

Rounding up the usual suspects

Towards inference of beyond-vacuum effects in LISA Global Fit

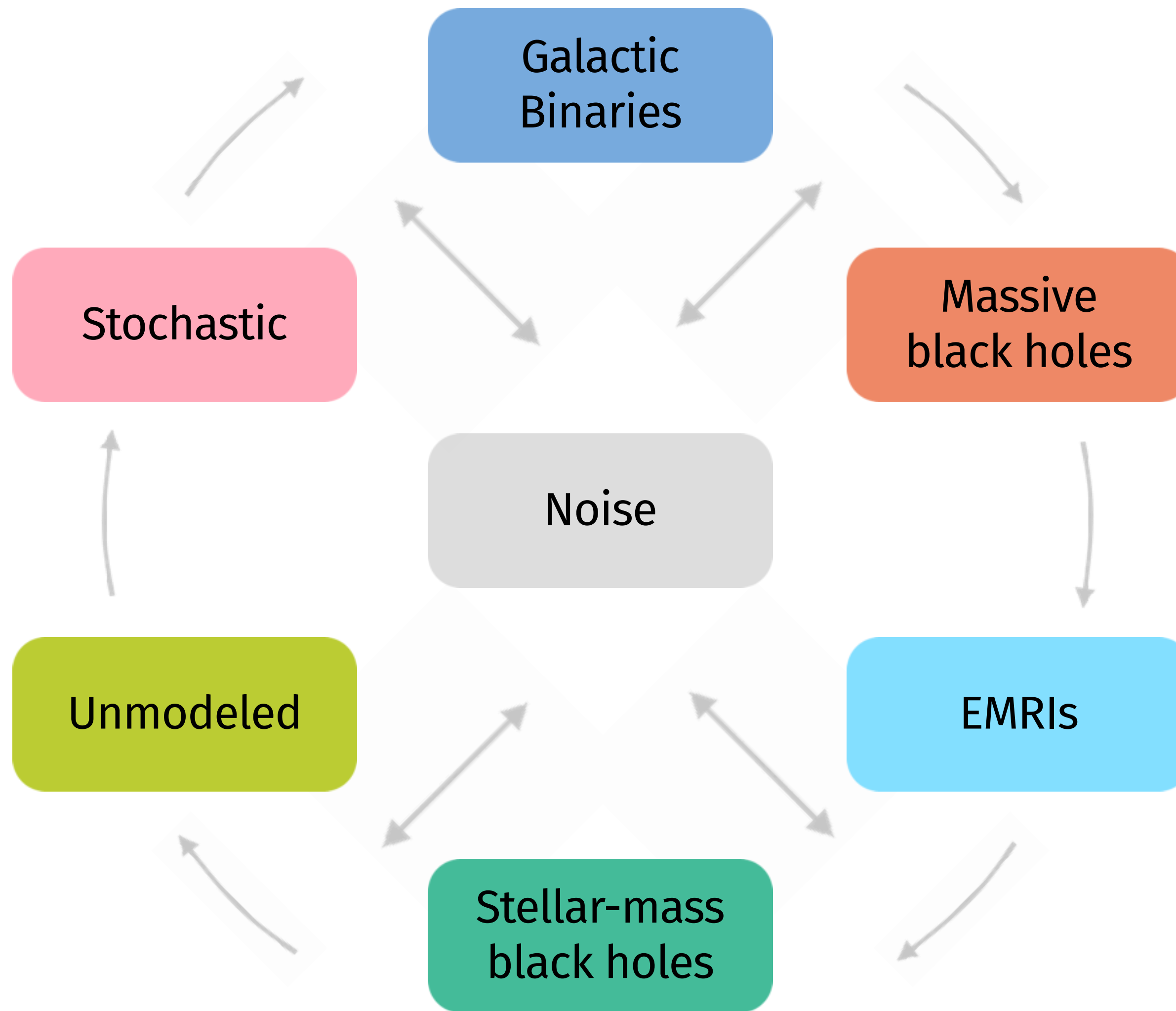


Francisco Duque

EMRI Search and Inference within the LISA Global Fit - Part II

MAX PLANCK INSTITUTE
FOR GRAVITATIONAL PHYSICS
(ALBERT EINSTEIN INSTITUTE)





X



Vacuum



Systematics



Accretion disks



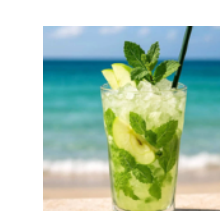
Dark matter



Triple Systems



Modified Gravity

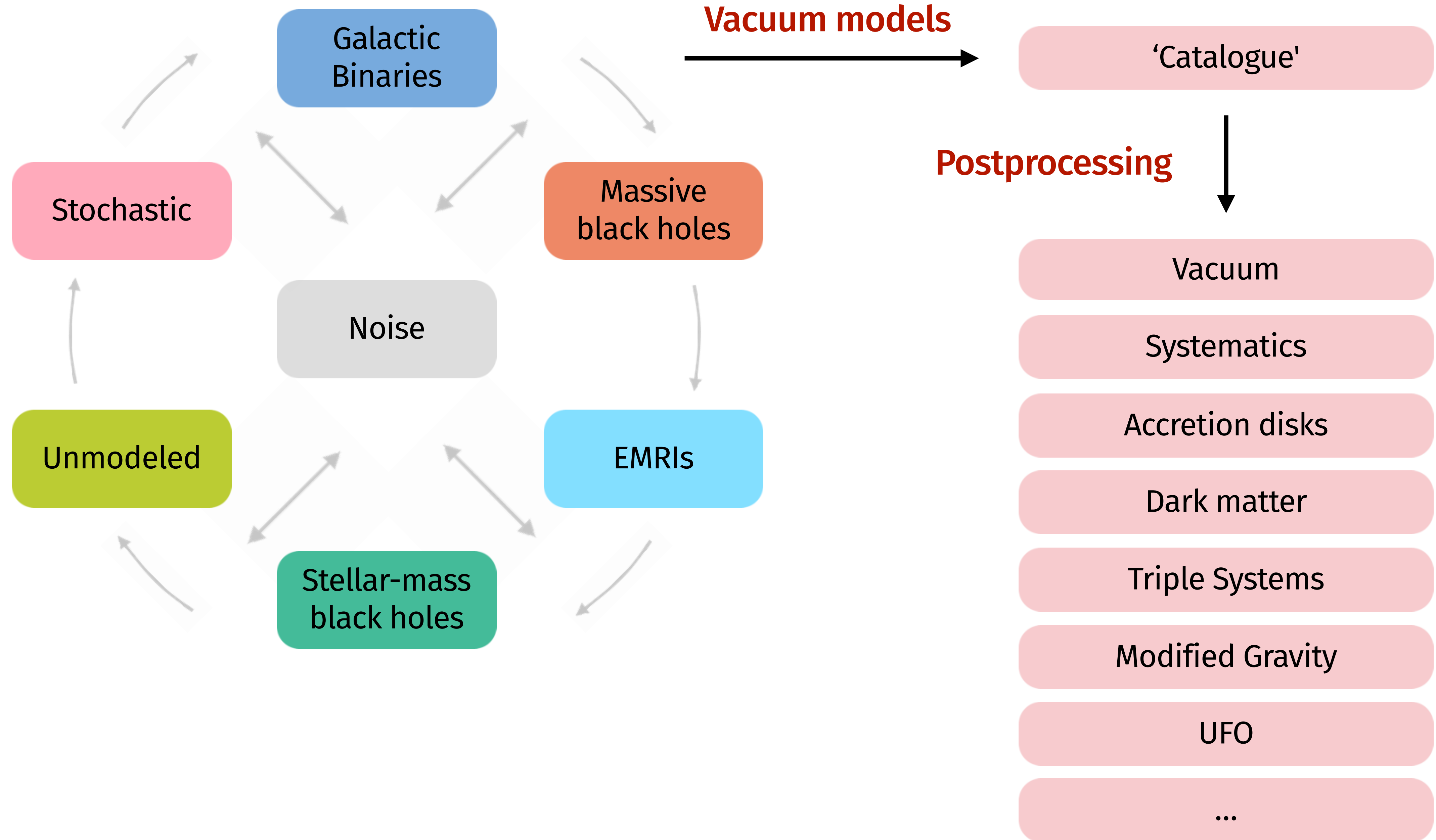


UFOs



...





Problem

Many signals in the data



Solution (?)

Down-scale global fit in follow-up tests of vacuum GR?

Problem

Many signals in the data



Solution (?)

Down-scale global fit in follow-up tests of vacuum GR?

Many possible models



1. Use *catch-all* agnostic models
2. Map them to physical models

Problem

Many signals in the data



Solution (?)

Down-scale global fit in follow-up tests of vacuum GR?

Duque & Gair, in prep

Many possible models



1. Use *catch-all* agnostic models
2. Map them to physical models

When we reanalyse a source of interest for a beyond-vacuum effect, some overlapping nuisance/confusion sources can absorb part of that effect in the first vacuum global fit, masking it. *Which ones*, so we know which to re-fit *jointly* with the source of interest instead of alone?

When we reanalyze
overlapping nuisance
first vacuum glo
joint



vacuum effect, some
of that effect in the
now which to re-fit
alone?

d

Data

$$d = n$$

Data **Noise**

$$d = n + h(\theta^{\text{v}}, \theta^{\text{b}})$$

Data **Noise** **Source(s) of interest**
 θ^{v} : vacuum GR parameters
 θ^{b} : beyond-vacuum GR params.

$$d = n + h(\theta^{\text{v}}, \theta^{\text{b}}) + h_{\text{c}}(\theta^{\text{c}})$$

Data **Noise** **Source(s) of interest** **Nuisance sources**

θ^{v} : vacuum GR parameters
 θ^{b} : beyond-vacuum GR params.

$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data Noise Source(s) of interest Nuisance sources

θ^v : vacuum GR parameters
 θ^b : beyond-vacuum GR params.

Data space

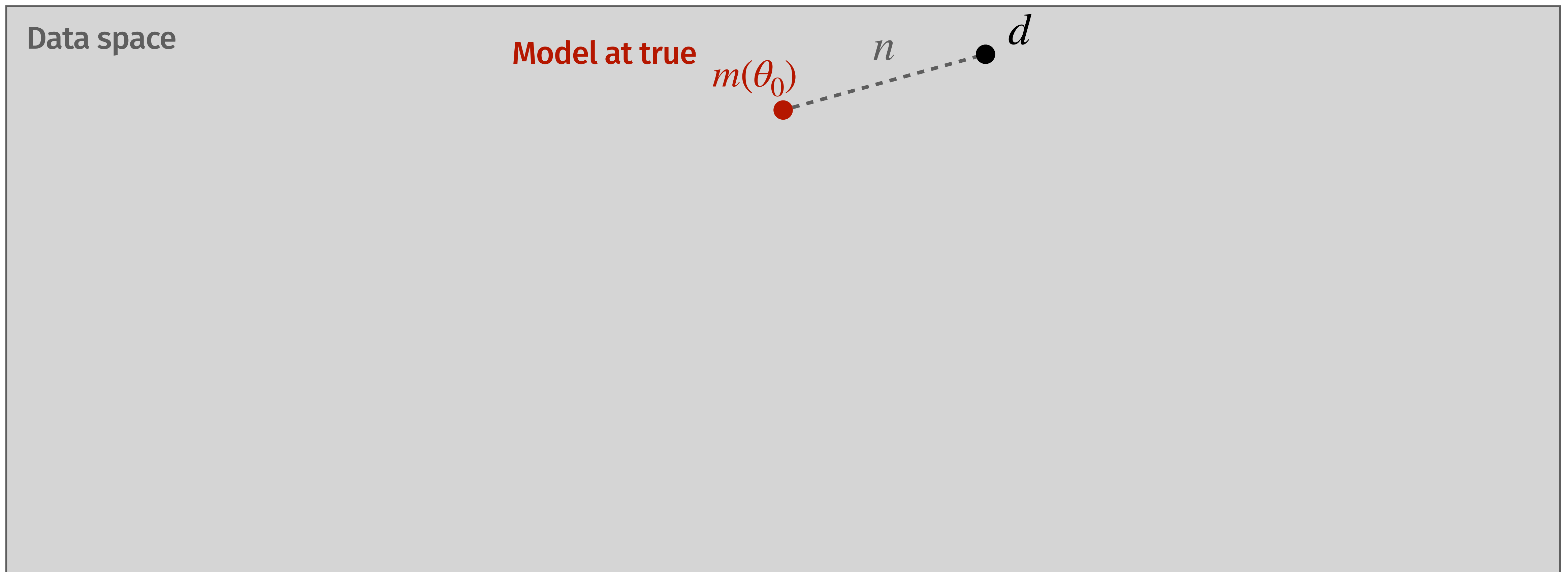
• d

$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data
Noise
Source(s) of interest
Nuisance sources

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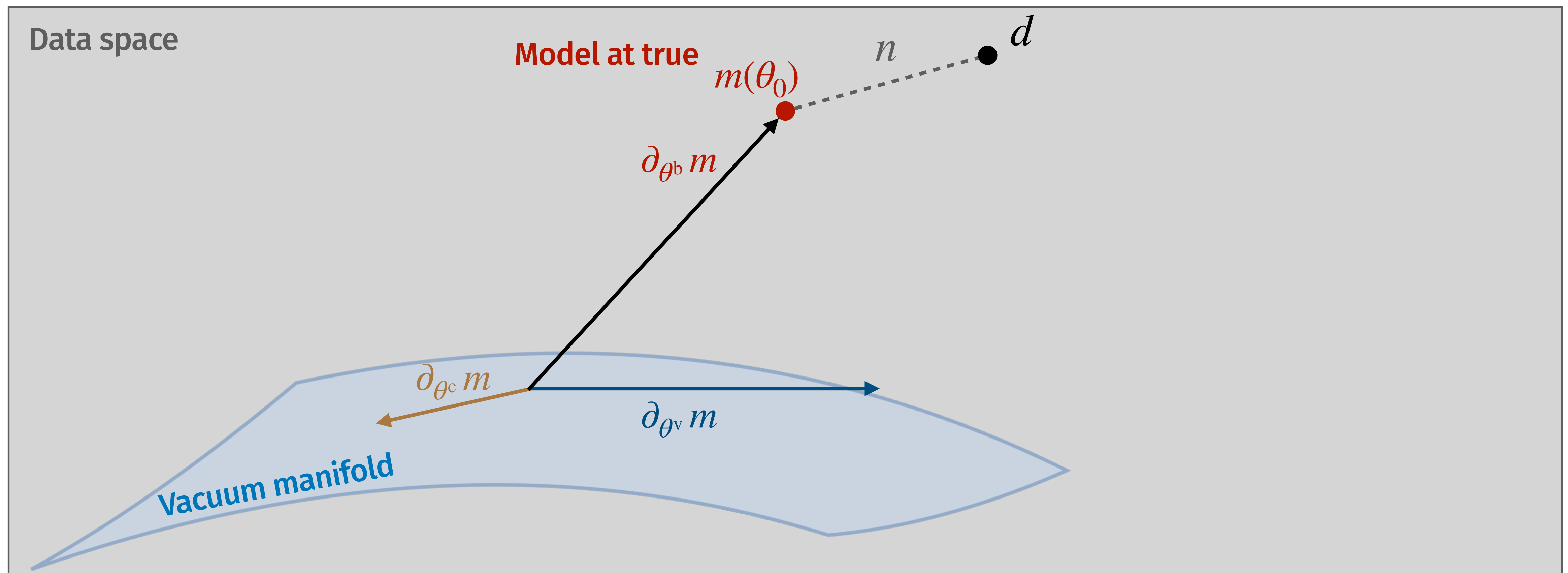


