

EMRI Search and Inference

within the LISA Global Fit

July 16, 2026 · Singapore



Probing vector hair of black holes with EMRIs

Kavli Institute for Astronomy and Astrophysics

Speaker: Lijing Shao (邵立晶)

Singapore 2026

Talk summary

1. Found all spherically symmetric BH solutions

Static spherical vacuum solutions in the bumblebee gravity model

#1

Rui Xu (Peking U., Beijing, KIAA), Dicong Liang (Peking U., Beijing, KIAA), Lijing Shao (Peking U., Beijing, KIAA and NAOC, Beijing) (Sep 6, 2022)

Published in: *Phys.Rev.D* 107 (2023) 2, 024011 • e-Print: [2209.02209](#) [gr-qc]

 pdf  DOI  cite  claim  reference search  104 citations

Probing vector hair of black holes with extreme-mass-ratio inspirals

#2

Dicong Liang (Peking U., Beijing, KIAA), Rui Xu (Peking U., Beijing, KIAA), Zhan-Feng Mai (Peking U., Beijing, KIAA), Lijing Shao (Peking U., Beijing, KIAA and NAOC, Beijing and Nankai, CIM) (Dec 19, 2022)

Published in: *Phys.Rev.D* 107 (2023) 4, 044053 • e-Print: [2212.09346](#) [gr-qc]

 pdf  DOI  cite  claim  reference search  100 citations



2. Limits from EMRIs, $\sim 10^3$ better than EHT images

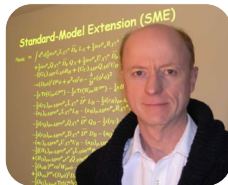
Xu, Liang, Shao [[PRD:2209.02209](#)]; Liang, Xu, Mai, Shao [[PRD:2212.09346](#)]

Bumblebee gravity model

$$S = \int \sqrt{-g} d^4x \left[\frac{1}{2\kappa} (R + \xi B^\mu B^\nu R_{\mu\nu}) - \frac{\alpha}{4} B^{\mu\nu} B_{\mu\nu} + \frac{\beta}{2} (D_\mu B_\nu) (D^\mu B^\nu) - V(B^\mu) \right] + S_m$$

■ As an illustrative example in **Standard Model Extension**

- **Gravity-bumblebee nonminimal coupling**
- **Lorentz-violating potential** $V(B^\mu) = V(B^\mu B_\mu \pm b^2)$
- **Stückelberg ghost** in Minkowski spacetime $\Rightarrow$ we set $\beta = 0$



Nambu 1968; Will & Nordtvedt 1972; Hellings & Nordtvedt 1973; Kostelecký & Samuel 1989

Jacobson & Mattingly [[PRD:gr-qc/0007031](#)]; Kostelecký [[PRD:hep-th/0312310](#)]; Bluhm & Kostelecký [[PRD:hep-th/0412320](#)]; Bailey & Kostelecký [[PRD:gr-qc/0603030](#)]

Bumblebee gravity model

Field Equations

$$G_{\mu\nu} = \kappa(T_m)_{\mu\nu} + \kappa(T_B)_{\mu\nu}$$

$$D^\mu B_{\mu\nu} - 2B_\nu \frac{dV}{d(B^\lambda B_\lambda)} + \frac{\xi}{\kappa} B^\mu R_{\mu\nu} = 0$$

where

$$(T_B)_{\mu\nu} = \frac{\xi}{2\kappa} \left[g_{\mu\nu} B^\alpha B^\beta R_{\alpha\beta} - 2B_\mu B_\lambda R_\nu{}^\lambda - 2B_\nu B_\lambda R_\mu{}^\lambda - \square_g (B_\mu B_\nu) - g_{\mu\nu} D_\alpha D_\beta (B^\alpha B^\beta) + D_\kappa D_\mu (B^\kappa B_\nu) + D_\kappa D_\nu (B_\mu B^\kappa) \right]$$

$$+ B_{\mu\lambda} B_\nu{}^\lambda - g_{\mu\nu} \left(\frac{1}{4} B^{\alpha\beta} B_{\alpha\beta} + V \right) + 2B_\mu B_\nu \frac{dV}{d(B^\lambda B_\lambda)}$$

