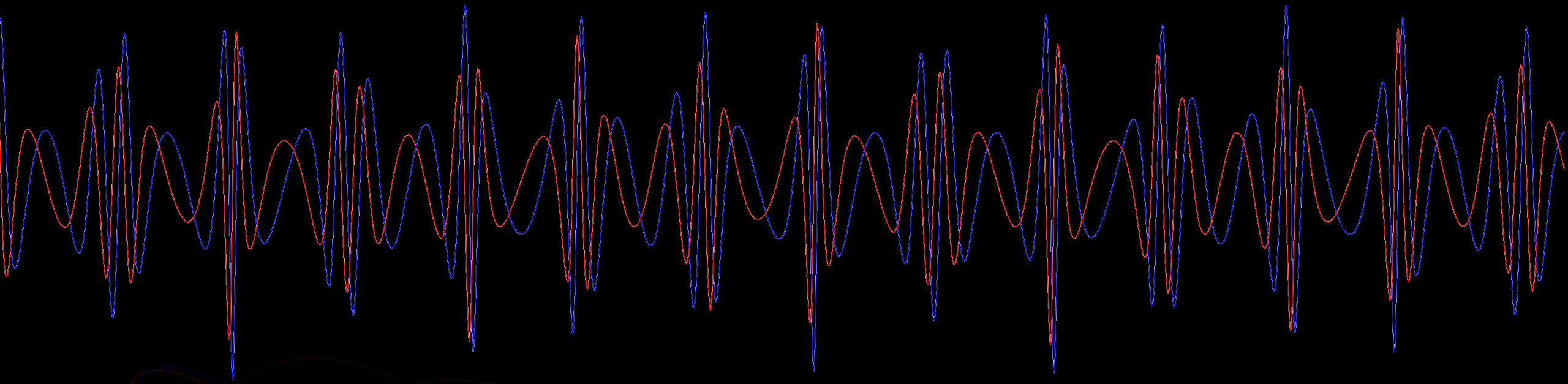
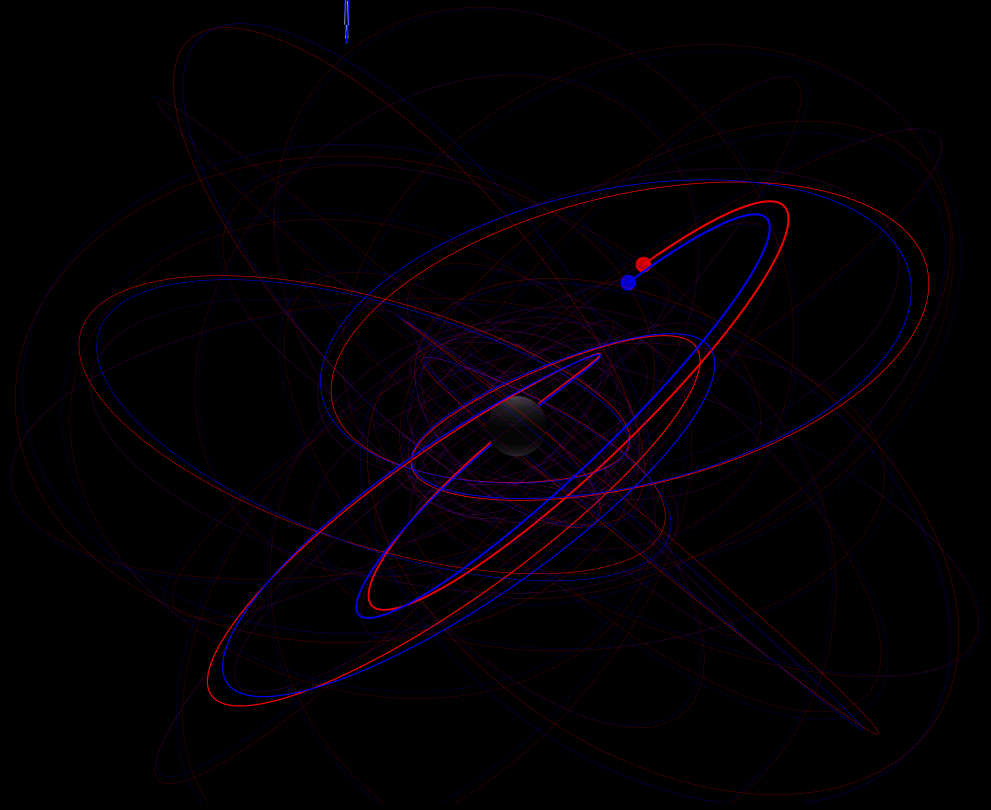




EMRI Aerodynamics

EMRI Search and Inference within the LISA Global Fit - Part II

Leif Lui

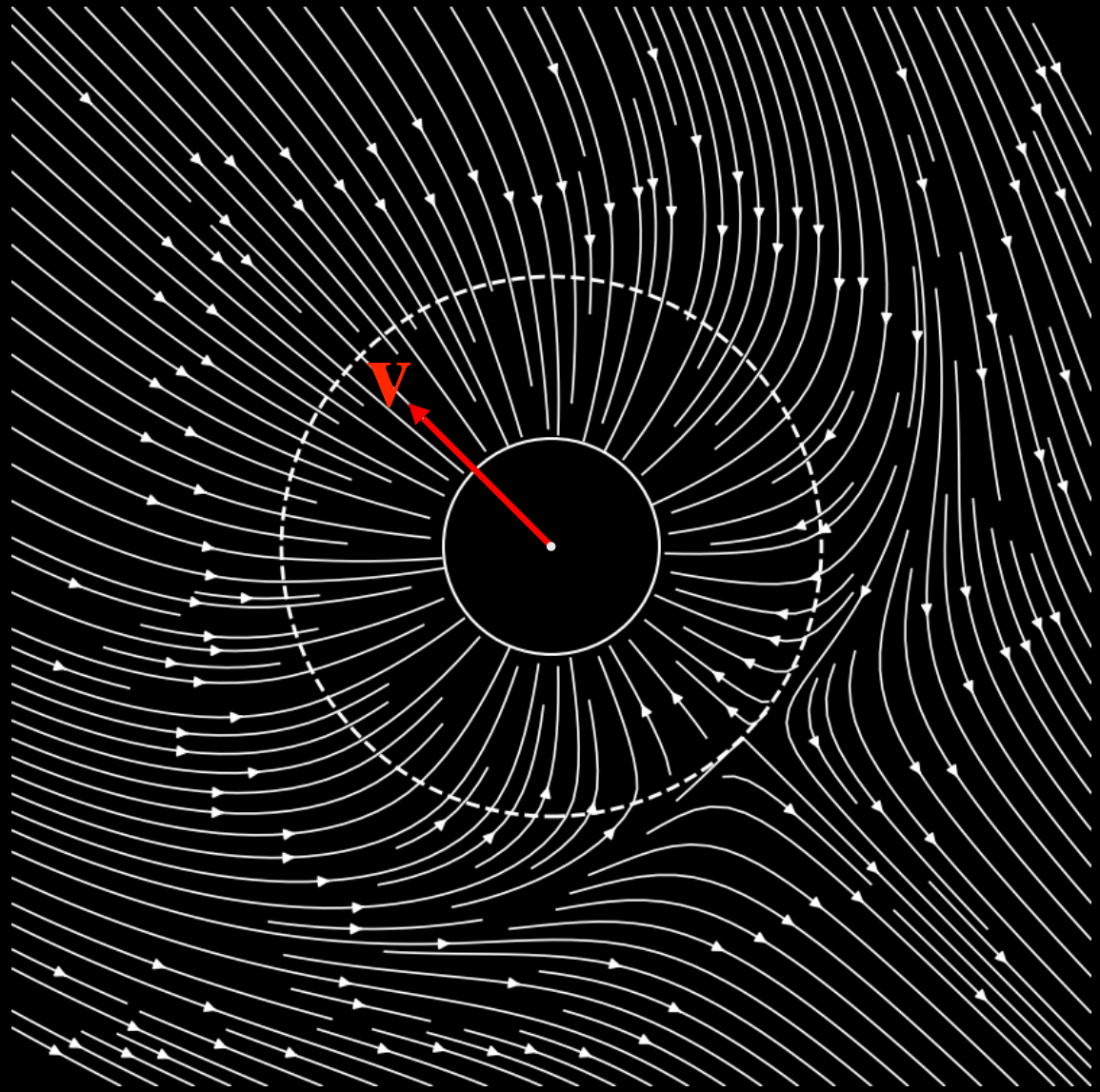
In collaboration with: Lisa Drummond and Alejandro Torres-Orjuela