$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data
Noise
Source(s) of interest
Nuisance sources

θ^v : vacuum GR parameters

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$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data

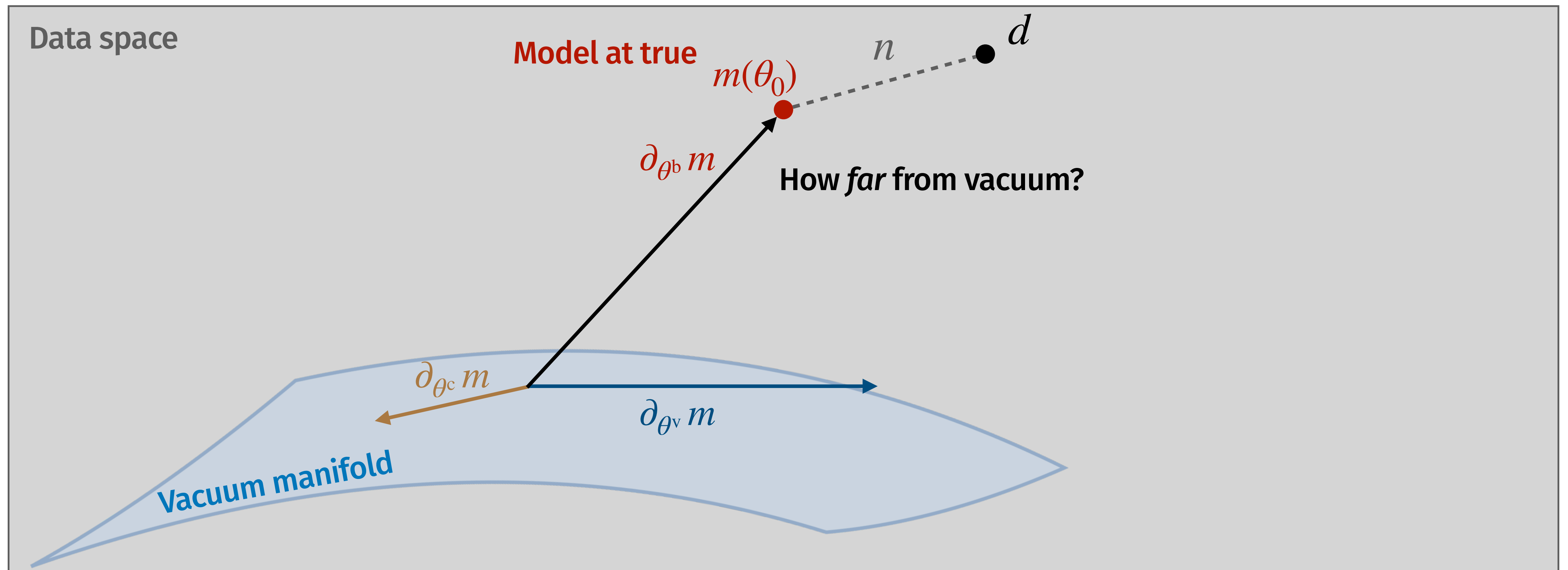
Noise

Source(s) of interest

Nuisance sources

θ^v : vacuum GR parameters

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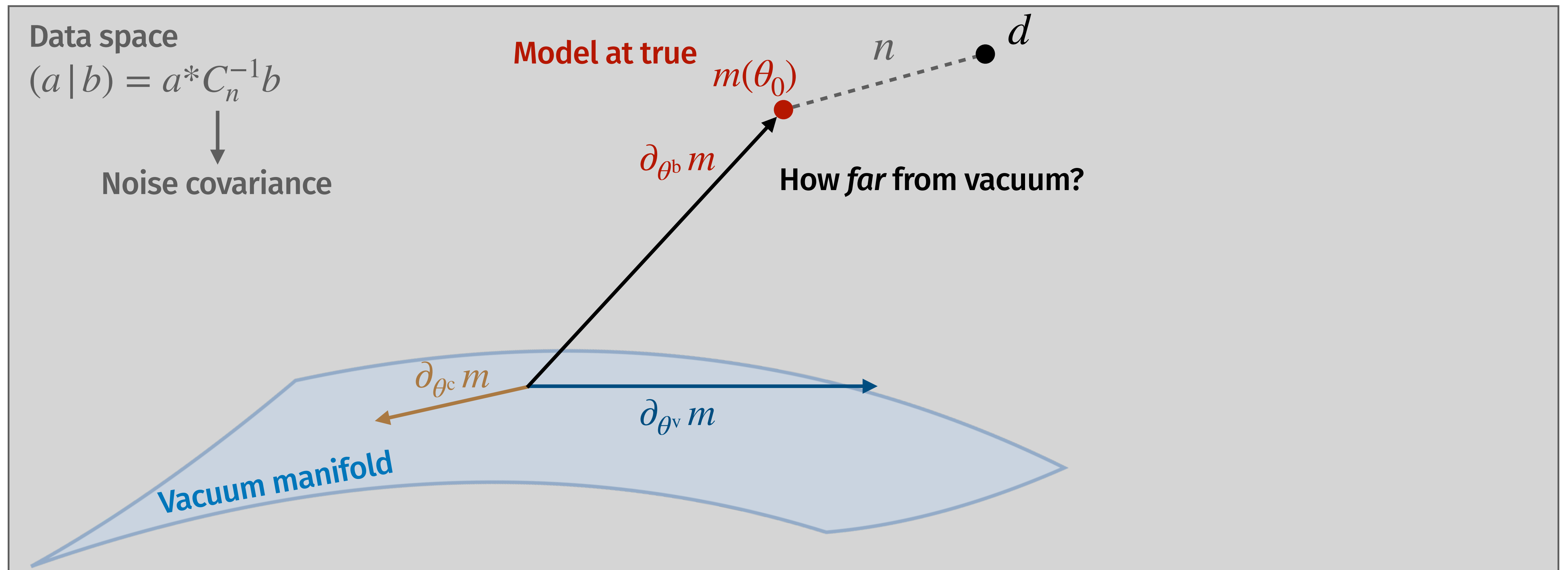


$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data
Noise
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$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data

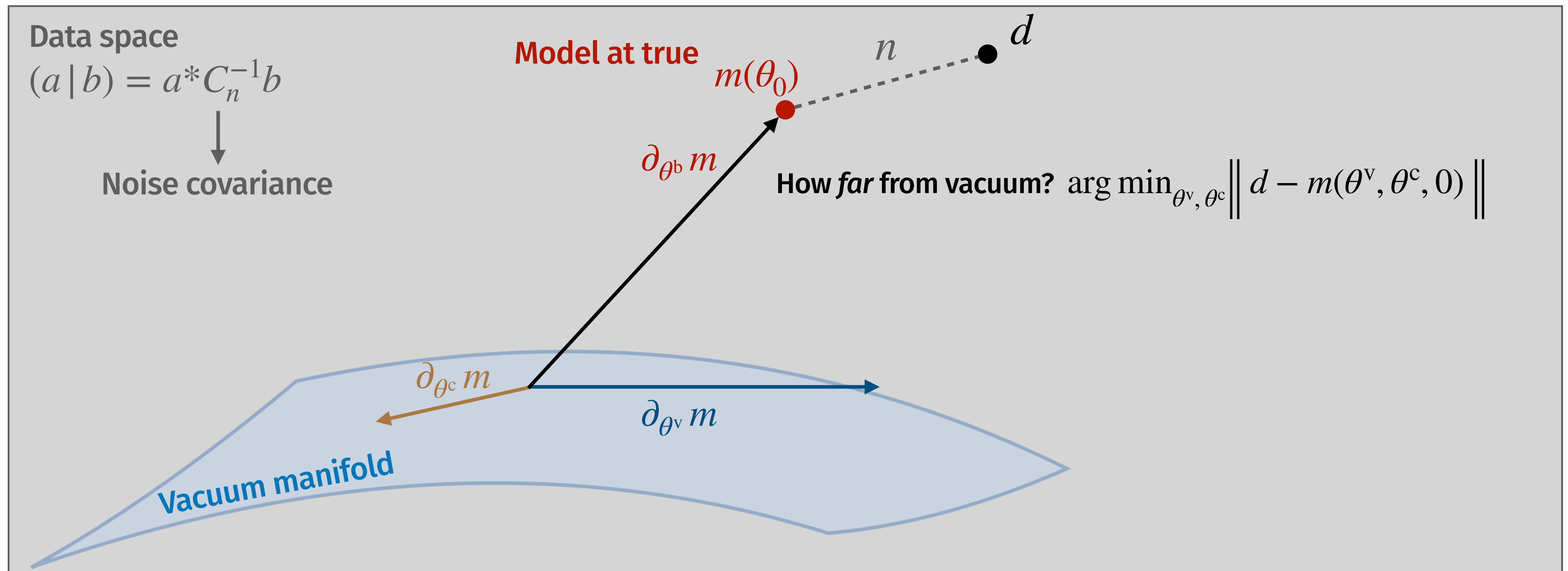
Noise

Source(s) of interest

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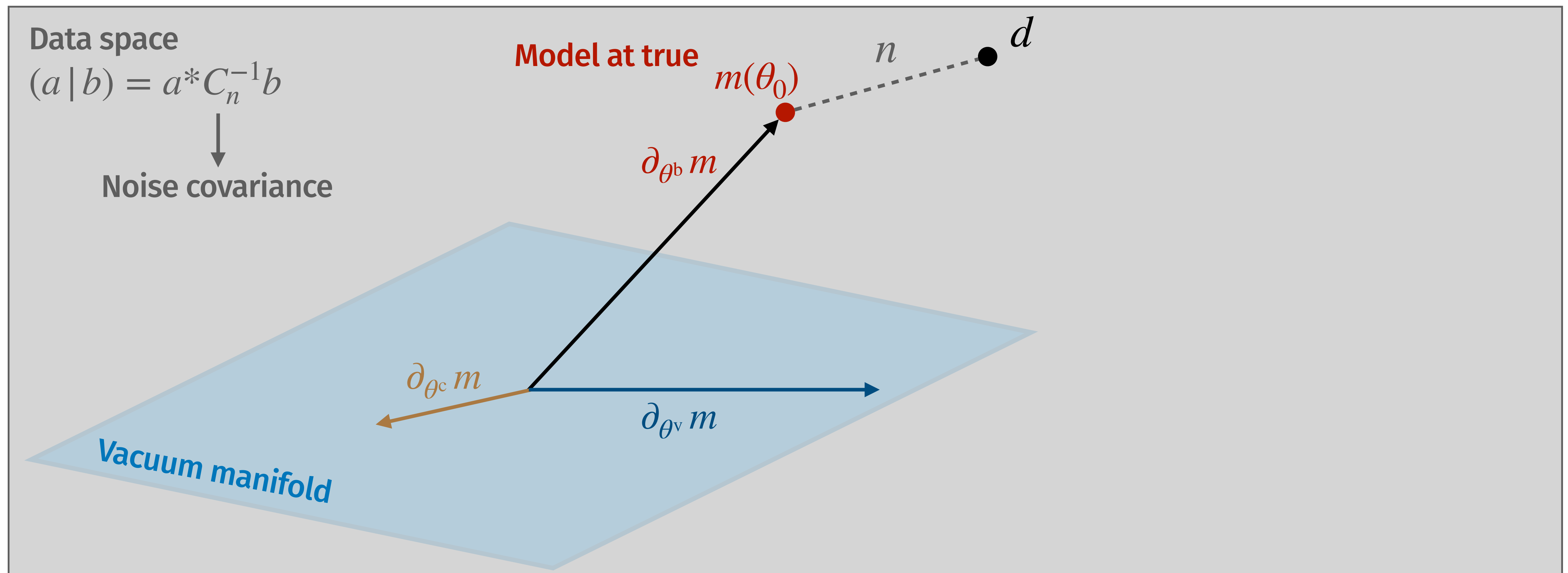