Kostelecký [PRD:hep-th/0312310]; Bailey & Kostelecký [PRD:gr-qc/0603030]; Xu, Liang, Shao [PRD:2209.02209]

Simple bumblebee BHs

$$S = \int \sqrt{-g} d^4x \left[\frac{1}{2\kappa} (R + \xi B^\mu B^\nu R_{\mu\nu}) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V(B^\mu) \right] + S_m$$

- Casana *et al.* (2018) found an exact Schwarzschild-like solution in bumblebee gravity model

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + (1 + \ell) \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\Omega^2$$

where $\ell = \xi b^2$ with b^μ having only a nonvanishing *radial* component

Casana, Cavalcante, Poulis, Santos [PRD:1711.02273]

Extending bumblebee BHs

$$S = \int \sqrt{-g} d^4x \left[\frac{1}{2\kappa} (R + \xi B^\mu B^\nu R_{\mu\nu}) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V(B^\mu) \right] + S_m$$

■ Ansatz

$$ds^2 = -e^{2\nu} dt^2 + e^{2\mu} dr^2 + r^2 d\Omega^2$$

$$b_\lambda = (b_t, b_r, 0, 0)$$

where μ , ν , b_t , and b_r are functions of r

Xu, Liang, Shao [[PRD:2209.02209](#)]

Extended analytical solution

$$\nu = \frac{1}{2} \ln \left(1 - \frac{2M}{r} \right), \quad \mu = \mu_0 - \frac{1}{2} \ln \left(1 - \frac{2M}{r} \right), \quad b_t = \lambda_0 + \frac{\lambda_1}{r}$$

$$b_r^2 = e^{2\mu_0} \left[\frac{1}{\xi} \frac{(e^{2\mu_0} - 1) r}{r - 2M} - \frac{\kappa \lambda_1^2}{3\xi M(r - 2M)} + \frac{\lambda_1^2(2r - M) + 6\lambda_0\lambda_1 Mr + 6\lambda_0^2 M^2 r}{3M(r - 2M)^2} \right]$$

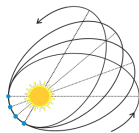
- **Four integral constants:** $M, \mu_0, \lambda_0, \lambda_1$
- $\lambda_0 = \lambda_1 = 0 \Rightarrow$ Casana et al. [[PRD:1711.02273](#)]
- $\mu_0 = 0$: Schwarzschild with a nontrivial $b_\lambda \Rightarrow$ Fan [[EPJC:1709.04392](#)]
- $\mu_0 = M = 0$: Minkowski metric, with a nontrivial b_λ

Xu [[CPT:2301.12666](#)]; Xu, Liang, Shao [[PRD:2209.02209](#)]

Bumblebee BHs

$$\nu = \sum_{n=1}^{\infty} \frac{\nu_n}{r^n}, \quad \mu = \sum_{n=0}^{\infty} \frac{\mu_n}{r^n}, \quad b_t = \sum_{n=0}^{\infty} \frac{\lambda_n}{r^n}$$

- Field equations give recurrence relations of ν_n , μ_n , and λ_n
- **Two families of horizon solutions** (g_{rr} diverging at r_h)
 - 1 a vanishing radial component of the vector field ($b_r = 0$)
 $\Rightarrow$ **3** free parameters $\{\mu_1, \lambda_0, \lambda_1\}$
 - 2 a vanishing radial component of the Ricci tensor ($R_{rr} = 0$)
 $\Rightarrow$ **5** free parameters $\{\mu_0, \mu_1, \mu_2, \lambda_0, \lambda_1\}$



**Perihelion advances
in Solar System rule
out $R_{rr} = 0$ family**

Xu, Liang, Shao [[PRD:2209.02209](#)]; Yang, Mai, Liang, Shao [[arXiv:2606.05801](#)]

