Superconducting Heart of Neutron Stars: Implications for Gravitational Wave Emission

Mayusree Das

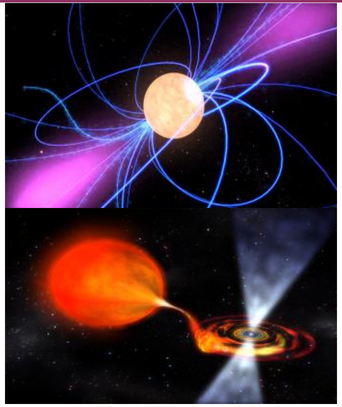
With Armen Sedrakian (FIAS/U.Wroclaw)
& Banibrata Mukhopadhyay (IISc)

Joint Astronomy Programme
Department of Physics
Indian Institute of Science, Bangalore

GWsNS 2026

Observation of Neutron stars (NSs)

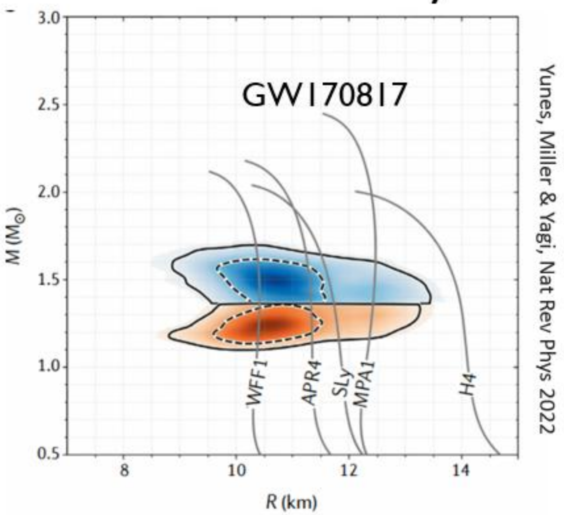
- **Electromagnetic radiation: Pulsars**
- **X-ray sources: Accretion-powered NSs**



Compact stars = extreme density and gravity -- cannot recreate on Earth all forces (strong-- weak-- gravitational--electromagnetic) act together!

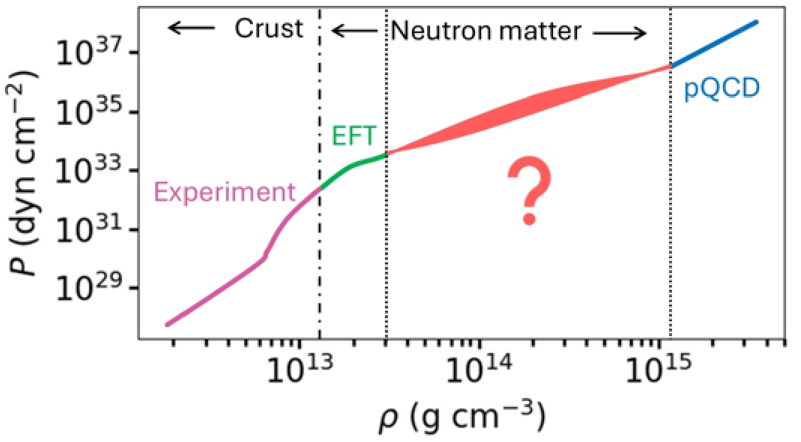
What are constituents? Equation of state (EoS)?

Can be constrained by GW



How to Detect 'Invisible' , isolated NSs?

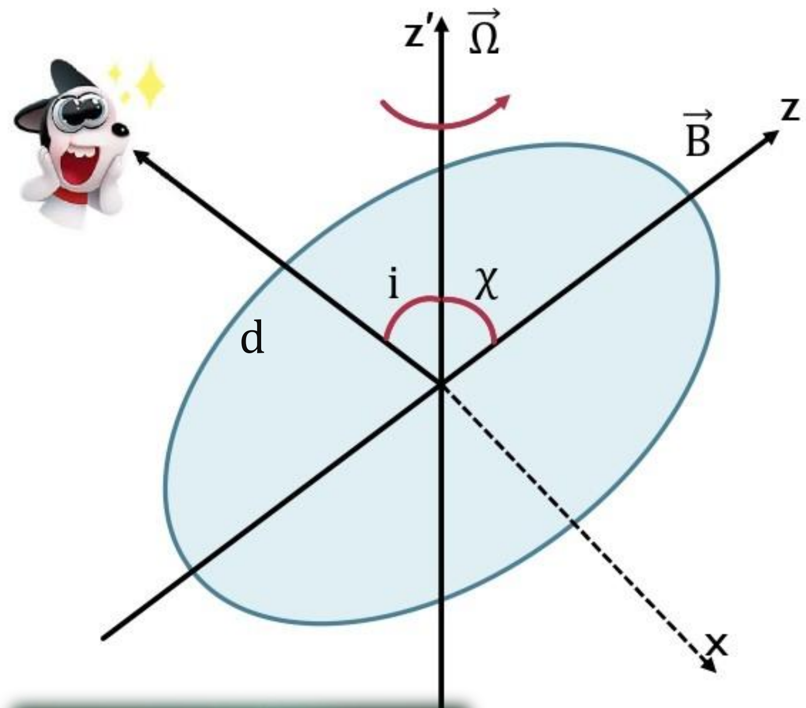
Gravitational waves (GWs)!