$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

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Noise
Source of interest
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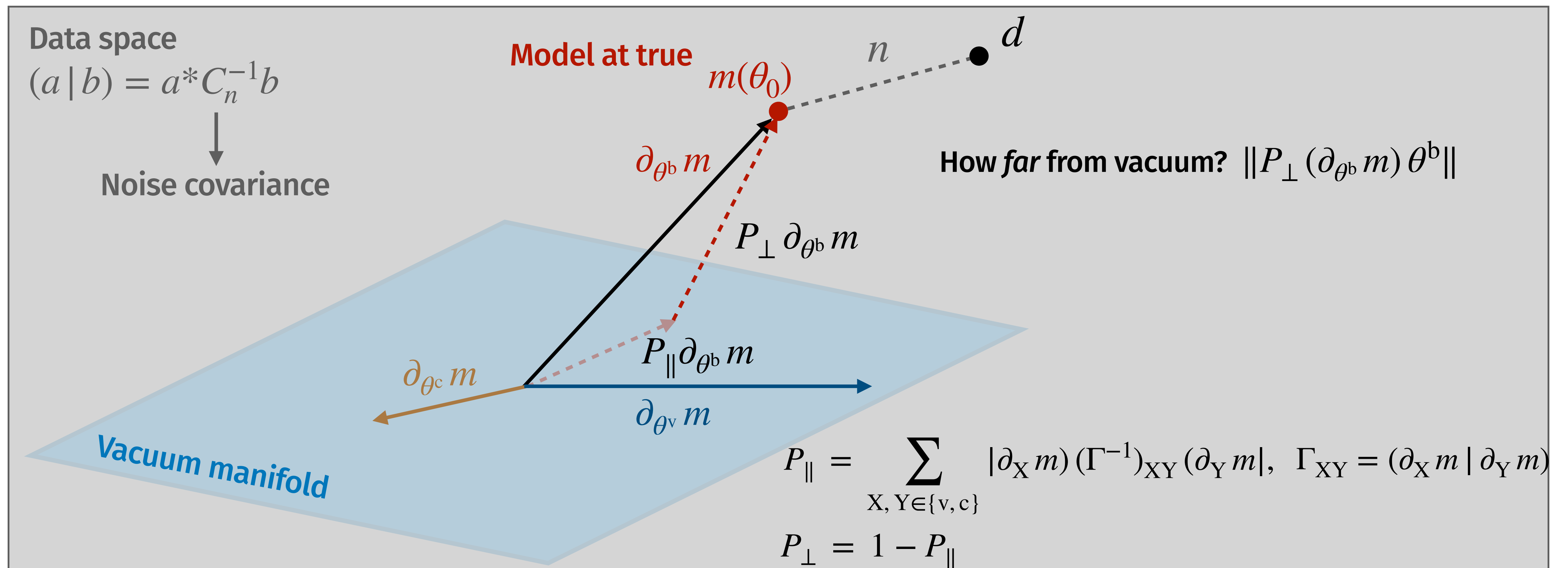


$$d = n + h(\theta^v, \theta^b) + h_c(\theta^c)$$

Data
Noise
Source of interest
Nuisance sources

θ^v : vacuum GR parameters

θ^b : beyond-vacuum GR params.



Ignoring nuisance sources & noise for now...

$$\begin{aligned} \|P_{\perp}(\partial_{\theta^b} m) \theta^b\|^2 &= (\theta^b)^T (\partial_{\theta^b} m | P_{\perp} | \partial_{\theta^b} m) \theta^b \\ &= (\theta^b)^T (\partial_{\theta^b} m | [1 - |\partial_i m\rangle (\Gamma^{-1})_{VV} \langle \partial_{\theta^V} m|] | \partial_{\theta^b} m) \theta^b \end{aligned}$$

Ignoring nuisance sources & noise for now...

$$\begin{aligned} \|P_{\perp}(\partial_{\theta^b} m) \theta^b\|^2 &= (\theta^b)^T (\partial_{\theta^b} m | P_{\perp} | \partial_{\theta^b} m) \theta^b \\ &= (\theta^b)^T (\partial_{\theta^b} m | [1 - (\partial_i m) (\Gamma^{-1})_{vv} (\partial_{\theta^v} m) | \partial_{\theta^b} m) \theta^b \\ &= (\theta^b)^T [(\partial_{\theta^b} m | \partial_{\theta^b} m) - (\partial_{\theta^b} m | \partial_{\theta^v} m) (\Gamma^{-1})_{vv} (\partial_{\theta^v} m | \partial_{\theta^b} m)] (\theta^b) \\ \Gamma_{ij} &= (\partial_i m | \partial_j m) \quad \curvearrowright \\ &= (\theta^b)^T [\Gamma_{bb} - \Gamma_{bv} (\Gamma^{-1})_{vv} \Gamma_{vb}] (\theta^b) \end{aligned}$$

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$$\begin{aligned}
 \|P_{\perp}(\partial_{\theta^b} m) \theta^b\|^2 &= (\theta^b)^T (\partial_{\theta^b} m | P_{\perp} | \partial_{\theta^b} m) \theta^b \\
 &= (\theta^b)^T (\partial_{\theta^b} m | \quad [1 - (\partial_{\theta^v} m | \partial_{\theta^b} m) (\Gamma^{-1})_{vv} (\partial_{\theta^b} m | \partial_{\theta^v} m)] \quad | \partial_{\theta^b} m) \theta^b \\
 &= (\theta^b)^T [(\partial_{\theta^b} m | \partial_{\theta^b} m) - (\partial_{\theta^b} m | \partial_{\theta^v} m) (\Gamma^{-1})_{vv} (\partial_{\theta^v} m | \partial_{\theta^b} m)] (\theta^b) \\
 \Gamma_{ij} = (\partial_i m | \partial_j m) &\quad \curvearrowright \\
 &= (\theta^b)^T \underbrace{[\Gamma_{bb} - \Gamma_{bv} (\Gamma^{-1})_{vv} \Gamma_{vb}]}_{\Gamma_{b|v}} (\theta^b)
 \end{aligned}$$

Ignoring nuisance sources & noise for now...

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 &= (\theta^b)^T \underbrace{[\Gamma_{bb} - \Gamma_{bv} (\Gamma^{-1})_{vv} \Gamma_{vb}]}_{\Gamma_{b|v}} (\theta^b)
 \end{aligned}$$

$$\begin{pmatrix} \Gamma_{vv} & \Gamma_{vb} \\ \Gamma_{bv} & \Gamma_{bb} \end{pmatrix}^{-1} = \begin{pmatrix} - & - \\ - & \Gamma_{b|v} \end{pmatrix}$$

Ignoring nuisance sources & noise for now...

$$\|P_{\perp}(\partial_{\theta^b} m) \theta^b\|^2 = (\theta^b)^T (\partial_{\theta^b} m | P_{\perp} | \partial_{\theta^b} m) \theta^b$$

$$= (\theta^b)^T (\partial_{\theta^b} m | \quad [1 - (\partial_{\theta^v} m) (\Gamma^{-1})_{vv} (\partial_{\theta^v} m)] \quad | \partial_{\theta^b} m) \theta^b$$

$$= (\theta^b)^T [(\partial_{\theta^b} m | \partial_{\theta^b} m) - (\partial_{\theta^b} m | \partial_{\theta^v} m) (\Gamma^{-1})_{vv} (\partial_{\theta^v} m | \partial_{\theta^b} m)] (\theta^b)$$

$$\Gamma_{ij} = (\partial_i m | \partial_j m)$$

$$= (\theta^b)^T [\Gamma_{bb} - \Gamma_{bv} (\Gamma^{-1})_{vv} \Gamma_{vb}] (\theta^b)$$

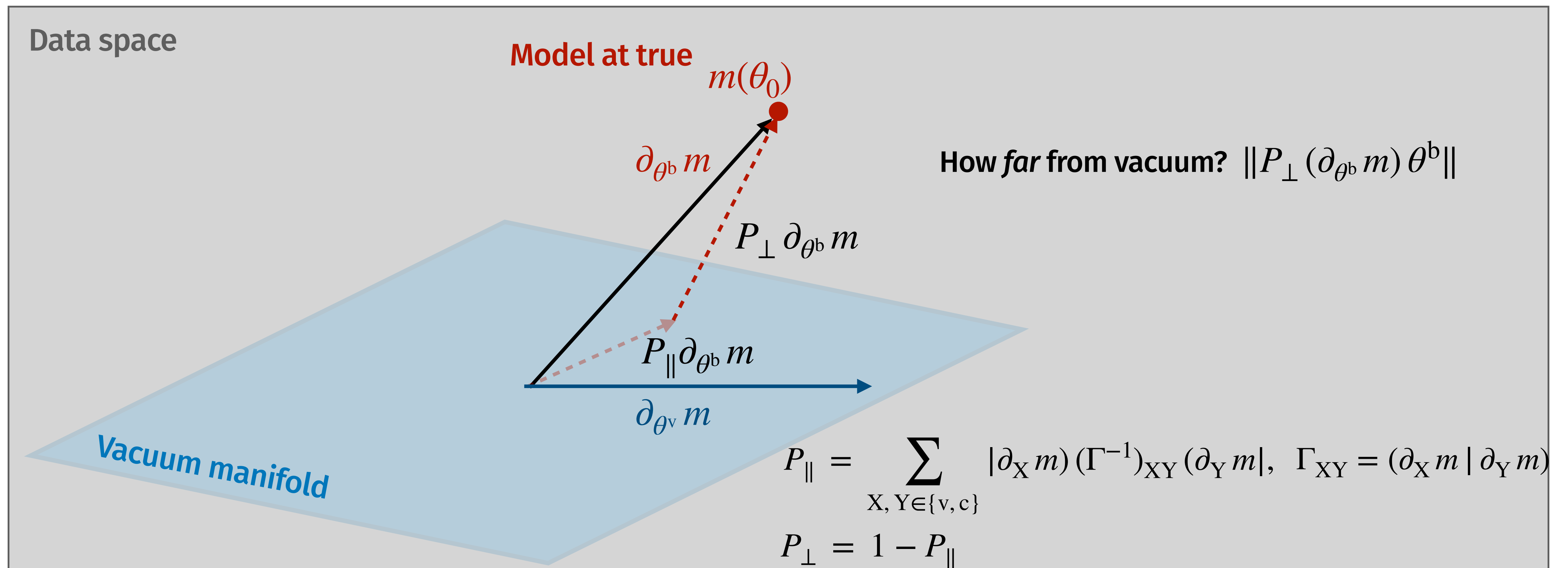
$$\underbrace{\hspace{10em}}_{\Gamma_{b|v}}$$

$$= (\theta^b)^T (\Gamma^{-1})_{bb} (\theta^b)$$

$$\begin{pmatrix} \Gamma_{vv} & \Gamma_{vb} \\ \Gamma_{bv} & \Gamma_{bb} \end{pmatrix}^{-1} = \begin{pmatrix} - & - \\ - & \Gamma_{b|v} \end{pmatrix}$$

Detection Statistic should be

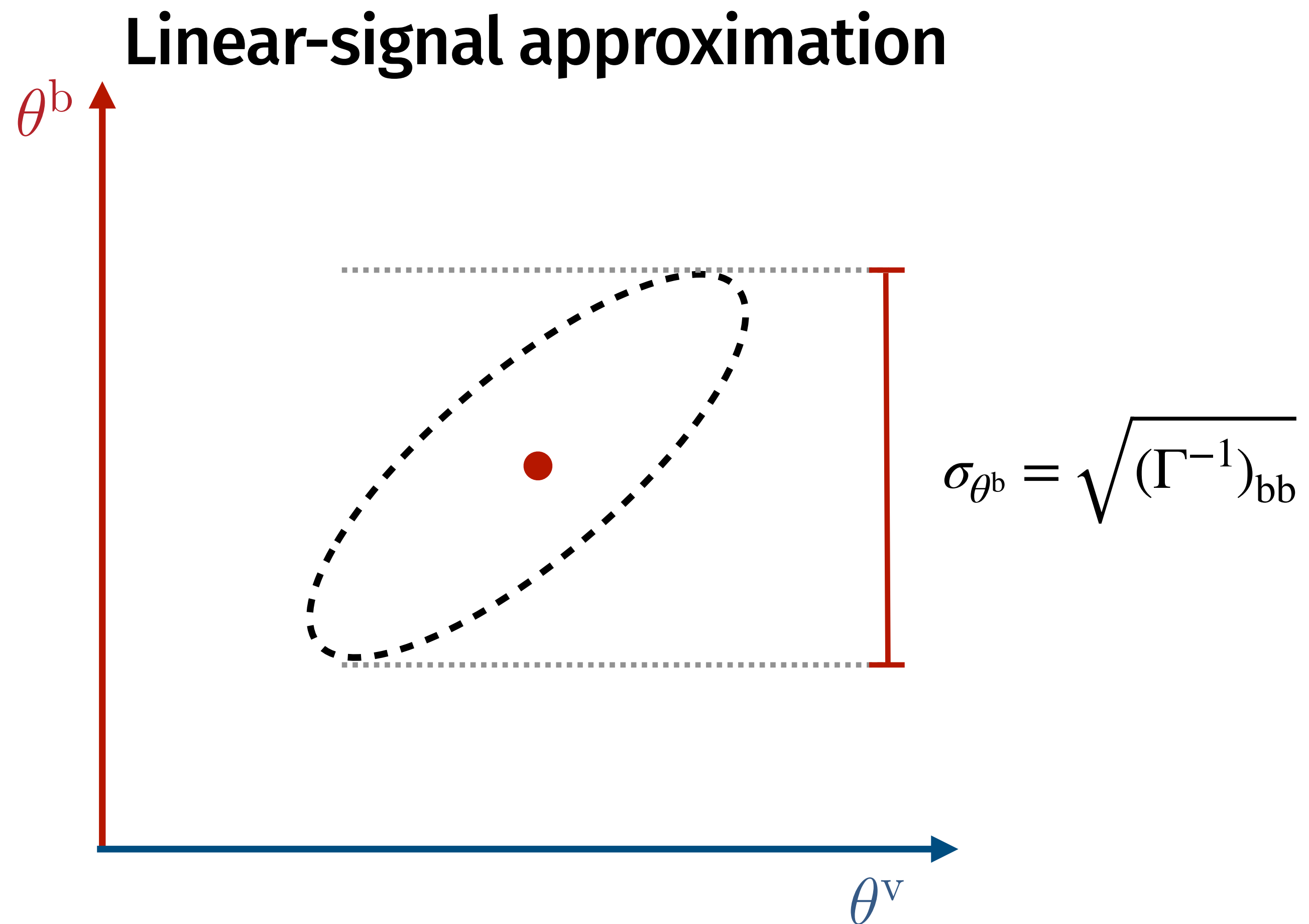
$$S_{\text{GF}} = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b}|\text{v}} (\theta^{\text{b}}) = (\theta^{\text{b}})^{\text{T}} (\Gamma^{-1})_{\text{bb}} (\theta^{\text{b}}) \gg \text{few}$$



