Resonant Excitation of Neutron Star Modes in NS-NS/NS-BH Coalescing Binaries

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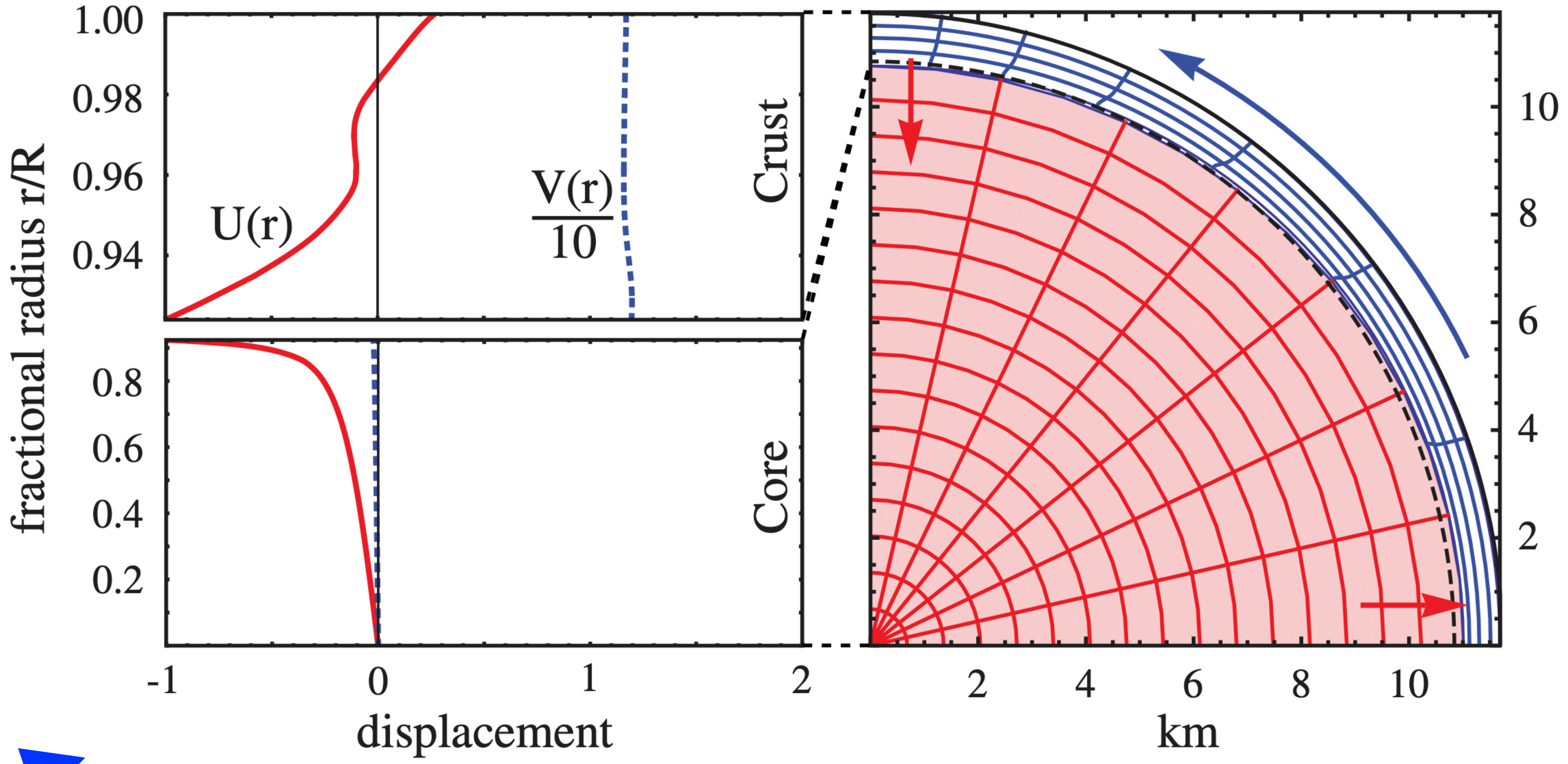
Thematic School GWsNS-2026: Gravitational Waves from Neutron Stars
Roscoff (France) — 1 July 2026

Neutron star (NS) oscillatory modes

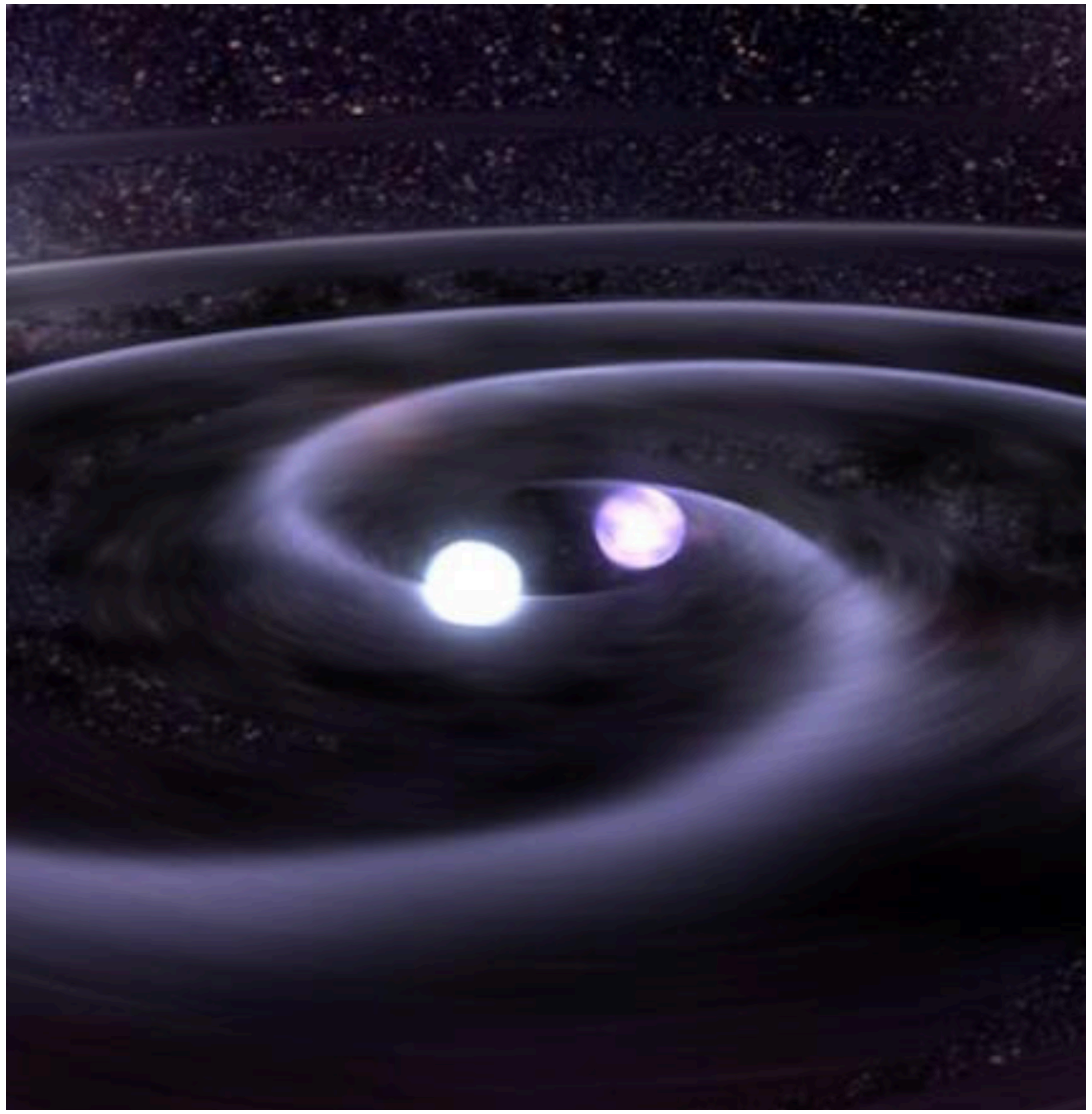
- ***Quasi-normal modes:*** NS oscillatory modes
- **Different kinds** with different **frequencies**
- Non-radial modes: **GW emission**
- **Example: *i*-modes**



