Relativistic Mean Field Approach with Chiral Symmetry Breaking and Quark Confinement in the light of LQCD and Astrophysical Constraints

Bikram Keshari Pradhan (**IP2I, Lyon**)

In collaboration with:
(in Alphabetical order according to the last name)

Guy Chanfray,
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Jean-Paul Ebran,
Hubert Hansen,
Elias Khan and
Jerome Margueron

Thematic school GWsNS-2026: Gravitational Waves from Neutron Stars

Jul 2, 2026

Lagrangian density for Nuclear matter

$$\mathcal{L} = \bar{\Psi}\, i\gamma^{\mu}\partial_{\mu}\Psi + \mathcal{L}_s + \mathcal{L}_{\omega} + \mathcal{L}_{\rho} + \mathcal{L}_{\delta} + \cancel{\mathcal{L}_{\pi}}$$

$$\mathcal{L}_s = -M_N(s)\bar{\Psi}\Psi - V(s) + \frac{1}{2}\partial^{\mu}s\partial_{\mu}s$$

- Chiral potential resulting from the bosonized NJL model

$$V(s) = \frac{m_s^2}{2}s^2 + \frac{m_s^2 - m_{\pi}^2}{2\,F_{\pi}}s^3 + \frac{m_s^2 - m_{\pi}^2}{8\,F_{\pi}^2}s^4$$

- m_{π} ~ 140 MeV, F_{π} ~ 93 MeV : less freedom, m_s controls *V(s)* .
- Nuclear Saturation requires $m_s << m_{\pi}$ (not physical) .

$$M_N(s) = M_N + g_s\,s + \boxed{\frac{1}{2}\kappa_{NS}\left(s^2 + \frac{s^3}{3\,F_{\pi}}\right)}$$

Nucleon polarization

- Nucleon response to the the (nuclear) scalar field: Re-adjustment of the quark wave function
- Restores the nuclear matter saturation.

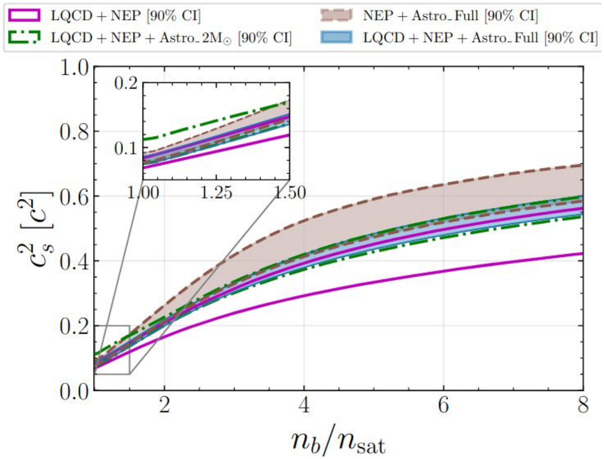
- **G. Chanfray, M. Ericson, EPJA 25, 151 (2005) ,**
- **G. Chanfray, H. Hansen and J. Margueron, EPJA 59, 11 (2023),**
- **Chanfray et al, Symmetry 17, 313 (2025)**

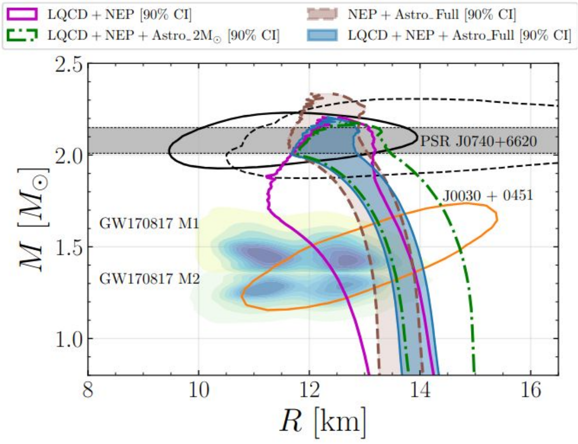
Chiral Confining Model : In light of Neutron Star and/or LQCD constraints

