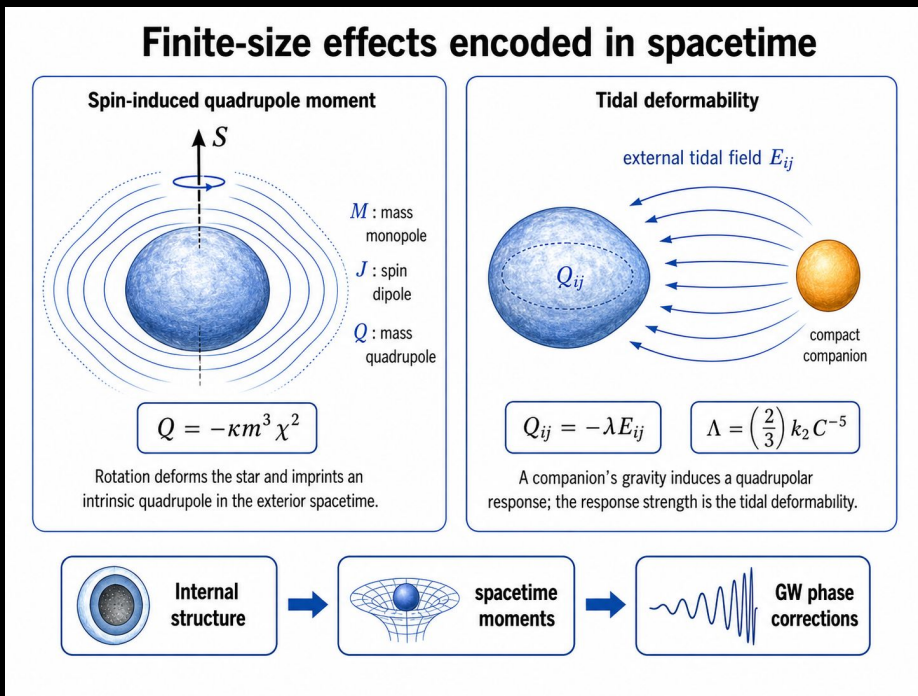
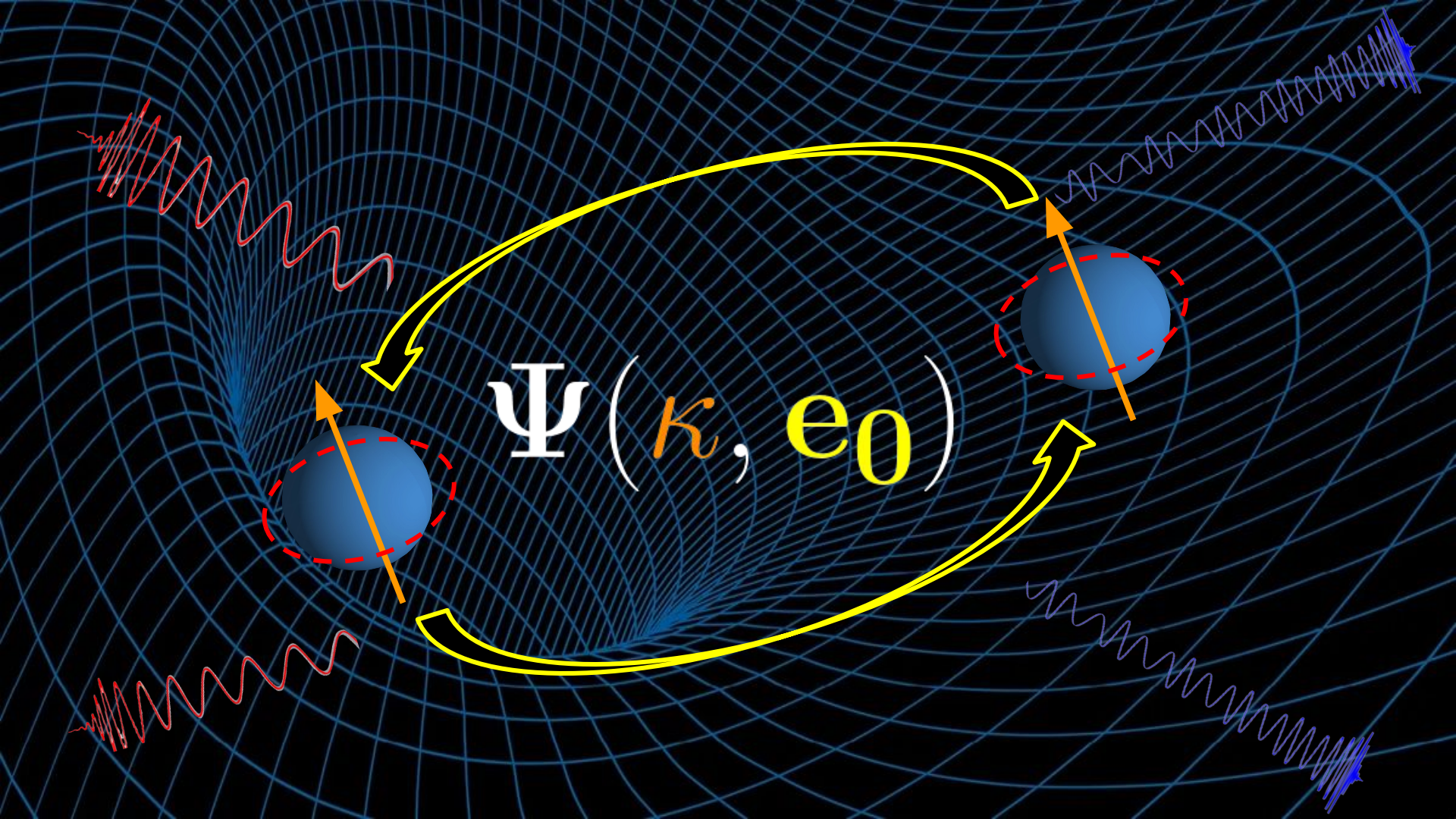


