

The impact of nuclear equations of state in numerical models of extreme explosions

A. Celati^{1,2,3}, M. Bugli^{4,5,3}, L. Del Zanna^{1,2,3}, M. Cusinato⁶ and M. Obergaulinger⁶

¹Dipartimento di Fisica e Astronomia, Università degli Studi di Firenze

²INFN - Sezione di Firenze

³INAF - Osservatorio di Arcetri

⁴Université Paris-Saclay, Université Paris Cité, CEA, CNRS, AIM 91191, Gif-sur-Yvette, France;

⁵Institut d'Astrophysique de Paris, 75014 Paris, France;

⁶Departament d'Astronomia i Astrofísica, Universitat de València, Spain

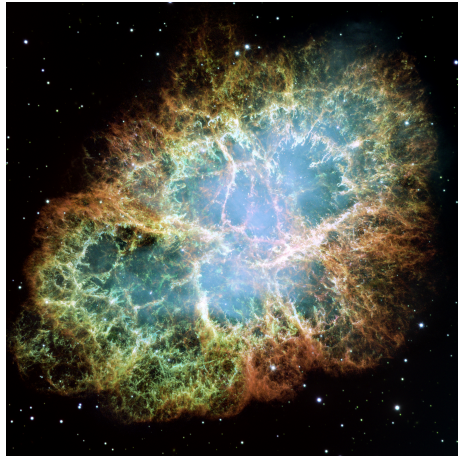
Thematic school GWsNS-2026: Gravitational Waves from Neutron Stars -
29/06/2026



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FIRENZE



- Scientific context of core-collapse supernovae;
- Numerical methods and initial conditions;
- Equation of State description;
- Main results discussion;
- Conclusion and future works.



Crab Nebula. Credit: NASA

Progenitor

- Core-collapse supernovae result from the collapse of massive stars ($\gtrsim 8M_{\odot}$)

The impact of CCSN on the universe

- Formation of **new stellar compact objects** (neutron stars or black holes)
- **Dynamic feedback** on host galaxy (energy injection)
- **Chemical evolution** of the Universe (explosive nucleosynthesis)

CCSN as multi-messenger sources

- **Multi-messenger sources** (emission of light, neutrinos and gravitational waves)
- Astrophysical laboratories with extreme physical conditions (GR, nuclear matter, astroparticles)



Cassiopea A. Credit: NASA

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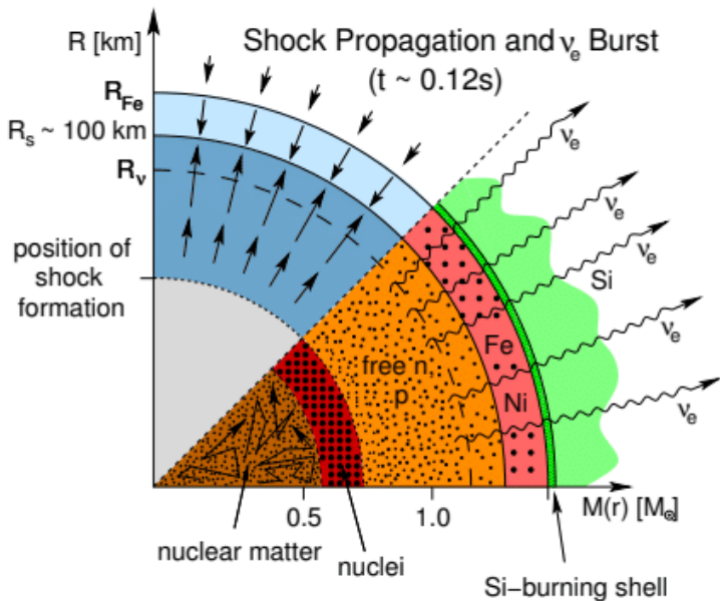
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Neutrino-driven supernovae

"Standard" CCSN

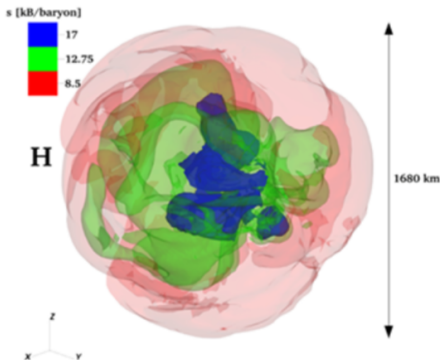
- 99% of CCSN
- $\sim 10^{58}$ neutrinos emitted during the whole collapse.
- $\sim 99\%$ of gravitational energy carried away by neutrinos.
- **Neutrino-heating** (Bethe 1985)

Modeling

- Relativistic hydrodynamic simulations (RHD);
- Spectral neutrino transport;
- Pseudo-Newtonian gravity;
- **Nuclear equations of state.**

Open Questions

- Can we **predict in advance** whether a given progenitor will **explode** and how?
- Can we determine **a priori** whether a progenitor will form a **neutron star** or a **black hole**?
- What is the **equation of state** of matter under these extreme conditions?
- How accurately can we model the **neutrino-matter interaction** and its impact on the explosion mechanism?



