# Beyond Circles

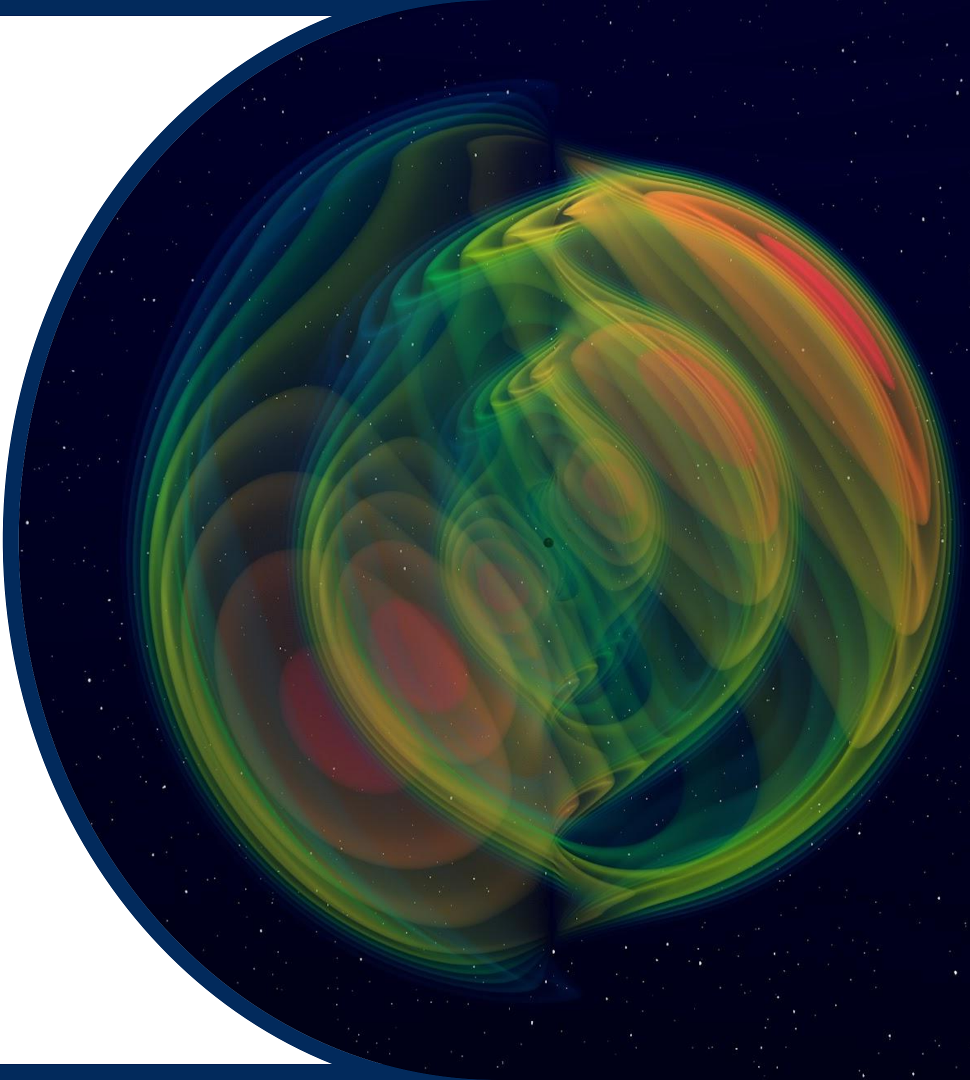
*Stationary axisymmetric black holes and the breaking of circularity*

**Jacopo Mazza**

*IJCLab, CNRS/IN2P3, Université Paris-Saclay*

**API GWOC**

*Obs. de Paris, Meudon*
*Dec. 12th, 2025*



# The talk, in a nutshell

Many well-established properties of stationary & axisymmetric BHs are not obvious beyond GR

**CIRCULARITY**

‘form’ of $g_{\mu\nu}$, i.e. coordinate choice

event horizon properties [Killing/not Killing]

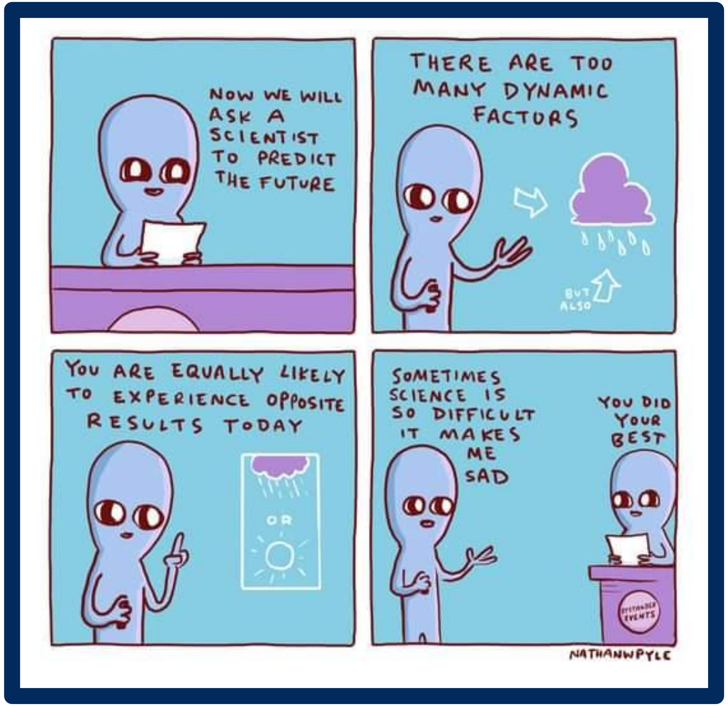
[ Keenan Crane [link] ]

# Outline

- What is circularity?
  - Definition(s) and interpretation
  - Why circularity?
  - Why *not* circularity?
- Moving beyond circularity
  - A metric of sufficient generality
    [*i.e. the most general rotating metric*]
  - 'Solving' circularity
- Properties of non-circular BHs
  - Rotosurface
  - Horizon mechanics
  - Towards thermodynamics

E. Babichev, **JM,** JCAP 10 (2025) 011 [2505.08880]

F. Del Porro, **JM,** [2511.02911]




[Nathan W. Pyle]

# What is circularity?

**Def**

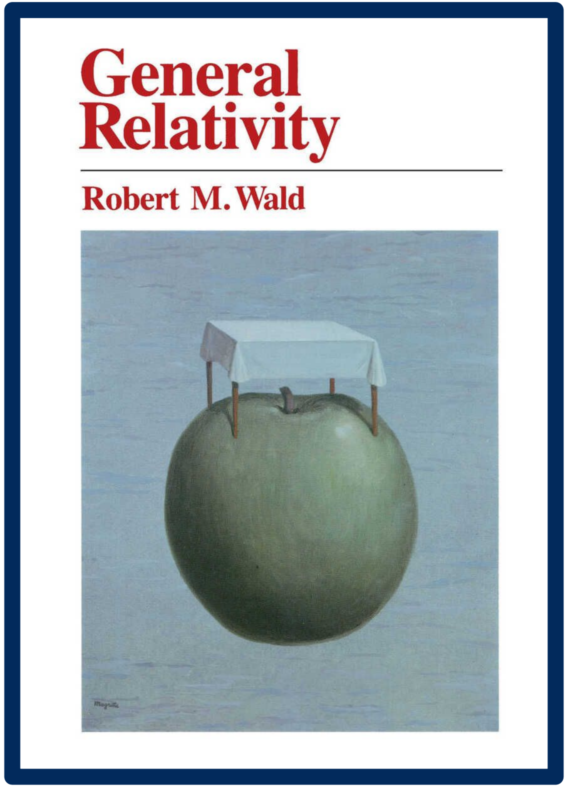
A stationary & axisymmetric spacetime
is said to be *circular*
if it can be foliated by codimension-2
surfaces orthogonal to the Killing vectors