The story of spinning Compact objects and Eccentricity



Syed Naqvi, Post-Doc
Institute of Nuclear Physics Polish Academy of Sciences

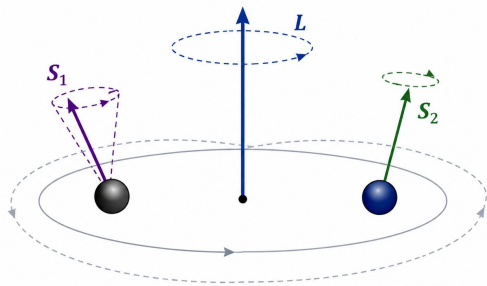


Theory: $h = Ae^{i\Psi}$

Physical effects entering the GW phase

$$\Psi(\mathbf{f}) = \Psi_{\text{pp}}(\mathbf{f}) + \Psi_{\text{SO}}(\mathbf{f}) + \Psi_{\text{SS}}(\mathbf{f}) + \delta\Psi_{\text{tidal}}(\mathbf{f}; \tilde{\Lambda})$$

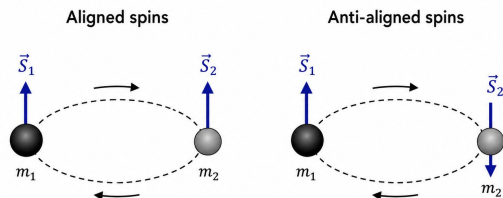
Spin-Orbit Effect



Coupling between orbital angular momentum and spins modifies the GW phase and can induce precession.

$$\mathbf{L} \cdot \mathbf{S}_1 / \mathbf{L} \cdot \mathbf{S}_2$$

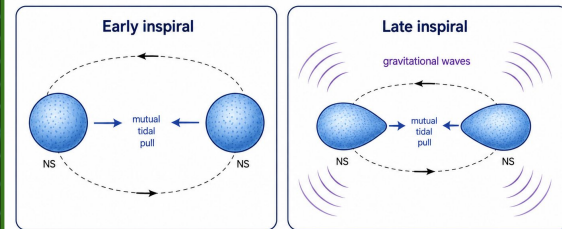
Spin-Spin Effect



Coupling between $\mathbf{S}_1$ and $\mathbf{S}_2$ changes the binding energy and gravitational-wave phasing.

$$\mathbf{S}_1 \cdot \mathbf{S}_2$$

Tidal Effect



★ Tidal effects enter at **5PN**, but become important near merger and carry **EOS information**.