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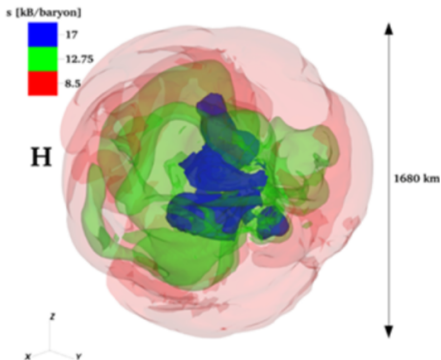
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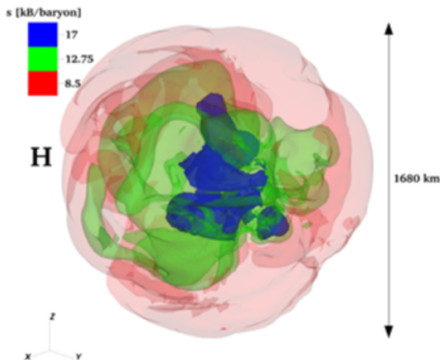
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Magnetorotational supernovae

Scientific interest in magnetorotational explosions

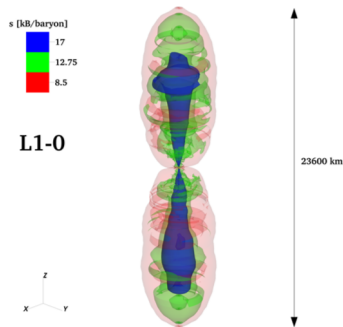
- 10 times more kinetic energy $\sim 10^{52}$ erg (**hypernovae**, **LGRBs**);
- Up to 100 times higher luminosity $\sim 10^{51}$ erg/s (**superluminous SN**);
- Linked to **r-process** elements;
- Can produce **stronger multimessenger signals** than neutrino-driven collapses.

Magnetorotational mechanism

- **Fast rotation** $\Rightarrow$ extra energy reservoir
- **Strong large-scale magnetic fields** to tap into it
- Rotating PNS can generate intense magnetic fields (dynamo processes).
- The magnetic field extracts rotational energy and accelerates **ejected material**, forming jets (Bisnovatyi et Kogan 1980)

The importance of the magnetorotational mechanism

- Formation of **relativistic jets**, driven by:
 - 1 Rotation of the **accretion disk** (Blandford–Payne effect);
 - 2 Rotation of the **spacetime metric** (Blandford–Znajek effect).
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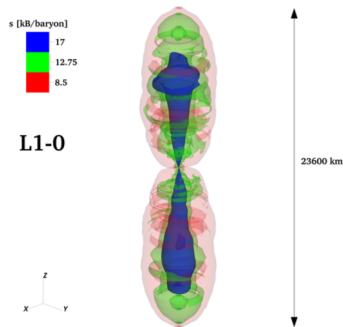
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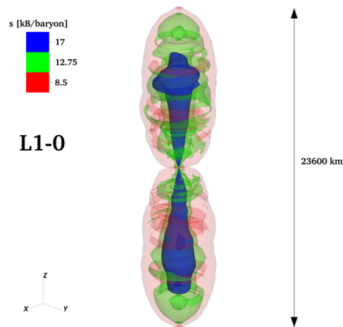
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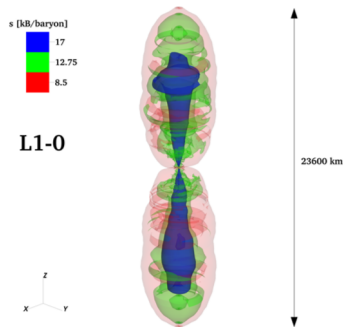
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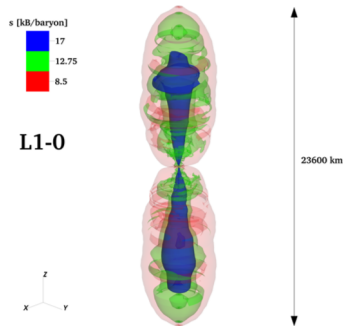
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Main goals

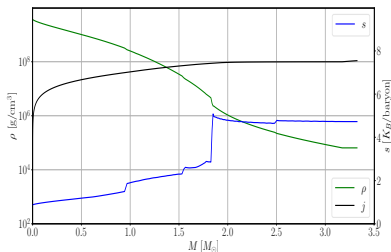
Investigate, for the **first time**, the impact of the **nuclear EoS** in **magnetorotational supernovae** on explosion **dynamics** and **multi-messenger** emissions (under revision for A&A);

Progenitor properties

- **ZAMS mass:** $20 M_{\odot}$ (Woosley and Heger 2007)
- **Iron core mass at collapse:** $1.54 M_{\odot}$
- **Oxygen shell mass at collapse:** $1.82 M_{\odot}$

Rotation and magnetic field

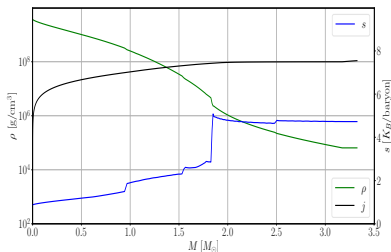
- **Rotation:** $\Omega(r) = \Omega_0 \frac{R_{\Omega}^2}{r^2 + R_{\Omega}^2}$
- **Magnetic field:** $A_{\phi}^{\text{dip}} = \frac{B_0}{2} \frac{r_0^3}{r^3 + r_0^3} r \sin \theta$
- $r_0 = R_{\Omega} = 10^3 \text{ km}$



Density, specific angular momentum, and entropy profile

Progenitor properties

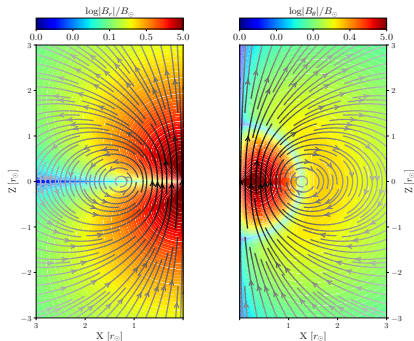
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Left: Absolute value of polar magnetic field normalized by $B_0 = 5 \times 10^{11} \text{ G}$. Right: Absolute value of radial magnetic field normalized by B_0 .

