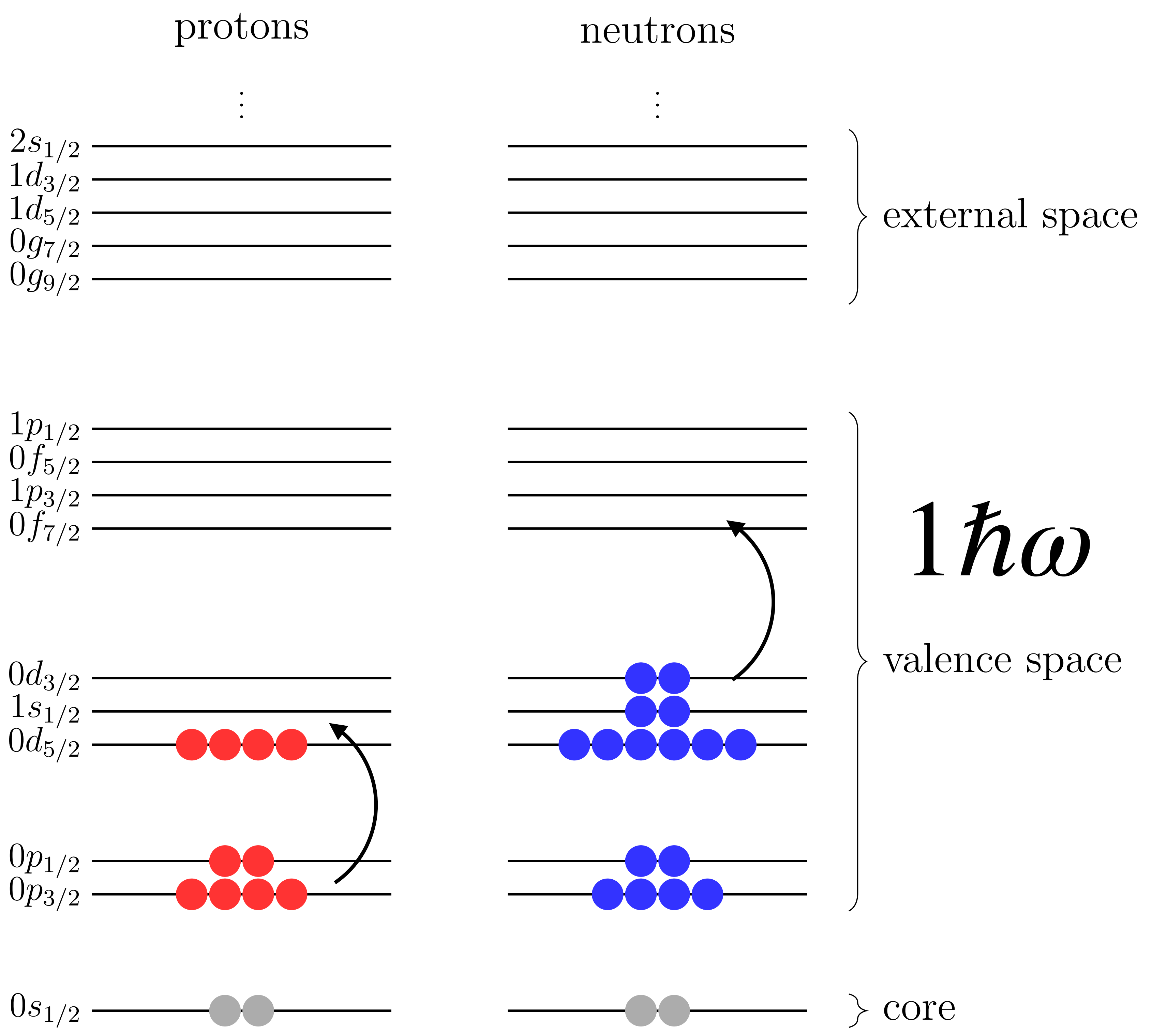
Valence space definition

$$\mathcal{H} \longrightarrow \mathcal{H}_{\text{valence}}$$



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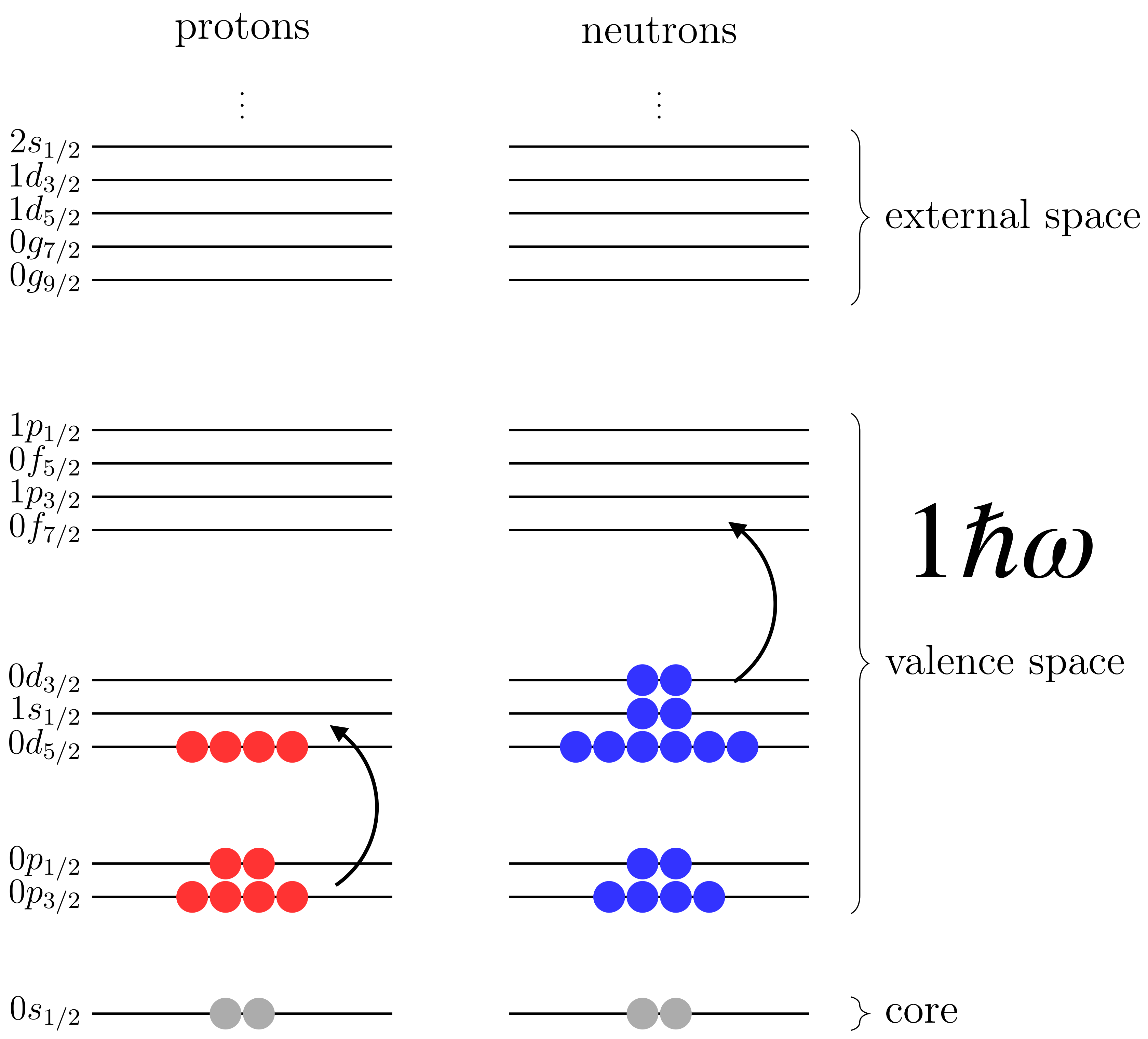

Effective Hamiltonian

$$H \longmapsto H_{\text{eff}}$$

p-shell nuclei: WBP interaction

E. K. Warburton and B. A. Brown,
Physical Review C 46, 923 (1992).

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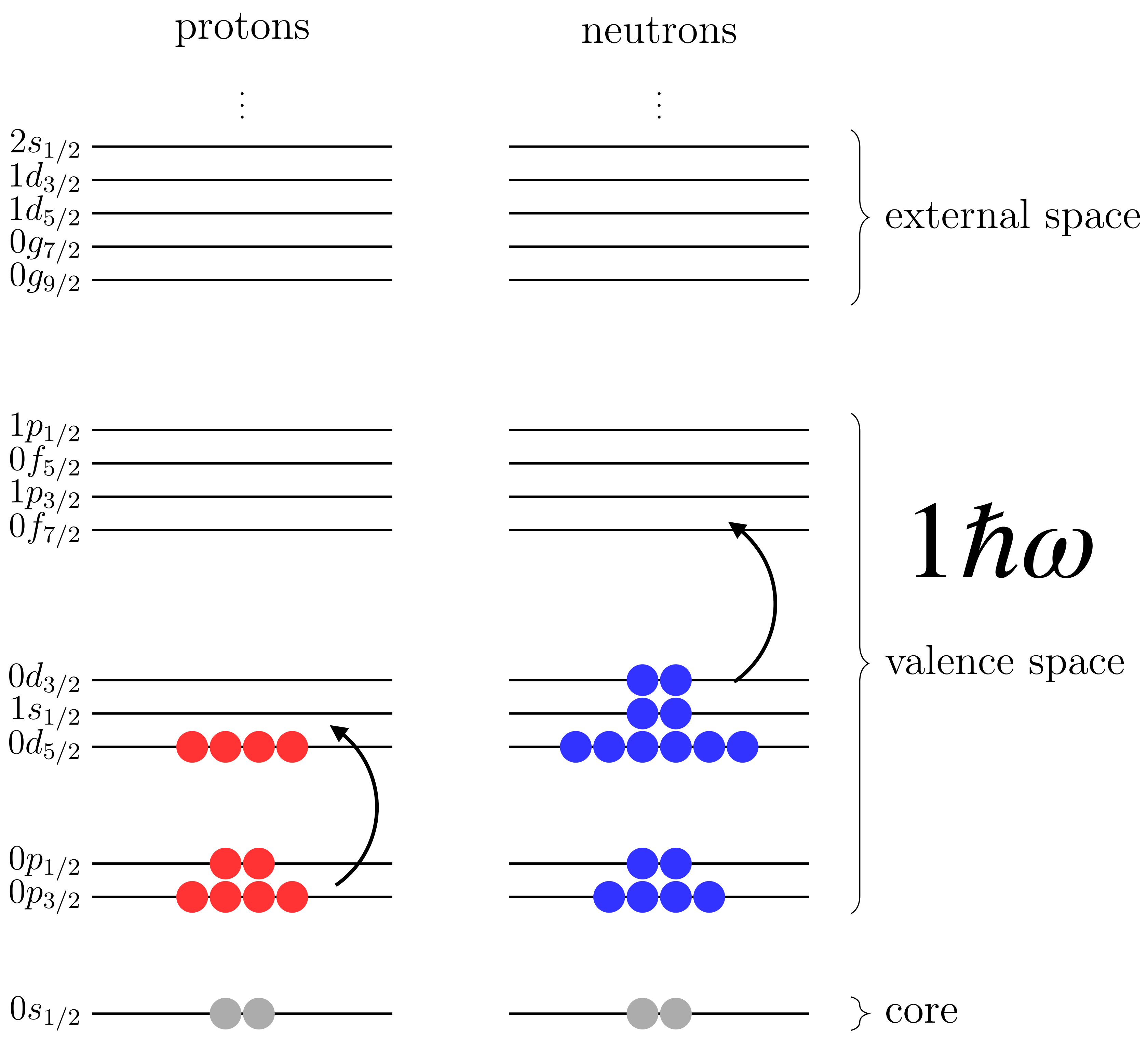

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Diagonalization procedure

Lanczos algorithm

Algorithm 1: Lanczos tri-diagonalization

Input: Initial vector $|\mathcal{L}_0\rangle$, Hamiltonian H, number of iterations k

Output: Tridiagonal matrix T_k

Normalize $|\mathcal{L}_0\rangle$, $\beta_{-1} = 0$, $|\mathcal{L}_{-1}\rangle = 0$

for $n = 0$ **to** k **do**

- $|\mathcal{Y}_{n+1}\rangle = H\,|\mathcal{L}_n\rangle - \beta_{n-1}\,|\mathcal{L}_{n-1}\rangle$;
- $\alpha_n = \langle \mathcal{L}_n | \mathcal{Y}_{n+1}\rangle$;
- $|\mathcal{Z}_{n+1}\rangle = |\mathcal{Y}_{n+1}\rangle - \alpha_n\,|\mathcal{L}_n\rangle$;
- $\beta_n = \| \mathcal{Z}_{n+1} \|$;
- $|\mathcal{L}_{n+1}\rangle = |\mathcal{Z}_{n+1}\rangle / \beta_n$;

return $\{ \, | \, \mathcal{L}_n \rangle \, \}, \{ \, \alpha_n \, \}, \{ \, \beta_n \, \}$.

Min 300 iterations in the following calculations

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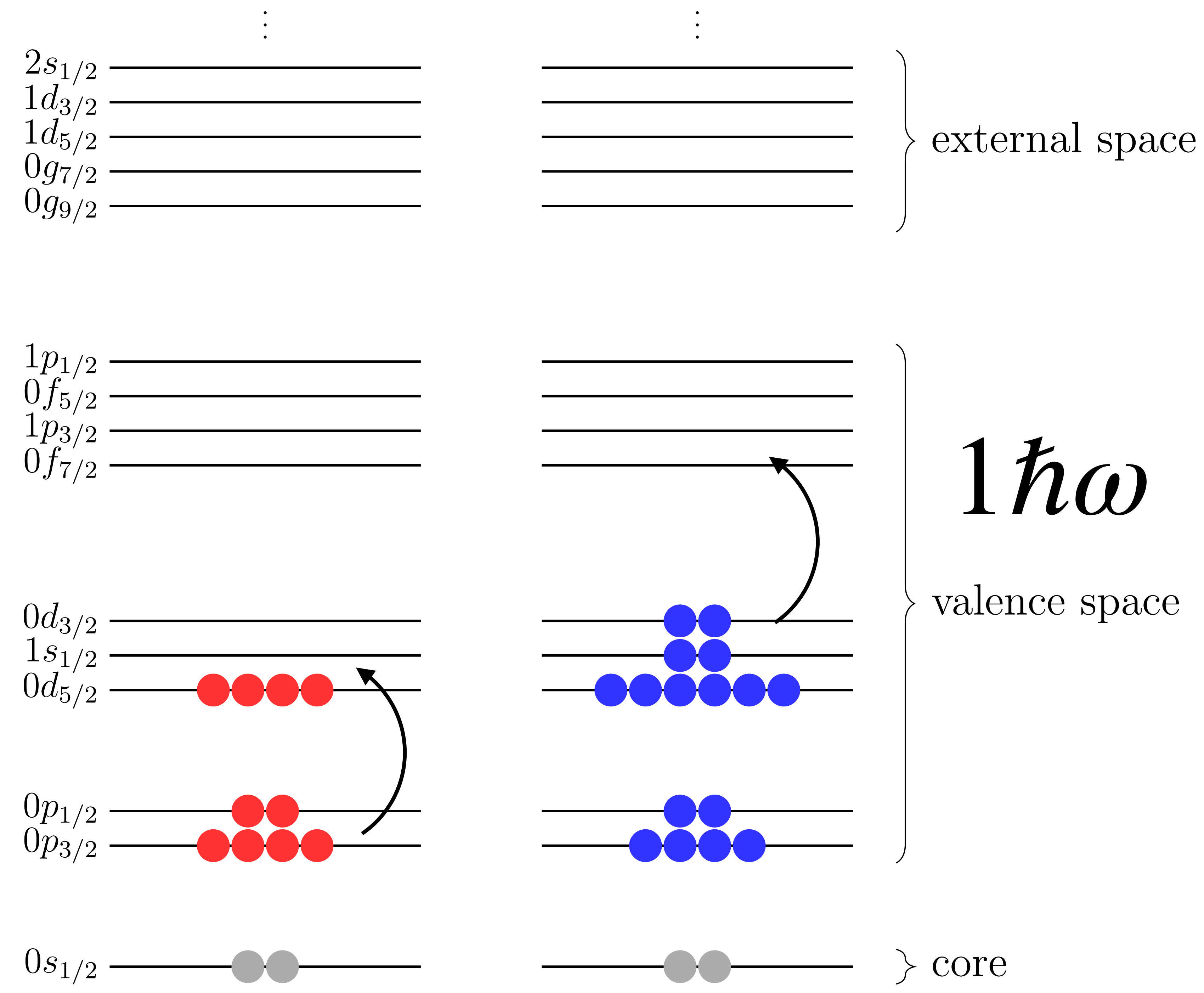

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+ Center-of-mass treatment: Gloeckner & Lawson's method

D. H. Gloeckner and R. D. Lawson, Physics Letters B 53, 313 (1974)

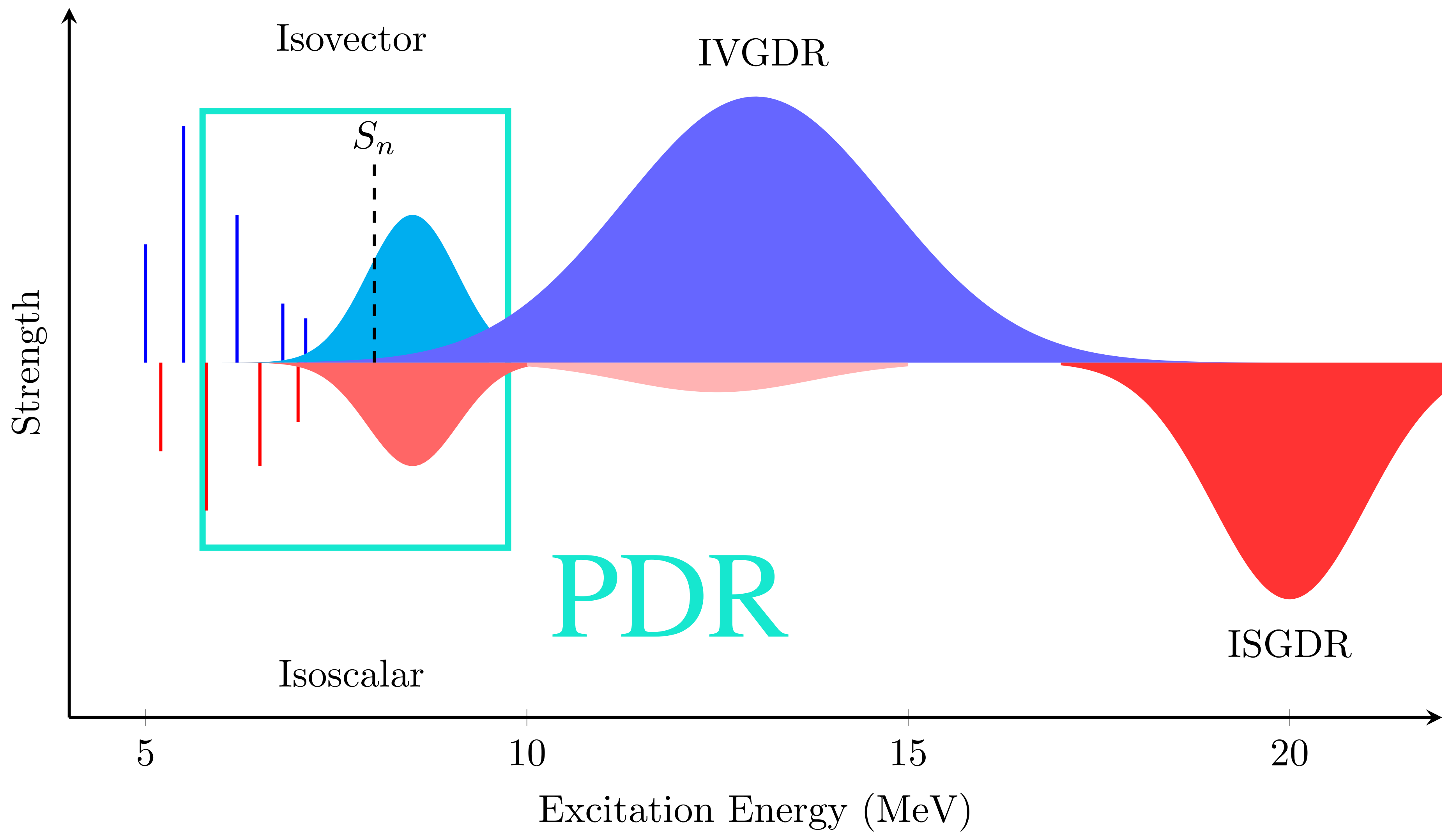
+ Renormalization of the dipole operator: E1 effective charge

III - A shell model view on the pygmy dipole resonance

Pygmy dipole resonance: definition

Observation:

Low-energy E1 peaks observed and predicted in neutron-rich nuclei.



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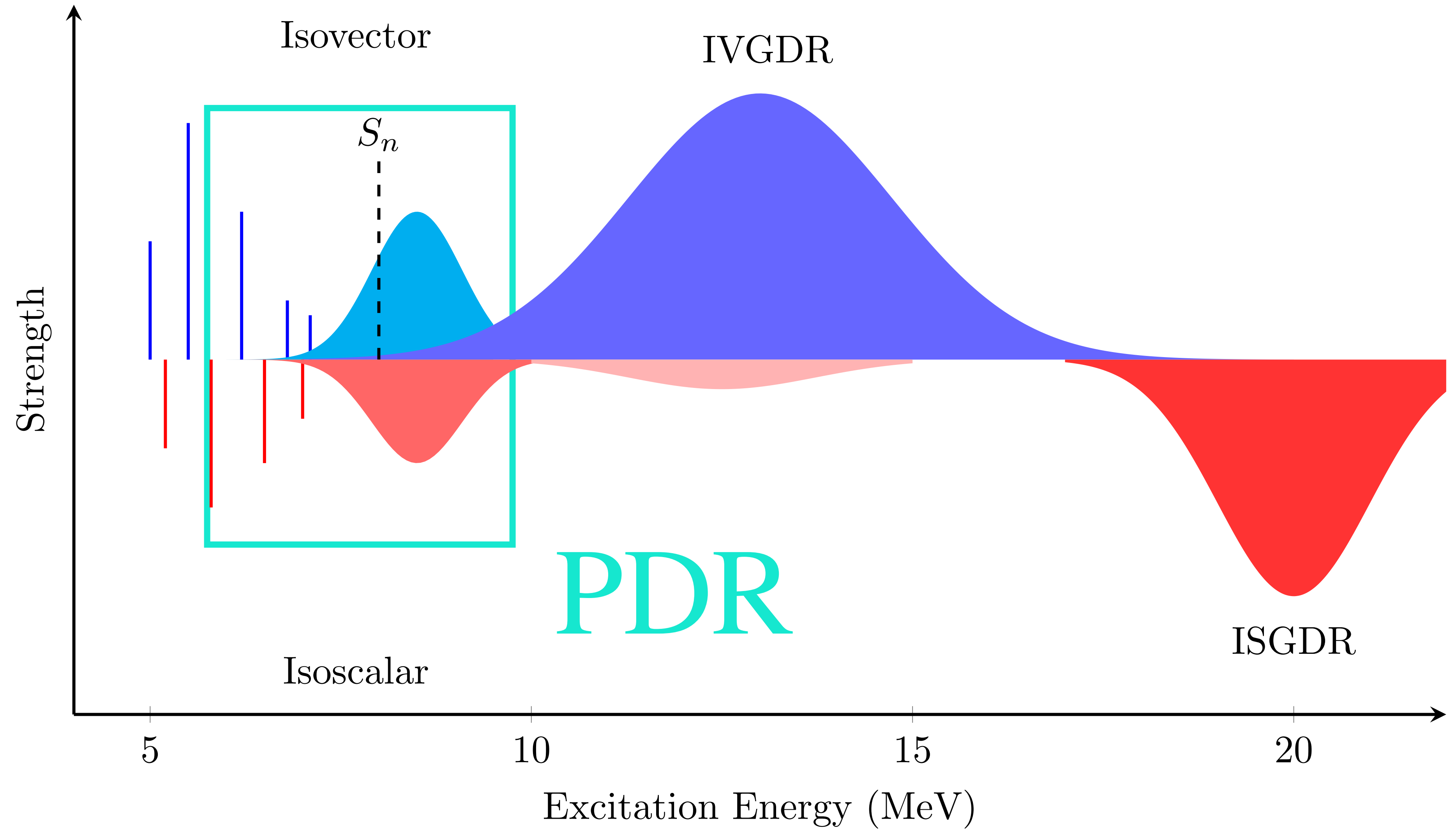

