Relativistic Liquid Crystals and more..

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For GWsNS-2026

Minutes 1-3:

Self-gravitating anisotropic fluids. I: Context and overview

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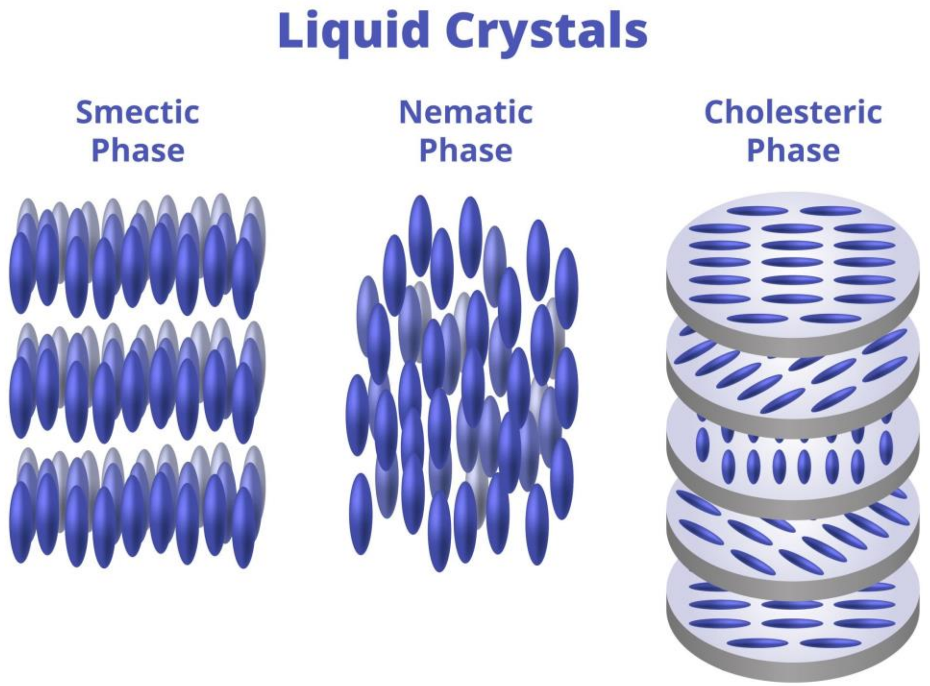
Minutes 3-5:

Relativistic Plasticity,
University of Southampton,
Nils Andersson

Liquid Crystals?

- Well established in soft-matter.
- Derived from a classical field theory[1].

- Hydrodynamic limit of pasta phases[2].
- Introduces an effective viscosity.
- Effective field theory version of "pion condensation".
- "Anisotropic" – argued to produce more compact bodies.



Ex: polymers (colloids), protein, bacteria.

[1] P. G. de Gennes, J. Prost, P. G. de Gennes, and J. Prost, *The Physics of Liquid Crystals*, Second Edition, Second Edition (Oxford University Press, Oxford, New York, 1995).

[2] D. N. Kobyakov and C. J. Pethick, Superfluid liquid crystals: pasta phases in neutron star crusts, J. Exp. Theor. Phys. **127**, 851 (2018).

Goal:

Promote the Newtonian liquid crystal action to general relativity.

Investigate the effects in a toy stellar model!