Equation of State

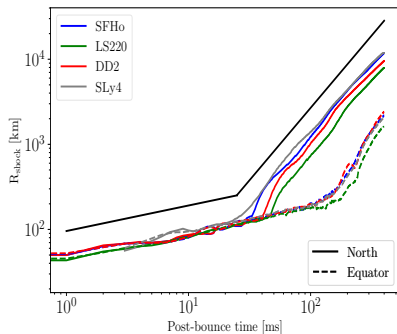
- **SFHo**: RMF EoS; statistical NSE treatment of nuclei (full ensemble).
- **DD2**: RMF EoS; statistical NSE treatment of nuclei (full ensemble).
- **SLy4**: Skyrme EoS; liquid-drop model, single nucleus approximation (SNA).
- **LS220**: Skyrme EoS; liquid-drop model, single nucleus approximation (SNA).

Property	SLy4	SFHo	LS220	DD2
Model type	Skyrme	RMF	Skyrme	RMF
Year	2017	2013	1991	2010
<i>K</i> [MeV]	230	245	220	242
<i>S</i> [MeV]	32.04	31.57	28.61	31.67
<i>L</i> [MeV]	46.0	47.10	73.80	55.03
<i>R</i> [km]	11.68	11.92	12.66	13.26

Equation of State comparison: shock and PNS radii

Property	SLy4	SFHo	LS220	DD2
Stiffness	Softest	Softer	Stiffer	Stiffest

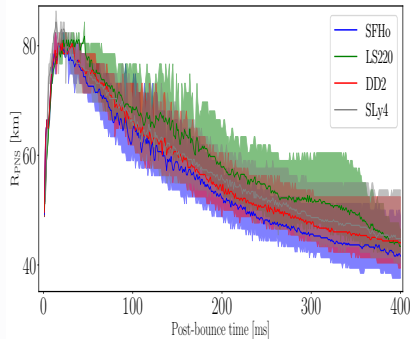
Explosion



Shock radius comparison

- Different revival times;
- Stiffer EoS models generally lead to less energetic explosions.

PNS



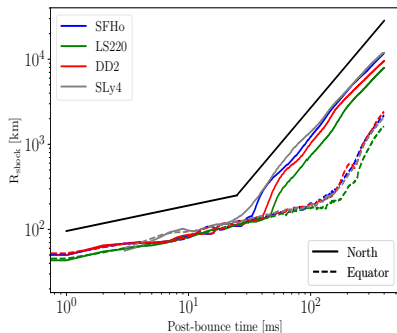
PNS radius comparison

- Dichotomous behavior observed;
- Within the observed dichotomy, stiffer EoS models lead to larger PNS radii.

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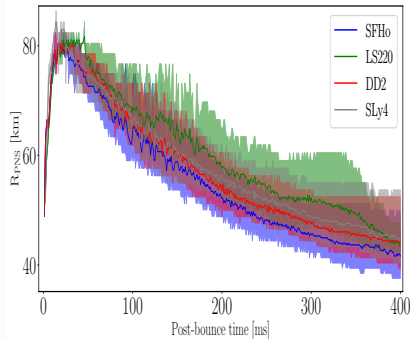
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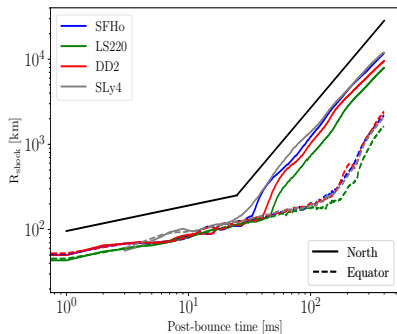
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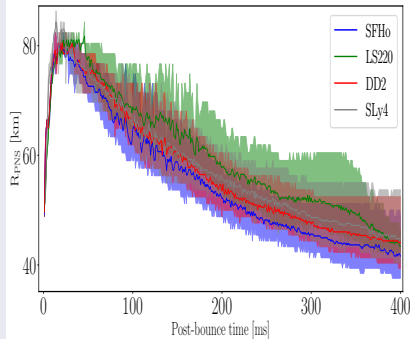
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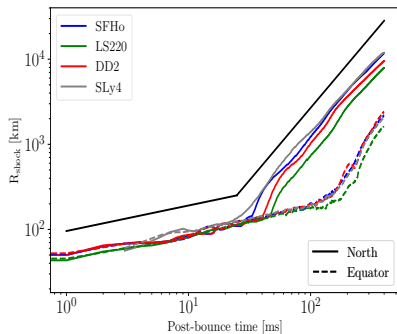
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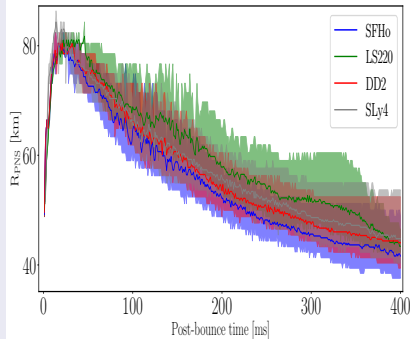
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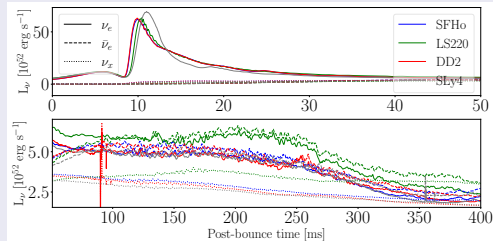
PNS