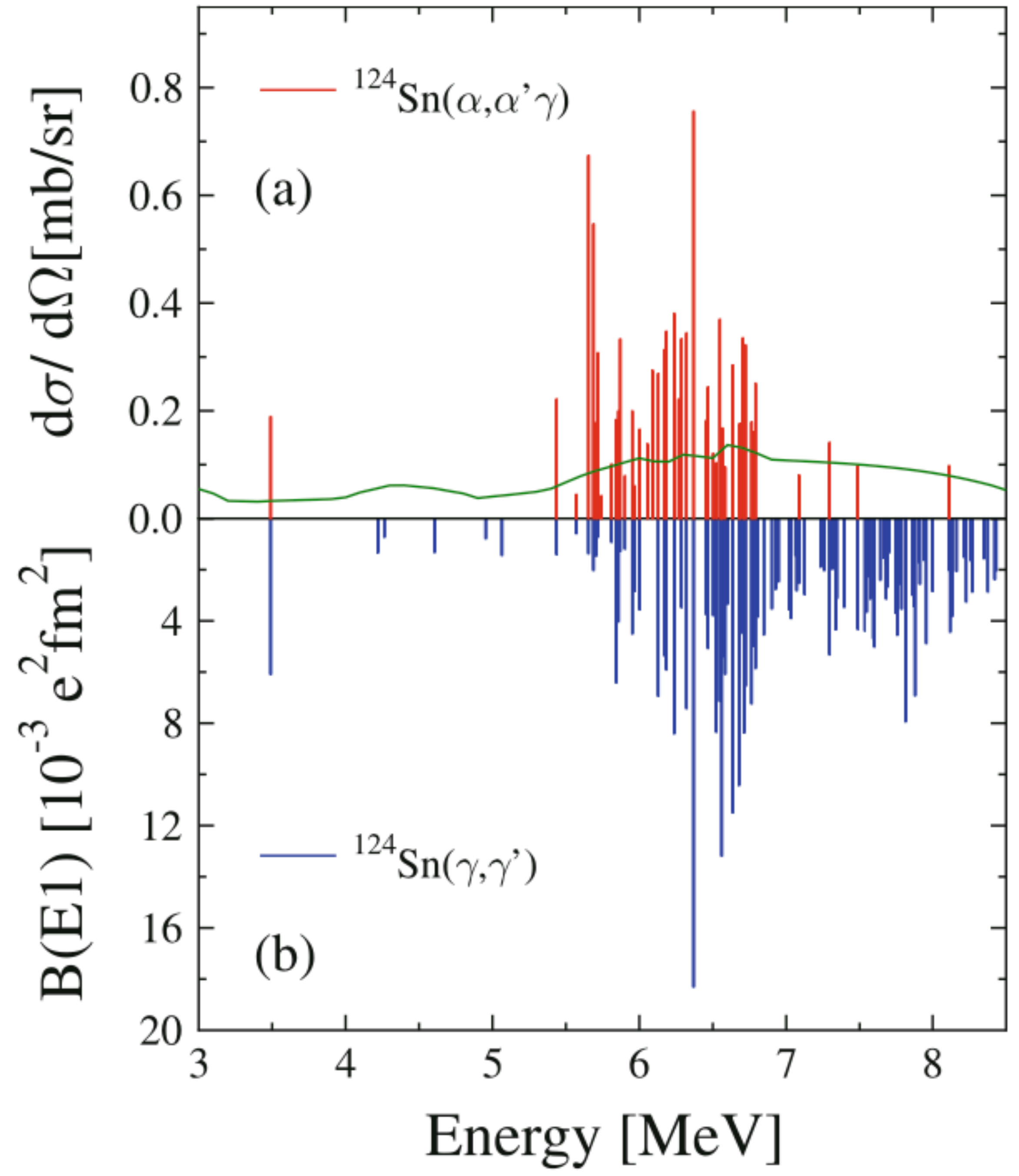

J. Endres et al., Phys. Rev. Lett. 105, 212503 (2010)

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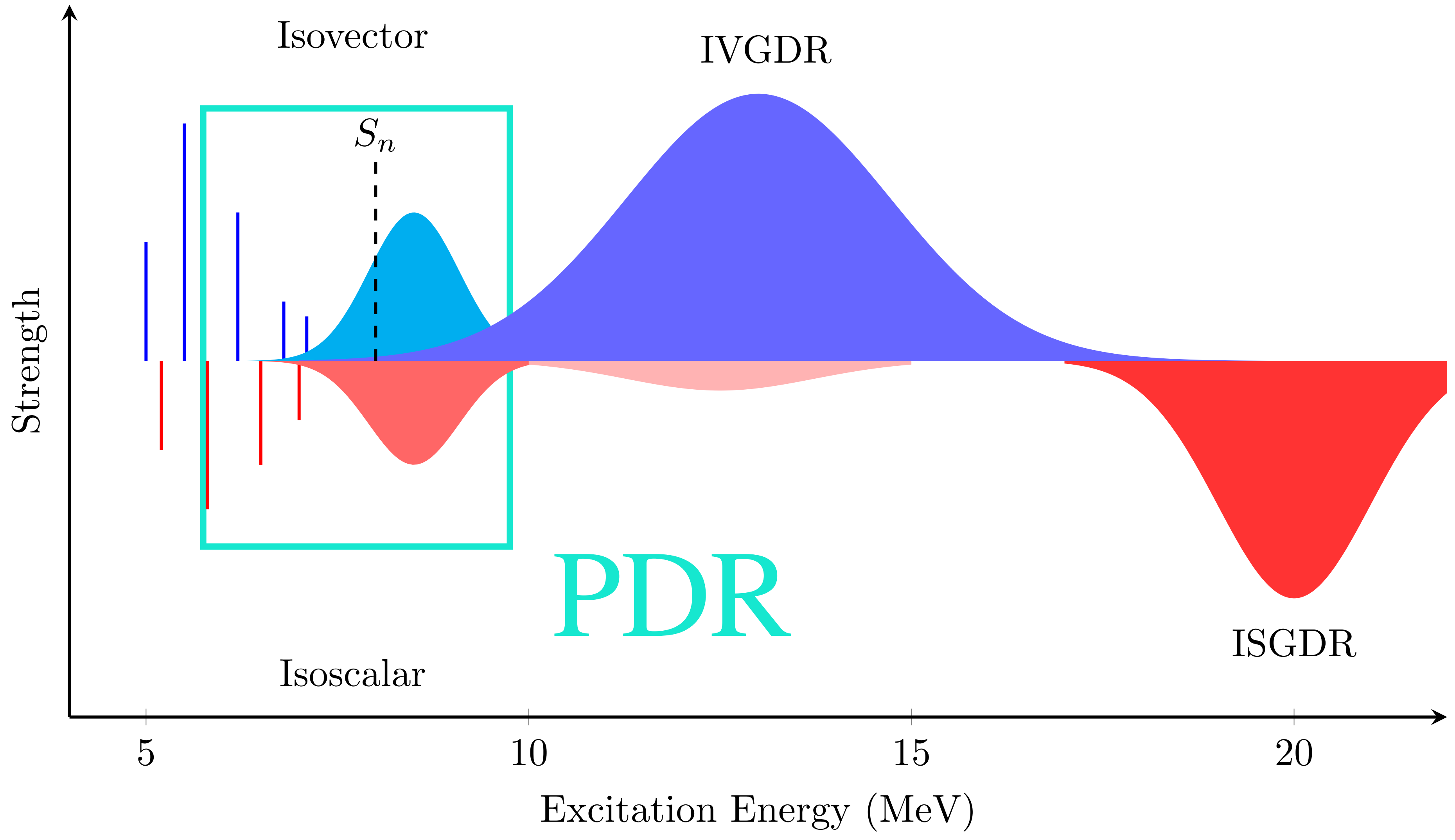
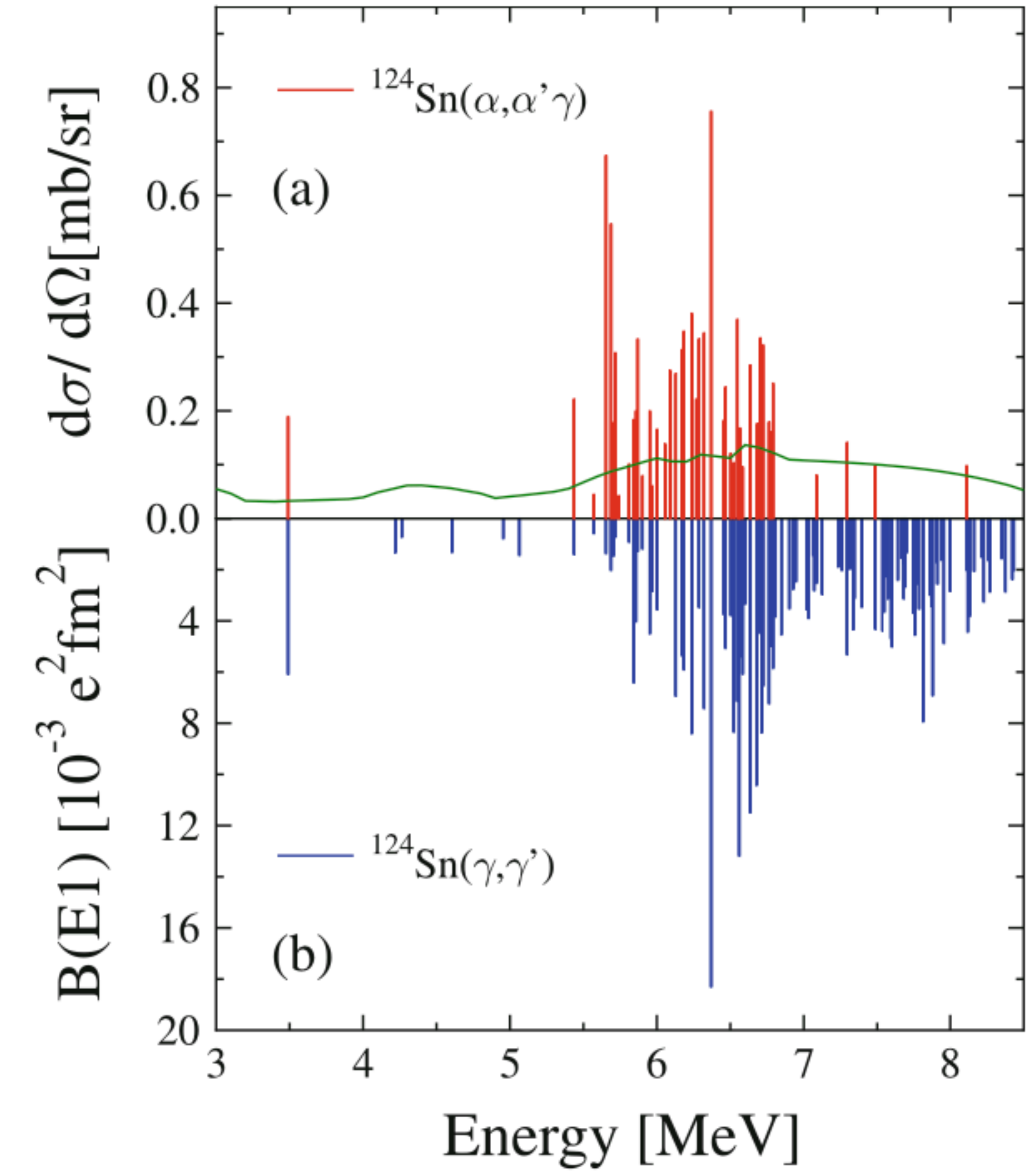
Our definition:

Low-energy E1 peaks sharing a common structure, which should be clearly distinct from that of the GDR.





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- **Degree of collectivity ?**
- **Coexistence of different low-energy E1 states ?**
- **Transition from PDR to GDR ?**
- **Universality of the isospin splitting ?**
- Effect of nuclear deformation ?
- Toroidal mode ?
- PDR built on top of excited states ?
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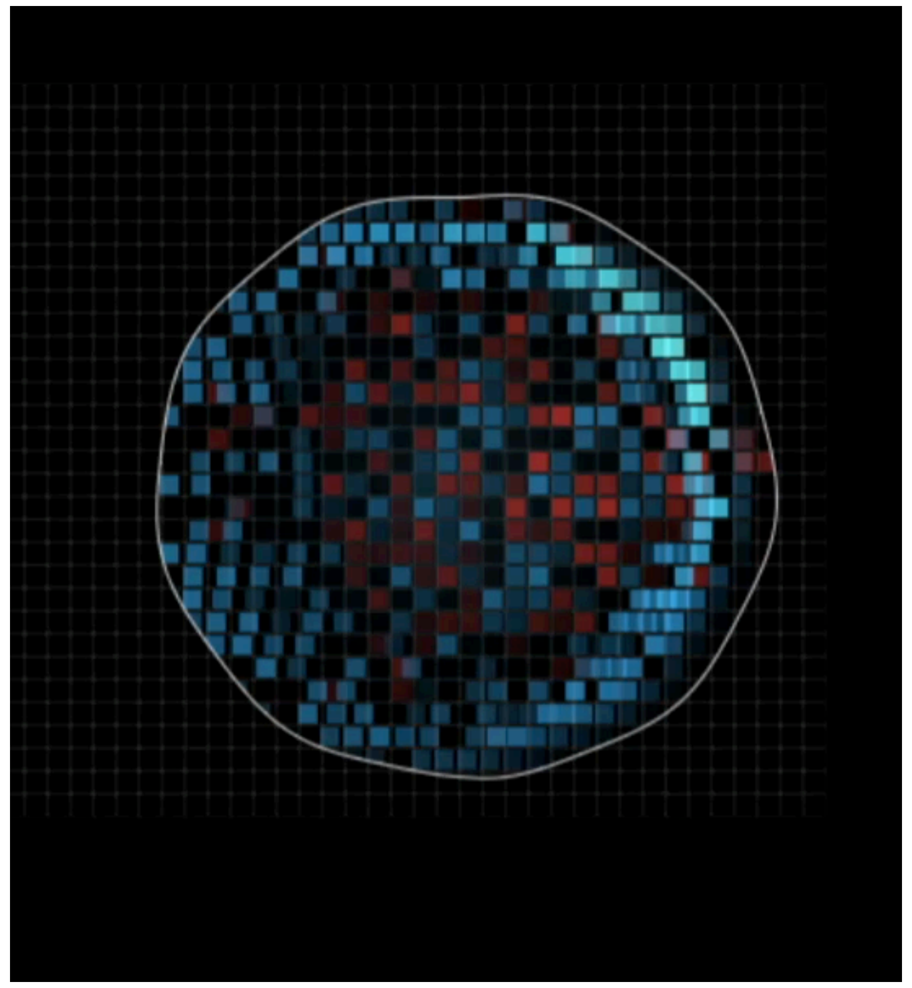

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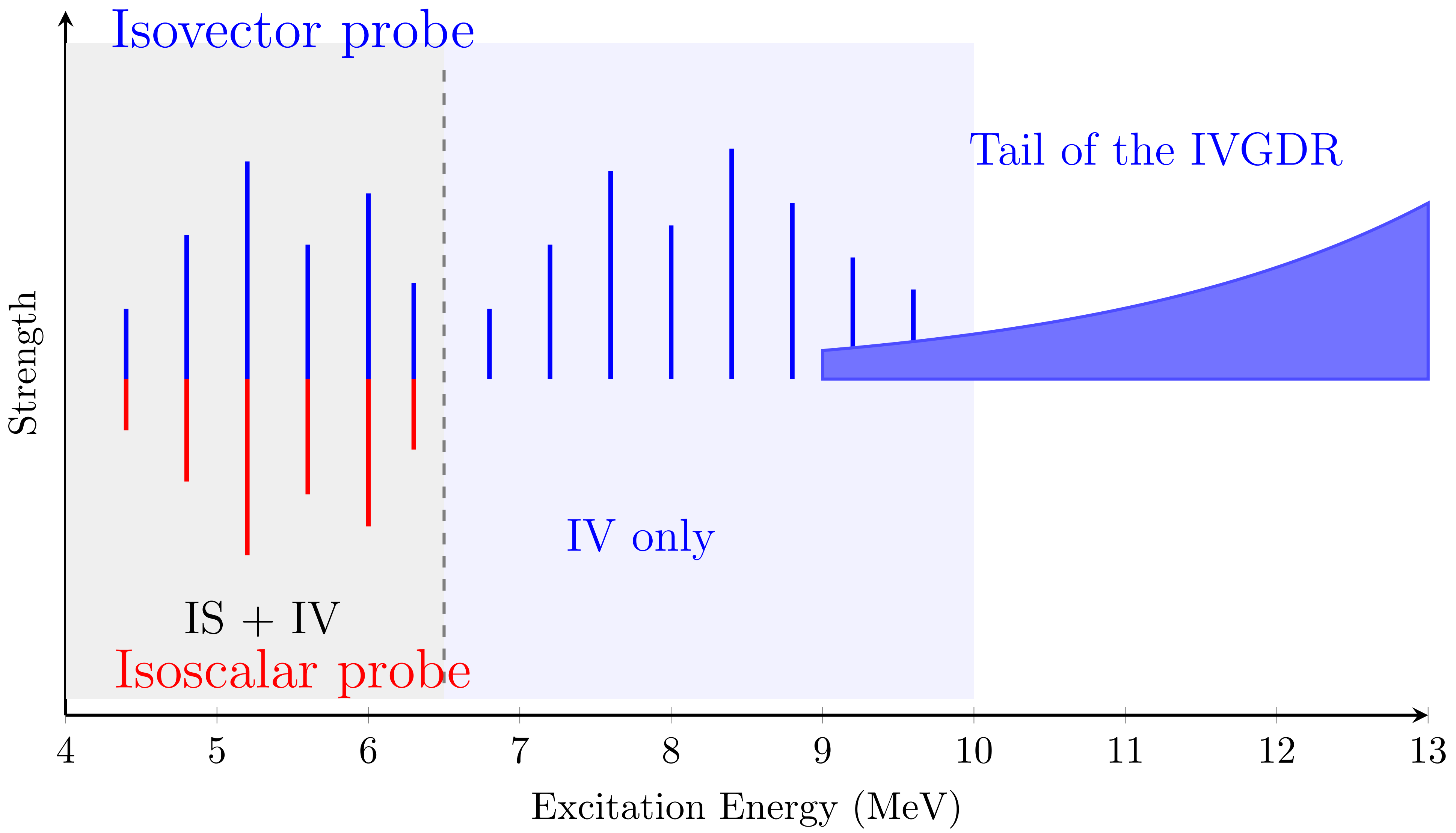

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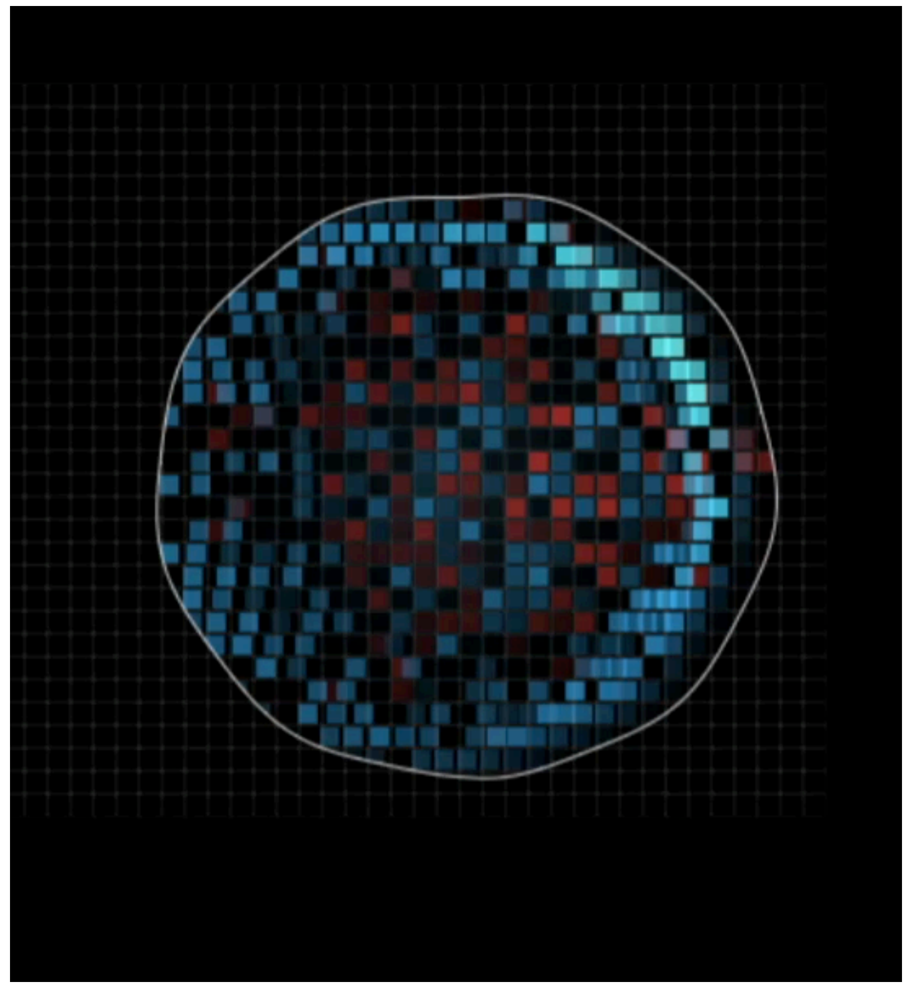
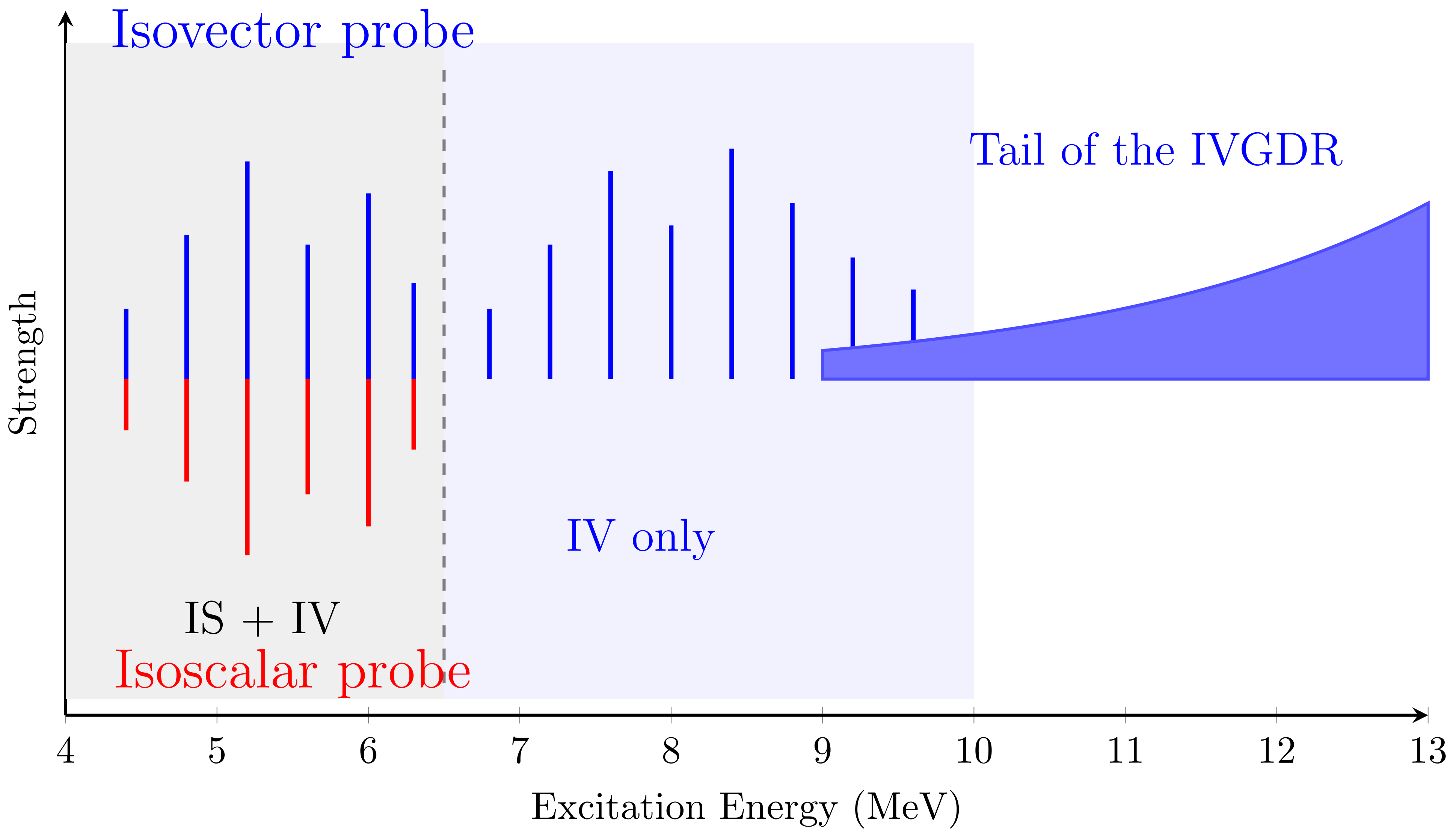
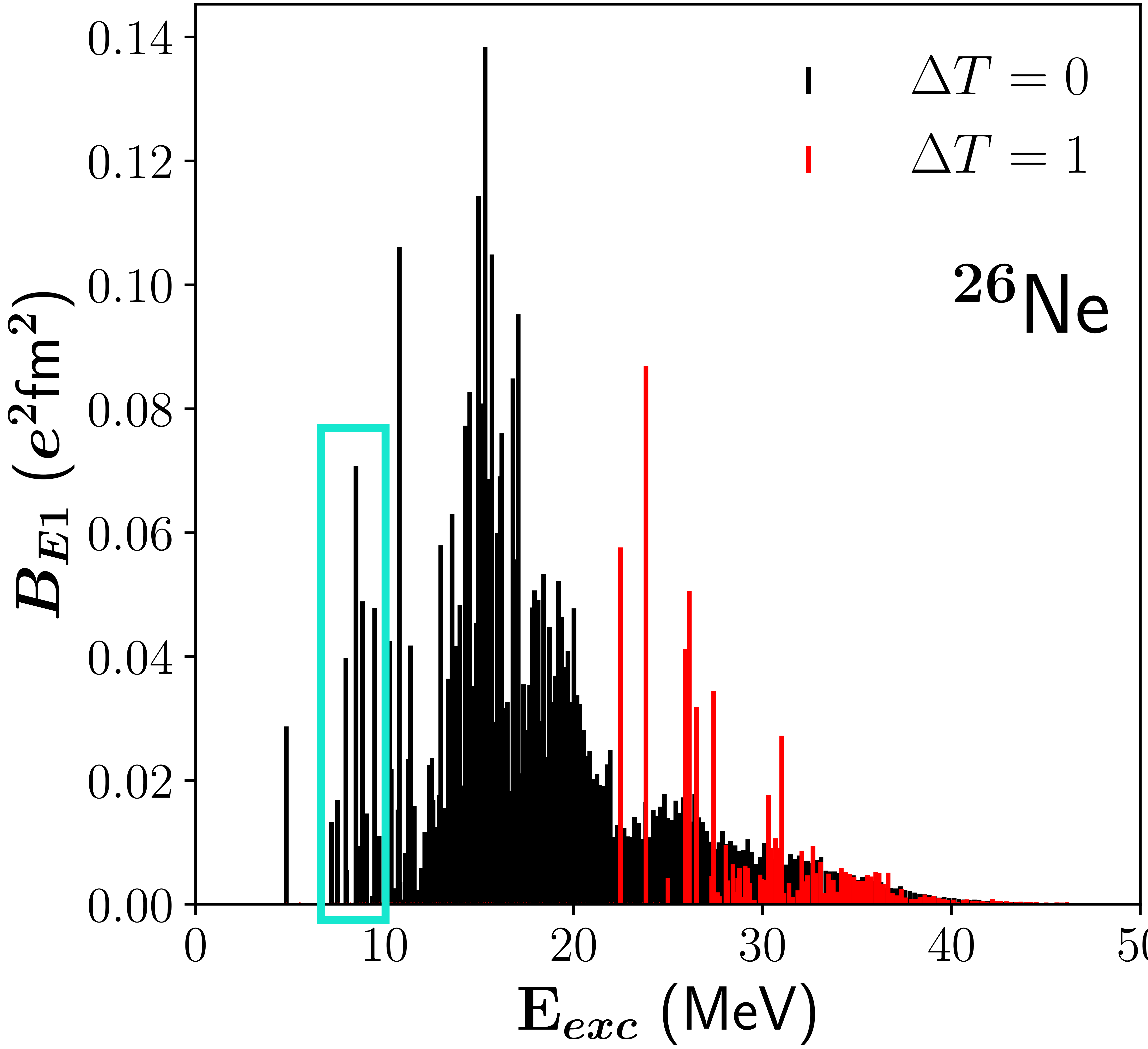
- …



Animation credit: Martin Bordeau



Pygmy dipole resonance: experiment-theory comparison



CISM

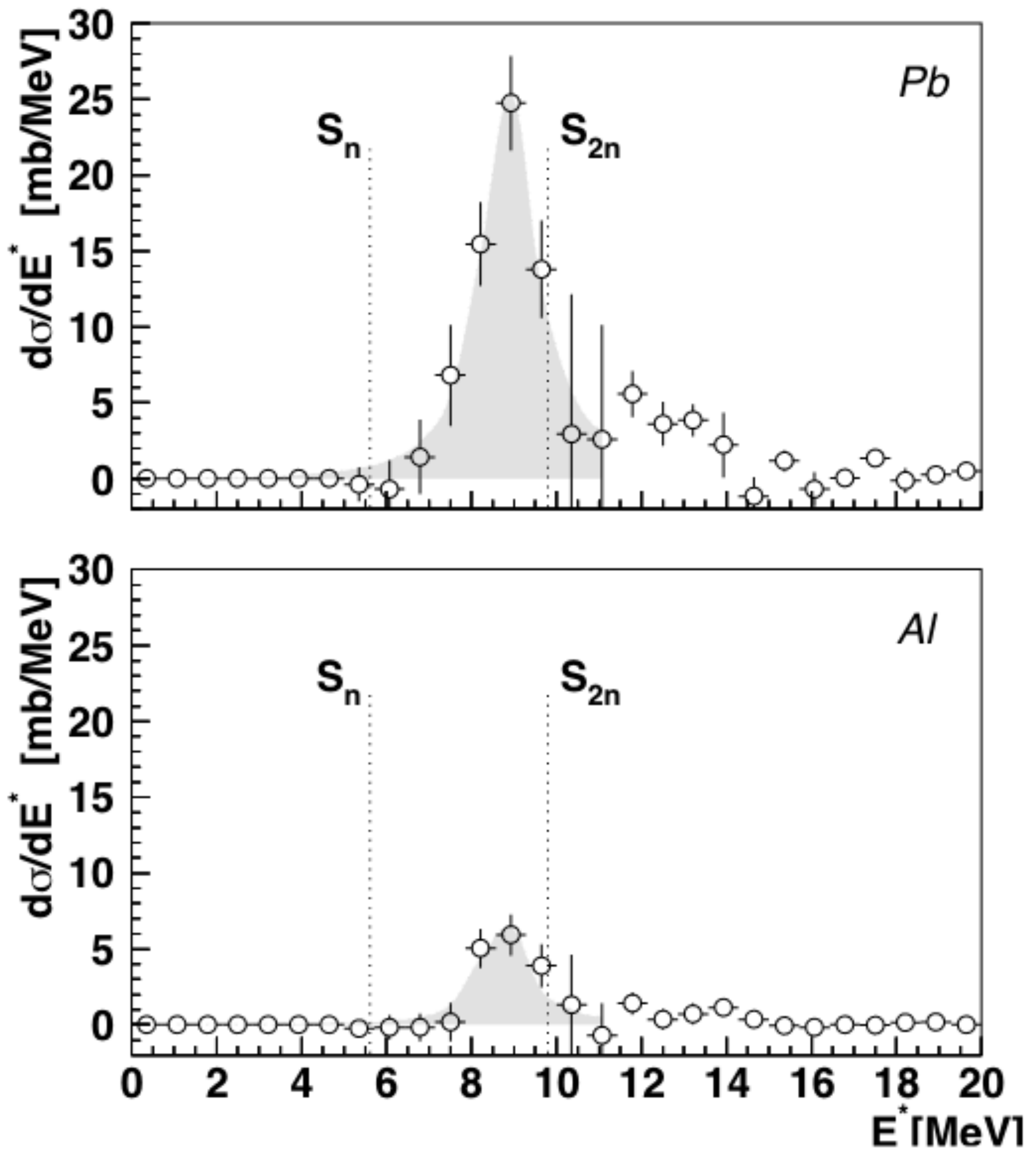
$$\bar{S}_{\text{PDR}} = 8.63 \text{ MeV}$$
$$S_0^{\text{PDR}} = 0.37 \; e^2\text{fm}^2$$

$\sim 4\%$ TRK

EXP

$$\bar{S}_{\text{PDR}} \approx 9 \text{ MeV}$$
$$S_0^{\text{PDR}} = 0.49 \pm 0.16 \; e^2\text{fm}^2$$

$^{26}\text{Ne} + \text{Pb} \longrightarrow {}^{26}\text{ Ne}^*$



Gibelin, J. et al. Nuclear Physics A 2007, 788, 153–158.

How to assess a degree of collectivity ?

