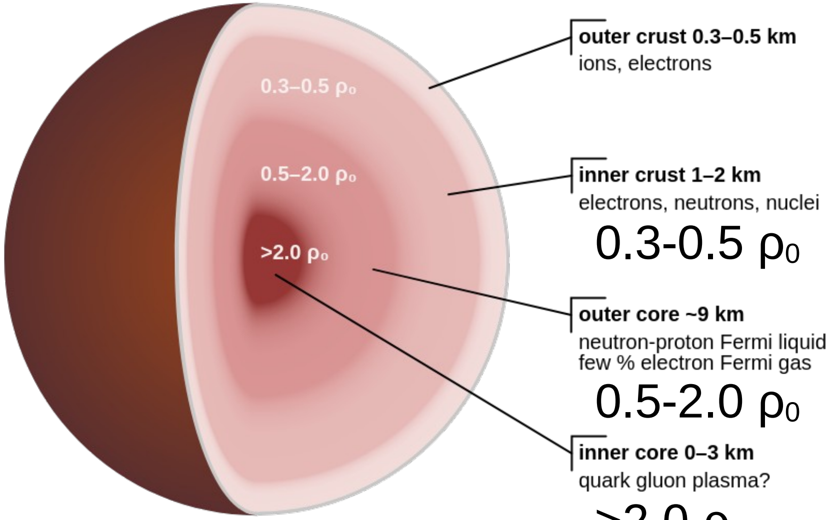
Théau DIVERRES

Neutron-star-crust elastic properties at zero temperature

2^{nd} year PhD student supervised by
Anthea Fantina

The crust



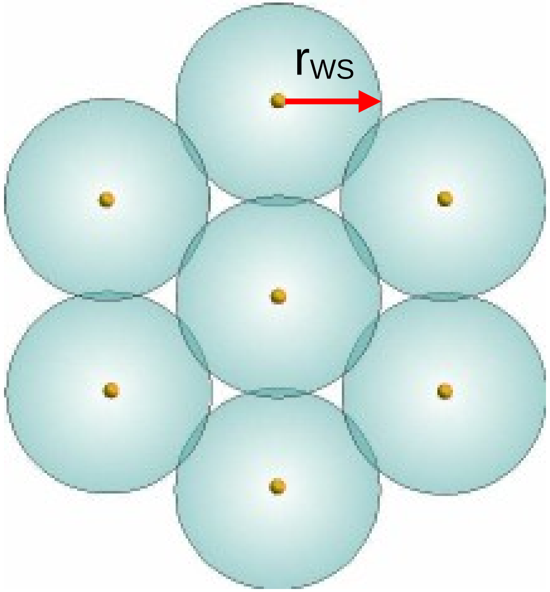
$0.3\text{-}0.5\ \rho_0$

$0.5\text{-}2.0\ \rho_0$

$>2.0\ \rho_0$

Neutron star - Wikipedia

T = 0: ions on a lattice surrounded by a gas of electrons and neutrons



N. Chamel, “Neutron star crust beyond the Wigner-Seitz approximation” (2007)

CLDM to treat non homogeneous many-body system

Elastic properties, shear modulus and speed:

$$\mu = 0.1194 \frac{(Ze)^2}{\frac{4}{3}\pi r_{\mathrm{ws}}^4} \qquad V_s = \sqrt{\frac{\mu}{\rho}}$$

Stromayer, The Astrophysics Journal (1991)

Important quantities for mountains, QPO, etc.