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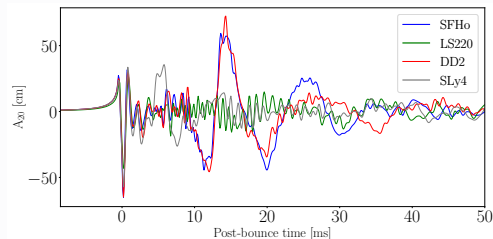
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Neutrino luminosity peak



- **SLy4** simulation produces a **higher-magnitude** deleptonization peak.
- **LS220** simulation produces a **higher** later neutrino emission

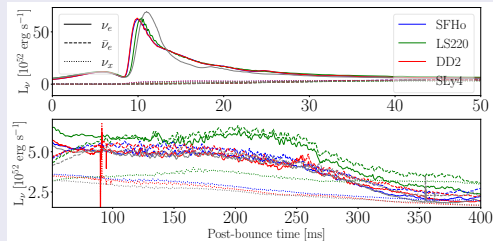
Gravitational wave early signal



Prompt convection signal

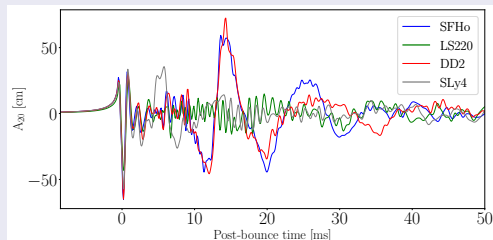
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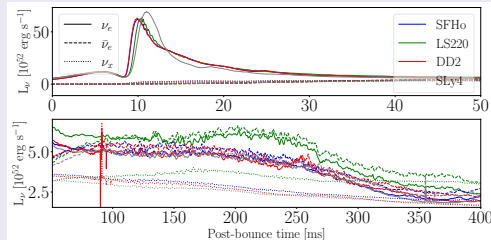
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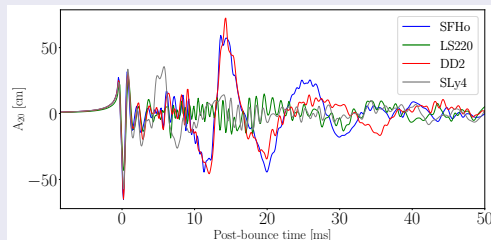
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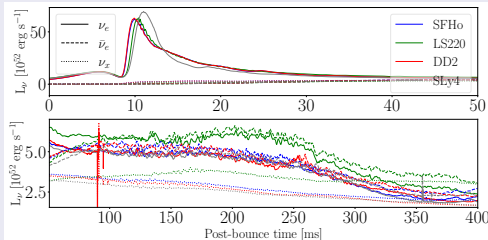
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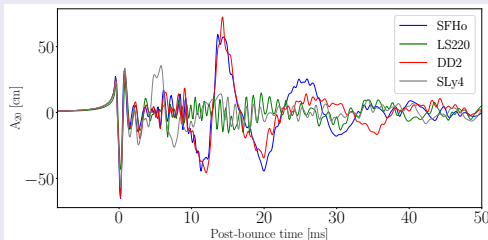
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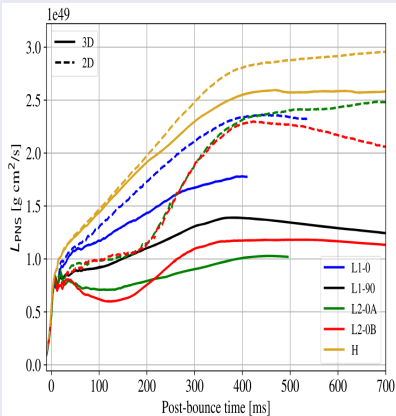
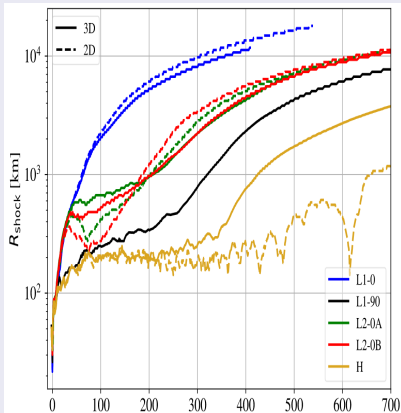
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- **S parameter** controls **collapse** and **explosion times**;
- **Magnetorotational efficiency** depends on the size of the **PNS**;
- **Stiffness alone** is not sufficient to characterize **PNS properties** or **multimessenger emission**;
- **Thermodynamic conditions** are crucial for explaining **dynamical properties**;
- **Microphysical variations** in the nuclear interaction are key to explaining the observed differences in **PNS radii** and **GW signals**;
- **GW emission** associated with **prompt convection** correlates with the **K parameter**.

3D models

- Extend simulations to **3D models**.

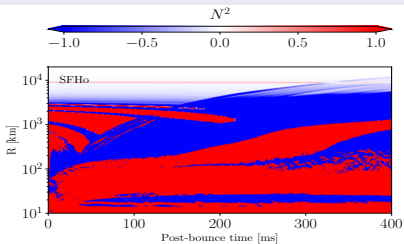


Credit: Bugli et al. 2021

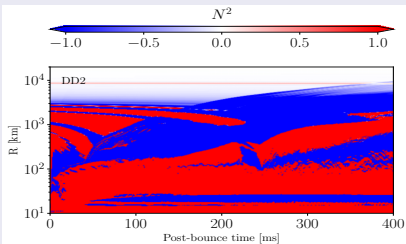
A vibrant cosmic background featuring a large, bright, yellow and orange nebula in the center, surrounded by smaller, reddish-orange and blue nebulae. A bright, blue, comet-like object streaks across the upper left, and a bright, white, comet-like object streaks across the lower right. The background is filled with numerous small, white stars.