Credit: D. Tsang et al. PRL 108, 011102 (2012)

NS mode resonant excitation in a NS-NS or NS-BH binary

- **Companion:** exerts a **tidal force** over the NS
- The **tidal force's frequency** can reach that of some NS modes, **resonantly exciting them**
- **Orbit -> mode energy transfers advance coalescence and leave phase imprints on GW waveforms**



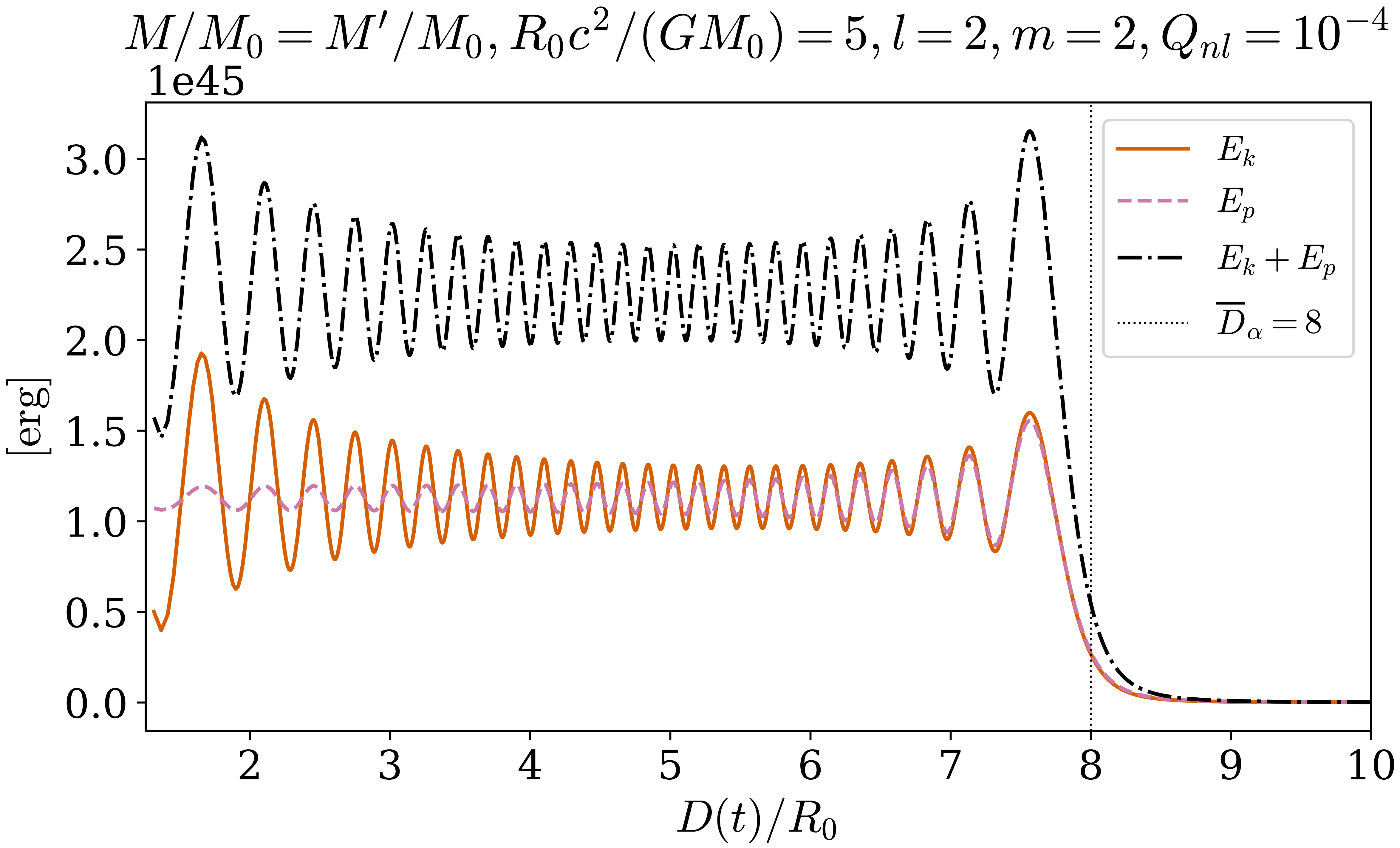
Credit: NASA / Goddard Space Flight Center

Resonant excitation of NS modes: approach

1. Computing **NS modes**

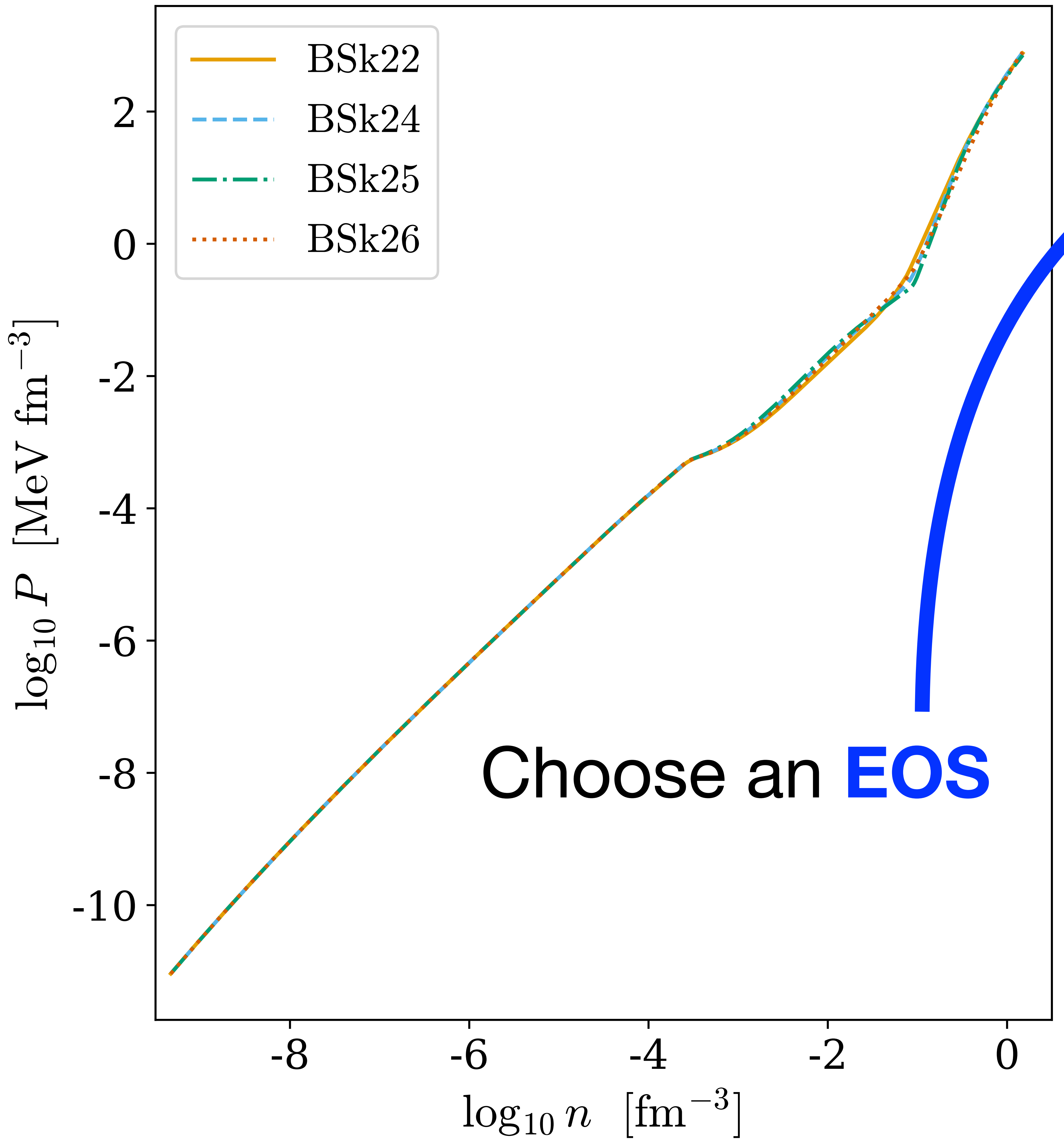
2. Quantifying the **resonant excitation** of NS modes