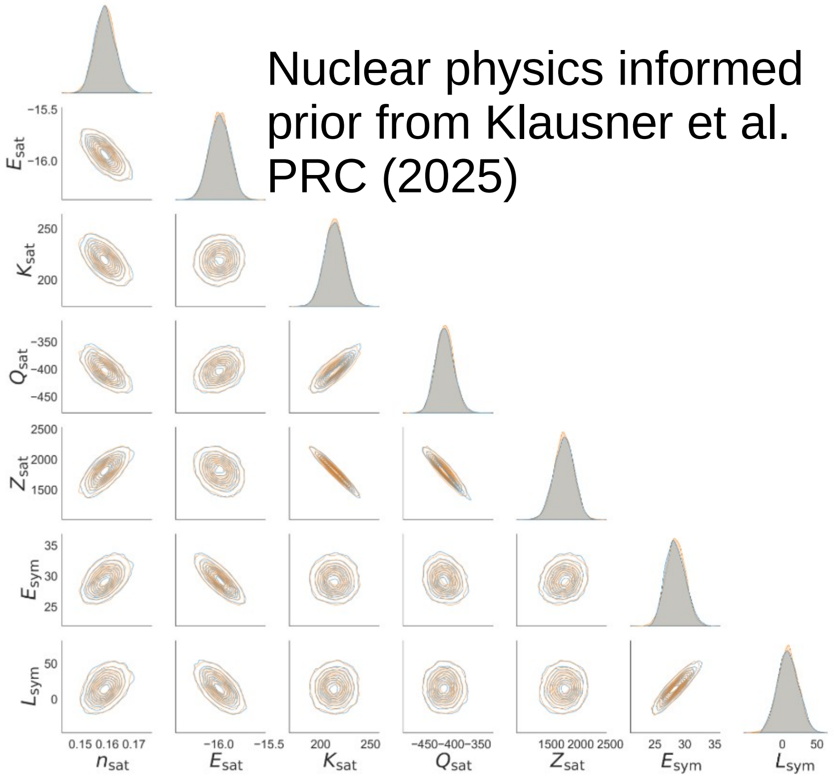
Bayesian analysis: two priors

(1): Flat prior

(2): Nuclear-physics informed prior →

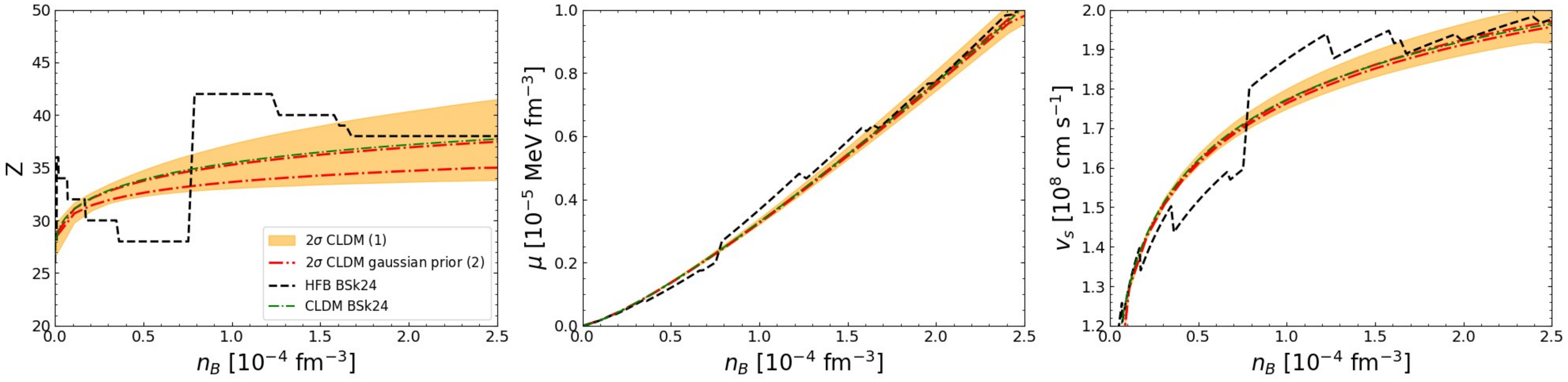
For both:

- Surface energy fitting (AME2020)
- Chiral EFT
- High density constraints
- Astro constraints: M_{max}, GW170817, NICER



Nuclear physics informed prior from Klausner et al. PRC (2025)

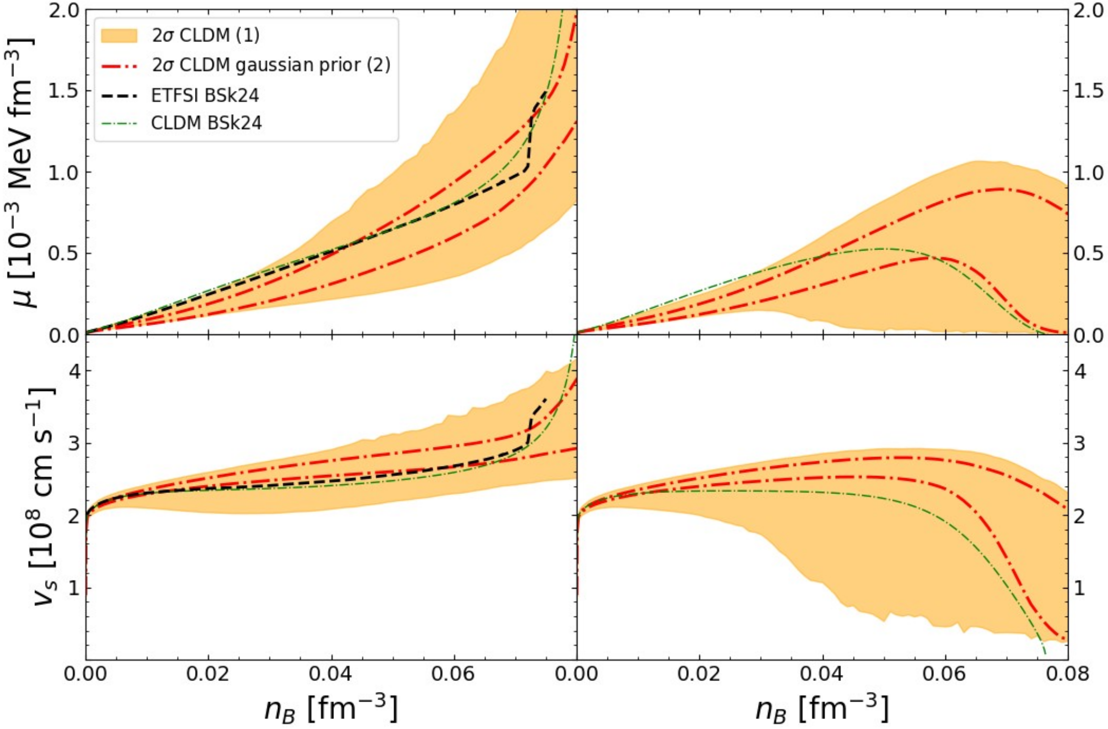
Elastic properties in the outer crust



- Uncertainties reduced in the outer crust
- Shell effects impact in more microscopic calculations

(Diverrès et al. A&A in press)

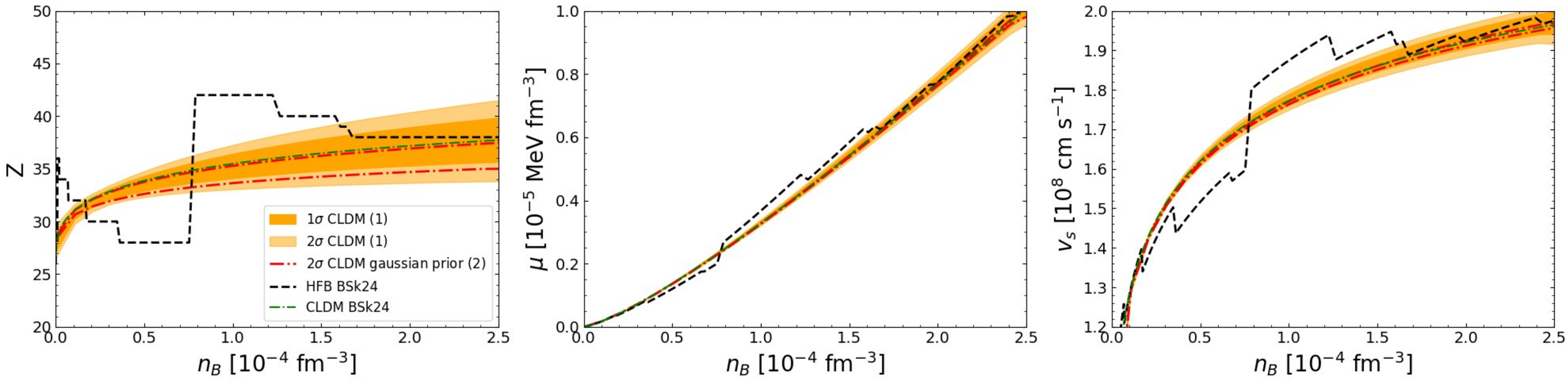
Elastic properties in inner crust



(Diverrès et al. A&A in press)