Thank you for your attention!

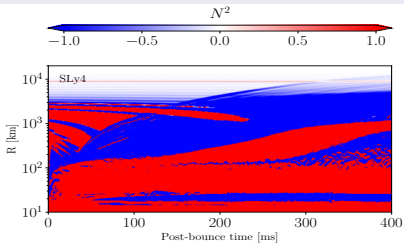
SFHo



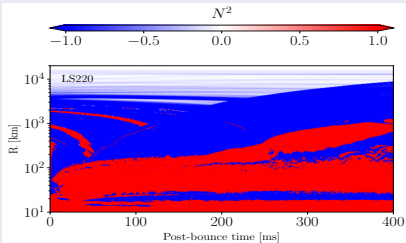
DD2



SLy4

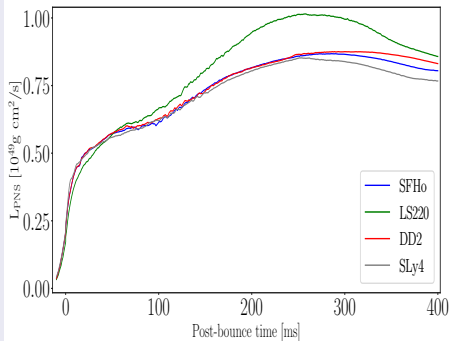


LS220



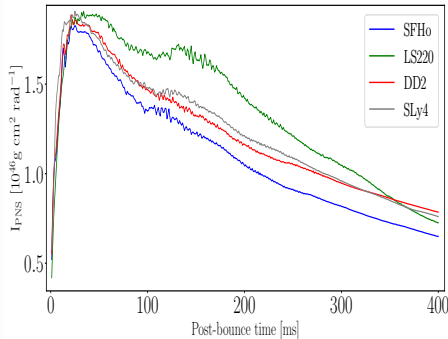
Equation of State comparison: PNS angular momentum and momentum of inertia

PNS angular momentum



- **LS220 deviates** significantly from the other EoS models;
- The remaining EoS models exhibit a **similar** behavior.

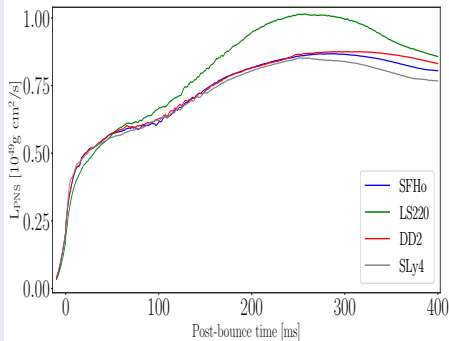
Momentum of inertia



- The momentum of inertia is **significantly different**;
- This results in a different **efficiency** for the extraction of **rotational energy** from the PNS.

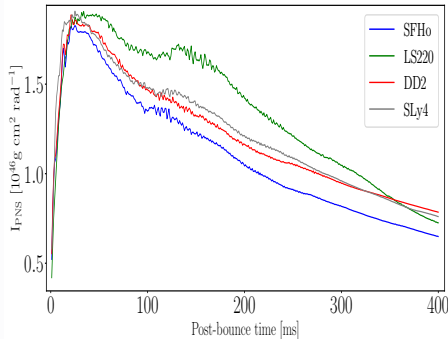
Equation of State comparison: PNS angular momentum and momentum of inertia

PNS angular momentum



- **LS220 deviates** significantly from the other EoS models;
- The remaining EoS models exhibit a **similar** behavior.

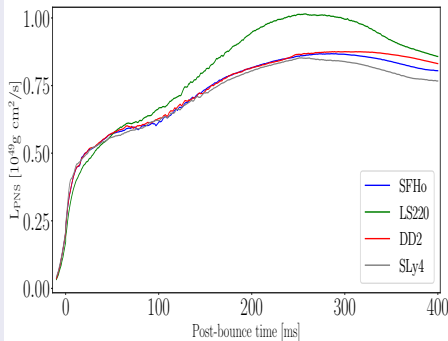
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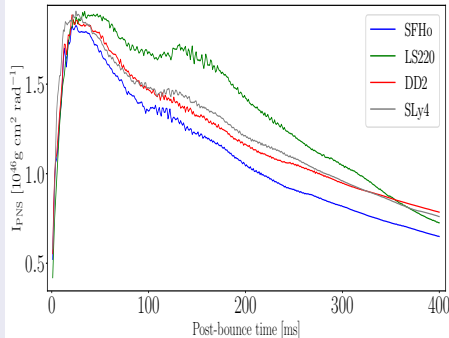
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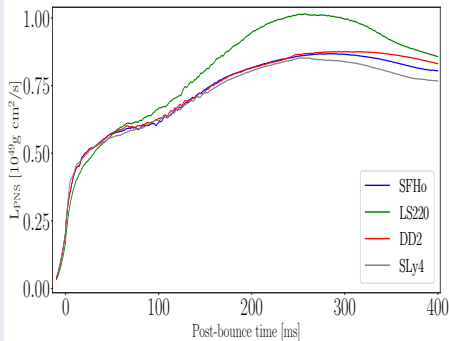
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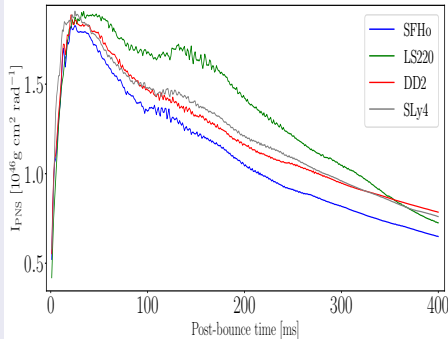
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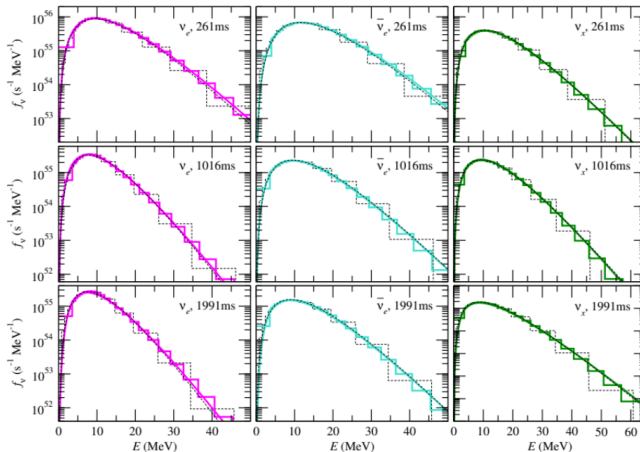