Killing vectors
$\xi^\mu$ ……… stationarity
$\psi^\mu$ ……… axisymmetry

by Frobenius' theorem

$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

[everywhere]



# Circularity and geometry

**Circularity conditions**

$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

geometric statement on Killing flows

Practically,
there exist coordinates such that

$$g_{\mu\nu} = \begin{pmatrix} * & 0 & 0 & * \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ * & 0 & 0 & * \end{pmatrix} \begin{matrix} t \\ r \\ \theta \\ \varphi \end{matrix}$$

$t, \varphi$.... Killing coordinates

$(t-\varphi)$ symmetry:
$t \to -t,\ \varphi \to -\varphi$

5 'free' functions,
$g_{\mu\nu} = g_{\mu\nu}(r,\theta)$

i.e. Boyer–Lindquist coordinates

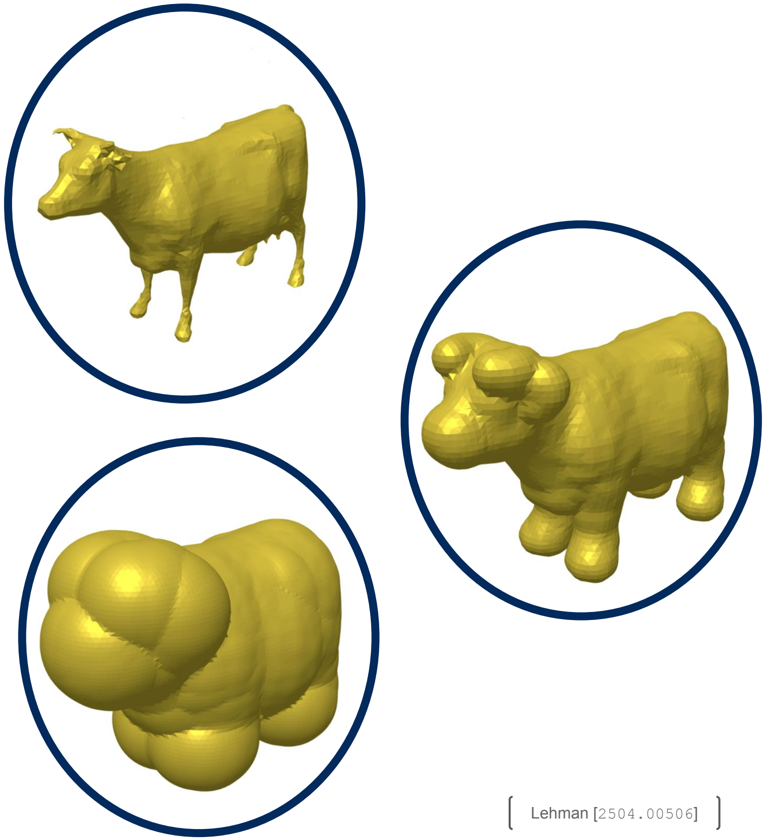
# Why circularity?

Circularity is quite ubiquitous, because:

- Many GR sol's are circular
- Some beyond-GR sol's are circular
- *'Square peg in a circular hole'*: beyond-GR sol's that are continuously connected to GR are circular

  [ Xie, Zhang, Silva, de Rham, Witek, Yunes, PRL 126 (2021) 241104 [2103.03925] ]

- Simplicity
  - block-diagonalisation of $g_{\mu\nu}$
  - and more…



[ Lehman [2504.00506] ]

# Why *not* circularity?

## Some beyond GR sol's are *not* circular

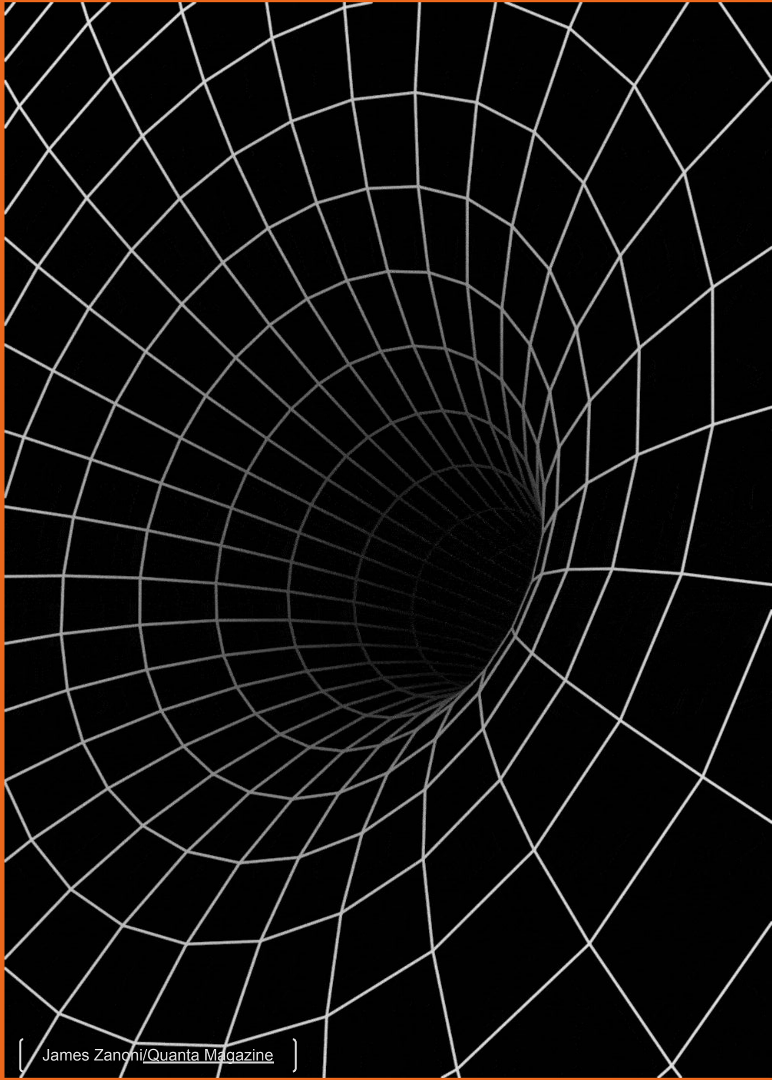
- Cubic Galileon [numerical] — Grandclement, CQG 41, 025012 (2024) [2308.11245] | Van Aelst, Gourgoulhon, Grandclément, Charmousis, CQG 37 035007 (2020) [1910.08451]
- Semiclassical gravity [numerical] — Fernandes, PRD 108 (2023) 6, 6 [2305.10382]
- Einstein–æther [numerical] — Adam, Figueras, Jacobson, Wiseman, Class. Quant. Grav. 39 (2022) 12, 125001 [2108.00005]
- Disformal Kerr [analytical] — Anson, Babichev, Charmousis, Hassaine, JHEP 2021 (2021) 18 [2006.06461] | Ben Achour, Liu, Motohashi, Mukohyama, Noui, JCAP 11 (2020) 001 [2006.07245]
- 'Locality principle' regular BHs [analytical-ish] — Eichhorn, Held, JCAP 05 (2021) 073 [2103.13163]

# Why *not* circularity? [cont.]

Beyond GR,
circularity is a choice
[often]

What happens if we
give it up?