- Larger uncertainties in the inner crust
- Inclusion of nuclear physics experimental data constrains the crust predictions
- Inclusion of finite size corrections impact results at the bottom of the crust
- Estimations of the QPOs are compatible with observations

Elastic properties in the outer crust



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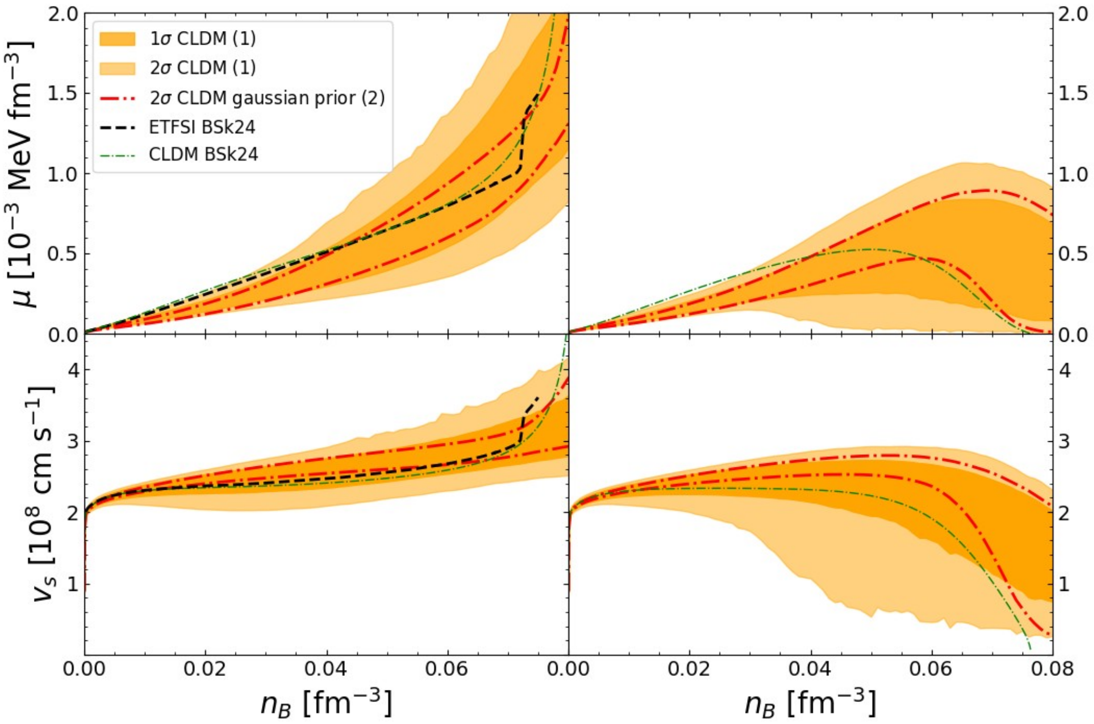
Elastic properties in inner crust



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(Diverrès et al. A&A in press)

Bayesian analysis: non-uniform prior

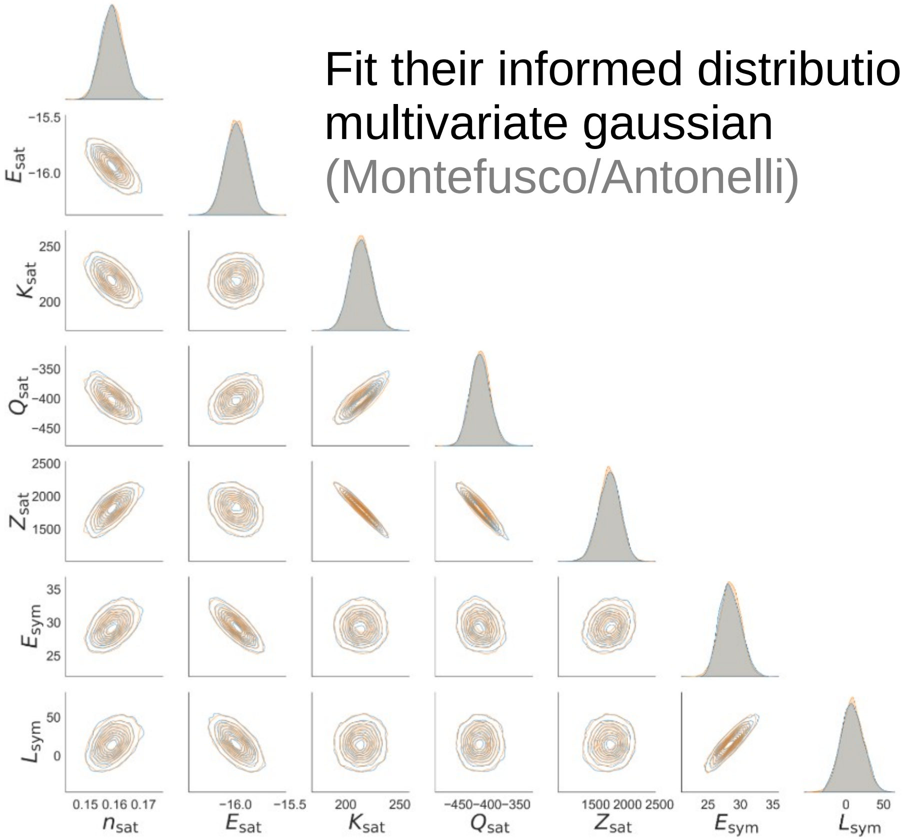
Porsterior from Klausner et al. PRC (2025)

