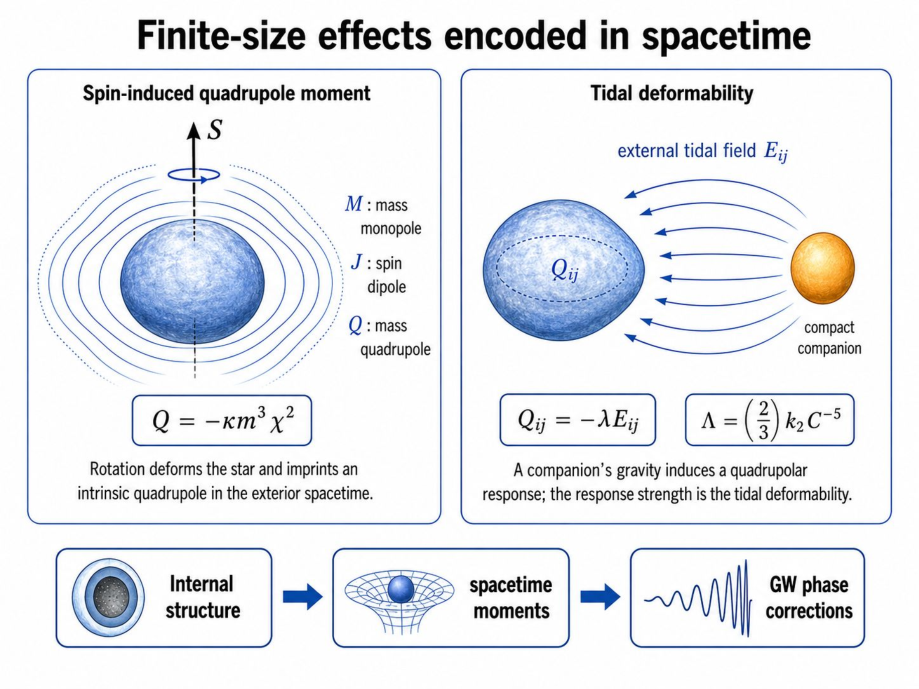
The story of spinning
Compact objects and Eccentricity



