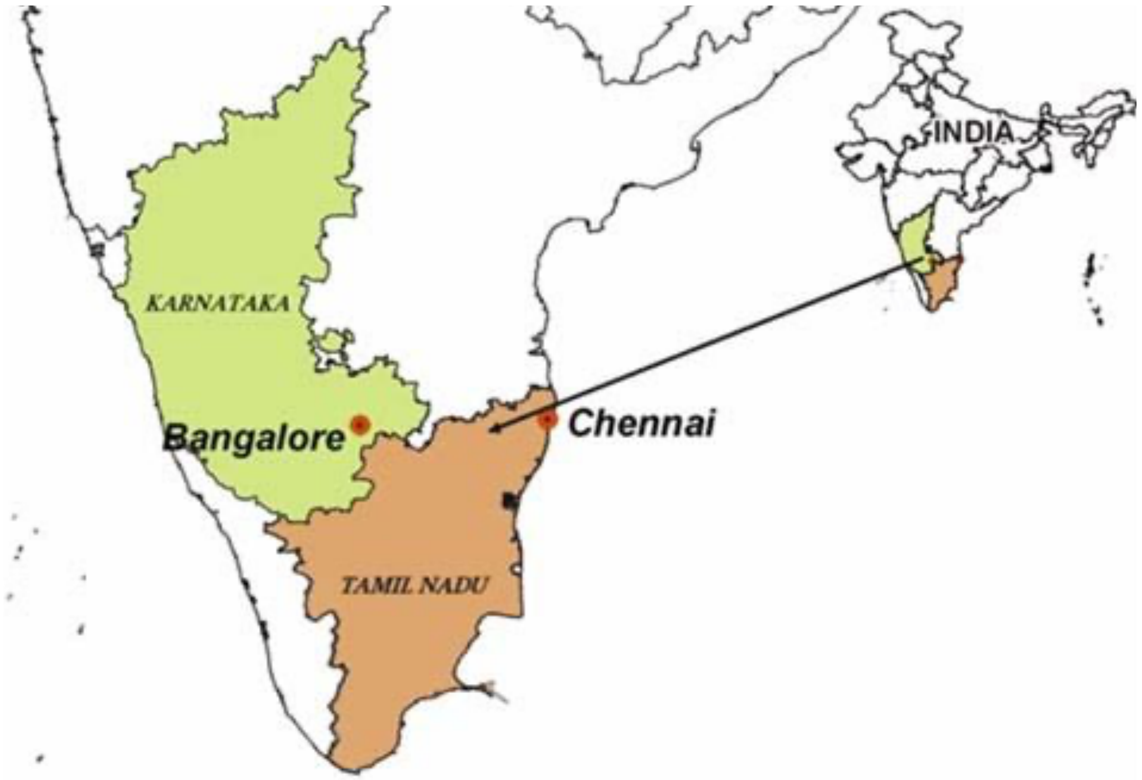
Constraining higher order tidal parameters in 3G era: role of parametrisations and universal relations



Chandra Kant Mishra
IIT Madras, Chennai (India)

Collaborator: Nidhi Biswas (IIT Madras)

Thematic school GWsNS-2026: Gravitational Waves from Neutron Stars
June 28-July 3, 2026

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The Model

Stationary phase approximation (Restricted Waveform)

$$\tilde{h}(f) = \mathcal{A}\, f^{-7/6} e^{i\Psi_{\mathrm{TaylorF2}}(f)}$$

$$\Psi_{\mathrm{TaylorF2}} = 2\,\pi\,f\,t_c - 2\,\phi_c - \frac{\pi}{4} + \frac{3}{128\nu v^5}\Bigg[\psi_{\mathrm{pp}}(v,\nu)$$

$$+\,\psi_{\mathrm{tidal}}(v,\nu,\tilde{\mu}_{\pm}^{(2)},\tilde{\sigma}_{\pm}^{(2)},\tilde{\mu}_{+}^{(3)})\Bigg]$$

Henry, Faye, Blanchet (arXiv:2005.13367v2)