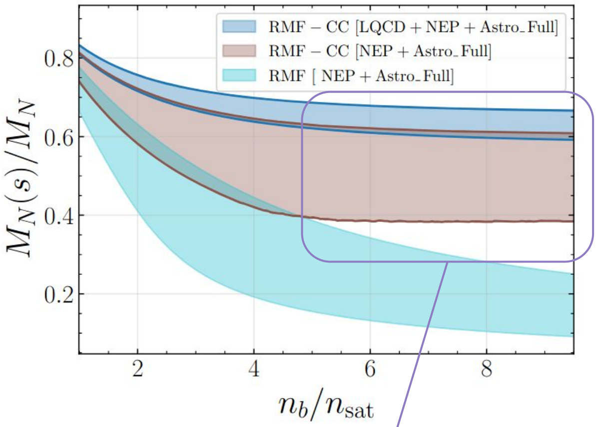
Lattice QCD (LQCD)

$$M_N(m_\pi^2) = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \cdots + \Sigma_\pi$$

$$\sigma_N \equiv m_\pi^2 \frac{dM_N}{d(m_\pi^2)} = a_2 m_\pi^2 + 2a_4 m_\pi^4 + \cdots + m_\pi^2 \frac{d\Sigma_\pi}{d(m_\pi^2)}$$







Confinement/polarization effect

- Confinement does not allow sufficient reduction in in-medium mass.
- For $2M_{sun}$, stiff EoSs are obtained with K_{sat} ~ 300 MeV.
- LQCD has strong constraints on V(s).
- Future works : Beyond mean field, inclusion of hyperons, deconfinement

Oscillation Modes in Binary Neutron Star Systems: Implications for Nuclear Physics and tidal corrections

Bikram Keshari Pradhan

In Collaboration with
Tathagata Ghosh, Dhruv Pathak and Debarati Chatterjee

- *Astrophys.J. 966 (2024) 1, 79,*

Neutron Star in Binary System and Oscillation modes :

- Tidal distortion dominated by f-mode.

$$\vec{\xi}(\mathbf{r},t) = \sum_n a_n(t)\vec{\xi}_n(r)$$

(Lai D., 1994, MNRAS, 270, 611)

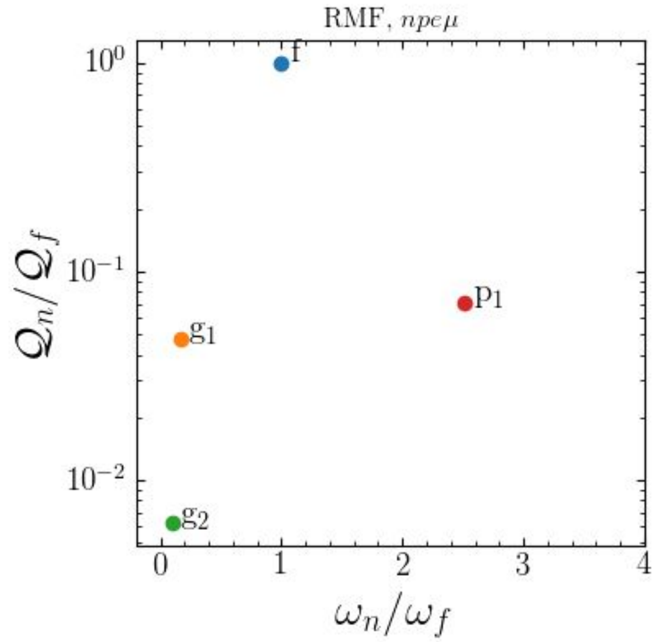
$$\ddot{a}_n + \omega_n^2 a_n = -\int d^3x \; \rho \; \vec{\xi}_{nl}^{\,*} \, . \, \nabla U = \frac{M_B W_{lm}}{D^{l+1}} Q_{nl} e^{-im\Phi(t)}$$

$$E_{tide}(t) = \frac{1}{2}\sum_n \left[|\dot{a}_n(t)|^2 + \omega_n^2 |a_n(t)|^2\right] = \sum_n E_n(t)$$

$$\mathcal{L}_N = \frac{1}{4\lambda_{2,A}\omega_f^2}\left[\frac{\mathrm{d}Q_{ij}}{\mathrm{d}t}\frac{\mathrm{d}Q_{ij}}{\mathrm{d}t} - \omega_f^2 Q_{ij}Q_{ij}\right] - \frac{1}{2}Q_{ij}\mathcal{E}_{ij}$$

$$Q_{ij} = -\lambda_2 \mathcal{E}_{ij}$$

Flanagan and Hinderer, *PRD 77 (2008) 021502*



- Complete GW: Orbital dynamics+internal dynamics (L_N)