Finite-size deformation is encoded through the tidal deformability Λ .

$$\Lambda \sim \text{Equation of State}$$

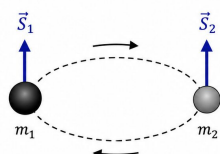
Theory: $h = Ae^{i\Psi}$

Physical effects entering the GW phase

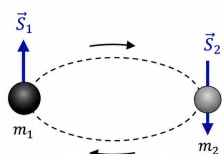
$$\Psi(\mathbf{f}) = \Psi_{\text{pp}}(\mathbf{f}) + \Psi_{\text{SO}}(\mathbf{f}) + \Psi_{\text{SS}}(\mathbf{f}) + \delta\Psi_{\text{tidal}}(\mathbf{f}; \tilde{\Lambda})$$

Spin-Spin Effect

Aligned spins



Anti-aligned spins



Coupling between S_1 and S_2 changes the binding energy and gravitational-wave phasing.

κ appears at 2PN, 3PN, 3.5PN

$\kappa = 1$, BH

$\kappa = [2-10]$ N.S

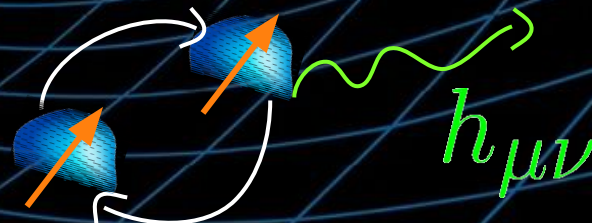
$\kappa = [50-100]$ Boson Star

Omkar et al:

PRD D 112, 024026

$$\mathbf{S}_1 \cdot \mathbf{S}_2$$

$$\Psi_{\text{spin-circ}} + \Psi_{\text{spin-ecc}}$$



Data: $h = \frac{\Delta L}{L}$

SIQM-Ecc : Population Level Analysis

1 Inputs

- ★ Mass range and population model
- 🌀 Redshift distribution
- 📊 Detector PSD $S_n(f)$
- 📡 Waveform model: eccentric + SIQM
- ✅ Detection threshold

2 Monte Carlo population draw

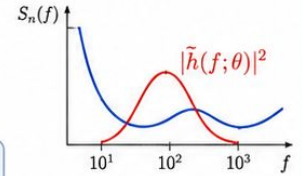
- Sample m_1, m_2
- Sample z
- Compute luminosity distance $D_L(z)$



3 Signal and SNR filter

- Build $h(f; \theta)$
- Compute optimal SNR
- Keep only detectable events

$$\rho_i^2 = 4 \int_{f_{\min}}^{f_{\max}} \frac{|\tilde{h}(f; \theta)|^2}{S_n(f)} df$$



- ✅ Keep detectable events
- ❌ Reject low-SNR events

4 Fisher matrix step

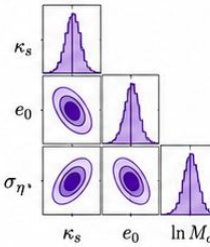
- Derivatives of waveform
- Construct $\Gamma_{ij} = \langle \frac{\partial h}{\partial \theta_i}, \frac{\partial h}{\partial \theta_j} \rangle$
- Invert to covariance $\Sigma = \Gamma^{-1}$

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1N} \\ \Gamma_{21} & \Gamma_{22} & \cdots & \Gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{N1} & \Gamma_{N2} & \cdots & \Gamma_{NN} \end{bmatrix} \rightarrow \Sigma = \Gamma^{-1}$$

Parameter errors are estimated here

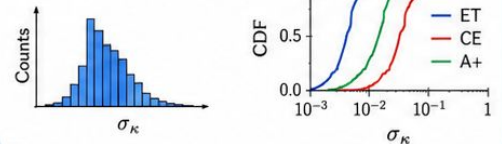
5 Extract parameters

- σ_{κ}
- σ_{e_0}
- Correlations
- Optionally $\sigma_{M_c}, \sigma_{\eta}$



6 Population results

- Save successful events
- Build histograms / CDFs
- Compare detector performance
- Final measurability of SIQM and eccentricity