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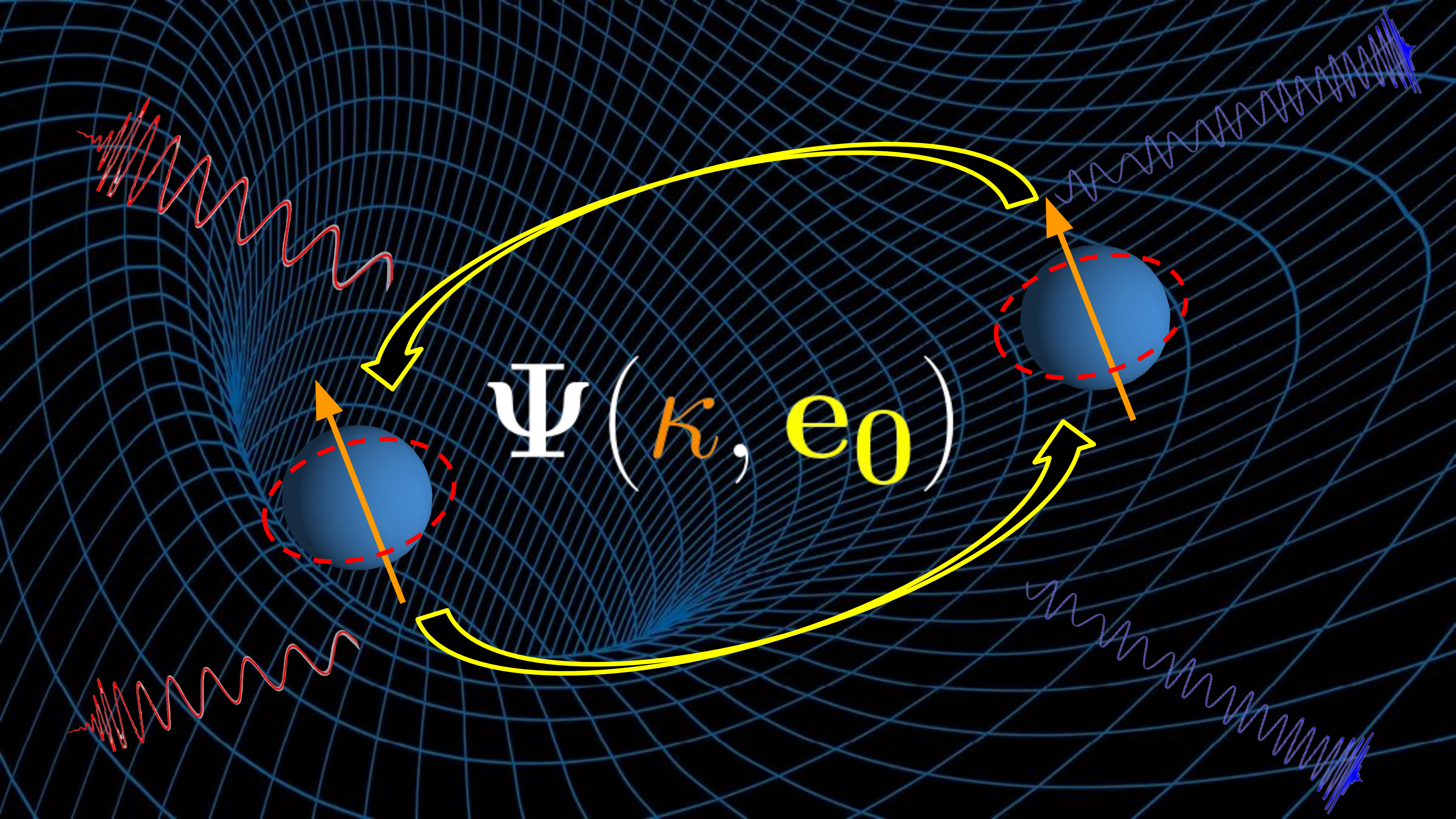
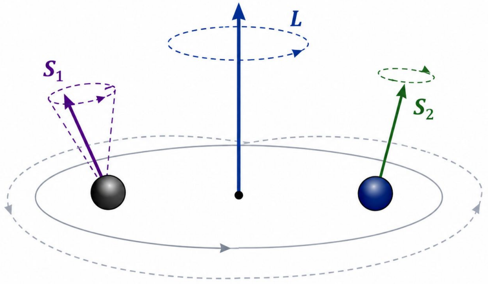

$\Psi(\kappa, \mathbf{e}_0)$

Theory: $h = Ae^{i\Psi}$

Physical effects entering the GW phase

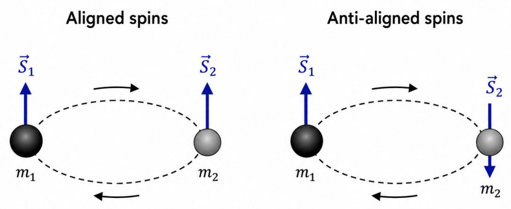
$$\Psi(\mathrm{f}) = \Psi_{\mathrm{pp}}(\mathrm{f}) + \Psi_{\mathrm{SO}}(\mathrm{f}) + \Psi_{\mathrm{SS}}(\mathrm{f}) + \delta\Psi_{\mathrm{tidal}}(\mathrm{f}; \tilde{\Lambda})$$

Spin–Orbit Effect



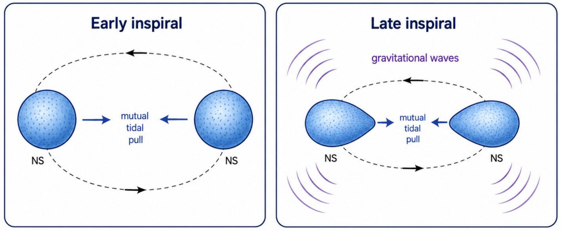
Coupling between orbital angular momentum and spins modifies the GW phase and can induce precession.

Spin–Spin Effect



Coupling between S_1 and S_2 changes the binding energy and gravitational-wave phasing.

Tidal Effect



★ Tidal effects enter at **5PN**, but become important near merger and carry **EOS information**.

Finite-size deformation is encoded through the tidal deformability Λ.