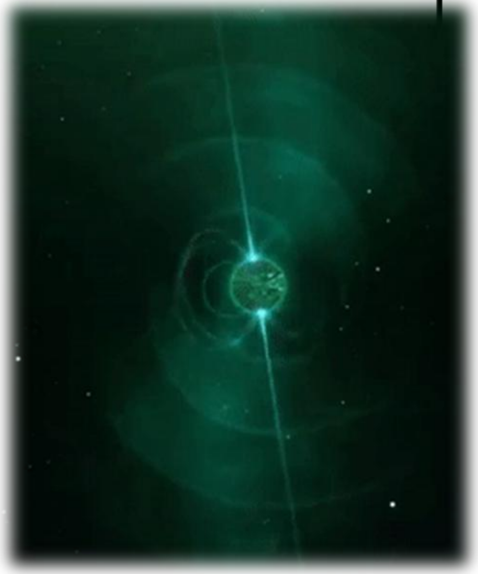


Fraga et al. 2016, ApJ

GW enlightens heart of compact stars

Current detectors: LIGO, Virgo, KAGRA
future: Einstein Telescope, LISA





Magnetic field

Rotation

Deformation

Deformation & Misalignment → Triaxial symmetry → Time varying, non zero quadrupole moment → **Continuous GW**

$$h_+ = h_0 \sin\chi \left[\frac{1}{2}\cos i \sin i \cos\chi \cos\Omega t - \frac{1+\cos^2 i}{2}\sin\chi \cos 2\Omega t\right]$$

$$h_\times = h_0 \sin\chi \left[\frac{1}{2}\sin i \cos\chi \sin\Omega t - \cos i \sin\chi \sin 2\Omega t\right]$$

$$h_0 = \frac{2G}{c^4}\Omega^2 \epsilon \frac{I_{xx}}{d}, \qquad \epsilon = \frac{I_{zz}-I_{xx}}{I_{xx}}$$

Bonazzola & Gourgoulhon 1996

LET'S FIND OUT SUPERCONDUCTING REGIONS!!

MODELLING NEUTRON STARS IN 2D (AXISYMMETRIC)

BY SOLVING

EINSTEIN-MAXWELL EQUATIONS

USING XNS

Pili et al. 2014

3+1 Metric: $ds^2 = -\alpha^2\, dt^2 + \psi^4 \left[dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta \left(d\emptyset + \beta^{\emptyset}\, dt\right)^2\right]$

$$\nabla_\mu(\rho u^\mu) = 0$$
$$\nabla_\mu T^{\mu\nu} = 0$$
$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu F^{*\mu\nu} = 0$$