Detection Statistic should be

$$S_{\text{GF}} = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b}|\text{v}}(\theta^{\text{b}}) = (\theta^{\text{b}})^{\text{T}} (\Gamma^{-1})_{\text{bb}}(\theta^{\text{b}}) \gg \text{few}$$

$$\Gamma_{\text{XY}} = (\partial_{\text{X}} m \mid \partial_{\text{Y}} m)$$

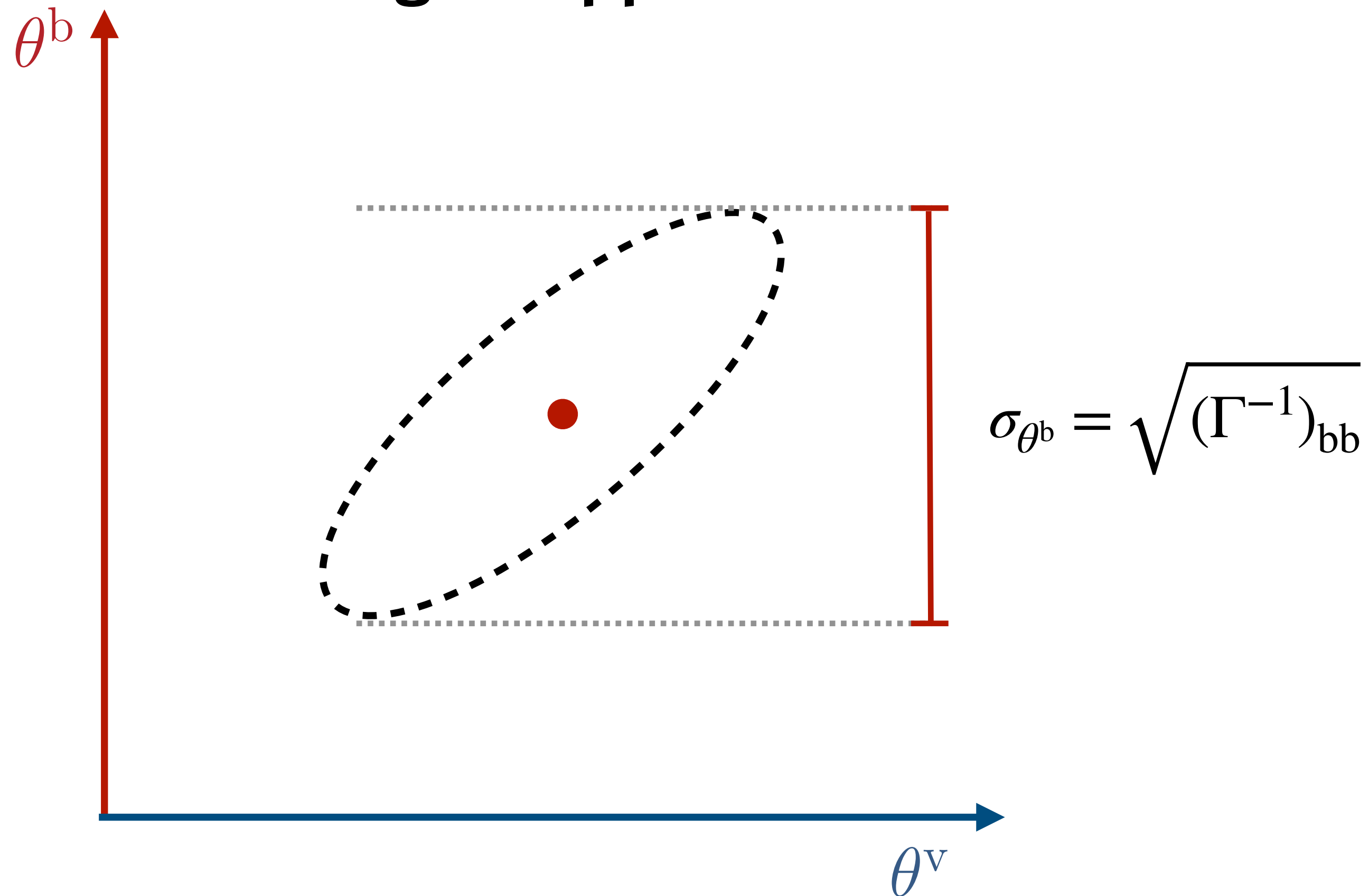


Detection Statistic should be

$$S_{\text{GF}} = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b}|\text{v}} (\theta^{\text{b}}) = (\theta^{\text{b}})^{\text{T}} (\Gamma^{-1})_{\text{bb}} (\theta^{\text{b}}) = (\theta^{\text{b}} / \sigma_{\theta^{\text{b}}})^2 \gg \text{few}$$

$$\Gamma_{\text{XY}} = (\partial_X m \mid \partial_Y m)$$

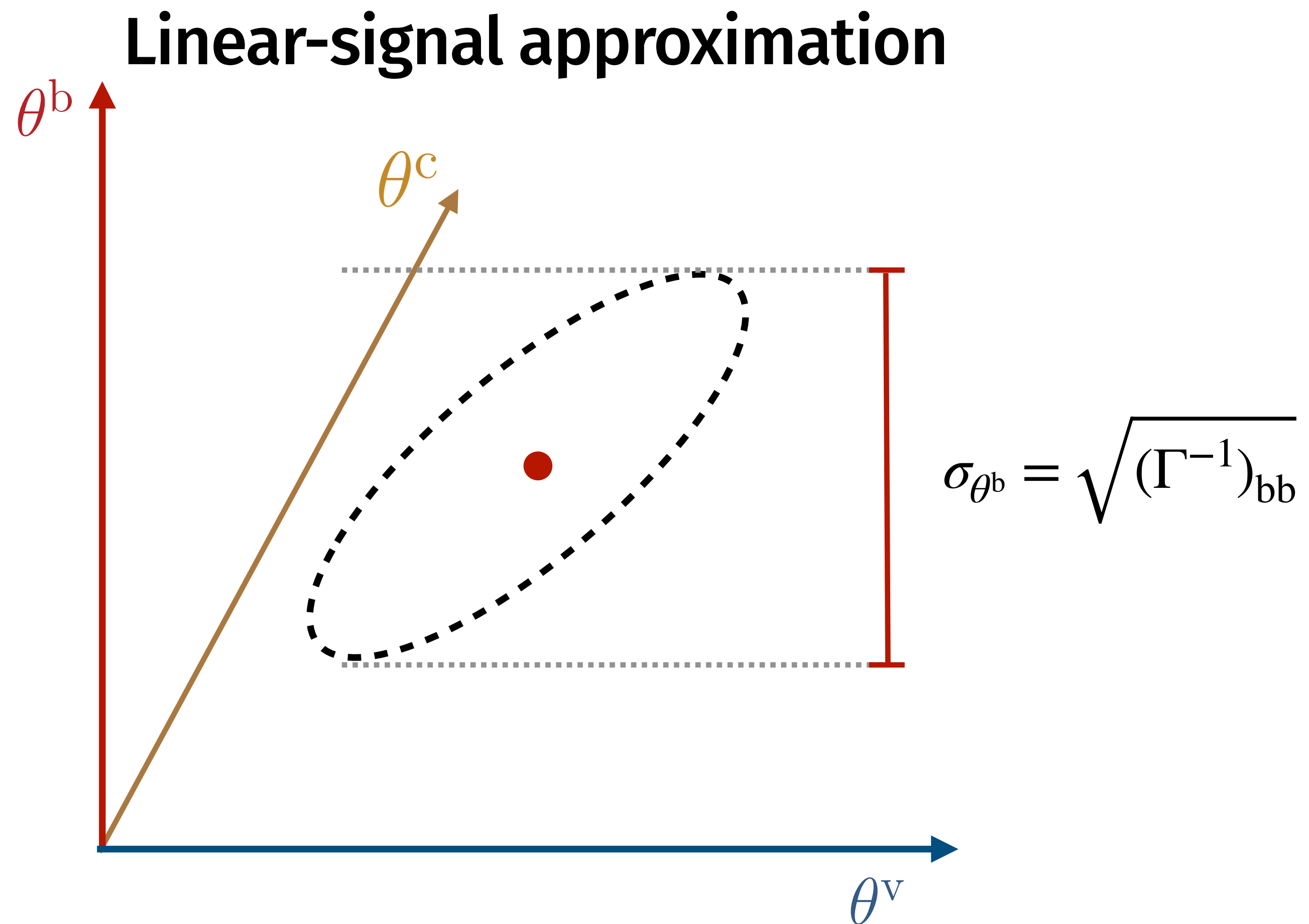
Linear-signal approximation



Detection Statistic should be

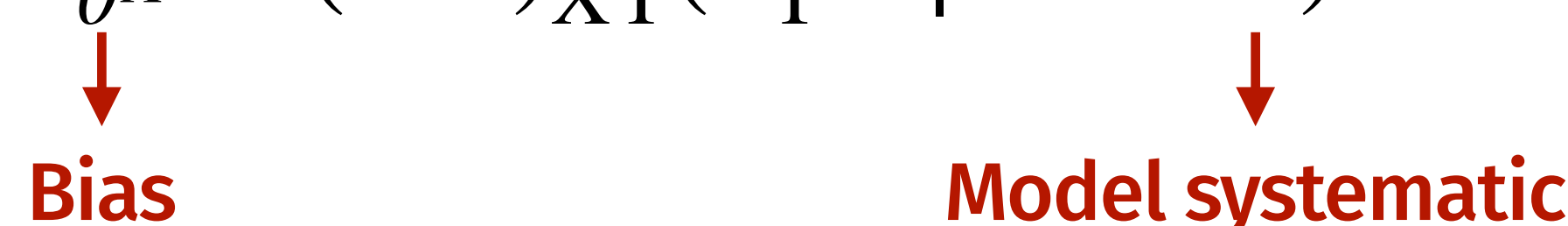
$$S_{\text{GF}} = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) = (\theta^{\text{b}})^{\text{T}} (\Gamma^{-1})_{\text{bb}} (\theta^{\text{b}}) = (\theta^{\text{b}} / \sigma_{\theta^{\text{b}}})^2 \gg \text{few}$$

$$\Gamma_{\text{XY}} = (\partial_X m \mid \partial_Y m)$$



Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$
$$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$$



Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) + N_{\text{b}}$$



Beyond-vacuum GR subspace dimensionality

$$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$$

 **Bias**  **Model systematic**

Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) + N_{\text{b}}$$



Beyond-vacuum GR subspace dimensionality

In post-processing

$$\langle S_{\text{PP}} \rangle = \langle (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}})^{\text{T}} \Gamma_{\text{b|v}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}}) \rangle$$

$$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$$

 **Bias**  **Model systematic**

Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) + N_{\text{b}}$$

↓
Beyond-vacuum GR subspace dimensionality

$$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$$

↓
Bias

↓
Model systematic

In post-processing

$$\langle S_{\text{PP}} \rangle = \langle (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}})^{\text{T}} \Gamma_{\text{b|v}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}}) \rangle$$

1. Global fit under vacuum models biases θ^{v} and θ^{c} . Compute $b_{\theta^{\text{c}}}$ with δm expanded around $\theta_{\text{bf}} = (\theta_{\text{bf}}^{\text{v}}, 0)$

$$b_{\theta^{\text{c}}} = (\Gamma_{\text{c|v}})^{-1} \Gamma_{\text{cb|v}} \theta_0^{\text{b}}$$

Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) + N_{\text{b}}$$

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Beyond-vacuum GR subspace dimensionality

$$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$$

↓
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↓
Model systematic

In post-processing

$$\langle S_{\text{PP}} \rangle = \langle (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}})^{\text{T}} \Gamma_{\text{b|v}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}}) \rangle$$

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$$b_{\theta^{\text{c}}} = (\Gamma_{\text{c|v}})^{-1} \Gamma_{\text{cb|v}} \theta_0^{\text{b}}$$

2. Subtract $h_{\text{c}}(\theta_0^{\text{c}} + b_{\theta^{\text{c}}})$ from data d and compute $b_{\theta^{\text{b}}}^{\text{PP}}$ on the residual from the 'imperfect' subtraction

Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = (\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}}) + N_{\text{b}}$$

Beyond-vacuum GR subspace dimensionality

$$b_{\theta^X} = (\Gamma^{-1})_{XY}(\partial_Y m \mid \overbrace{n + \delta m}^{\text{Residual}})$$

$\downarrow$
Bias
 $\downarrow$
Model systematic

In post-processing

$$\langle S_{\text{PP}} \rangle = \langle (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}})^{\text{T}} \Gamma_{\text{b}|\text{v}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}}) \rangle = \boxed{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b}|\text{cv}} \Gamma_{\text{b}|\text{v}}^{-1} \Gamma_{\text{b}|\text{cv}} (\theta^{\text{b}})} + N_{\text{b}}$$

1. **Global fit under vacuum models biases θ^v and θ^c . Compute b_{θ^c} with δm expanded around $\theta_{\text{bf}} = (\theta_{\text{bf}}^v, 0)$**

$$b_{\theta^c} = (\Gamma_{\mathbf{c}|\mathbf{v}})^{-1} \Gamma_{\mathbf{cb}|\mathbf{v}} \theta_0^b$$

2. Subtract $h_c(\theta_0^c + b_{\theta^c})$ from data d and compute $b_{\theta^b}^{\text{PP}}$ on the residual from the ‘imperfect’ subtraction

Detection Statistic with noise and nuisance sources

$$S_{\text{GF}} = (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{GF}}) \gg \text{few}$$

$$\langle S_{\text{GF}} \rangle = \boxed{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} (\theta^{\text{b}})} + N_{\text{b}}$$

↓
Beyond-vacuum GR subspace dimensionality

$b_{\theta^{\text{x}}} = (\Gamma^{-1})_{\text{XY}} (\partial_{\text{Y}} m | \overbrace{n + \delta m}^{\text{Residual}})$

↓
Bias

↓
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In post-processing

$$\langle S_{\text{PP}} \rangle = \langle (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}})^{\text{T}} \Gamma_{\text{b|v}} (\theta^{\text{b}} + b_{\theta^{\text{b}}}^{\text{PP}}) \rangle = \boxed{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \Gamma_{\text{b|v}}^{-1} \Gamma_{\text{b|cv}} (\theta^{\text{b}})} + N_{\text{b}}$$

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2. Subtract $h_{\text{c}}(\theta_0^{\text{c}} + b_{\theta^{\text{c}}})$ from data d and compute $b_{\theta^{\text{b}}}^{\text{PP}}$ on the residual from the 'imperfect' subtraction