$L.S_1/L.S_2$

$S_1.S_2$

$\Lambda \sim$ Equation of State

Theory: $h = Ae^{i\Psi}$

Physical effects entering the GW phase

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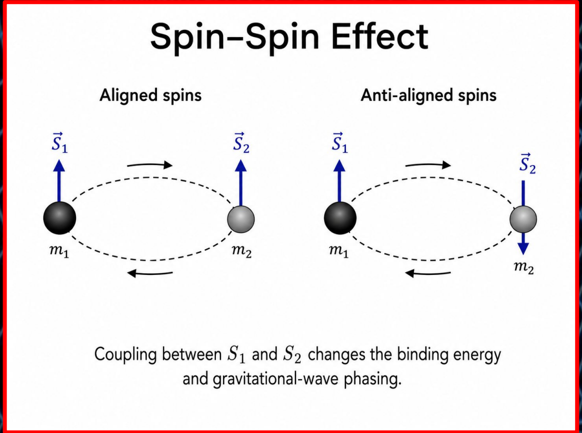


{κ appears at 2PN, 3PN, 3.5PN}

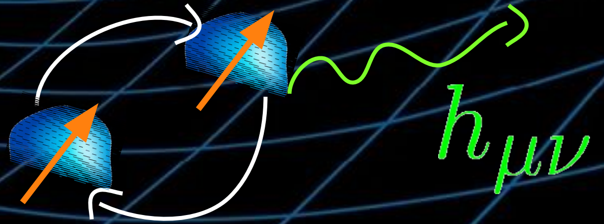
- κ = 1, BH
- κ = [2-10] N.S
- κ = [50-100] Boson Star

Omkar et al:

PRD D 112, 024026

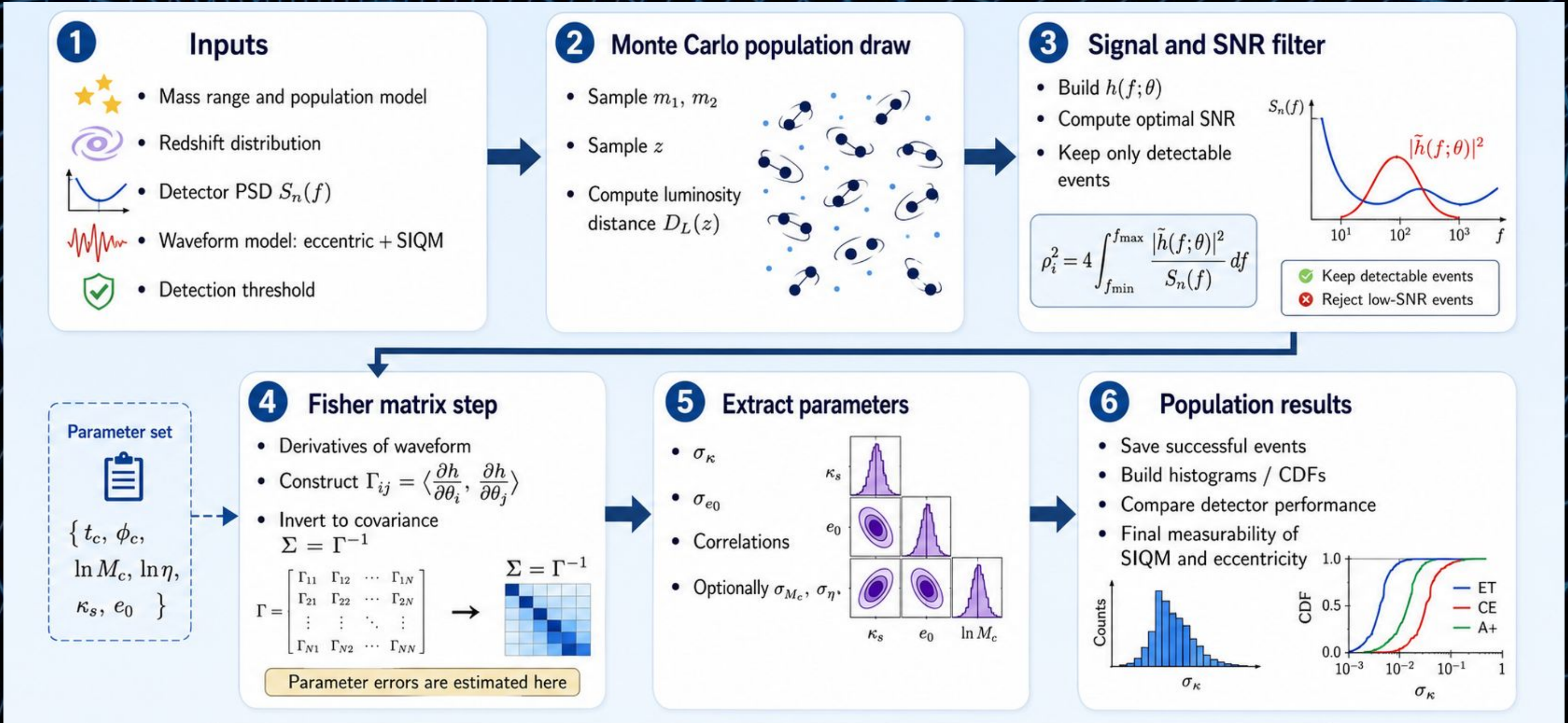
$\mathbf{S_1 \cdot S_2}$

$$\Psi_{\mathrm{spin-circ}} + \Psi_{\mathrm{spin-ecc}}$$



Data: $h = \frac{\Delta L}{L}$

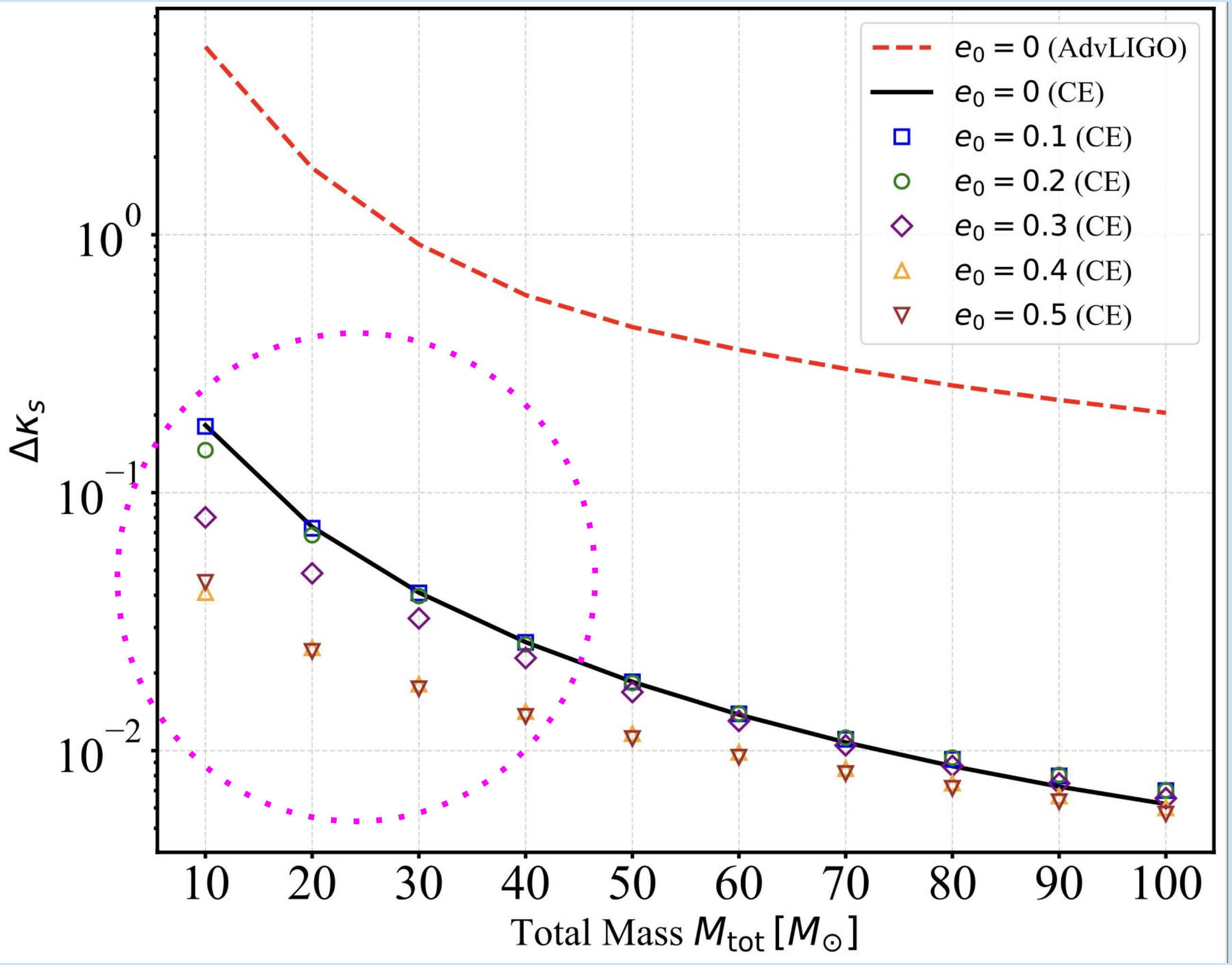
SIQM-Ecc : Population Level Analysis



SIQM-Ecc Waveform: e0 [8] for CE vs advLIGO

For higher eccentric case: Large number of PN cycles => allows eccentric binaries to imprint additional harmonics over many cycles.

Additional phase information, leading to slightly smaller uncertainties compared to the circular case.



PN Waveform: General overview + where **SIQM** Enters

GW Waveform (Freq. Domain)

$$\tilde{h}(f) = A\psi(f)$$

$$\Psi(f) = \frac{3}{128\eta v^5}\left\{1 + v^2\left[\ \text{1 PN}\ \right] + v^3\left[\ \text{1.5PN } (\beta)\ \right] + v^4\left[\ \text{2 PN } (\sigma)\ \right] + v^5(\ldots)\right.$$

$$v = (\pi M f)^{1/3}$$

$$\eta = \mu/M$$

M = Total Mass
η = symm mass-ratio

Spin-Orbit: $\beta \sim \sum_A \hat{\mathbf{L}}.\chi_{\mathbf{A}}$

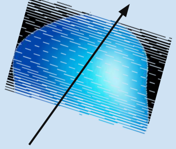
Spin-Spin: $\sigma = \sigma_{ss} + \boxed{\sigma_{qm}}$

$\sigma_{ss} \sim \chi_{\mathbf{1}} \cdot \chi_{\mathbf{2}}$