↓

Our nuclear-informed prior

Parameter	min	max
$E_{\rm sat}$ [MeV]	~~-17~~	~~-15~~
$n_{\rm sat}$ $[{\rm fm}^{-3}]$	~~0.15~~	~~0.17~~
$K_{\rm sat}$ [MeV]	~~190~~	~~270~~
$Q_{\rm sat}$ [MeV]	~~-1000~~	~~1000~~
$Z_{\rm sat}$ [MeV]	~~-3000~~	~~3000~~
$E_{\rm sym}$ [MeV]	~~26~~	~~38~~
$L_{\rm sym}$ [MeV]	~~10~~	~~80~~
$K_{\rm sym}$ [MeV]	~~-400~~	~~200~~
$Q_{\rm sym}$ [MeV]	~~-2000~~	~~2000~~
$Z_{\rm sym}$ [MeV]	~~-5000~~	~~5000~~
$Q^{\star}_{\rm sat}$ [MeV]	-2000	2000
$Z^{\star}_{\rm sat}$ [MeV]	-5000	5000
$Q^{\star}_{\rm sym}$ [MeV]	-3000	3000
$Z^{\star}_{\rm sym}$ [MeV]	-5000	5000
$m^{\star}_{\rm sat}/m$	~~0.6~~	~~1.2~~
$\Delta m^{\star}_{\rm sat}/m$	~~0.0~~	~~2.0~~
b	1	10

12 parameters constrained by nuclear physics experiments



Fit their informed distribution as a multivariate gaussian (Montefusco/Antonelli)

The Compressible Liquid Drop Model

Sharp separation between the nucleus and the gas

$$\mathcal{E}_{\mathrm{CLDM}} = \mathcal{E}_e + \mathcal{E}_{\mathrm{g}}(1 - \frac{V_{\mathrm{N}}}{V_{\mathrm{WS}}}) + \frac{E_i}{V_{\mathrm{WS}}}$$

with

$$E_i = M_i c^2 + E_{\mathrm{bulk}} + E_{\mathrm{Coul}} + E_{\mathrm{surf+curv}}$$

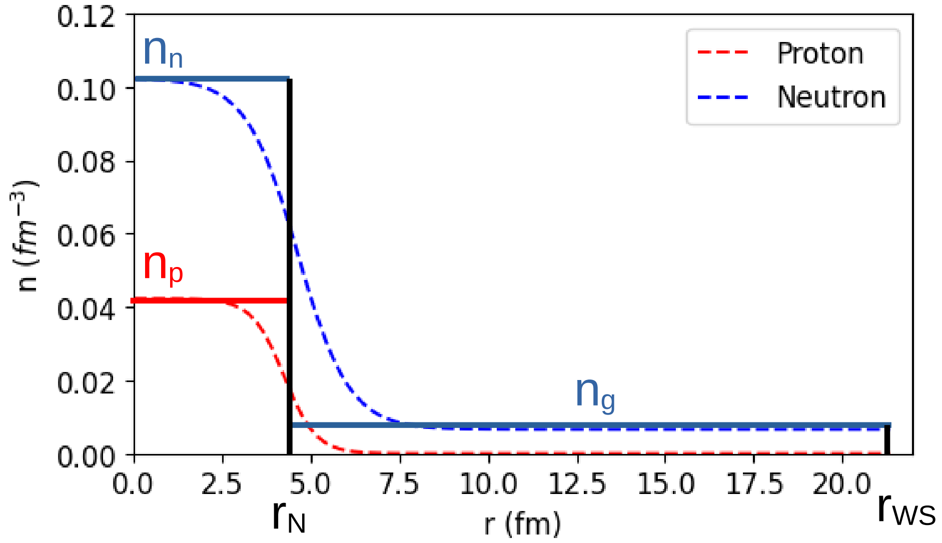
- E_{bulk} : meta model approach (Taylor expansion around n_{sat})

$$E_{\mathrm{bulk}} = V_{\mathrm{N}} \mathcal{E}_B(n = n_{\mathrm{N}}, \delta = \delta_{\mathrm{N}})$$

$$\mathcal{E}_B(n, \delta) = \mathcal{E}_{\mathrm{kin}}(n, \delta) + \mathcal{V}_{\mathrm{MM}}(n, \delta)$$

$$\mathcal{V}_{\mathrm{MM}}^N(n, \delta) = \sum_{k=0}^{N} \frac{n}{k!}(v_k^{\mathrm{is}} + v_k^{\mathrm{iv}} \delta^2) x^k u_k^N(x)$$

E_{sat}, K_{sat}, Q_{sat}... E_{sym}, L_{sym}, K_{sym}...



$$n_N = n_n + n_p$$

$$x = \frac{n - n_{\mathrm{sat}}}{3 n_{\mathrm{sat}}}$$

$$\delta_N = \frac{n_n - n_p}{n_n + n_p}$$

Margueron, Hoffmann, Guilminelli, PRC (2018)

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$$\mathcal{E}_{\mathrm{CLDM}} = \mathcal{E}_e + \mathcal{E}_{\mathrm{g}}\left(1 - \frac{V_{\mathrm{N}}}{V_{\mathrm{WS}}}\right) + \frac{E_i}{V_{\mathrm{WS}}}$$