Based on: Phys.Rev.Lett. 137 (2026) 12, 121403

北京雁栖湖
应用数学研究院
Beijing Institute
of Mathematical
Sciences and Applications

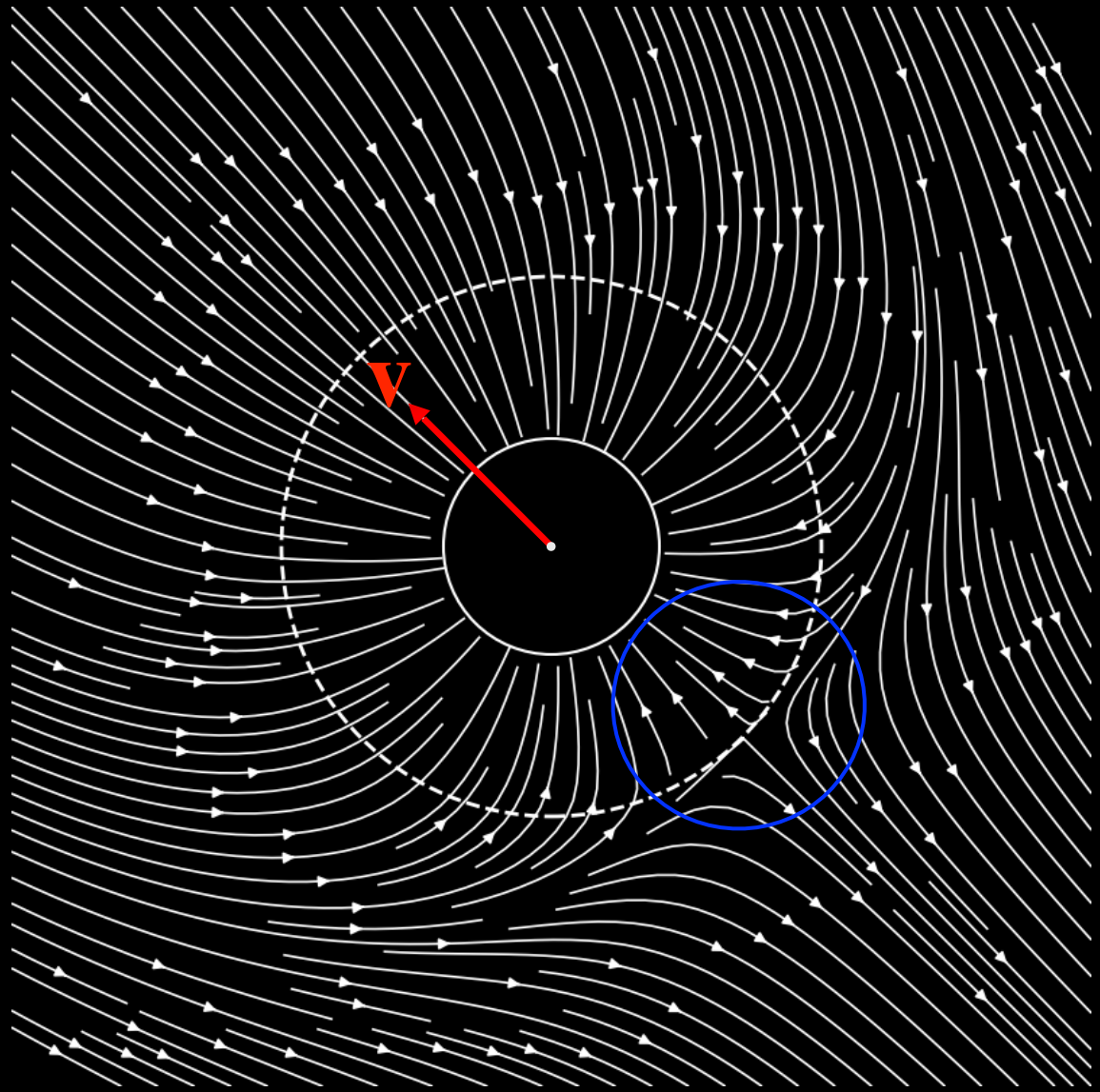
NUS
National University
of Singapore

Dynamical Friction



Dynamical Friction

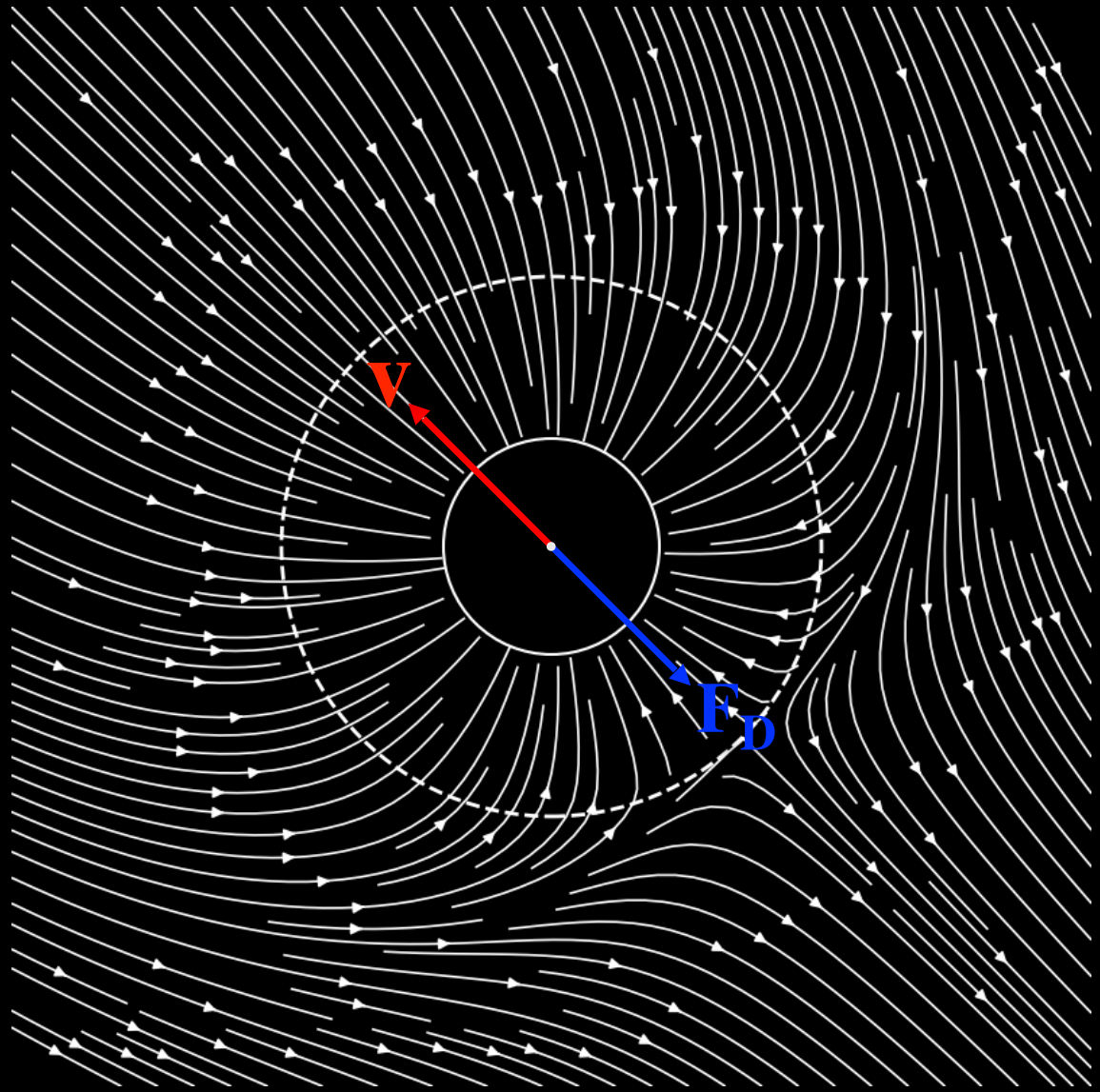
- Over-density develops behind BH.
- Over-density causes gravitational acceleration.
- Acceleration tugs the BH.