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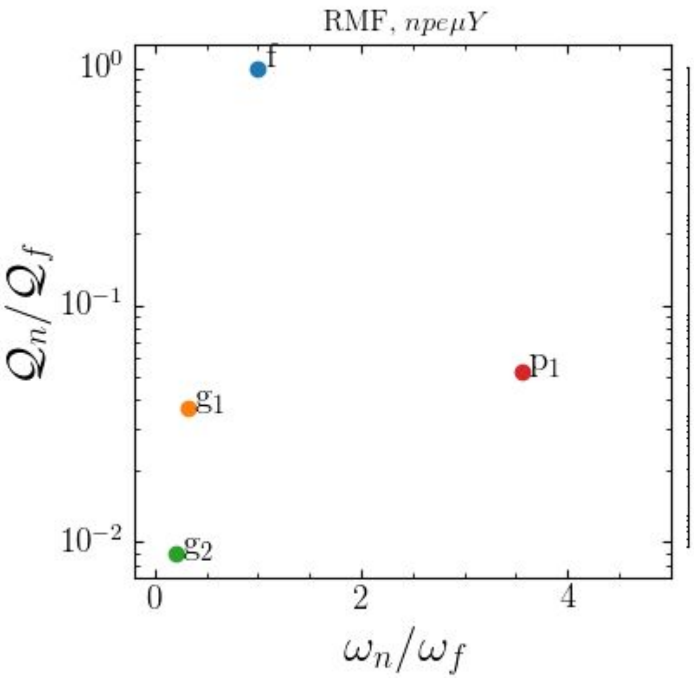

- Adiabatic limit: $1/\omega_f << 1/\omega_{GW}$.
- Finite frequency corrections to f-mode tidal correction : dynamical f-mode tide (P. Schmidt, & T. Hinderer 2019, PRD, 100,021501(R))
- Interface Modes in Inspiralling Neutron Stars. A.R. Counsell, F. Gittins+, *PRL*. 135 (2025) 8, 081402

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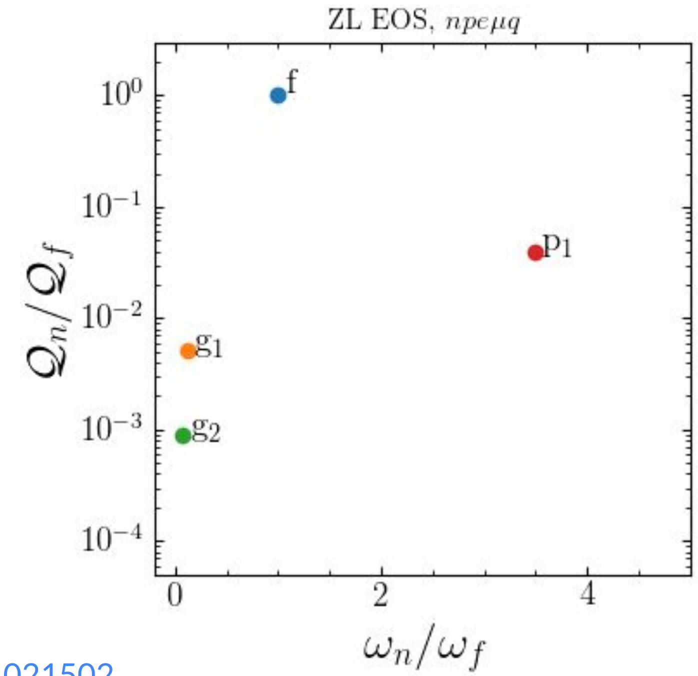