The Masking factor

$$\mathcal{R} = \frac{\langle S_{\text{PP}} \rangle}{\langle S_{\text{GF}} \rangle} = \frac{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \Gamma_{\text{b|v}}^{-1} \Gamma_{\text{b|cv}} (\theta^{\text{b}})}{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \theta^{\text{b}}}$$

The ~~Masking~~ Mojito factor

$$\mathcal{R} = \frac{\langle S_{\text{PP}} \rangle}{\langle S_{\text{GF}} \rangle} = \frac{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \Gamma_{\text{b|v}}^{-1} \Gamma_{\text{b|cv}} (\theta^{\text{b}})}{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \theta^{\text{b}}}$$

The ~~Masking~~ Mojito factor

Because Fisher matrices are (semi)positive definite

$$\mathcal{R} = \frac{\langle S_{\text{PP}} \rangle}{\langle S_{\text{GF}} \rangle} = \frac{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \Gamma_{\text{b|v}}^{-1} \Gamma_{\text{b|cv}} (\theta^{\text{b}})}{(\theta^{\text{b}})^{\text{T}} \Gamma_{\text{b|cv}} \theta^{\text{b}}} \in [0, 1]$$


Postprocessing always hides extra physics

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{\text{vac}}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s(\theta_s) = (\partial_{\theta_b} m \mid m(\theta_s))$

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{vac}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s(\theta_s) = (\partial_{\theta_b} m \mid m(\theta_s))$

Computed once @ best-fit estimates

Can be saved during first inference

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{\text{vac}}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s(\theta_s) = (\partial_{\theta_b} m \mid m(\theta_s))$

Computed once @ best-fit estimates

Can be saved during first inference

Under linear-signal approximation $\text{Cov}[s] = \sum_{j,k} \Gamma_{bj} (\Gamma)_{jk}^{-1} \Gamma_{kb} \text{ w/ } j, k = c, v$

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{\text{vac}}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s(\theta_s) = (\partial_{\theta_b} m \mid m(\theta_s))$

Computed once @ best-fit estimates

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Under linear-signal approximation $\text{Cov}[s] = \sum_{j,k} \Gamma_{bj} (\Gamma)_{jk}^{-1} \Gamma_{kb}$ w/ $j, k = c, v$

$$\Gamma_{b|cv} = \Gamma_{bb} - \text{Cov}[s]$$

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{\text{vac}}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s(\theta_s) = (\partial_{\theta_b} m \mid m(\theta_s))$

Computed once @ best-fit estimates

Can be saved during first inference

Under linear-signal approximation $\text{Cov}[s] = \sum_{j,k} \Gamma_{bj} (\Gamma)_{jk}^{-1} \Gamma_{kb} \text{ w/ } j, k = c, v$

$\Gamma_{b|cv} = \Gamma_{bb} - \text{Cov}[s]$ **Suffers from Monte-Carlo errors**

How to compute $\mathcal{R}$ (re)using a global fit posterior

$$\mathcal{R} = \frac{(\theta^b)^T \Gamma_{b|cv} \Gamma_{b|v}^{-1} \Gamma_{b|cv} (\theta^b)}{(\theta^b)^T \Gamma_{b|cv} \theta^b}$$

$\{\theta_s\} \sim p(\theta_c, \theta_v \mid d, M_{vac}) \longrightarrow$ **Global Fit posterior**

For each sample compute $s_{\perp}(\theta_s) = (P_{\perp}^v \partial_{\theta_b} m \mid m(\theta_s))$

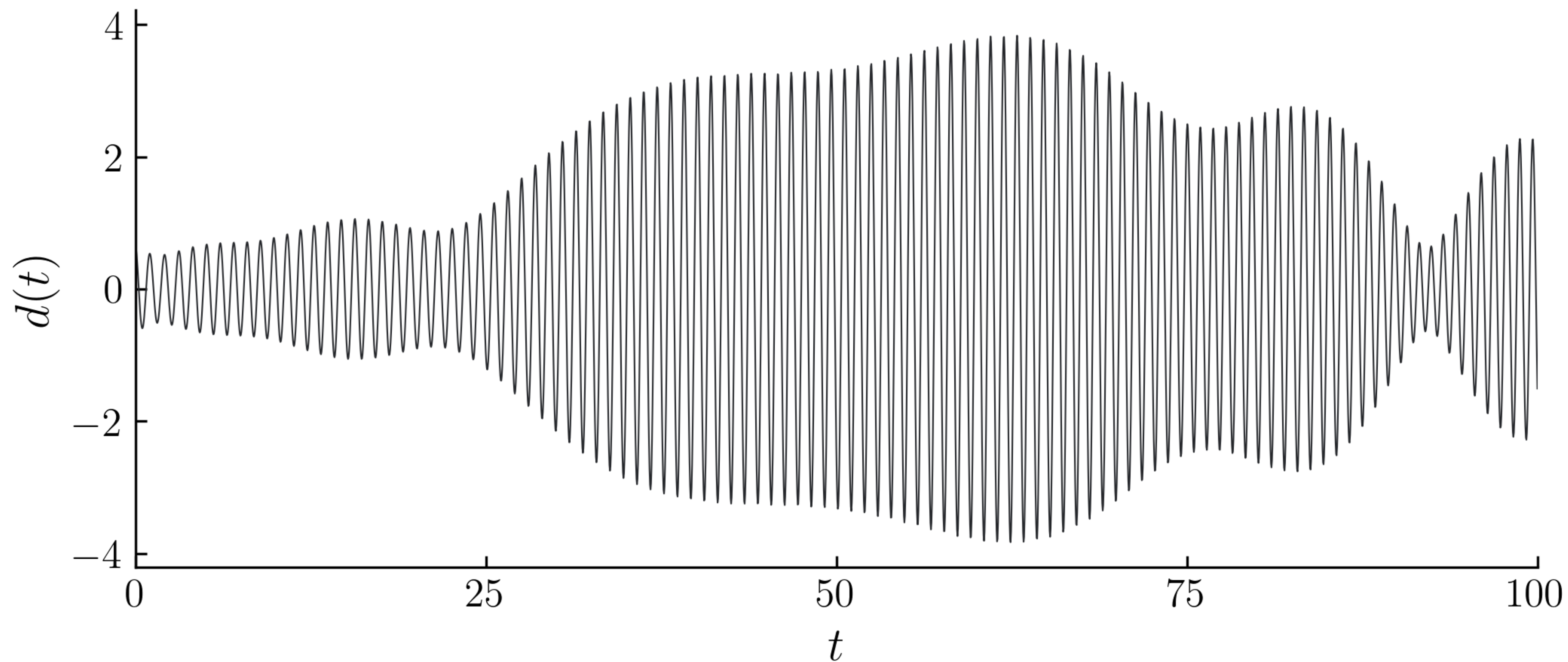
Computed once @ best-fit estimates

Can be saved during first inference

Under linear-signal approximation

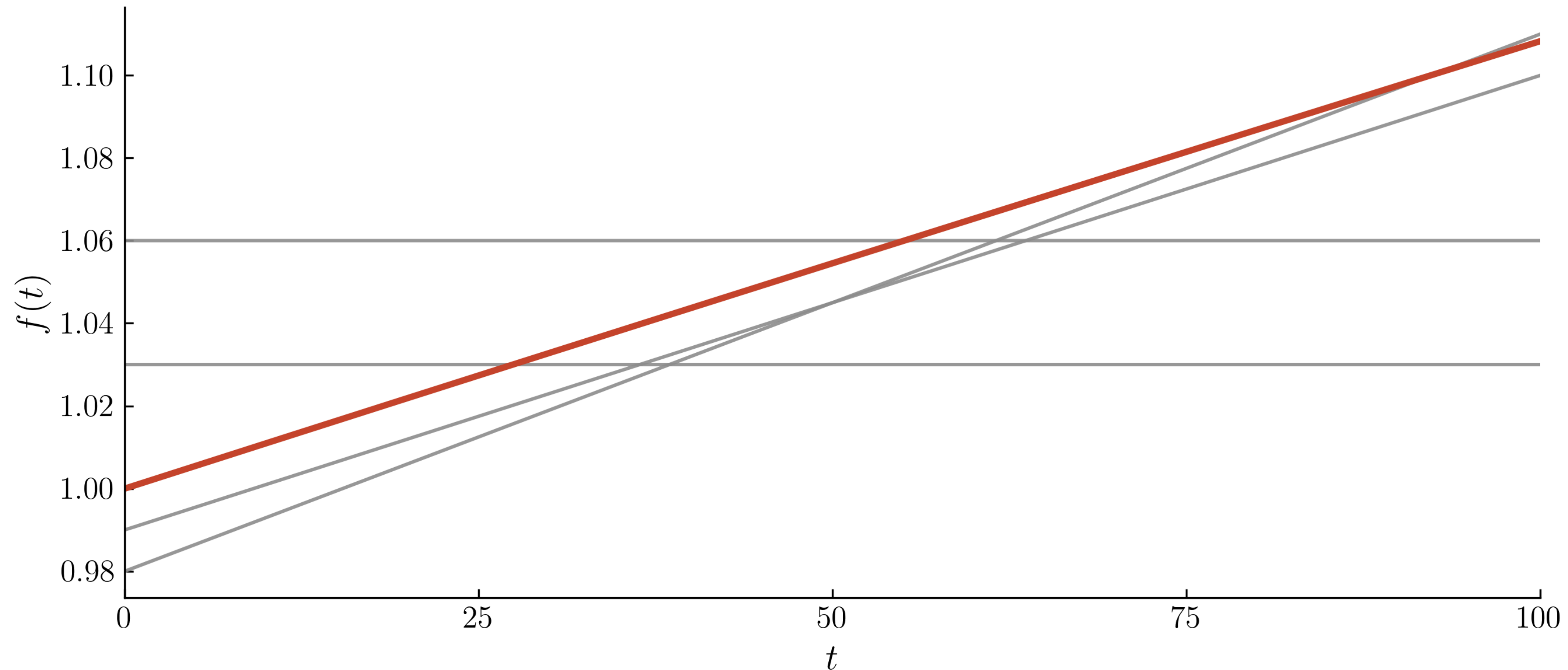
$$\Gamma_{b|cv} = \Gamma_{b|v} - \text{Cov}[s_{\perp}]$$

Moshito: my s***** common dataset



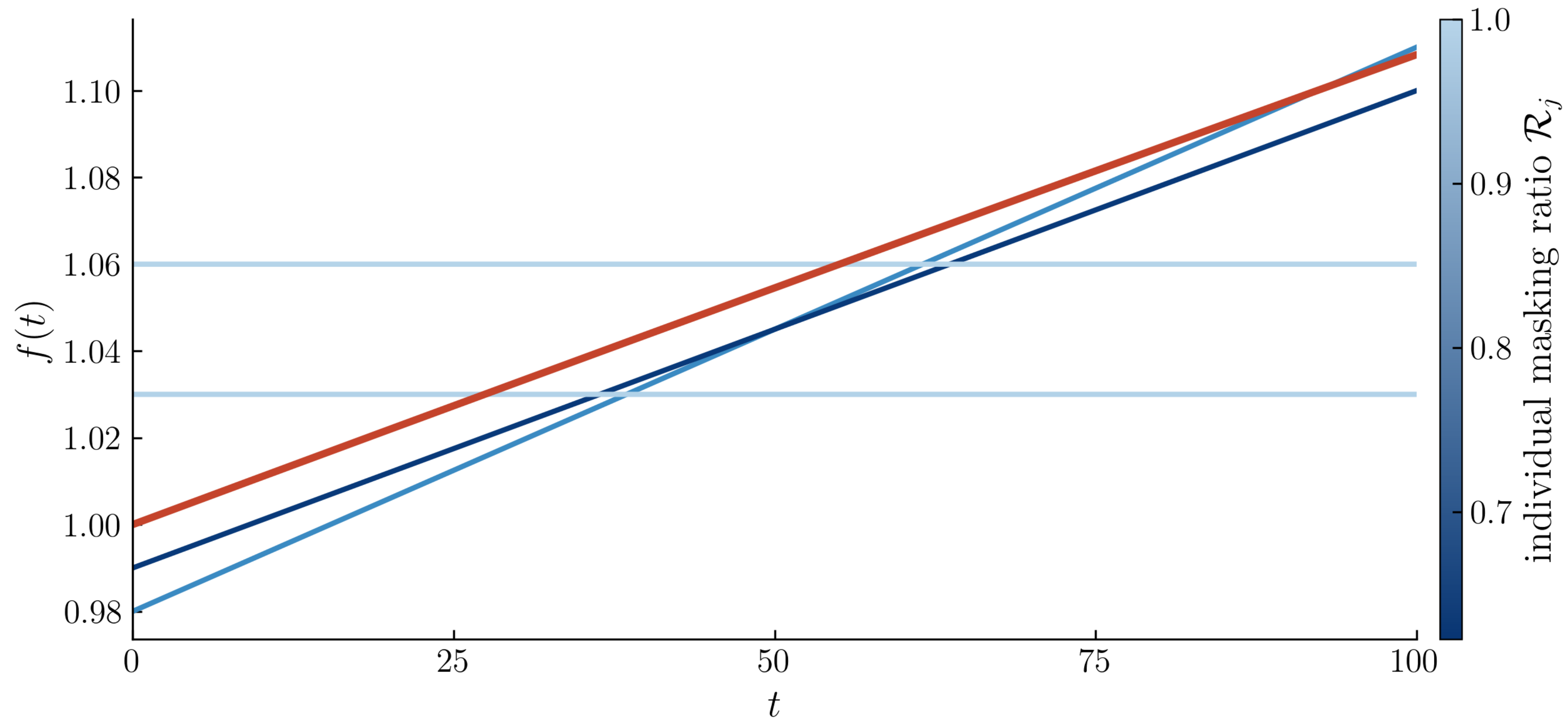
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$$h(t) = A \sin [2\pi f(t) t + \varphi_0] \quad \frac{df}{dt} = \dot{f}_0 \left[1 + \varepsilon \left(\frac{f}{f_0} \right)^{n_r} \right] \quad \begin{array}{l} \varepsilon = 0 \\ \varepsilon \neq 0 \end{array}$$

