Magnetic Evolution of Superconducting Neutron Star Cores

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Superfluidity/Superconductivity in NSs cores

* **Neutrons /protons** become **superfluid/ superconducting** relatively early in the stellar life (Migdal 1959)

* London penetration depth:

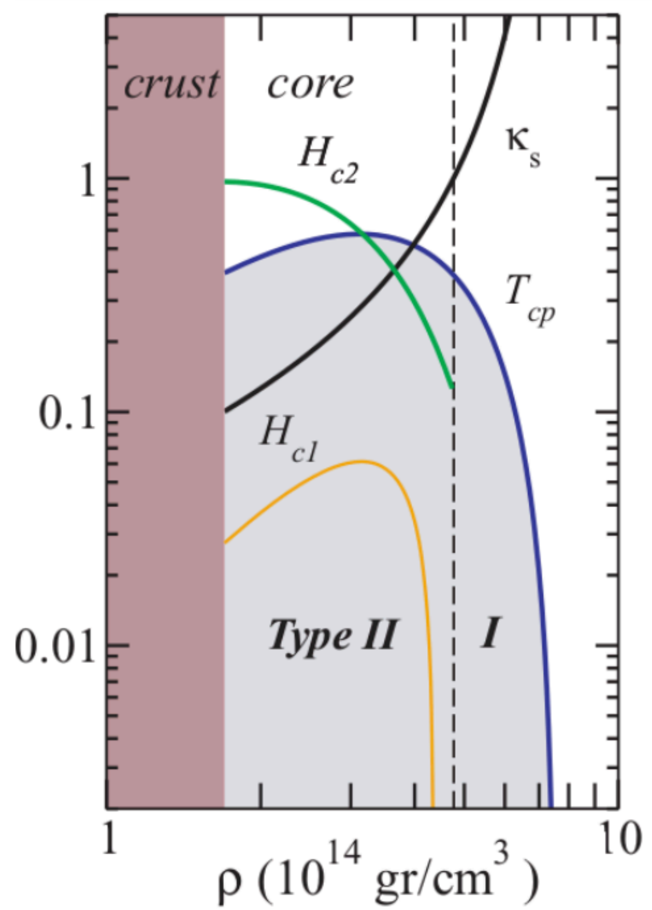
$$\lambda_p \sim 9 \times 10^{-12} \left(\frac{m_p}{m_p^*}\right)^{-1/2} \rho_{14}^{-1/2} \left(\frac{x_p}{0.1}\right)^{-1/2} \text{cm}$$

*Coherence lenghth:

$$\xi_p \sim 5 \times 10^{-12} \left(\frac{m_p}{m_p^*}\right) \rho_{14}^{1/3} \left(\frac{x_p}{0.1}\right) \left(\frac{10^9\,\text{K}}{T_{cp}}\right) \text{cm}$$

* The type of superconductivity depends on κ_s

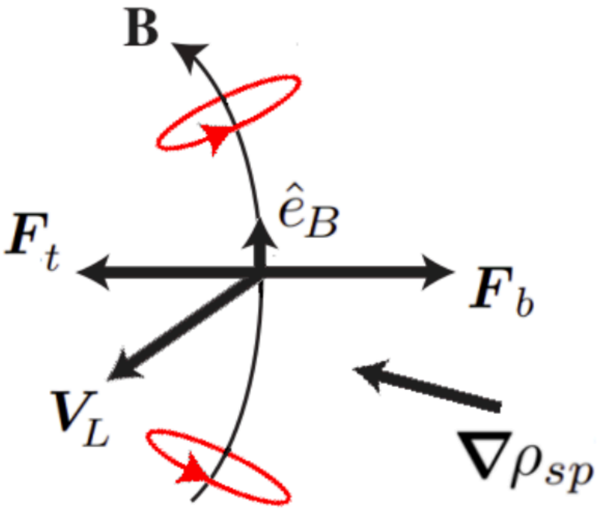
$$\boxed{\kappa_s = \frac{\xi_p}{\sqrt{2}\lambda_p} \rightarrow \begin{cases} > 1, & \text{Type I}, \\ < 1, & \text{Type II}. \end{cases}}$$