E. Babichev, **JM,** JCAP 10 (2025) 011 [2505.08880]



# Moving beyond circularity

To understand consequences
of circularity breaking,
need simple 'toy' examples

[then maybe generalise]

circularity conditions are PDEs for metric components

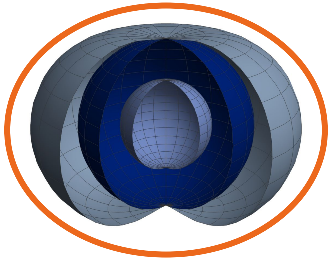
$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

- checking circularity for given metric ---- easy
- constructing 'simple' non-circular metrics ---- hard

**idea**

'solve' conditions,

translate them into algebraic relations



need a gauge…

# General stationary & axisymmetric metric

What is *the most general* stationary and axisymmetric metric?

No one really knows…

[our answer]

Kerr-like gauge

‘Kerr’ as in ‘Kerr ingoing coordinates’
[not Kerr solution]

$$g_{\mu\nu} = \begin{pmatrix} \circledast & \circledast & 0 & \circledast \\ * & 0 & 0 & \circledast \\ 0 & 0 & \circledast & 0 \\ * & * & 0 & \circledast \end{pmatrix}$$

$v \quad r \quad \theta \quad \varphi$

Killing coordinates

non-Killing coordinates

6 ‘free’ functions

$$\begin{cases} g_{vv}(r,\theta) \\ g_{vr}(r,\theta) \\ g_{r\phi}(r,\theta) \\ g_{\phi\phi}(r,\theta) \\ g_{\theta\theta}(r,\theta) \end{cases}$$

existence not obvious, [see paper]

gauge fixing not complete

Comments:

1. Important open problem!
2. Beware of claims saying that BL-like metrics are general, they are not!

# ‘Solving’ circularity

[Idea: ‘solve’ circularity conditions]

plug Kerr-like guage into conditions…

$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

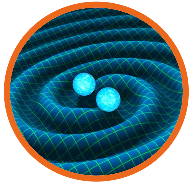
$\longleftrightarrow$

$$g^{vr} = f(r)g^{rr}$$
$$g^{r\phi} = h(r)g^{rr}$$

used them to construct two examples, both simple deformations of Kerr

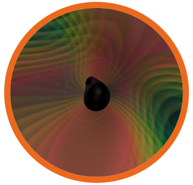
‘Minimal’ breaking

$$g^{vr} = g^{vr}_{\text{Kerr}} + \delta(r,\theta)$$



‘Not-so-minimal’ breaking

$$M \mapsto m(r,\theta)$$



E. Babichev, **JM,** JCAP 10 (2025) 011 [2505.08880]

# Properties of non-circular BHs

1. Non-separability of geodesics (and Klein–Gordon), no Killing tensor

not really *due to* circularity breaking

2. (Near-)horizon structure is different

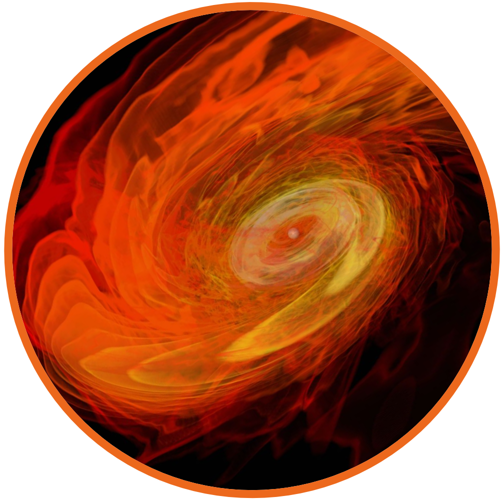
new relevant surface outside *H*: rotosurface

phenomenological consequences
?

event horizon not Killing horizon

thermodynamics
?

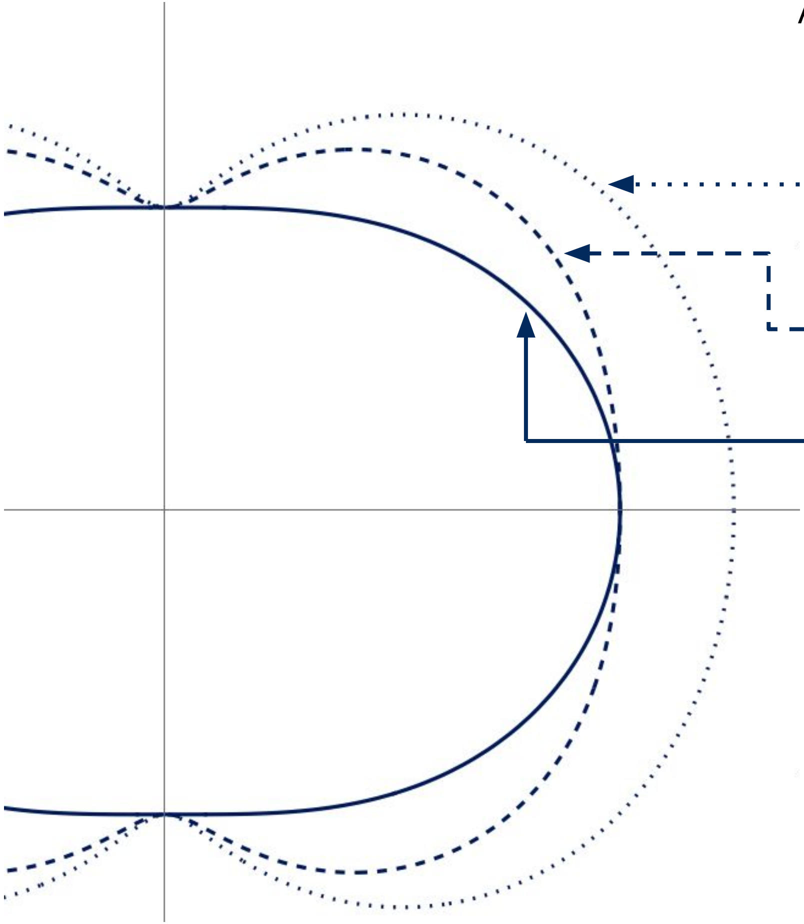
F. Del Porro, **JM,** [2511.02911]



[NASA/AEI/ZIB/M. Koppitz and L. Rezzolla]

# The rotosurface

A consequence of circularity breaking: a ‘new’ surface appears



$$\xi_\mu \xi^\mu = 0$$ ergosurface, static limit

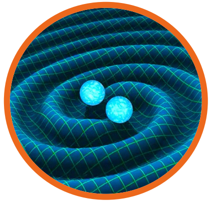
$$(\xi_\mu + \Omega\,\psi_\mu)(\xi^\mu + \Omega\,\psi^\mu) = 0$$ rotosurface, stationary limit

event horizon

in Kerr they coincide but in general they are different

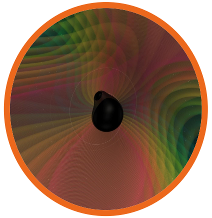
**pheno consequences**
of rotosorfaces?