Newtonian formalism [D. Lai (1994)]

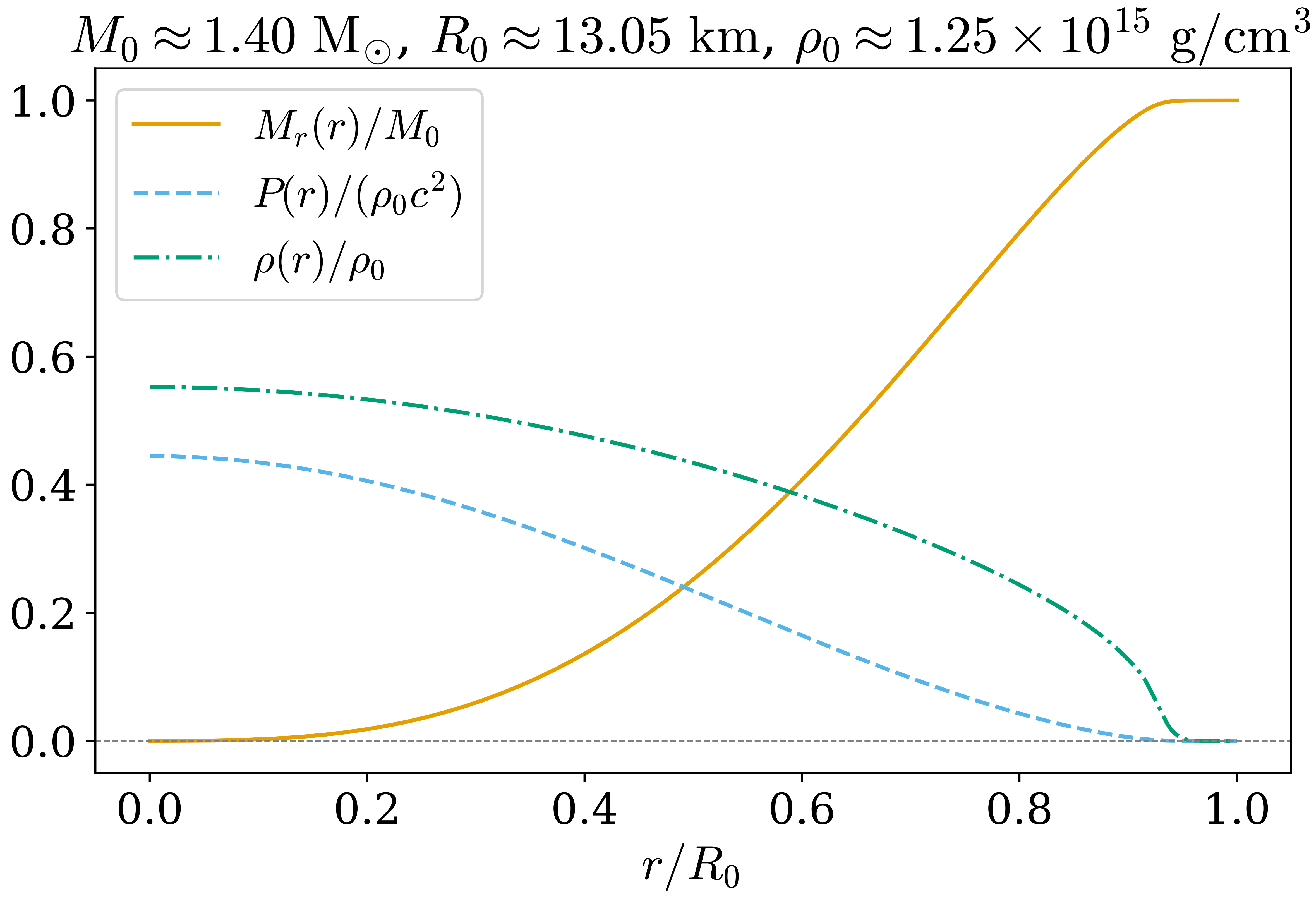


3. Estimating the **detectability** of NS mode resonances

1. EOS and NS background model



Solve the **TOV equations**



1. Compute NS modes via spectral collocation

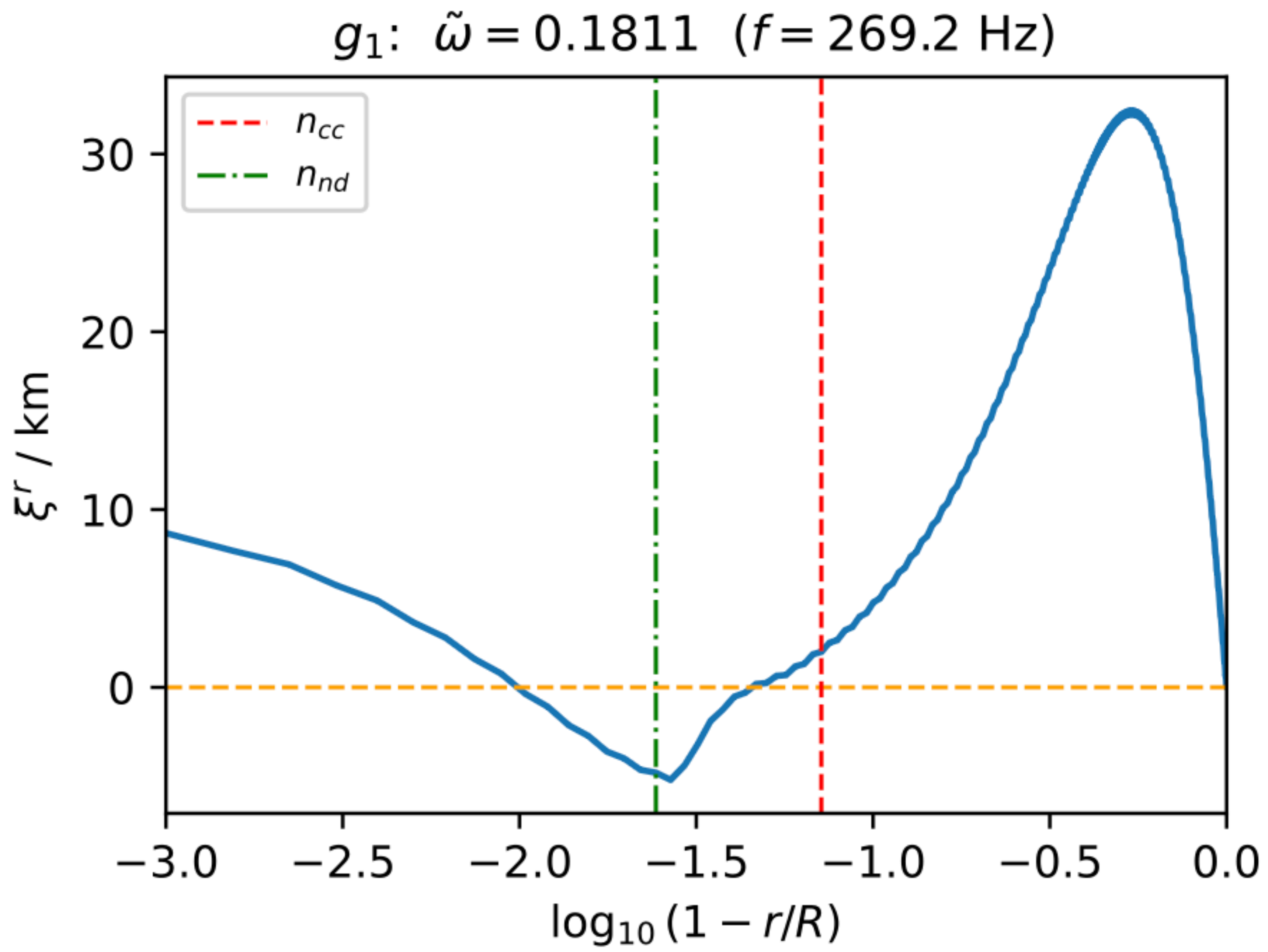
- **Linear adiabatic perturbations** of the **continuity and momentum conservation equations**
- **Cowling approximation:** no metric perturbations
- **Spectral collocation:** solve an **eigenvalue problem**