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Neutrino spectra (Janka et al. 2017)

$$f_{\alpha}(E) \propto \left(\frac{E}{\langle E \rangle} \right)^{\alpha} \exp \left[-(\alpha + 1) \frac{E}{\langle E \rangle} \right],$$

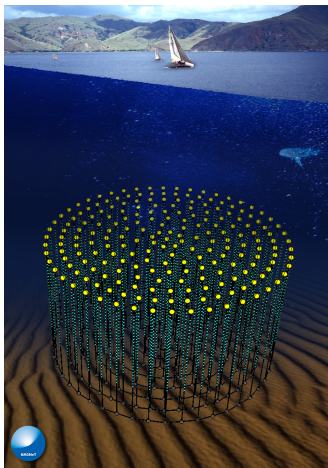
Neutrino Observatories

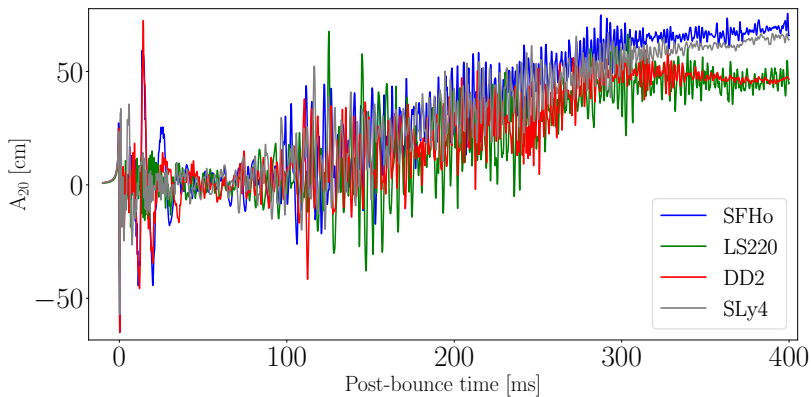
Current Observatories

- ORCA (KM3NeT)
- IceCube
- Super-Kamiokande

Future Observatories

- DUNE
- DarkSide
- Hyper-Kamiokande





Numerical Approximations

Neutrino Spectral Resolution

- **High**-resolution spectra: $N_E = 12$
- **Low**-resolution spectra: $N_E = 6$

Correction to TOV Potential

- **Case A**: Adding of **metric term** in the enclosed mass;
- **Case B**: Neglect the internal energy and radiation moments.

$$|\Phi_A| > |\Phi_B|$$

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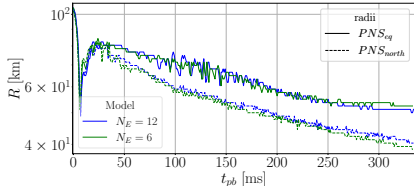
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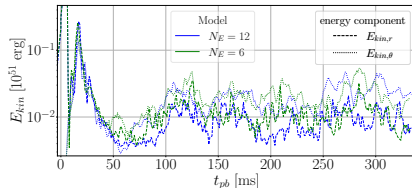
Numerical approximations comparison: dynamical differences

PNS



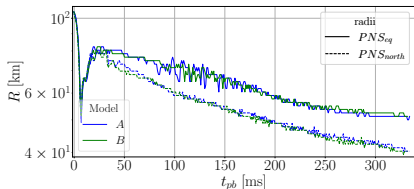
Low spectral-resolution simulation leads to a **more oblate** PNS.

Kinetic energy



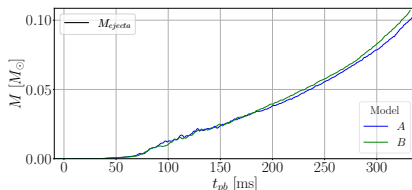
Low spectral-resolution simulation results in **stronger convection** within the PNS.

PNS



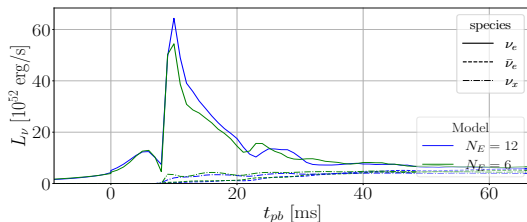
Case B simulation leads to a **marginally more oblate** PNS.