# Rotosurface and examples



**Minimal**

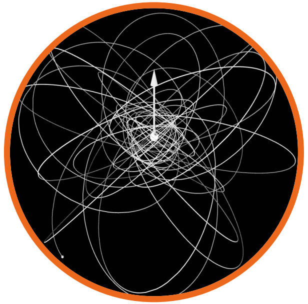
Rotosurface and horizon coincide, horizon is a 'sphere'



**Non-minimal**

Rotosurface and horizon differ, Everything known analytically

Have fun!



[ Maarten van de Meent [link] ]

# Killing horizons are nice!

Usually
[i.e. vacuum GR]
BH event horizons are also Killing horizons [rigidity theorem]

i.e. they are generated by Killing vectors

Several consequnces:

- $\Omega_H$ constant over $H$
- different notions of surface gravity coincide
- surface gravity $\kappa$ constant over $H$

Four laws of 'BH mechanics' ⟷ Four laws of 'BH thermodynamics'

0th law $\kappa = \text{const.}$

1st law $\delta M = \frac{\kappa}{8\pi G}\delta A_H + \Omega_H \delta J$

2nd law $\delta A_H \geq 0$

3rd law $\kappa = 0$ unattainable

$\kappa \leftrightarrow T$

$A_H \leftrightarrow S$

$M \leftrightarrow E$

[**with circularity**]

# Non-Killing horizons are not nice!

[ **without circularity** ]

Horizons not Killing, in general

Usually
[i.e. vacuum GR]
BH event horizons are also Killing horizons

[ rigidity theorem ]

i.e. they are generated by Killing vectors

Several consequnces:

- $\Omega_H$ constant over $H$
- surface gravity $\kappa$ constant over $H$
- different notions of surface gravity coincide

?

Four laws of 'BH mechanics' ? ⟷ Four laws of 'BH thermodynamics' ??

0th law $\kappa = \text{const.}$

1st law $\delta M = \frac{\kappa}{8\pi G}\delta A_H + \Omega_H \delta J$

2nd law $\delta A_H \geq 0$

3rd law $\kappa = 0$ unattainable

$\kappa \leftrightarrow T$
$A_H \leftrightarrow S$
$M \leftrightarrow E$
???

F. Del Porro, **JM**, [2511.02911]

# Surface gravities

Assume the horizon is $r = H(\theta)$ then define $\zeta_\mu := \frac{\alpha}{g^{vr}} \partial_\mu \left[ r - H(\theta) \right]$

inaffinity surface gravity $$\zeta^\nu \nabla_\nu \zeta^\mu \Big|_{r=H} = \kappa_{\mathrm{i}} \zeta^\mu \Big|_{r=H}$$

normal surface gravity $$\nabla_\mu \left( \zeta_\nu \zeta^\nu \right) \Big|_{r=H} = 2\kappa_{\mathrm{n}} \zeta_\mu \Big|_{r=H}$$

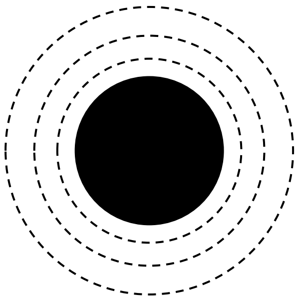
on-horizon surface gravities

compute them both

$$\kappa_{\mathrm{n}} = \frac{\alpha}{2g^{vr}} \left[ \partial_r g^{rr} + (H')^2 \partial_r g^{\theta\theta} \right] \Big|_{r=H}$$

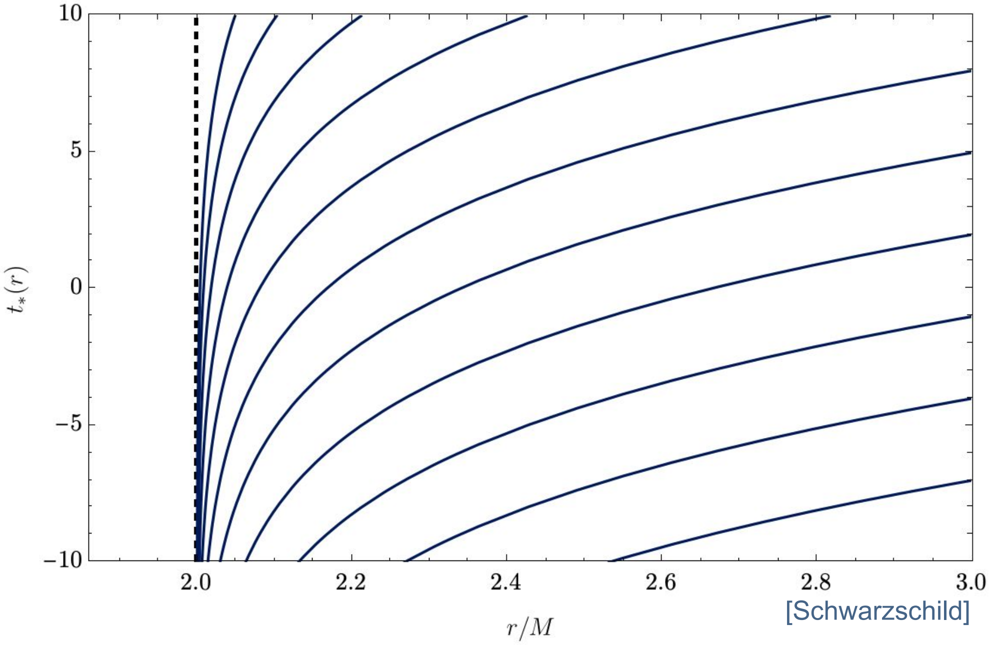
$$\kappa_{\mathrm{i}} = \kappa_{\mathrm{n}} + \zeta^\mu \partial_\mu \log\left( \frac{\alpha}{g^{vr}} \right) \Big|_{r=H}$$

depend on position on *H*



# Surface gravities – cont.

Outgoing causal geodesics “peel off” of the horizon




[Schwarzschild]

$$t_* \sim \frac{1}{\kappa_\mathrm{p}} \log(r)$$

peeling surface gravity

despite the lack of geodesic separability, we can compute:

$$\kappa_\mathrm{p} = \kappa_\mathrm{n}$$

( w/ good choice of normalisation )