Conserved quantities

Komar Mass

Conserved current $J_M^\mu \equiv K_\nu R^{\mu\nu} = D_\nu D^\mu K^\nu$, with $K^\mu = (1, 0, 0, 0)$

$$M_K \equiv -\frac{1}{4\pi} \int d^3x \sqrt{-g} J_M^t = e^{-\mu_0} \mu_1$$

Bumblebee Charge

Conserved current $J_Q^\mu \equiv \xi b_\nu R^{\mu\nu} / \kappa = -D_\nu b^{\nu\mu}$

$$Q \equiv -\frac{1}{4\pi} \sqrt{\frac{\kappa}{2}} \int d^3x \sqrt{-g} J_Q^t = \sqrt{\frac{\kappa}{2}} e^{-\mu_0} \lambda_1$$

Xu, Liang, Shao [[PRD:2209.02209](#)]

Horizon r_h in family $b_r = 0$

Family $b_r = 0$

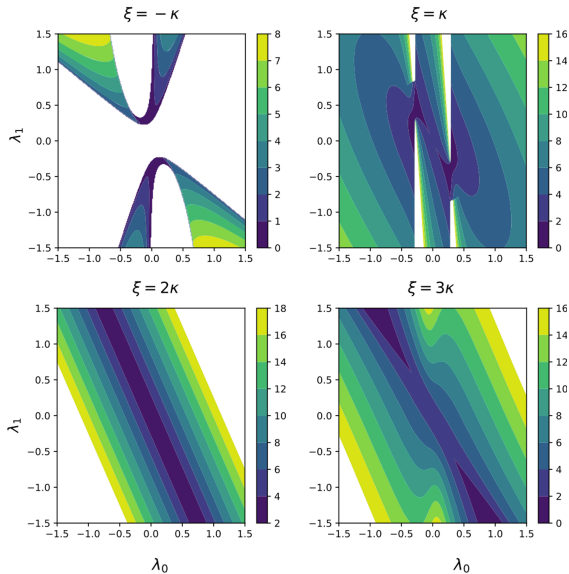
Parameters: $\mu_1, \lambda_0, \lambda_1$

When $\xi = 2\kappa$,

$\lambda_1 = -2\lambda_0$ gives

Schwarzschild metric

Xu, Liang, Shao [PRD:2209.02209]



r_h in unit of $\mu_1 = \text{ADM mass} = \text{Komar mass}$ in family $b_r = 0$

BHs in family $b_r = 0$

Family $b_r = 0$

Parameters: $\mu_1, \lambda_0, \lambda_1$

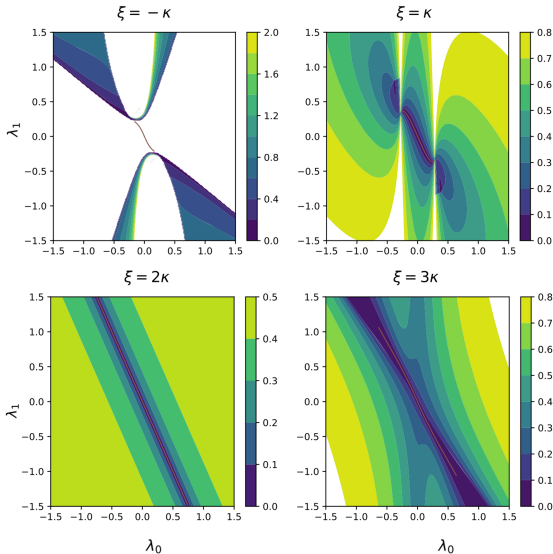
Demanding $g_{tt}|_{r_h} = 0$
gives BH solutions

(brown curves in λ_1 - λ_0 plane)