NS cross section. $H/10^{16}$G & $T_{cp} = 10^{10}$K
Source: Glampedakis et al. (2011)

Single fluid MHD

Following the model by Gusakov+2020



Credits: Vasily Dommes

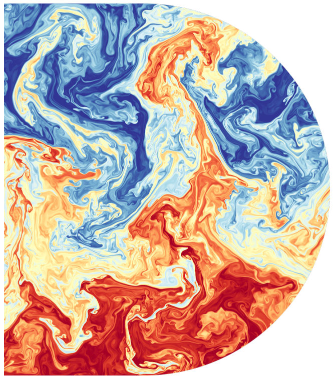
$$\frac{n_c\mu}{c^2}\frac{\partial \vec{v}_c}{\partial t} = \frac{(\vec{\nabla}\times\vec{H}_{c1})\times\vec{B}}{4\pi} - n_c\vec{\nabla}\,\delta\mu_c^\infty + \nu_e\vec{f}_{\nu_e}$$

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}_L\times\vec{B}) + \eta\nabla^2\vec{B}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\cdot(n_c\vec{v}_c) = 0$$

$$\int_{\mathscr{V}} \delta\mu_c^\infty\, d\mathscr{V}_{\rm core} = 0$$



Dedalus code

$$\boxed{\vec{v}_L = \vec{v}_c + \frac{c\nabla\times\vec{H}_{c1}}{4\pi e_p n_c} + \frac{\mathscr{D}_e}{\mathscr{D}_e'^2}\frac{(\vec{\nabla}\times\vec{H}_{c1})\times\vec{B}}{4\pi}}$$

where $\vec{H}_{c1} = H_{c1}(r)\,\hat{e}_B,\ n_p = n_e$, the terms in red operate on time-scales $\gtrsim 10^7\,{\rm yrs}$

Equilibrium configurations

* Equilibrium configurations found by Lander 2013 & Lander+2014 in axial symmetry in absence of entrainment

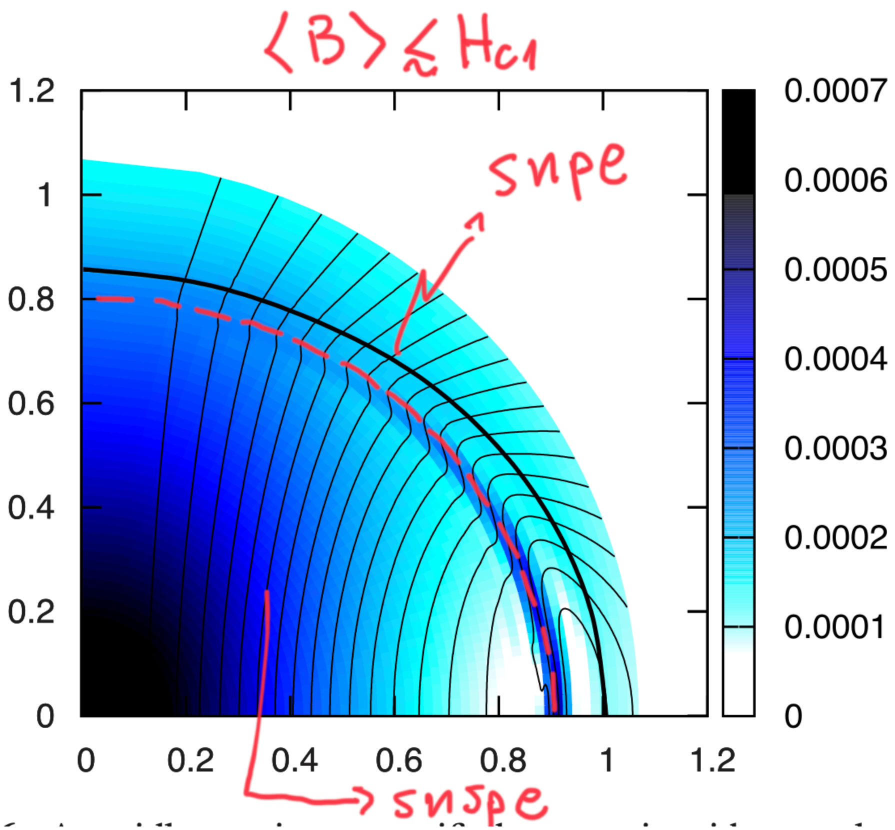


Image from Lander+2014

*Core **snspe-matter**

$$\frac{\left(\vec{\nabla} \times \vec{H}_{c1}\right) \times \vec{B}}{4\pi} = n_c \vec{\nabla} \delta\mu_c^{\infty}$$