# Smarr’s formula

We can prove this geometric identity, no field equations

$$M_H = \frac{\overline{\kappa}}{4\pi G}\mathcal{A}_H + 2\overline{\sigma}_H J_H + \eta$$

Komar mass

horizon area

Komar angular momentum

normal surface gravity

angular velocity [sort of]

averages over $H$

dissipation [?]

vanishes in minimal case $H' = 0$

# Hawking radiation

The peeling entails Hawking radiation

Via tunnelling method, we find:
an observer at infinity detects particles with spectral density

$$\langle \hat{\mathcal{N}}_{\omega m} \rangle = \frac{\upsilon(\omega, m)}{\exp\left[\frac{2\pi}{\kappa_{\mathrm{p}}}(\omega - m\sigma_H)\right] - 1}$$

Bose–Einstein-ish
distribution w/

$$T_H = \frac{\kappa_{\mathrm{p}}}{2\pi} \quad \text{and} \quad \mu_H = m\sigma_H$$

[AI generated]

# Towards thermodynamics

Comments on the four laws of mechanics of non-Killing horizons:

**[0th law]** constancy of $\kappa$: does not hold

**[1st law]** energy conservation: [w/ caveats] holds in averaged form

**[2nd law]** horizon's area cannot decrease: holds under the same assumptions as usual

**[3rd law]** unattaibility of extremal state: same as usual

Open questions:

- Thermodynamic interpretation?
- Out of equilibrium / local (i.e. non global) equilibrium / ?
- [...]

# Recap & Food for thought

Circularity
'accidental' symmetry

[might not hold beyond GR]

1. found most general stationary & axisymmetric metric
2. solved circularity conditions
3. constructed simple examples

[ E. Babichev, **JM,** JCAP 10 (2025) 011 [2505.08880] ]

main consequences

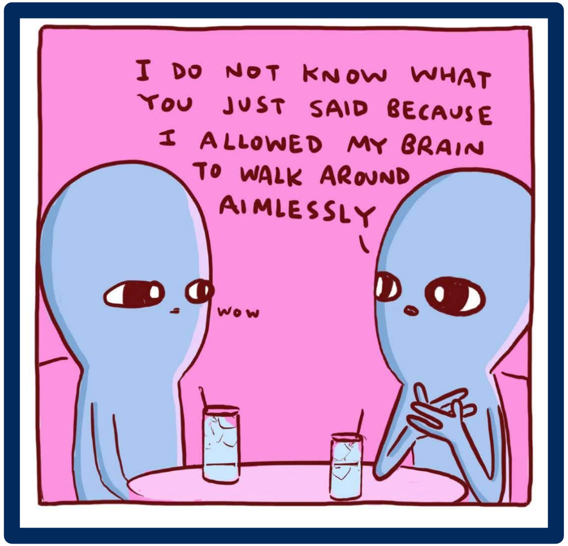
rotosurface separate from horizon

event horizon not Killing

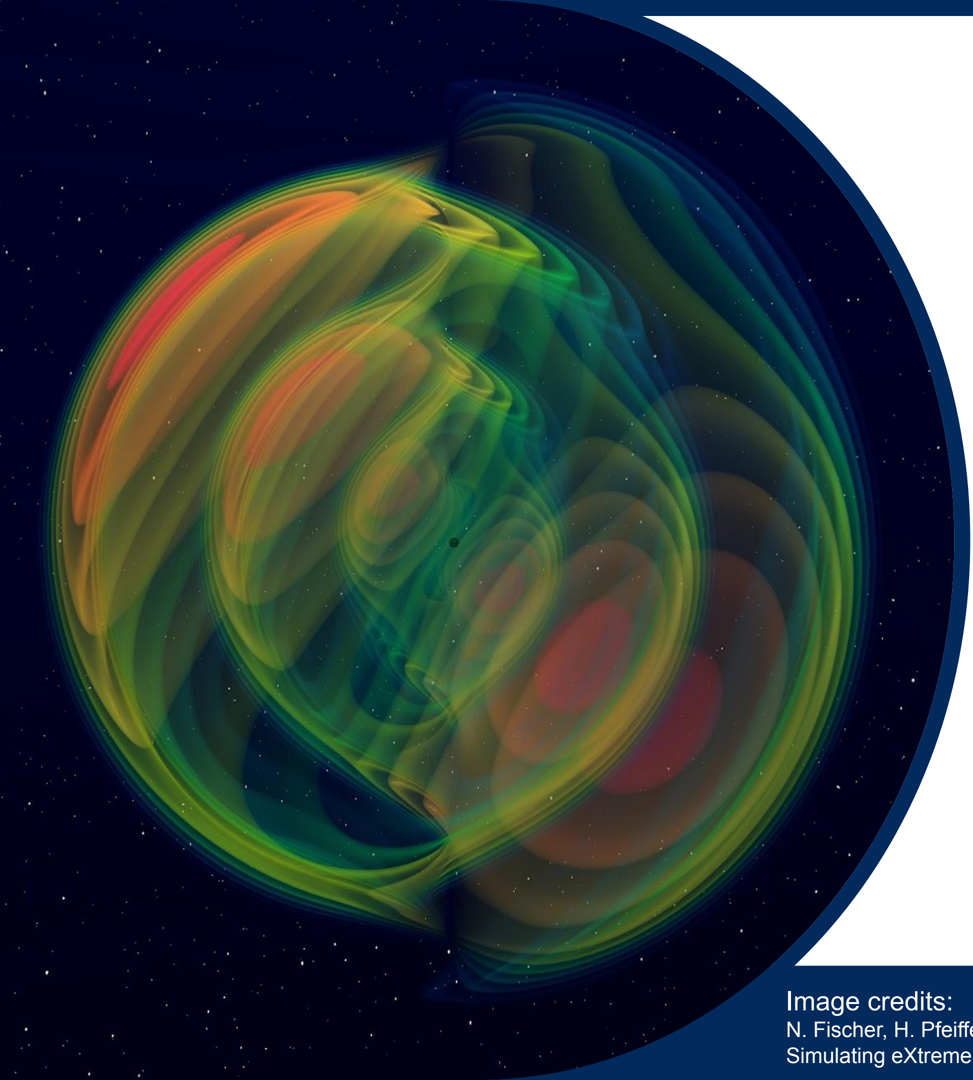
surface gravities are different and not constant

yet, Hawking radiation; thermodynamics?