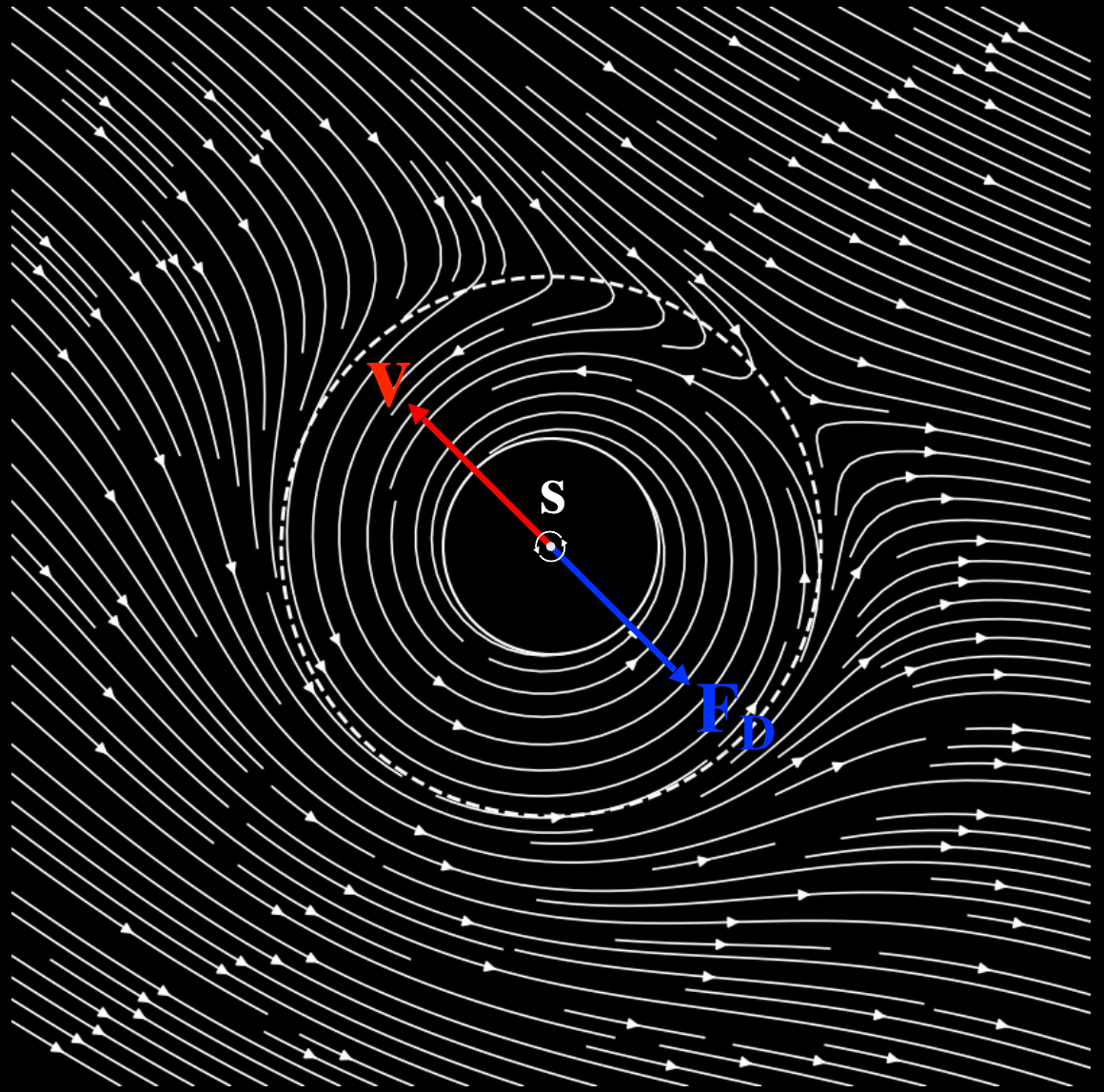
Dynamical Friction

- Over-density develops behind BH.
- Over-density causes gravitational acceleration.
- Acceleration tugs the BH.
- Producing a dynamical friction.

[Chandrasekhar. (1943). Ostriker. (1999). Escala, et al. (2004). Kim & Kim (2007). Traykova, et al. (2021) & (2023).]