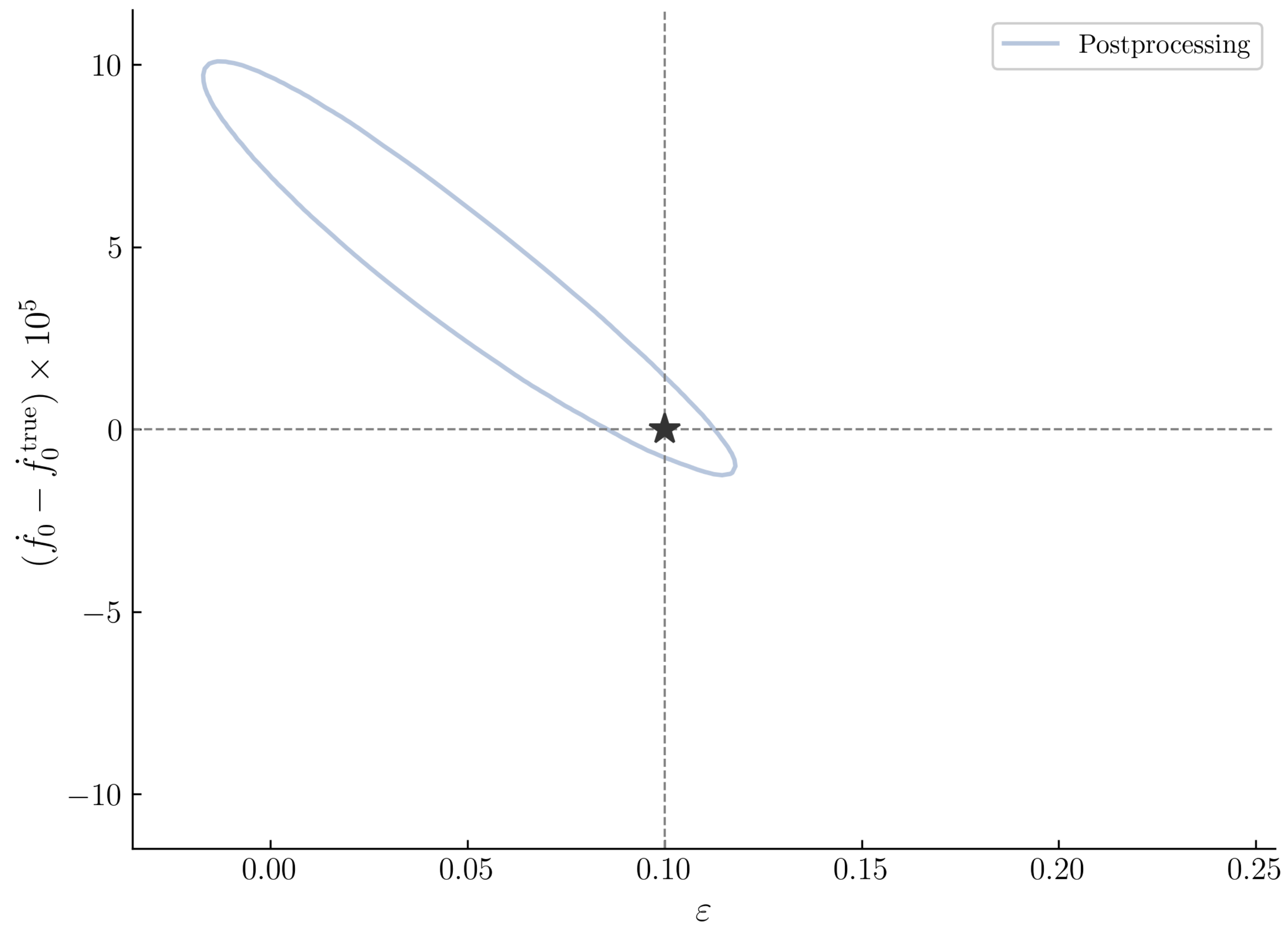


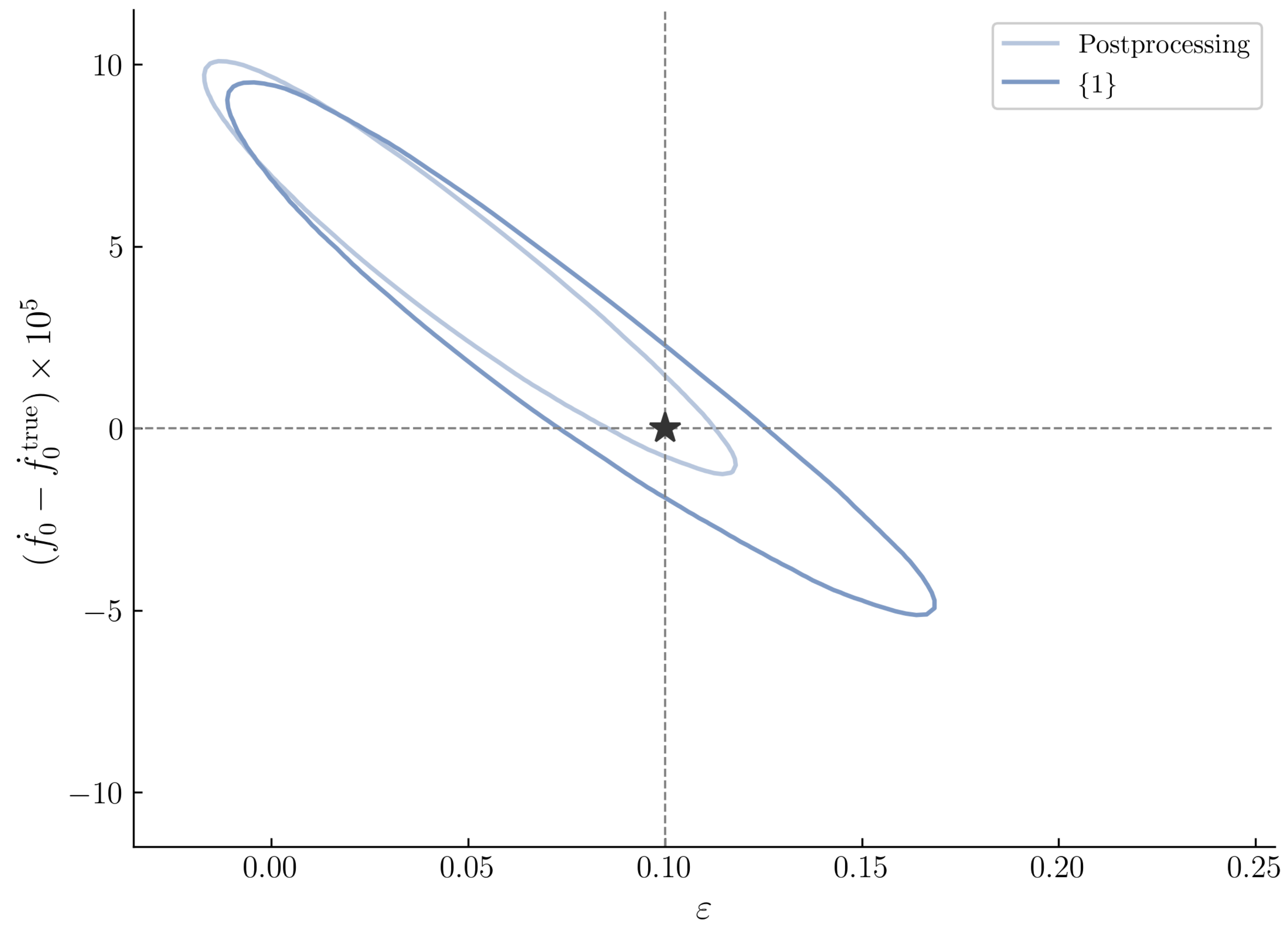
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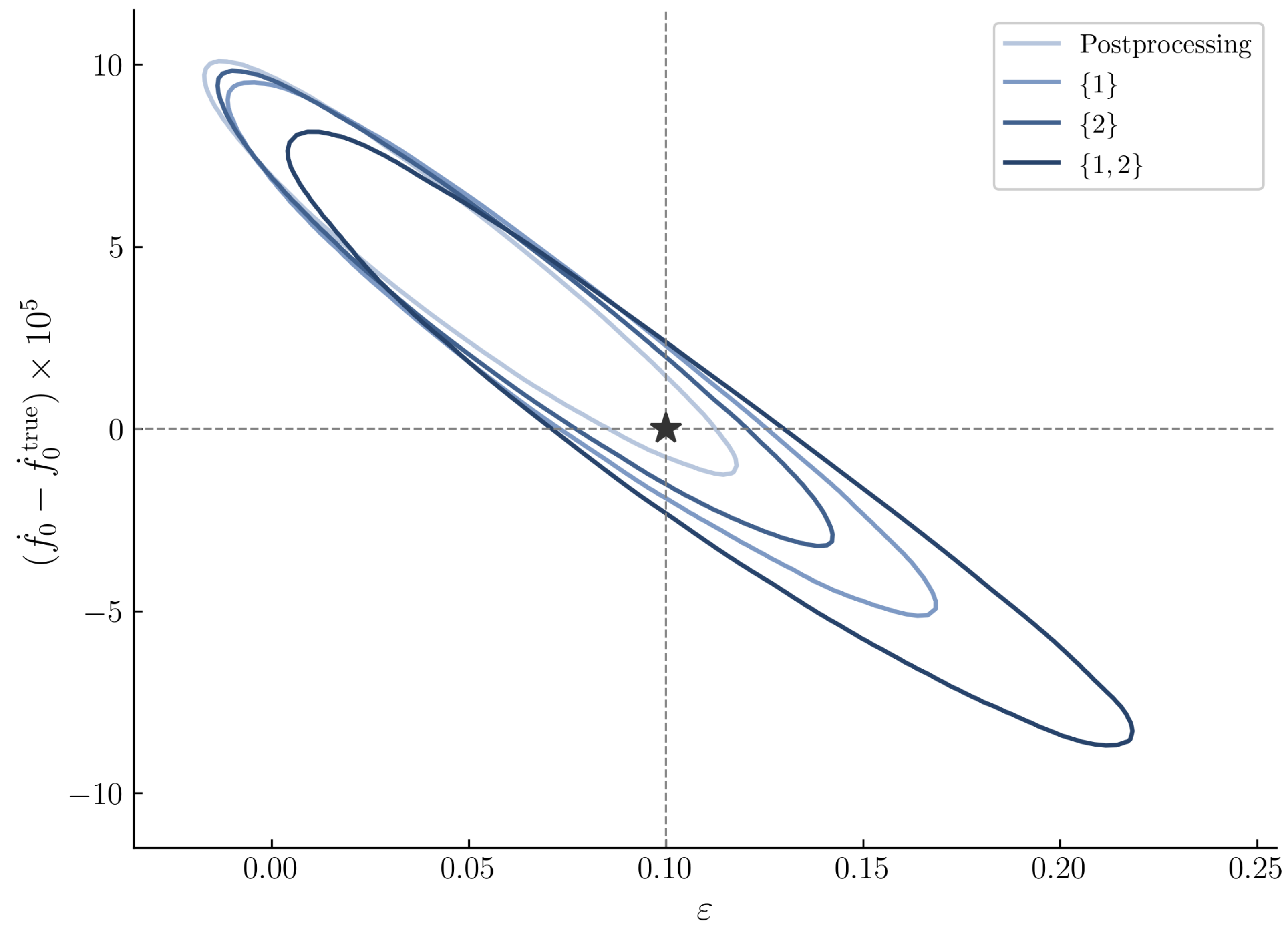
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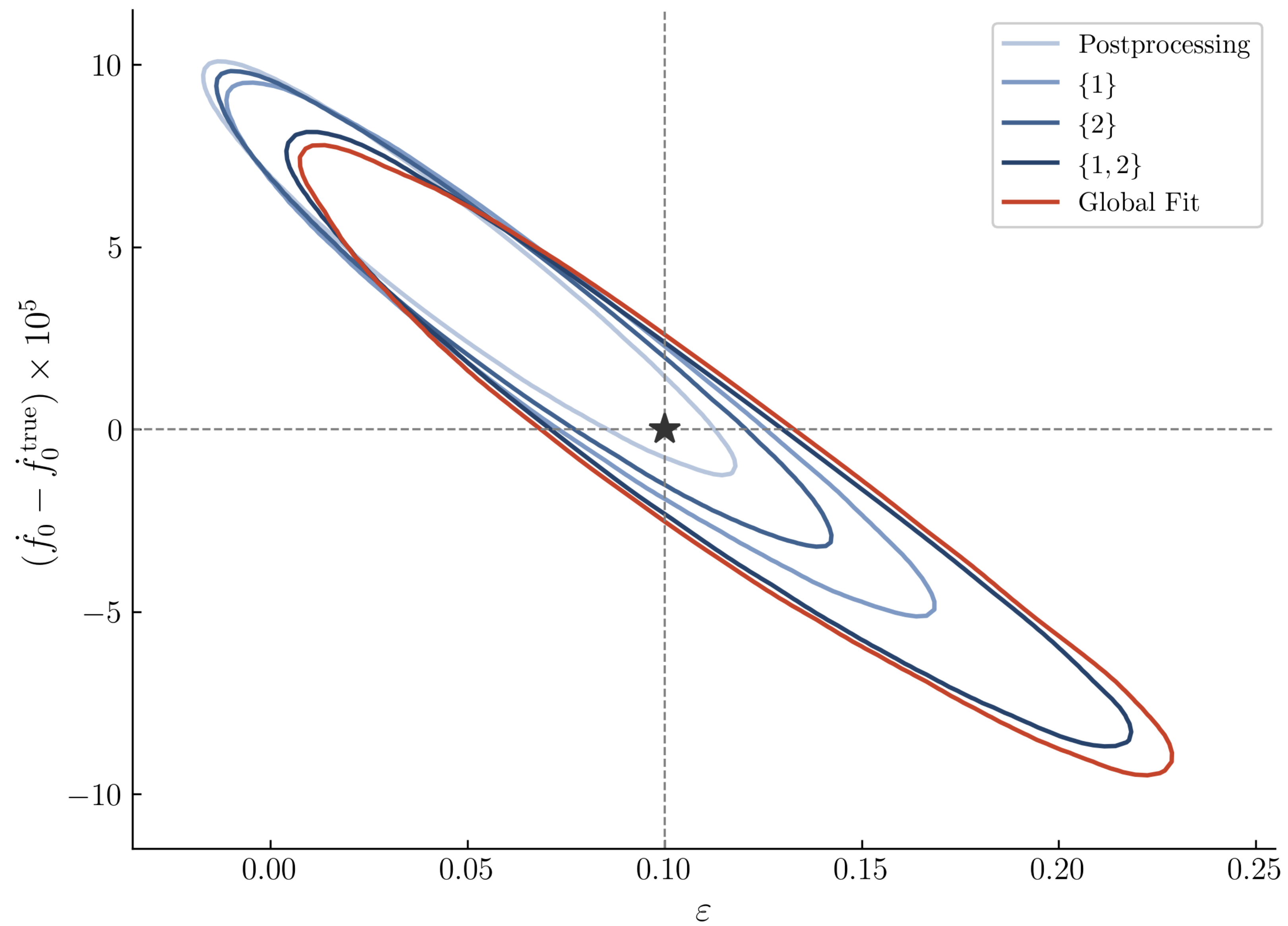


j	$\mathcal{R}_j$	S	$\mathcal{R}_S$ (Fisher)	$\mathcal{R}_S$ (samples)
1	0.632	$\{1\}$	0.632	0.672
2	0.803	$\{1, 2\}$	0.477	0.463
3	0.992	$\{1, 2, 3\}$	0.467	0.452
4	0.999	$\{1, 2, 3, 4\}$	0.465	0.449