- « Strong » response
- Regularity over broad region of the nuclear chart
- Involvement of many nucleons in coherent way
- Similar structural wave function features

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Sum rule strength

$$S_q = \sum_\nu E_\nu^q |\langle \nu | O | 0 \rangle|^2$$

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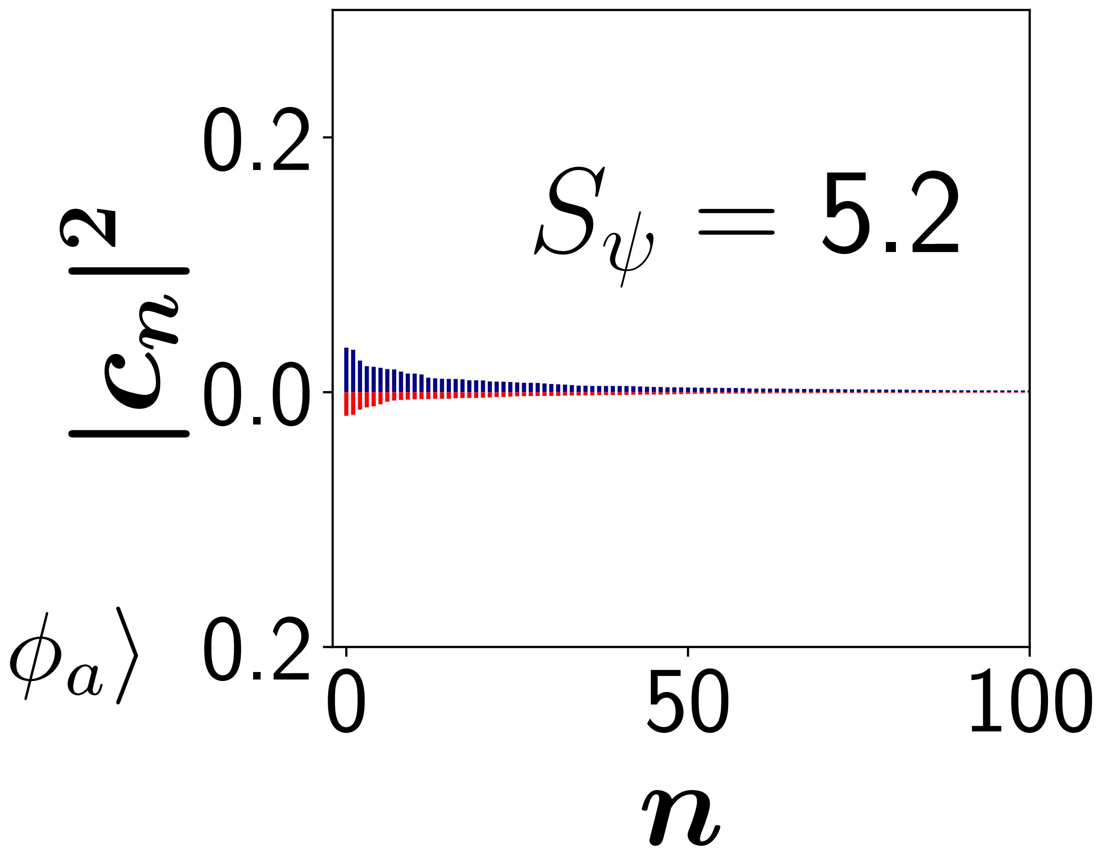
$$\mathcal{S}_\nu = -\sum_a |c_a^\nu|^2 \ln |c_a^\nu|^2$$

$$|\nu\rangle = \sum_a c_a^\nu |\phi_a\rangle$$



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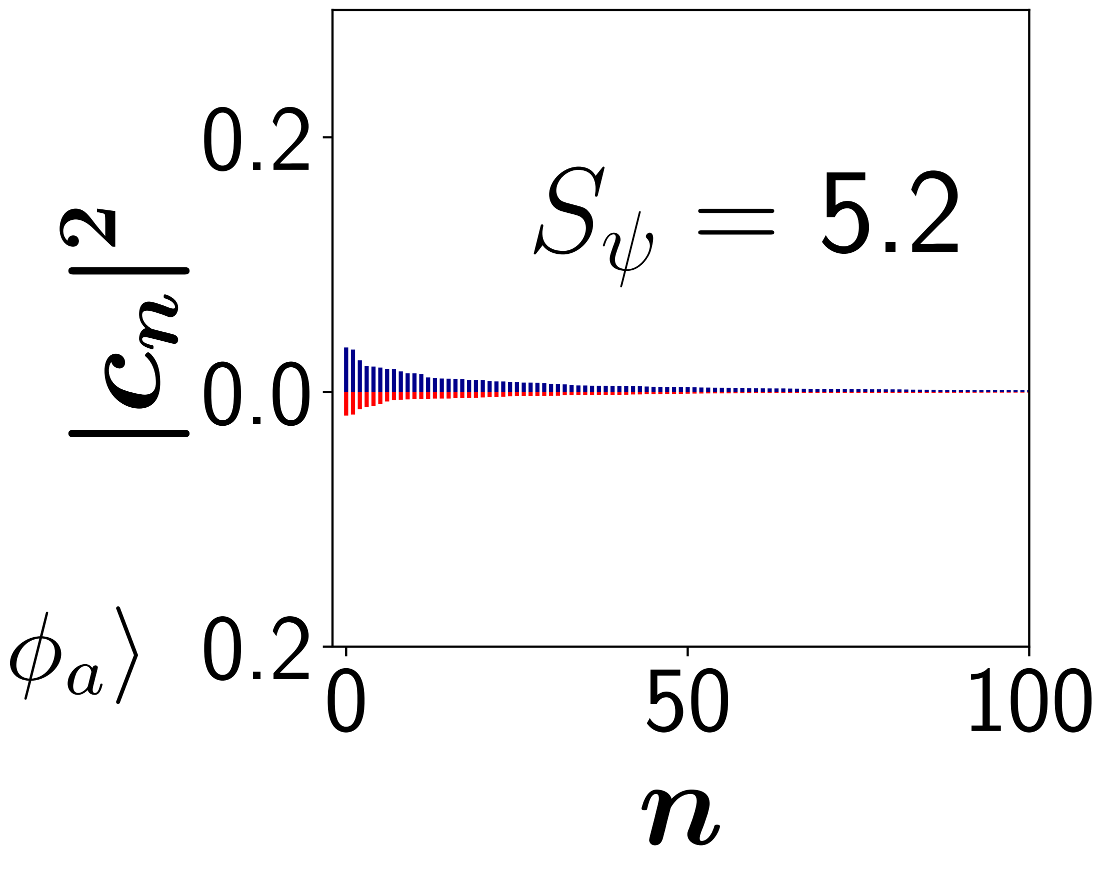
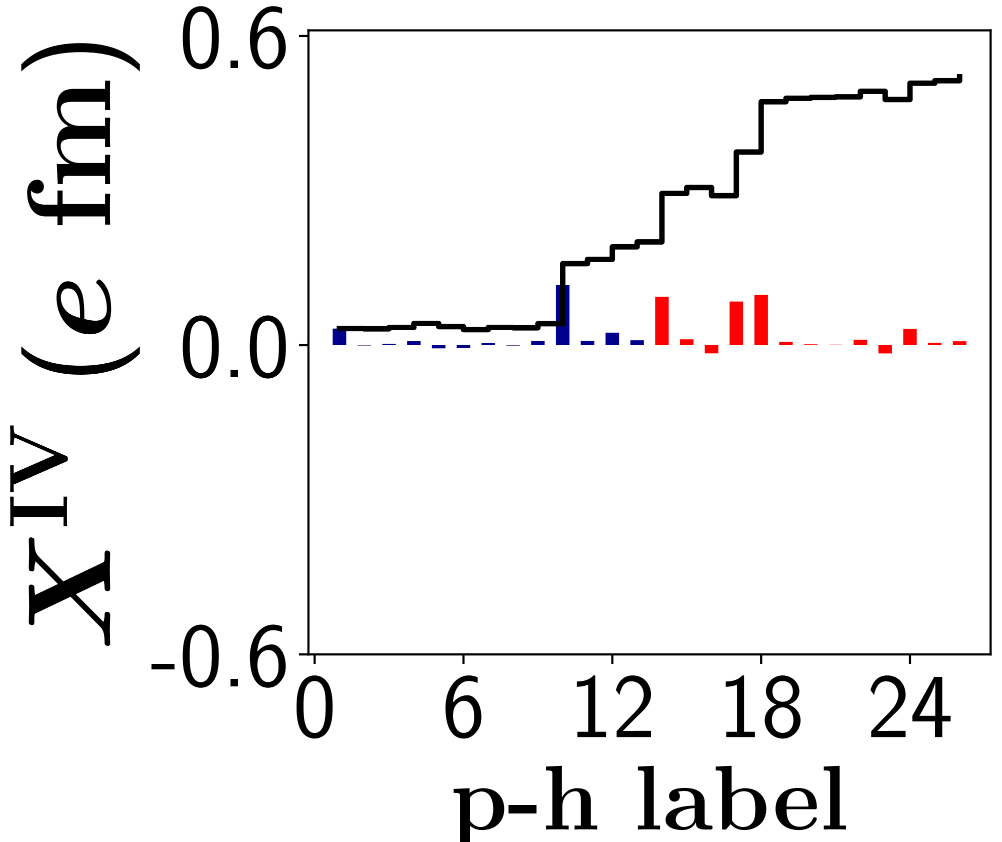


Coherence analysis

$$X_\lambda^{\nu 0} \equiv \frac{\langle \nu | \, |O_\lambda| \, |0\rangle}{\sqrt{2J_0+1}} = \sum_{k_\alpha k_\beta} \sum_{\tau = p,n} X^{\nu 0}_{k_\alpha k_\beta \lambda, \tau}$$

$$k_\alpha = (n_\alpha l_\alpha j_\alpha)$$



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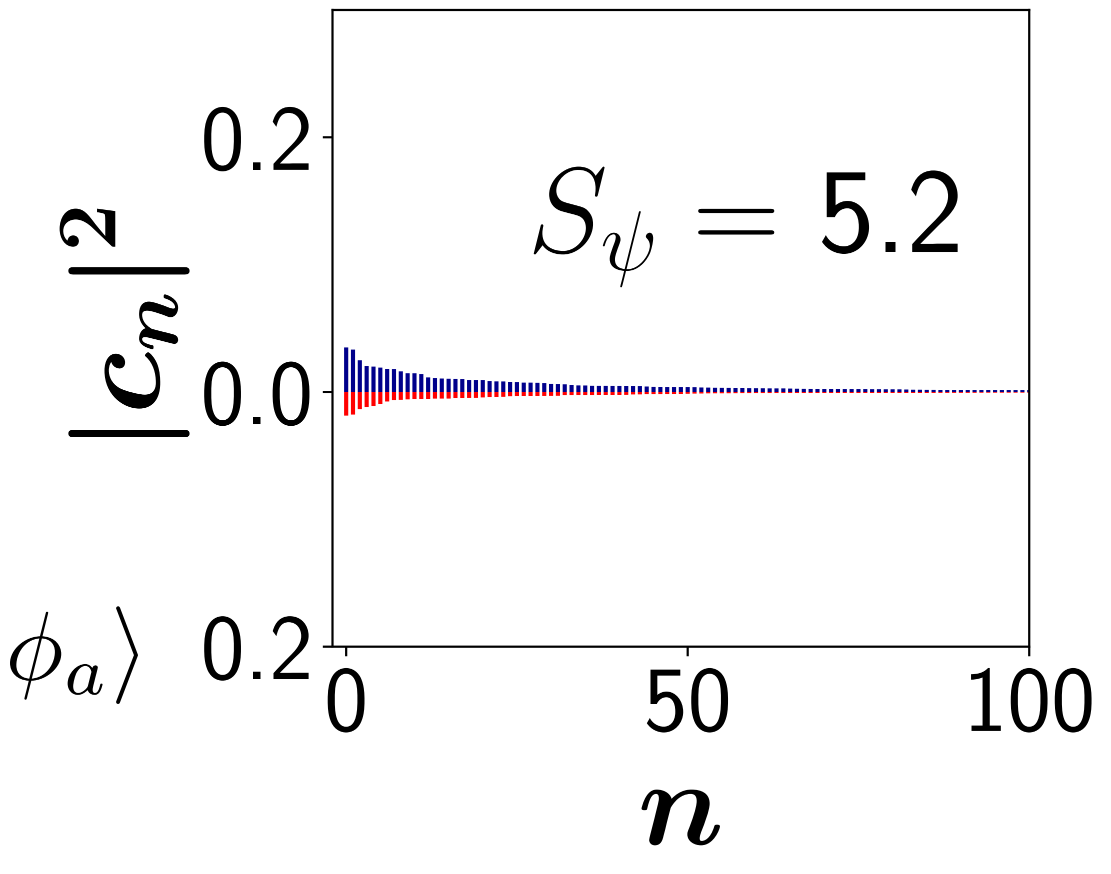

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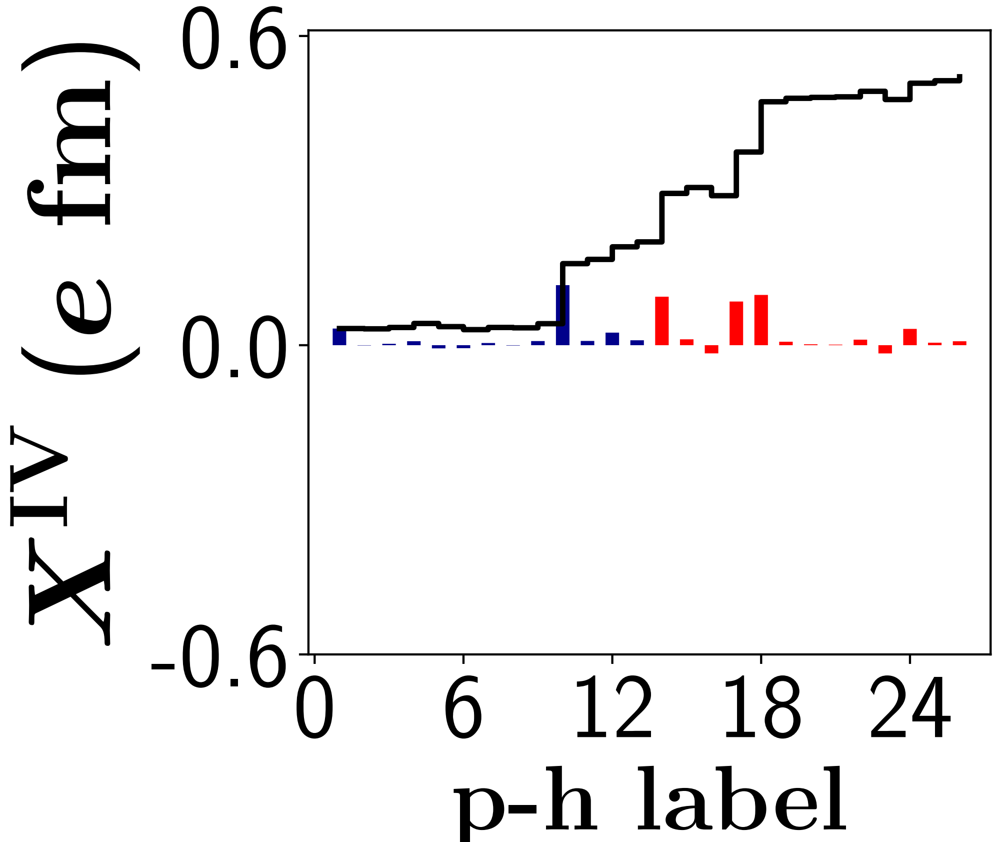

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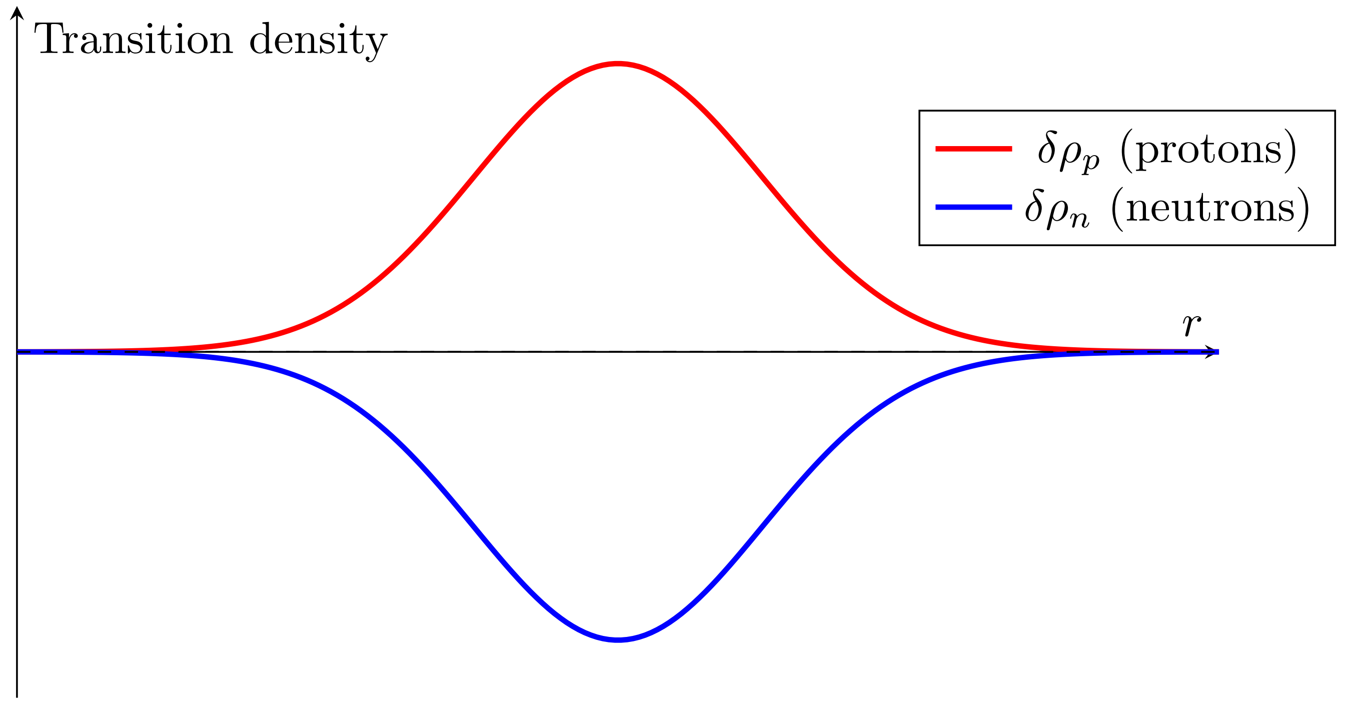
Transition density

$$\delta\rho_{\nu 0}(\mathbf{r}) = \langle \nu | \, \hat{\rho}(\mathbf{r}) \, | 0 \rangle$$



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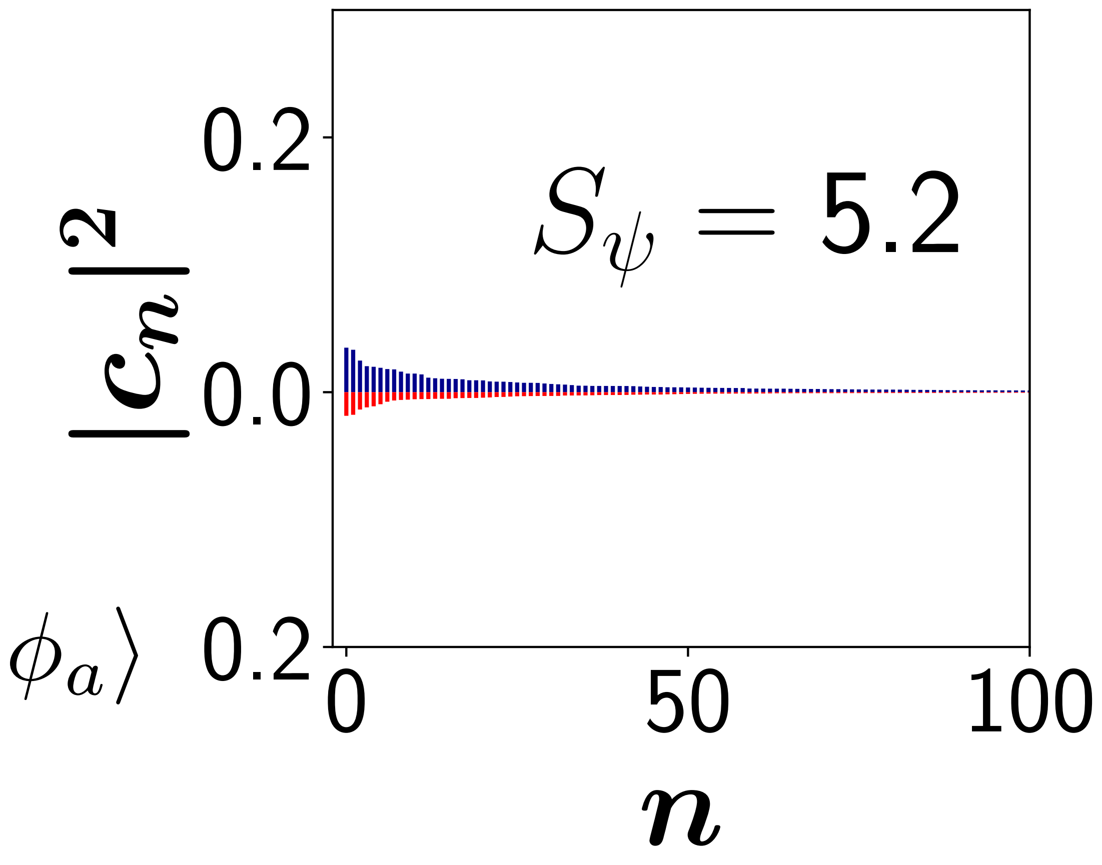

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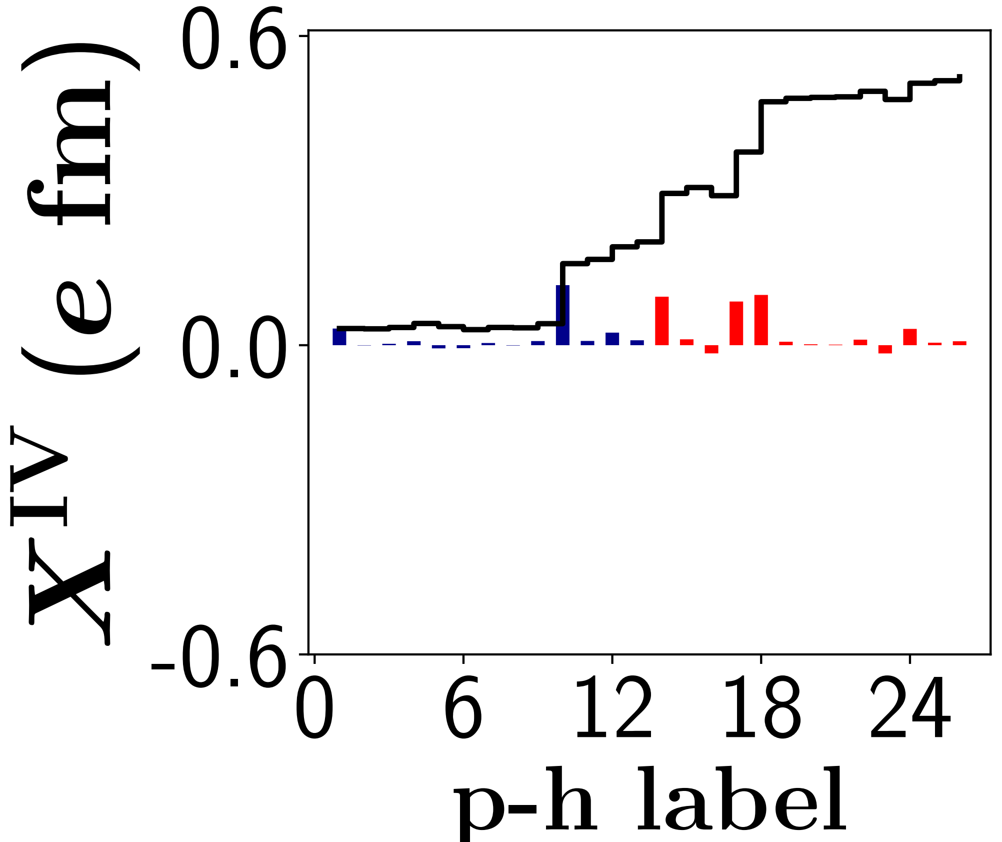
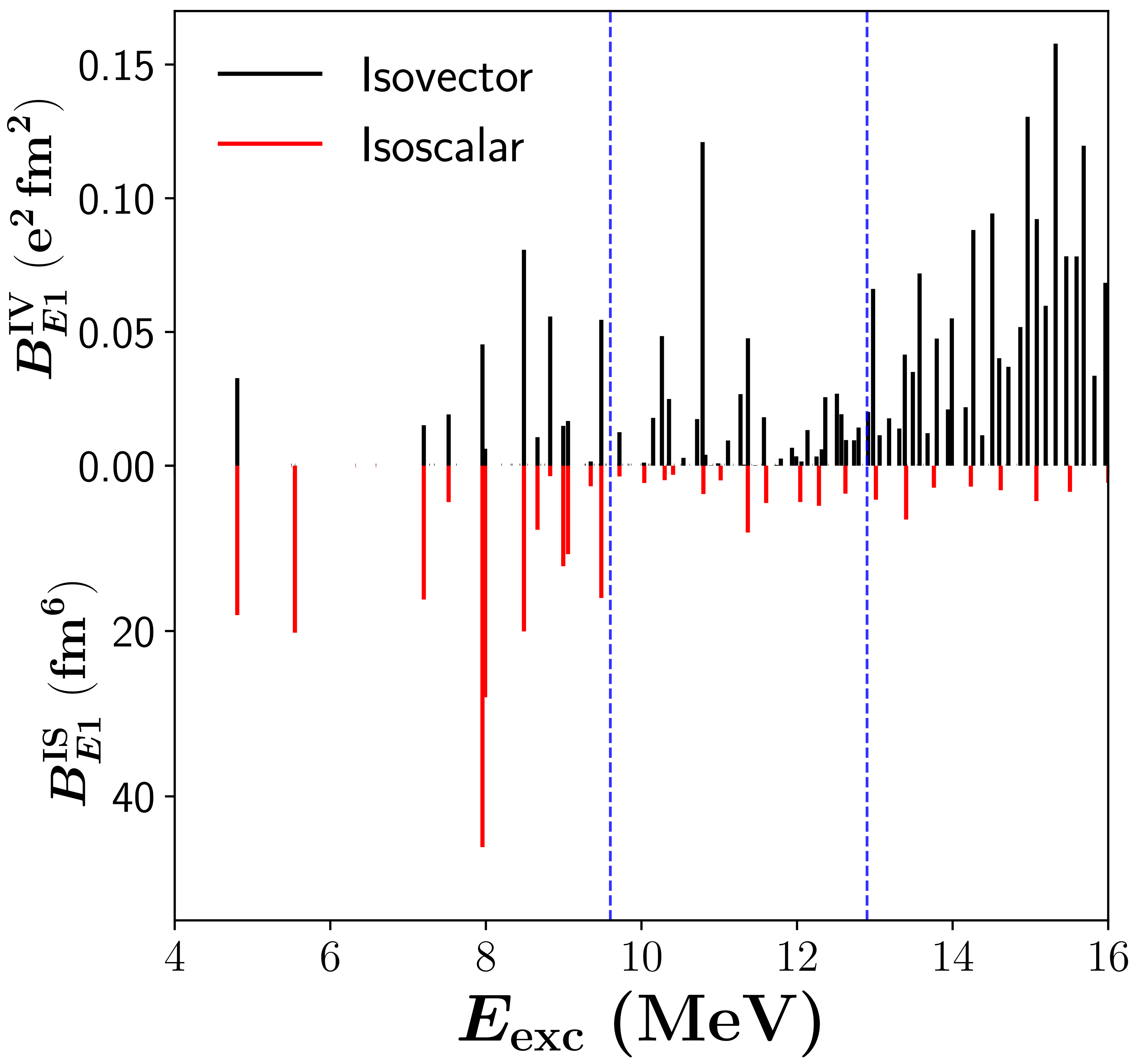