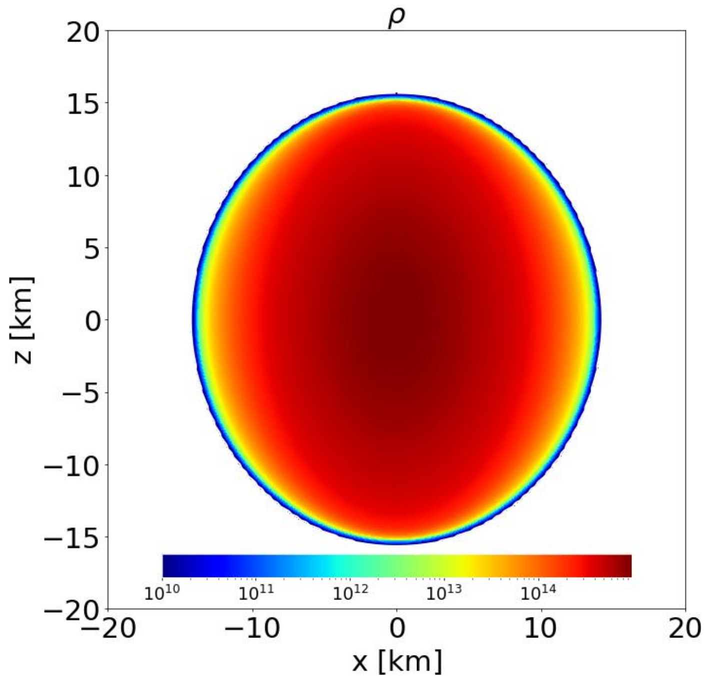
Input: EoS, Central density,

Maximum magnetic field and geometry, rotation

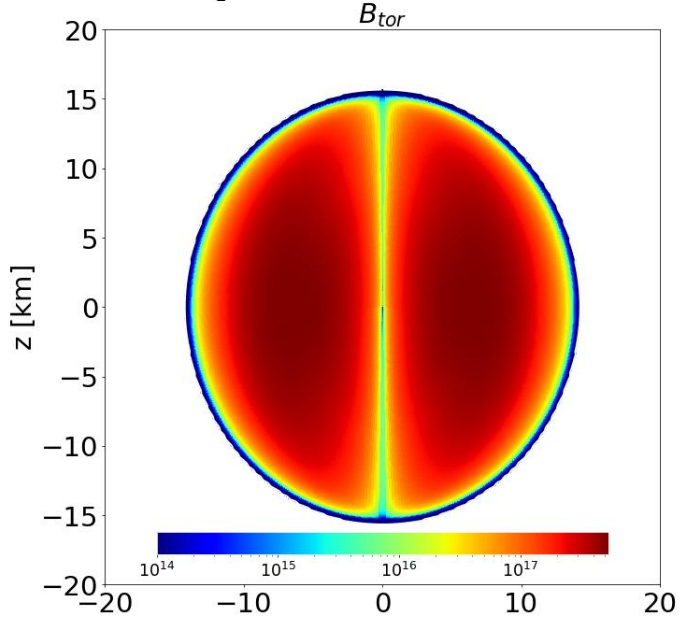
Output: $\mathbf{M}, \mathbf{R}, \mathbf{I_{xx}}, \mathbf{I_{zz}}$, magnetic field profile

Deformation

Density isocontour



Magnetic field isocontour

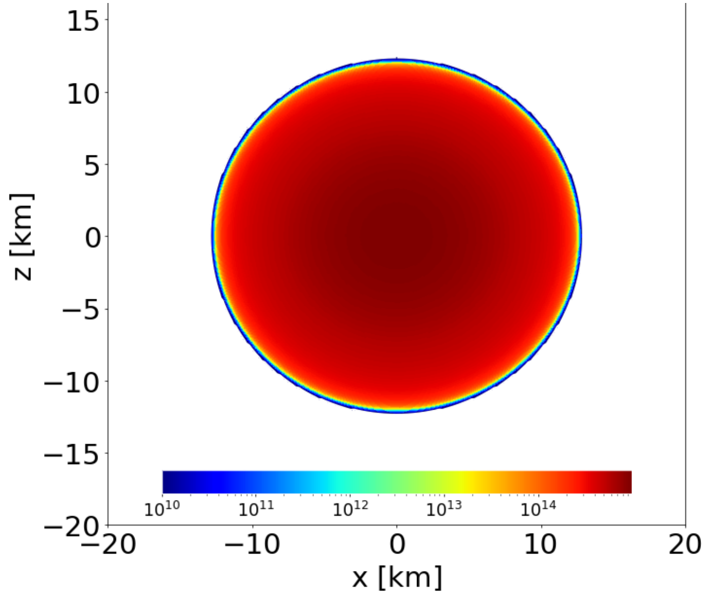


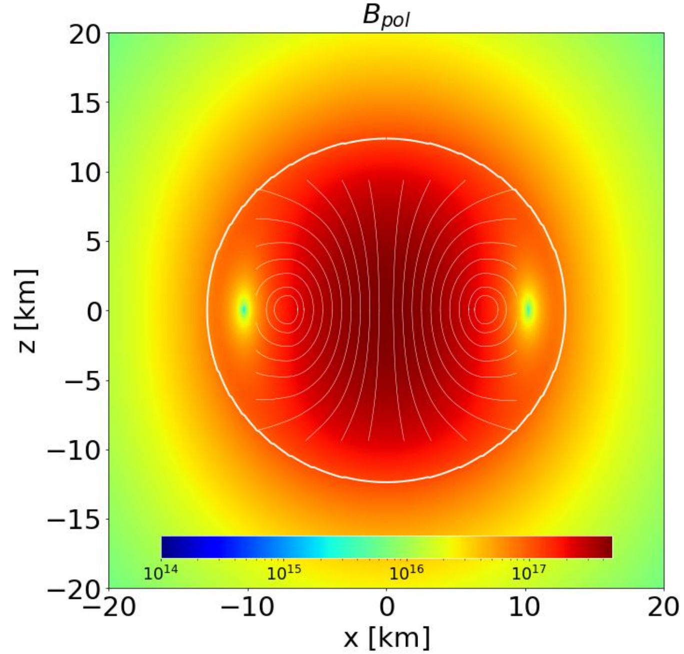
Toroidal Magnetic field ($\vec{B} = B_\phi \hat{\phi}$)

$$B_{max} = 10^{17} G$$

Poloidal Magnetic field ($\vec{B} = B_r \hat{r} + B_\theta \hat{\theta}$)

$$B_{max} = 10^{17} G$$





MD & Mukhopadhyay, ApJ (2023)