Meditations and possible directions

1. Is this useful/interesting as it is?

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$$\langle S_{\text{GF}} \rangle = \frac{1}{2} \left[(\theta_0^{\text{b}})^T \Gamma_{\text{b|cv}} \theta_0^{\text{b}} + N_{\text{b}} \right] + (\theta_0^{\text{b}})^T Z_{\delta m}^{\text{GF}} + \frac{1}{2} (Z_{\delta m}^{\text{GF}})^T \Gamma_{\text{b|cv}}^{-1} Z_{\delta m}^{\text{GF}}$$

$$Z_{\delta m}^{\text{GF}} = \left(\partial_{\theta^{\text{b}}} h \mid \delta m \right) - \left(\Gamma_{\text{bc}} \quad \Gamma_{\text{bv}} \right) \begin{pmatrix} \Gamma_{\text{cc}} & \Gamma_{\text{cv}} \\ \Gamma_{\text{vc}} & \Gamma_{\text{vv}} \end{pmatrix}^{-1} \begin{pmatrix} \left(\partial_{\theta^{\text{c}}} m \mid \delta m \right) \\ \left(\partial_{\theta^{\text{v}}} m \mid \delta m \right) \end{pmatrix}$$

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false positive term

$$\langle S_{\text{GF}} \rangle = \frac{1}{2} \left[(\theta_0^{\text{b}})^T \Gamma_{\text{b|cv}} \theta_0^{\text{b}} + N_{\text{b}} \right] + (\theta_0^{\text{b}})^T Z_{\delta m}^{\text{GF}} + \boxed{\frac{1}{2} (Z_{\delta m}^{\text{GF}})^T \Gamma_{\text{b|cv}}^{-1} Z_{\delta m}^{\text{GF}}}$$

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Problem

Many signals in the data



Solution (?)

Down-scale global fit in follow-up tests of vacuum GR?

Duque & Gair, in prep

Many possible models



1. Use *catch-all* agnostic models
2. Map them to physical models

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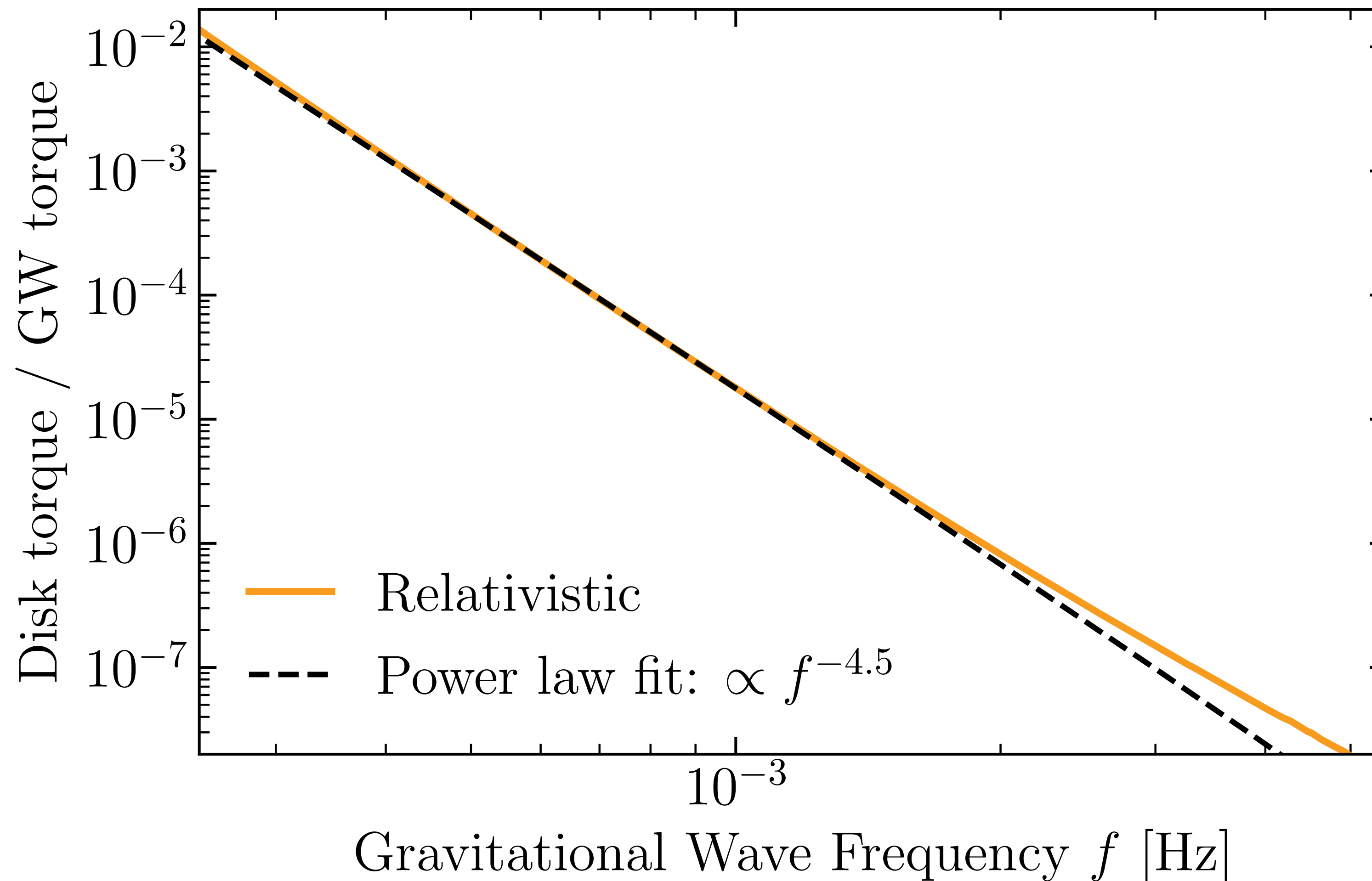
Santos, Duque, Toubiana & Gair, in prep

parametric tests of General Relativity

$$h = A_{\text{GW}}(1 + \delta A)e^{i\Phi_{\text{GW}}(1 + \delta\Phi_{\text{GW}})}$$

$$\dot{E} = \dot{E}_{\text{GW}} (1 + \delta\dot{E})$$

$$\delta\dot{E} = A f^{-2n_r/3}$$



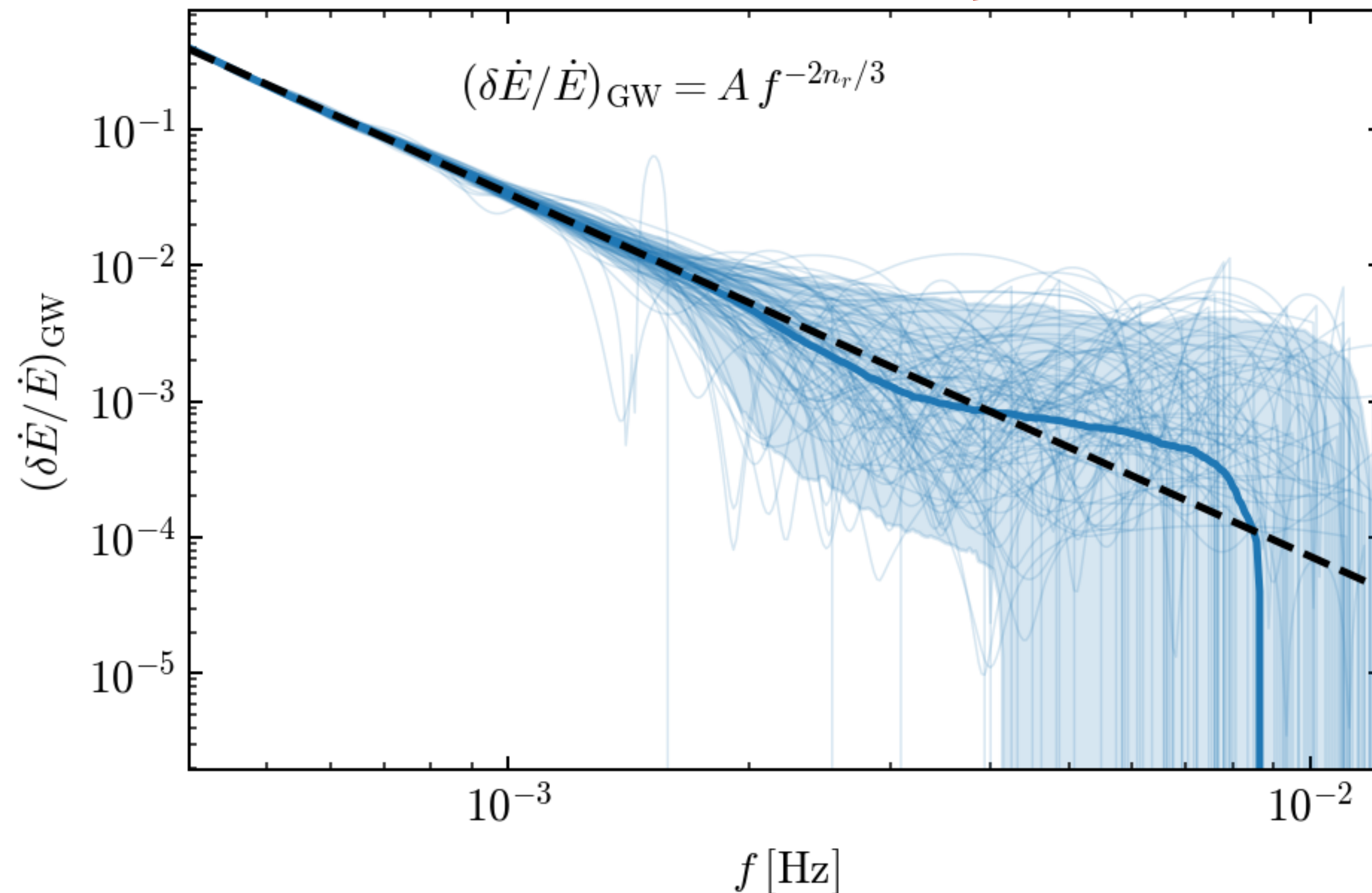
Non-parametric tests of General Relativity

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$$\delta\dot{E} = \text{Splines}$$

1 flexible envelope



System parameters

$$p(\theta_\gamma, \gamma | d)$$

Spline parameters

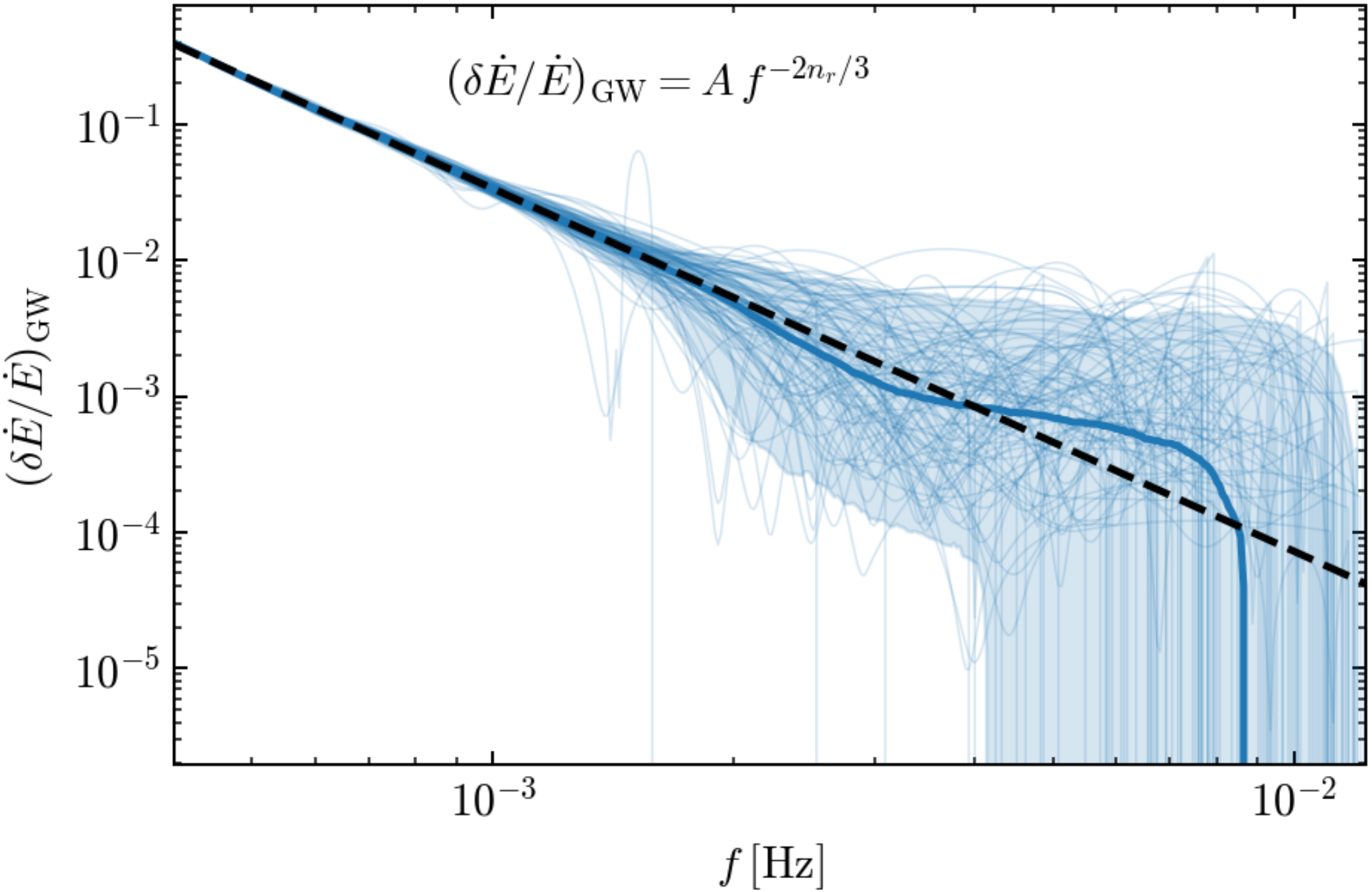
$$\gamma = (\bar{B}(\gamma), \bar{\mathcal{O}}(\gamma))$$