Mass $M_K = \mu_1$

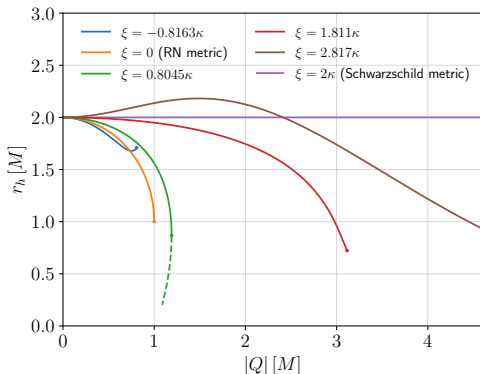
Charge $Q = \sqrt{\kappa/2}\lambda_1$

Xu, Liang, Shao [[PRD:2209.02209](#)]



r_h in unit of $\mu_1 = \text{ADM mass} = \text{Komar mass}$ in family $b_r = 0$

BHs in family $b_r = 0$



$\xi < 0: Q^{(\max)} < M$

$\xi = 0$: Reissner-Nordström

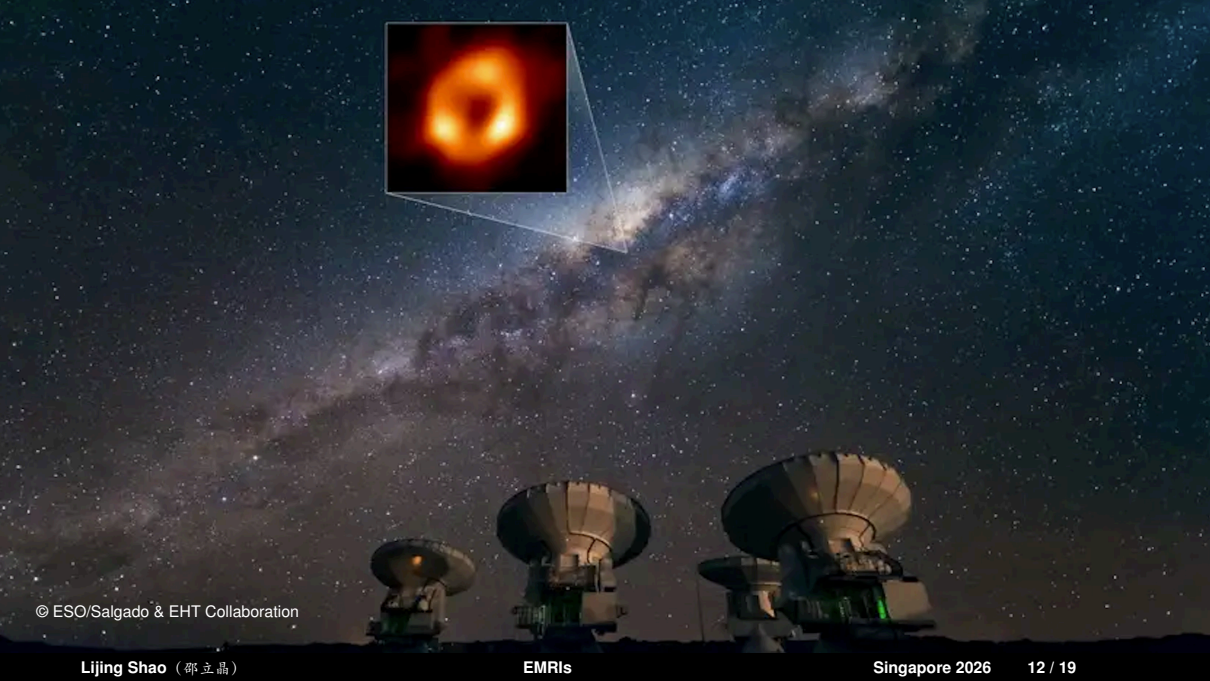
$\xi > 0: Q^{(\max)} > M$

$\xi < 2\kappa: r_h^{(\max)} = 2M$

$\xi = 2\kappa$: stealth Schwarzschild

$\xi > 2\kappa: r_h^{(\max)} > 2M$

Xu, Liang, Shao [PRD:2209.02209]; Xu, Liang, Shao [ApJ:2302.05671]