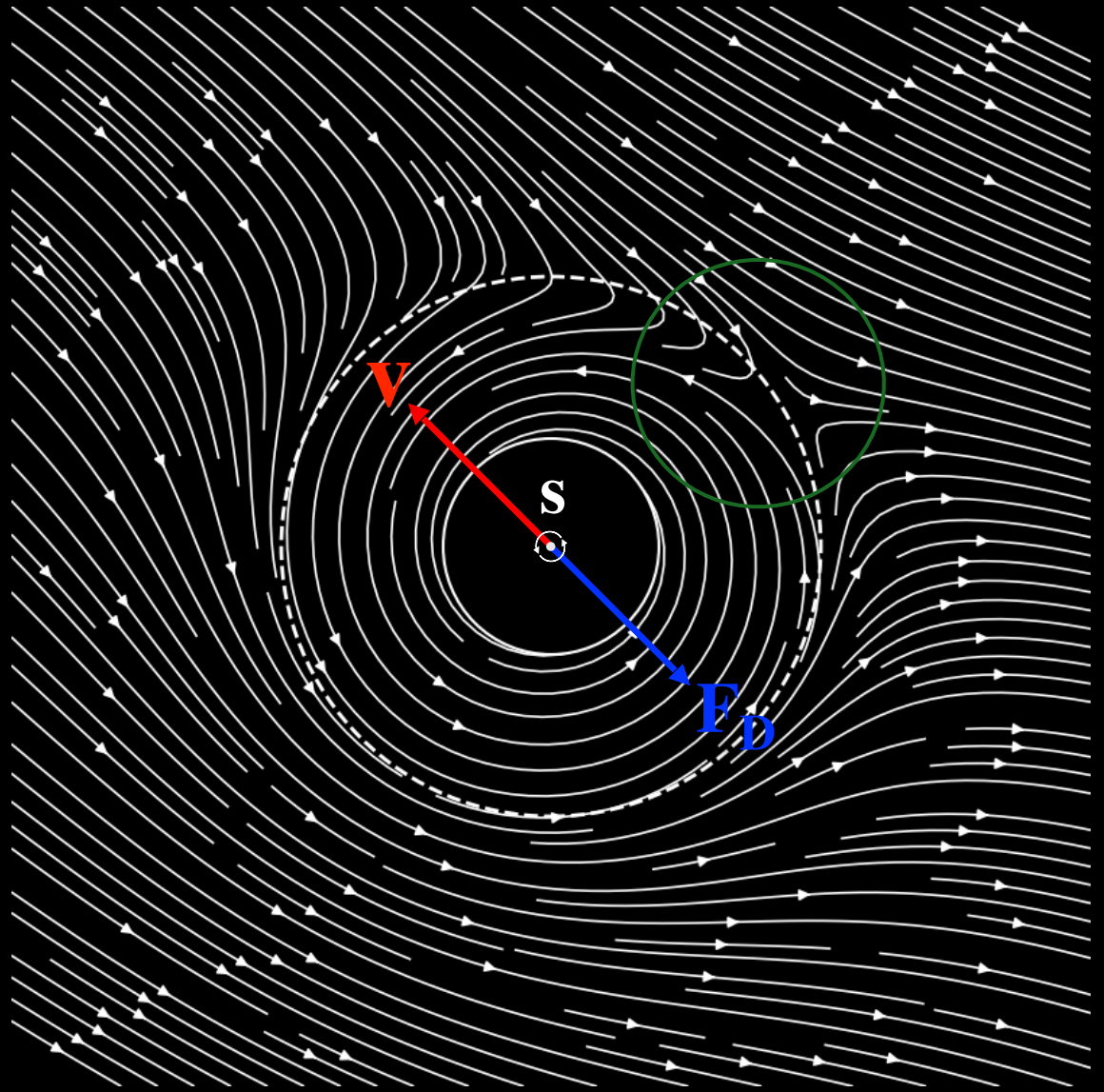


Magnus Effect



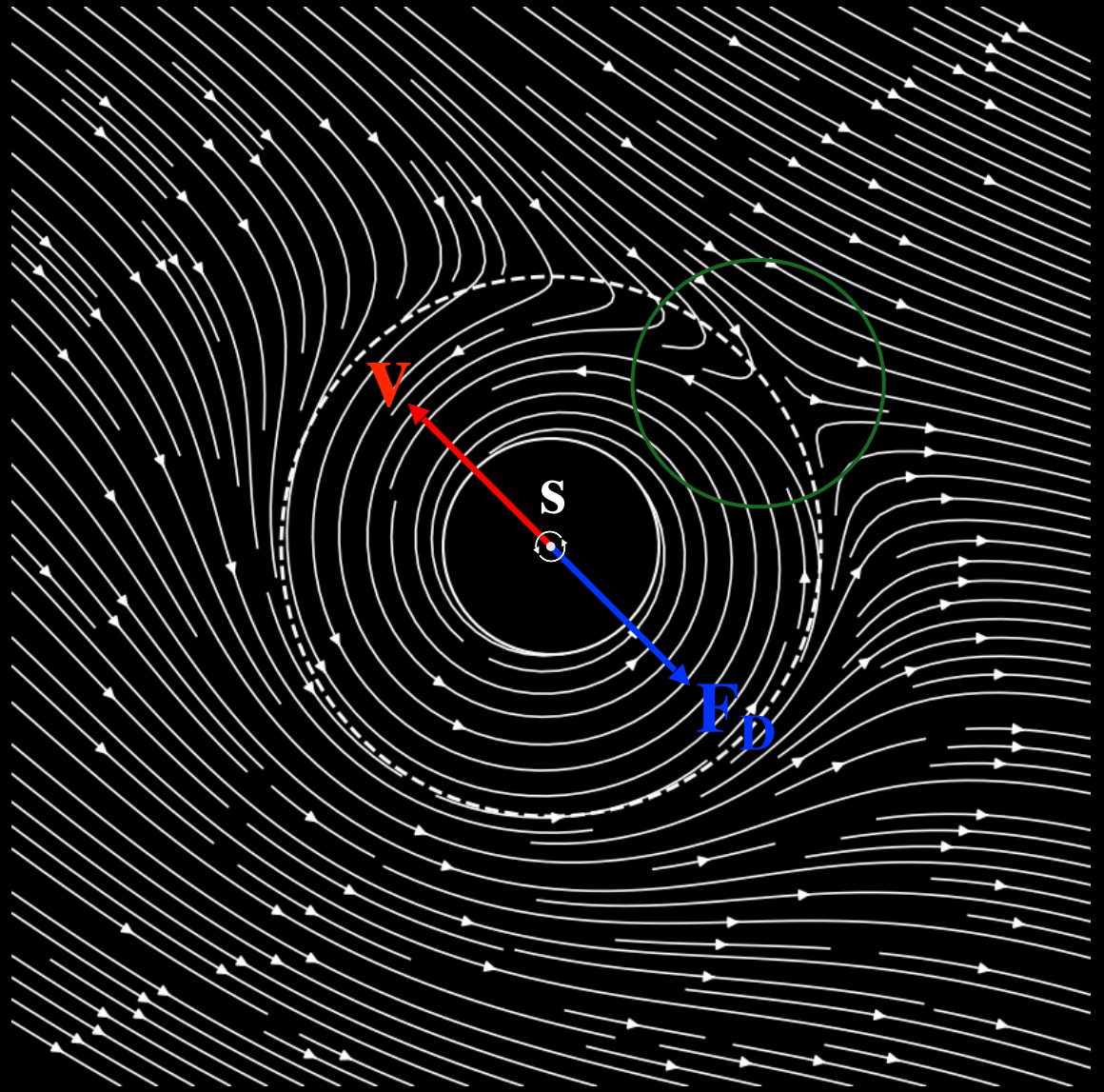
Magnus Effect

- Overdensity develops in the top half of hemisphere.
- Overdensity generates a gravitational tug.



Magnus Effect

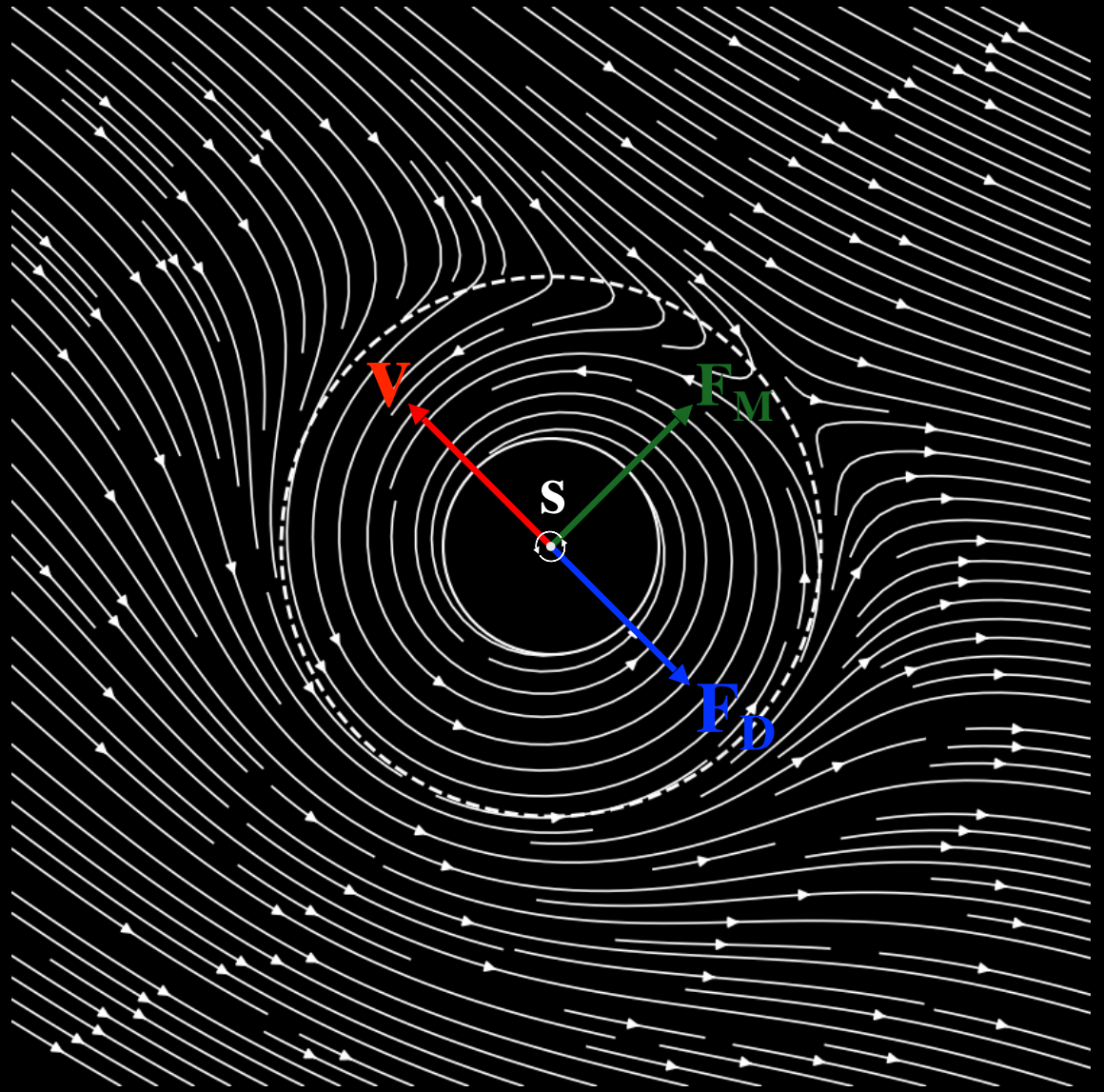
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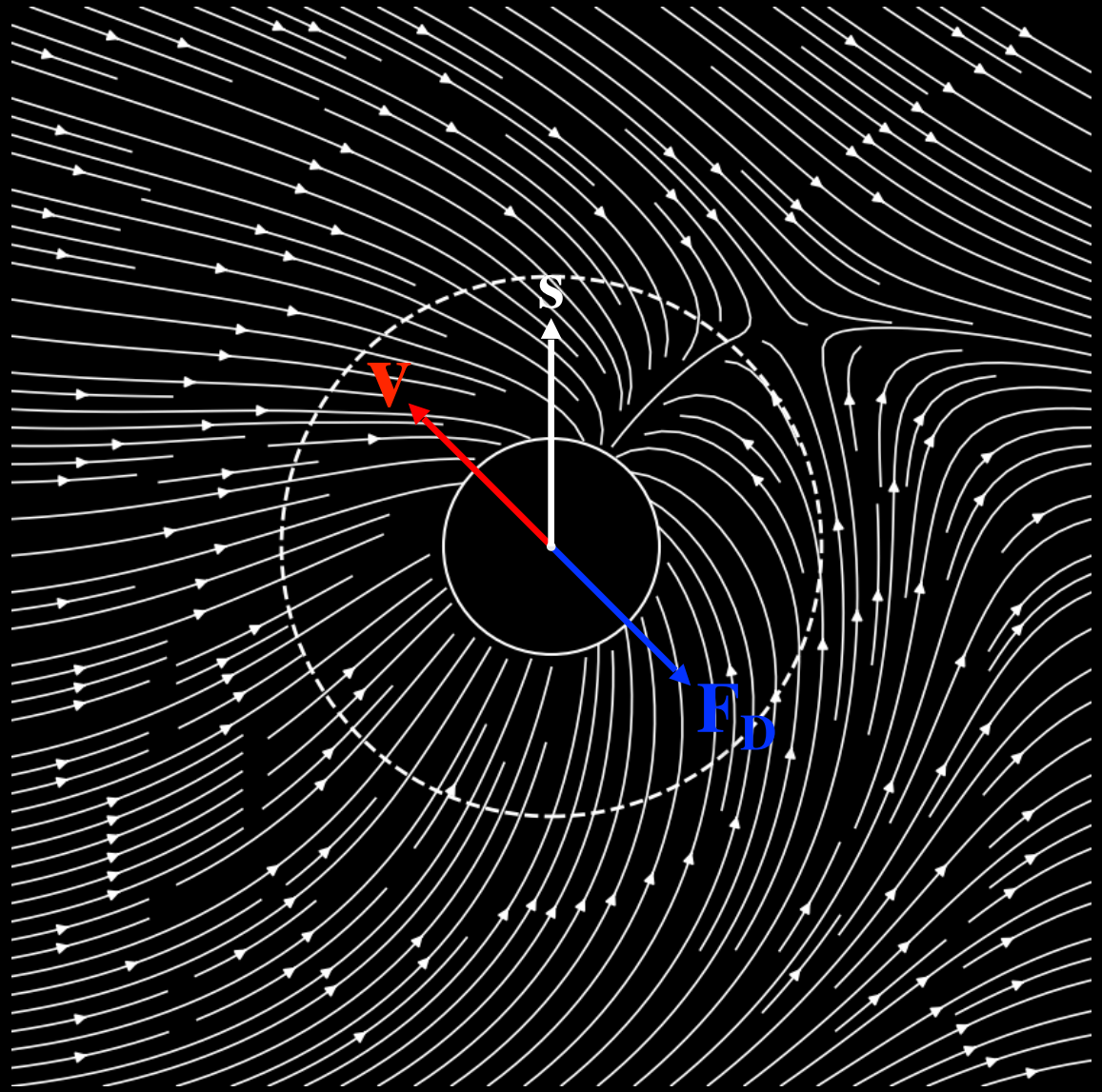
Magnus Effect

- Overdensity develops in the top half of hemisphere.
- Overdensity generates a gravitational tug.
- Tug gives rise to a Magnus force!