κ **appears at 2PN, 3PN, 3.5PN**

$$Q_A = -\kappa\chi_A^2 m_A^3$$

GWs: The case for SIQM



The spinning motion of companion A creates a distortion in its mass distribution => creates a distortion in the gravitational field outside the star, measured by $(Q_A)^{ij}$

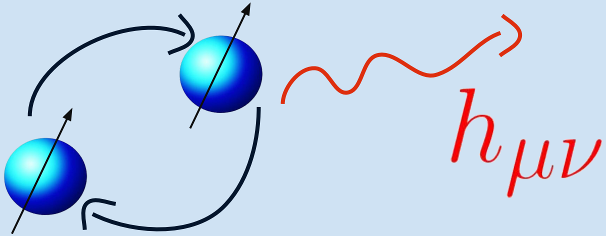
The quadrupole term, $(Q_A)^{ij}$, in the gravitational potential affects the orbital motion of the companions, and it affects also the emission of GWs.

Poisson '98, Kidder '95

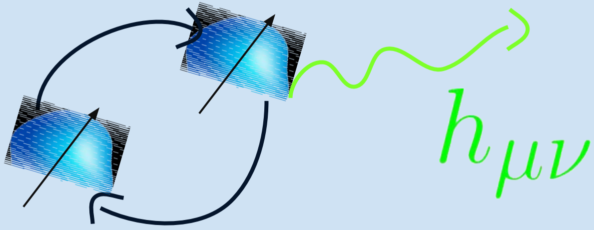
$$\left\{ Q_A = -\kappa \chi_A^2 m_A^3 \right\}$$

$$\left\{ \begin{array}{l} \kappa = 1, \text{ BH} \\ \kappa = [2\text{-}10] \text{ N.S} \\ \kappa = [50\text{-}100] \text{ Boson Star} \end{array} \right\}$$

Kappa: Spin-induced moment constant



SIQM:Testing the Compact binary nature



Inspiral Waveform: Adding Kappa-Ecc Terms*

- Inspiral Waveform : an update to the 3PN eccentric phasing (**Moore et al**)
- **Omkar et al** aligned-spin effects to the small eccentricity expanded time and freq domain phases
- For $e0^2$ valid till e0=0.3 . . . $e0^8$ valid till e0=0.5