Ejecta mass



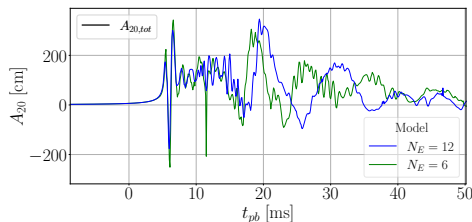
The **ejecta mass is greater** in Case B.

Neutrino luminosity peak



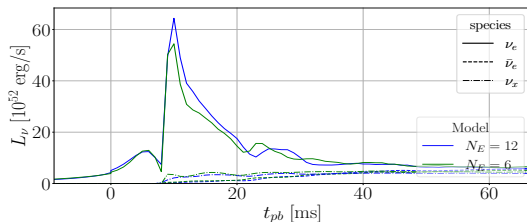
Low-spectral resolution simulation produces a lower-magnitude deleetonization peak.

Gravitational wave early signal



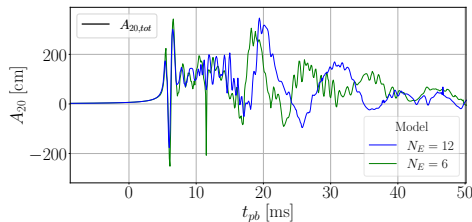
Low-spectral resolution simulation produces an anticipated prompt convection signal.

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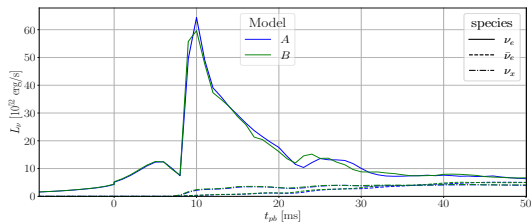
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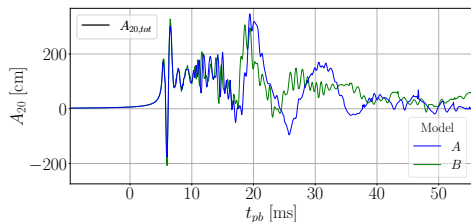
Gravitational potential correction comparison: multi-messenger signal

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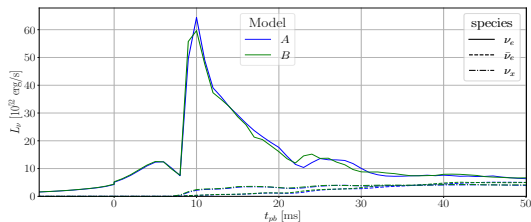
Gravitational wave early signal



Case B simulation produces an **anticipated** prompt convection signal.

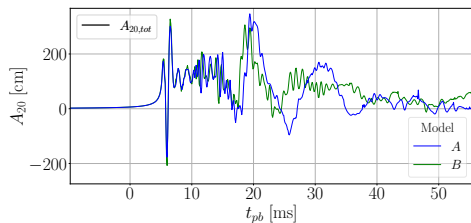
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Gravitational wave early signal



Case B simulation produces an **anticipated** prompt convection signal.

$$\Phi_{\text{TOV}}(r) = -4\pi \int_r^\infty \frac{dr'}{r'^2} \left(\frac{m_{\text{TOV}}}{4\pi} + r'^3(P + p_\nu) \right) \frac{1}{(\Gamma^2)} \left(\frac{\rho + e + P}{\rho} \right)$$

$$m_{\text{TOV}}(r) = 4\pi \int_0^r dr' r'^2 \left(\rho + e + E + \frac{vF}{\Gamma} \right)$$

$$\Gamma = \sqrt{1 + v^2 - \frac{2m_{\text{TOV}}}{r}}$$

$$\tilde{A}_{+/\times}(F) = \int_{-\infty}^{\infty} A_{+/\times}(t) H(t - \tau) e^{-2\pi i F t} dt$$

Parameters

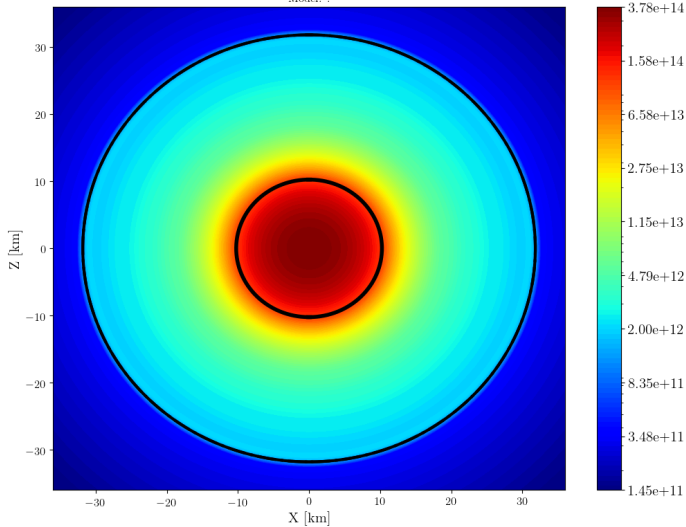
- $\text{nperseg} = 200$;
- $\text{overlap} = 100$;
- $f_s = 10^4$ Hz

