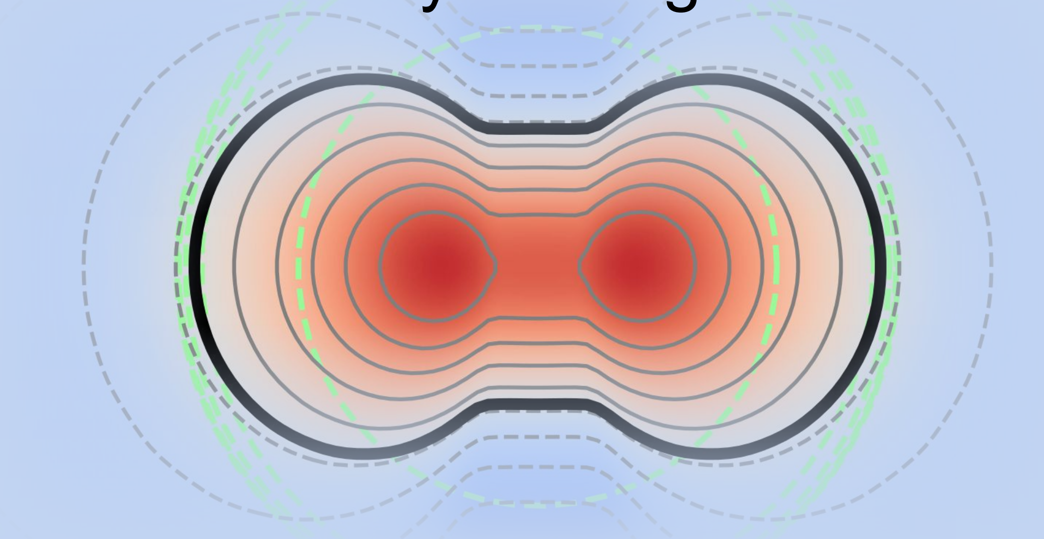
Numerical simulations of oscillating and differentially rotating neutron stars



Based on [SJ, Novak, Oertel, A&A 708, A133 (2026)]

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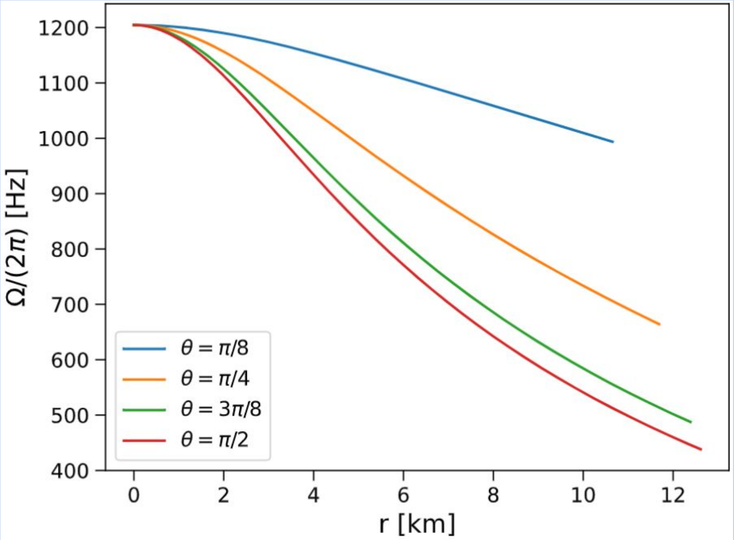
Observatoire astronomique de Strasbourg | ObAS

Motivation

- We simulate perturbed, isolated rotating neutron stars
  → BNS post-merger dynamics.
- ROXAS: **R**elativistic **O**scillations of non-a**X**isymmetric neutron st**A**r**S** (Servignat, Novak 2025, available at https://zenodo.org/records/14849547).
  - Code for dynamical relativistic evolution.
  - Lightweight: runs in laptops, typical simulation ~1-2 days.
  - Spectral methods, conformal flatness (CFC), primitive variables (Servignat et al. 2023).
  - Based on LORENE framework (C++).
- SJ, Novak, Oertel 2026
  → implement and test differentially rotating stars.