Analysis of the dipole response in ^{26}Ne



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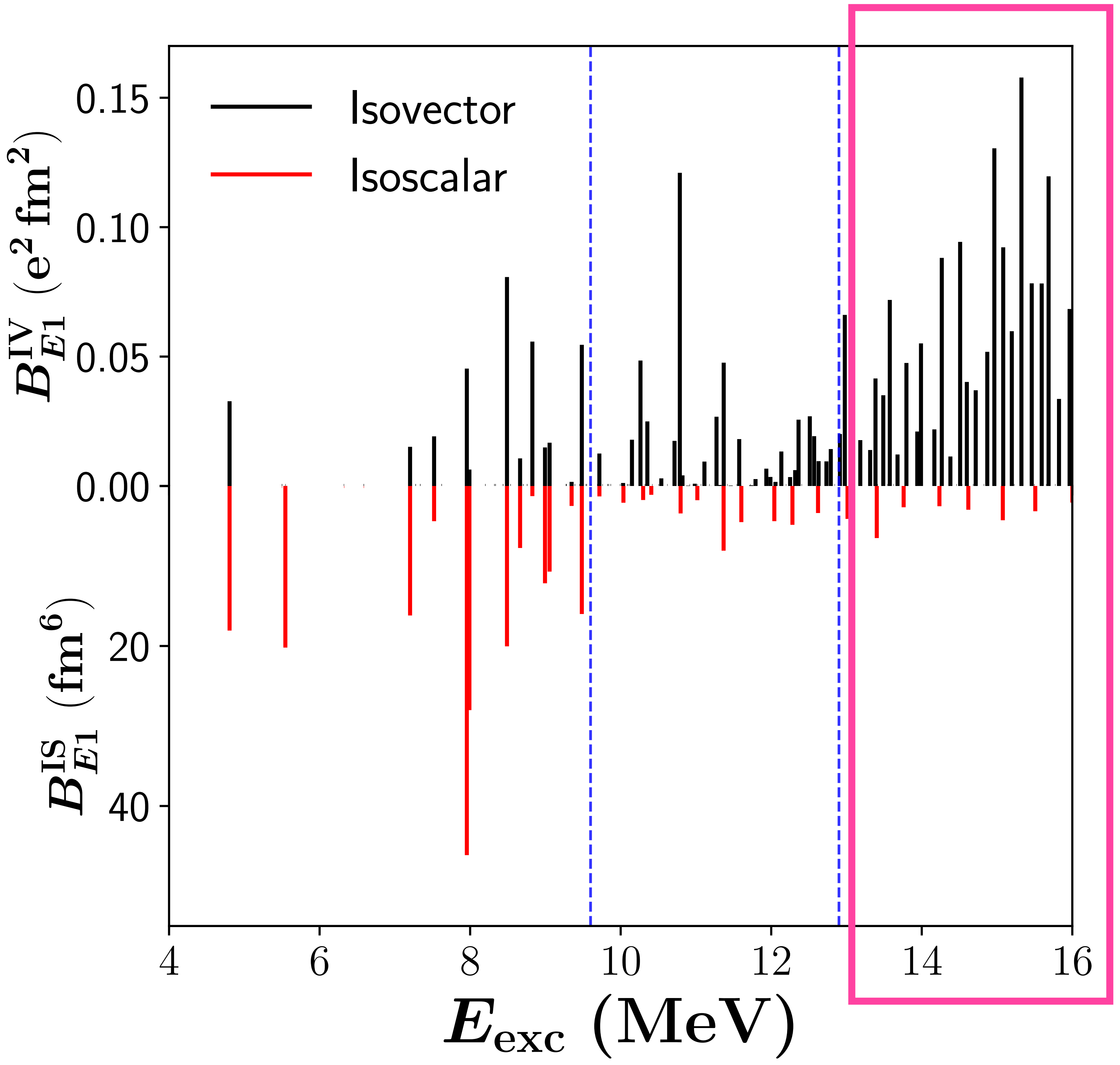
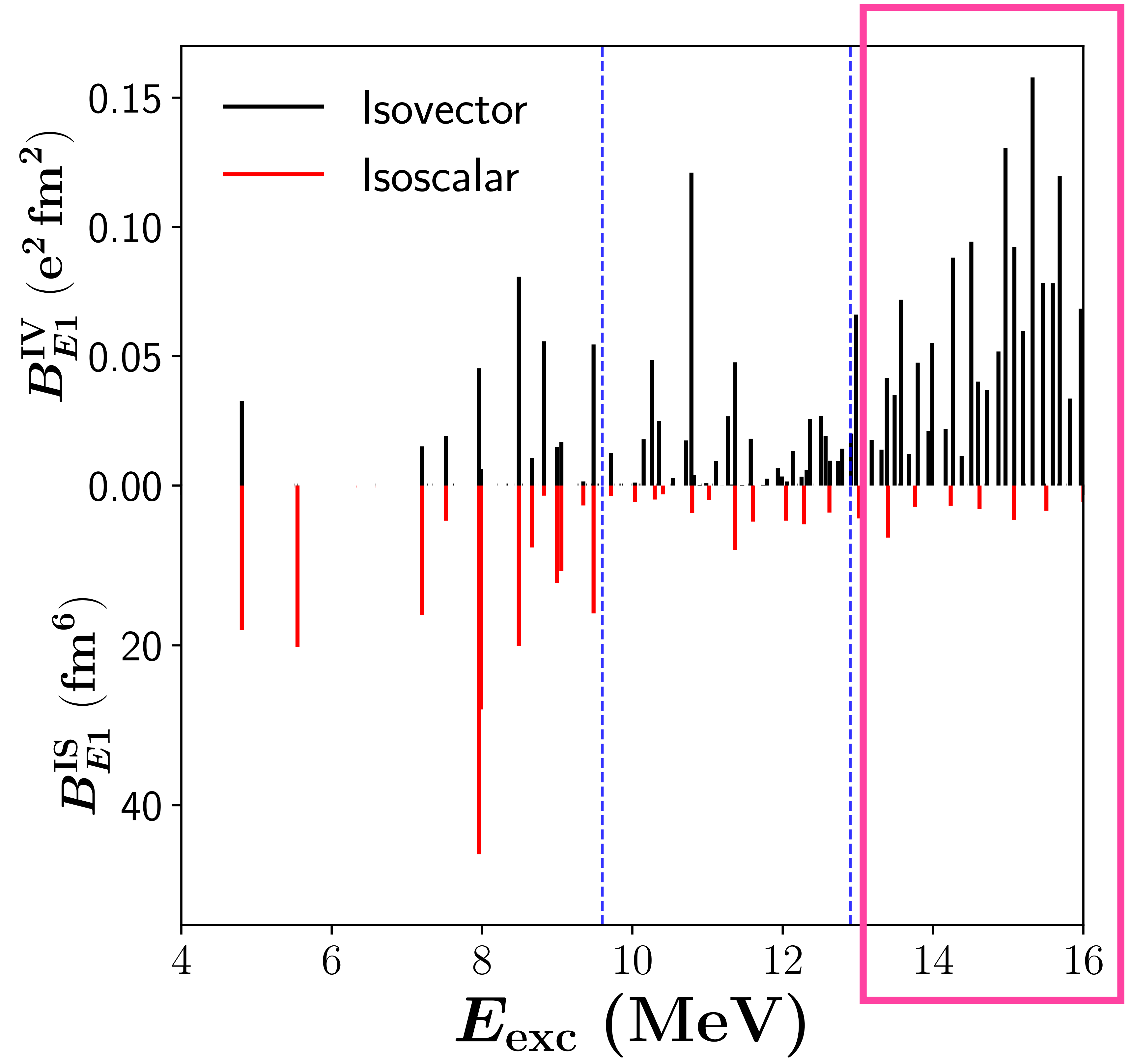


Above 13 MeV: GDR region

Analysis of the dipole response in ^{26}Ne

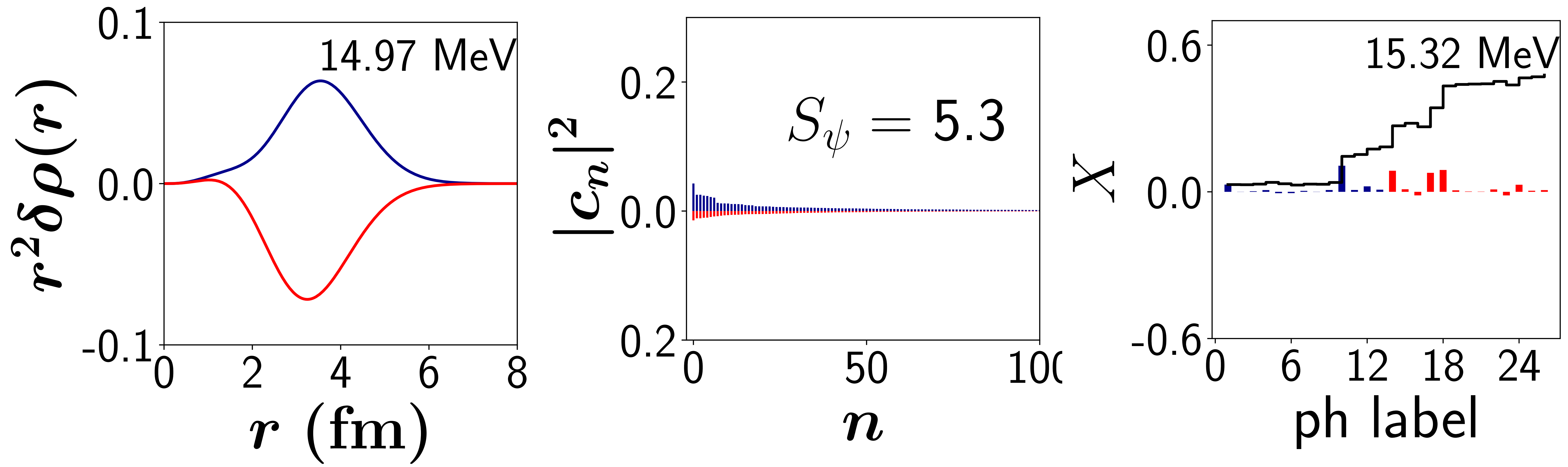


Above 13 MeV: GDR region

⟶ Paradigmatic example of a purely collective state: reference point for collectiveness.

⟶ All states above 13 MeV are similar.

⟶ Many nucleons participating coherently.



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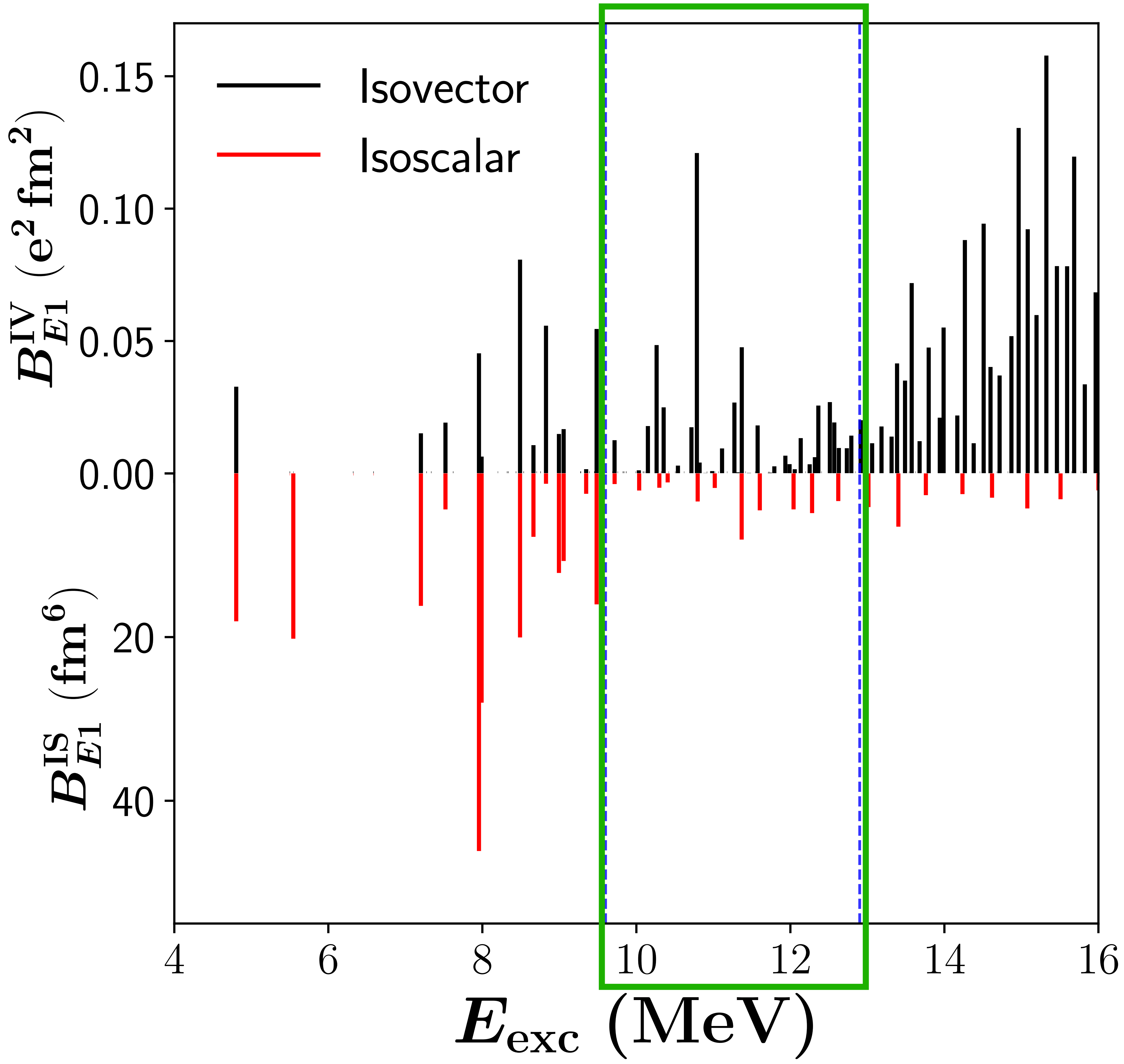
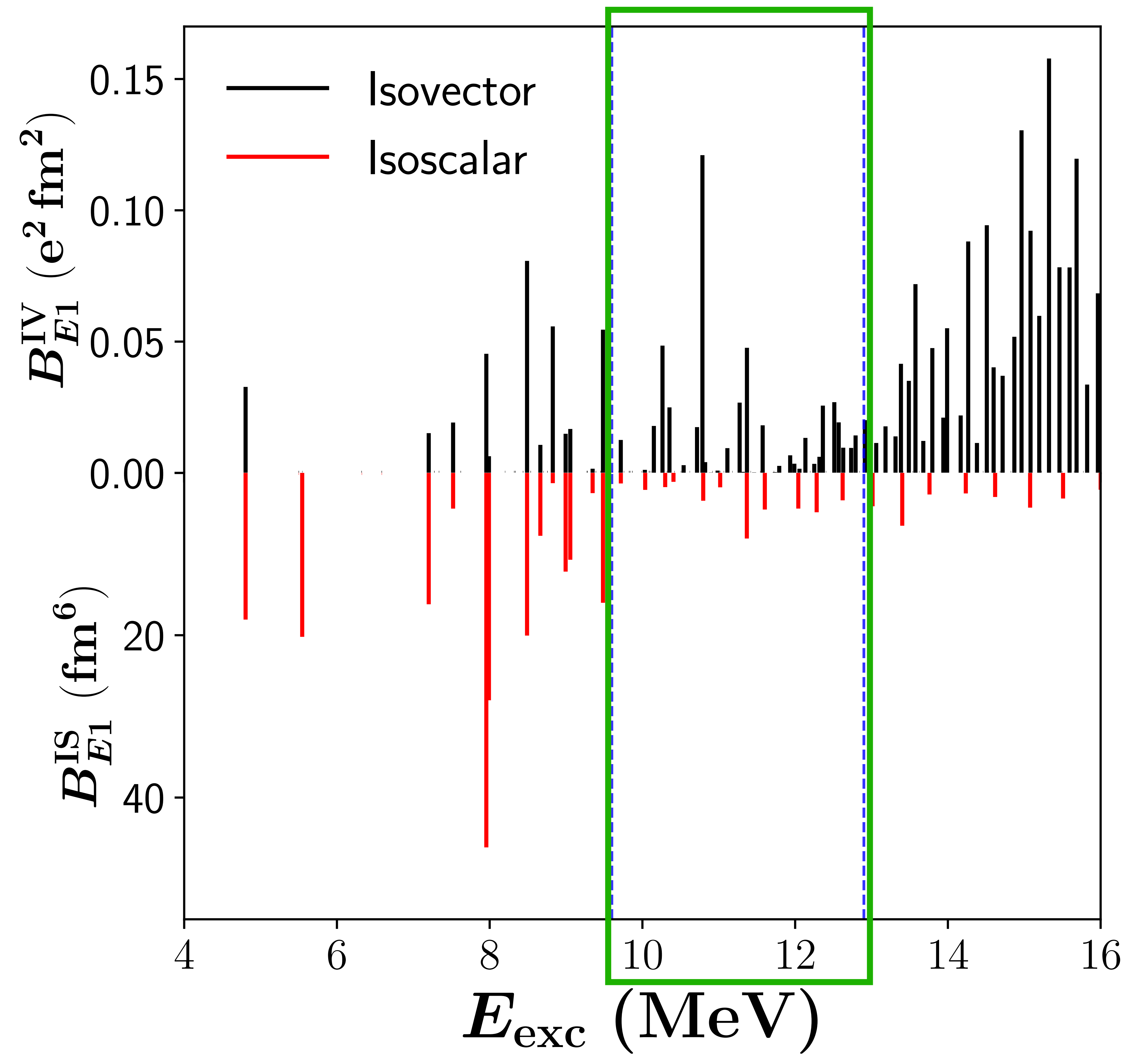


Between 9.7 and 13 MeV: GDR tail

Analysis of the dipole response in ^{26}Ne



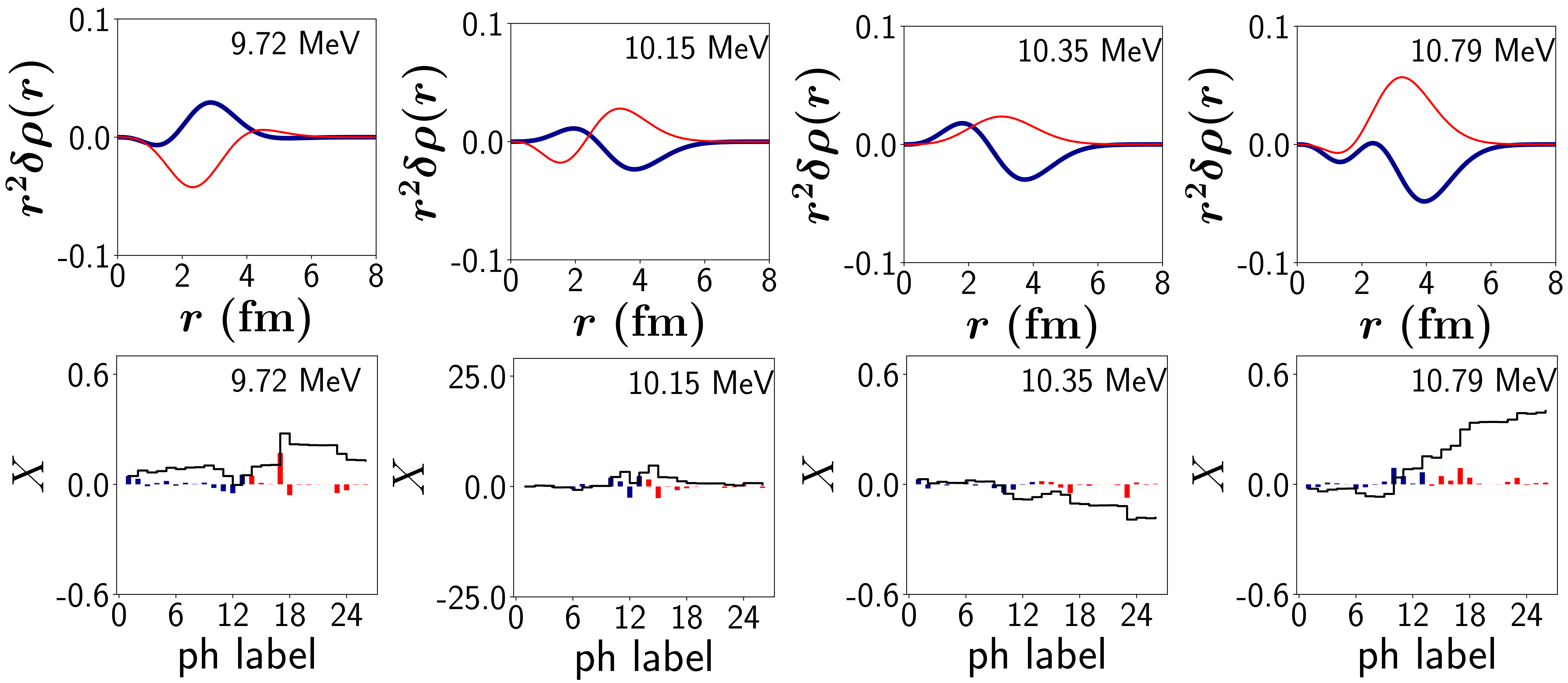
Between 9.7 and 13 MeV: GDR tail

$\longrightarrow$ Mostly IV

$\longrightarrow$ Not typical GDR

$\longrightarrow$ Weak coherence

$\longrightarrow$ Many different modes



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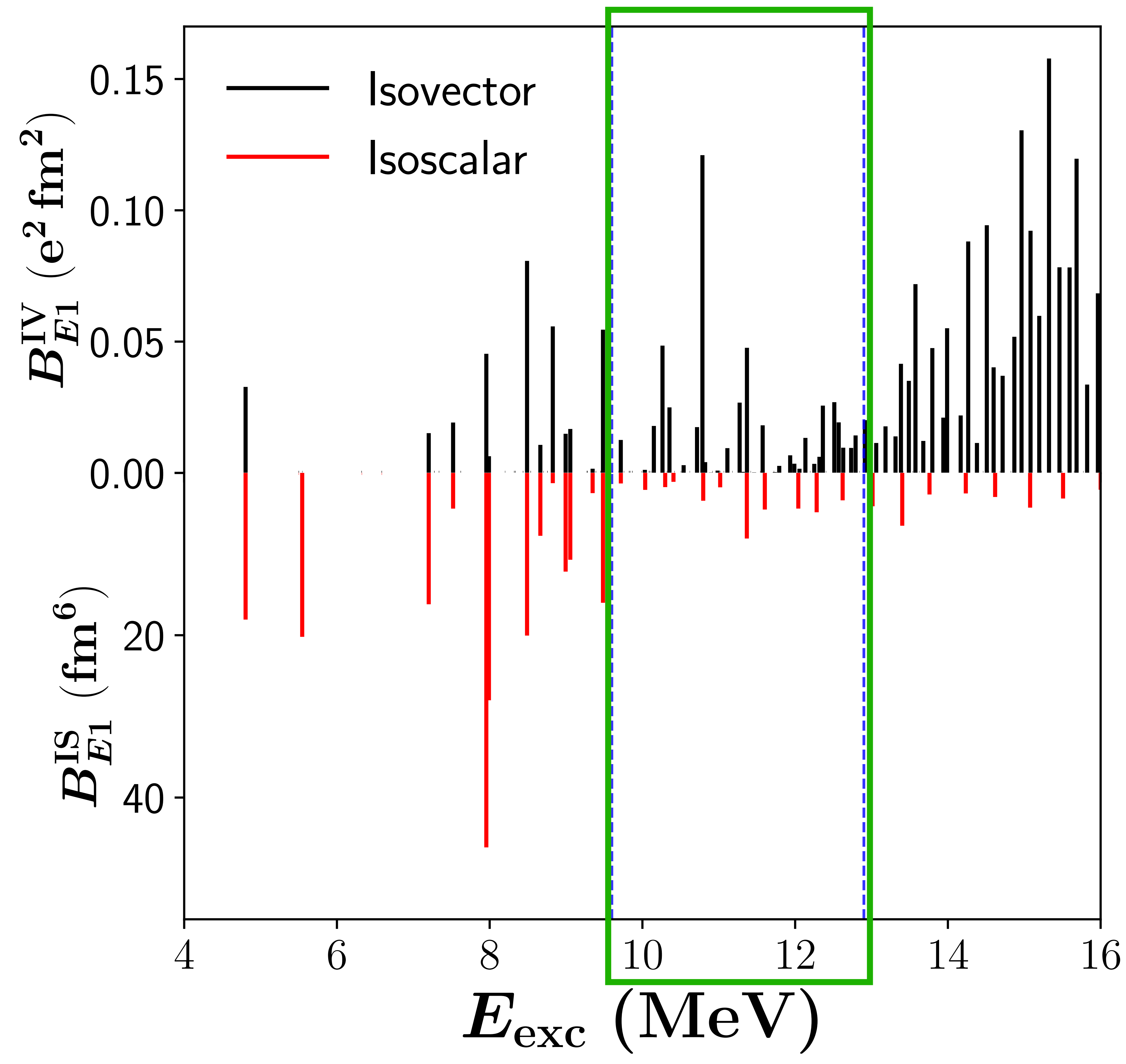


⟶ Transitional region: from PDR to GDR ≡ Tail of the GDR

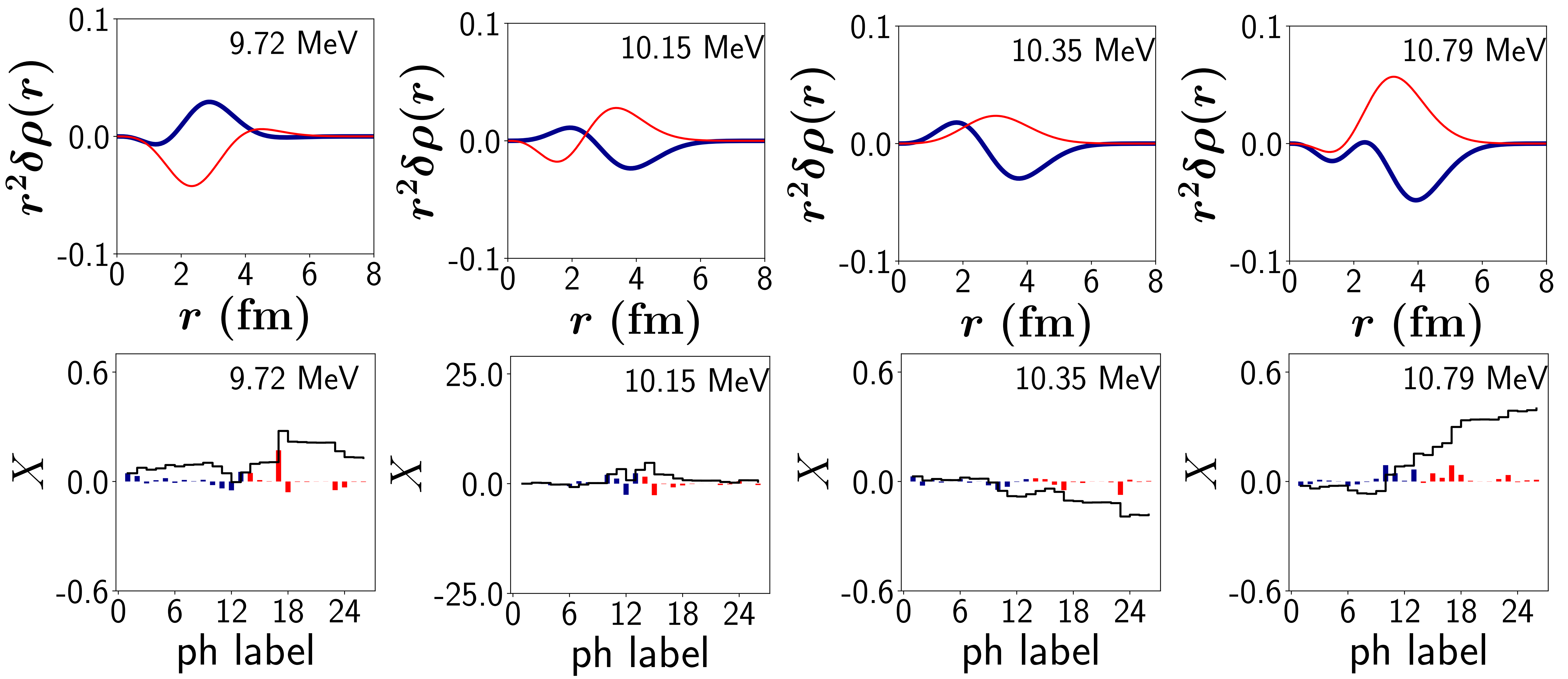
Between 9.7 and 13 MeV: GDR tail

⟶ Mostly IV

⟶ Not typical GDR

⟶ Weak coherence

⟶ Many different modes



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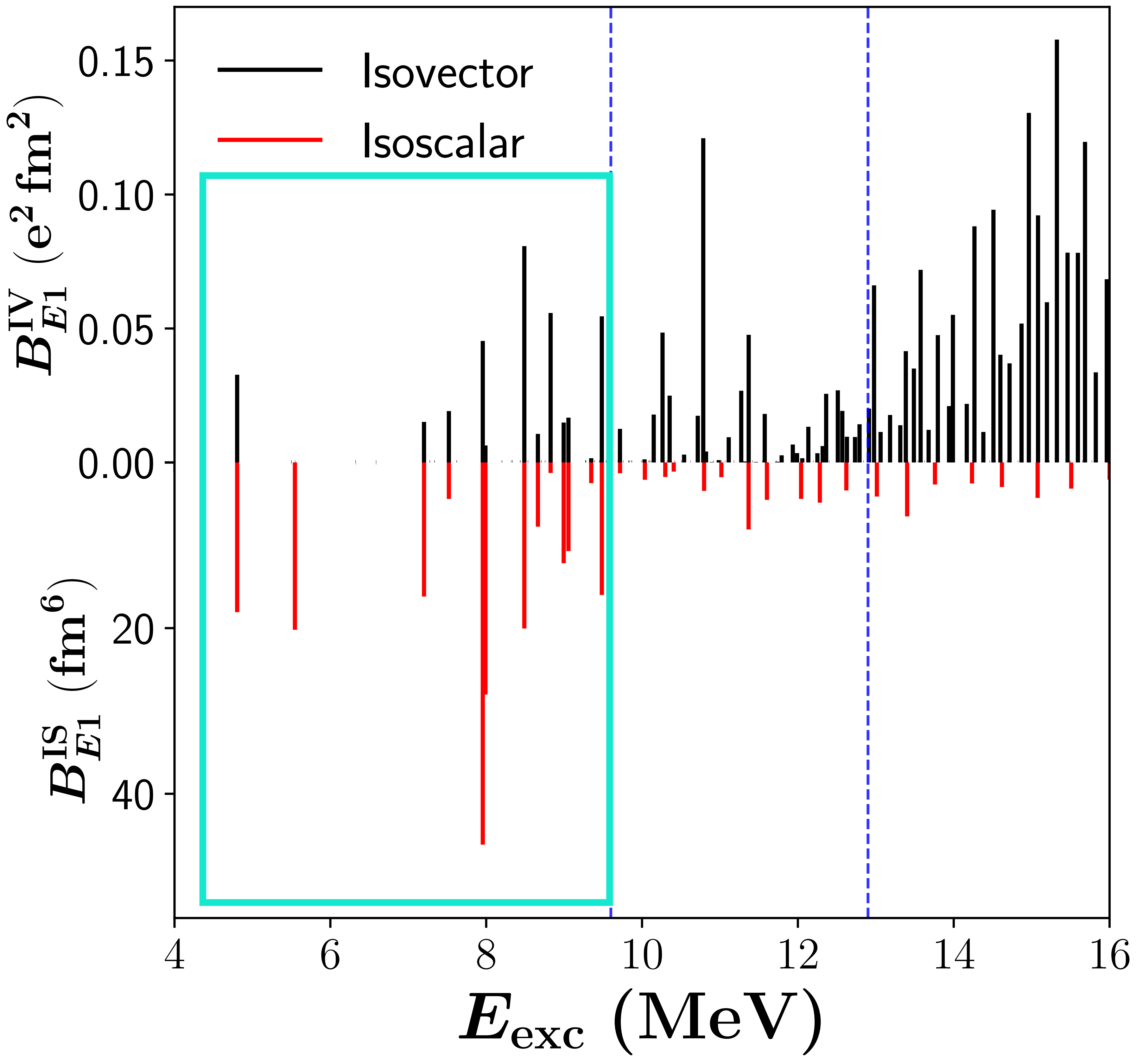