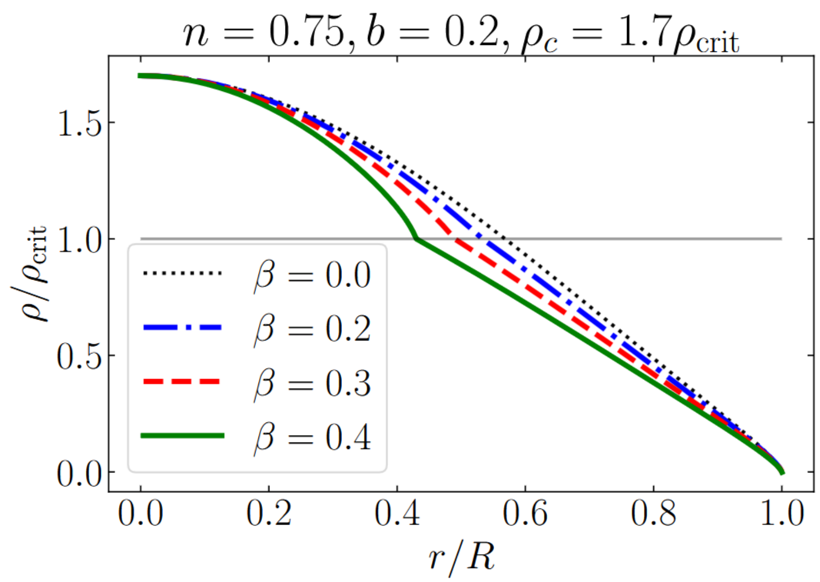
Modifications: $O(\beta^2)$

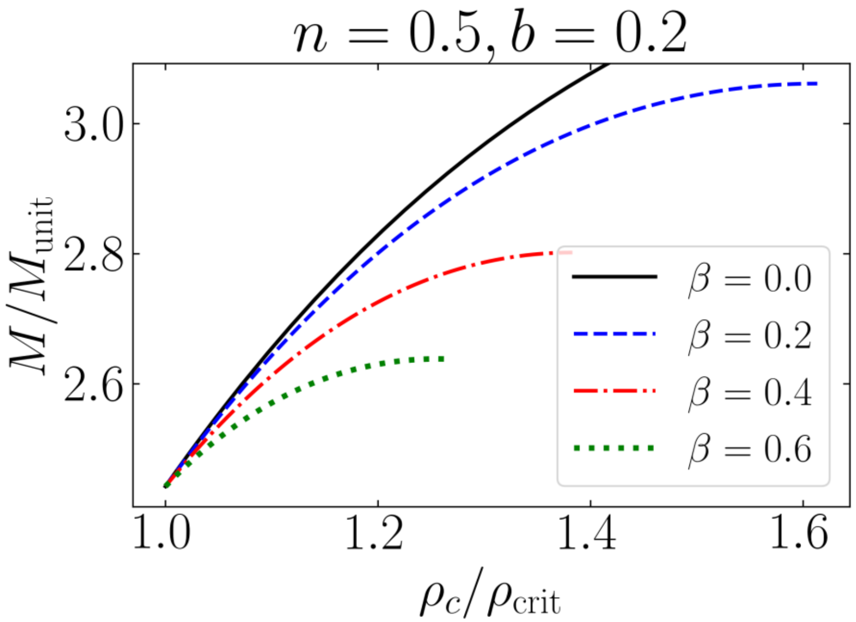
Requires a phase transition.

Generally, leads to **less compactness.**

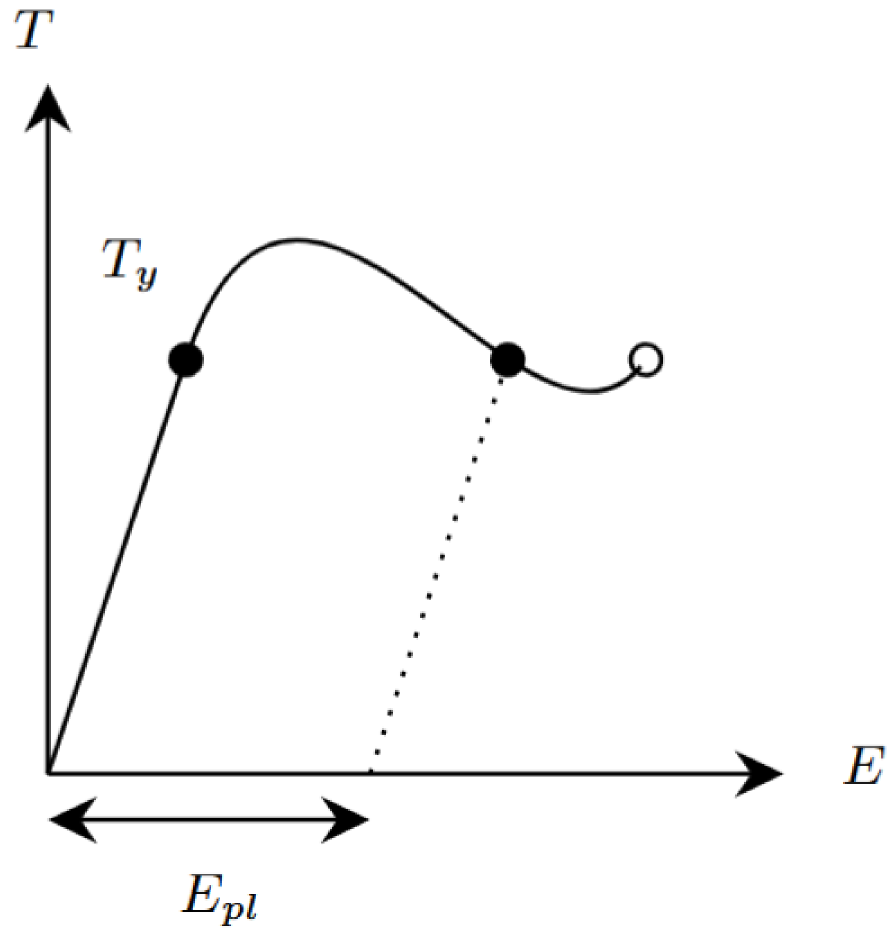
Appears to possess a new dynamical instability.