Parameter set



$\{t_c, \phi_c,$
 $\ln M_c, \ln \eta,$
 $\kappa_s, e_0\}$

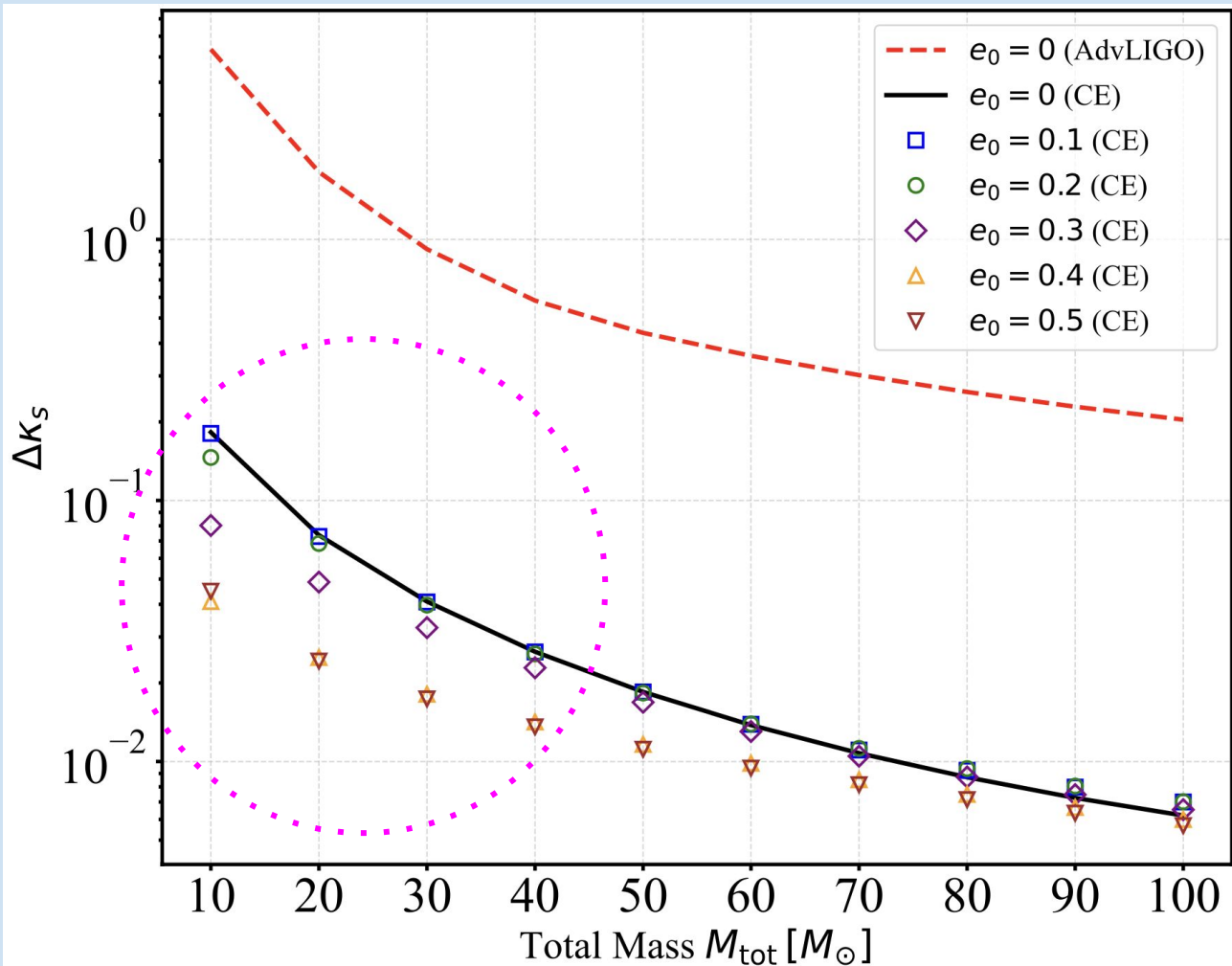
SIQM-Ecc

Waveform: e_0^8

for CE vs advLIGO

For higher eccentric case:
Large number of PN cycles =>
allows eccentric binaries to
imprint additional harmonics
over many cycles.