Below 9.7: PDR and more

Analysis of the dipole response in ^{26}Ne

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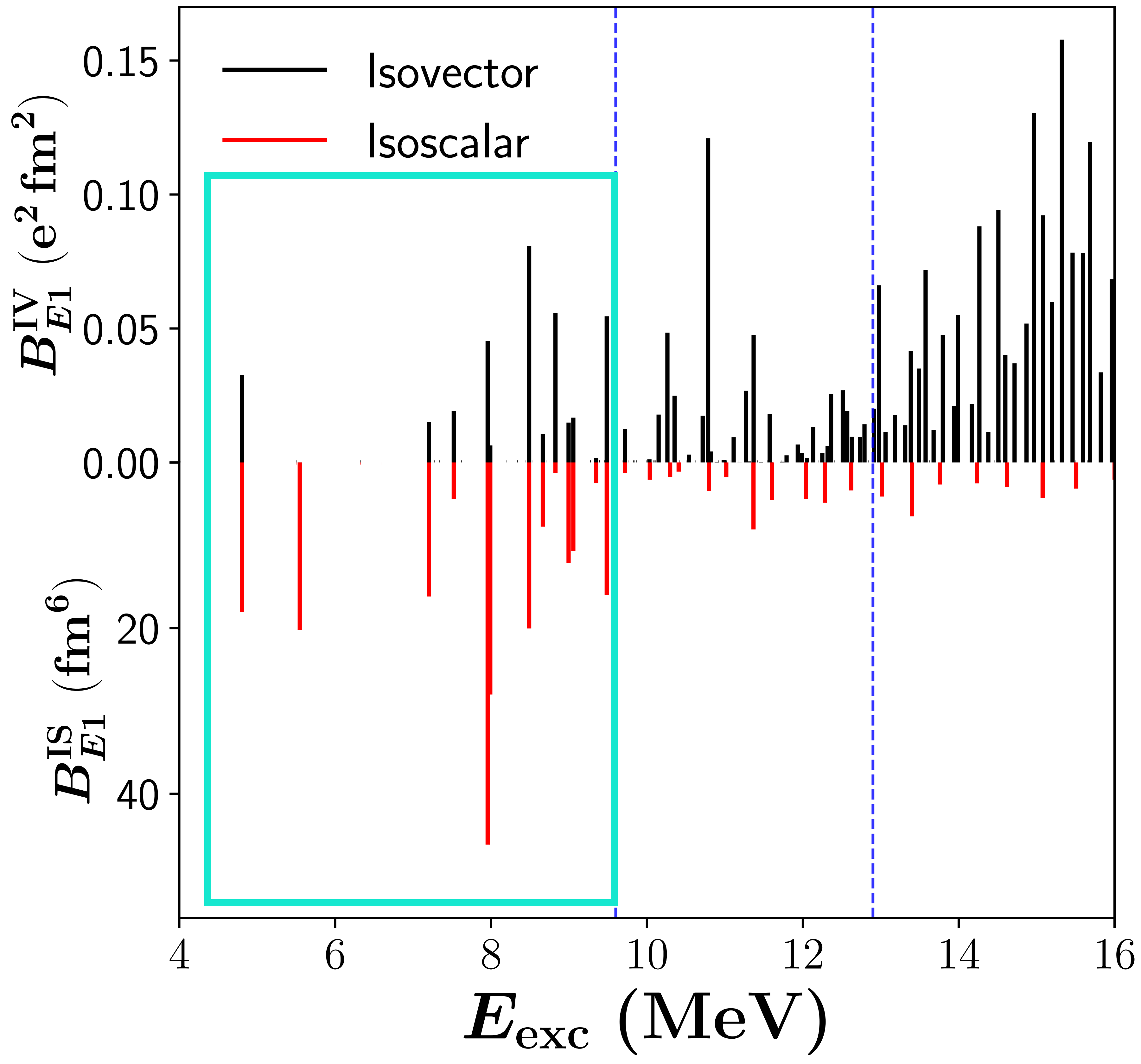

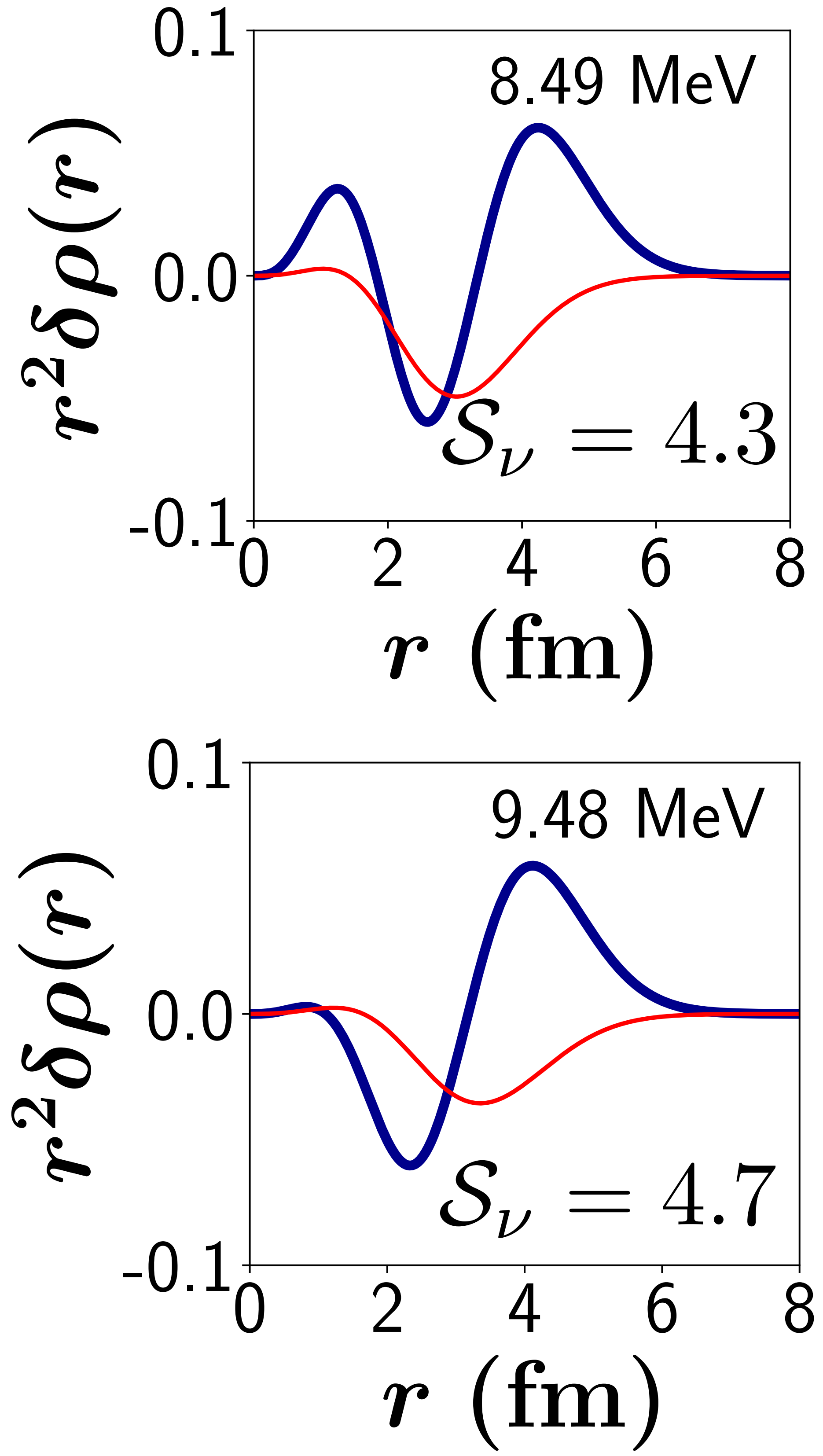

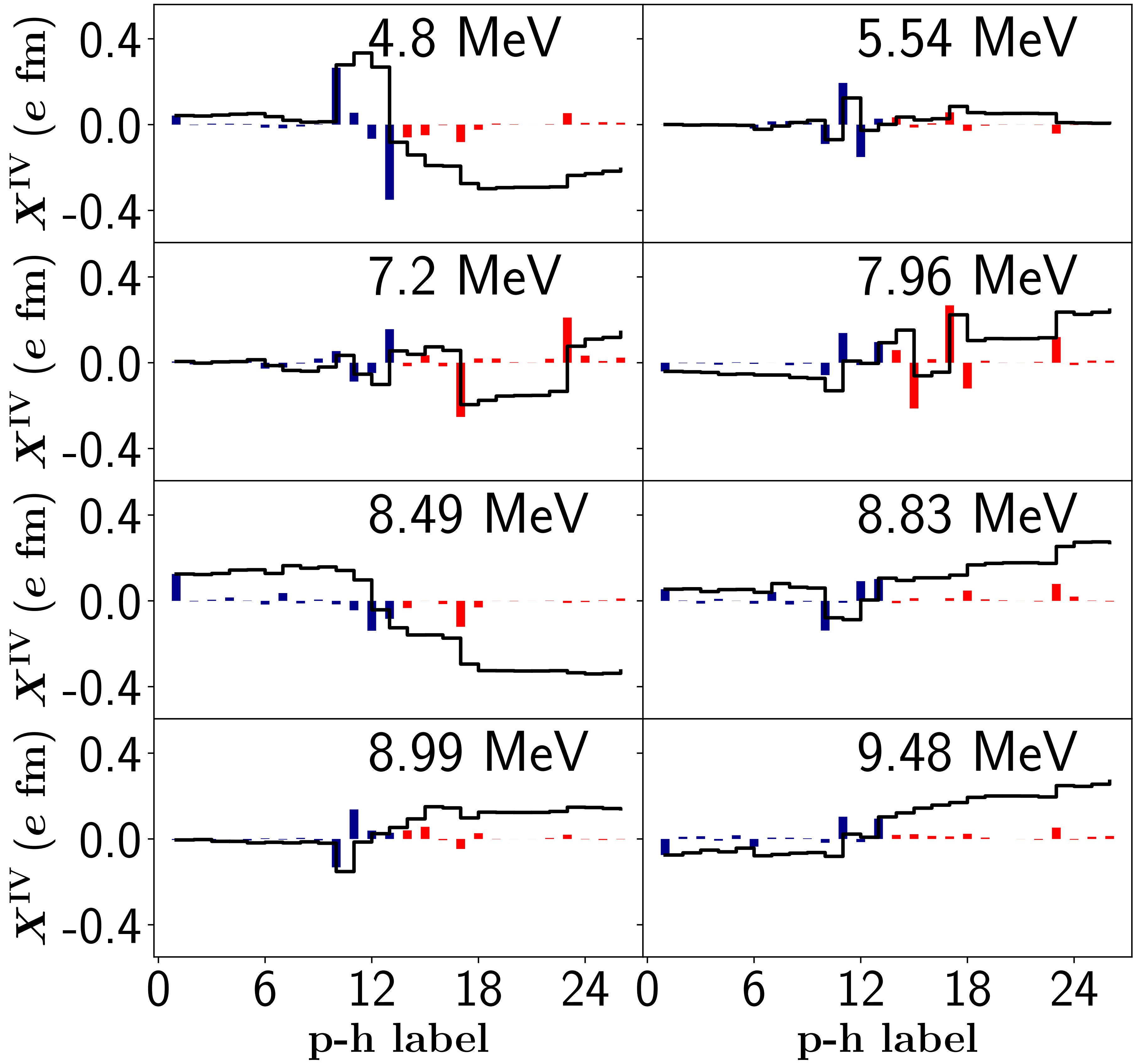

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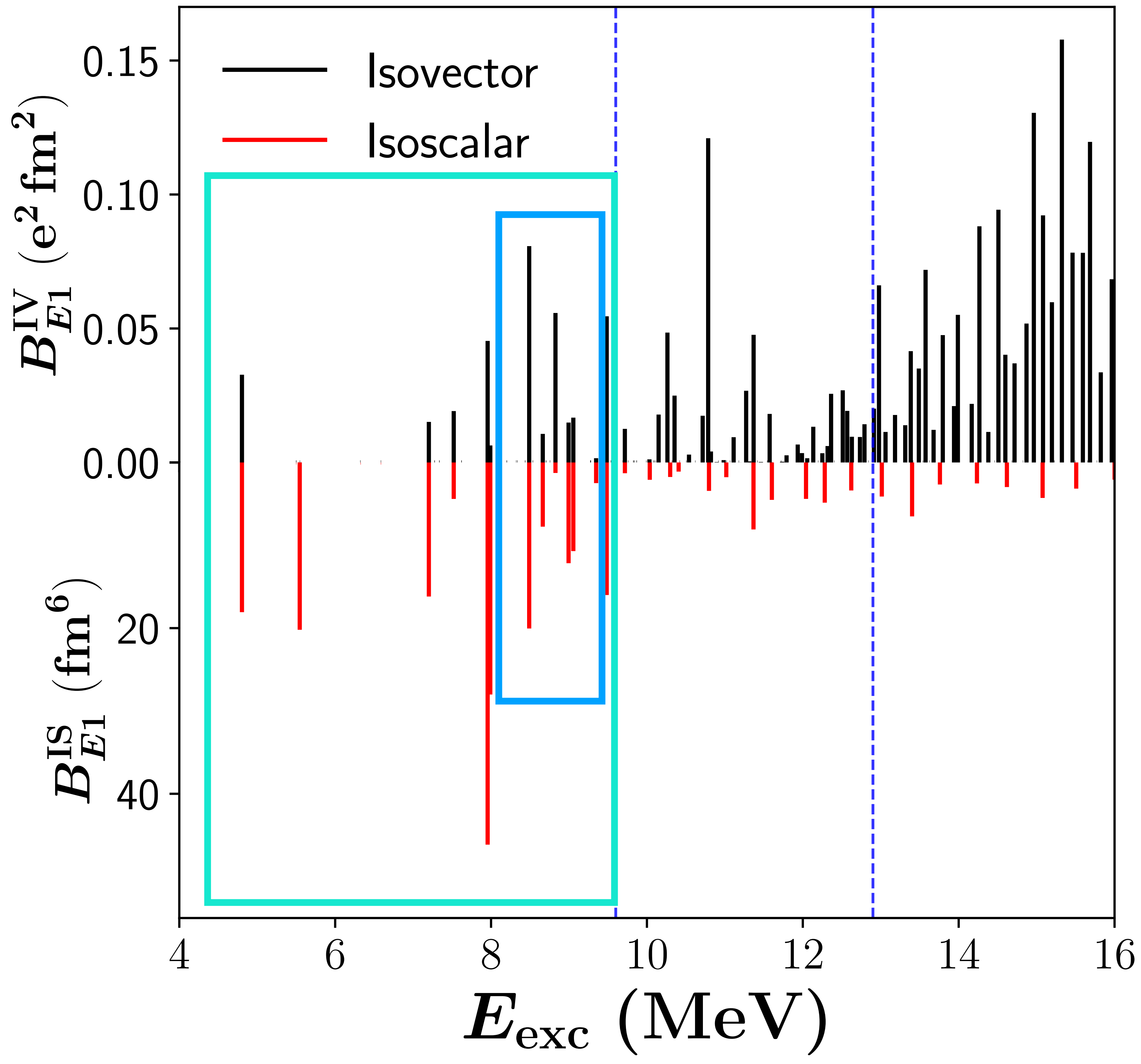

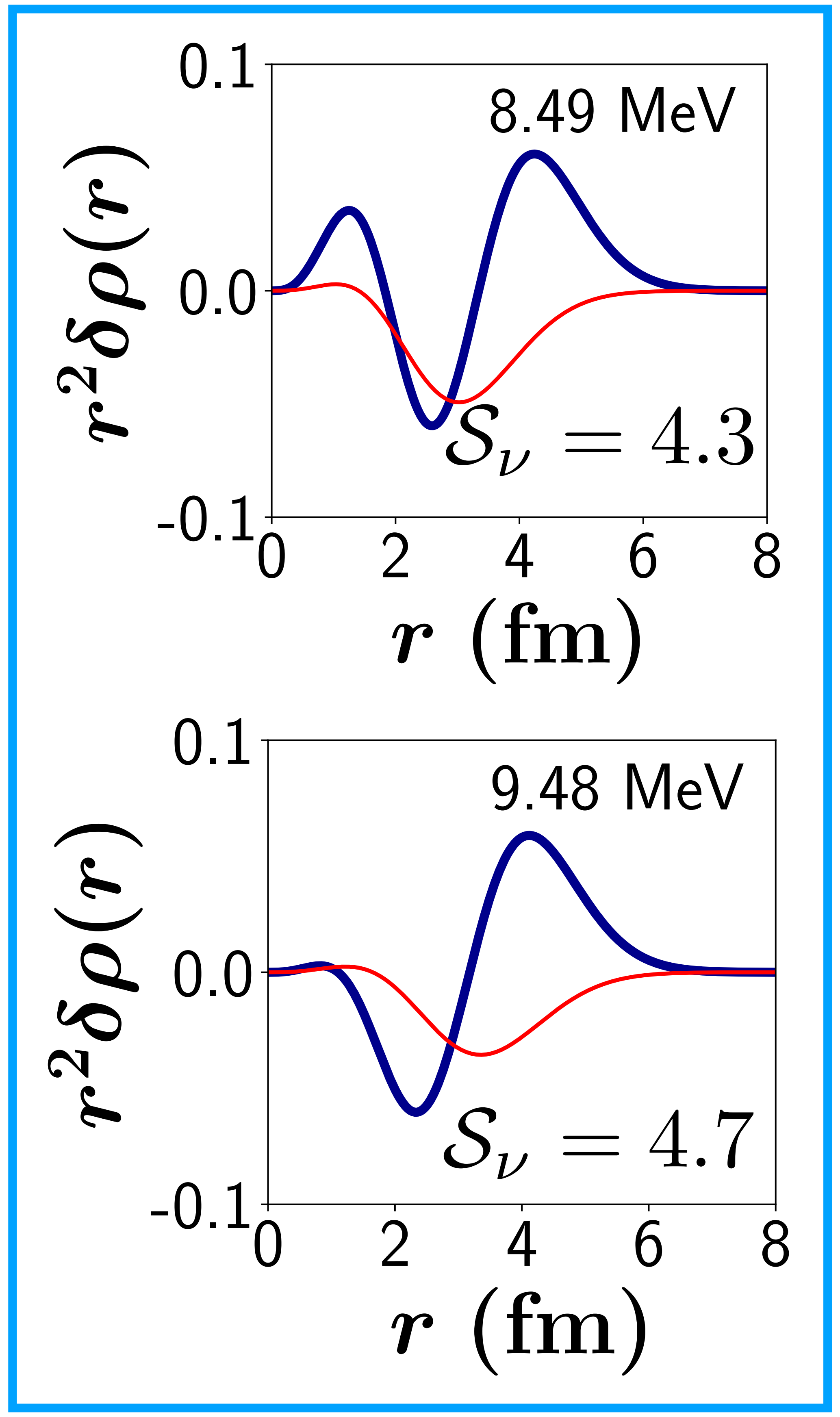

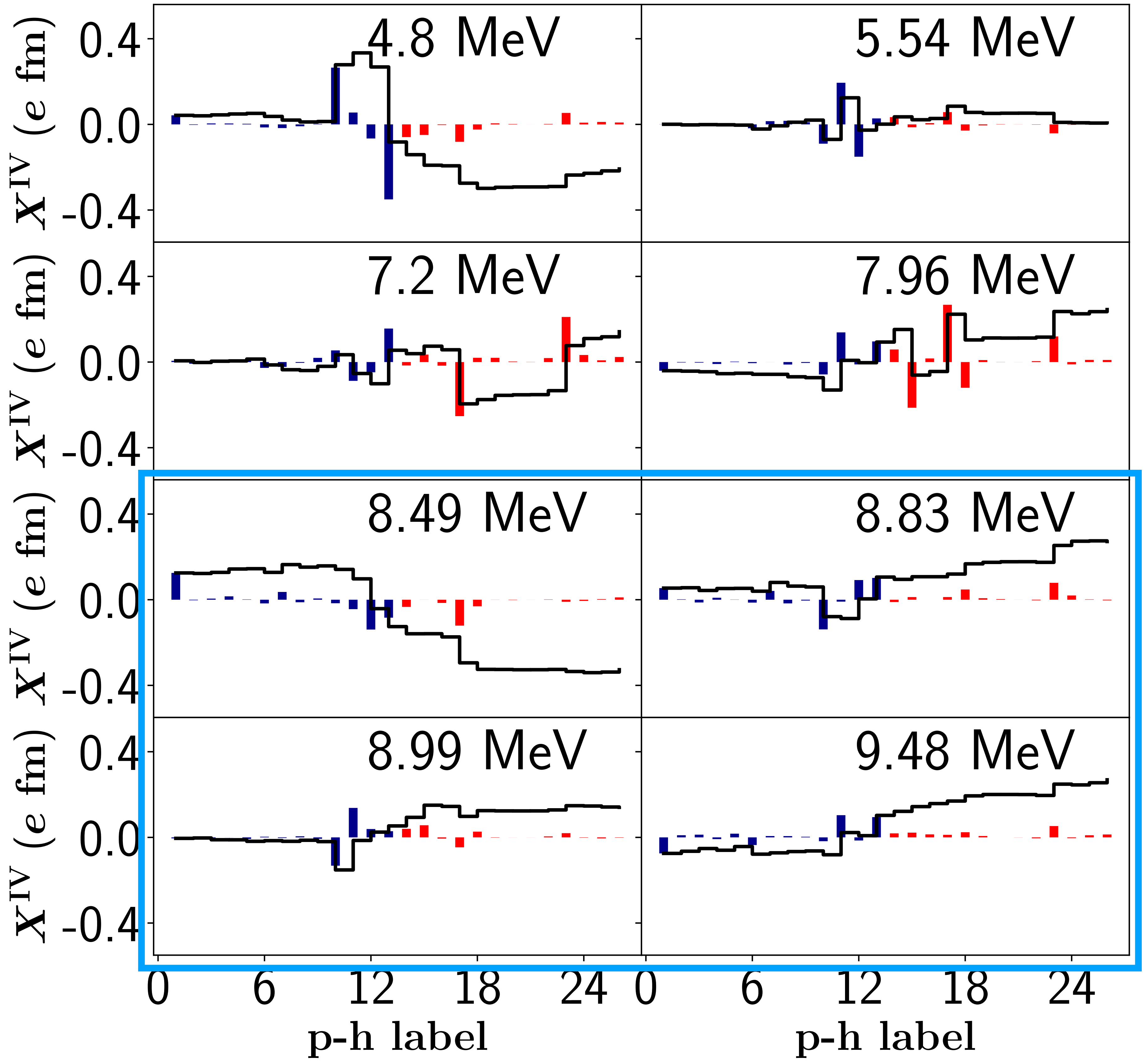

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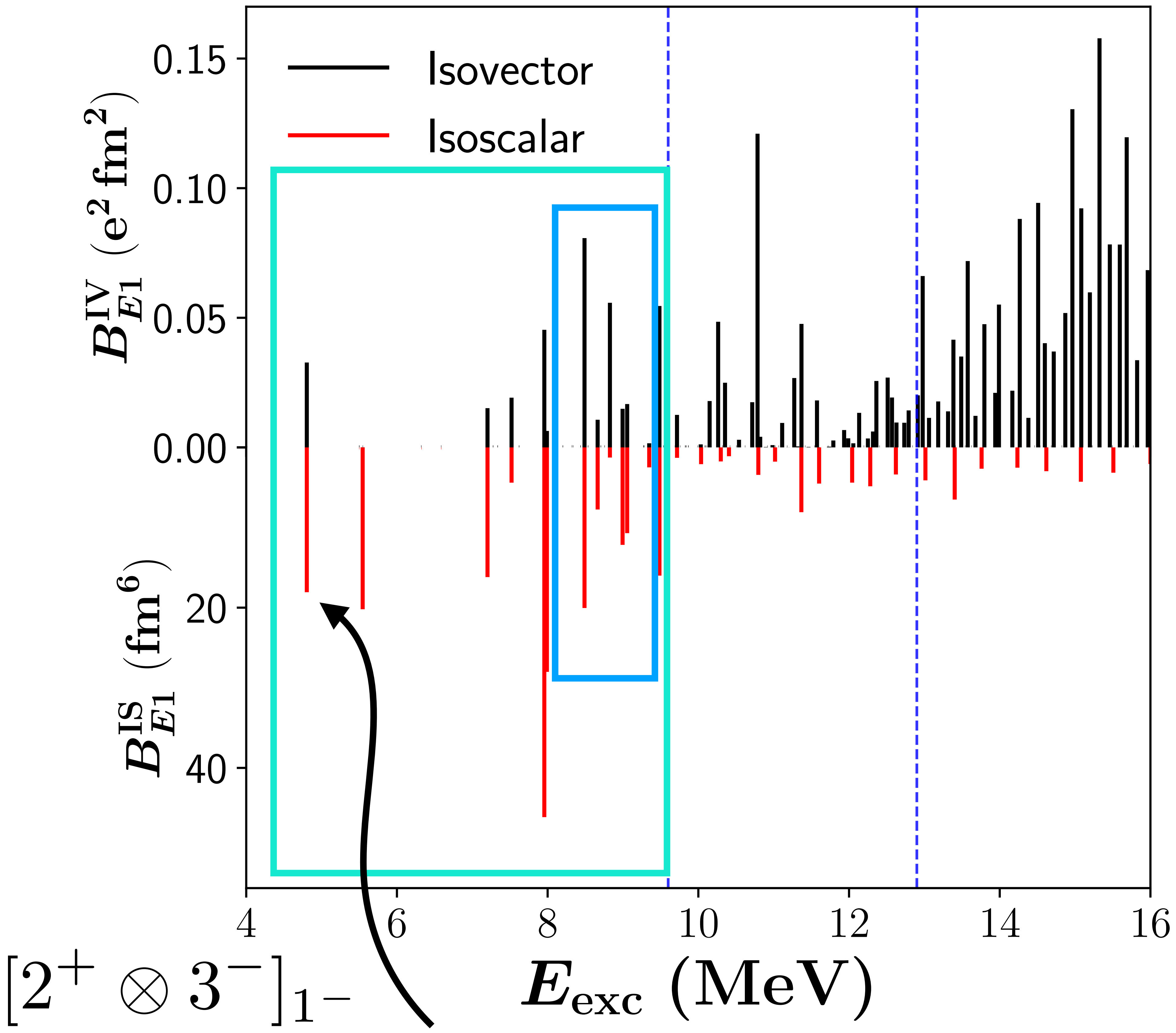