Fluid perturbations couple to liquid crystal perturbations.





Relativistic Plasticity



- Dissipative – formally interesting, physically relevant
- Yield stress of the crust is often reached in simple tidal models.
- Thought to play a role in producing magnetar flares.
- Permanent deformations, sustained Mountains

Our Aim:

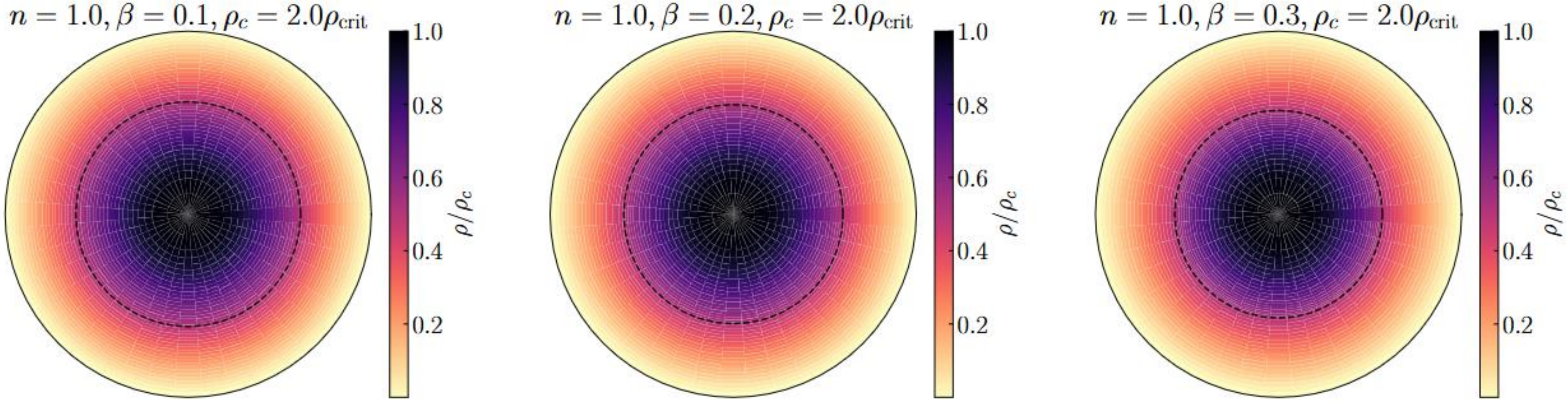
1. Modify the established relativistic elastic action to account for plastic strains.

2. Connect the macroscopic stresses to the equivalent microscopic models.

3. Couple the mechanics to simple magnetic field models.

…See what happens!

Thank you!