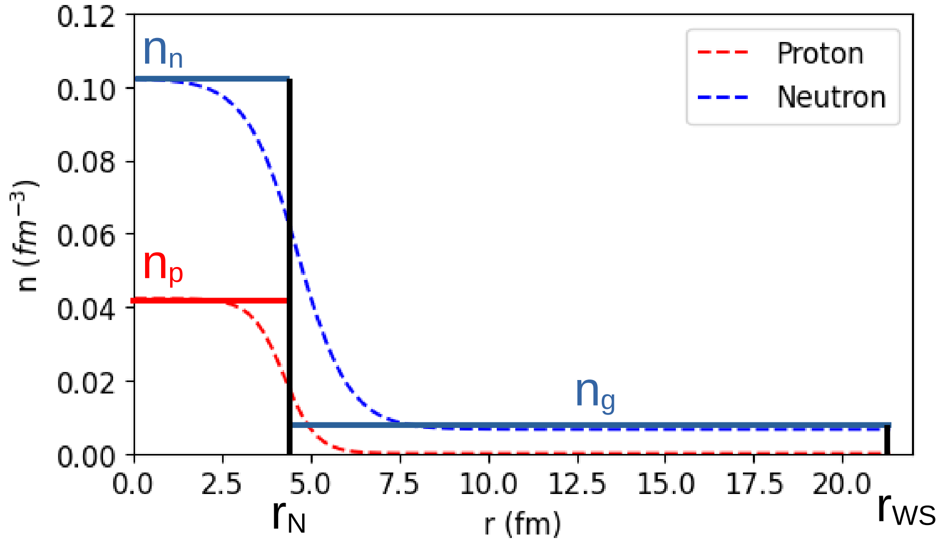
with

$$E_i = M_i c^2 + E_{\mathrm{bulk}} + E_{\mathrm{Coul}} + \boxed{E_{\mathrm{surf+curv}}}$$

- $E_{\mathrm{surf+curv}}$: parametrised and fitted to experimental data (AME2020)

$$E_{\mathrm{surf+curv}} = 4\pi\sigma_s r_N^2 + 8\pi\sigma_c r_N$$

J. M. Lattimer and F. Douglas Swesty, NPA (1991)



The shear modulus

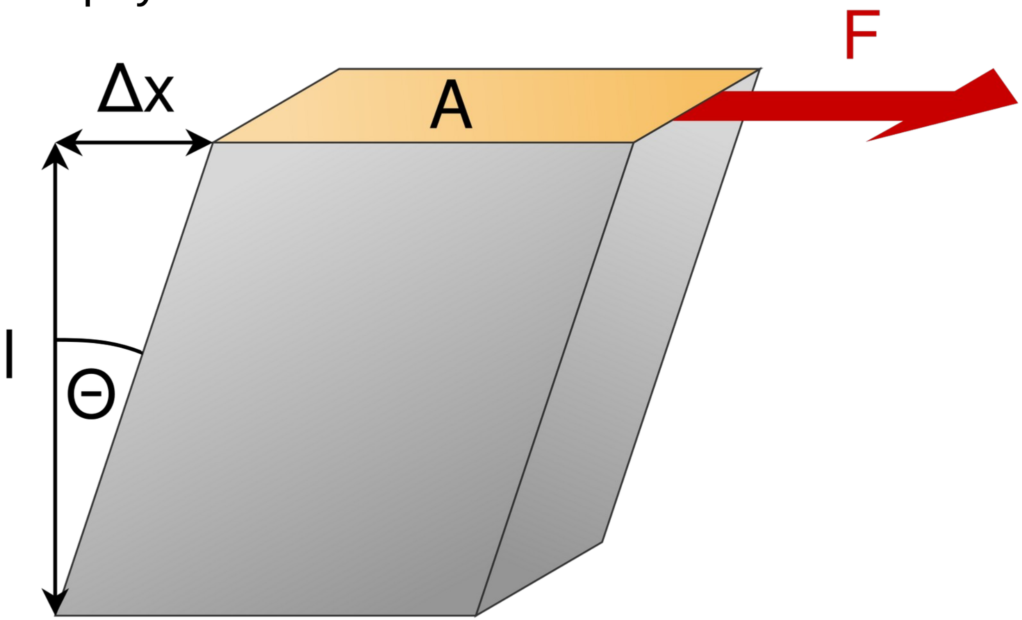
The shear modulus is an input for different astrophysical applications (QPO, montains, etc.)

The shear modulus is an intrinsic property of the material related to its ability to be deformed under such forces F

Fit to Monte Carlo calculations :

$$\mu = 0.1194 \frac{(Ze)^2}{\frac{4}{3}\pi r_{\mathrm{ws}}^4}$$

Stromayer, The Astrophysics Journal (1991)
Condensed Matter Physics, Marder (2000)



The shear velocity

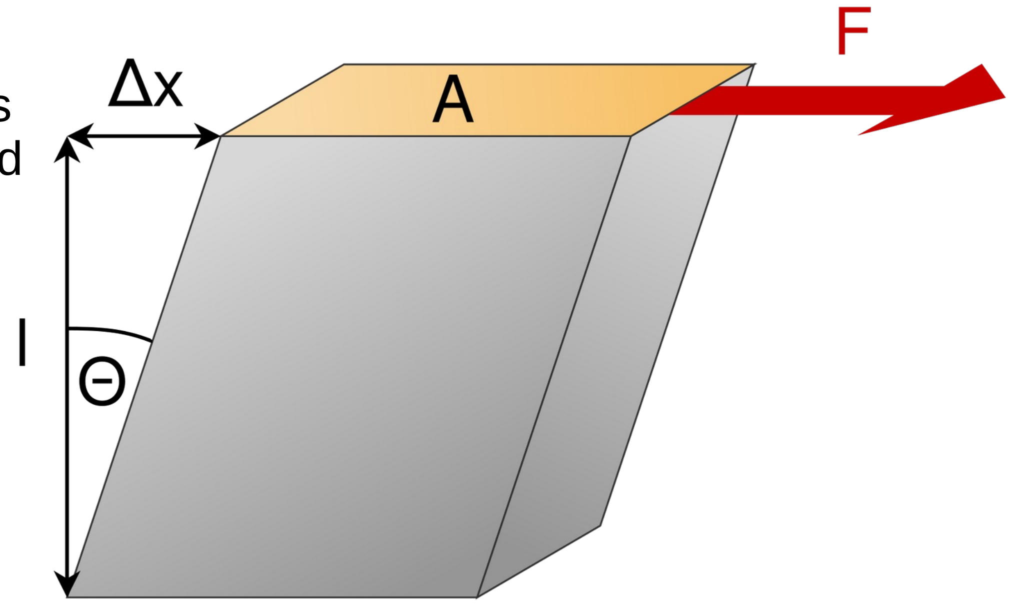
The shear deformations propagate at a speed V_S