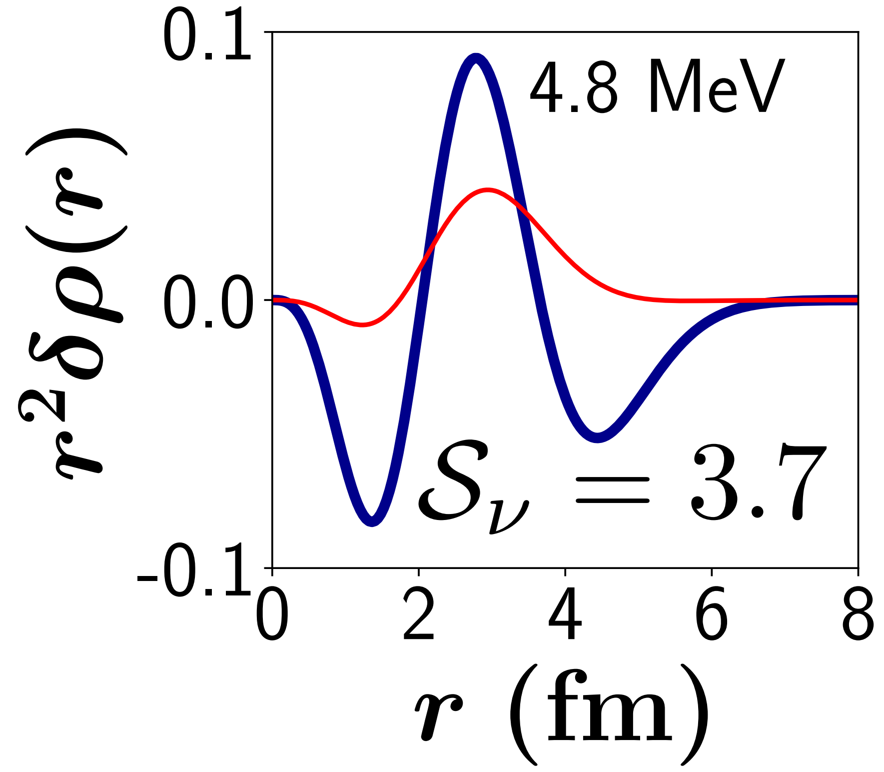

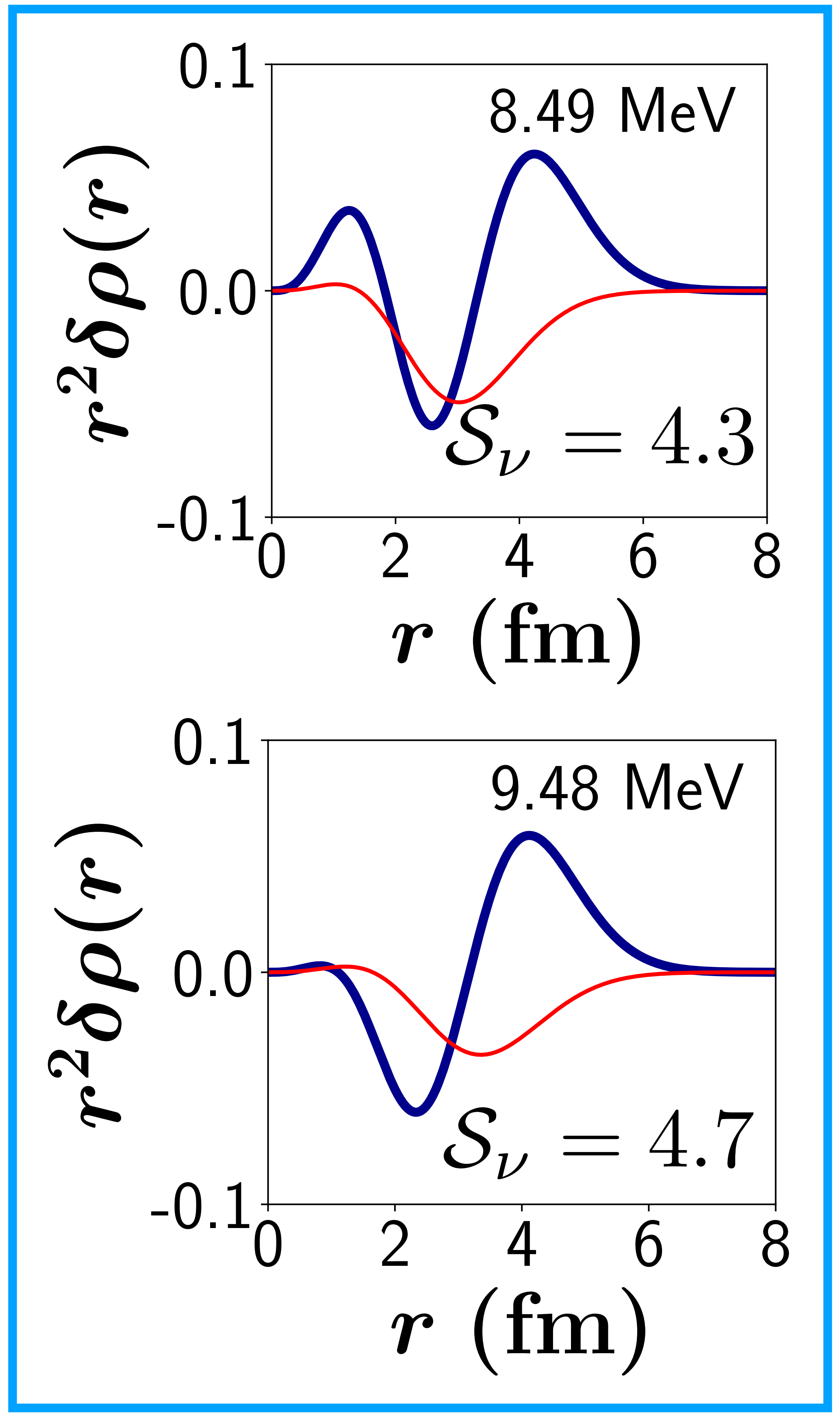

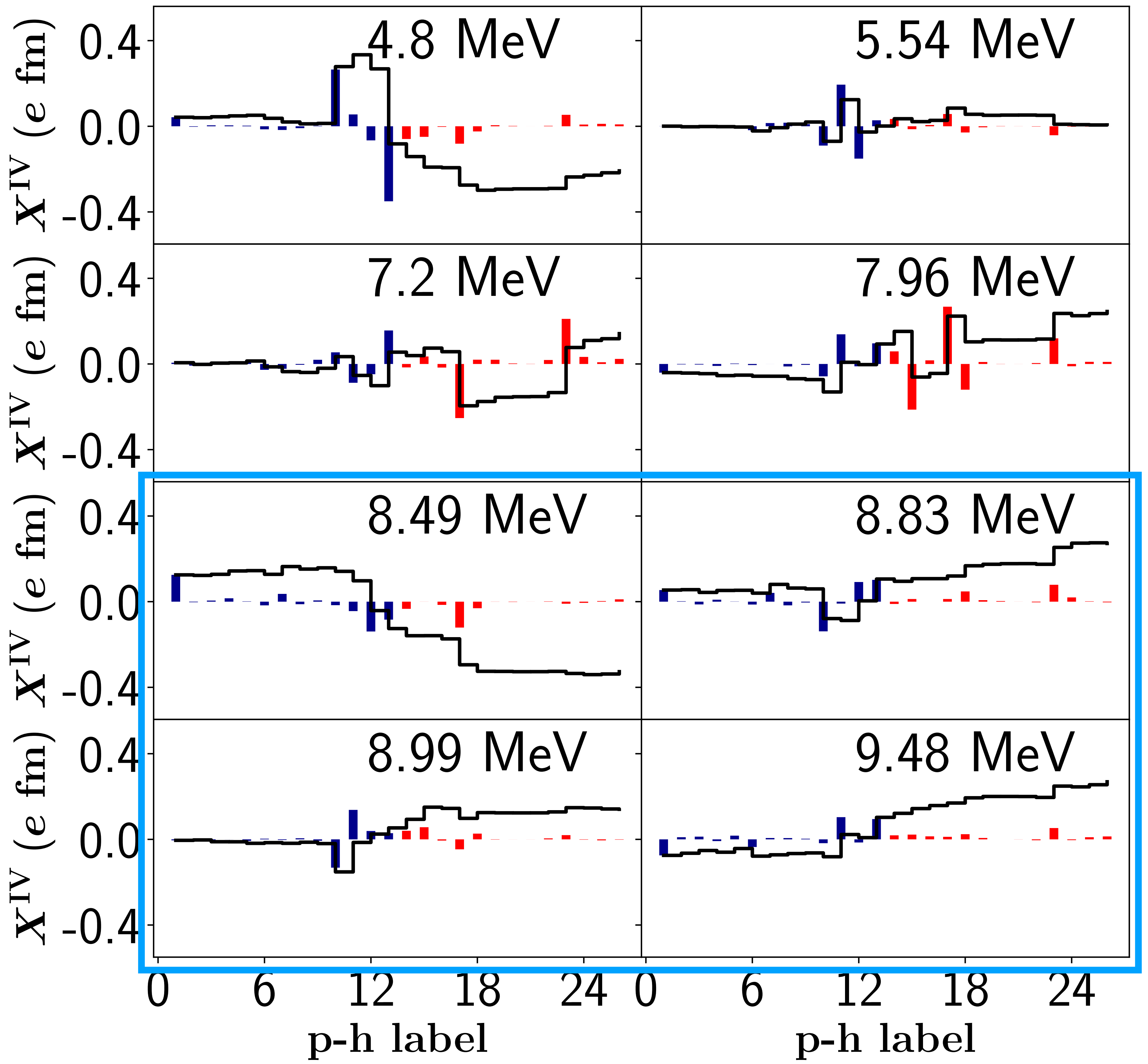

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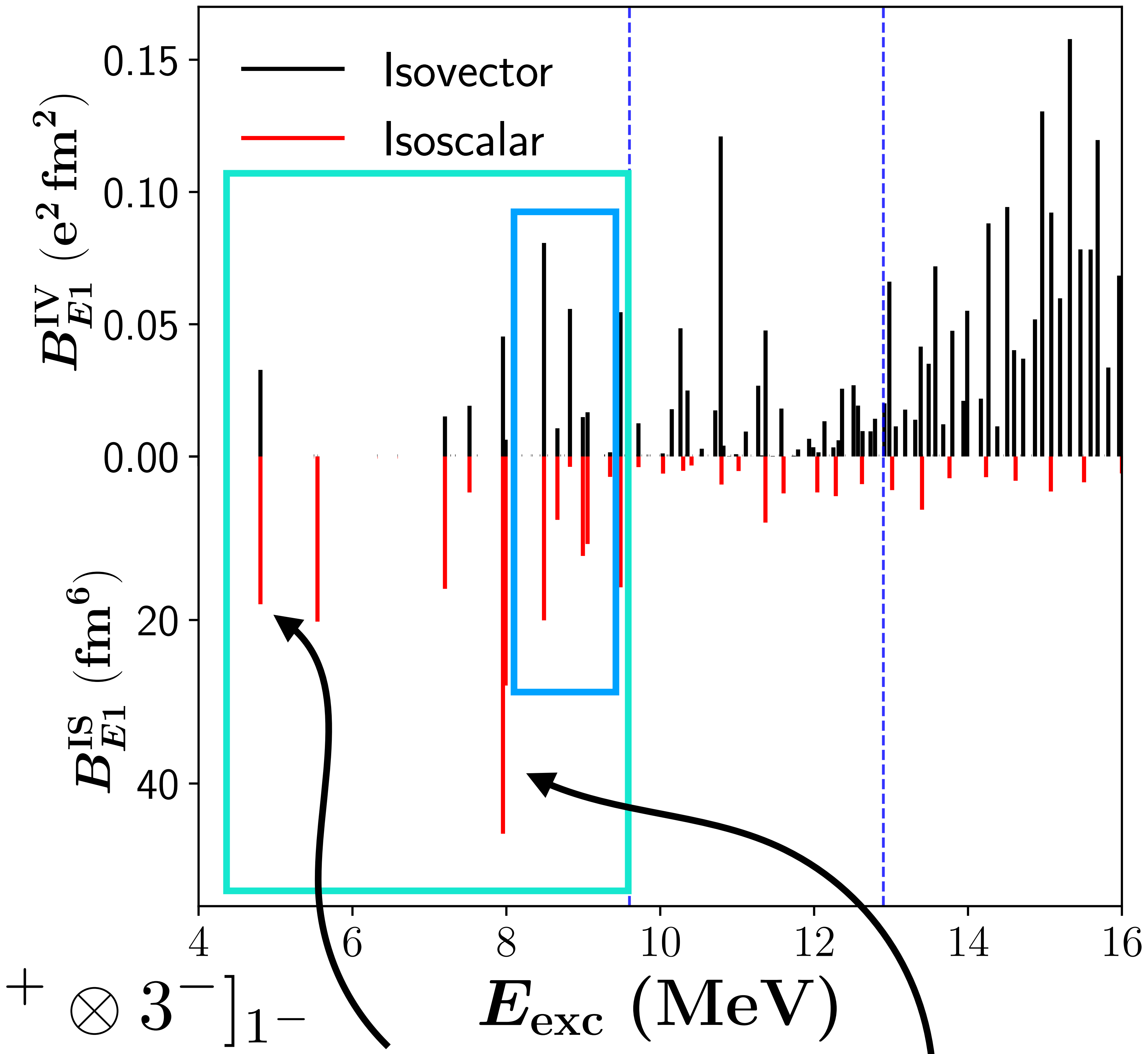
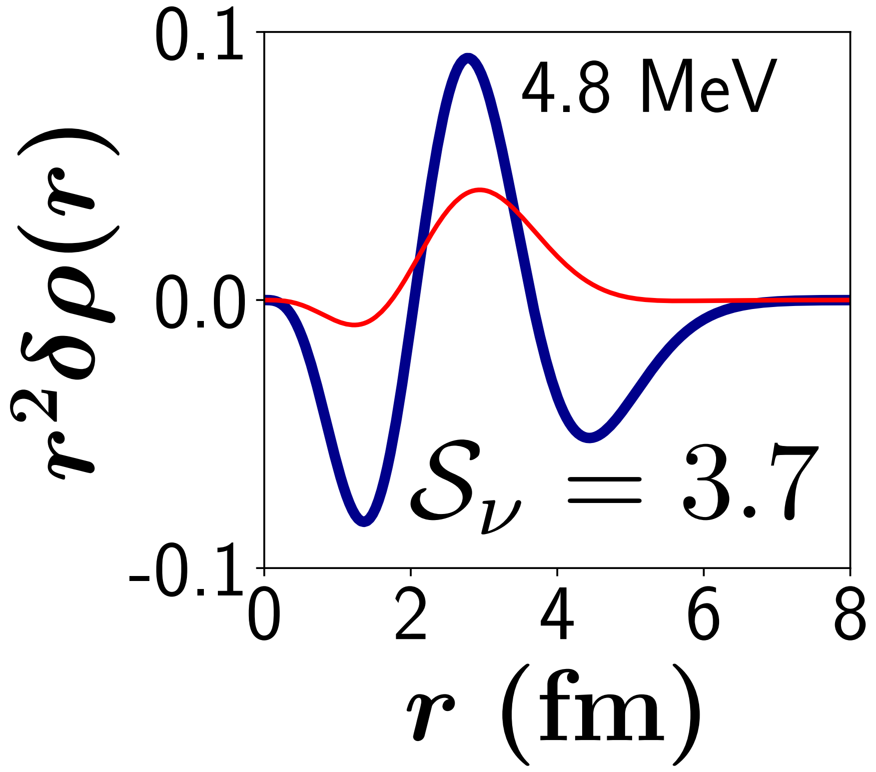
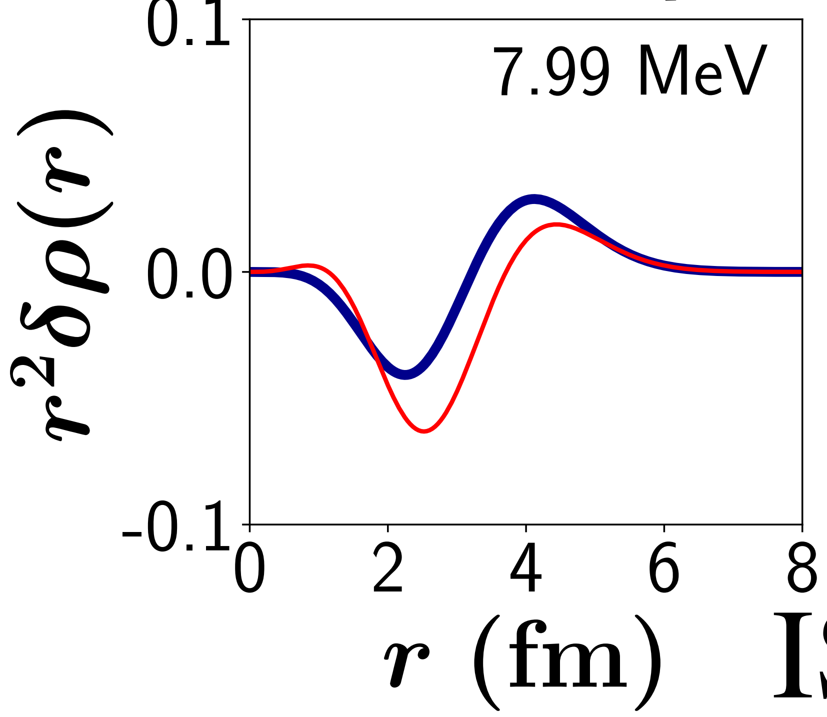
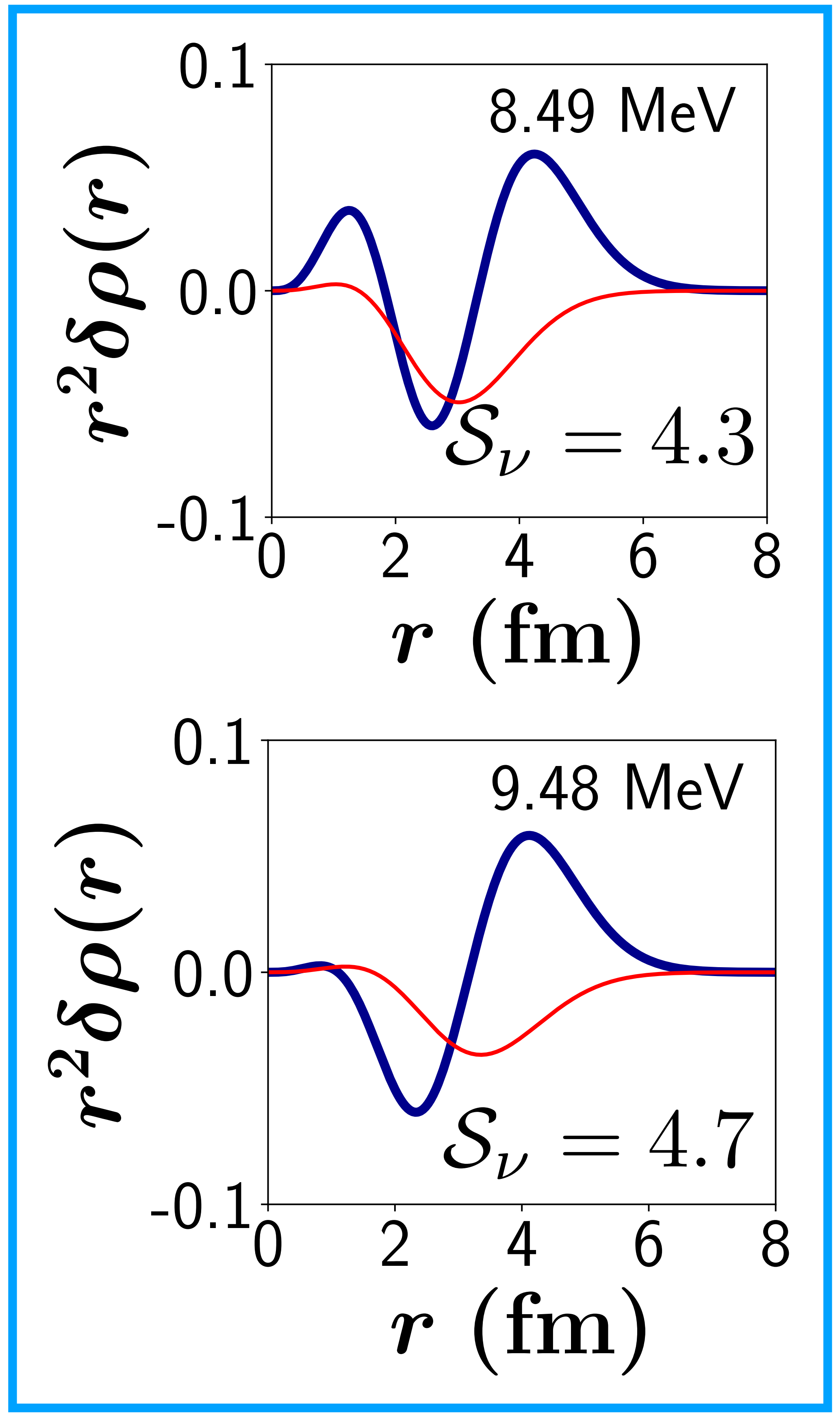


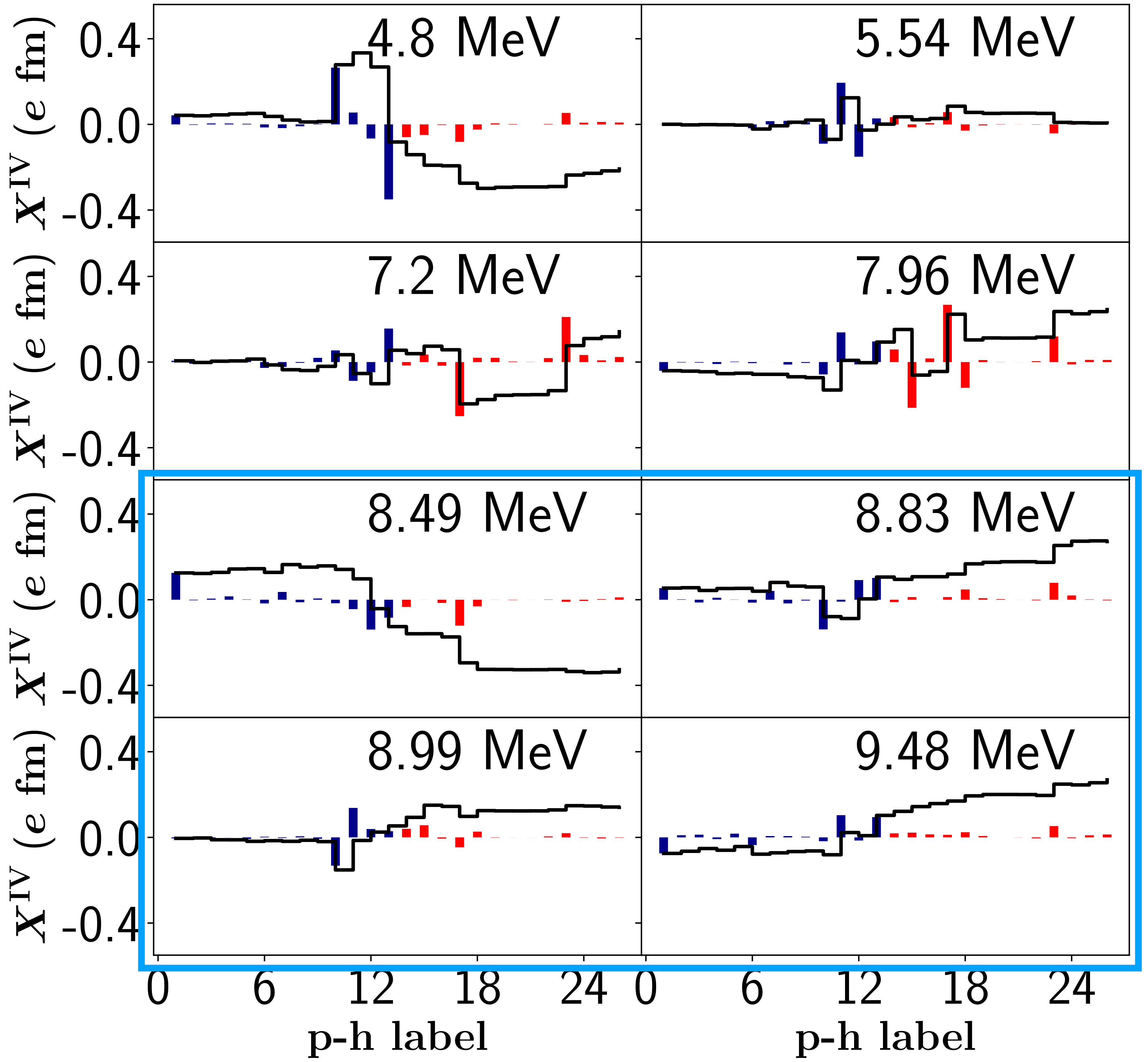
$[2^+ \otimes 3^-]_{1^-}$





IS-LED





Analysis of the dipole response in ^{26}Ne

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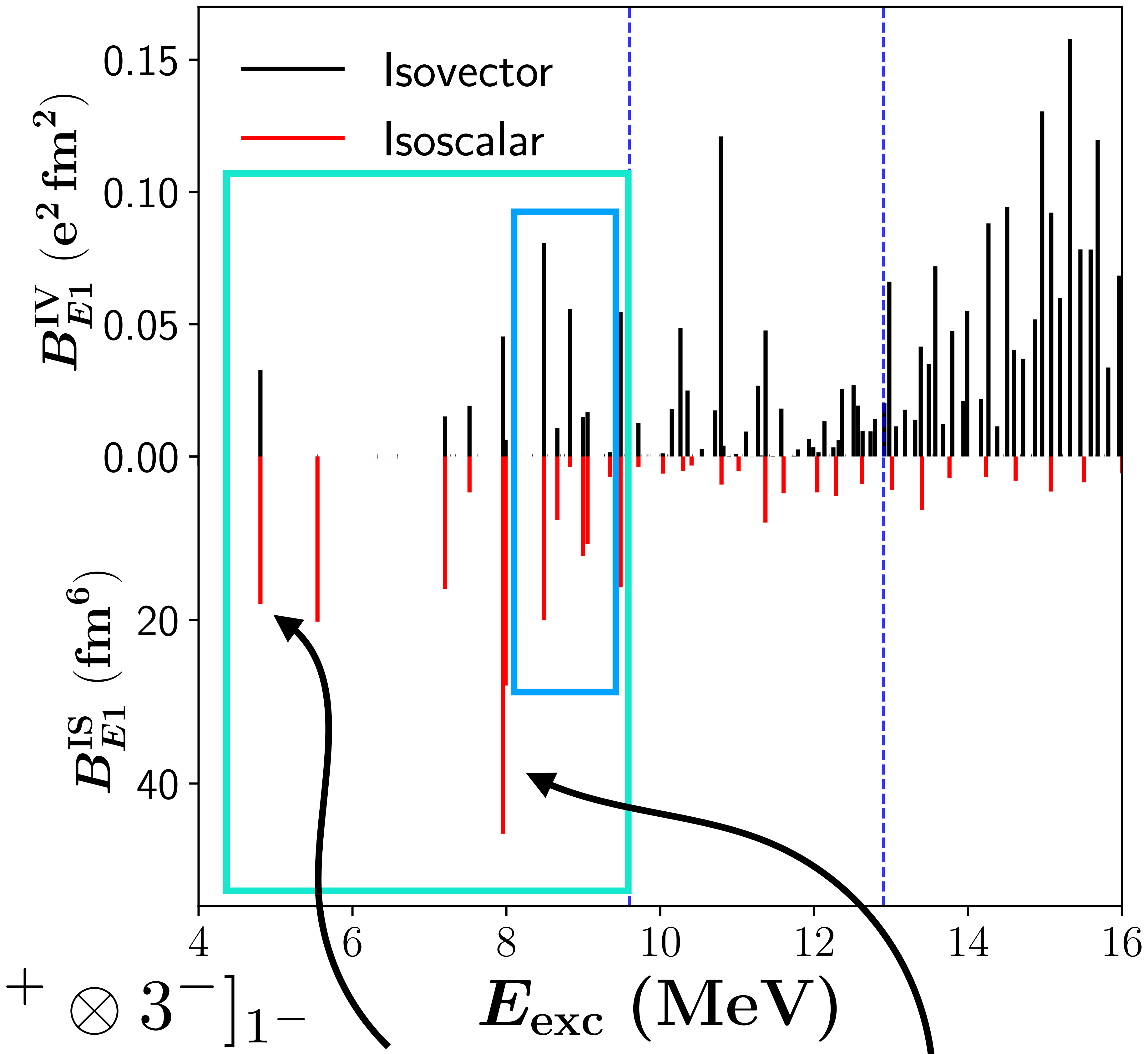