[ F. Del Porro, **JM,** [2511.02911] ]




[Nathan W. Pyle]



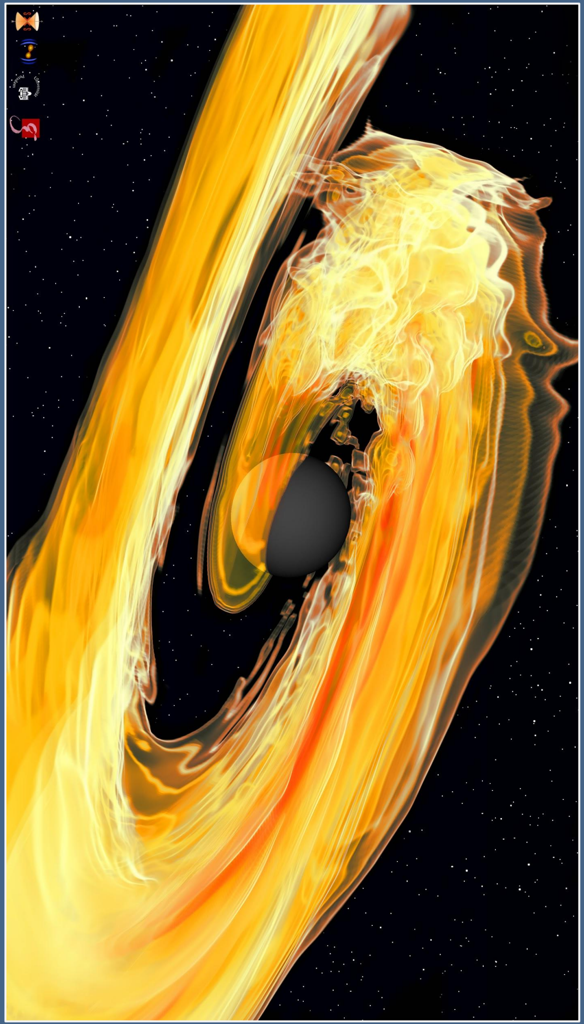
# Thanks!

*Get in touch*

jacopo.mazza@ijclab.in2p3.fr

Image credits:
N. Fischer, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics), Simulating eXtreme Spacetimes (SXS) Collaboration

# Backup Slides



# [BckUp] Circularity and (*more*) geometry

Since you liked…

$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

[everywhere]

… you might also like

$$\xi^\mu R_{\mu[\nu}\psi_\rho\xi_{\sigma]} = 0$$
$$\psi^\mu R_{\mu[\nu}\psi_\rho\xi_{\sigma]} = 0$$

[everywhere]

+

$$\xi_{[\mu}\psi_\nu\partial_\rho\xi_{\sigma]} = 0$$
$$\xi_{[\mu}\psi_\nu\partial_\rho\psi_{\sigma]} = 0$$

[at least at one point]

equivalent formulations
in ‘normal’ situations

(use Killing eq.’s and
some assumptions)

# [BckUp] Circularity and matter

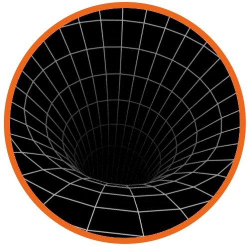
$$\xi^\mu R_{\mu[\nu}\psi_\rho\xi_{\sigma]} = 0$$
$$\psi^\mu R_{\mu[\nu}\psi_\rho\xi_{\sigma]} = 0$$

[everywhere]

statement on symmetry of sources via field equations

Einstein spaces automatically circular

Kerr–Newman
(A)dS, etc.

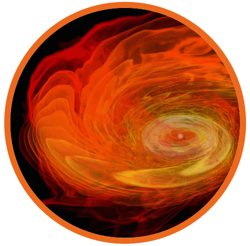


[James Zanoni/Quanta Magazine]

SET of a fluid: velocity

$$u^\mu = \alpha\,\xi^\mu + \beta\,\psi^\nu$$

$$u^{[\mu}\xi^\nu\psi^{\rho]} = 0$$



[NASA/AEI/ZIB/M. Koppitz and L. Rezzolla]

# [BckUp] Square peg in circular hole

Y. Xie, J. Zhang, H.O. Silva, C. de Rham, H. Witek and N. Yunes, PRL 126 (2021) 241104 [2103.03925]

*'Square peg in a cirucla hole'*

beyond-GR theories like

$$\mathcal{L}_0\left(g_{\mu\nu},\varphi\right)+\alpha\mathcal{L}_M\left(g_{\mu\nu},\varphi\right)$$

operators up to dim 4

higher order

that allow perturabtive treatment

$$g_{\mu\nu}=g^{(0)}_{\mu\nu}+\sum_n \alpha^n g^{(n)}_{\mu\nu}$$

$$\varphi=\varphi^{(0)}+\sum_n \alpha^n \varphi^{(n)}$$

theorem

| if | then |
|---|---|
| $\varphi$ scalar …… $\xi^\mu\partial_\mu\varphi=0$ | $g_{\mu\nu}$ circular order by order in $\alpha$ |
| $\varphi$ vector …… $\varphi$ circular | |
| […] | |

# [BckUp] Kerr-like gauge

Kerr-like gauge condition

$$0 = g_{v\theta} = g_{\tilde{v}\tilde{v}} \frac{\partial V}{\partial \theta} + g_{\tilde{v}\tilde{r}} \frac{\partial R}{\partial \theta} + g_{\tilde{v}\tilde{\theta}} \frac{\partial \Theta}{\partial \theta} + g_{\tilde{v}\tilde{\phi}} \frac{\partial \Phi}{\partial \theta}$$

$$0 = g_{\phi\theta} = g_{\tilde{\phi}\tilde{v}} \frac{\partial V}{\partial \theta} + g_{\tilde{\phi}\tilde{r}} \frac{\partial R}{\partial \theta} + g_{\tilde{\phi}\tilde{\theta}} \frac{\partial \Theta}{\partial \theta} + g_{\tilde{\phi}\tilde{\phi}} \frac{\partial \Phi}{\partial \theta}$$

$$\begin{aligned} 0 = g_{r\theta} = {} & [g_{v\theta}] \frac{\partial V}{\partial r} + [g_{\phi\theta}] \frac{\partial \Phi}{\partial r} \\ & + \left[ g_{\tilde{v}\tilde{r}} \frac{\partial R}{\partial r} + g_{\tilde{v}\tilde{\theta}} \frac{\partial \Theta}{\partial r} \right] \frac{\partial V}{\partial \theta} + \left[ g_{\tilde{r}\tilde{\phi}} \frac{\partial R}{\partial r} + g_{\tilde{\theta}\tilde{\phi}} \frac{\partial \Theta}{\partial r} \right] \frac{\partial \Phi}{\partial \theta} \\ & + \left[ g_{\tilde{r}\tilde{r}} \frac{\partial R}{\partial r} + g_{\tilde{r}\tilde{\theta}} \frac{\partial \Theta}{\partial r} \right] \frac{\partial R}{\partial \theta} + \left[ g_{\tilde{r}\tilde{\theta}} \frac{\partial R}{\partial r} + g_{\tilde{\theta}\tilde{\theta}} \frac{\partial \Theta}{\partial r} \right] \frac{\partial \Theta}{\partial \theta} \end{aligned}$$