The Tidal Phase

$$\psi_{\text{tidal}} \equiv \psi_{\text{tidal}}\left(v, \nu, \tilde{\Lambda}^{(2)}, \delta\tilde{\Lambda}^{(2)}, \tilde{\Sigma}^{(2)}, \delta\tilde{\Sigma}^{(2)}, \tilde{\Lambda}^{(3)}, \delta\tilde{\Lambda}^{(2)}_{\text{tail}}, \delta\tilde{\Lambda}'^{(2)}\right)$$

$$\begin{aligned}
\psi_{\text{tidal}} = v^{10}\Bigg[& -\frac{39}{2}\tilde{\Lambda}^{(2)} + v^2\Bigg[-\frac{3115}{64}\tilde{\Lambda}^{(2)} - \frac{185}{3}\tilde{\Sigma}^{(2)} \\
& + \frac{6595}{364}\Delta\delta\tilde{\Lambda}^{(2)}\Bigg] + \frac{39}{2}\pi v^3\tilde{\Lambda}^{(2)} \\
& + v^4\Bigg[-\frac{379931975}{2286144}\tilde{\Lambda}^{(2)} - \frac{467690}{1701}\tilde{\Sigma}^{(2)} - \frac{250}{9}\nu\tilde{\Lambda}^{(3)} \\
& + \Delta\Big[\frac{181283435}{2122848}\delta\tilde{\Lambda}'^{(2)} - \frac{134165}{440559}\delta\tilde{\Sigma}^{(2)}\Big]\Bigg] \\
& + \pi v^5\Big[\frac{2137}{28}\tilde{\Lambda}^{(2)} - \frac{1613}{182}\Delta\delta\tilde{\Lambda}^{(2)}_{\text{tail}}\Big]\Bigg]
\end{aligned}$$

[Order counting terminology : nPN := v^{2n} i.e. v^2 is 1PN, v^3 is 1.5PN etc.**]**

Biswas+ (in prep)

The Method

Fisher Matrix Approach — high SNR, stationary & gaussian noise, semi-analytical

$$\Gamma_{ab} \equiv (h_a|h_b) = 2\int_0^\infty \frac{\tilde{h}_a^*(f)\tilde{h}_b(f) + \tilde{h}_a(f)\tilde{h}_b^*(f)}{S_h(f)} \; ; \quad h_a \equiv \frac{\partial h_a}{\partial a}$$

Detector Noise

Covariance Matrix

$$\Sigma^{ab} \equiv \langle \Delta\theta^a \Delta\theta^b \rangle = (\Gamma^{-1})^{ab}$$