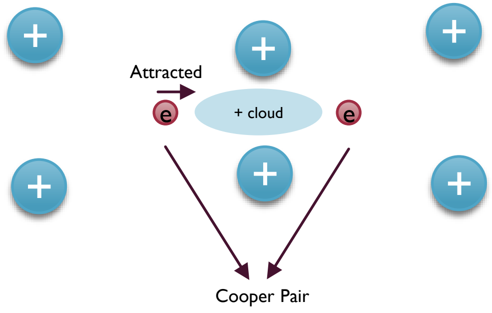
NS's

Superconductivity

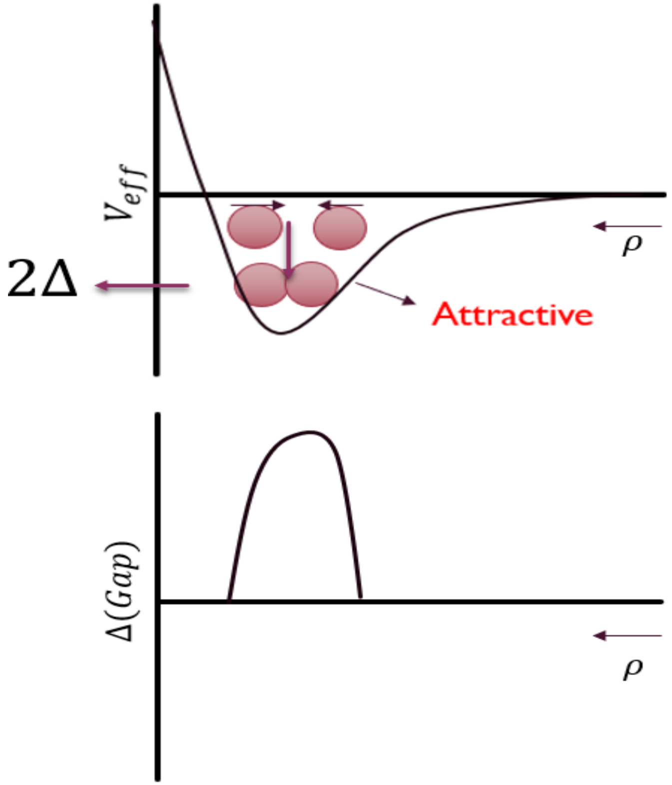
How neutron stars NSs become superconductor

- BCS theory: low temperature degenerate fermions can form Cooper pairs if there is an attractive interaction.

- Laboratory superconductor: Electrons form Cooper pairs via lattice vibrations (phonons) where one electron distorts the lattice, attracting another electron.

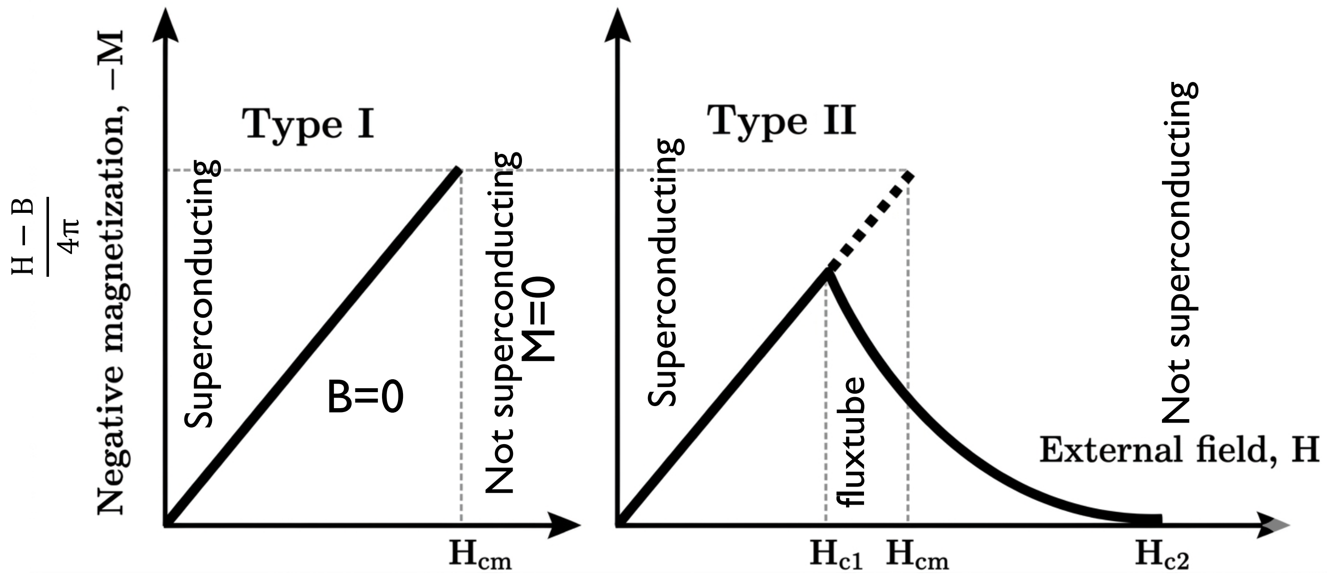


- Unlike the laboratory superconductor, protons form a cooper pair in NSs rather than electrons, as electron's critical temperature is far below NS's temperature($\sim 10^9$ K) as they are highly relativistic.



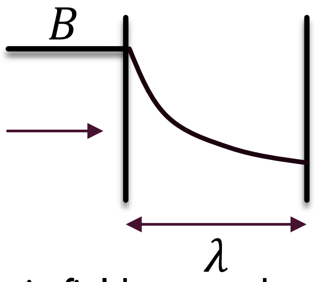
- NS: the attraction between protons is coming from the attractive nature of strong nuclear.