Considering the shear deformation propagates as a plane wave, the shear speed is expressed

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

ρ is the mass density of nucleons moving with the lattice

$$\rho = \rho_B - \rho_g$$



Chamel PRC (2012) / Almirante & Urban arXiv:2503.21635 (2025)

Bayesian Analysis (1) : uniform prior

The simplicity and computation efficiency of the CLDM is adapted for bayesian analysis

Start from a set of empirical parameters

Going through several filters

$$\mathcal{L}(\text{model}|\mathbf{c}) \propto \mathcal{L}(\text{model}) \prod_{\mathrm{k}} \mathcal{L}(\mathrm{c_k}|\text{model})$$

Replace the original Qs and Zs at densities > n_{sat}

Parameter	min	max
E_{sat} [MeV]	-17	-15
n_{sat} [fm^{-3}]	0.15	0.17
K_{sat} [MeV]	190	270
Q_{sat} [MeV]	-1000	1000
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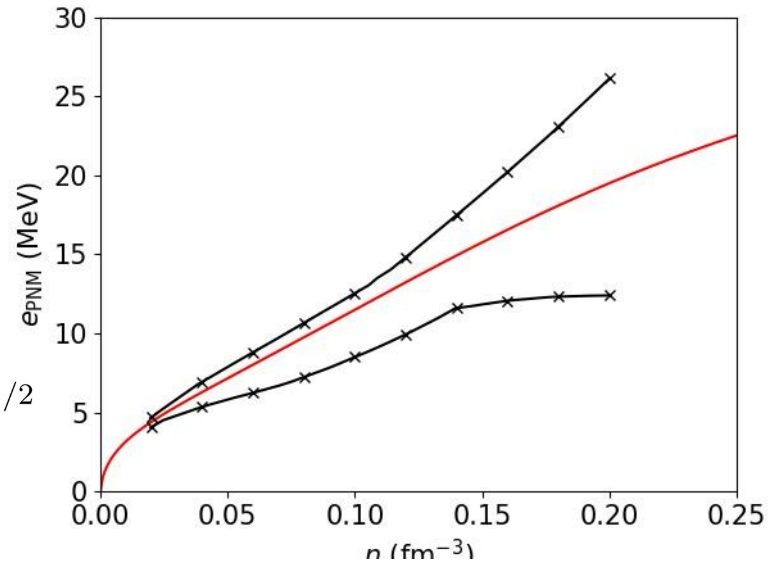
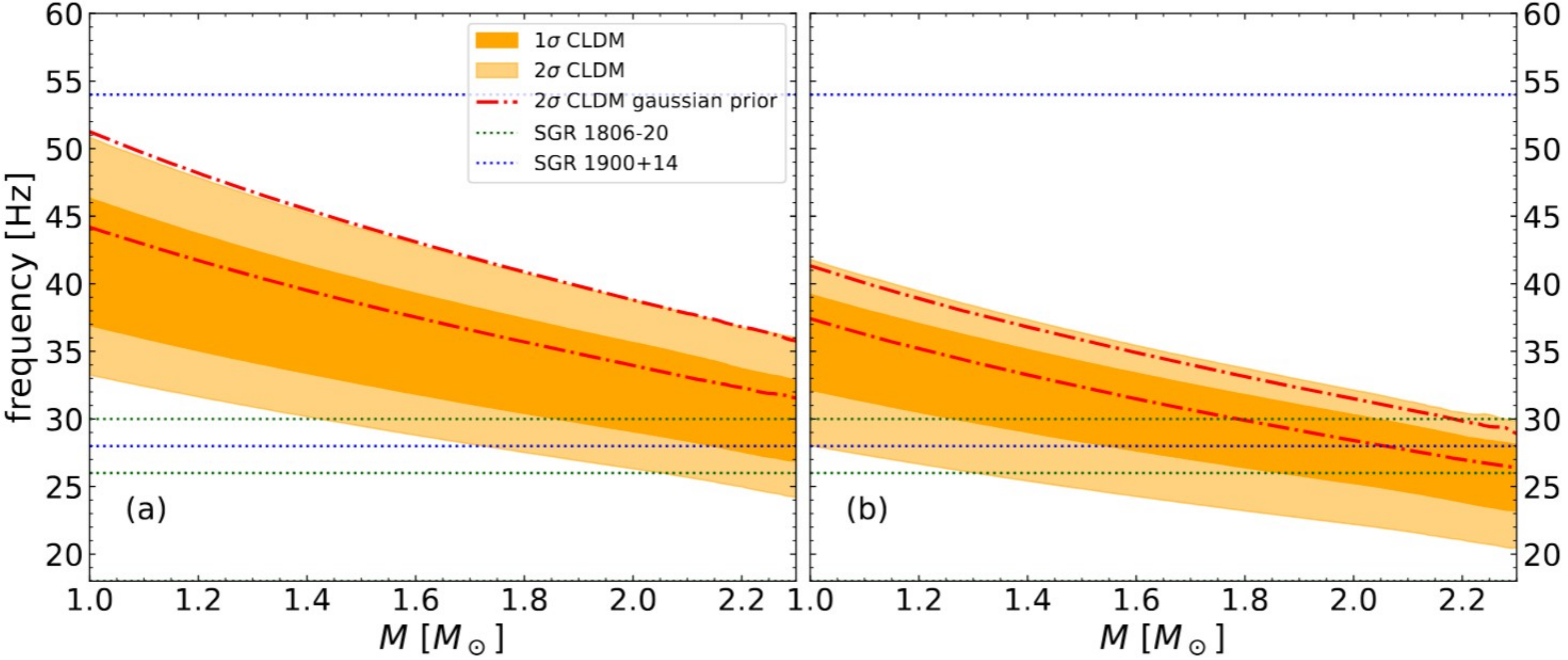
Bayesian filters

$$\mathcal{L}(\text{model}) = \prod_{\text{filter}} \mathcal{L}_{\text{filter}}(\text{model})$$