$$\begin{aligned} 0 = g_{rr} = {} & g_{\tilde{v}\tilde{v}} \left( \frac{\partial V}{\partial r} \right)^2 + g_{\tilde{r}\tilde{r}} \left( \frac{\partial R}{\partial r} \right)^2 + g_{\tilde{\theta}\tilde{\theta}} \left( \frac{\partial \Theta}{\partial r} \right)^2 + g_{\tilde{\phi}\tilde{\phi}} \left( \frac{\partial \Phi}{\partial r} \right)^2 \\ & + 2 g_{\tilde{v}\tilde{r}} \frac{\partial V}{\partial r} \frac{\partial R}{\partial r} + 2 g_{\tilde{v}\tilde{\theta}} \frac{\partial V}{\partial r} \frac{\partial \Theta}{\partial r} + 2 g_{\tilde{v}\tilde{\phi}} \frac{\partial V}{\partial r} \frac{\partial \Phi}{\partial r} \\ & + 2 g_{\tilde{r}\tilde{\theta}} \frac{\partial R}{\partial r} \frac{\partial \Theta}{\partial r} + 2 g_{\tilde{r}\tilde{\phi}} \frac{\partial R}{\partial r} \frac{\partial \Phi}{\partial r} + 2 g_{\tilde{\theta}\tilde{\phi}} \frac{\partial \Theta}{\partial r} \frac{\partial \Phi}{\partial r} \end{aligned}$$

perform coordinate change

[tilt Cauchy surface]

$$r \to r\,, \quad \theta \to \theta + r$$

$$\frac{\partial U_i}{\partial \theta} \to \frac{\partial U_i}{\partial \theta}\,, \qquad \frac{\partial U_i}{\partial r} \to \frac{\partial U_i}{\partial r} + \frac{\partial U_i}{\partial \theta}$$

Cauchy–Kovalvskaya form

$$\frac{\partial U_i}{\partial \theta} = F\left( g_{\tilde{\mu}\tilde{\nu}}, \frac{\partial U_i}{\partial r}, \partial U_j \right)$$

# [BckUp] Remarks

Indeces up

$$\frac{g^{vr}}{g^{rr}} = f(r)$$

$$\frac{g^{r\phi}}{g^{rr}} = h(r)$$

nice

Indeces down

$$\frac{g_{vr}g_{\phi\phi} - g_{r\phi}g_{v\phi}}{g_{v\phi}^2 - g_{vv}g_{\phi\phi}} = f(r)$$

$$\frac{g_{r\phi}g_{vv} - g_{vr}g_{v\phi}}{g_{v\phi}^2 - g_{vv}g_{\phi\phi}} = h(r)$$

not (so) nice

Solved conditions are existence conditions for coordinate change to 'Boyer–Lindquist form'

$$g_{\mu\nu} = \begin{pmatrix} g_{vv} & 0 & 0 & g_{v\phi} \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\theta\theta} & 0 \\ * & 0 & 0 & g_{\phi\phi} \end{pmatrix}$$

# [BckUp] 'Minimal' deformation

Solved circularity conditions

$$g^{vr} = f(r) g^{rr}$$
$$g^{r\phi} = h(r) g^{rr}$$

'soft' breaking

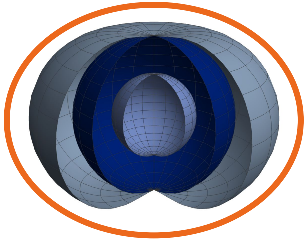
$$g^{vr} = f_{\text{Kerr}} g^{rr}_{\text{Kerr}} + f(r,\theta)_{\text{deform}}$$
$$g^{r\phi} = h_{\text{Kerr}} g^{rr}_{\text{Kerr}}$$

$$g^{\mu\nu} = g^{\mu\nu}_{\text{Kerr}} + \frac{\delta(r,\theta)}{\Sigma} \delta^{\mu}_{\ v} \delta^{\mu}_{\ \phi}$$
$$g_{\mu\nu} = \left(\text{Quite-A-Mess}\right)_{\mu\nu}$$