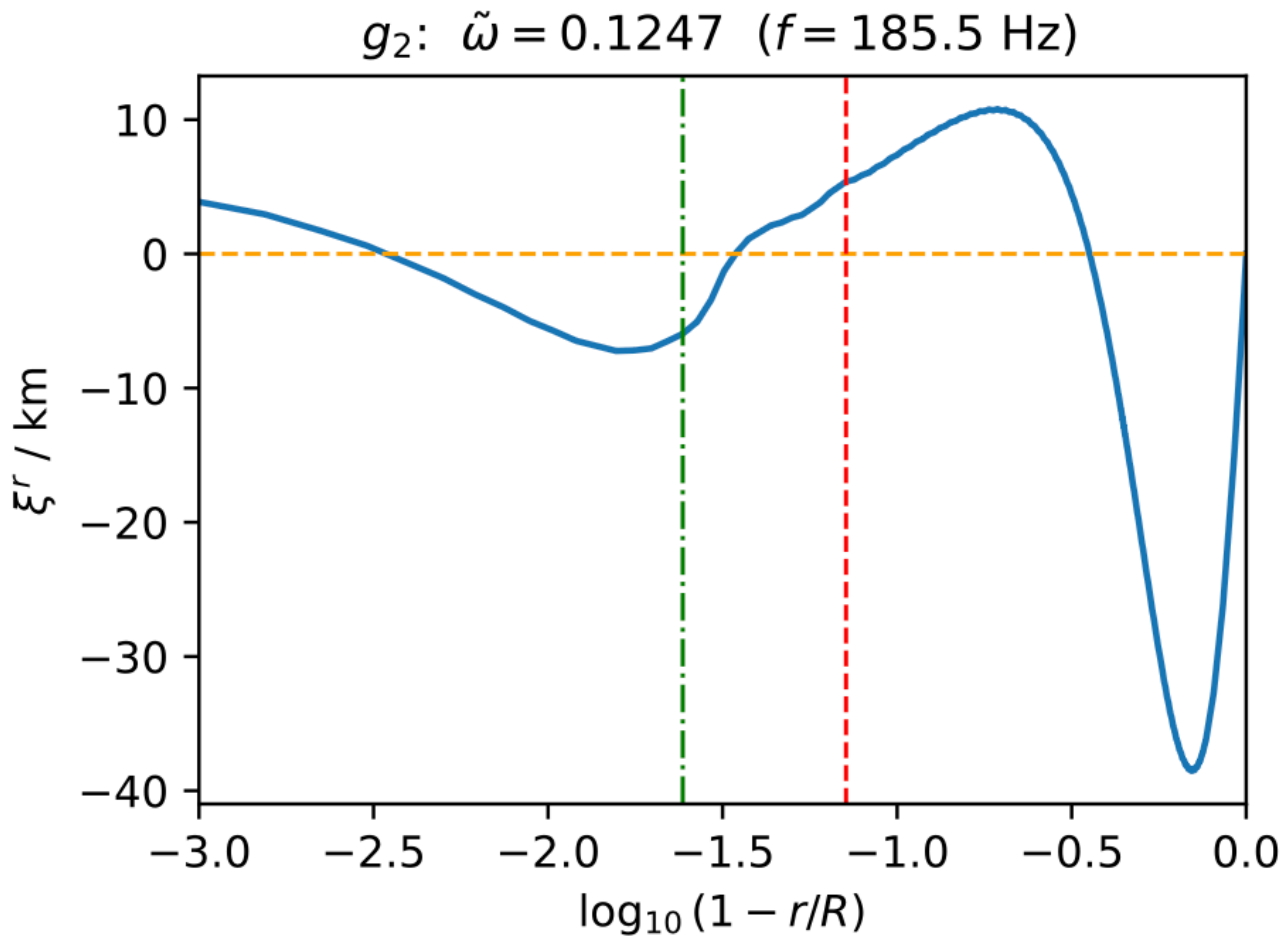
$$\boldsymbol{A}^{2\times 2}\cdot\boldsymbol{\xi}^{(2)} = \sigma^2 \boldsymbol{B}^{2\times 2}\cdot\boldsymbol{\xi}^{(2)} \qquad + \qquad \textbf{boundary conditions}$$

- Formalism described by **D. Tseneklidou et al. PRD 112, 103032 (2025)**

1. Compute NS modes: obtain NS mode spectra

- **g-modes:**





(...)

- **p-modes, i-modes...**

resemble modes in
Counsell, A. R., et al. *MNRAS* **536.2**, 1967-1979 (2025)
(without spectral collocation)