Type I — Type of Superconductivity — Type II



Meissner effect: magnetic field expulsion
Field decays exponentially in

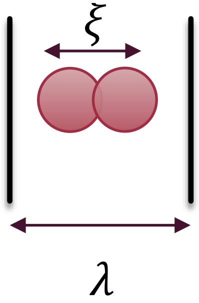
London penetration depth (λ)



- In NS, magnetic field cannot be expelled into normal matter as the conductivity is very high.
- Timescale of expulsion>universe's age.
- Magnetic field will be present in Type I superconducting matter in a discontinuous manner even if $B < H_{Cm}$.

Magnetic field can **partially penetrate** in Type II superconductor:

B can be confined inside Cooper pairs
which are **smaller in size (ξ) than λ** (approximate)
Cooper pair with magnetic fields: quantum fluxtube



Magnetic field will also exist in Type II superconducting matter, but in **ordered array of quantum fluxtubes.**

Critical temperature

$T_c \sim \Delta/1.76k_B$

Critical magnetic fields

Type II: $\boldsymbol{\kappa} = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$

Type I: $\boldsymbol{\kappa} < \frac{1}{\sqrt{2}}$

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$

$$H_{c1} \sim \frac{\Phi_0}{2\pi\lambda^2}$$

$H_{c1} = \frac{\Phi_0}{2\pi\lambda^2}(\ln\kappa + 0.5 + (1 + \ln 2)/(2\kappa - \sqrt{2} + 2))$

$$H_{cm} = \frac{\kappa\Phi_0}{2\sqrt{2}\pi\lambda^2} = \frac{H_{c2}}{\sqrt{2}\kappa}$$

Quantum flux of fluxtubes: $\Phi_0 = \frac{\pi\hbar c}{e}$

$\lambda \propto \rho^{-\frac{1}{2}}, \xi \propto \rho^{\frac{1}{3}}/\Delta$

Type II partially superconducting (fluxtube):

$$\boldsymbol{T < T_c \ \& \ H_{c1} < B < H_{c2} \ \& \ \kappa > 1/\sqrt{2}}$$

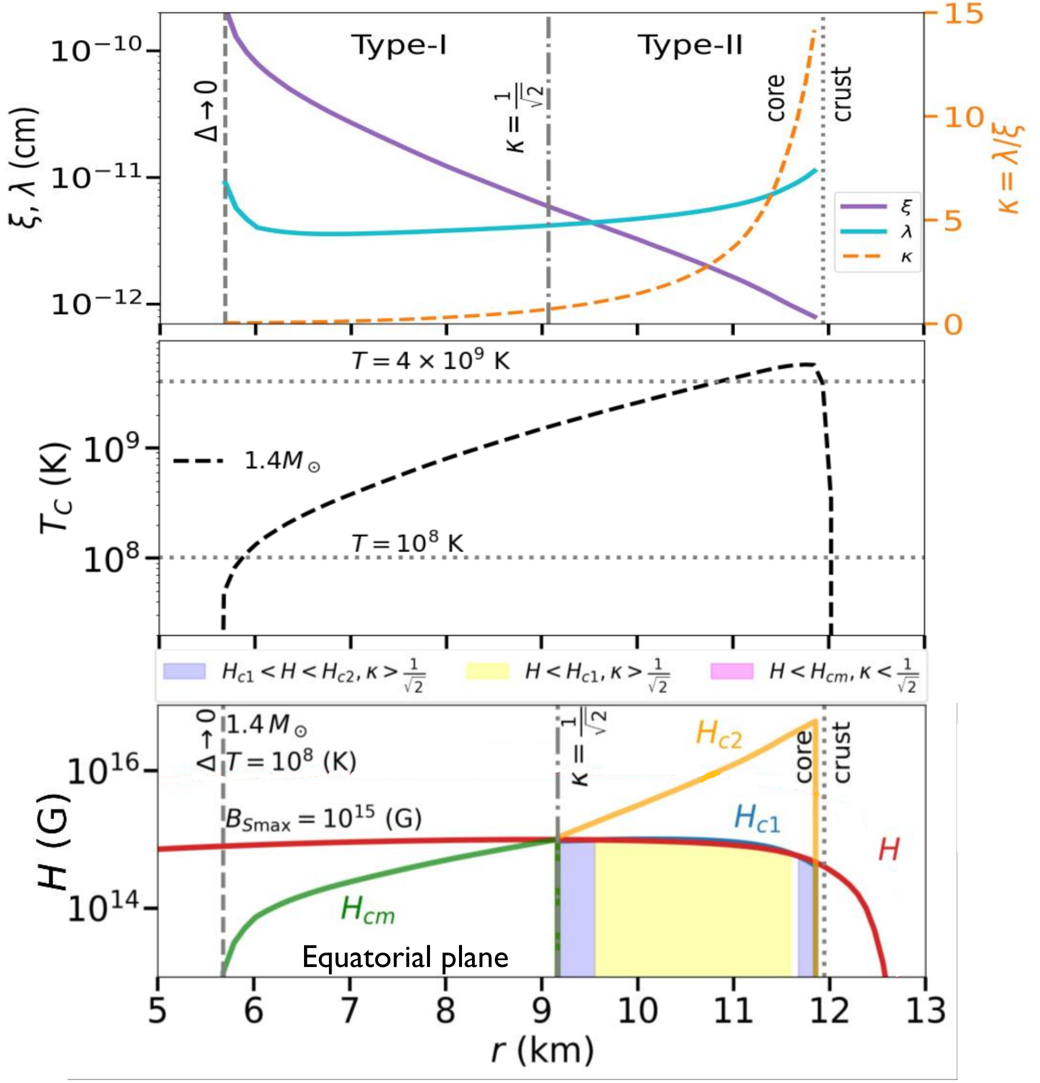
Type II completely superconducting (paired protons):