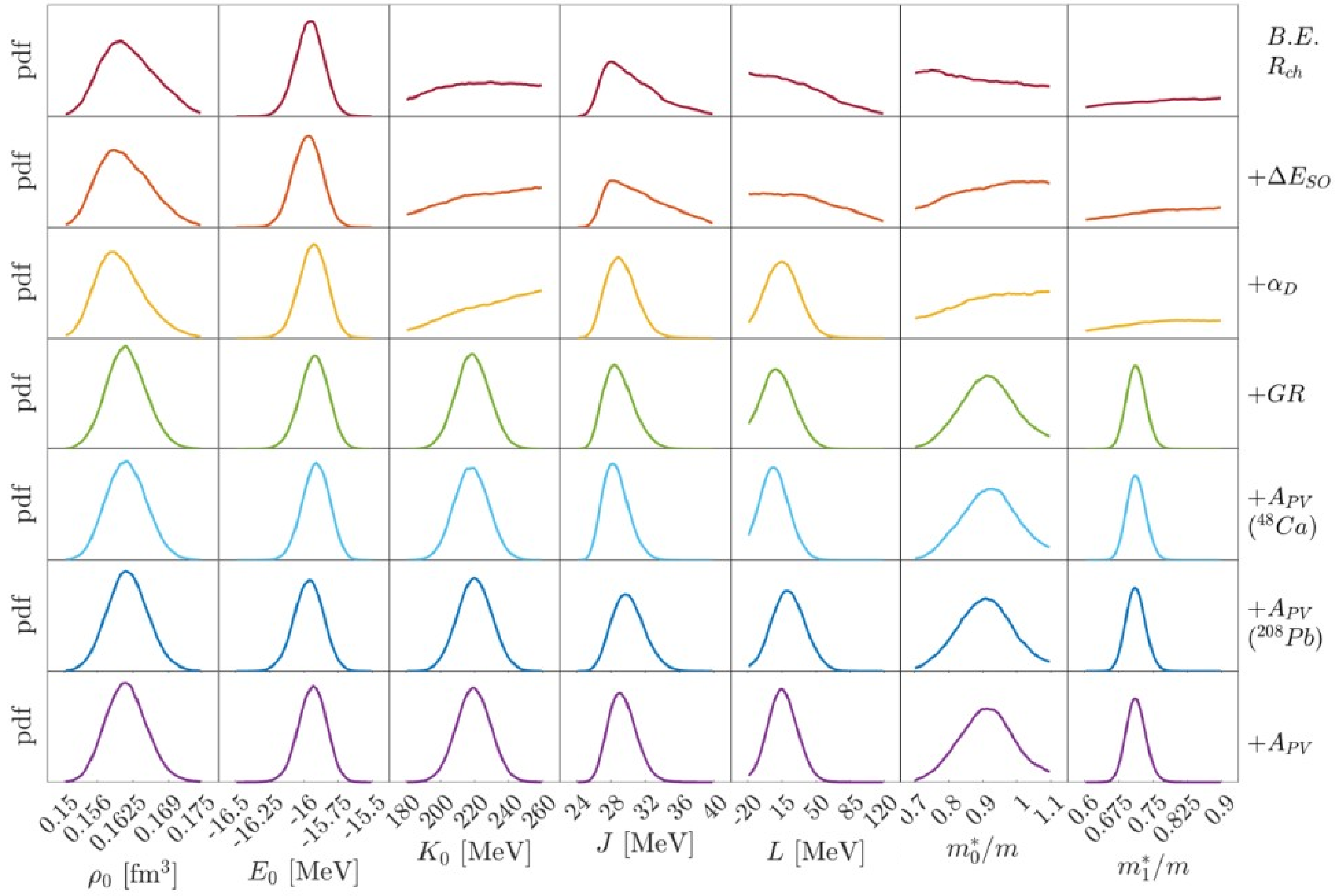
The filters:

- Surface energy fitting (AME2020) $\mathcal{L}_{\text{fit}}(\text{model}) \propto \mathrm{e}^{-\chi^2(\text{model})/2}$
- low density chiral EFT
- causality, thermodynamic stability, non-negative symmetry energy at all densities
- capability of producing a neutron star with a mass > 2 solar masses
- constraints from tidal deformability (GW170817) and NICER



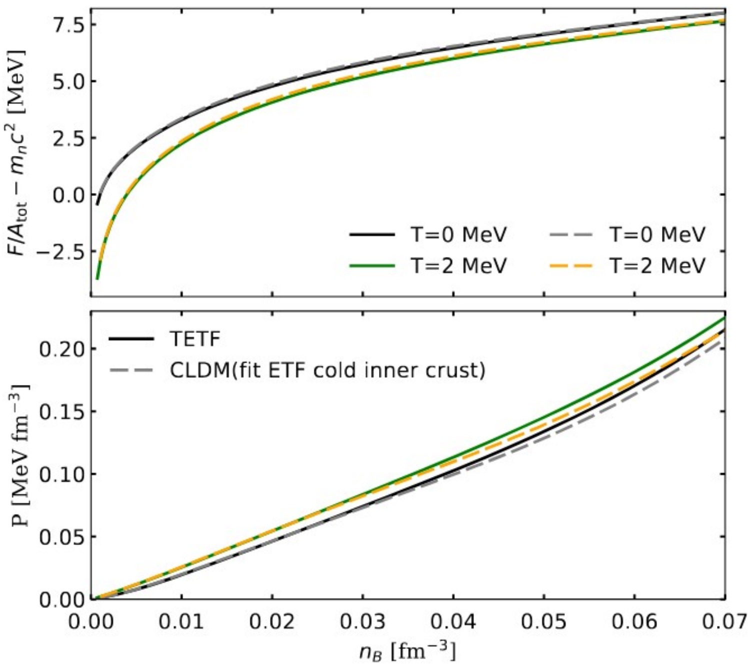
$$\omega_0 \approx \frac{e^{2\nu} v_s^2 (l-1)(l+2)}{2RR_{\rm cc}}$$