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BH Shadows: family $b_r = 0$

- Angular diameter of a **BH shadow**

$$d = 2\theta_g (\sigma_{\text{LR}}/M)$$

where σ_{LR} is the **critical impact parameter** for photons, and $\theta_g \equiv M/D$ with D the distance to BH

- **Event Horizon Telescope** results

M87* (EHT 2019)

Akiyama et al. [[ApJL:1906.11238](#)]

$$d = 42 \pm 3 \mu\text{as}$$

$$\theta_g = 3.62 \pm 0.60 \mu\text{as}$$

Sgr A* (EHT 2022)

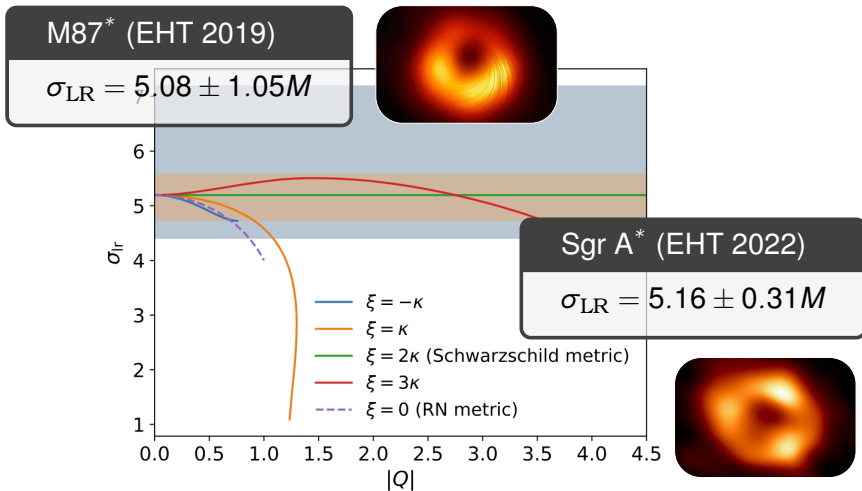
Akiyama et al. [[ApJL:2311.08680](#)]

$$d = 51.8 \pm 2.3 \mu\text{as}$$

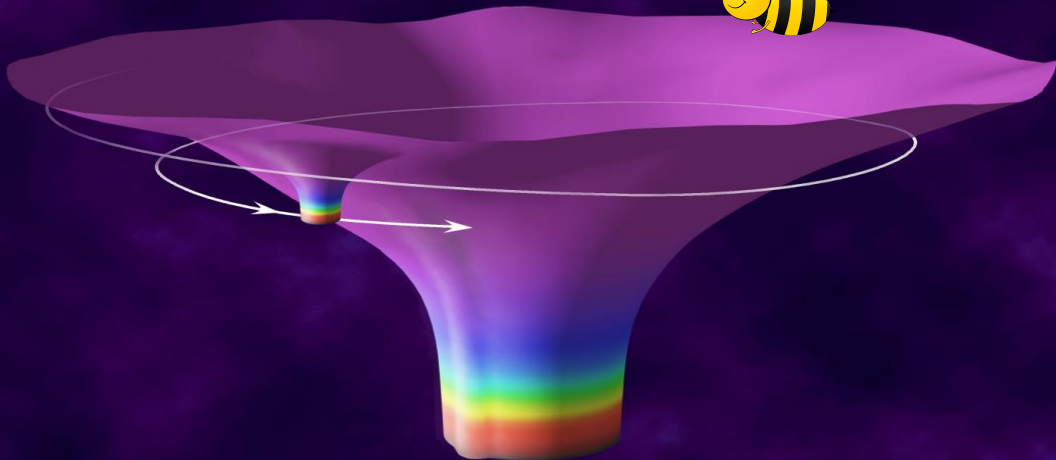
$$\theta_g = 5.02 \pm 0.20 \mu\text{as}$$