Assumptions

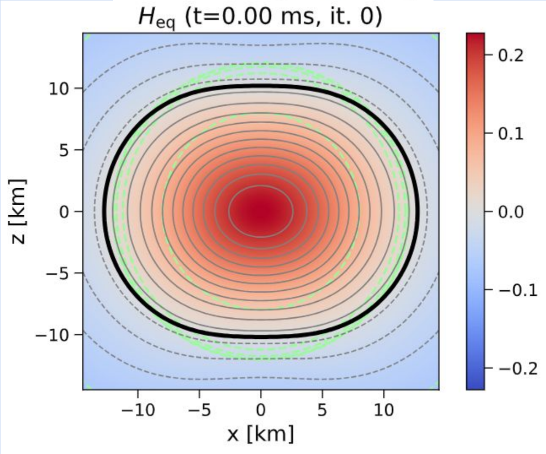
- Polytropic equation of state (EoS), $p=\kappa\rho^{\gamma}$.
- Equilibrium stars differentially rotating, $\vec{v}(r,\theta) = \Omega(r,\theta)\, r \sin\theta\, \hat{\varphi}$
- A differential rotation profile must be chosen: KEH (Komatsu, Eriguchi, Hachisu 1989) or *j-constant* law.



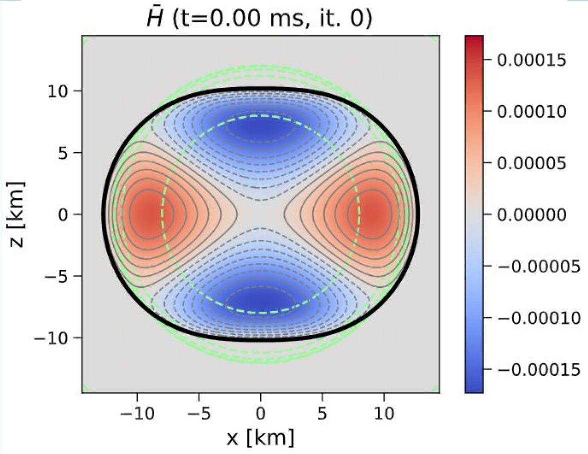
Numerical simulations

- We find configurations of differentially rotating NSs in equilibrium with LORENE.
- A perturbation combining l=m=0 and l=m=2 modes is added, and evolved with ROXAS.

$$H = \ln\left(\frac{e+p}{m_B n_B}\right)$$



$$\delta H = \varepsilon\left(x^2 - y^2 - \frac{2}{3}r^2\right)\left(1 - \frac{r^2}{R_S(\theta)^2}\right)$$