Additional phase information,
leading to slightly smaller
uncertainties compared to the
circular case.



PN Waveform: General overview + where **SIQM** Enters

GW Waveform (Freq. Domain)

$$\tilde{h}(f) = A\psi(f)$$

$$\Psi(f) = \frac{3}{128\eta v^5} \left\{ 1 + v^2 \left[\text{1 PN} \right] + v^3 \left[\text{1.5PN } (\beta) \right] + v^4 \left[\text{2 PN } (\sigma) \right] + v^5(\dots) \right\}$$

$$v = (\pi M f)^{1/3}$$

$$\eta = \mu/M \quad \left\{ \begin{array}{l} M = \text{Total Mass} \\ \eta = \text{symm mass-ratio} \end{array} \right\}$$

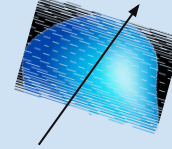
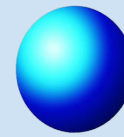
Spin-Orbit: $\beta \sim \sum_A \hat{\mathbf{L}} \cdot \chi_A$

Spin-Spin: $\sigma = \sigma_{SS} + \sigma_{qm}$
 $[\sigma_{SS} \sim \chi_1 \cdot \chi_2]$

κ appears at 2PN, 3PN, 3.5PN

$$\left\{ Q_A = -\kappa \chi_A^2 m_A^3 \right\}$$

GWs: The case for SIQM



The spinning motion of companion A creates a distortion in its mass distribution => creates a distortion in the gravitational field outside the star, measured by $(Q_A)^{ij}$

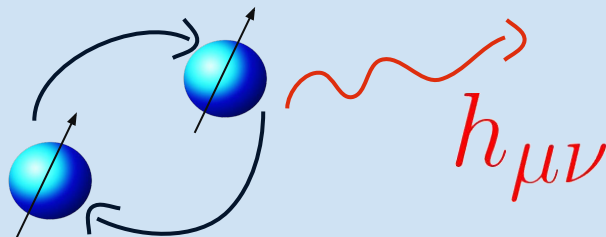
The quadrupole term, $(Q_A)^{ij}$, in the gravitational potential affects the orbital motion of the companions, and it affects also the emission of GWs.

Poisson '98, Kidder '95

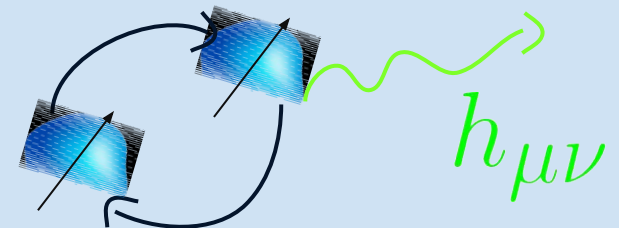
$$\left\{ Q_A = -\kappa \chi_A^2 m_A^3 \right\}$$

$$\left\{ \begin{array}{l} \kappa = 1, \text{ BH} \\ \kappa = [2-10] \text{ N.S} \\ \kappa = [50-100] \text{ Boson Star} \end{array} \right\}$$

Kappa: Spin-induced moment constant



SIQM: Testing the Compact binary nature



Inspiral Waveform: Adding Kappa-Ecc Terms*

- Inspiral Waveform : an update to the 3PN eccentric phasing (**Moore et al**)
- **Omkar et al** aligned-spin effects to the small **eccentricity** expanded time and freq domain phases
- For e_0^2 valid till $e_0=0.3$... e_0^8 valid till $e_0=0.5$

$$\Psi = \frac{3}{128\nu y^5} \left\{ \Psi_{\text{circ}} + \frac{650}{731} e_0^2 \left(\frac{y_0}{y} \right)^{19/3} \Psi_{\text{ecc+kappa}} + e_0^4(\dots) \right\}$$