Neutron Star in Binary System and Oscillation modes

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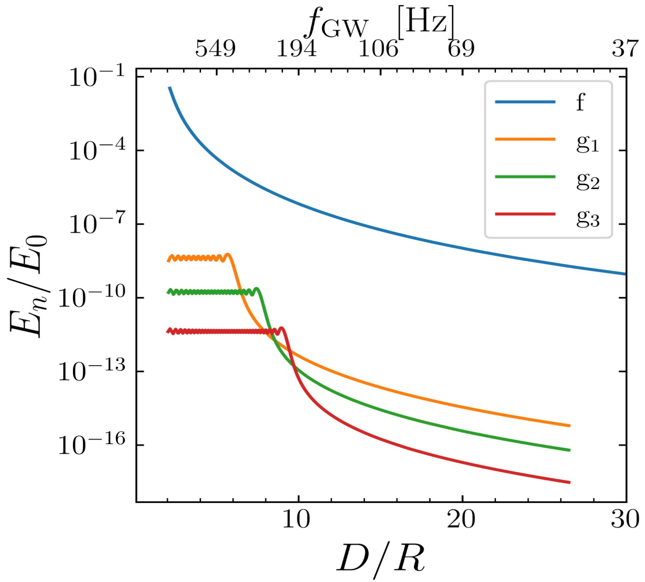
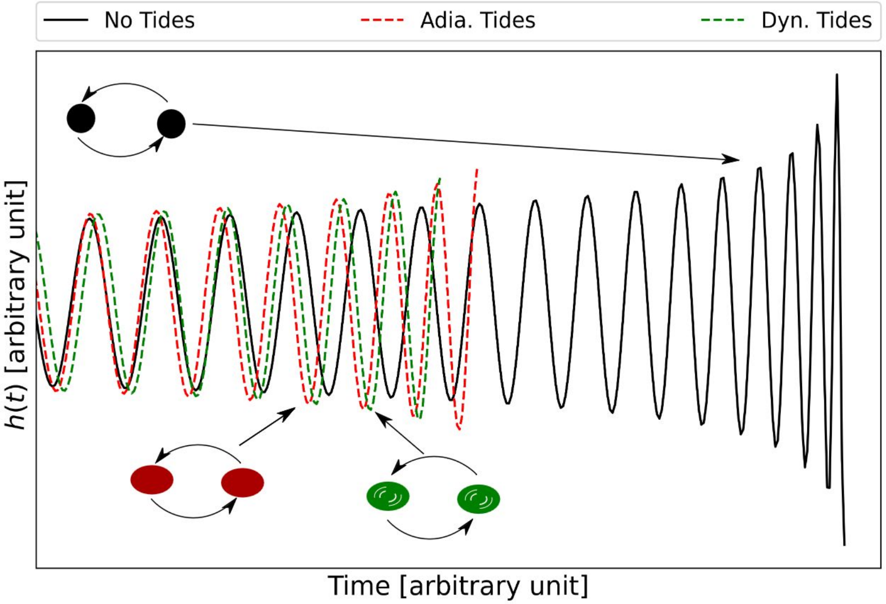


- ❑ **Presentation by Roque Márquez Rodríguez**

- f-mode tidal energy is dominated so other modes are ignored
- Adiabatic limit: *f*> *1* kHz. Implies $1/\omega_f << 1/\omega_{GW}$.
- Dynamical tide: Finite freq. Correction
- Interface Modes in Inspiralling Neutron Stars. A.R. Counsell, F. Gittins+, *PRL*. 135 (2025) 8, 081402
- Nuclear EoS depends on the inferred Tidal parameters and Hence on the GW waveform model