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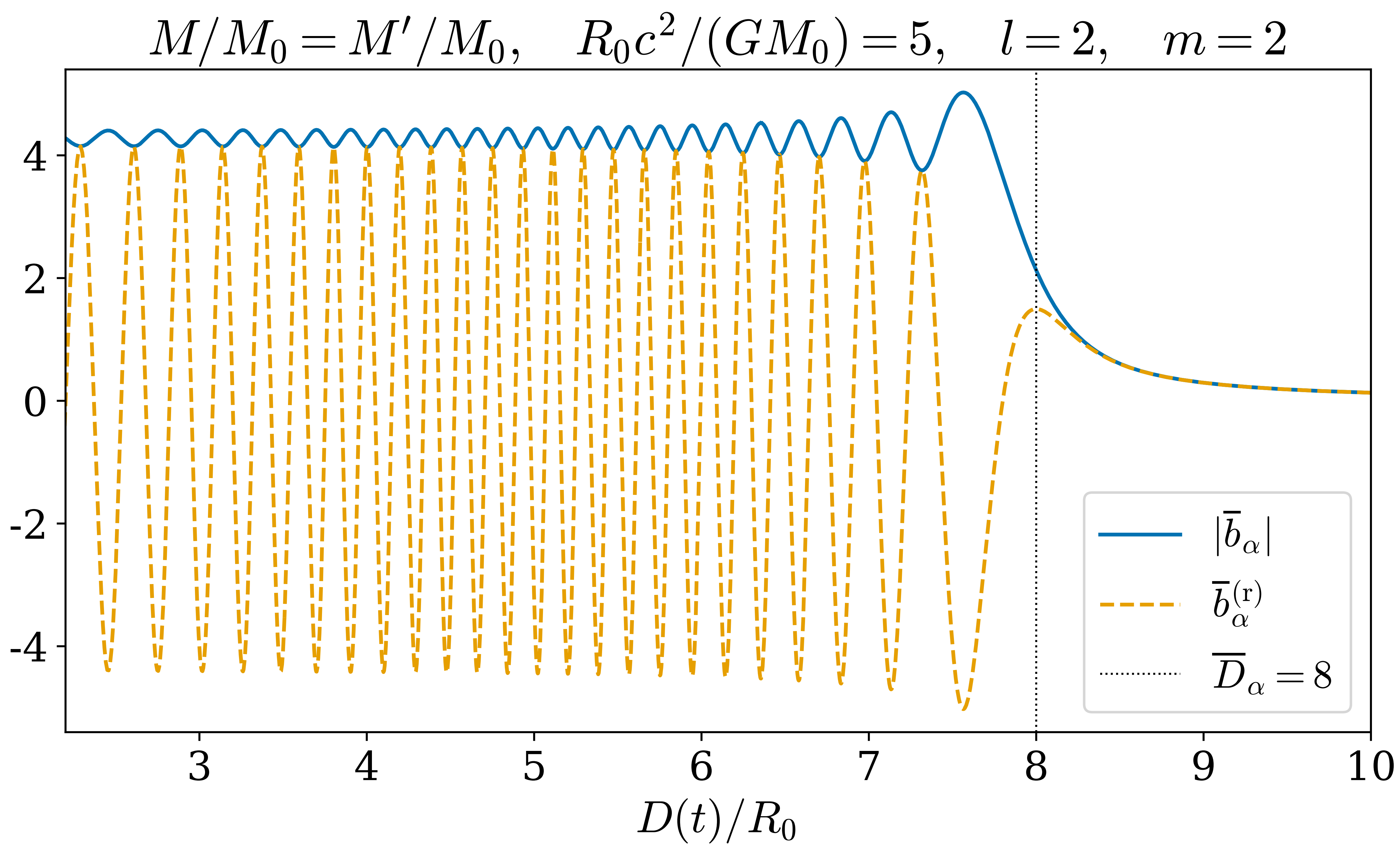
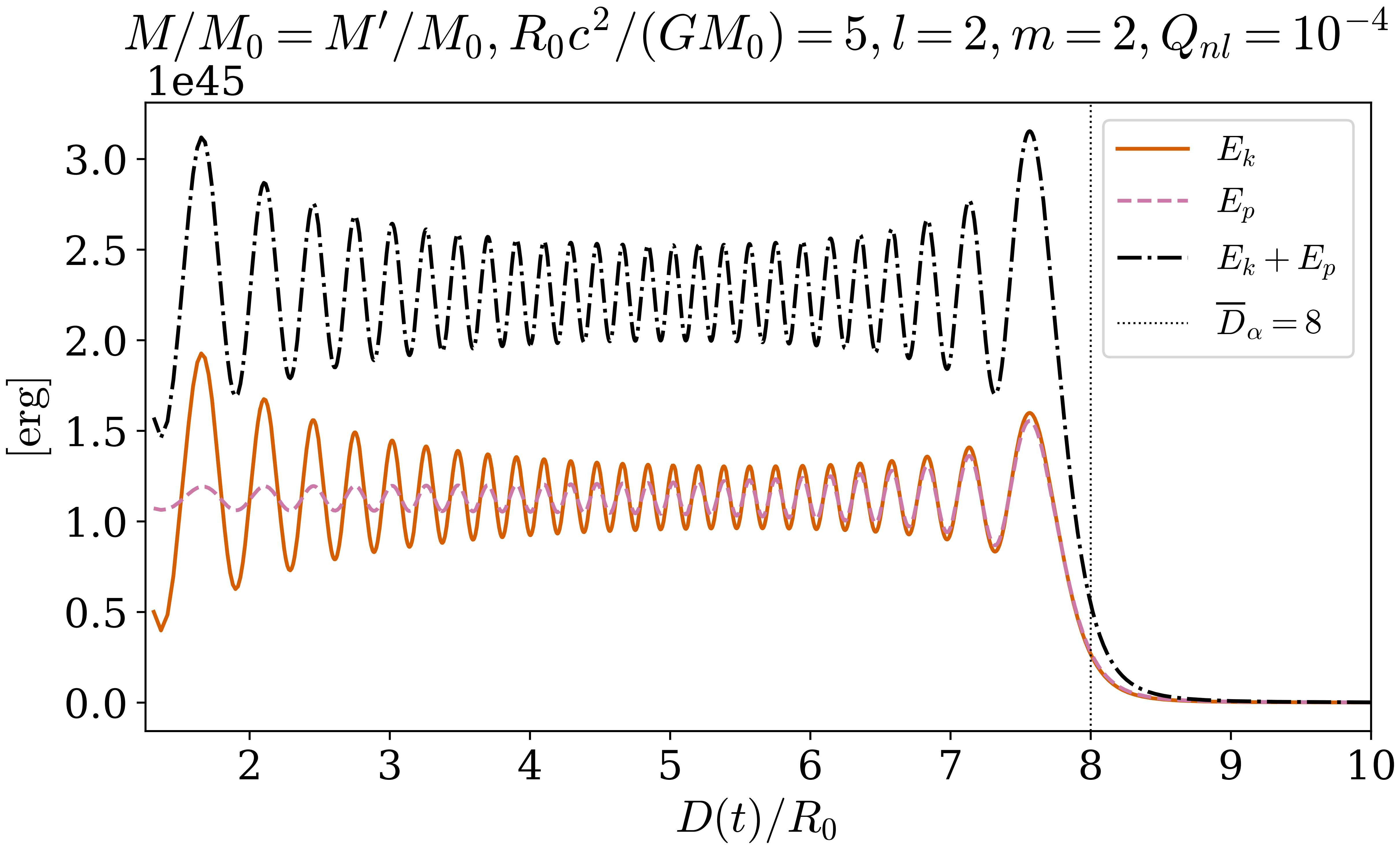
2. Quantifying the excitation of NS modes



Mode amplitudes



Mode energies

Key points

- **NS modes** can be **resonantly excited** during **NS-NS/NS-BH inspirals**
- This happens when the **frequency of the companion's tidal force** reaches that of a given **mode**
- We have used **spectral collocation** methods to compute **NS modes**
- We have computed numerically the **time evolution of NS mode amplitudes** following D. Lai (1994)
- **Next steps:** 3. detectability of resonances in GW waveforms following **A. Revilla Peña et al. (2026)**

Thank you very much for your attention! :)

Back-up slides

1. Spectral collocation methods

- **Linear adiabatic perturbations** of the **continuity and momentum conservation equations**
- **Cowling approximation:** no metric perturbations
- Solve the **eigenvalue problem** $\boldsymbol{A}^{2\times 2}\cdot\boldsymbol{\xi}^{(2)} = \sigma^2 \boldsymbol{B}^{2\times 2}\cdot\boldsymbol{\xi}^{(2)}$ with

$$A_{00}^{2\times2} = \frac{\alpha^2}{\psi^4}\mathcal{B}\mathcal{G}_{\mathcal{P}}, \qquad B_{00}^{2\times2} = 1,$$

$$B_{01}^{2\times2} = -r\partial_r - \left[1 + r\mathcal{G}_{\mathcal{P}}\left(1 - \frac{1}{c_s^2}\right) + r\left(\frac{\rho'}{\rho} + \frac{h'}{h} + 4\frac{\psi'}{\psi}\right)\right],$$

$$A_{10}^{2\times2} = -r\partial_r - 2 - \frac{r}{c_s^2}\mathcal{G}_{\mathcal{P}} - 6r\frac{\psi'}{\psi},$$

$$A_{11}^{2\times2} = l(l+1), \qquad B_{11}^{2\times2} = \frac{r^2\psi^4}{\alpha^2 c_s^2}.$$