Samuelsson & Andersson MNRAS (2007)



Klausner et Al. PRC (2025)

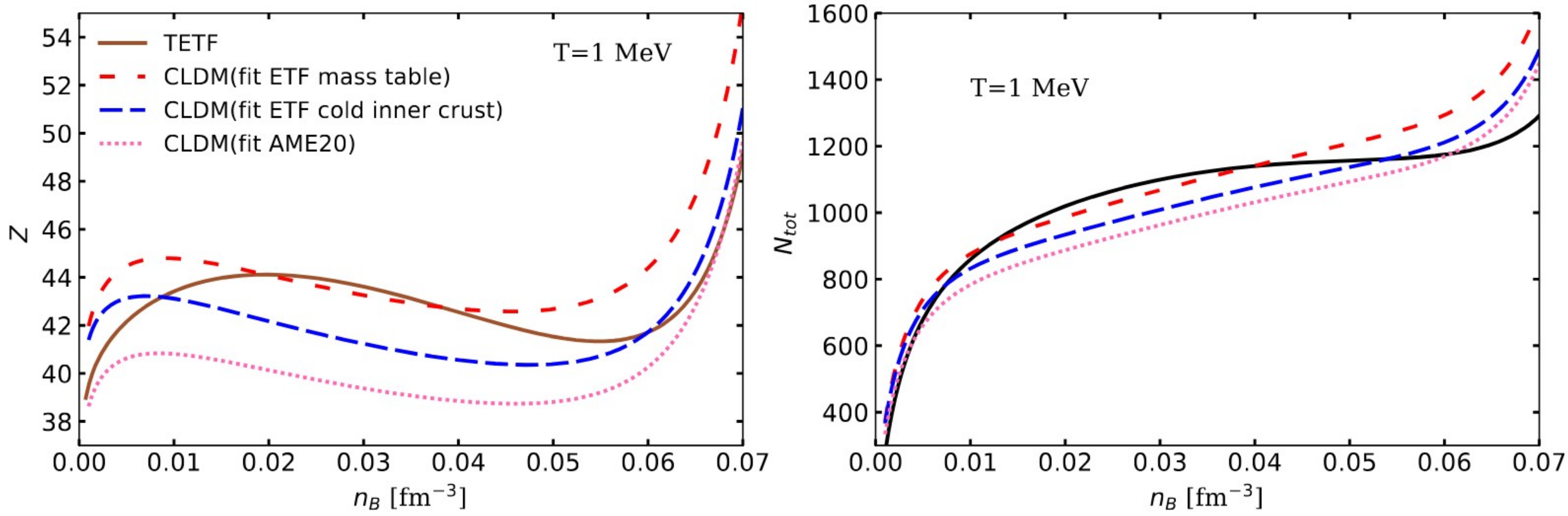
Comparison ETF/CLDM : thermodynamical quantities



Very good agreement between the ETF and CLDM on thermodynamical quantities in the inner crust

Grams, Shchechilin, Diverres, et al., Universe (2025)

Comparison ETF/CLDM : composition



Similar trend but the composition is more sensitive to the model

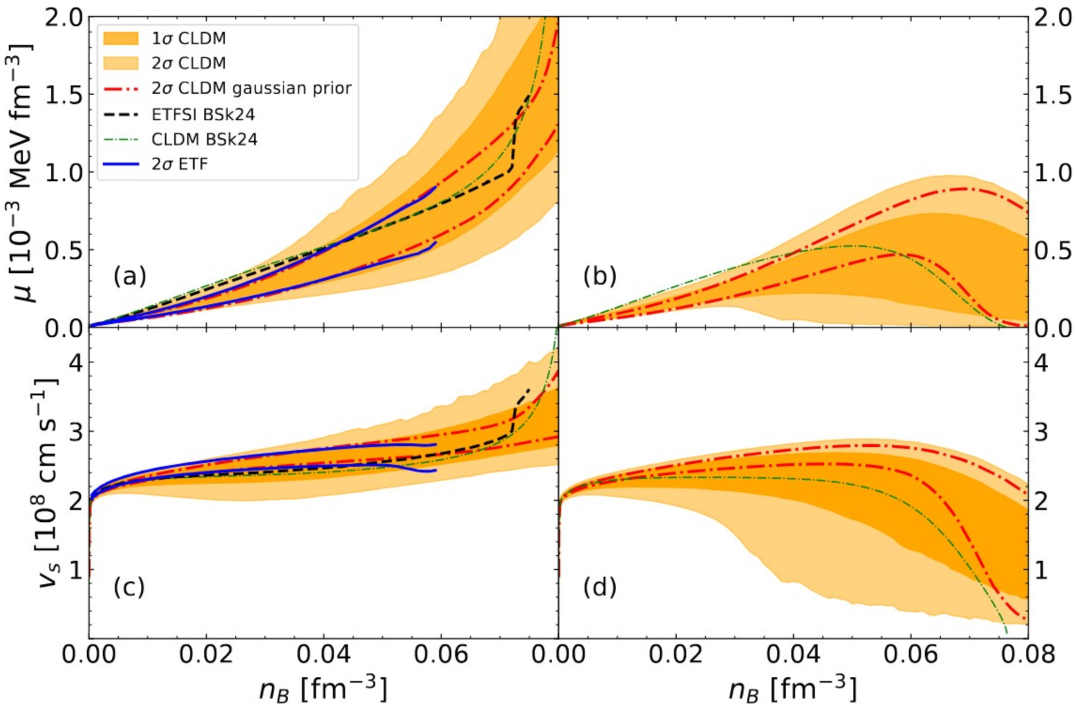
Grams, Shchechilin, Diverres, et al., Universe (2025)

Bayesian Analysis of the shear modulus

$$\mu_{\rm fsc} = C_{\rm fsc}\mu$$

$$C_{\rm fsc} = 1 - \frac{u^{5/3}}{2 + 3u - 4u^{1/3}}$$

$$u = \frac{V_N}{V_{WS}}$$



Zemlyakov & Chugunov, Universe (2023)

Parameters distribution