$$\boldsymbol{T < T_c \ \& \ B < H_{c1} \ \& \ \kappa > 1/\sqrt{2}}$$

Type I superconducting:

$$\boldsymbol{T < T_c \ \& \ B < H_{cm} \ \& \ \kappa < 1/\sqrt{2}}$$

Toroidal field

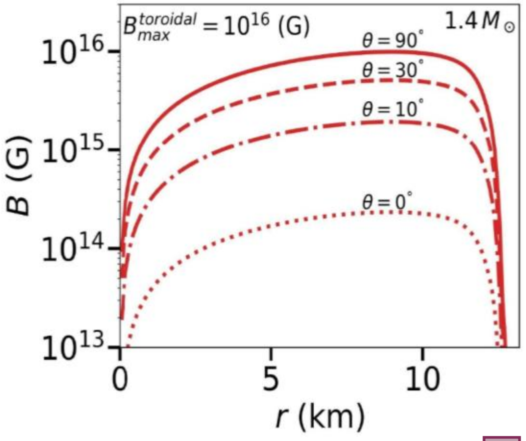


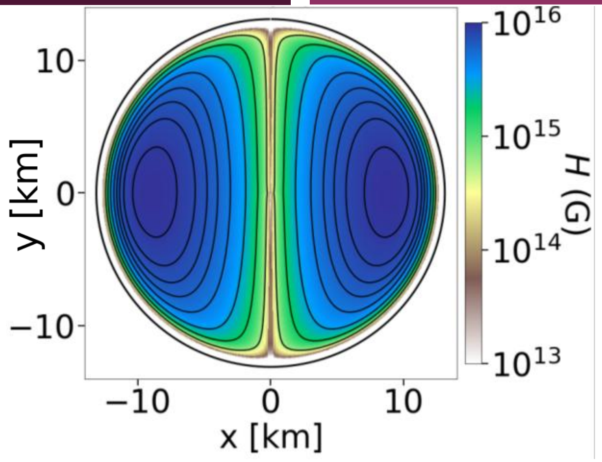
MD, Sedrakian & Mukhopadhyay, PRD letter (2025)