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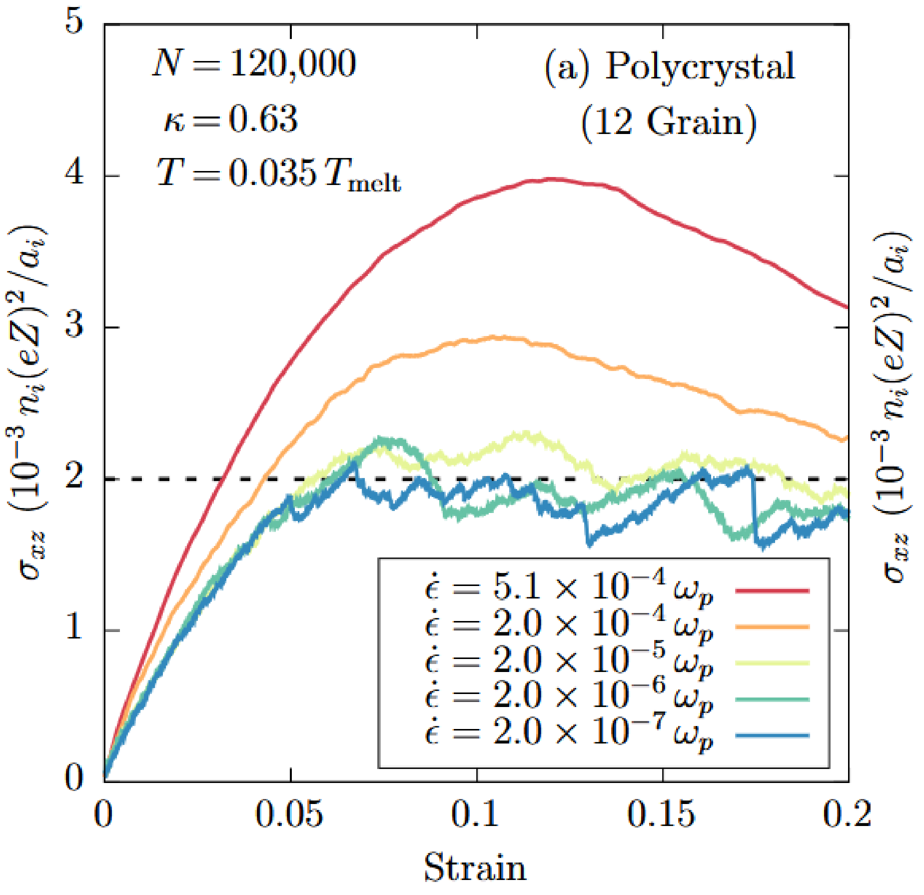
Relativistic Plasticity



M. E. Caplan, N. T. Smith, A. J. Bransgrove, and C. J. Horowitz, *Plasticity of Neutron Star Crusts*, arXiv:2606.06706.

Liquid Crystal Equations

$$S_{\text{fluid}} := -\int_{\mathcal{M}} \left[\mu\left(1 - \tfrac{1}{2}w^2\right) + \tfrac{1}{2}\kappa\Xi - \varphi u_\alpha c^\alpha\right] dV$$

$$\downarrow$$

$$\nabla_\beta J^{\beta\alpha} = \varphi u^\alpha. \qquad \nabla_\beta T^{\alpha\beta} = 0.$$

Liquid Crystal Equations

$$T^{\alpha\beta} := \left[\mu(1-\tfrac{3}{2}w^2)+\tfrac{1}{2}\kappa\Xi\right]u^\alpha u^\beta + \mu w^\alpha w^\beta + \left[p(1-\tfrac{1}{2}w^2)+\tfrac{1}{2}\lambda\Xi\right]P^{\alpha\beta} + 2\varphi u^{(\alpha}c^{\beta)}$$
$$+ \mu u^{(\alpha}c^{\beta)}{}_\gamma w^\gamma + 2u^{(\alpha}J^{\beta)}{}_\gamma w^\gamma - J^{(\alpha}{}_\gamma c^{\beta)\gamma} + J^{\gamma(\alpha}c_\gamma{}^{\beta)} - \nabla_\gamma J^{\gamma\alpha\beta}$$

$$J^{\gamma\alpha\beta} := c^\alpha J^{[\gamma\beta]} + c^\beta J^{[\gamma\alpha]} + c^\gamma J^{(\alpha\beta)},$$

$$J_{\alpha\beta} := \mu u_\alpha w_\beta - \kappa c_{\alpha\beta}$$