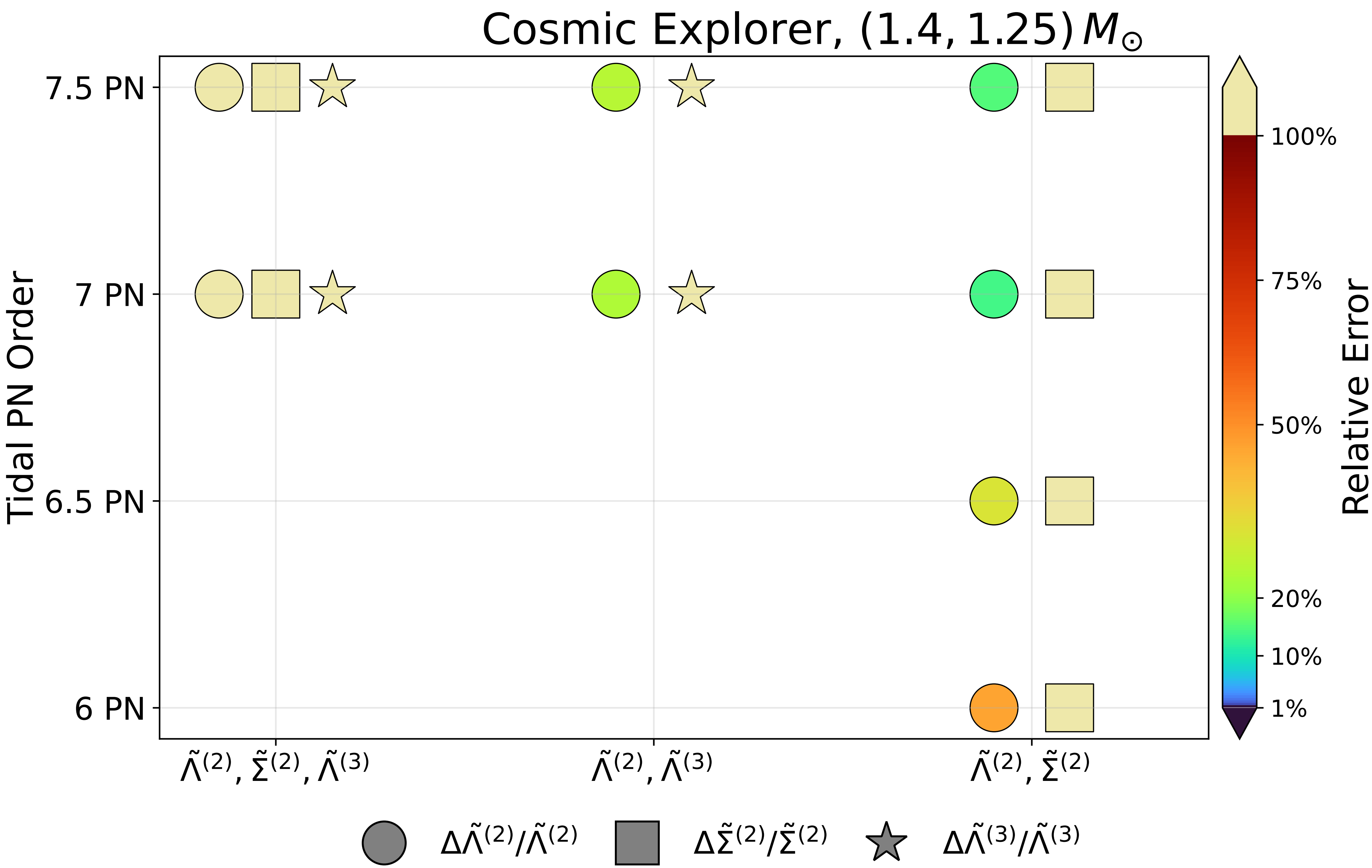
Parameter Estimation Errors

$$\sigma_a = \langle (\Delta\theta^a)^2 \rangle^{1/2} = \sqrt{\Sigma^{aa}}$$

Parameter Space

$$p \equiv \left\{ \ln \mathcal{A}, t_c, \phi_c, \ln \mathcal{M}_c, \Delta, \chi_s, \chi_a, \tilde{\Lambda}^{(2)}, \delta\tilde{\Lambda}^{(2)}, \tilde{\Sigma}^{(2)}, \tilde{\Lambda}^{(3)} \right\}$$

The spoiler



Cosmic Explorer, $(1.4, 1.25)\,M_\odot$
Tidal PN Order
7.5 PN
7 PN
6.5 PN
6 PN
$\tilde{\Lambda}^{(2)}, \tilde{\Sigma}^{(2)}, \tilde{\Lambda}^{(3)}$
$\tilde{\Lambda}^{(2)}, \tilde{\Lambda}^{(3)}$
$\tilde{\Lambda}^{(2)}, \tilde{\Sigma}^{(2)}$
Relative Error
100%
75%
50%
20%
10%
1%
$\Delta\tilde{\Lambda}^{(2)}/\tilde{\Lambda}^{(2)}$
$\Delta\tilde{\Sigma}^{(2)}/\tilde{\Sigma}^{(2)}$
$\Delta\tilde{\Lambda}^{(3)}/\tilde{\Lambda}^{(3)}$

Re-parameterization

Universal Relations — Newtonian limit, uniform density

$$\bar{\lambda}_2^{\mathrm{N},(n=1)} = \frac{15-\pi^2}{3\pi^2}\frac{1}{C^5}\,,$$

$$\bar{\lambda}_3^{\mathrm{N},(n=1)} = \frac{21-2\pi^2}{9\pi^2}\frac{1}{C^7}\,, \qquad \Longrightarrow \qquad \tilde{\Sigma}^{(2)}\left(\tilde{\Lambda}^{(2)}, \delta\tilde{\Lambda}^{(2)}\right) \quad \& \quad \tilde{\Lambda}^{(3)}\left(\tilde{\Lambda}^{(2)}, \delta\tilde{\Lambda}^{(2)}\right)$$