\+ **boundary conditions**

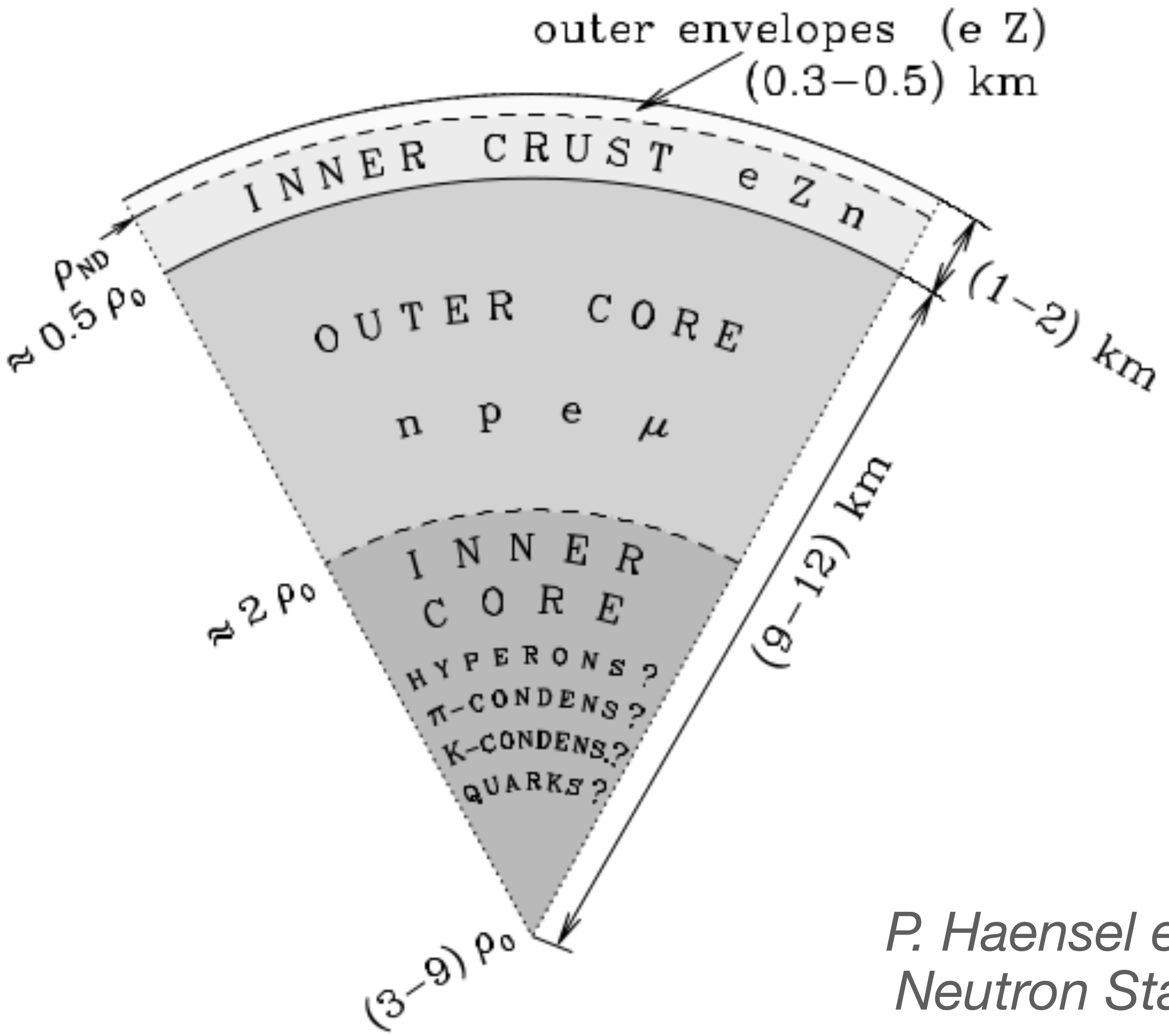
D. Tseneklidou et al.
PRD 112, 103032 (2025)

Example: *i*-modes and NS crust shattering

- The excitation of **crust-core *i-modes*** leading to NS **crust shattering** has been proposed as a mechanism driving **precursor SGRBs**

- **Multimessenger** scenario

- D. Tsang et al. *PRL* **108**, 011102 (2012)



P. Haensel et al. Neutron Stars I

2. Quantifying the excitation of NS modes

- Formalism by D. Lai. *MNRAS* ***270***, 611-629 (1994)
- **Companion:** Newtonian gravitational potential
- Newtonian **mode-sum expansion** $\boldsymbol{\xi}(\boldsymbol{r}, t) = \sum_{lm} a_{lm}(t)\boldsymbol{\xi}_{lm}(\boldsymbol{r})$
- **Time evolution of the amplitudes** (solve numerically):

$$\ddot{a}_{lm} + \omega_{lm}^2 a_{lm} = \frac{GM' W_{lm} Q_{lm}}{D^{l+1}} e^{-im\Phi(t)}$$

- Q_{lm} is **mode and EOS-dependent:** quantifies the **orbit-mode coupling** strength

2. D. Lai (1994) formalism

- Consider:
  - NS of **mass** M and **radius** R orbiting a (point-like) companion of mass M'
  - **Coord. system** (r, θ, ϕ) centered in M
  - $D(t)$: distance from the center of M to M'
  - **Assume:** non-rotating M, no magnetic fields...
- Newtonian **gravitational potential** (companion):

$$U = -GM' \sum_{lm} W_{lm} \frac{r^l}{D(t)^{l+1}} e^{-im\Phi(t)} Y_{lm}(\theta, \phi)$$

  - Focus on (dominant) $l = 2$ term, for which

$$W_{20} = -\left(\frac{\pi}{5}\right)^{1/2}, \quad W_{2\pm1} = 0 \quad \text{and} \quad W_{2\pm2} = \left(\frac{3\pi}{10}\right)^{1/2}$$

2. D. Lai (1994) formalism

- **Perturbations** are described in terms of **Lagrangian displacements** $\boldsymbol{\xi}$ that satisfy

$$\left(\rho \frac{\partial^2}{\partial t^2} + \mathcal{L}\right) \boldsymbol{\xi} = -\rho \nabla U$$

  - $\mathcal{L}$ contains information about **restoring forces** (depends on NS structure, EOS...)
  - $-\rho \nabla U$ acts as a **driving force**
  - We can expand $\boldsymbol{\xi}$ in terms α-modes (NMs) with **mode amplitudes** $a_\alpha(t)$:

$$\boldsymbol{\xi}(\boldsymbol{r}, t) = \sum_\alpha a_\alpha(t) \boldsymbol{\xi}_\alpha(\boldsymbol{r})$$

2. D. Lai (1994) formalism

- The previous equation leads us to

$$\ddot{a}_\alpha + \omega_\alpha^2 a_\alpha = \frac{GM' W_{lm} Q_{nl}}{D^{l+1}} e^{-im\Phi(t)}$$

- Q_{nl} and $\Phi(t)$ are the so-called **tidal coupling coefficients** and the **orbital phase**:

$$Q_{nl} = \int d^3x \, \rho \boldsymbol{\xi}^*{}_{nlm} \cdot \nabla \left[r^l Y_{lm}(\theta, \phi) \right] \quad , \qquad \Phi(t) = \int^t dt \, \Omega_{orb}(t)$$

2. D. Lai (1994) formalism

- Assume $D(t)$ given by the **quadrupole formula for the gravitational radiation of 2 point masses**:

$$\dot{D}(t) = -\frac{64G^3}{5c^5}\frac{MM'(M+M')}{D^3(t)}$$

with

$$\Omega^2(t) = \frac{G(M+M')}{D^3} \quad \text{and} \quad \dot{\Omega}(t) = -\frac{3\dot{D}(t)}{2D(t)}\Omega(t)$$

- The **resonance** occurs at a $D = D_\alpha$ for which $\omega_\alpha \approx 2\Omega_{orb}$; Ω_{orb} is the orbital ang. freq.