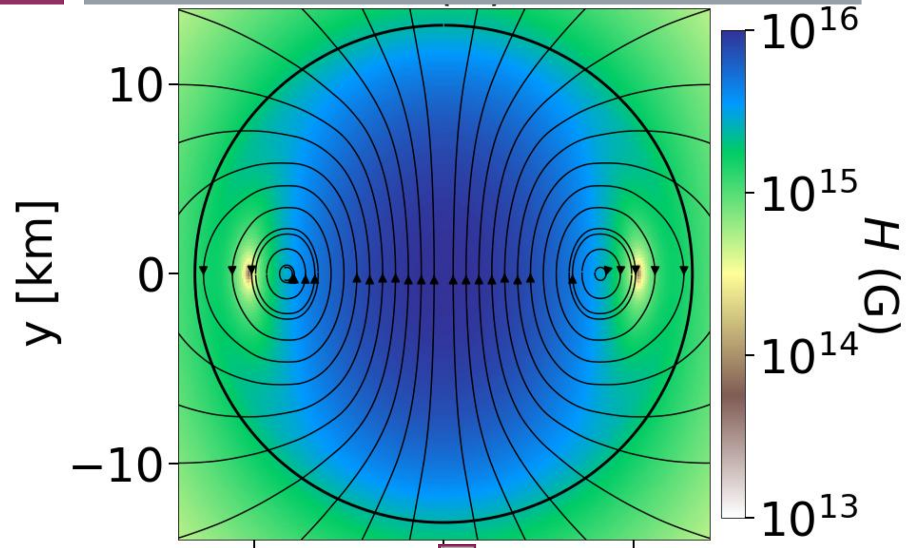
Toroidal field

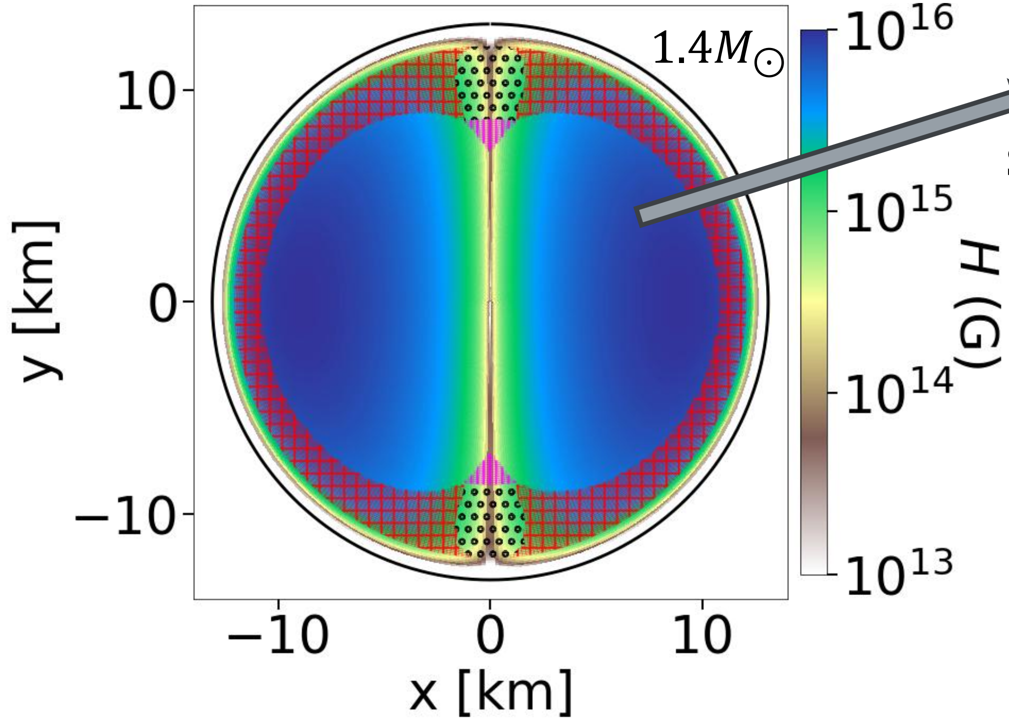
Superconducting regions

Poloidal field

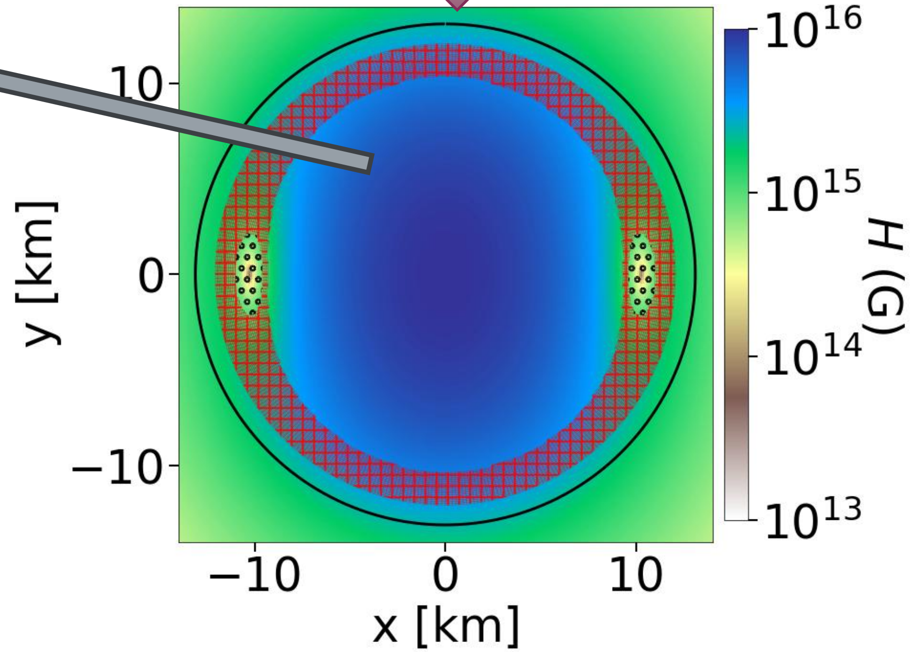








Void of superconductivity



Type I: $B < H_{cm}$

Type II: $B < H_{c1}$

Type II: $H_{c1} < B < H_{c2}$

MD, Sedrakian & Mukhopadhyay, PRD letter (2025)

MD, Sedrakian & Mukhopadhyay, PRD (2026)