*Crust **snpe-matter**

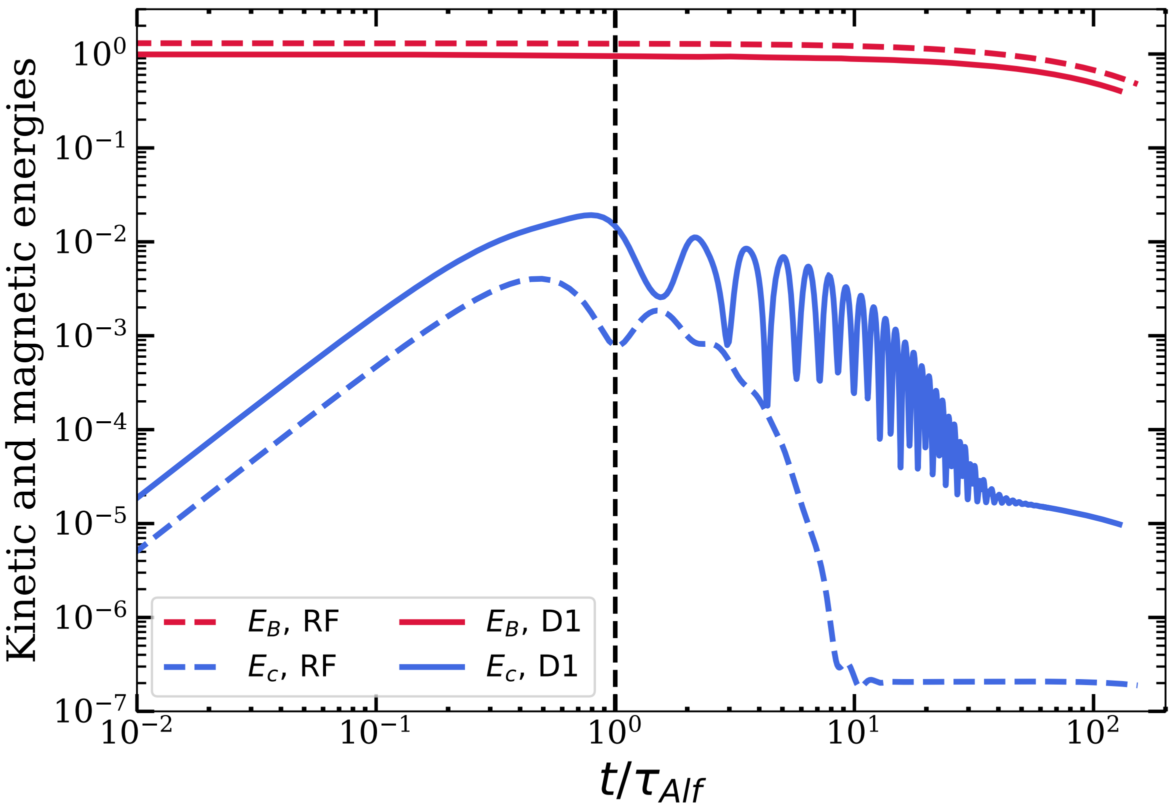
$$\frac{\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}}{4\pi} = n_c \vec{\nabla} \delta\mu_c^{\infty}$$

*Crust -core interphase

$$\frac{\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}}{4\pi} = \frac{\left(\vec{\nabla} \times \vec{H}_{c1}\right) \times \vec{B}}{4\pi}$$