[Costa, et al. (2018). Wang, et al. (2021). Dyson, et al. (2024).]

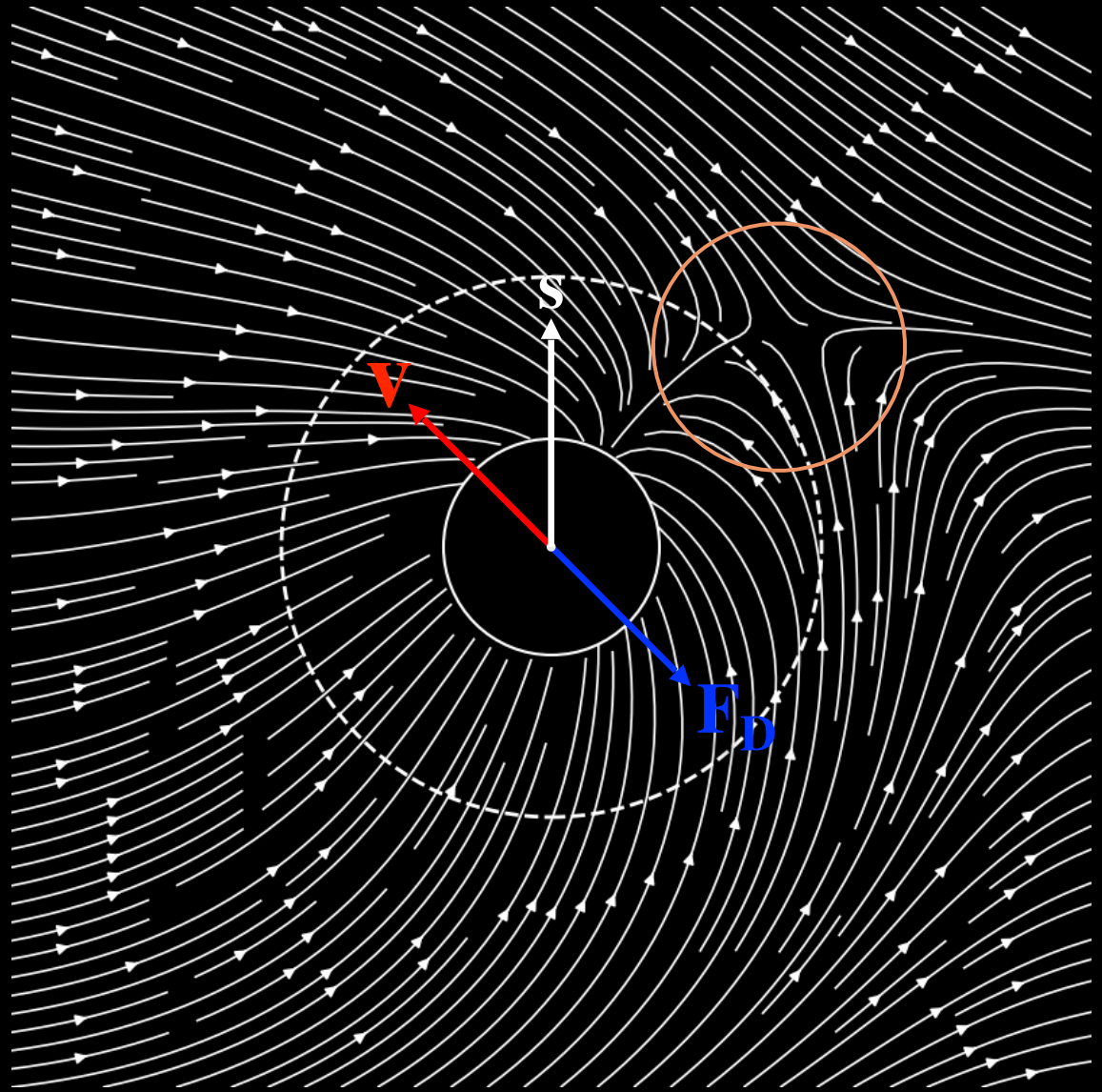


Lift Force



Lift Force

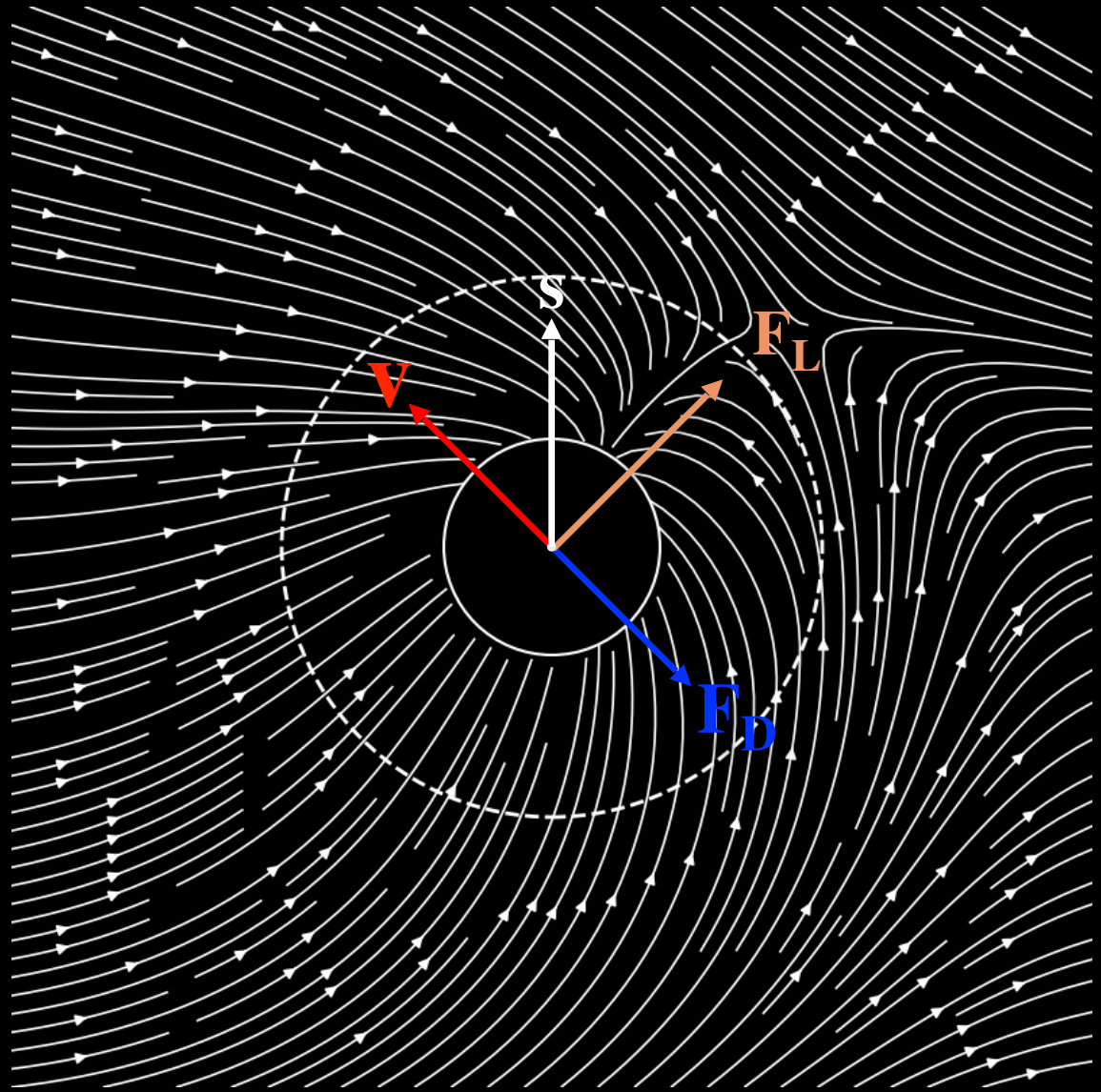
- Breaks axisymmetry in gravitational wake.
- Creating a density asymmetry.



Lift Force

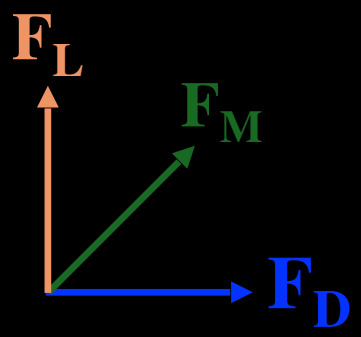
- Breaks axisymmetry in gravitational wake.
- Creating a density asymmetry.
- Generating a lift force!