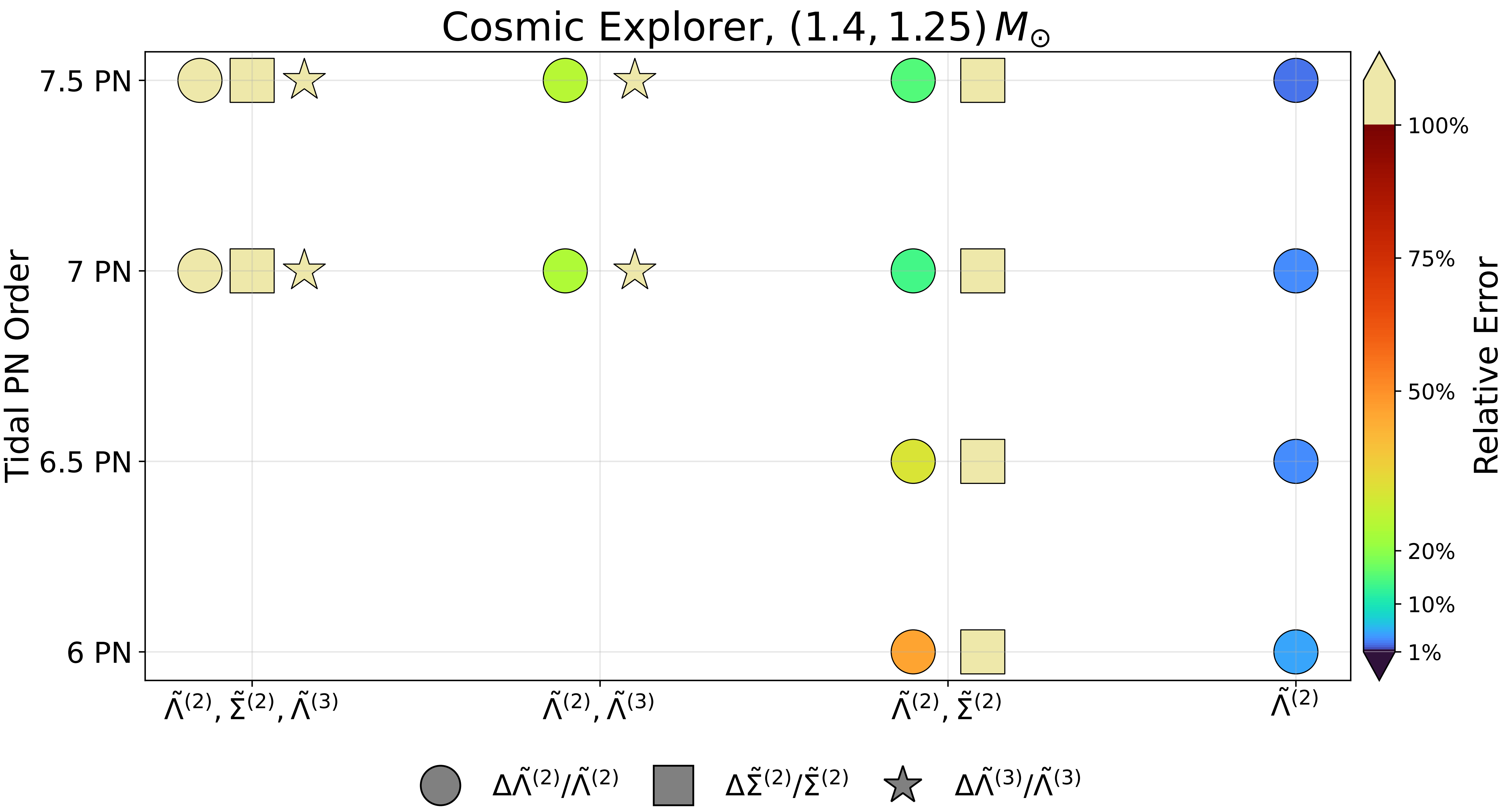
$$\bar{\sigma}_2^{\mathrm{1PN},(n=1)} = -\frac{1}{60}\left(1-\frac{20}{\pi^2}+\frac{120}{\pi^4}\right)\frac{1}{C^4}$$