Enhanced CGW from Superconductivity

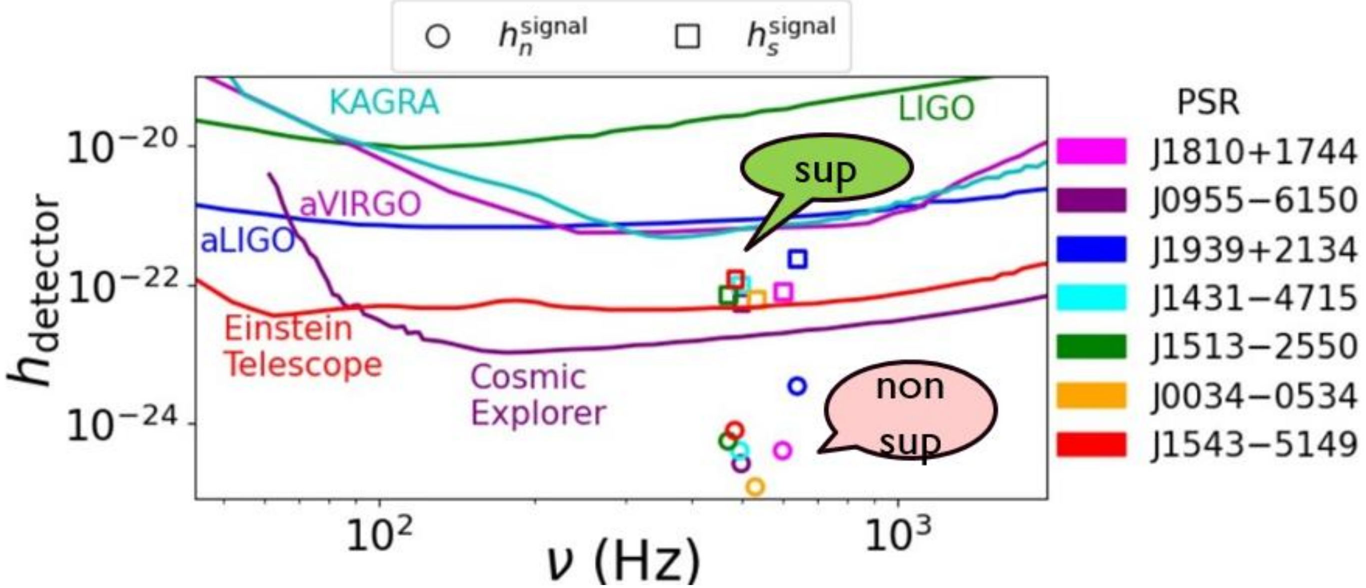
Observational imprint in gravitational wave from Millisecond pulsars (MSPs)

Highly rotating NSs with weak (observed) magnetic field

Observed						Simulated							
MSP PSR	ν (Hz)	d (kpc)	M ($M_\odot$)	$B_P^{\rm pole}$ (G)	$\epsilon_{95\%}$ ($\times 10^{-8}$)	$B_{\max}$ (G)	V_s (%)	$\mathcal{F}$	ϵ_n	ϵ_s	$h_n^{\rm signal}$	$h_s^{\rm signal}$
J1810+1744	601.4	2.36	2.13	2×10^{8}	2.3	1×10^{13}	55	203	5.5×10^{-11}	1×10^{-8}	4×10^{-25}	8×10^{-23}
J0955−6150	500.2	2.17	1.71	2×10^{8}	3.6	1×10^{13}	65	214	5.4×10^{-11}	1×10^{-8}	3×10^{-25}	5.5×10^{-23}
J1939+2134[a]	641.9	4.8	1.4[b]	7×10^{8}	6.1	3.5×10^{13}	71	74	6.7×10^{-10}	4.9×10^{-8}	3×10^{-24}	2.5×10^{-22}
J1431−4715	497.0	1.55	1.4[b]	2×10^{8}	2.3	1×10^{13}	71	258	5.5×10^{-11}	1×10^{-8}	4×10^{-25}	1×10^{-22}
J1513−2550	471.9	3.69	1.4[b]	4×10^{8}	5.2	2×10^{13}	71	129	2.2×10^{-10}	2.8×10^{-8}	5×10^{-25}	7×10^{-23}
J0034−0534	532.7	1.35	1.4[b]	1×10^{8}	1.2	5×10^{12}	71	520	1.3×10^{-11}	6.8×10^{-9}	1×10^{-25}	6×10^{-23}
J1543−5149	486.2	1.15	1.35	3×10^{8}	1.3	1.5×10^{13}	72	164	1×10^{-10}	1.6×10^{-8}	8×10^{-25}	1.2×10^{-22}

$$h_{superconducting} \rightarrow h_{non-superconducting} \times \frac{H_{c1}}{B}$$

detectable   Not detectable



MD, Sedrakian & Mukhopadhyay, PRD (2026)

THANK YOU

Lorenz force

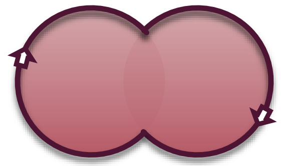
Normal matter:

$$F_{norm} = \frac{1}{4\pi} B^i [\nabla_j (B^i) - \nabla^i (B_j)]$$

Superconducting matter (Type II):

modified due to presence of arrays of fluxtubes floating inside superconducting matter, which confines high magnetic field inside them. The current loop (J) is generated on the fluxtube surface.

$$F_{sup} = \frac{1}{4\pi} B^i [\nabla_j (H_{C1} \hat{B}^i) - \nabla^i (H_{C1} \hat{B}_j)] - \frac{\rho_P}{4\pi} \nabla^i (B \frac{\partial H_{C1}}{\partial \rho_P})$$



Glampedakis K., Andersson N., Samuelsson L., 2011

Fluxtube pin to neutron vortices via mutual friction, restricting their motion

$$\boxed{\frac{|F_{sup}|}{|F_{norm}|} \sim \frac{H_{C1}}{B}}$$

As high magnetic is confined in fluxtube,
Lorenz force is expected to enhance if $B < H_{C1}$