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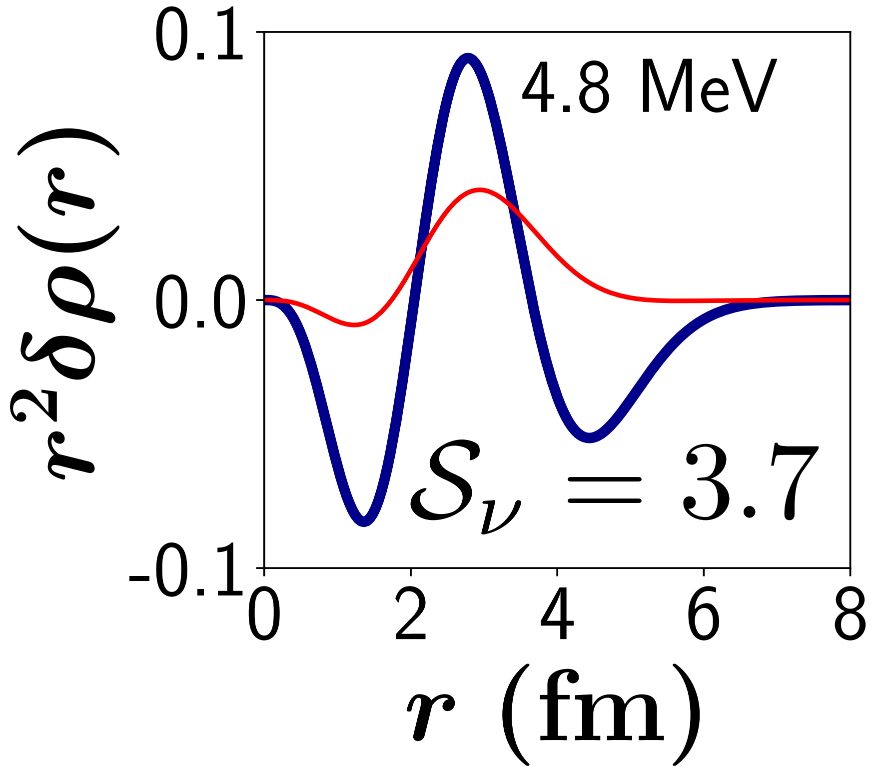

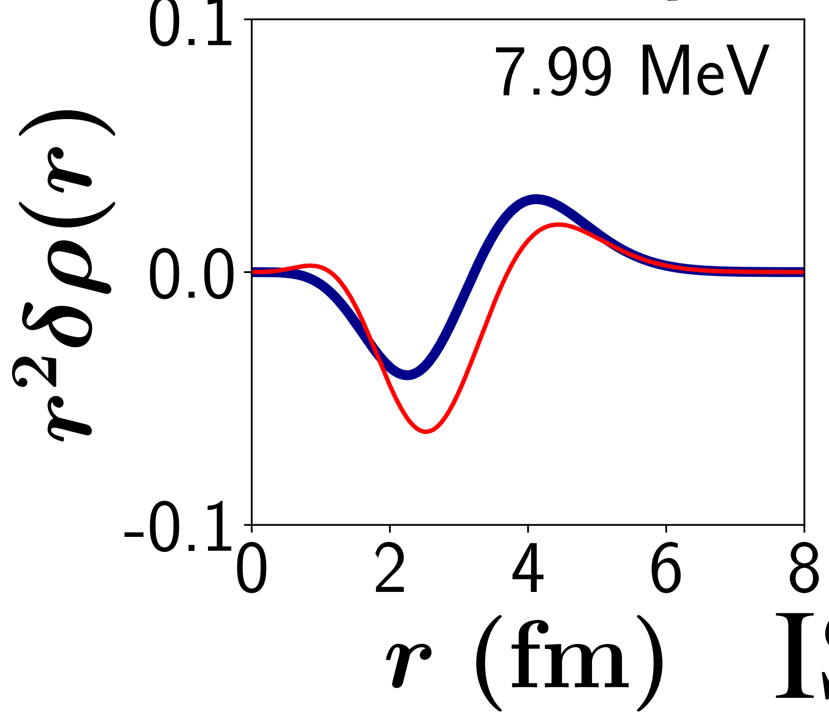

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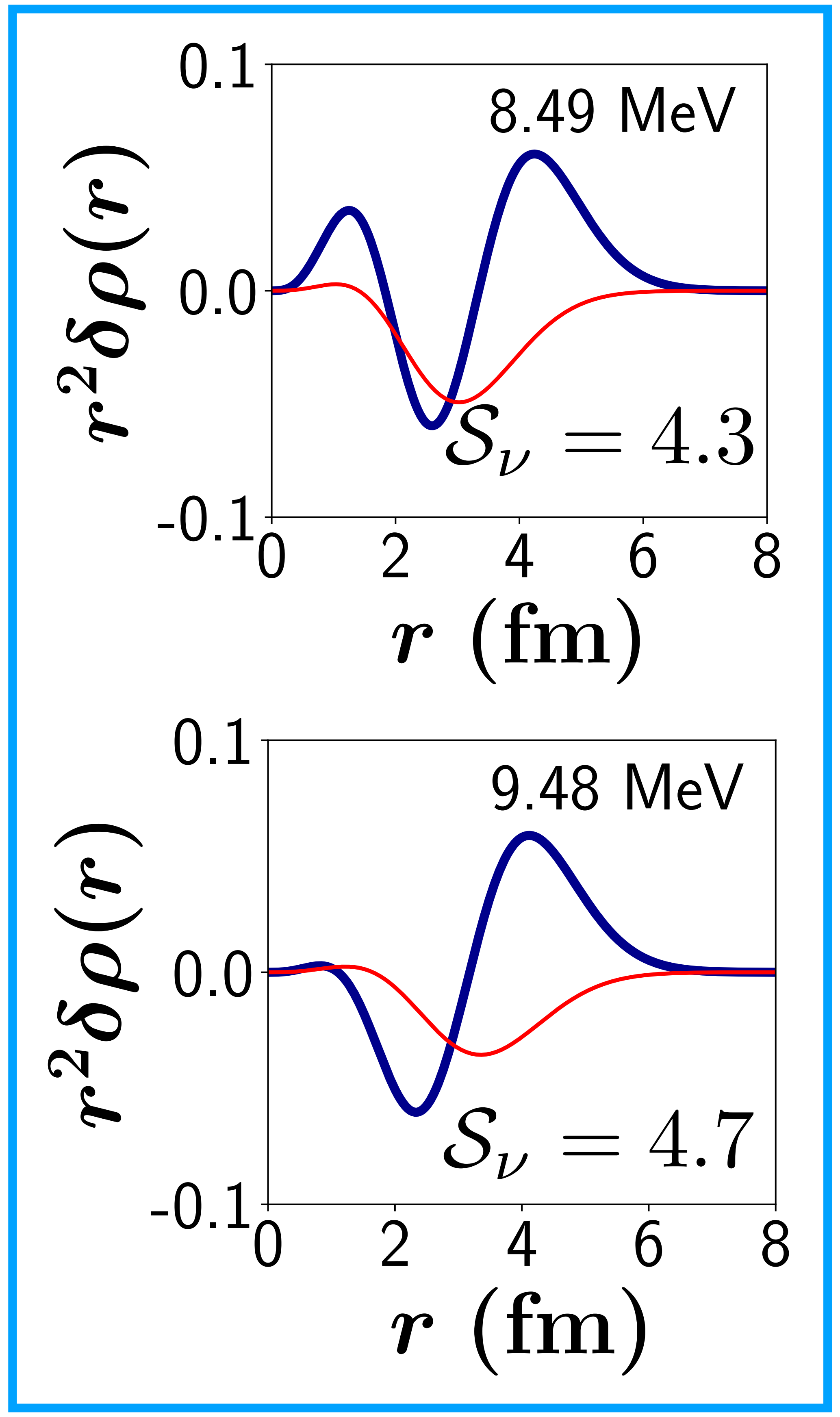
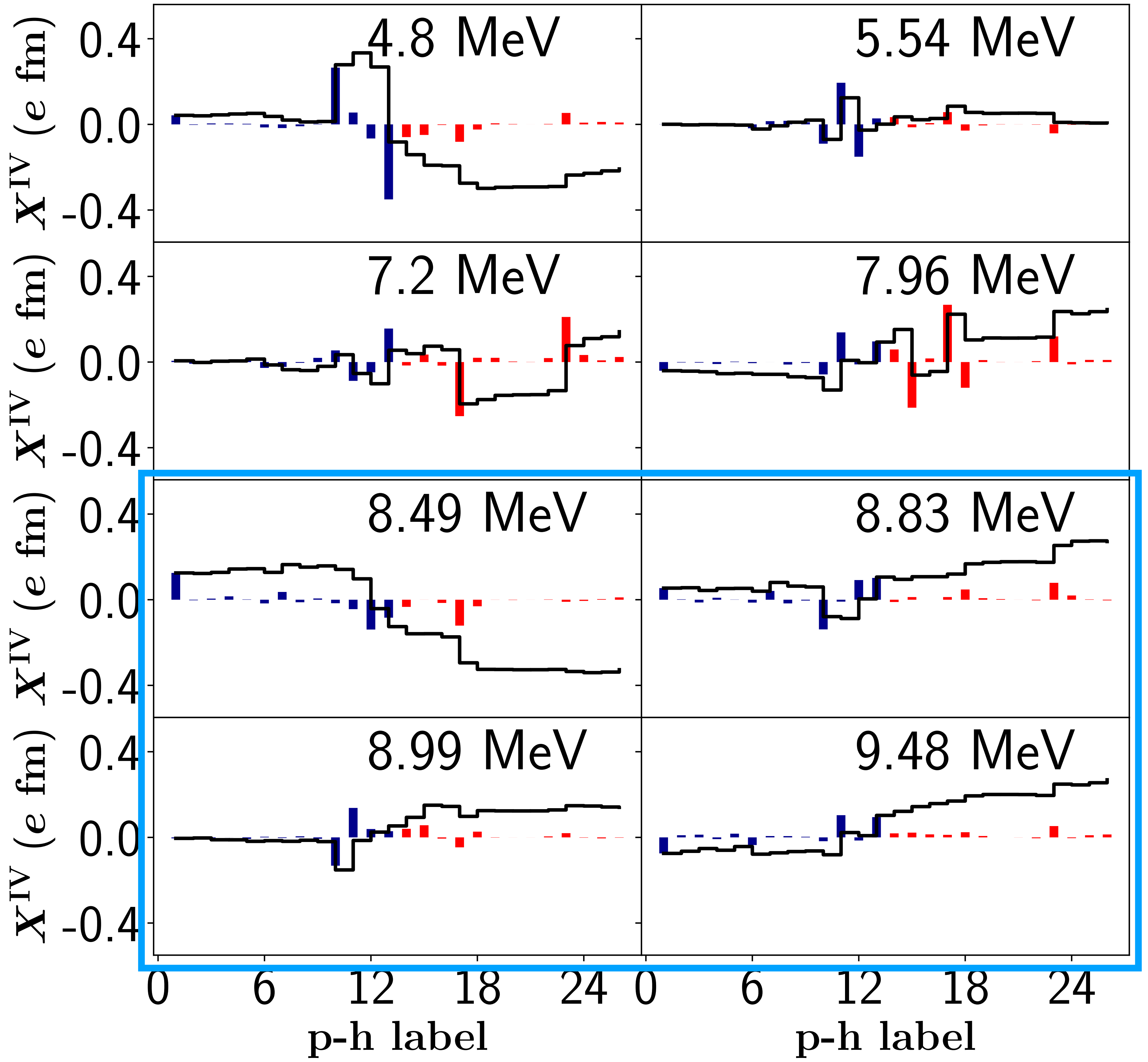




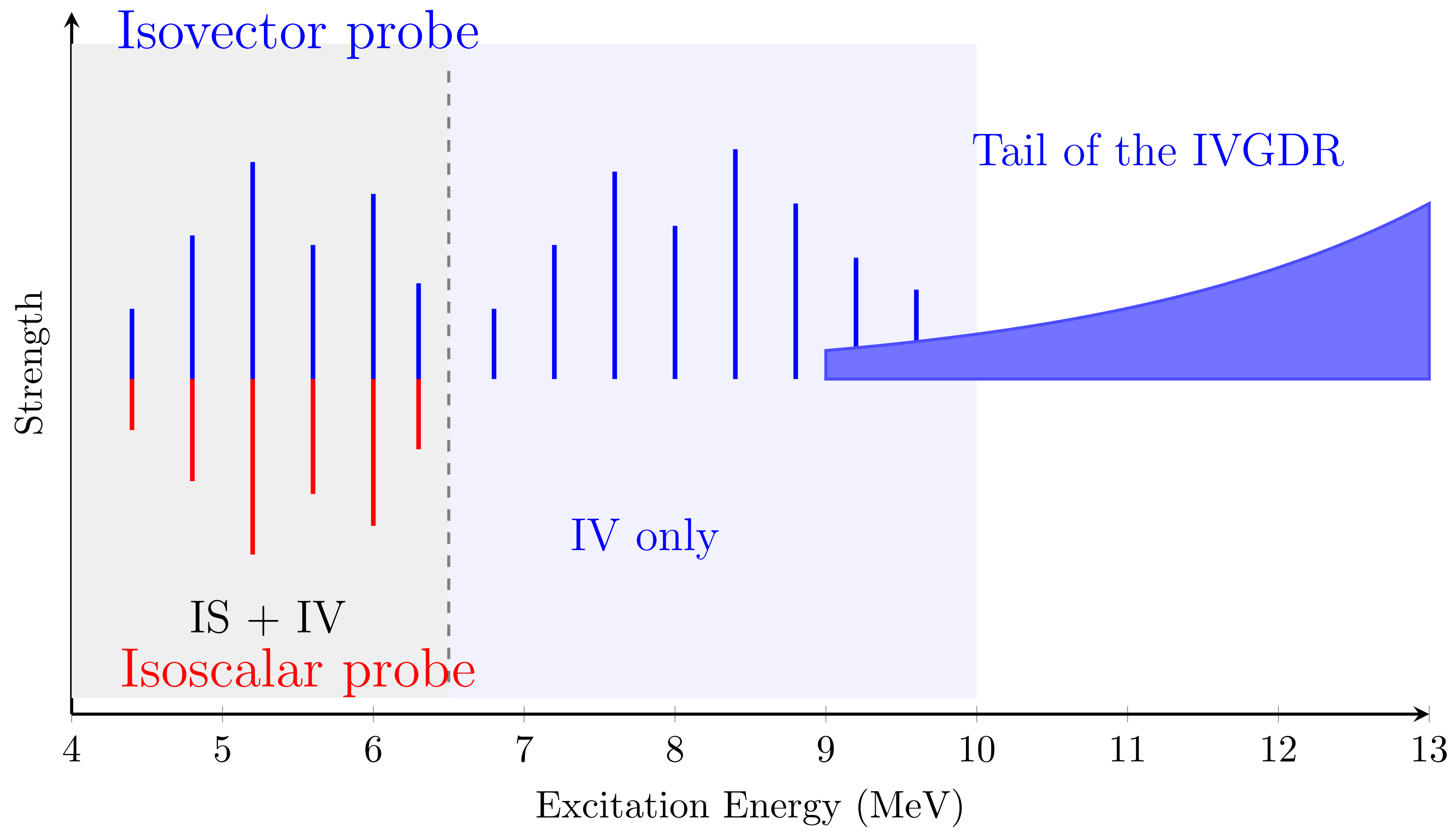
$\longrightarrow$ The PDR is a collective mode.

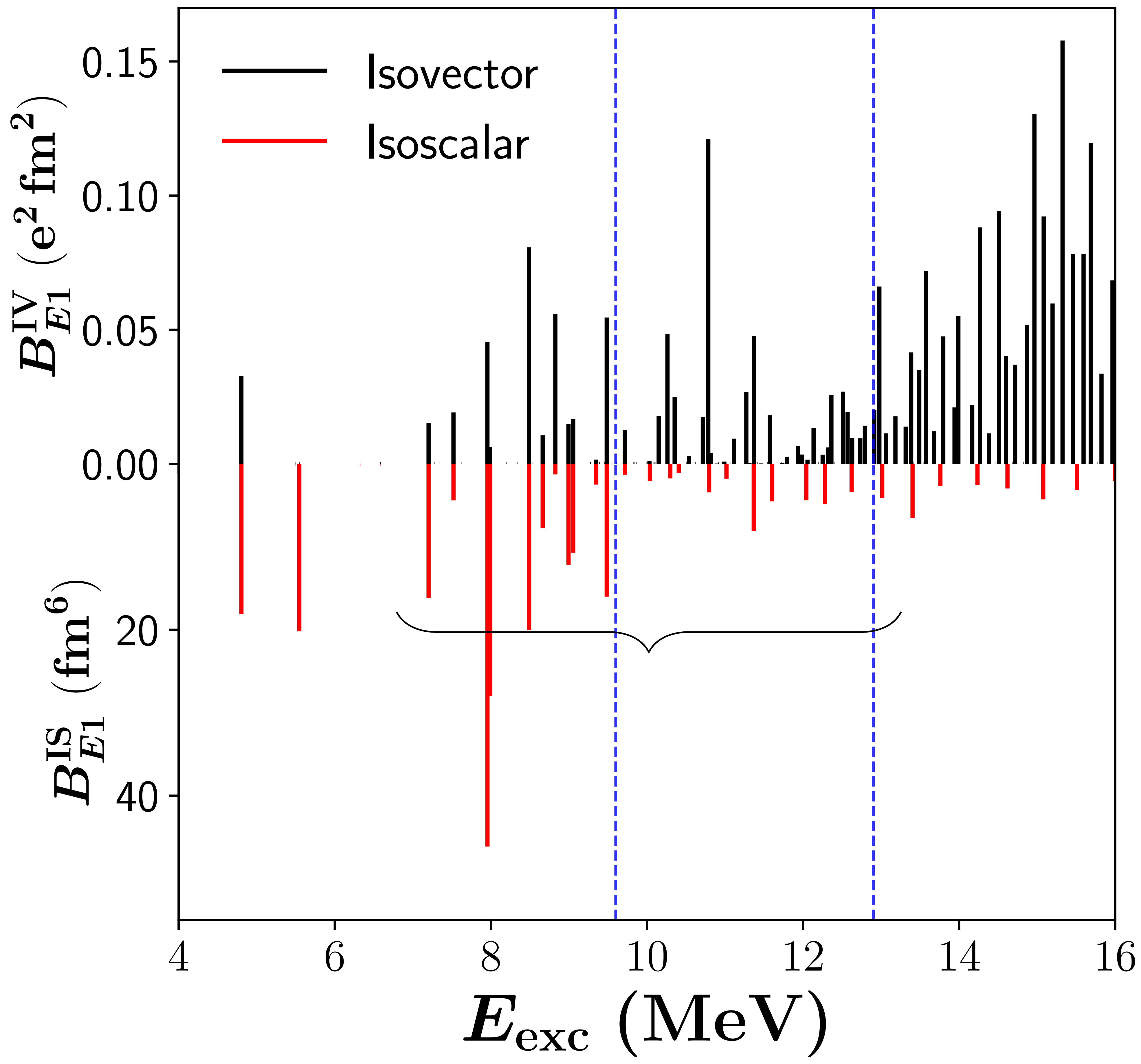
$\longrightarrow$ Neutron skin interpretation valid.

$\longrightarrow$ New definition of a PDR in a CI-SM picture.

Analysis of the dipole response in ^{26}Ne

Isospin-splitting of the PDR





New theoretical definition of a PDR

Low-lying $E1$ resonance appearing in neutron-rich nuclei with the following characteristics:

$\longrightarrow$ pronounced neutron response

$\longrightarrow$ neutron-skin oscillation

$\longrightarrow$ isospin mixing

$\longrightarrow$ significant coherence

$\longrightarrow$ first prediction of an isospin-splitting below S_n

$\longrightarrow$ Isospin-splitting of the low-energy dipole response but not of the PDR

Conclusion

[1] O. Le Noan and K. Sieja, Physical Review C 111, 064308 (2025)

[2] O. Le Noan, S. Goriely, E. Khan, and K. Sieja, Physical Review C 113, 044319 (2026)

[3] R. W. Fearick, **O. Le Noan**, H. Matsubara, P. von Neumann-Cosel, K. Sieja, and A. Tamii, Physical Review C 113, 064311 (2026)

[4] O. Le Noan and K. Sieja, Isoscalar E1 excitations in the configuration-interaction shell model framework. In preparation (2026)

[5] O. Le Noan, PhD manuscript (2026).

Provide reliable PSFs for applications in reaction modeling

⟶ p and sd-shell $E1$ PSFs for all isotopes (first systematic prediction of dipole response using the CI-SM framework). Included in TALYS database. **[1][3]**

⟶ Results are competitive with state-of-the-art microscopic linear-response models (without empirical adjustments on $\overline{S}$ and ΔS). **[1][2]**

⟶ Different centroid trend prediction for exotic isotopes between CI-SM and QRPA based models **[1][2]**

⟶ Moderate impact on the propagation of a ^{40}Ca UHECR seed. **[2]**

⟶ First investigation of E1 isoscalar excitation in CI-SM. **[4]**

Give a shell model view on the PDR

⟶ Good agreement with the available experimental value. **[1]**

⟶ Clear description of the transition from PDR to GDR. **[5]**

⟶ Several distinct dipole state types: quadrupole-octuple coupled state, IS-LED and PDR. **[1][5]**

⟶ Confirmed the interpretation as a neutron-skin oscillation. **[1][5]**

⟶ The PDR is a collective mode in a CI-SM point of view **[1][5]**

⟶ Isospin splitting observed but this concept is questioned **[4][5]**

Perspectives

Provide reliable PSFs for applications in reaction modeling

⟶ CI-SM E1 predictions for *pf*-shell isotopes.

⟶ Microscopic dressing of E1 operator

Give a shell model view on the PDR

⟶ Toroidal nature of the PDR ?

⟶ Possibility of building PDR on top of excited states ?

⟶ Effect of ground-state deformation on the PDR ?

Back up

Theoretical framework: CI-SM

Lanczos Strength function

Step 1: compute the GS $\Longrightarrow \quad |0\rangle$

For a 0^+ GS we diagonalize H_{eff} in the $M = 0$ subspace using Lanczos algorithm

Step 2: compute the sum rule state $|\tilde{O}\rangle = \frac{1}{\sqrt{S_0}} O\,|0\rangle$ where the sum rule of order q reads $S_q = \sum_\nu E_\nu^q |\langle \nu | O | 0 \rangle|^2$

Step 3: Lanczos calculation using $|\mathcal{L}_0\rangle \equiv |\tilde{O}\rangle$

After k Lanczos iterations, we obtain the transformation matrix $U_{\nu n} = \langle \nu | \mathcal{L}_n \rangle$

First column of this change of basis matrix $\langle \nu | \mathcal{L}_0 \rangle = \frac{1}{\sqrt{S_0}} \langle \nu |\, O\, | 0 \rangle$

Building the strength function

$$S(E) = \sum_{\nu=1}^{k} S_0 U_{\nu 0}^2 \frac{2}{\pi} \frac{\Gamma E^2}{\left(E^2 - E_\nu^2\right)^2 + \Gamma^2 E^2}$$

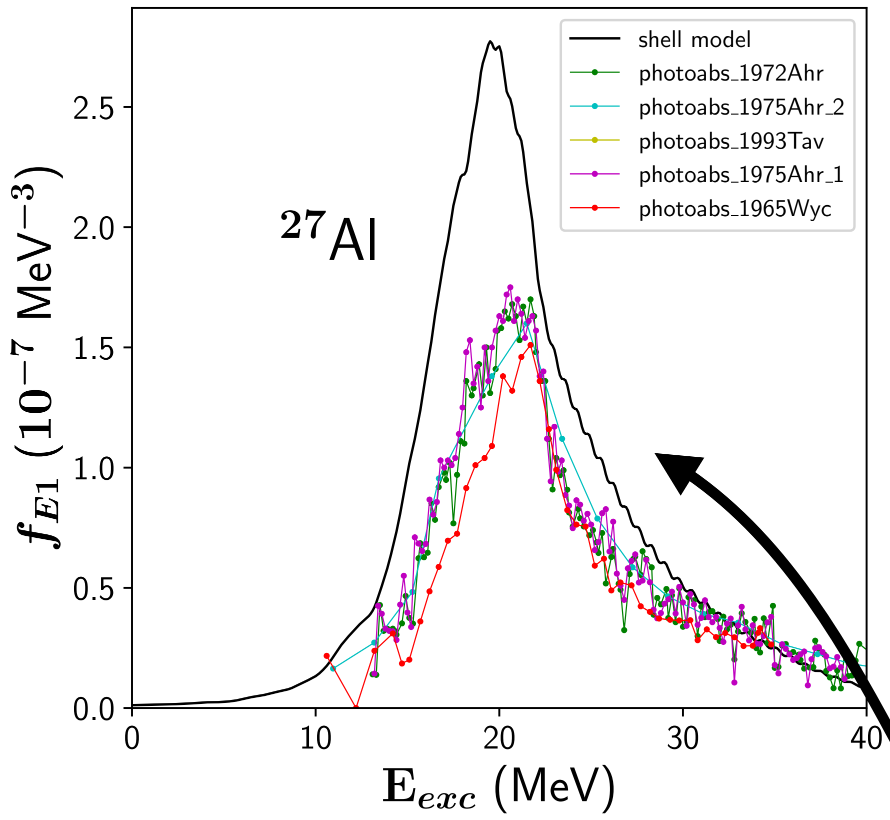
D. H. Gloeckner and R. D. Lawson, Physics Letters B 53, 313 (1974)

Theorem: The first $2k - 1$ moments of the strength function are exact, even though all eigenstates are not converged

Theoretical framework: CI-SM

Renormalization of the isovector $E1$ operator

Observation: systematic overestimated PSFs in the CI-SM framework



Conceptually, we need a mapping for operators.

$$\begin{aligned} \mathcal{H} &\longrightarrow \mathcal{H}_{\text{valence}} \\ O &\longmapsto O_{\text{eff}} \end{aligned}$$