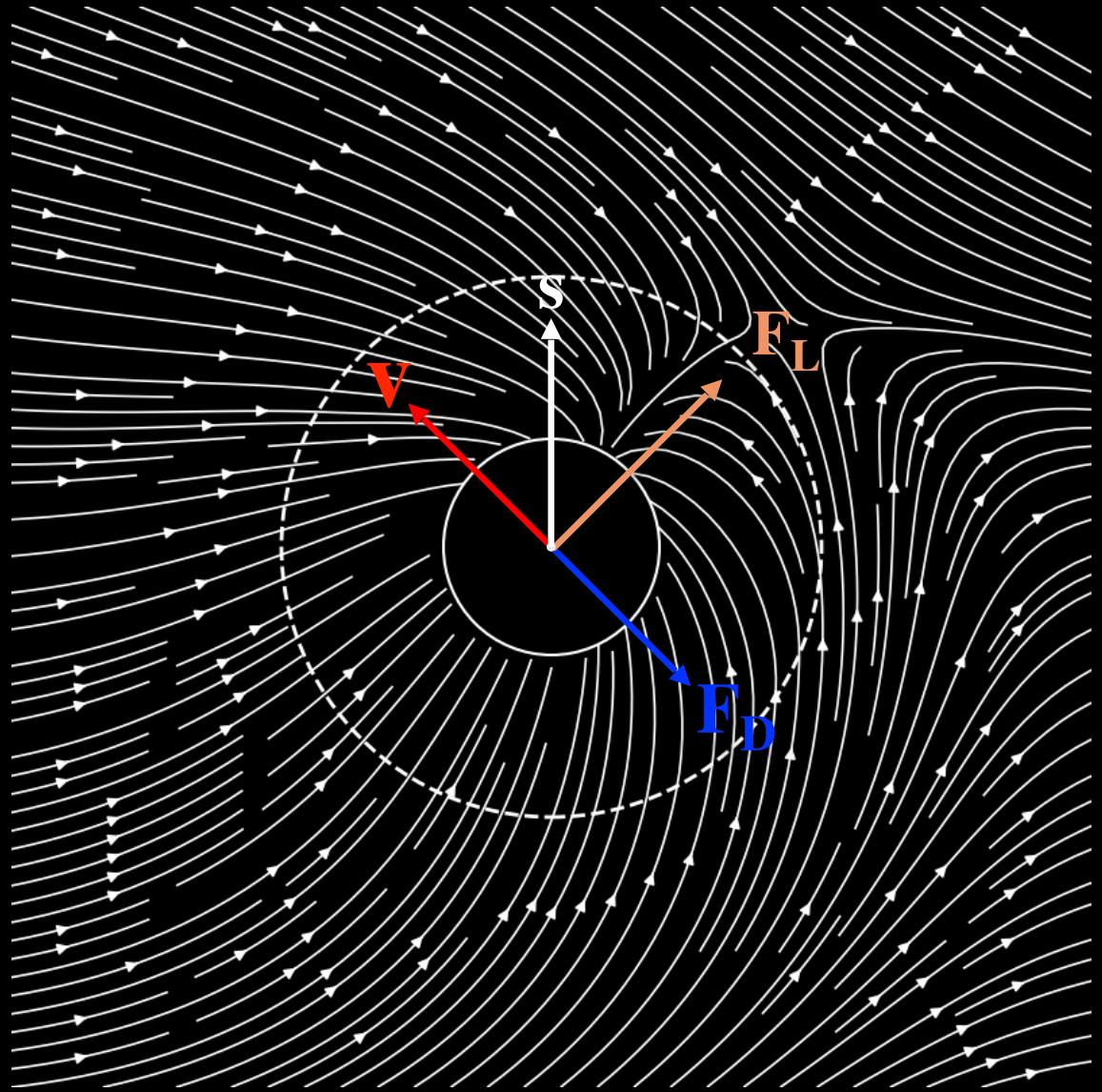
[Dyson, et al. (2024).]



Lift Force

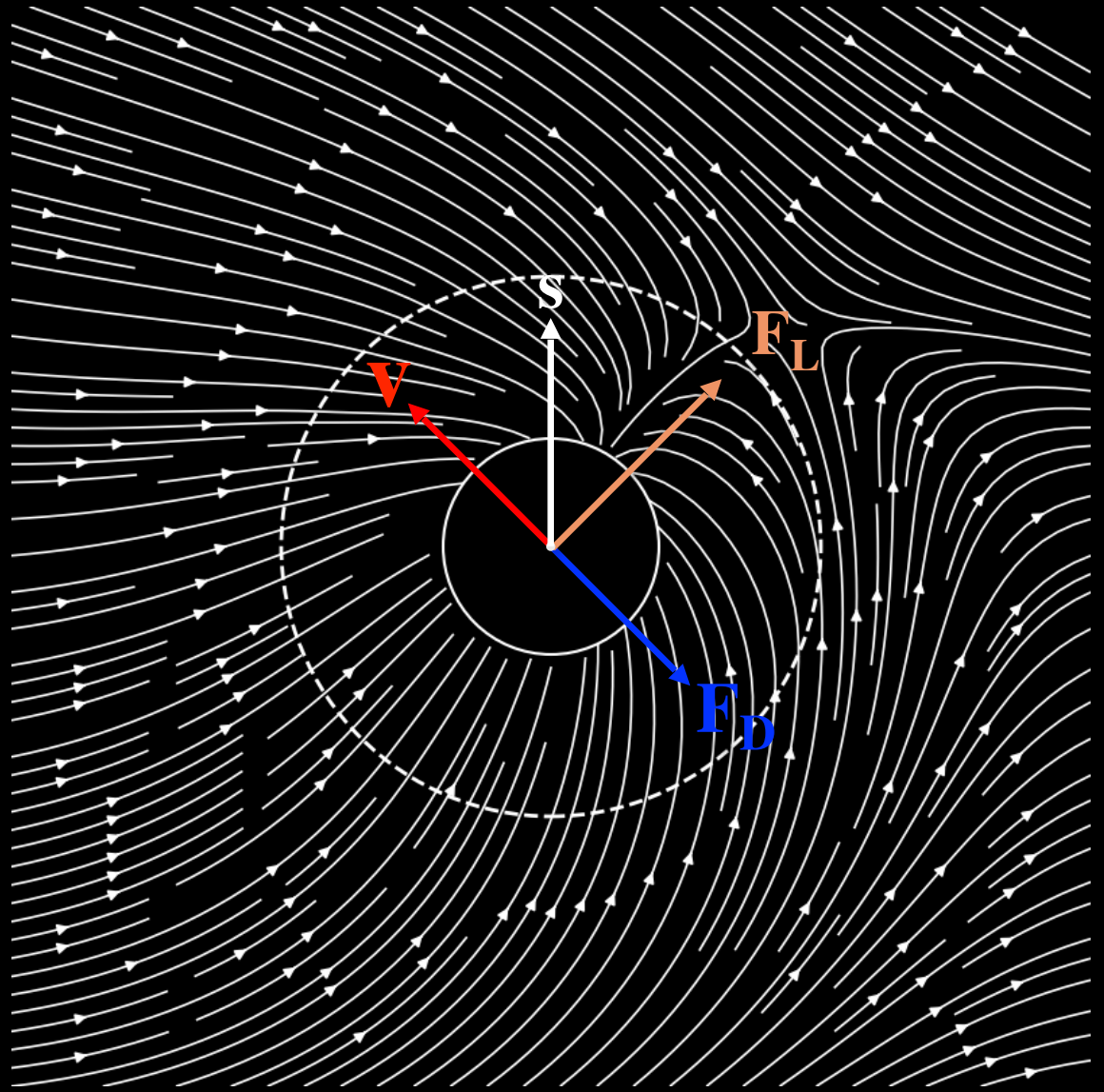
- Breaks axisymmetry in gravitational wake.
- Creating a density asymmetry.
- Generating a lift force!
- Summary:

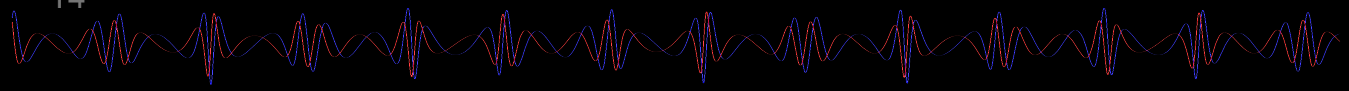




Some research questions

- What can we learn about the BH environment?
- Can we probe the spin of the secondary body?
- Can the environmental effects “magnify” the spin effects?





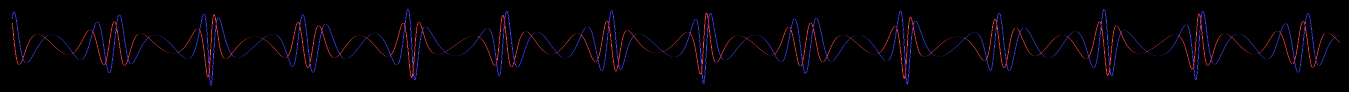
Point particle Spinning particle

Spinning Bodies in Kerr Spacetime

- If the secondary has spin

$$\frac{\mathrm{D}x^{\mu}}{\mathrm{d}\lambda} = -\frac{1}{2}R^{\mu}_{\nu\rho\sigma}S^{\nu\rho}u^{\sigma}, \qquad \frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\lambda} = p^{[\mu}u^{\nu]}, \qquad S^{\mu}u_{\mu} = 0.$$

[Papapetrou. (1951)., Dixon. (1970)., Tulczyjew (1959). Mathisson. (2010).]