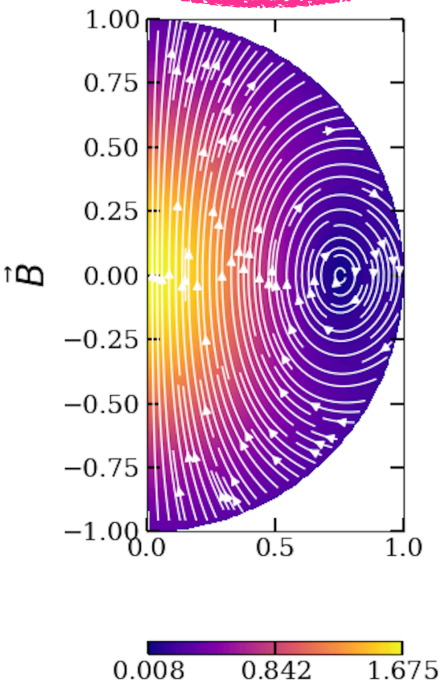
→ Lander results matched those of the Roberts field (Roberts 1981)

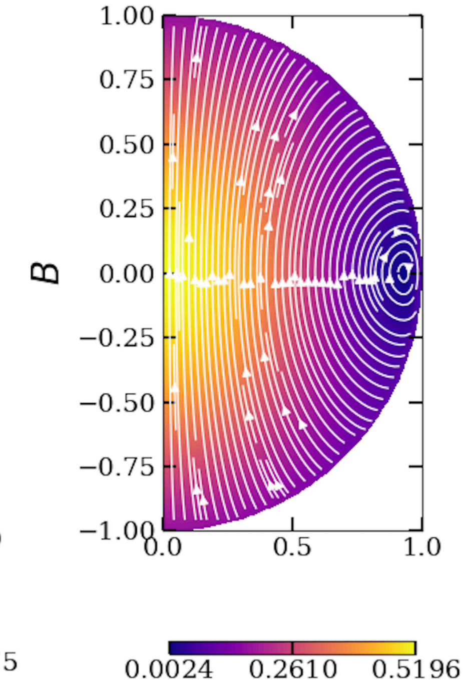
Results: comparing RF vs D1



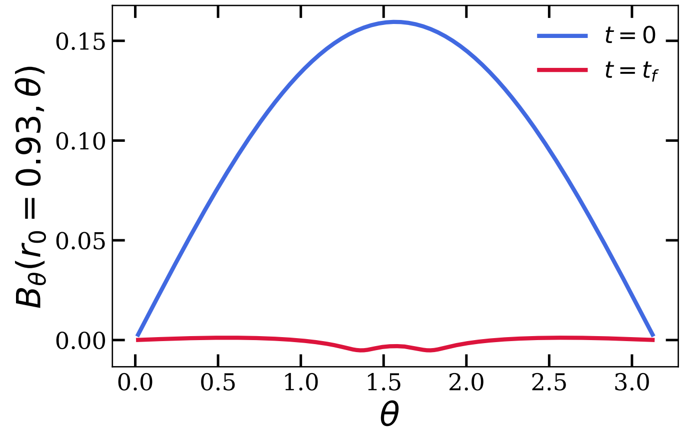
$* \, t = 0$

$* \, t = 30\tau_{Alf}$





$$E_B = \int_{\mathscr{V}_{core}} \frac{H_{c1} B}{2\pi} d\mathscr{V} \qquad E_c = \int_{\mathscr{V}_{core}} \frac{n_c \mu \, \vec{v}_c^{\,2}}{2c^2} d\mathscr{V}$$