$\longrightarrow$ Lee-Suzuki, IM-SRG

In practice: determine a reduction factor (effective charge) Q

$$Q^2_{\text{exp}} \equiv \frac{S_1^{\text{exp}}}{S_1^{\text{SM}}} \sim 0.64$$

$\kappa = 0.14$ from ^{27}Al measurements

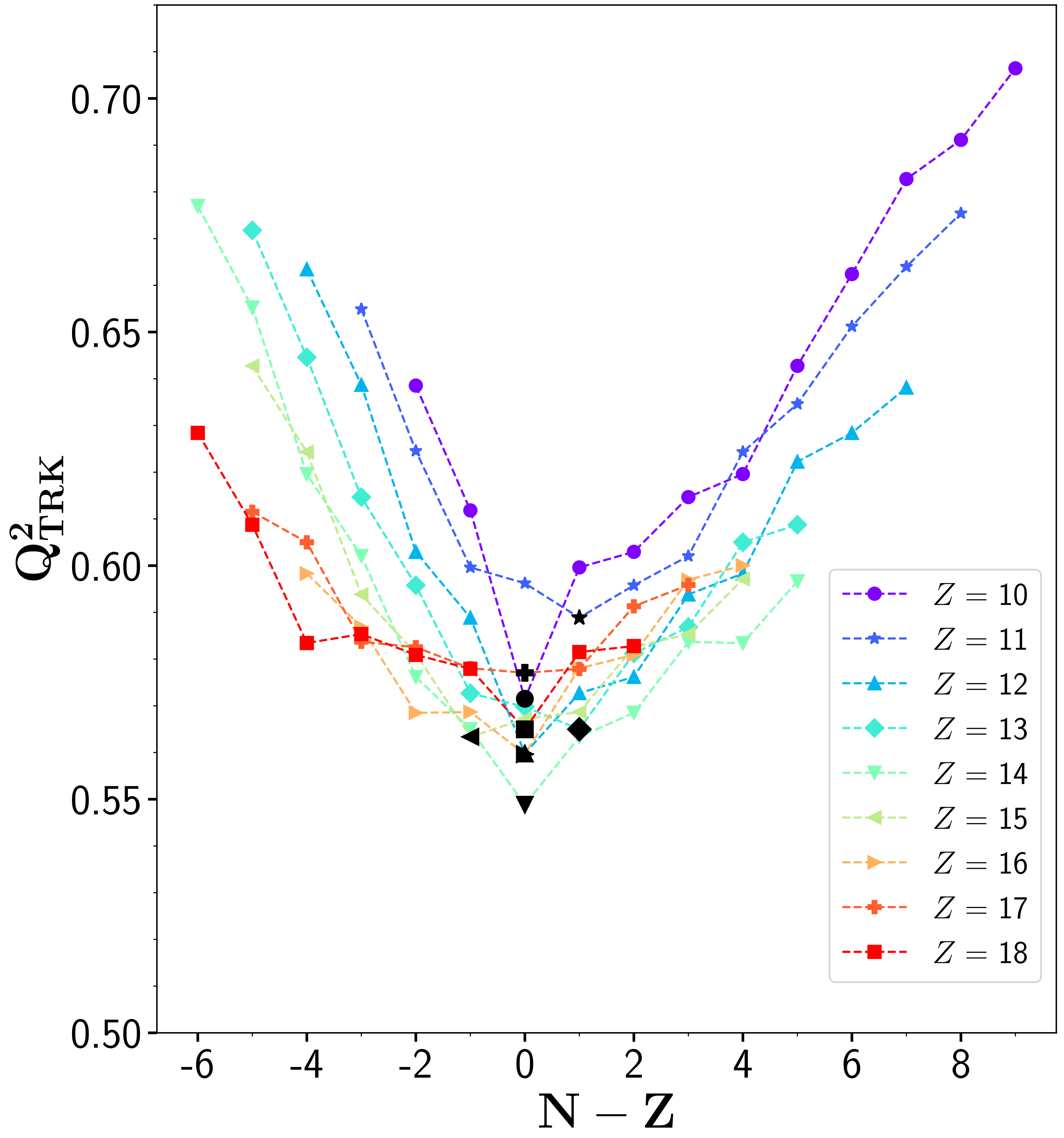
$$S_{\text{TRK}} = \frac{9\hbar}{8\pi m}\frac{NZ}{A}$$

$$S_1 = S_{\text{TRK}}(1+\kappa)$$

sd-shell effective charge

$$Q^2(N,Z) = Q^2_{\text{TRK}}(1+\kappa)$$

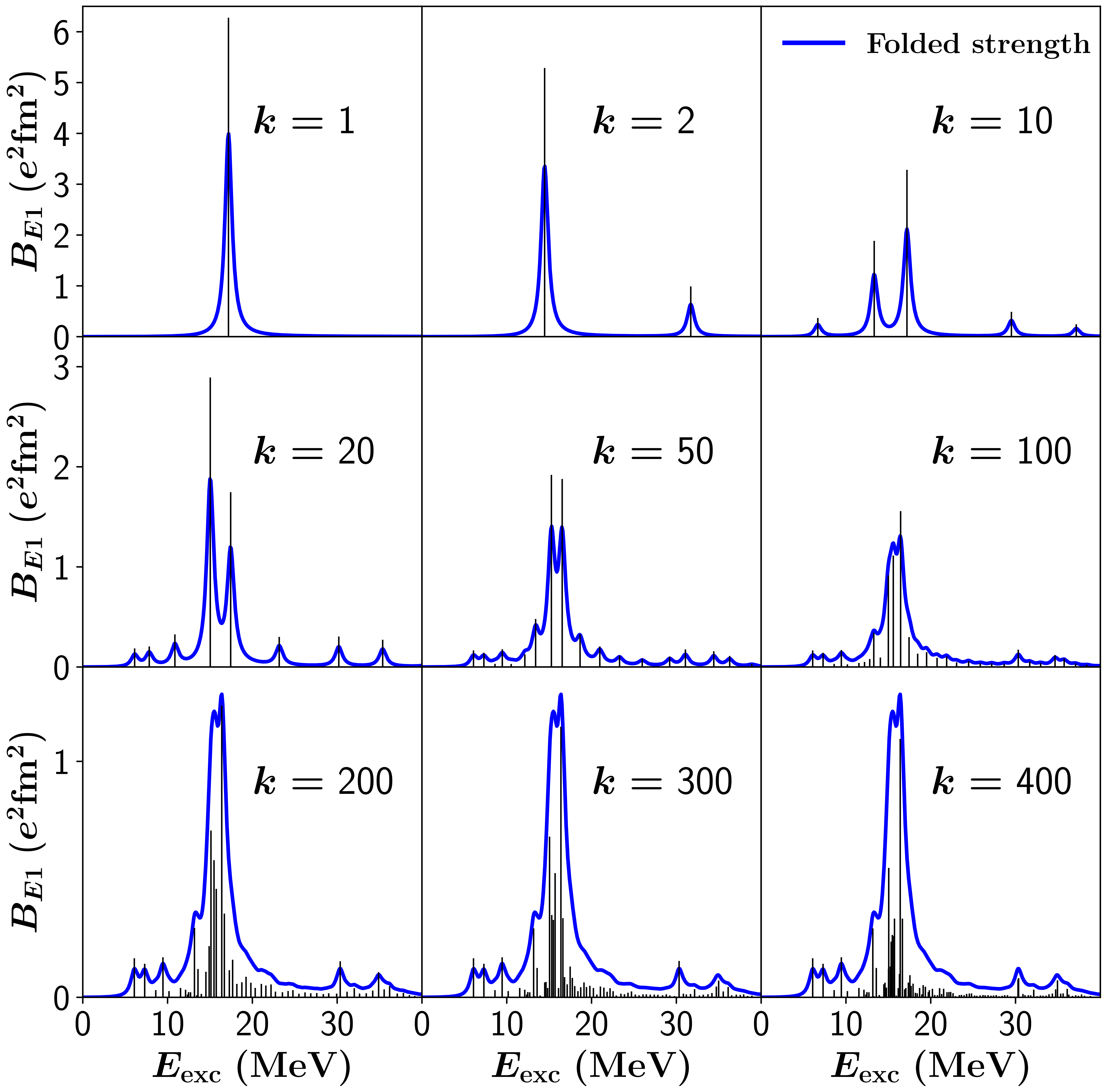
$$Q^2_{\text{TRK}} \equiv \frac{S_1^{\text{TRK}}}{S_1^{\text{SM}}}$$



LSF convergence

$$S(E) = \sum_{\nu=1}^{k} S_0 U_{\nu 0}^2 \delta(E - E_\nu)$$

Theorem: The first $2k-1$ moments of the strength function are exact, even though all eigenstates are not converged

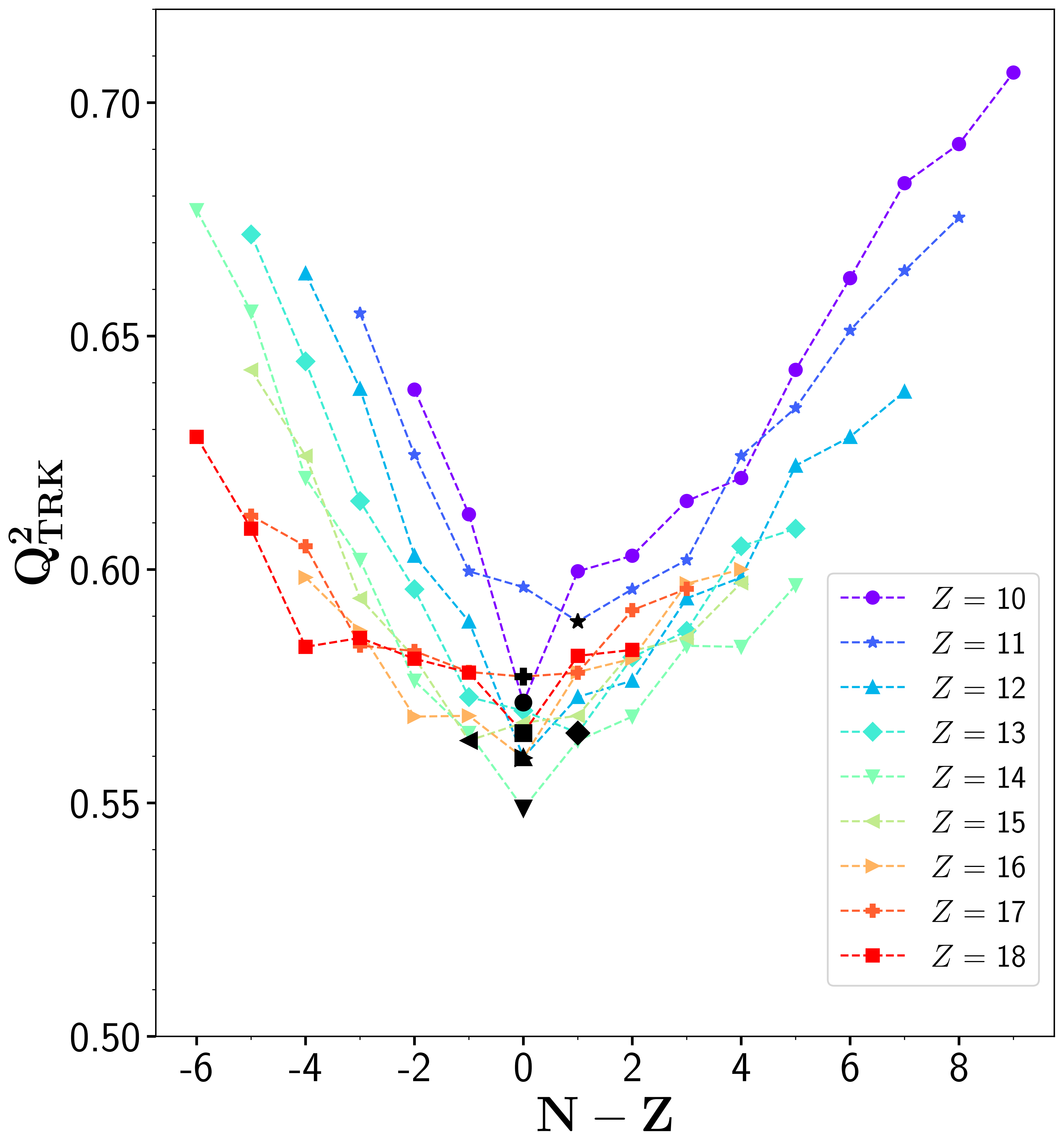


E1 effective charge trend

$$|0\rangle = \alpha_0 |0\rangle_{0\hbar\omega} + \alpha_2 |0\rangle_{2\hbar\omega}$$

$$S_0 = \langle 0| O^\dagger O |0\rangle = |\alpha_0|^2 S_0^{0\hbar\omega} + \mathcal{O}(|\alpha_2|^2)$$

$$Q^2 \sim |\alpha_0|^2$$



Since the contribution of $2\hbar\omega$ configurations in the ground state is expected to be more significant for $N = Z$ nuclei, one has $|\alpha_2(N = Z)| > |\alpha_2(N \neq Z)|$, which in turn implies $Q^2(N = Z) < Q^2(N \neq Z)$.

Application to UHECR propagation

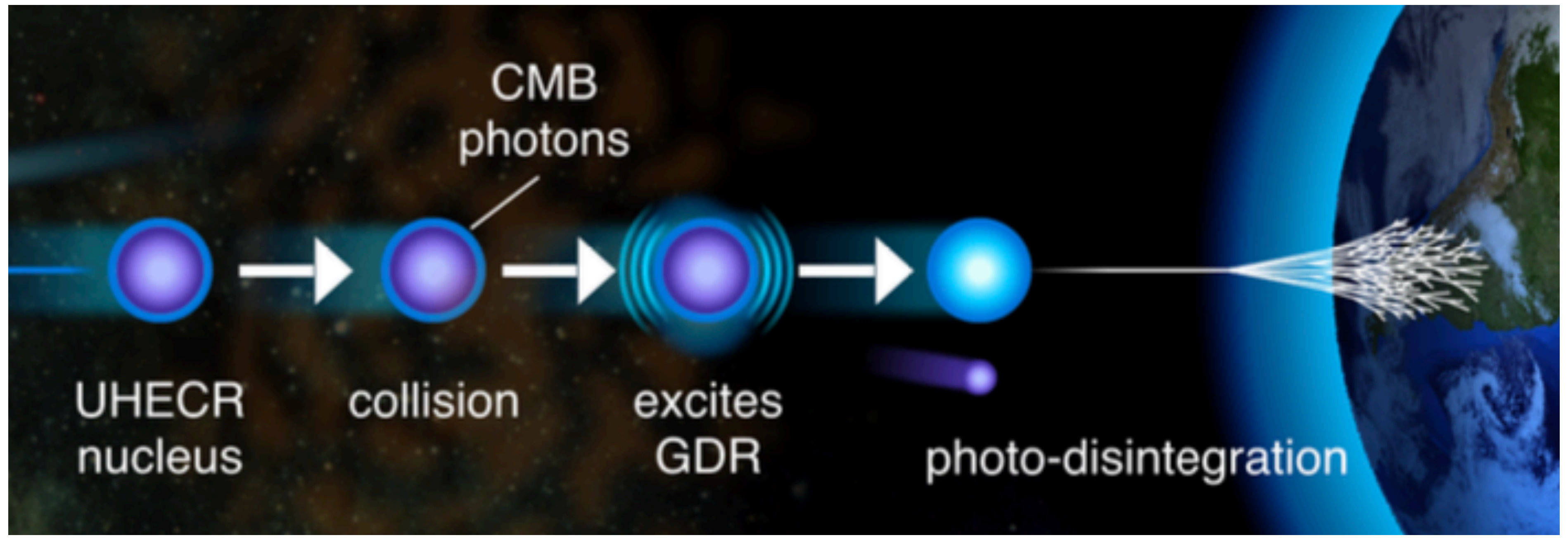


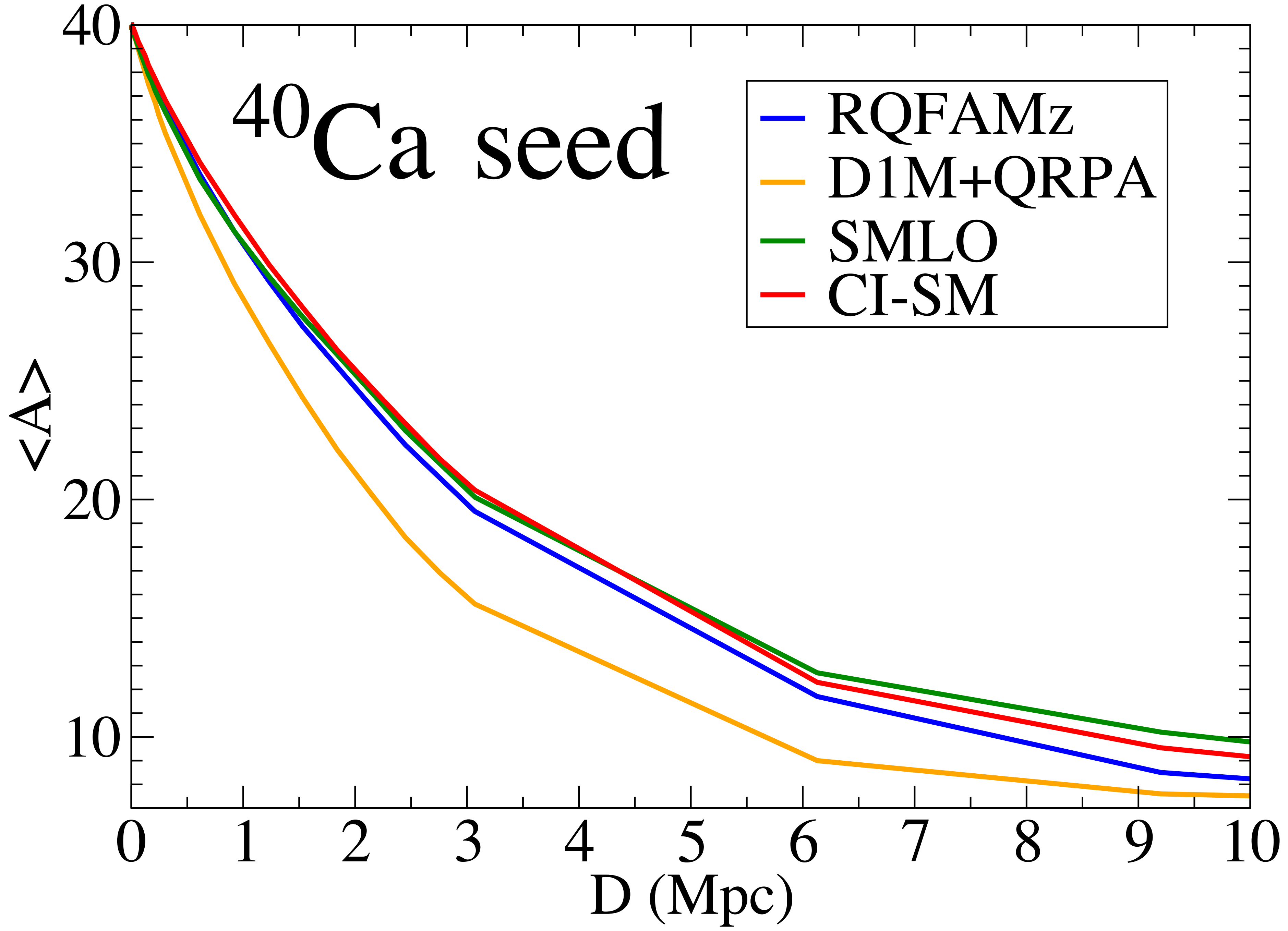
Image credit: Atsushi Tamii, 9th Workshop on Nuclear Level Density and Gamma Strength, 2024, Oslo.

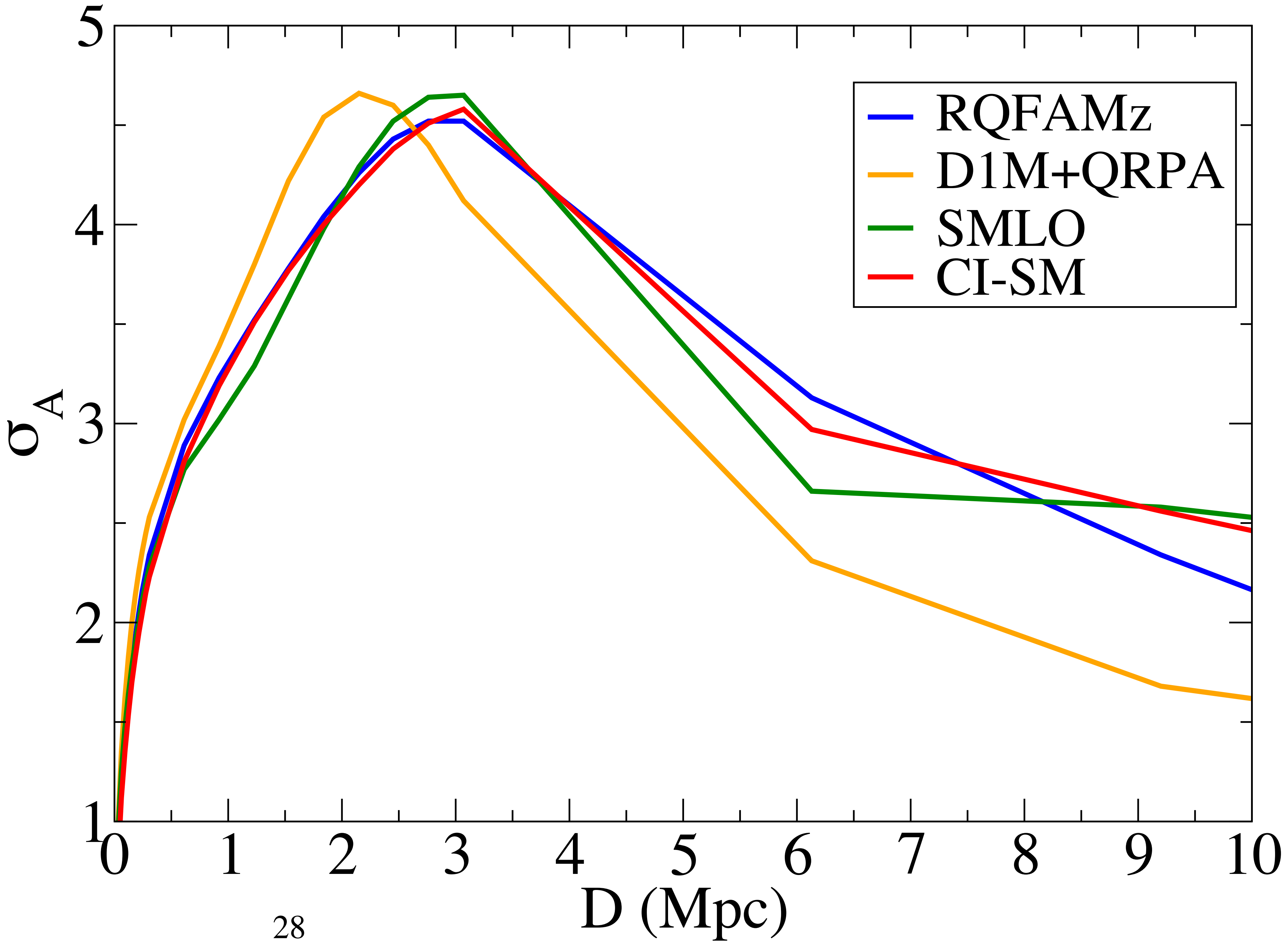
$$
\begin{aligned}
\frac{dN_{Z,A}}{dt} = & \; N_{Z+1,A}\lambda_{\beta}^{Z+1,A} + N_{Z-1,A}\lambda_{\beta}^{Z-1,A} \\
& + N_{Z,A+1}\lambda_{\gamma,n}^{Z,A+1} + N_{Z+1,A+1}\lambda_{\gamma,p}^{Z+1,A+1} \\
& + N_{Z+2,A+4}\lambda_{\gamma,\alpha}^{Z+2,A+4} + N_{Z,A+2}\lambda_{\gamma,2n}^{Z,A+2} \\
& + N_{Z+2,A+2}\lambda_{\gamma,2p}^{Z+2,A+2} + N_{Z+4,A+8}\lambda_{\gamma,2\alpha}^{Z+4,A+8} \\
& + N_{Z+1,A+2}\lambda_{\gamma,np}^{Z+1,A+2} + N_{Z+2,A+5}\lambda_{\gamma,n\alpha}^{Z+2,A+5} \\
& + N_{Z+3,A+5}\lambda_{\gamma,p\alpha}^{Z+3,A+5} \\
& - N_{Z,A}\left[\lambda_{\beta}^{Z,A} + \sum_{x}\lambda_{\gamma,x}^{Z,A}\right] ,
\end{aligned}
$$

$$\lambda_{\gamma,x} = \int n(\epsilon)\; \sigma_{\gamma,x}(\epsilon)\; c\; d\epsilon$$

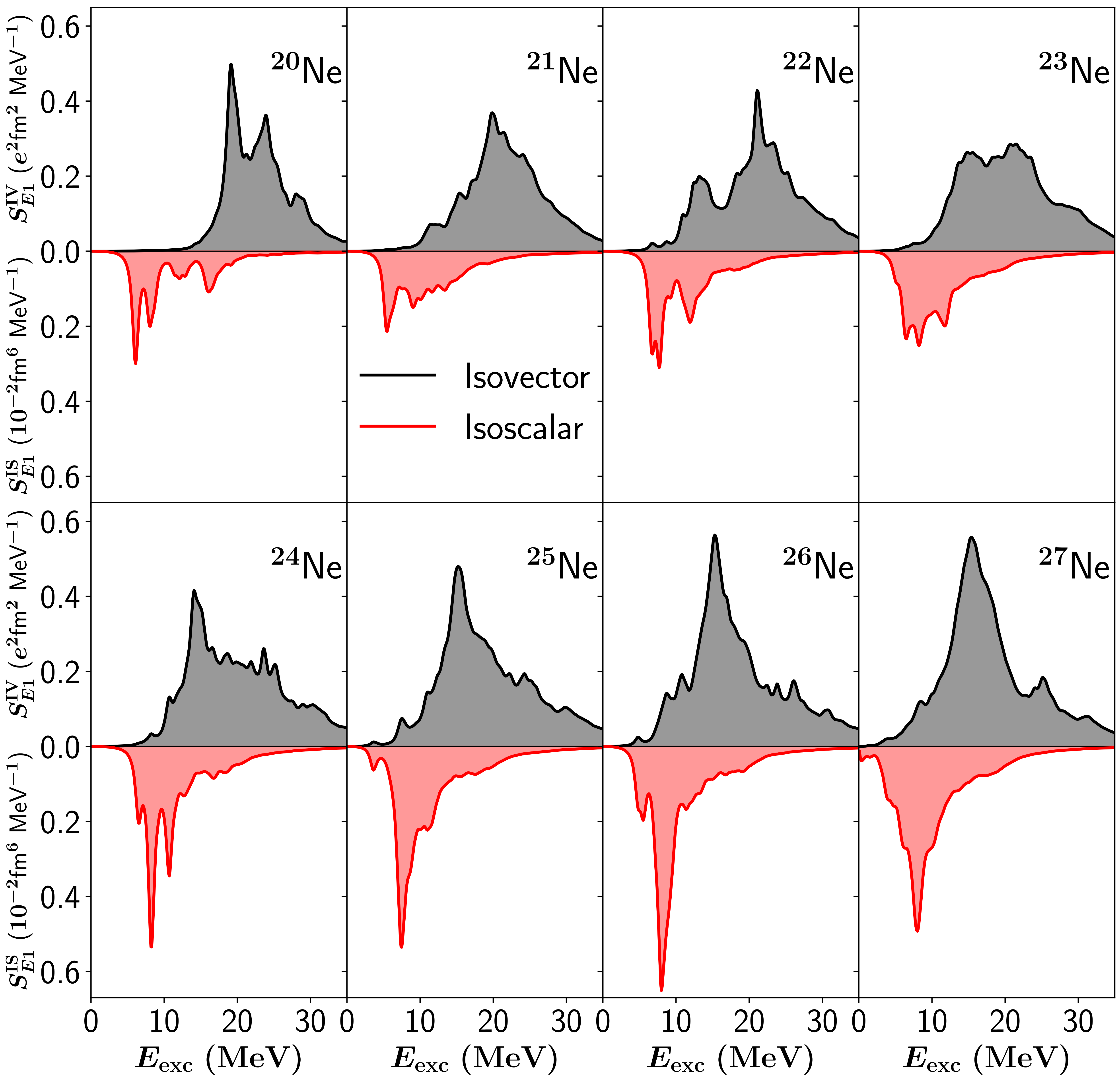
TALYS

CI-SM
PSFs





O. Le Noan, S. Goriely, E. Khan, and K. Sieja, Physical Review C 113, 044319 (2026)



^{20}Ne
^{21}Ne
^{22}Ne
^{23}Ne
^{24}Ne
^{25}Ne
^{26}Ne
^{27}Ne
Isovector
Isoscalar
S^{IV}_{E1} (e^2fm^2 MeV^{-1})
S^{IS}_{E1} (10^{-2}fm^6 MeV^{-1})
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0
10
20
30
E_{exc} (MeV)

J_i^π	$E_{\rm exc}$ (MeV)	$B_{E1}^{\rm IV}$ (e^2fm^2)	$B_{E1}^{\rm IS}$ (fm^6)	$\mathcal{S}_\psi$	f_N	f_Z	State type
1_1^-	4.80	0.033	19.23	3.7	0.94	0.06	$[2^+ \otimes 3^-]_{1^-}$
1_2^-	5.54	3×10^{-5}	19.48	4.1	0.96	0.04	?
1_3^-	7.20	0.015	15.83	4.7	0.53	0.47	IS-LED
1_4^-	7.52	0.019	4.25	4.5	0.65	0.35	PDR
1_5^-	7.96	0.045	45.32	4.3	0.38	0.62	IS-LED
1_6^-	7.99	0.006	26.39	4.7	0.38	0.62	IS-LED
1_7^-	8.49	0.081	19.94	4.3	0.91	0.09	PDR
1_8^-	8.66	0.011	7.46	4.6	0.75	0.25	PDR
1_9^-	8.83	0.056	1.45	4.4	0.82	0.18	PDR
1_{10}^-	8.99	0.015	11.95	4.7	0.66	0.34	PDR
1_{11}^-	9.06	0.017	10.51	4.4	0.80	0.20	PDR
1_{12}^-	9.35	0.002	2.30	4.7	0.62	0.38	PDR
1_{13}^-	9.48	0.055	15.69	4.7	0.70	0.30	PDR
1_{14}^-	9.72	0.013	1.31	5.0	0.63	0.37	GDR
1_{16}^-	10.15	0.018	0.20	4.8	0.45	0.55	One-node IV
1_{17}^-	10.26	0.048	0.52	5.0	0.69	0.31	GDR
1_{18}^-	10.35	0.025	2.13	4.8	0.75	0.25	Mixed
1_{20}^-	10.71	0.017	0.39	4.6	0.78	0.22	GDR
1_{21}^-	10.79	0.121	2.05	5.0	0.62	0.38	Mixed, GDR-like
1_{26}^-	11.27	0.027	1.35	4.9	0.63	0.37	Mixed, GDR-like
1_{28}^-	11.37	0.048	1.76	4.9	0.61	0.39	Mixed, GDR-like
1_{29}^-	11.46	2×10^{-4}	4.57				PDR-like
1_{30}^-	11.57	0.018	1.44	5.0	0.55	0.45	GDR
1_{37}^-	12.13	0.013	2.28	4.9	0.57	0.43	Mixed, GDR-like
1_{40}^-	12.36	0.026	0.14	5.1	0.72	0.28	GDR
1_{41}^-	12.50	0.021	0.03	4.9	0.65	0.35	One-node IV
1_{42}^-	12.56	0.029	0.10	5.0	0.58	0.42	GDR
1_{43}^-	12.67	0.011	$\sim10^{-4}$	5.1	0.69	0.31	One-node IV
1_{44}^-	12.78	0.023	$\sim10^{-2}$	5.2	0.54	0.46	Mixed, GDR-like
1_{45}^-	12.94	0.049	$\sim10^{-1}$	5.1	0.61	0.39	GDR
$1_{\rm GDR}^-$	~ 16	$\sim 3\times10^{-1}$	$\sim10^{-2}$	~ 5.3	~ 0.63	~ 0.37	Textbook GDR