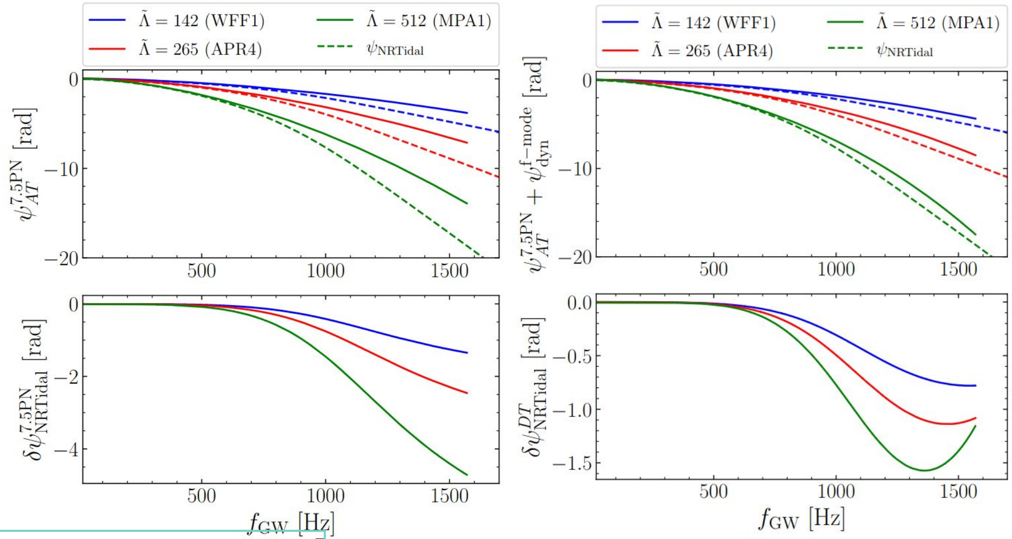
f-mode Dynamical tides in Binary Neutron Star System



$$h(f) = A(f)e^{i(\Psi_{pp}+\Psi_{AT}+\Psi_{dyn})}$$

$$\Psi_{AT}(\Lambda_2, ..) \text{ or, } \Psi_{AT}(\tilde{\Lambda}, ..)$$

$$\Psi_{dyn}(\Lambda_2, m\omega_2)$$

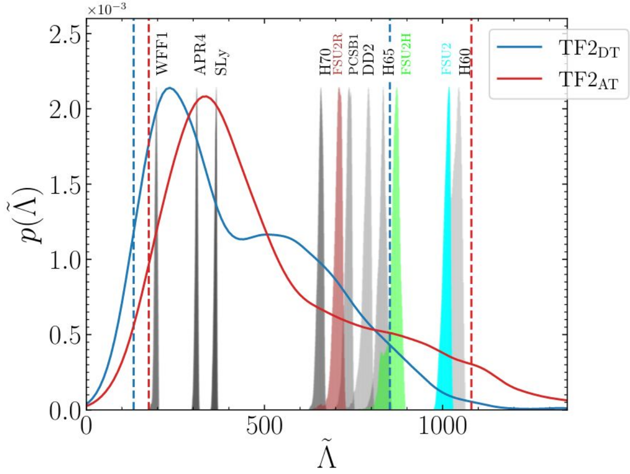


- GW waveform model used : Taylor F2
- F-mode dyn. correction, P. Schmidt, & T. Hinderer 2019, PRD, 100,021501(R),

$$\boxed{\tilde{\Lambda} = \frac{8}{13}\left[(1+7\eta-31\eta^2)(\Lambda_{2,1}+\Lambda_{2,2}) + \sqrt{1-4\eta}(1+9\eta-11\eta^2)(\Lambda_{2,1}-\Lambda_{2,2})\right]}$$

Thesis, B.K. Pradhan

GW170817



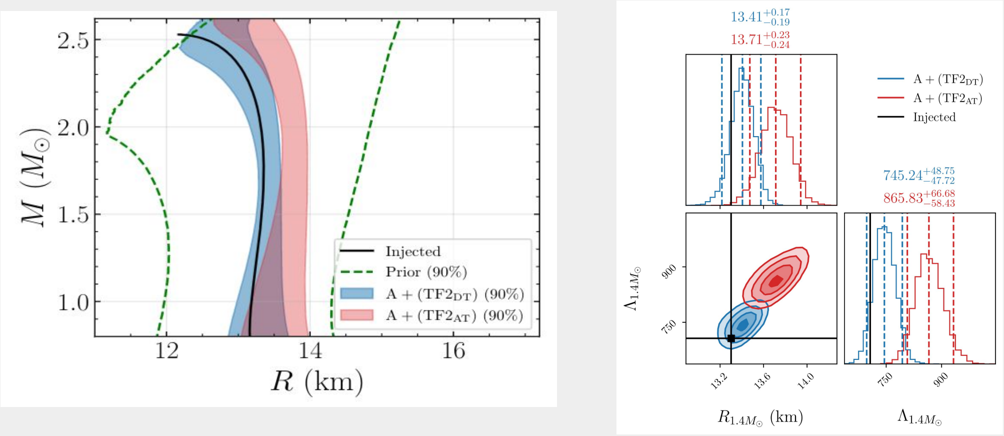
- ❖ **For GW170817, no significant impact.**
- ❖ **Important for future observations.**
- ❖ **<u>Higher-order tidal (electric and magnetic) corrections :</u>**