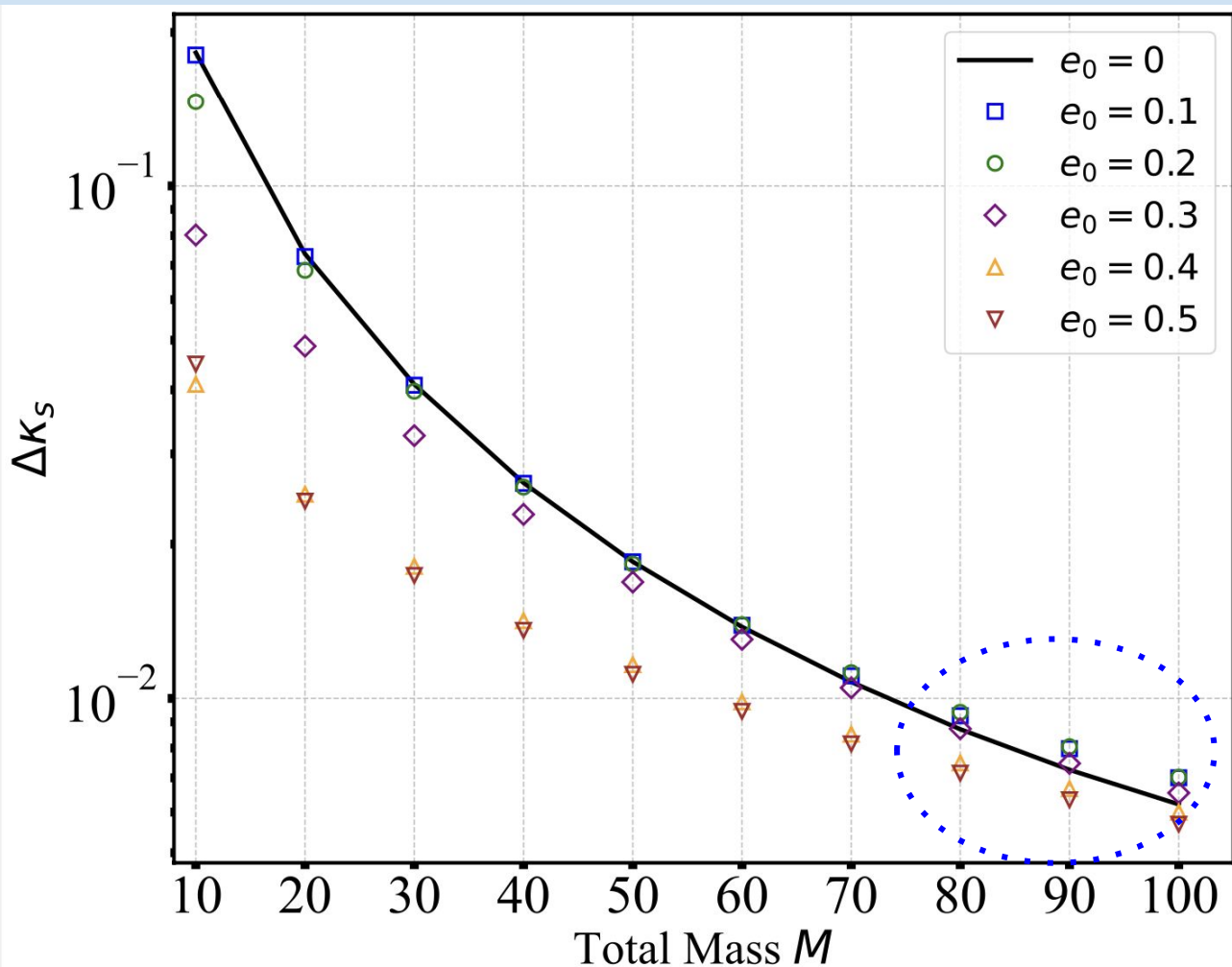
Note: A green oval containing κ_1, κ_2 is connected by dotted lines to the $\Psi_{\text{ecc+kappa}}$ term and the e_0^2 term in the equation.

Aim

- Kappa errors for ET and Cosmic Explorer and compare with adv. LIGO
- Define symmetric and anti-symm combination: $\kappa_S = (\kappa_1 + \kappa_2)/2$
(For BBH: $\kappa_S=1, \kappa_A=0$)

SIQM-Ecc Waveform: e_0^8 for CE

Inspiral contributes only a few PN cycles before merger, leaving little opportunity for eccentricity-driven harmonics to accumulate in the waveform.



Comparison of σ_K CDFs: ET vs CE (Circular vs Eccentric)