κ_1, κ_2

$$\Psi = \frac{3}{128\nu y^5}\left\{\Psi_{\rm circ} + \frac{650}{731}\boxed{e_0^2}\left(\frac{y_0}{y}\right)^{19/3} \Psi_{\text{ecc+kappa}} + e_0^4(\ldots)\right.$$

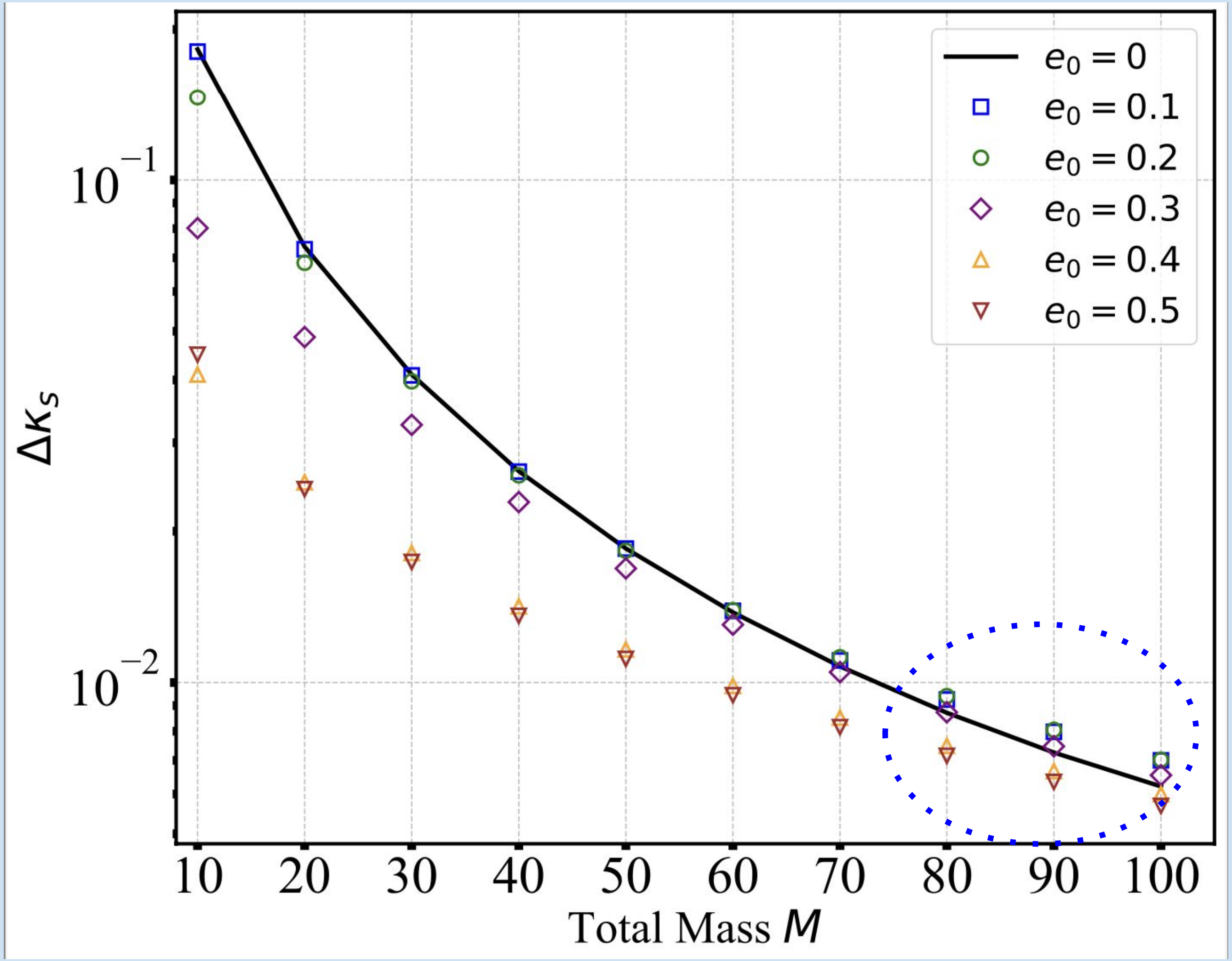
Aim

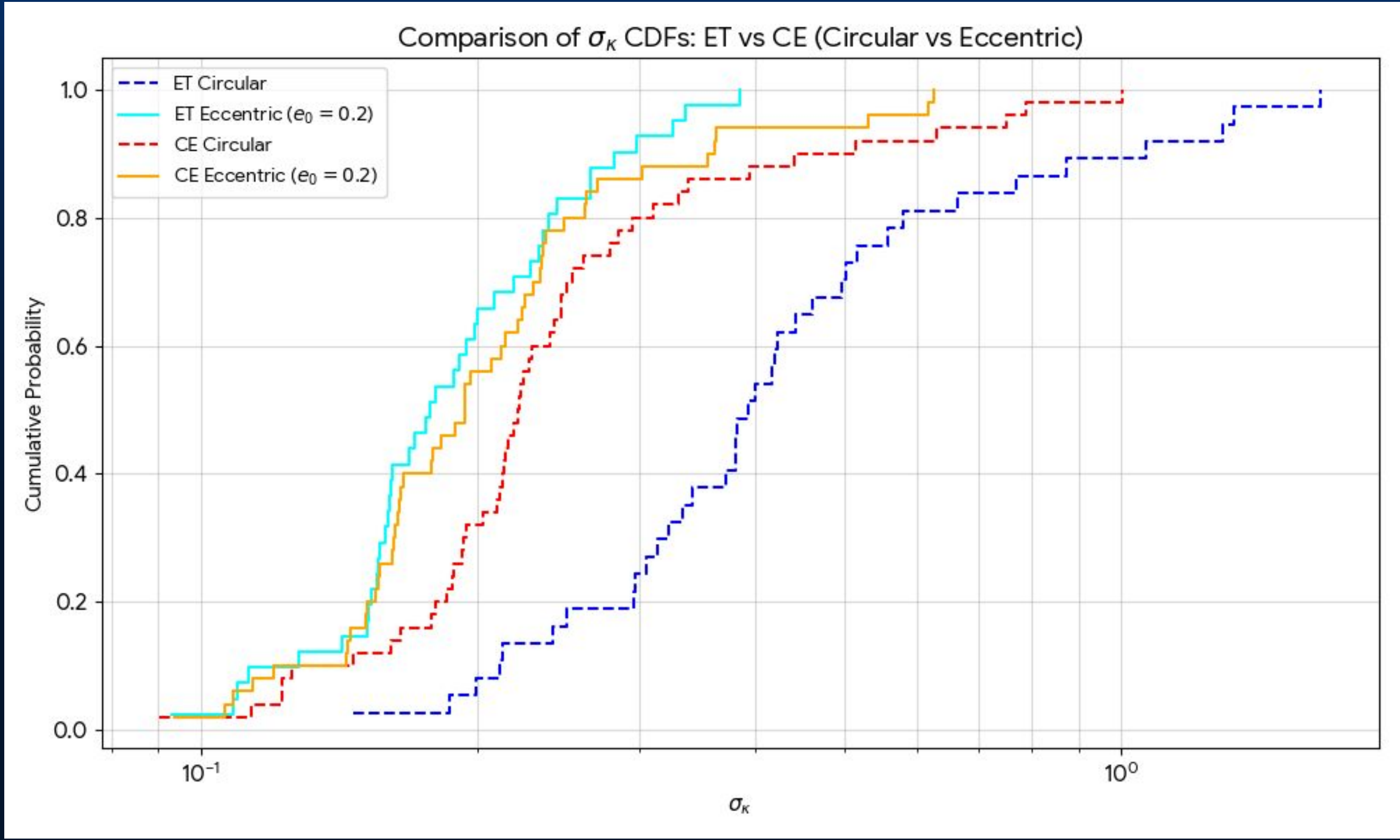
- Kappa errors for ET and Cosmic Explorer and compare with adv. LIGO
- Define symmetric and anti-symm combination: $\kappa_S = (\kappa_1 + \kappa_2)/2$
  (For BBH: κ_S=1, κ_A=0)

*Credit: Omkar, Soham Kaushik 2024

SIQM-Ecc Waveform: $e0^8$ for CE

Inspiral contributes only a few PN cycles before merger, leaving little opportunity for eccentricity-driven harmonics to accumulate in the waveform.





Comparison of σ_κ CDFs: ET vs CE (Circular vs Eccentric)
ET Circular
ET Eccentric ($e_0 = 0.2$)
CE Circular
CE Eccentric ($e_0 = 0.2$)
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
10^{-1}
10^0
σ_κ