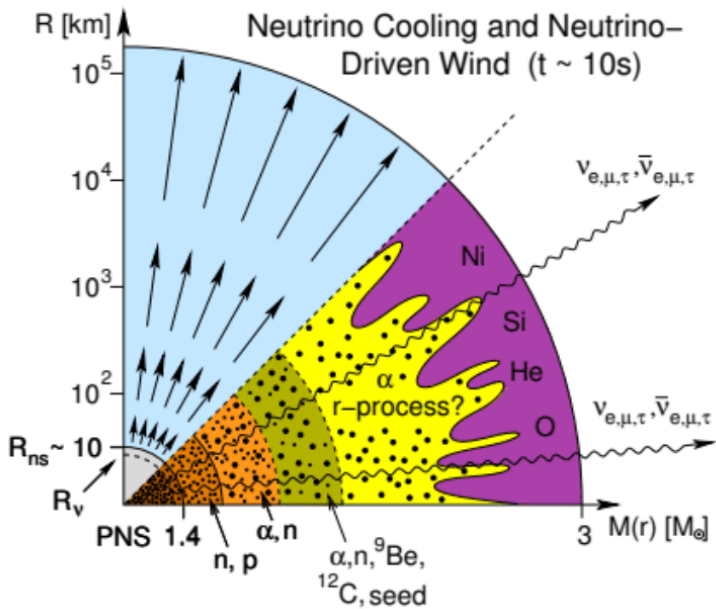
$$\Delta f = \frac{f_s}{\text{nperseg}} = \frac{f_s}{200} \simeq 20 \text{ Hz.}$$

Aspect	Skyrme (e.g., LS220)	RMF (e.g., SFHo)
Theory	Non-relativistic, empirical	Relativistic mean field
Interaction	Contact (effective potential)	Meson-exchange (σ, ω, ρ)
High-density regime	Less accurate	More reliable
Modern astrophysics use	Legacy model, still used	Widely adopted
Predicted properties	Binding energy, compressibility	Mass, radius, isospin symmetry
Flexibility	Limited	High (more parameters)

Comparison between Skyrme and RMF models

Density, Time= 268, 328x136x1
Model: .





RMHD equations

$$\partial_t \vec{u} + \vec{\nabla} \cdot \mathbf{f} = \vec{s}$$

$$\vec{u} = \begin{pmatrix} D \\ \vec{S} \\ U \\ \vec{B} \\ DY_e \end{pmatrix}, \quad \mathbf{f} = \alpha \begin{pmatrix} D\vec{v} \\ \omega\Gamma^2 \vec{v}\vec{v} - \vec{b}\vec{b} + p_{\text{tot}}\mathbb{I} \\ \vec{S} \\ \vec{B}\vec{v} - \vec{v}\vec{B} \\ DY_e \vec{v} \end{pmatrix},$$

$$\vec{s} = \begin{pmatrix} 0 \\ \alpha \vec{Q}_M - D\vec{\nabla}\Phi \\ \alpha Q_E + \alpha \vec{v} \cdot \vec{Q}_M + \vec{S} \cdot \vec{\nabla}\Phi \\ 0 \\ \alpha Q_N \end{pmatrix}$$

Neutrinos transport equations

$$\begin{aligned} \partial_t E + \partial_t(v_i F^i) + \nabla_i[\alpha(F^i + v^i E)] - (\nabla_i \alpha + \dot{v}_i) \left[\partial_\epsilon(\epsilon F^i) - F^i \right] \\ - \nabla_i(\alpha v_j) \left[\partial_\epsilon(\epsilon P^{ij}) - P^{ij} \right] = \alpha C^0, \end{aligned} \quad (1)$$

$$\begin{aligned} \partial_t(F^i + v_j P^{ij}) + \nabla_j(\alpha P^{ij} + v^j F^i) + \dot{v}^i E + \alpha F^j \nabla_j v^i + (E + P) \nabla^i \alpha \\ - \partial_\epsilon(\epsilon P_j^i) \dot{v}^j - \alpha \partial_\epsilon(\epsilon U_j^{ki}) \nabla_k v^j - \partial_\epsilon(\epsilon P^{ij}) \nabla_j \alpha = \alpha C^{(1),i}, \end{aligned} \quad (2)$$

Neutrino transport is modeled with the M1 algebraic closure.

$$X_{M_1}(f) = \frac{3 + 4f^2}{5 + 2\sqrt{4 - 3f^2}}$$

$$\begin{aligned} q_{M_1}(f) = f - \frac{(f - 2 + a)(f + 2 - a)}{2f(a - 2)^5} \left[12 \ln \left(\frac{f - 2 + a}{f + 2 - a} \right) (f^4 + 2af^2 - 7f^2 - 4a + 8) \right. \\ \left. + 48f^3 - 9af^3 - 80f + 40af \right] \end{aligned}$$

The amplitude is the second time derivative of the quadrupole moment.

- Radiative quadrupole moment

$$M_{20}^{E2} = \frac{G}{c^4} \cdot \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^t dz \int_0^\infty dr^3 \rho(r, z, t) r^2 \left(\frac{3}{2} z^2 - \frac{1}{2} \right)$$

- First time derivative is given by continuity equation

$$N_{20}^{E2} = \frac{G}{c^4} \cdot \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^t dz \int_0^\infty dr^3 \rho r \left[v_r \left(\frac{3}{2} z^2 - 1 \right) - 3v_\theta z(1 - z^2) \right]$$

- Second time derivative is given by MHD momentum equation

$$A_{20}^{E2} = \frac{G}{c^4} \cdot \frac{32\pi^{3/2}}{\sqrt{15}} \int_0^t dz \int_0^\infty dr^3 [f_{rr} (3z^2 - 1) + f_{\theta\theta} (2 - 3z^2) - f_{\phi\phi} - 6f_{r\theta} z(1 - z^2) - r \frac{\partial \Phi}{\partial r} (3z^2 - 1) + 3 \frac{\partial \Phi}{\partial \theta} z(1 - z^2)]$$

where

$$f_{ij} = \rho v_i v_j - b_i b_j$$