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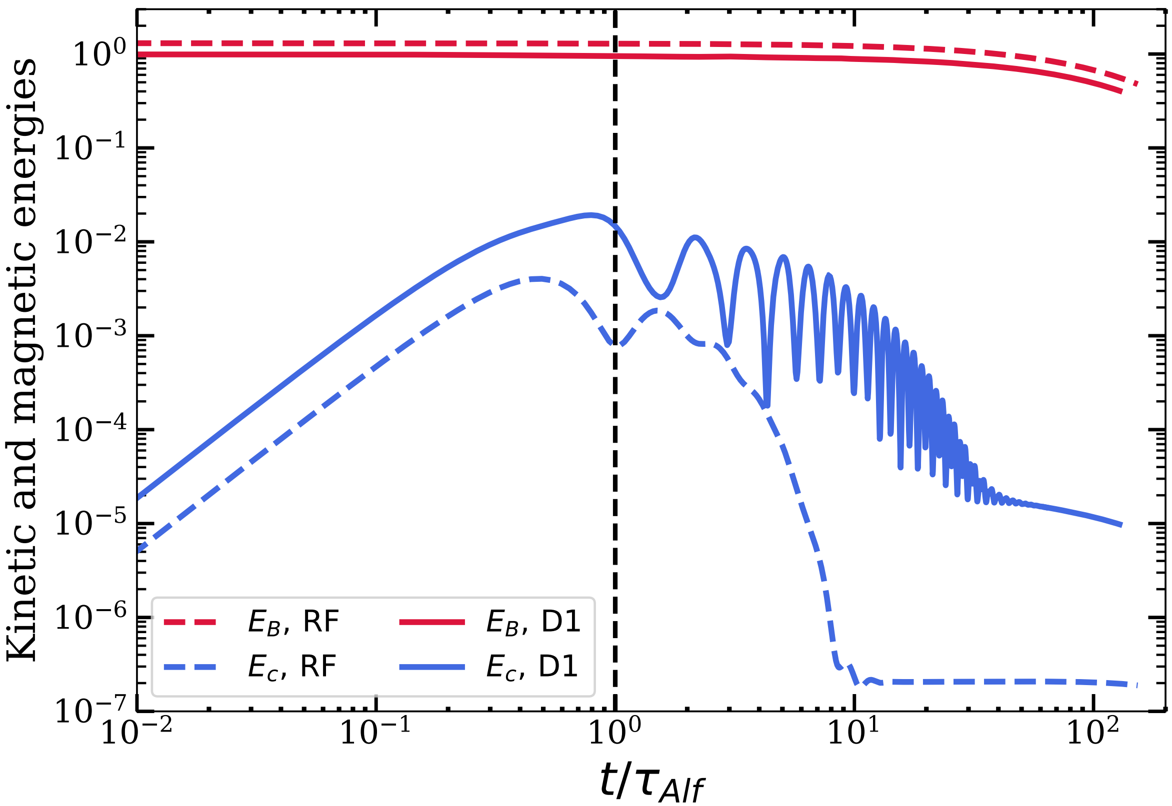