➢ *B. K. P, A. Vijaykumar, and D. Chatterjee, [PRD 107 (2023) 2, 023010](),*

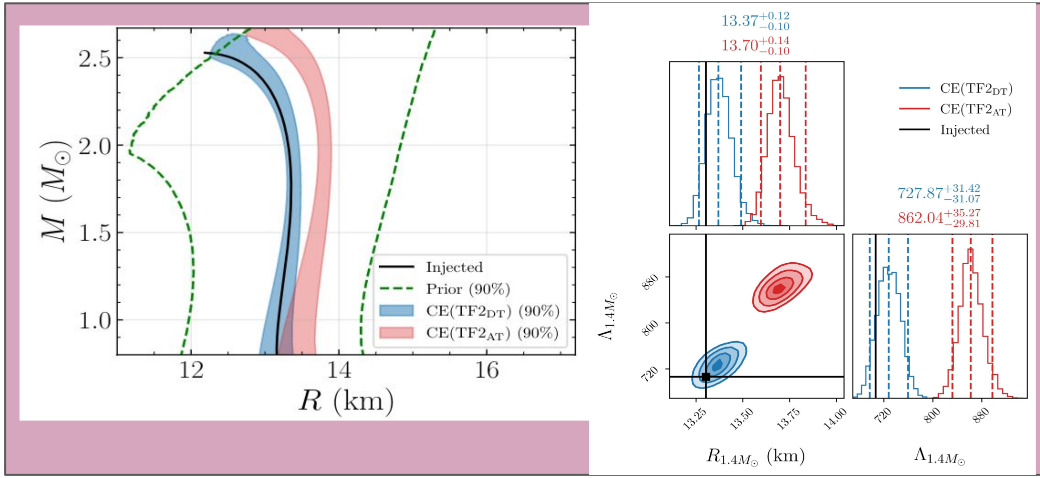
➢ *For tidal inference, Waveform models with dynamical tidal correction are important,*

- ○ ***SEOBV4NRT***
- ○ ***IMRPhenomXP_NRTidalv3 (talk by Presented by Adrian G. Abac)***

Future Events



LVK(O5)