Xu, Liang, Shao [[ApJ:2302.05671](#)]

Bounds from BH Shadows



Xu, Liang, Shao [[ApJ:2302.05671](#)]

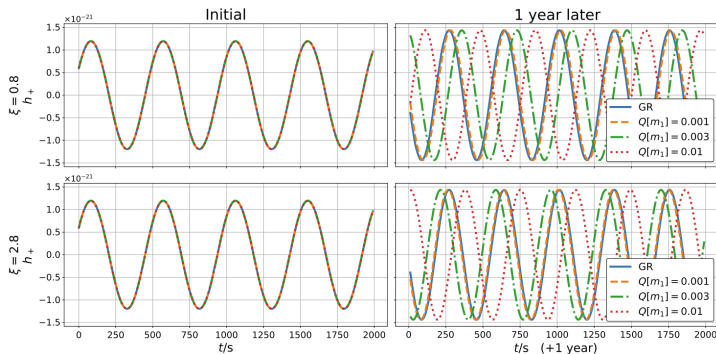


Extreme mass ratio inspirals (EMRIs)

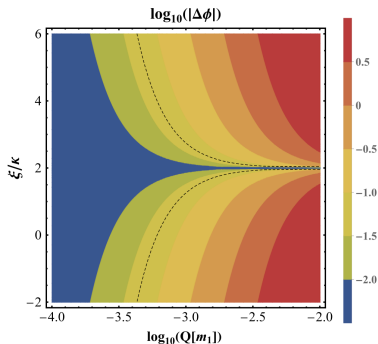
Bumblebee EMRIs

■ **Geodesics (+tensorial energy loss) change** w.r.t. GR EMRIs

■ e.g., $(10^6, 10) M_\odot$ EMRIs at $D_L = 100$ Mpc, starting at $5R_S$

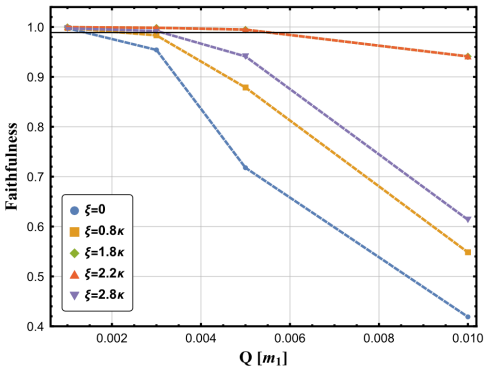


Liang, Xu, Mai, Shao [PRD:2212.09346]



Phase difference

Bumblebee EMRIs in LISA



Liang, Xu, Mai, Shao [PRD:2212.09346]

Faithfulness

$$\mathcal{F}[s_1, s_2] = \max_{\{t_c, \varphi_c\}} \frac{\langle s_1 | s_2 \rangle}{\sqrt{\langle s_1 | s_1 \rangle \langle s_2 | s_2 \rangle}}$$

with inner product

$$\langle s_1 | s_2 \rangle = 2 \int_{f_{\min}}^{f_{\max}} \frac{\tilde{s}_1(f) \tilde{s}_2^*(f) + \tilde{s}_1^*(f) \tilde{s}_2(f)}{S_n(f)} df$$

$$\mathcal{F}_{\text{threshold}} = 0.989 \text{ for SNR } \rho = 30 \Rightarrow Q \sim \mathcal{O}(10^{-3})$$

Timing of radio pulsars around Sgr A*
give comparable (slightly worse)
limits of EMRIs



Hu, Shao, Xu, Liang, Mai [JCAP:2312.02486]

Summary

- We found **two families of non-spinning BH solutions** in the bumblebee gravity model
- **Family $R_{rr} = 0$** is tightly constrained by perihelion advance observations in the Solar system
- **Family $b_r = 0$** is weakly constrained by the EHT BH images and instability
- **Family $b_r = 0$** will be tightly bounded by EMRIs and Pulsars