Point particle Spinning particle

Spinning Bodies in Kerr Spacetime

- If the secondary has spin

$$\frac{\mathrm{D}x^\mu}{\mathrm{d}\lambda} = -\frac{1}{2}R^\mu_{\nu\rho\sigma}S^{\nu\rho}u^\sigma, \qquad \frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\lambda} = p^{[\mu}u^{\nu]}, \qquad S^\mu u_\mu = 0.$$

- Physics: Replaces point particle with spinning dipole.
- There exists an analytic solution to $\mathcal{O}(s)$

$$\mathscr{C} = \mathscr{C}_{\mathrm{g}} + s\delta\mathscr{C}_{\mathrm{s}}, \qquad x^\mu = x^\mu_{\mathrm{g}} + s\delta x^\mu_{\mathrm{s}}\,.$$

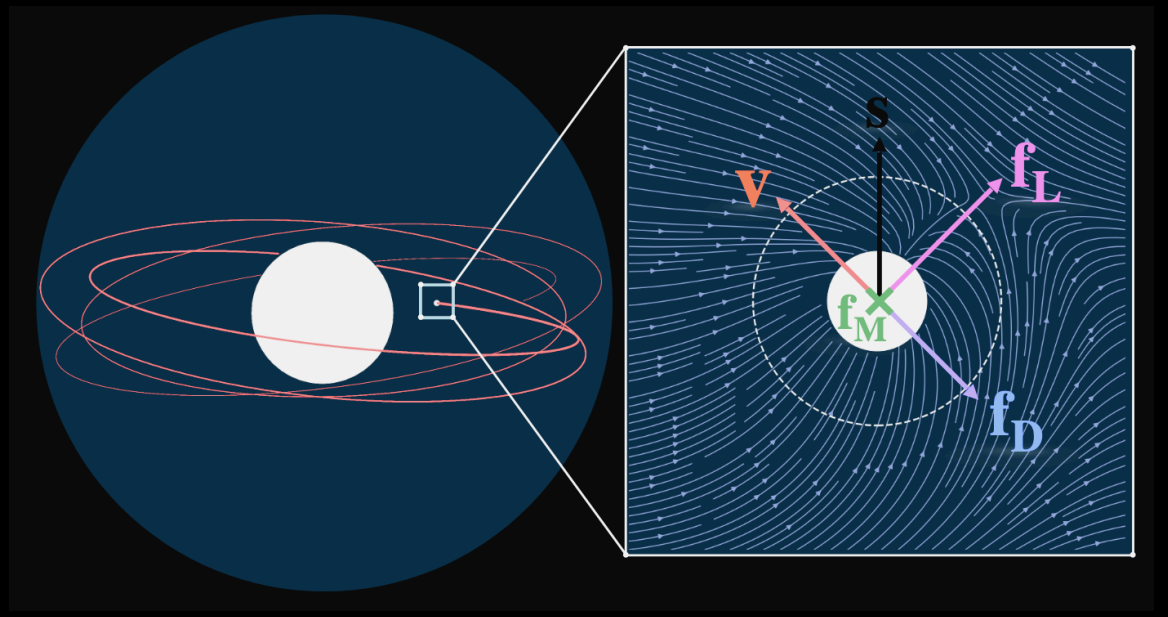
[Skoupỳ and Witzany. (2025).]

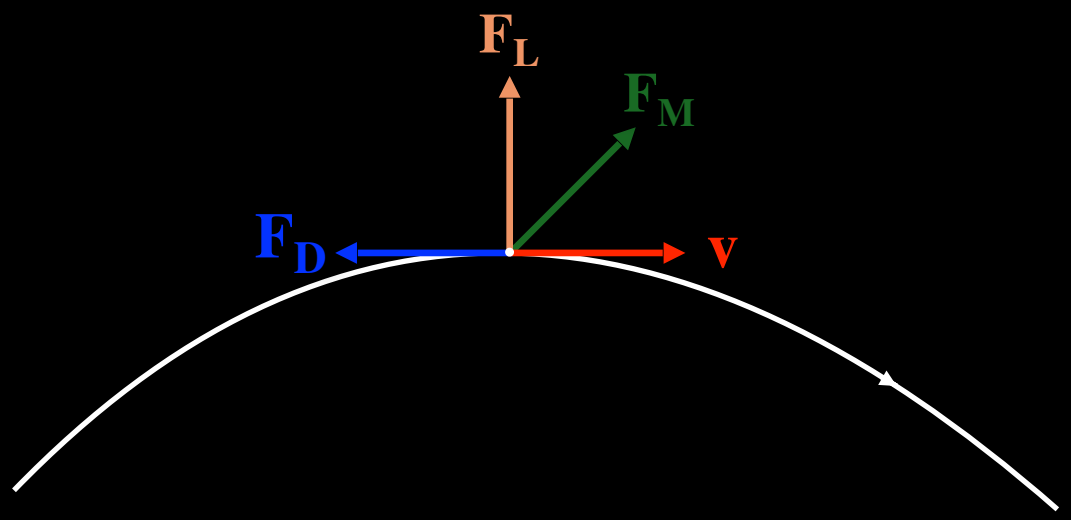
Physics of different frames

- Previously, we discussed the stationary frame of the BH.
- The force in this LIF is denoted as f^A .
- The 4-force in BL coordinates is denoted as f^μ .
- The forces in these frames are related by

$$f^\mu = \hat{e}^\mu_A f^A, \qquad \eta_{AB} = g_{\mu\nu}\hat{e}^\mu_A \hat{e}^\nu_B .$$

[Marck. (1983). van de Meent. (2020).]





Including Dissipative Effects

- EMRIs emit GW radiation.
- The aerodynamic force change the secondaries momentum.

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}(\mathscr{E},\mathscr{L},\mathscr{K}) = (-g_{t\mu}f^{\mu}, g_{\varphi\mu}f^{\mu}, -2\mathscr{K}_{\mu\nu}u^{\mu}f^{\mu})\,.$$

$$f^{\mu} = q\left[f^{\mu}_{(1,0,0)} + sf^{\mu}_{(1,1,0)} + \epsilon\left(f^{\mu}_{(1,0,1)} + sf^{\mu}_{(1,1,1)}\right)\right] + \mathcal{O}(q^2, s^2, \epsilon^2),$$

- Force: 0PA GSF, 0PA/1PA Spin-correction, Drag, Magnus/Lift.

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Calculated using pybhpt/FEW:

$$\langle \dot{\mathscr{E}}_p \rangle = \sum_{\ell mnk} \frac{|Z_{\ell mnk}|^2}{4\pi\omega_{nkm}^2}, \quad \langle \dot{\mathscr{L}}_p \rangle = \sum_{\ell mnk} \frac{m|Z_{\ell mnk}|^2}{4\pi\omega_{nkm}^2}\,.$$

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Calculated using KerrSpinningFluxes:

$$\langle \dot{\mathscr{E}}_{\mathrm{GW}} \rangle = \langle \dot{\mathscr{E}}_{\mathrm{p}} \rangle + s\langle \dot{\mathscr{E}}_{\mathrm{s}} \rangle, \quad \langle \dot{\mathscr{L}}_{\mathrm{GW}} \rangle = \langle \dot{\mathscr{L}}_{\mathrm{p}} \rangle + s\langle \dot{\mathscr{L}}_{\mathrm{s}} \rangle\,.$$

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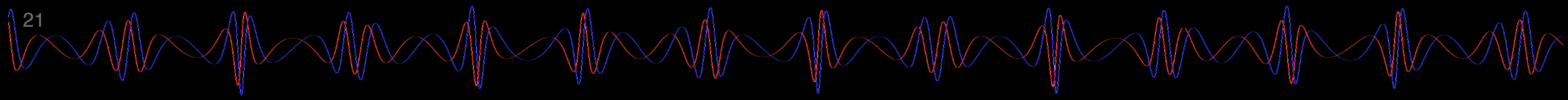
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Calculated using KerrSpinningFluxes:

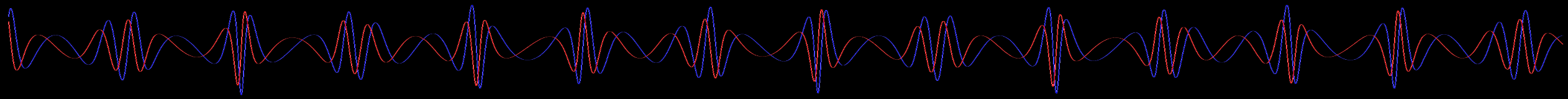
$$\langle \dot{\mathscr{E}}_{\mathrm{GW}} \rangle = \langle \dot{\mathscr{E}}_{\mathrm{p}} \rangle + s\langle \dot{\mathscr{E}}_{\mathrm{s}} \rangle, \quad \langle \dot{\mathscr{L}}_{\mathrm{GW}} \rangle = \langle \dot{\mathscr{L}}_{\mathrm{p}} \rangle + s\langle \dot{\mathscr{L}}_{\mathrm{s}} \rangle\,.$$