Kent Yagi (arXiv:1311.0872)

Parameter Space

$$p \equiv \left\{\ln\mathcal{A}, t_c, \phi_c, \ln\mathcal{M}_c, \Delta, \chi_s, \chi_a, \tilde{\Lambda}^{(2)}, \delta\tilde{\Lambda}^{(2)}\right\}$$

The Measurement



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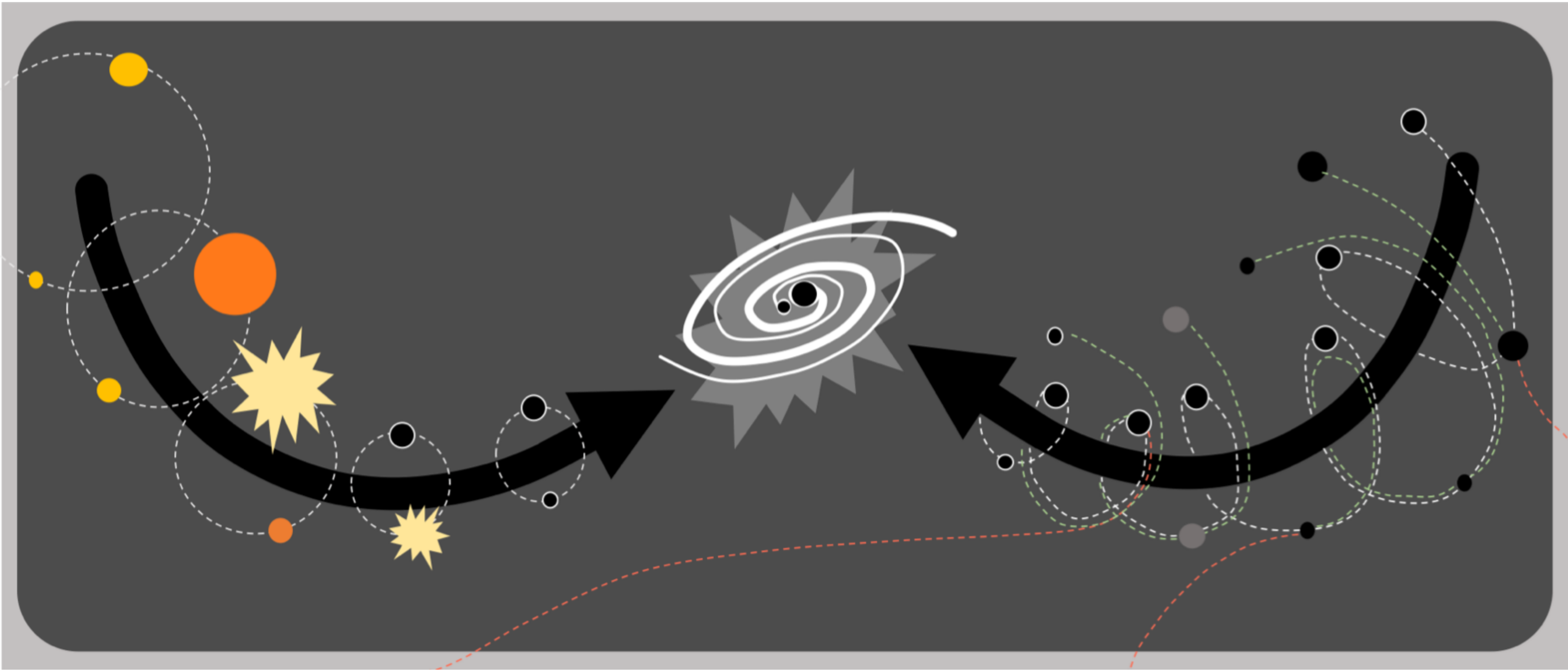
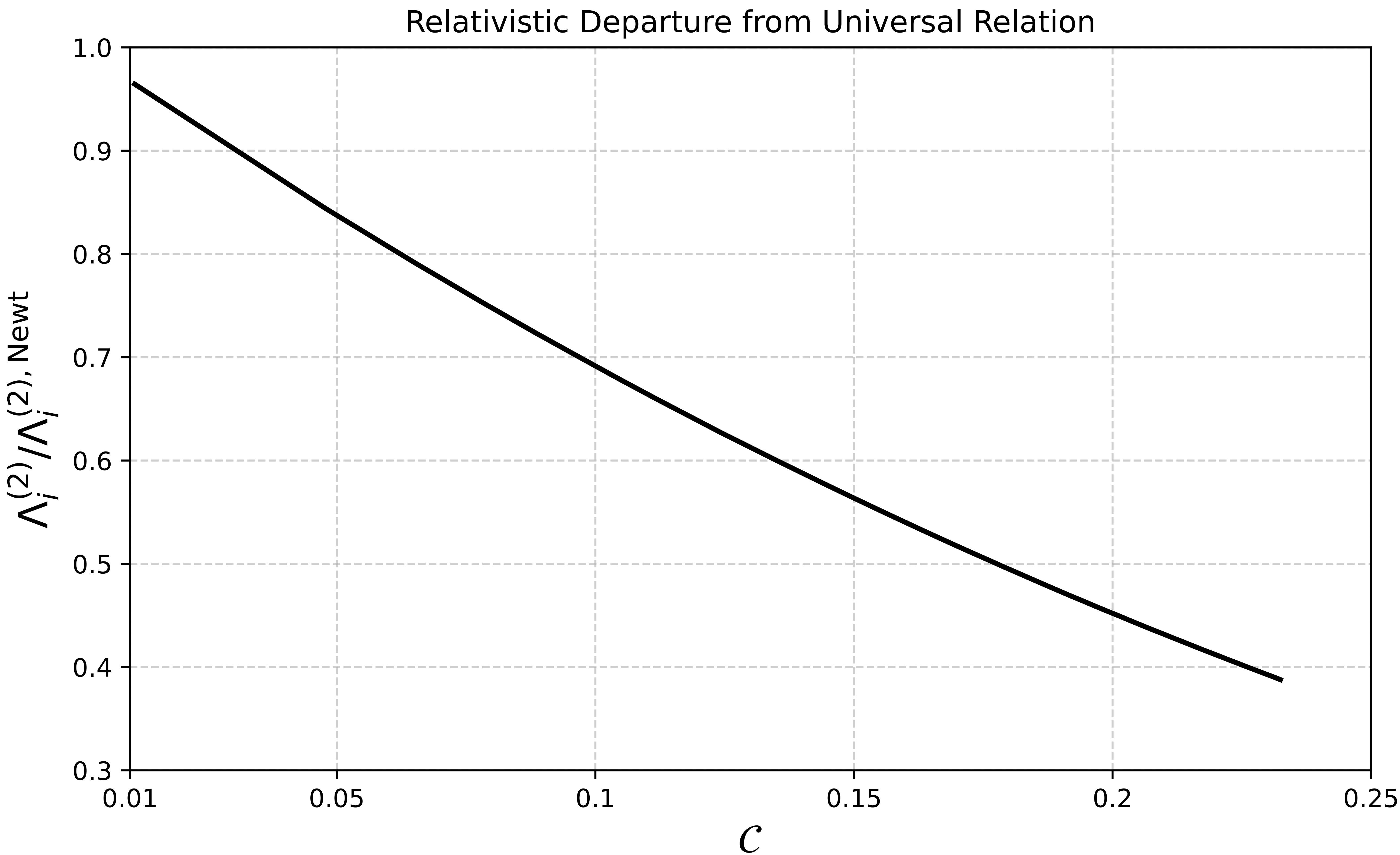
The Trojan Horse