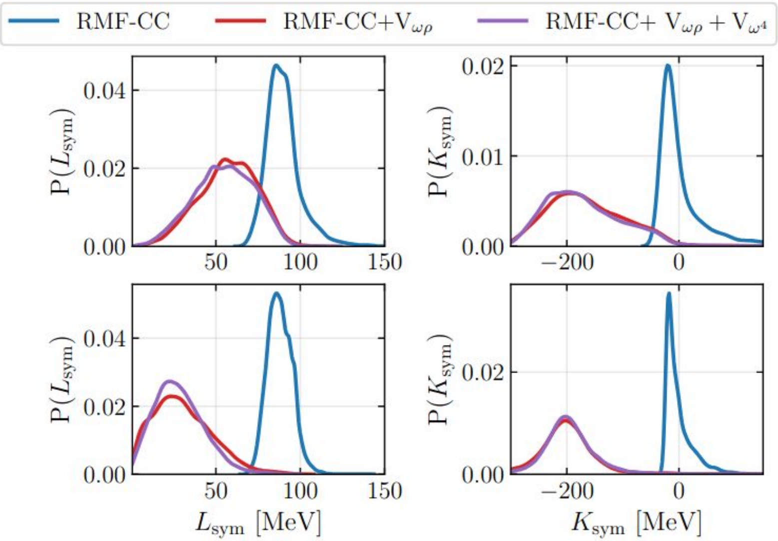


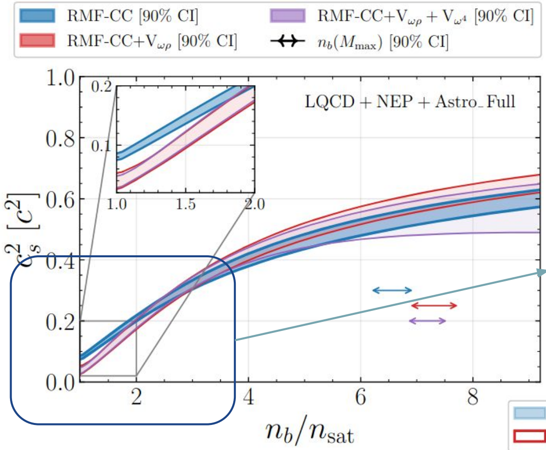
CE-CE

Thank You

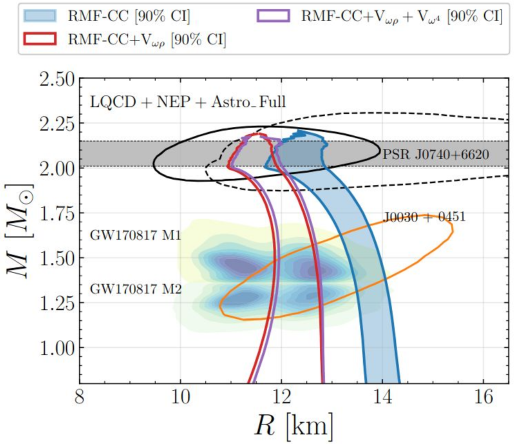
Improvement of the model compatibility with GW170817 : Phenomenologically

$$-\lambda_{\omega\rho}(g_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu)(g_\omega^2 \omega_\mu \omega^\mu)$$

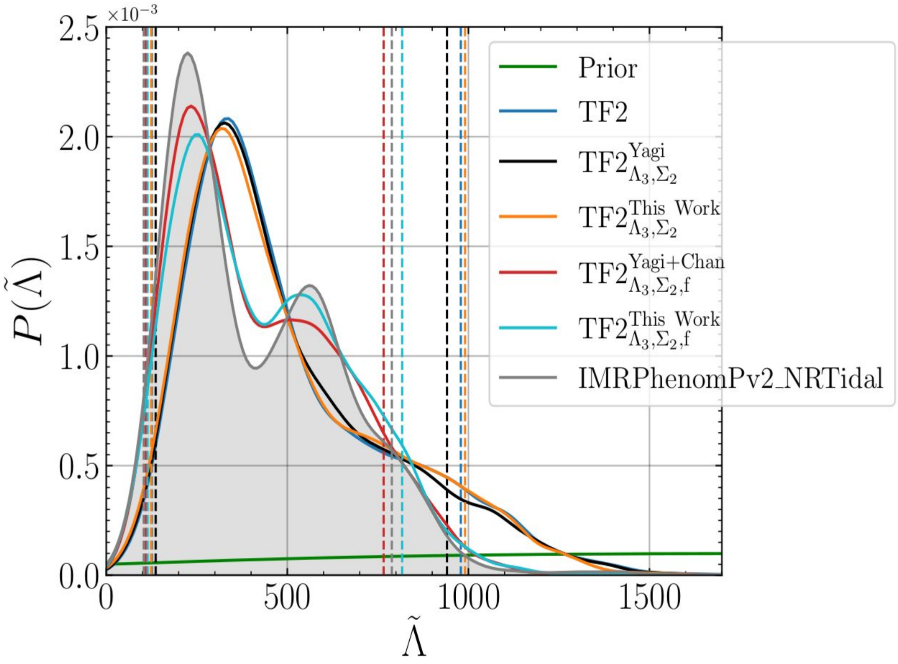




Changes the slope and stiffness



Results: GW170817



- Inclusion of f-mode dynamical phase lowers the 90% upper bound of \tilde{\Lambda} by ~15%.
- Λ_3 and $\mathbf{\Sigma}_2$ or the choice of multipole Love relation has no significant effect