2. D. Lai (1994) formalism

- **Define** (for numerical purposes):

$$b_\alpha(t) = \frac{a_\alpha(t)}{GM' W_{lm} Q_{nl}} e^{im\Phi(t)}$$

- Need to **solve numerically**:

$$\ddot{b}_\alpha - 2im\Omega\dot{b}_\alpha + (\omega_\alpha^2 - m^2\Omega^2 - im\dot{\Omega})b_\alpha = D^{-(l+1)}$$

(separating **real** and **imaginary** components)

2. D. Lai (1994) formalism

Resonant excitations produce **orbit-mode energy transfers**:

- **Kinetic** energy:

$$E_k(t) = \frac{1}{2}\sum_{\alpha} |\dot{a}_\alpha|^2 = \frac{1}{2}\sum_{\alpha} \left(M' W_{lm} Q_\alpha\right)^2 |\dot{b}_\alpha - im\Omega b_\alpha|^2 \ ,$$

- **Potential** energy:

$$E_p(t) = \frac{1}{2}\sum_{\alpha} \omega_\alpha^2 |a_\alpha|^2 = \frac{1}{2}\sum_{\alpha} \left(M' W_{lm} Q_\alpha\right)^2 \omega_\alpha^2 |b_\alpha|^2 \ .$$

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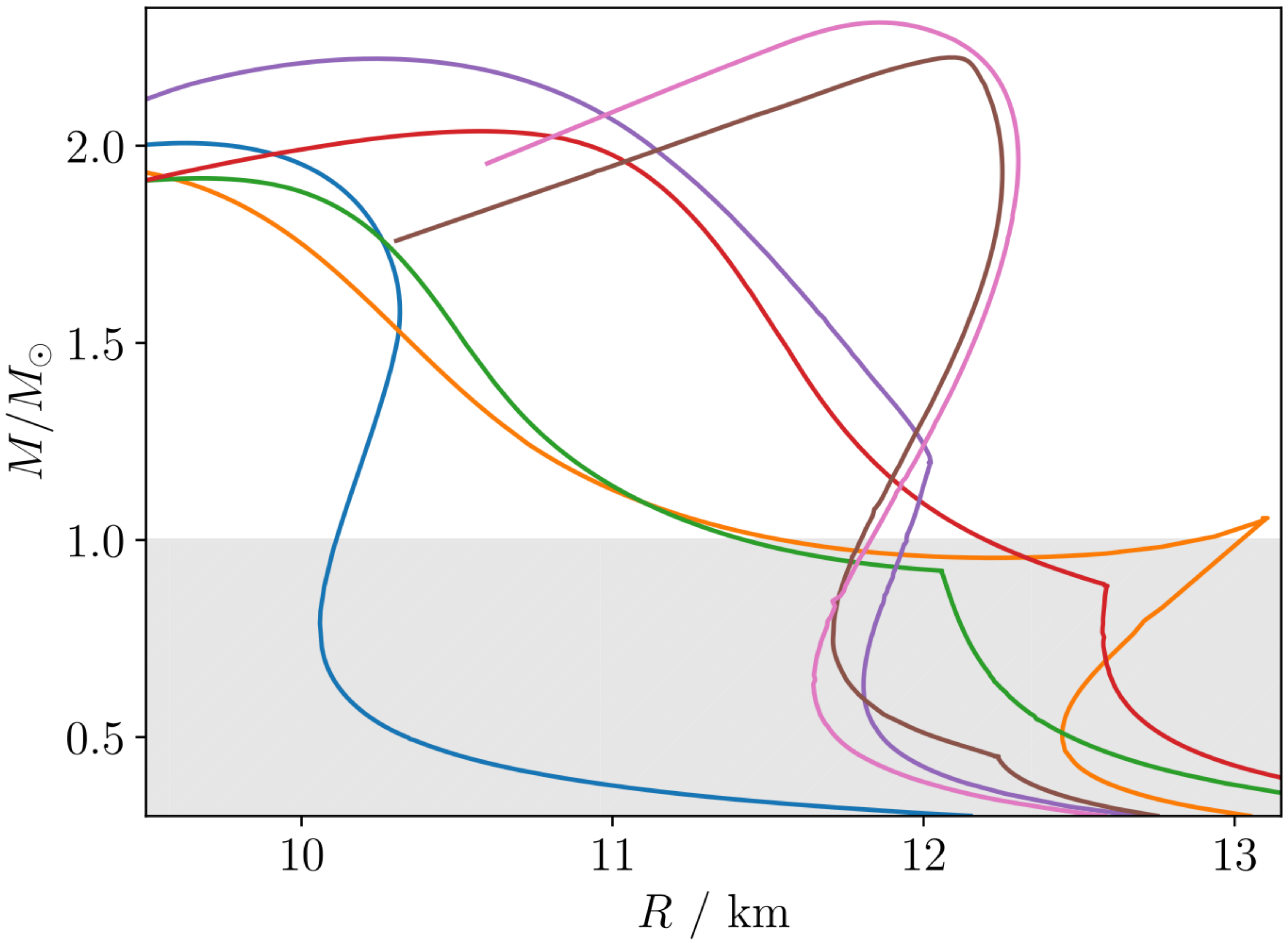
Example: *i*-modes and first-order phase transitions in NSs

- **Phase transitions to deconfined quark matte**r can occur in extremely dense NS interiors
- Associated ***i*-modes**
- Their resonant excitation: **GW probes of quark matter phase transitions**?



A. R. Counsell et al. PRL 135, 081402 (2025)

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2. Quantifying the excitation of NS modes

Recent work at the ICCUB:

Rethinking Resonance Detectability during Binary Neutron Star Inspiral: Accurate Mismatch Computations for Low-lying Dynamical Tides