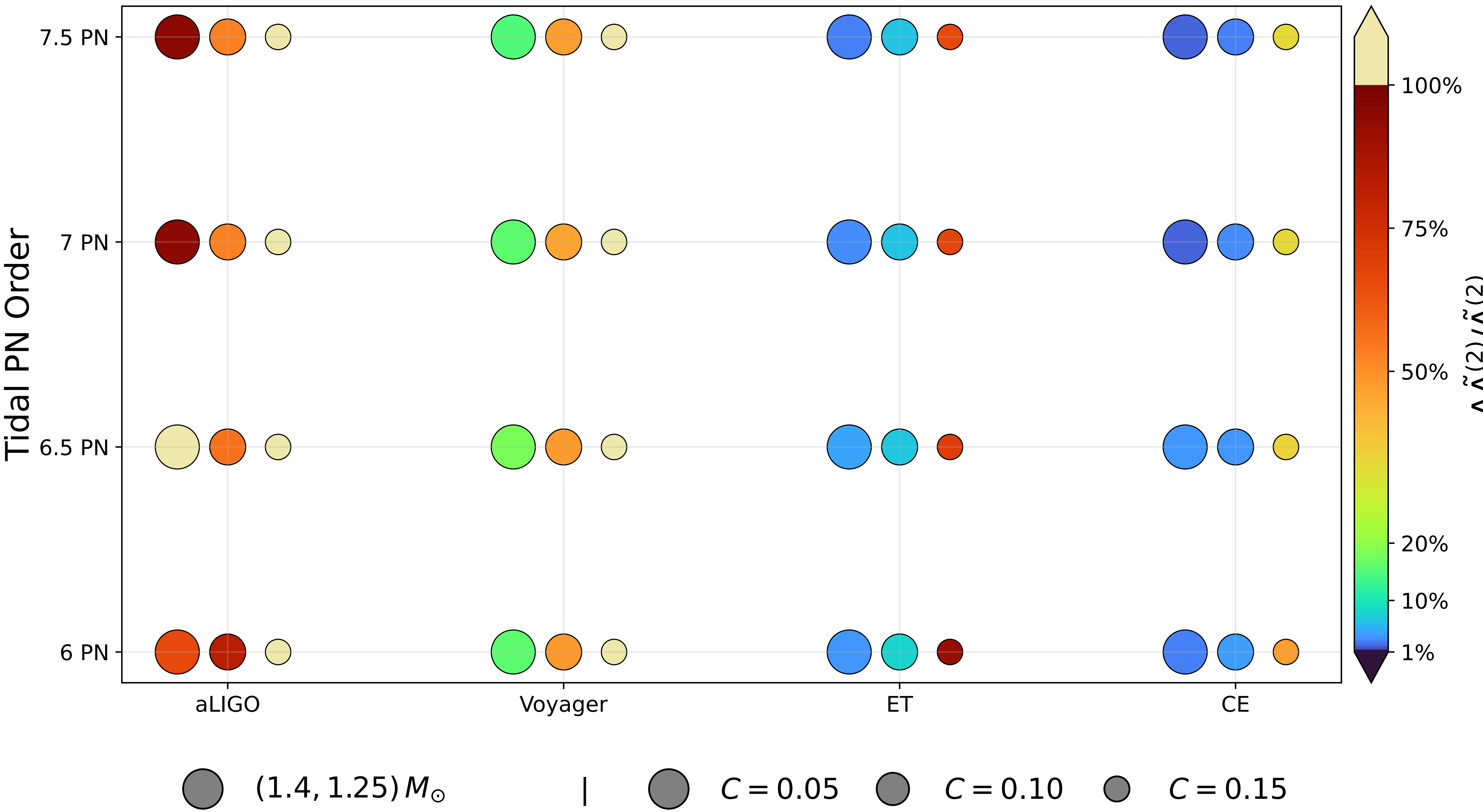
chandra.mishra@ligo.org

[Image Credits : Isobel Romero-Shaw]