surfaces @ same location

$$r_{\text{erg}} = M + \sqrt{M^2 - a^2 \cos^2\theta}$$

$$r_{\text{H}} = r_{\text{rot}} = M + \sqrt{M^2 - a^2}$$



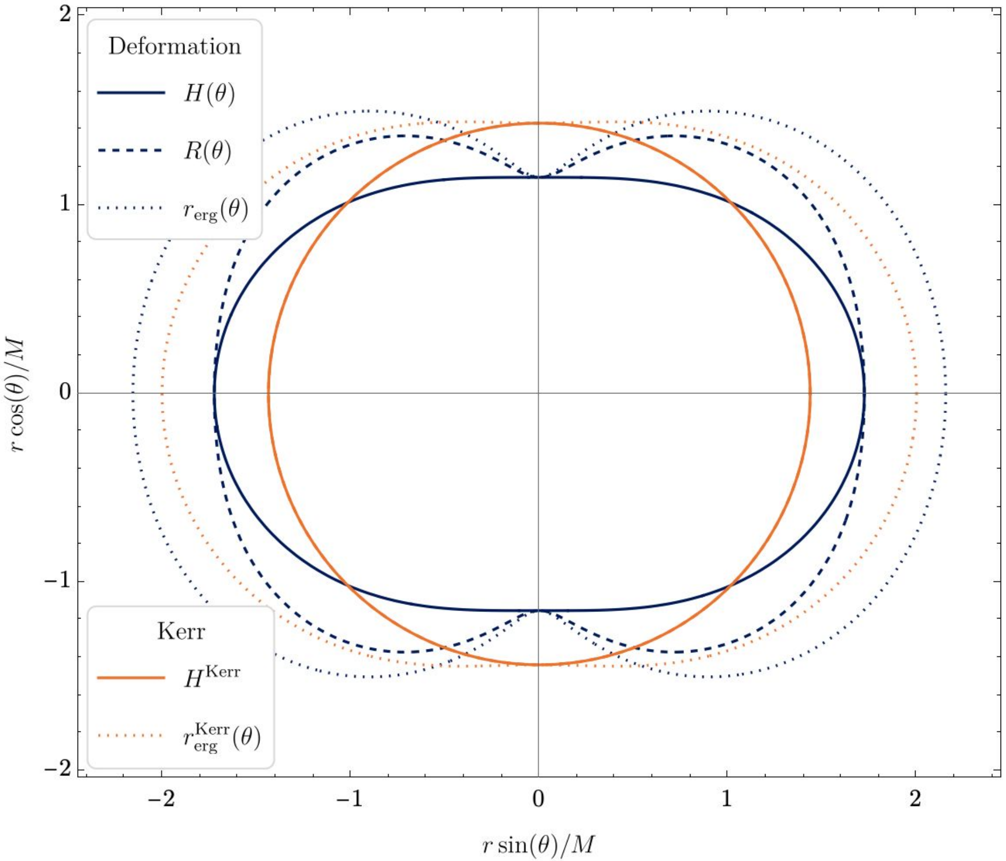
but $H$ not Killing, and

$$\kappa = \left.\frac{r-M}{2rM+\delta(r,\theta)}\right|_{r_H}$$

# [BckUp] 'Not-so-minimal' deformation

Choose horizon profile $H(\theta)$,
then reverse-engineer the metric

horizon, rotosurface, ergosphere
known analytically



# [BckUp] Horizon generators

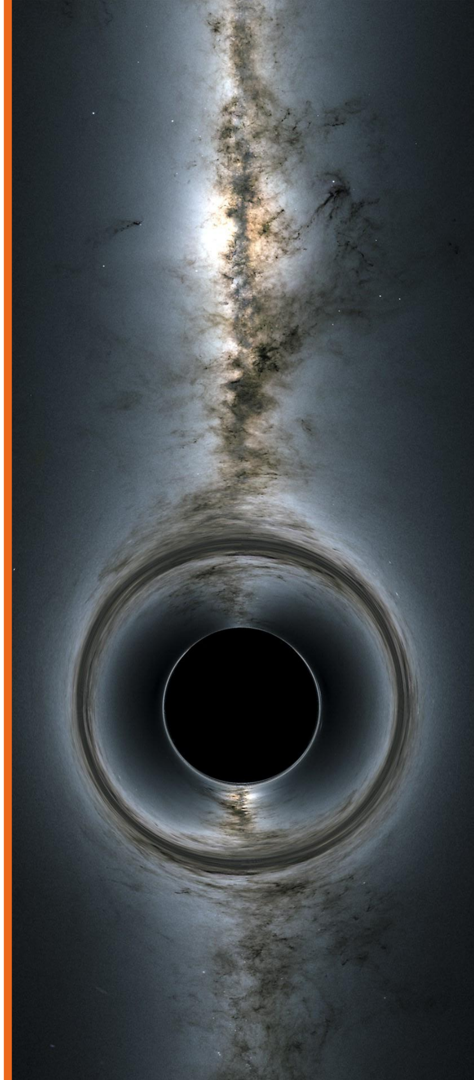
Horizon

- surface: $$r = H(\theta)$$
- normal: $$\zeta_\mu := \frac{\alpha}{g^{vr}} \partial_\mu \left[ r - H(\theta) \right]$$
- generator: $$\zeta_\mu \zeta^\mu \Big|_{r=H} = 0$$

$$\zeta^\mu \Big|_{r=H} = \alpha \left[ \Xi^\mu - \frac{H' g^{\theta\theta}}{g^{vr}} \tau^\mu \right] \Big|_{r=H}$$

$\Xi^\mu$: would-be Killing vector

$\tau^\mu$: tangent to H



# Smarr’s formula

We can prove this geometric identity, no field equations

$$M_H = \frac{\overline{\kappa}}{4\pi G}\mathcal{A}_H + 2\overline{\sigma}_H J_H + \eta$$

Komar mass

horizon area

Komar angular momentum

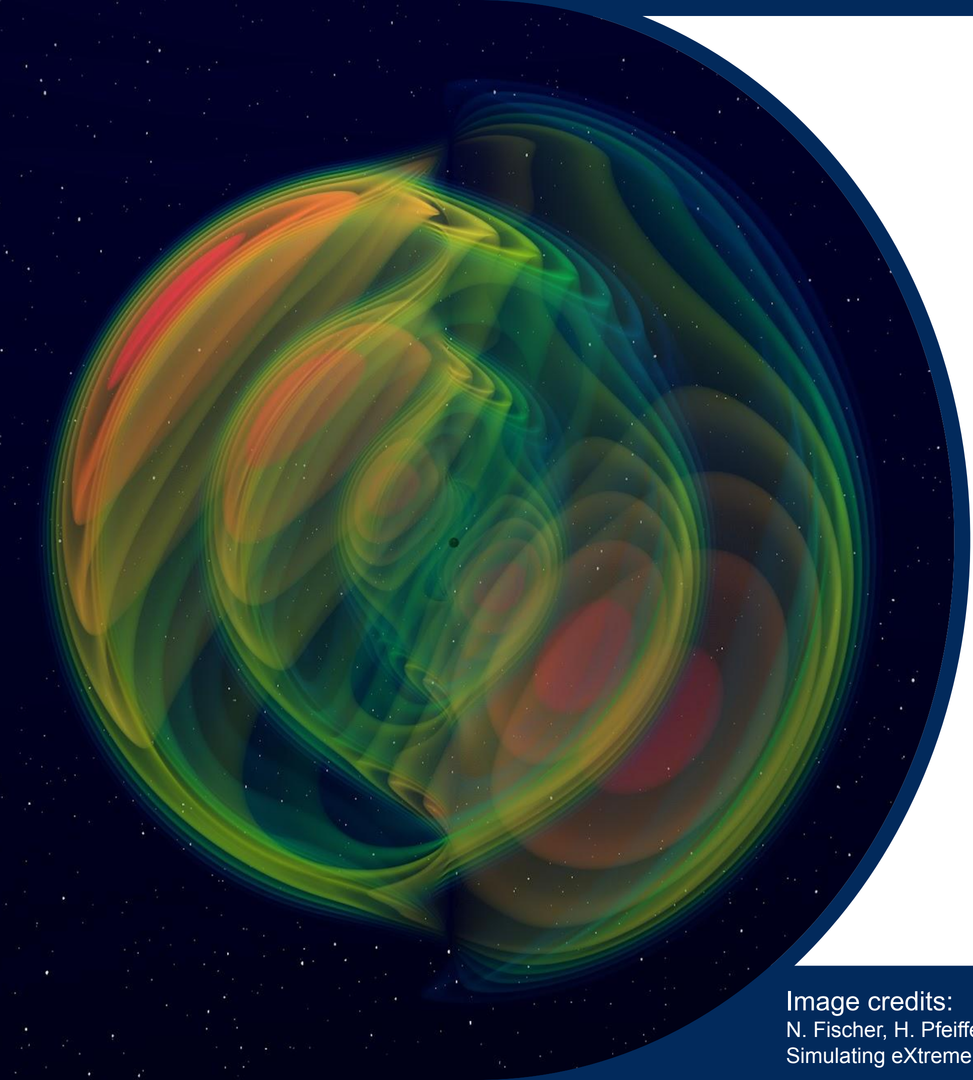
normal surface gravity

angular velocity [sort of]

averages over *H*

dissipation [?]

vanishes in minimal case $H' = 0$



# Thanks!

*Get in touch*

jacopo.mazza@ijclab.in2p3.fr

Image credits:
N. Fischer, H. Pfeiffer, A. Buonanno (Max Planck Institute for Gravitational Physics),
Simulating eXtreme Spacetimes (SXS) Collaboration

Friday - 12 Decembre 2025 - https://indico.in2p3.fr/event/37628/overview

Timing: 23 min + 5 min

## Beyond circles: stationary axisymmetric black holes and the breaking of circularity

Circularity is an accidental symmetry of the Kerr metric, one that is widely assumed when searching for rotating black hole solutions in modified gravity as well as when constructing models of Kerr mimickers. Though extremely enticing, circularity is often an excessively restrictive assumption, and understanding the consequences of its loss is thus crucially relevant. In this seminar, I wish to present some recent results on the subject: After describing in detail what this symmetry entails, I will show how to construct stationary and axisymmetric spacetimes exhibiting a controlled breaking of circularity; then, I will describe the impact of circularity breaking on the hole's horizon, focusing in particular on the laws of black hole mechanics. This discussion is thus going to be pertinent for anyone with an interest in compact astrophysical objects and their phenomenology, in general relativity and beyond.