Numerical simulations

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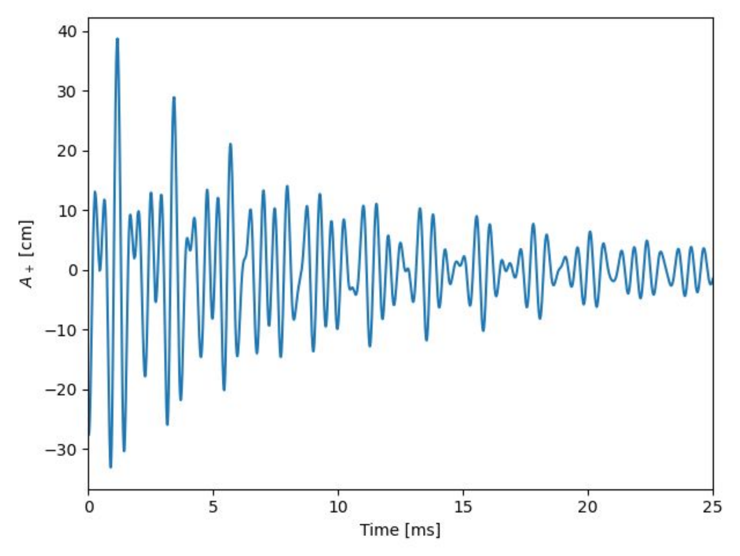
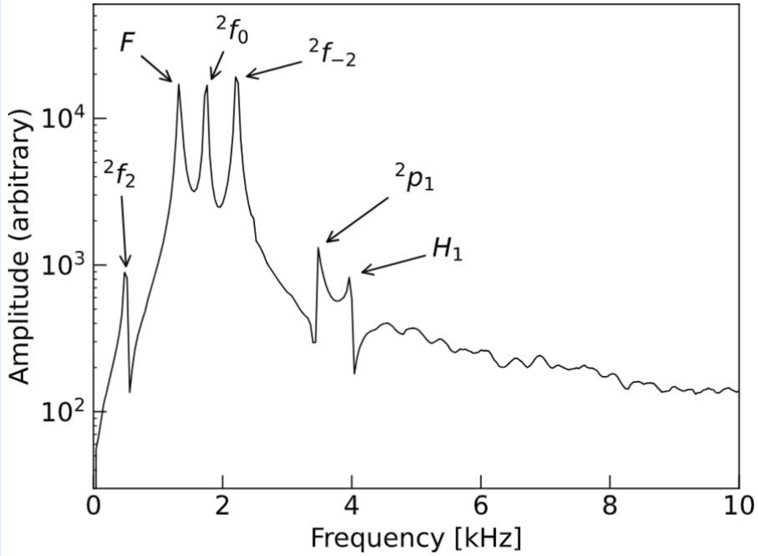
- Waveforms are extracted and Fourier-transformed to get oscillation mode frequencies.





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- A perturbation combining l=m=0 and l=m=2 modes is added, and evolved with ROXAS.
- Waveforms are extracted and Fourier-transformed to get oscillation mode frequencies.
- Two types of simulations: with Cowling approximation (constant metric) and without it.

Results: equilibrium configurations

- We study the standard B sequence of previous works.
- Equilibrium star properties consistent with Dimmelmeier et al. 2006.

Model	f_c (Hz)	f_e (Hz)	M ($M_\odot$)	r_p/r_e	T/W	δf_e (%)	δM (%)	$\delta(r_p/r_e)$ (%)	$\delta(T/W)$ (%)
B0	0	0	1.400	1.000	0.000	0.00	0.01	0.00	0.00
B1	582	215	1.437	0.950	0.012	0.00	0.03	0.00	3.85
B2	832	305	1.478	0.900	0.026	0.06	0.02	0.00	1.02
B3	1030	375	1.525	0.850	0.040	0.02	0.03	0.01	0.58
B4	1205	434	1.578	0.800	0.055	0.07	0.02	0.01	0.67
B5	1366	486	1.640	0.750	0.070	0.09	0.02	0.01	0.81
B6	1521	534	1.713	0.700	0.087	0.16	0.01	0.03	0.24
B7	1675	579	1.798	0.650	0.105	0.17	0.00	0.03	0.03
B8	1836	622	1.900	0.600	0.124	0.23	0.03	0.07	0.01
B9	2014	665	2.022	0.549	0.144	0.30	0.08	0.09	0.05

Results: simulations with Cowling approximation

- We study the standard B sequence of previous works.
- Equilibrium star properties consistent with Dimmelmeier et al. 2006.
- Mode frequencies with Cowling consistent with Stergioulas et al. 2004 (axisymmetric) and Krüger et al. 2010 (non-axisymmetric).