additional slides

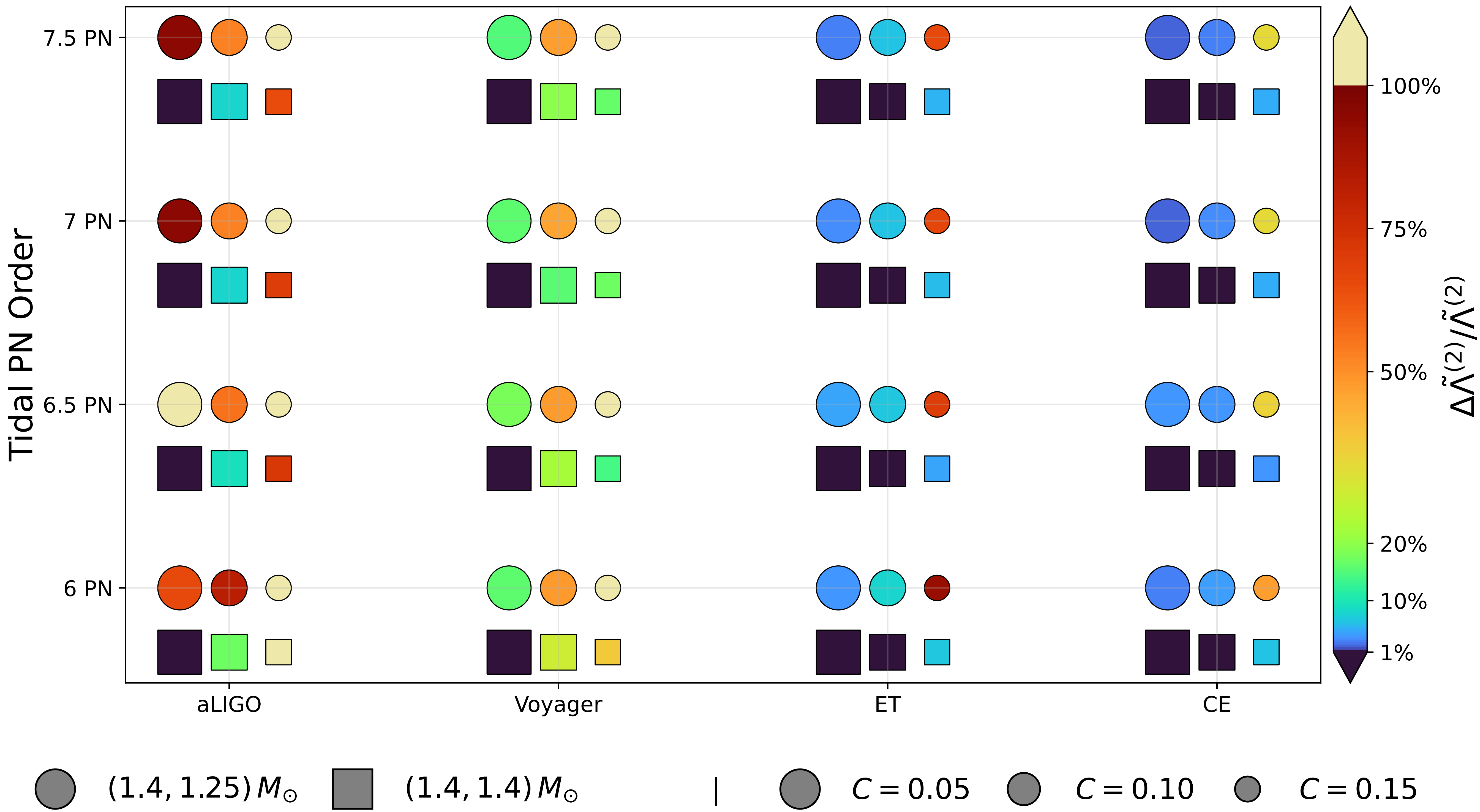

Relativistic Departure from Universal Relation
$\Lambda_i^{(2)}/\Lambda_i^{(2),\,\mathrm{Newt}}$
$\mathcal{C}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.01
0.05
0.1
0.15
0.2
0.25

The Measurement



Biswas+ (in prep)

The Measurement



Biswas+ (in prep)

The tidal parameters

$$\tilde{\Lambda}^{(2)} = \frac{8}{13}\left[(1+7\nu-31\nu^2)\left(\Lambda_1^{(2)}+\Lambda_2^{(2)}\right)\right.$$

$$\left.+\sqrt{1-4\nu}(1+9\nu-11\nu^2)\left(\Lambda_1^{(2)}-\Lambda_2^{(2)}\right)\right]$$

$$\delta\tilde{\Lambda}^{(2)} = \frac{1}{2}\left[\Delta\left(1-\frac{13272\nu}{1319}+\frac{8944\nu^2}{1319}\right)\left(\Lambda_1^{(2)}+\Lambda_2^{(2)}\right)\right.$$

$$\left.+\left(1-\frac{15910\nu}{1319}+\frac{32850\nu^2}{1319}+\frac{3380\nu^3}{1319}\right)\left(\Lambda_1^{(2)}-\Lambda_2^{(2)}\right)\right]$$

The Detector