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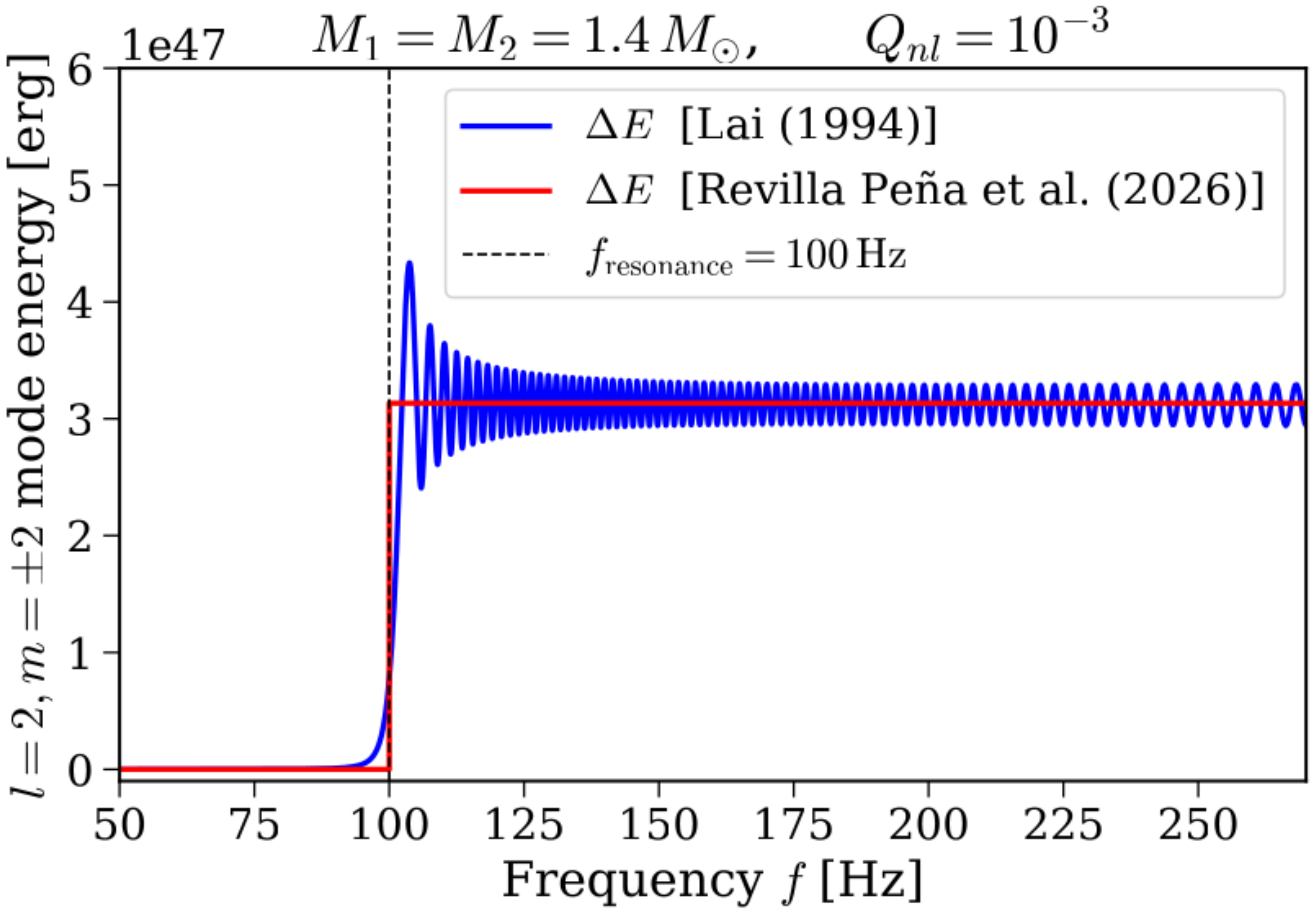
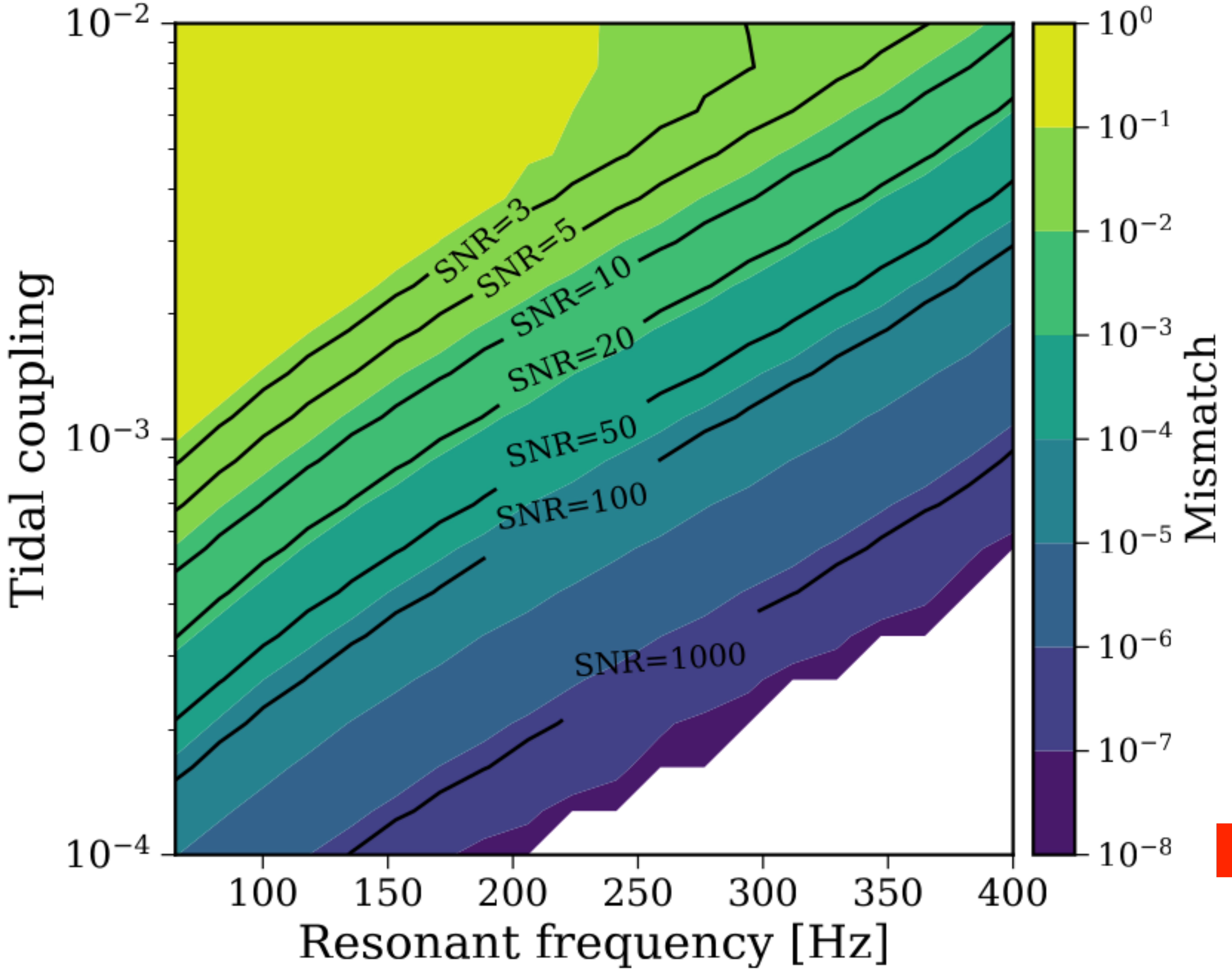
(Dated: February 3, 2026)

We compute deviations from observed gravitational wave signals, where the amplitude of the signal is unchanged. As an example, we consider the detectability of low lying dynamical tides in binary neutron star or neutron star black hole mergers. Tidal forces can excite oscillatory modes of one or both of the stars in the binary when the orbital frequency of the binary system sweeps through the resonant mode frequency dissipating energy into the vibrational mode. The orbital energy loss to the vibrational mode extracts energy from the orbital motion, advancing the time to merger. The inspiral then continues with an excess phase and a time advance. Both will cause a mismatch when fitting to a system that has not gone through the resonance. To resolve this effect, we compute the mismatch for current and planned detectors using both a quasi-analytical approach that relies on the computation of moment integrals and an optimized version of the standard numerical match function. We conclude that detectability can occur for time advances of the order of 1 ms with advanced LVK detectors for an excess energy-flux that is a few percent of the gravitational wave emission. Our results contrast with previous work, which model this effect solely as a phase shift of the waveform or by using the difference in the number of cycles induced by the resonant behavior. We show that tidal resonance effects primarily cause a time advance of the merger, rather than a phase difference, and that the single-frequency approximation commonly used in the literature significantly overestimates the detectability of this effect.



A. Revilla Peña et al. (2026)
https://arxiv.org/abs/2601.21086

3. Detectability of NS mode resonances



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