	$F_{\rm II}$	F	H_1	${}^2f_0$	${}^2p_1$	${}^2f_2$	${}^2f_{-2}$	$\delta F_{\rm II}$	δF	δH_1	$\delta\,{}^2f_0$	$\delta\,{}^2p_1$	$\delta\,{}^2f_2$	$\delta\,{}^2f_{-2}$	t_{3D}
B0	2.680	2.680	4.557	1.881	4.116	1.881	1.881	0.95	0.95	0.23	1.87	0.38	0.16	0.16	25.0
B1	2.663	2.663	4.487	1.917	4.109	1.439	2.242	1.35	0.17	0.93	1.98	0.18	0.74	0.13	25.0
B2	2.642	2.642	4.471	1.922	4.081	1.201	2.364	3.16	0.18	1.12	1.14	0.22	0.22	0.03	25.0
B3	2.555	2.649	4.443	1.953	4.041	1.011	2.450	1.18	0.65	0.85	2.10	0.11	0.40	0.03	25.0
B4	2.552	2.678	4.448	1.960	3.971	0.840	2.522	1.85	1.74	1.01	1.86	0.30	0.07	0.28	25.0
B5	2.519	2.680	4.480	1.960	3.909	0.679	2.563	1.29	1.84	1.32	1.62	0.04	0.95	0.15	25.0
B6	2.481	2.684	4.488	1.959	3.804	0.517	2.623	0.88	1.93	1.18	1.77	0.63	2.42	0.46	21.1
B7	2.451	2.701	4.519	1.933	3.718	0.331	2.651	1.15	2.49	1.61	1.24	0.07	2.12	0.10	21.2
B8	2.437	2.717	4.507	1.920	3.629	0.174	2.694	1.79	2.70	1.41	1.61	0.07	7.12	0.43	17.2
B9	2.398	2.720	4.442	1.884	3.543	-	2.750	1.62	2.54	0.65	0.92	0.67	-	0.88	13.2

Mode frequencies in kHz, relative errors in %.

Results: simulations in CFC

- We study the standard B sequence of previous works.
- Equilibrium star properties consistent with Dimmelmeier et al. 2006.
- Mode frequencies with Cowling consistent with Stergioulas et al. 2004 and Krüger et al. 2010.
- Axisymmetric mode frequencies w/o Cowling mostly consistent with Dimmelmeier et al. 2006.

Model	F	H_1	${}^2f_0$	${}^2p_1$	${}^2f_2$	${}^2f_{-2}$	δF	δH_1	$\delta\,{}^2f_0$	$\delta\,{}^2p_1$	t_{ax}	t_{3D}
B0	1.443	3.953	1.595	3.714	1.564	1.564	1.06	0.46	0.58	0.31	25.0	25.0
B1	1.429	3.915	1.634	3.701	1.118	1.945	1.57	0.32	0.34	0.32	25.0	25.0
B2	1.392	3.914	1.674	3.647	0.881	2.072	1.37	0.33	0.27	0.52	25.0	25.0
B3	1.359	3.953	1.718	3.572	0.681	2.159	1.99	0.27	0.51	0.33	25.0	25.0
B4	1.327	4.003	1.759	3.482	0.514	2.216	3.12	0.27	0.70	0.22	25.0	25.0
B5	1.287	4.073	1.800	3.395	0.320	2.276	4.03	0.02	0.63	0.14	25.0	25.0
B6	1.246	4.122	1.840	3.292	0.138	2.312	5.74	0.10	1.18	0.35	25.0	24.4
B7	1.210	4.193	1.881	3.199	-	2.353	12.36	0.33	1.48	0.36	25.0	19.3
B8	1.172	4.224	1.921	2.990	0.235	2.392	8.54	0.28	1.10	3.65	24.1	17.1
B9	1.130	4.177	1.960	2.912	0.471	2.459	19.60	6.54	2.23	3.83	20.4	14.9

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- We study the standard B sequence of previous works.
- Equilibrium star properties consistent with Dimmelmeier et al. 2006.
- Mode frequencies with Cowling consistent with Stergioulas et al. 2004 and Krüger et al. 2010.
- Axisymmetric mode frequencies w/o Cowling mostly consistent with Dimmelmeier et al. 2006.
- Non-axisymmetric ones reported for the first time.

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Mode frequencies in kHz, relative errors in %.

Summary

- ROXAS: lightweight code to simulate oscillating, rotating neutron stars.
    - Primitive variables, spectral methods, CFC.
- SJ, Novak, Oertel 2026: update for differentially rotating stars.
- Mode frequencies: great agreement with the literature in all cases.
- Non-axisymmetric modes without Cowling reported for the first time.
- Relevant to characterize BNS post-merger behaviour.
- Current work: thermal effects, physically motivated EoS.