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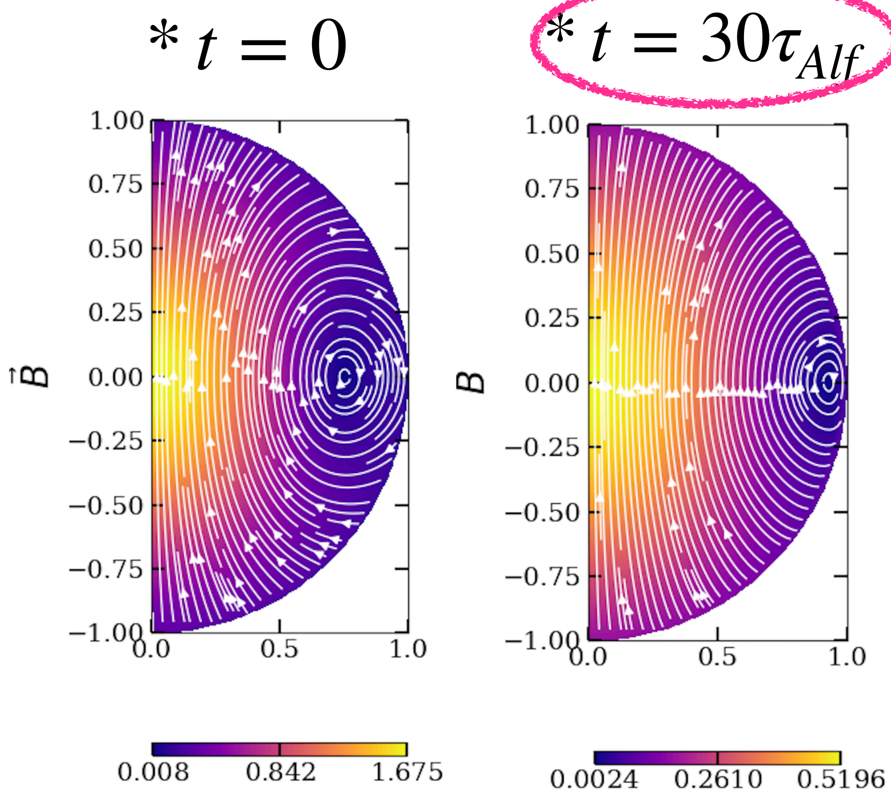
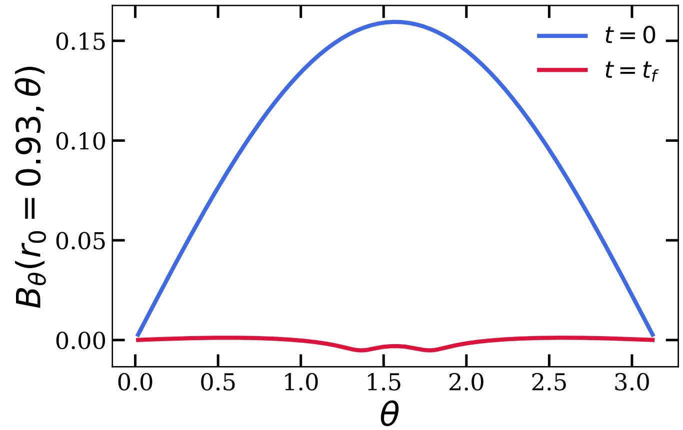
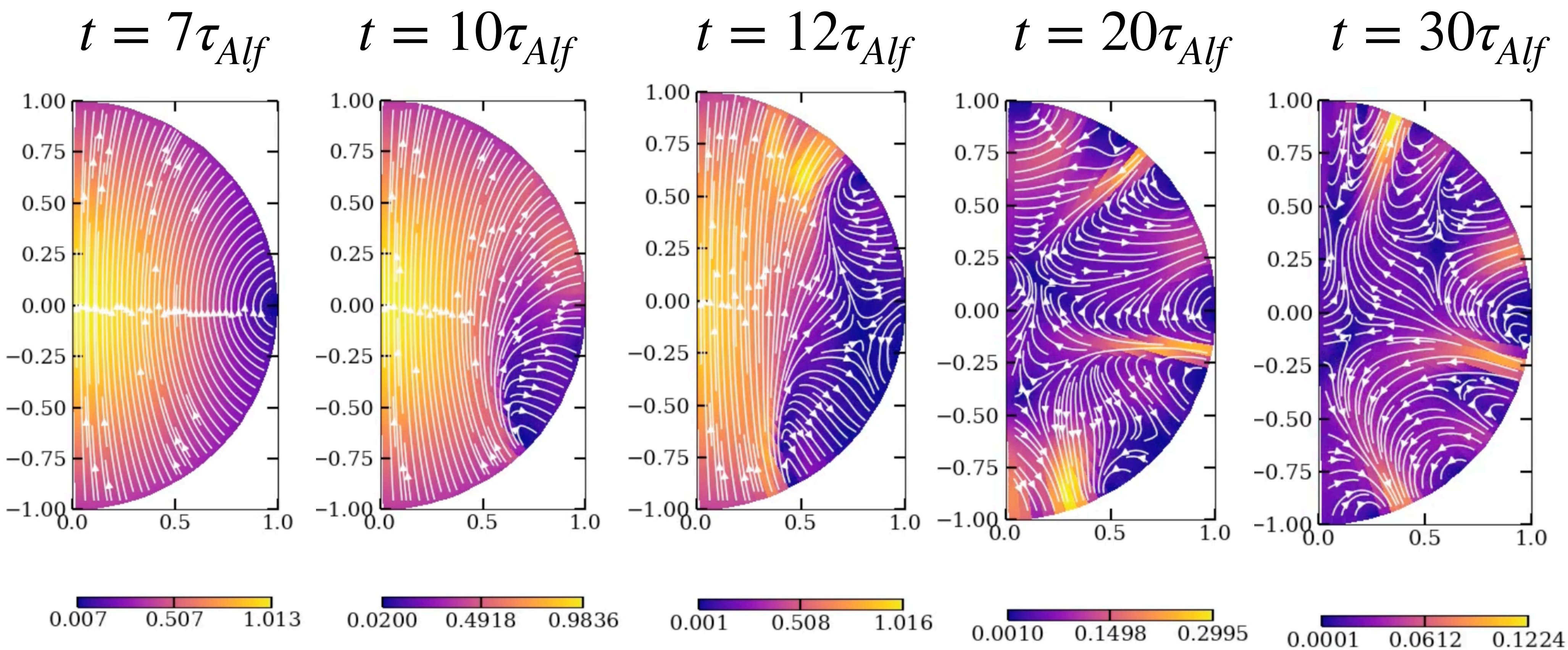