Knots locations

“Observable”
e.g. $\delta\dot{E}$

Non-parametric tests of General Relativity

1 flexible envelope $p(\theta_\gamma, \gamma | d)$



Many physical models $p(\theta_\lambda, \lambda, \xi | d)$

System params

Beyond-vacuum params

Remapping params

Non-parametric tests of General Relativity

Flexible-envelope likelihood

Remapping kernel ('loss function')

$$p(d | \lambda, \theta_\lambda, \xi) = \int \overbrace{p(d | \theta_\gamma, \gamma)}^{\text{Flexible-envelope likelihood}} \overbrace{p(\theta_\gamma, \gamma | \lambda, \theta_\lambda, \xi)}^{\text{Remapping kernel ('loss function')}} d\theta_\gamma d\gamma$$

1 flexible envelope $p(\theta_\gamma, \gamma | d)$



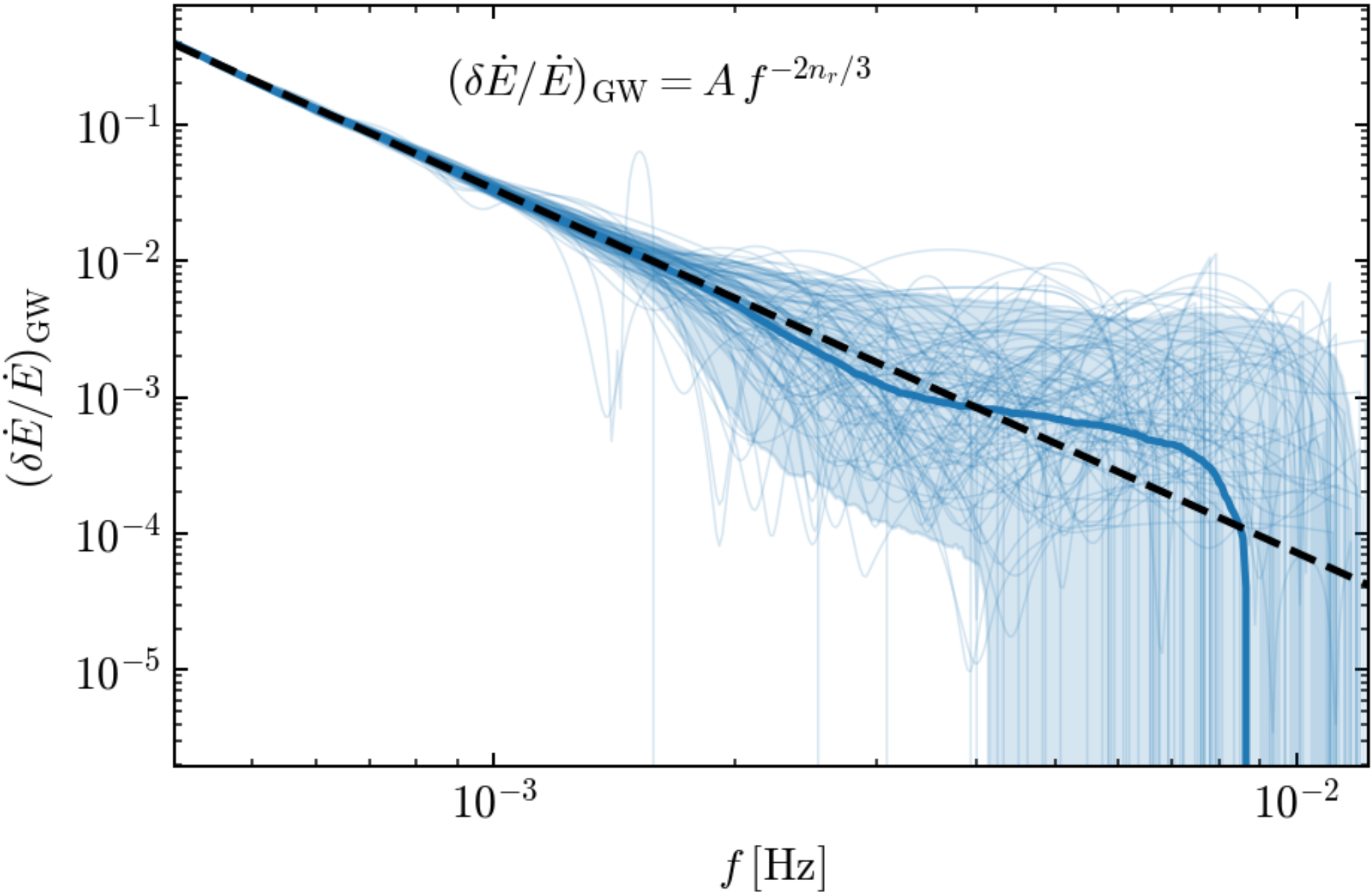
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System params



Beyond-vacuum params

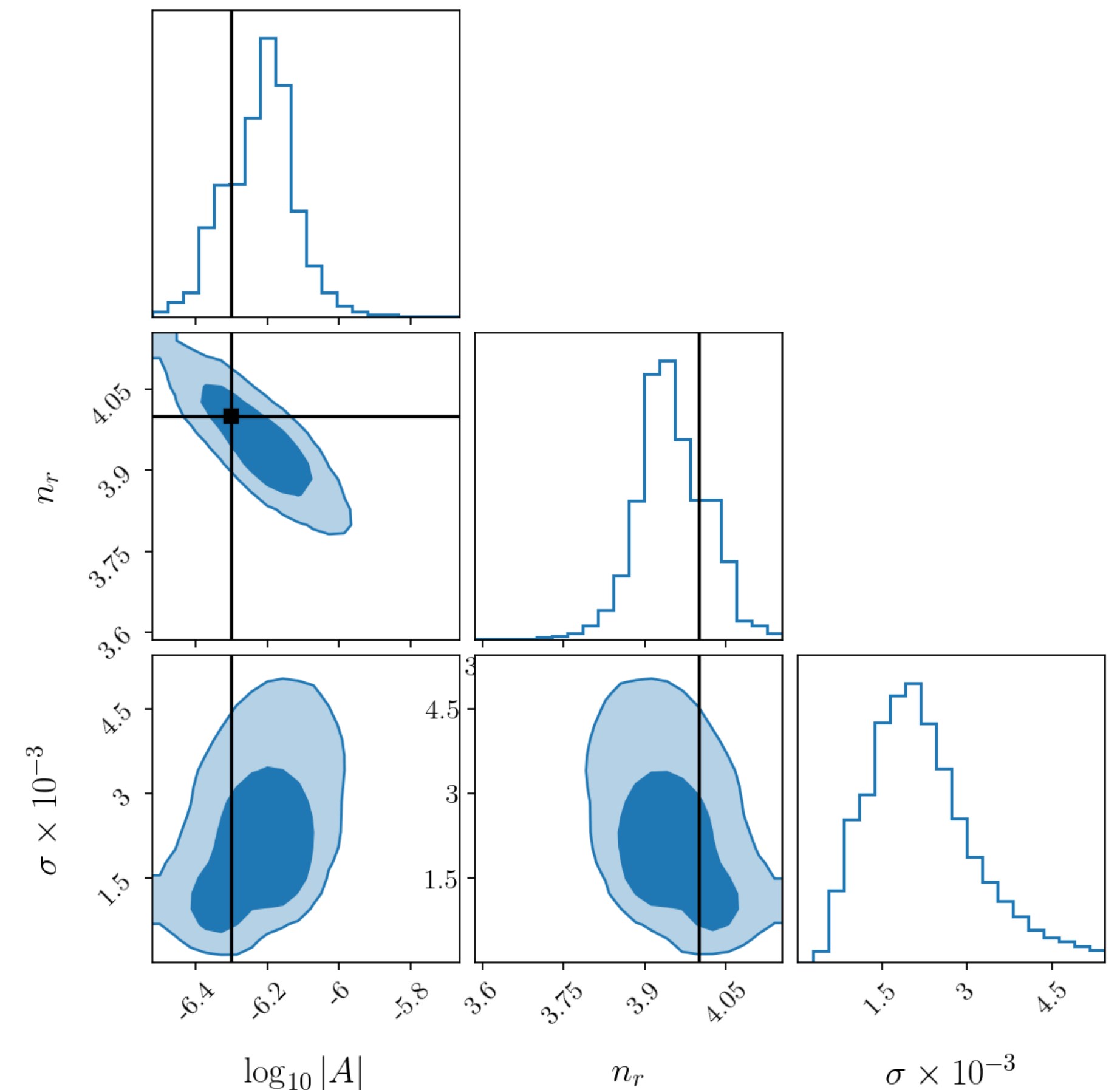
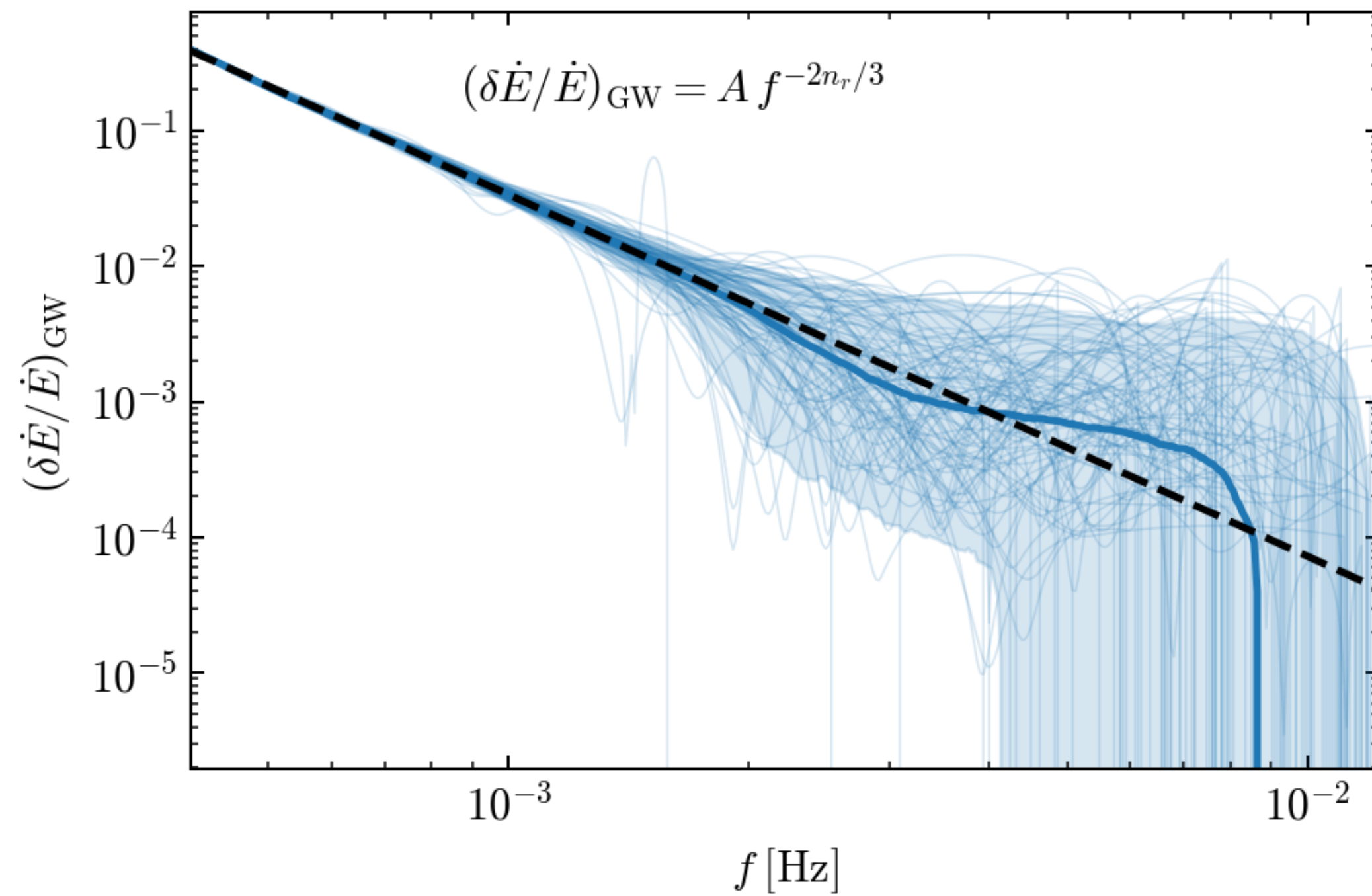
Remapping params



Non-parametric tests of General Relativity

E.g. $p(\theta_\gamma, \gamma | \lambda, \theta_\lambda, \xi) \sim \exp \left[- \left(\mathcal{O}(\theta_\lambda, \lambda) - \mathcal{O}(\gamma) \right)^2 / (2\sigma_\xi^2) \right]$

↓
‘Goodness of fit’



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Machine Learning approach to residual analysis

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5. Non-parametric analysis of individual events → population analysis