Results: RF stability in 3D



Summary ✓

* We implemented a new numerical scheme that reproduces the **GS-equilibrium** in axial symmetry

$$\boxed{\left.\frac{(\vec{\nabla}\times\vec{H}_{c1})\times\vec{B}}{4\pi}\right|_{\rm pol} = n_c\vec{\nabla}\,\delta\mu_c^{\infty}}$$

* Preliminarily, the Roberts field seems to be unavoidable

$$\boxed{\vec{B}\times\hat{n}\,\big|_{r=R_{cc}} = 0}$$

* **The Roberts field is unstable in 3D**

How to relate this with GW

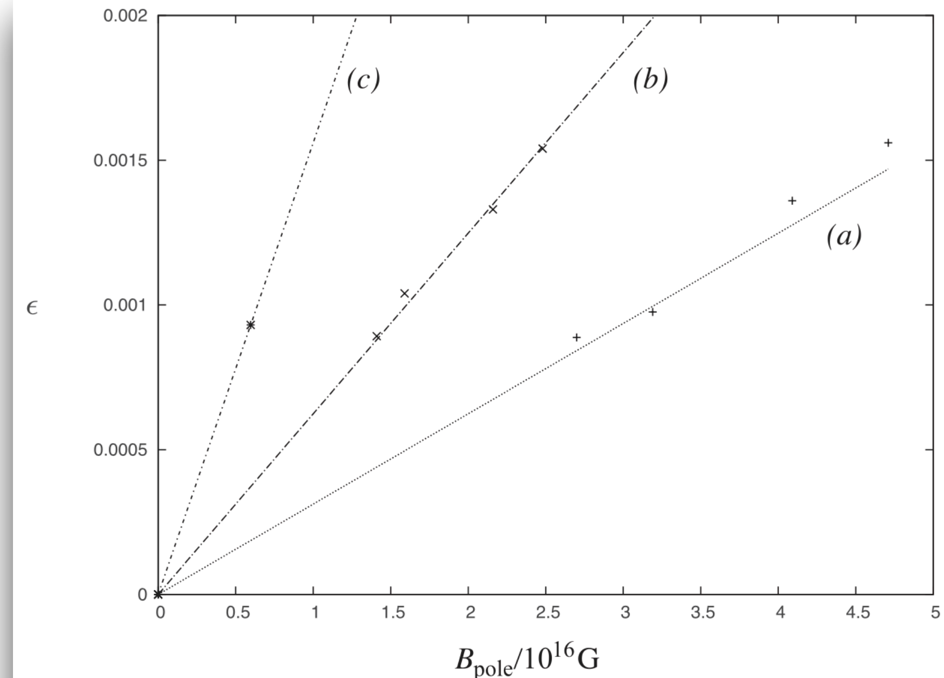


Figure 7. Scaling of magnetically induced ellipticity ϵ with field strength at the polar cap $B_{\rm pole}$. Lines (a), (b) and (c) are for models with a central critical field $H_{c1}(0) = 1, 2$ and 5×10^{16} G, respectively; these strong fields are needed to produce non-rotating stars with a sufficiently large distortion to be resolved on our numerical grid. Our results agree well with the expected scaling $\epsilon \sim H_{c1}B$.

Image from Lander+2014

WORK IN PROGRESS…