Calculated using Traykova, et al. (2021) and Dyson, et al. (2024). :

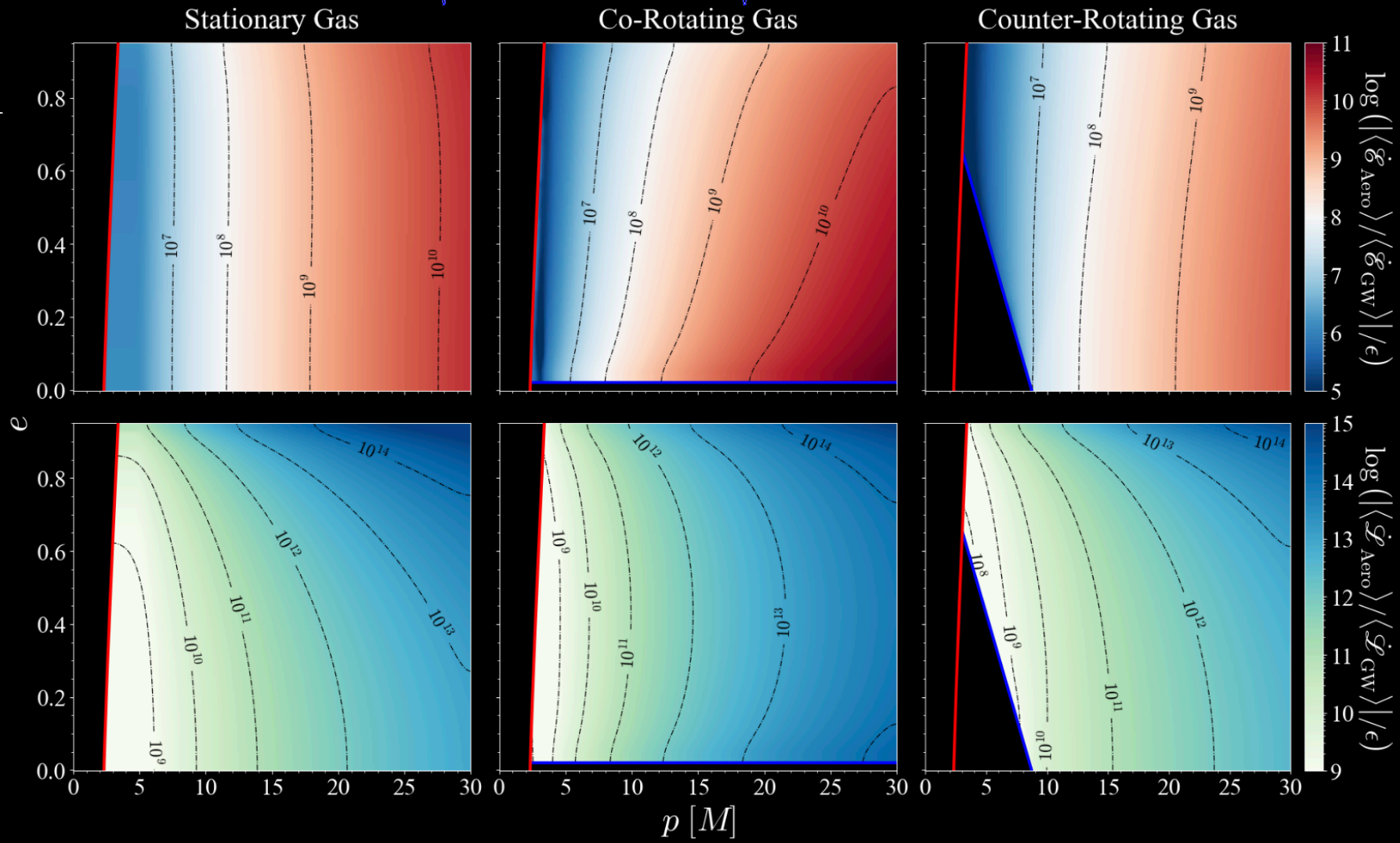
$$\begin{pmatrix} f_{\mathrm{D}} \\ f_{\mathrm{M}} \\ f_{\mathrm{L}} \end{pmatrix} = \gamma\nu\rho q^2 \begin{pmatrix} \Delta\tilde{p}^z\cos\beta - \Delta\tilde{p}^y\sin\beta \\ \Delta\tilde{p}^x \\ \Delta\tilde{p}^y\cos\beta + \Delta\tilde{p}^z\sin\beta \end{pmatrix},$$



Fluxes without Spin



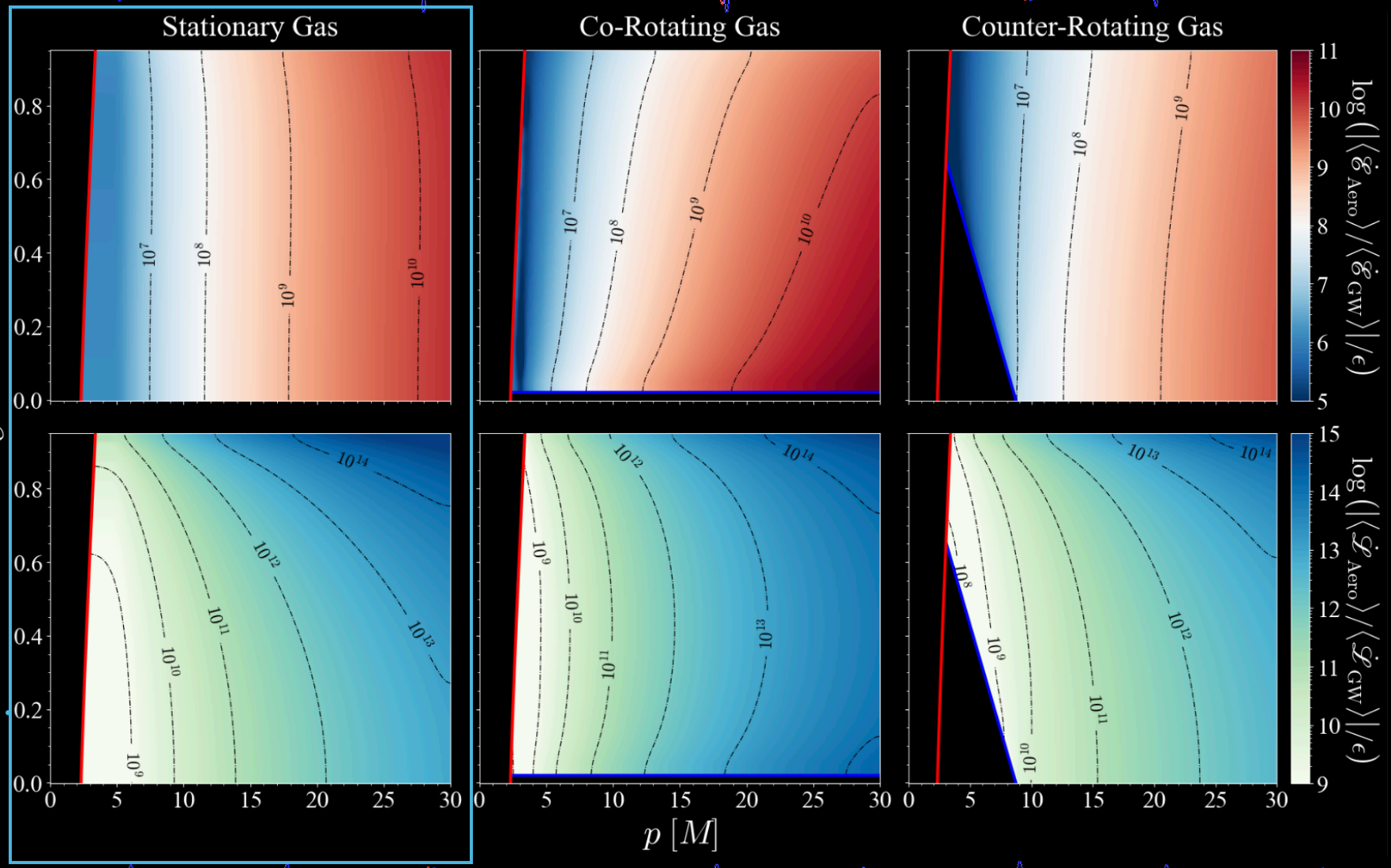
Fluxes without Spin



Fluxes without Spin

Case 1: Prograde EMRI, Stationary gas.

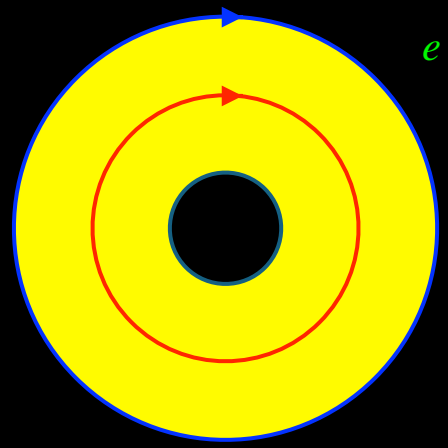
- For $M = 10^6 M_\odot$, $\epsilon \sim 10^{-6}$ for water.
- Depending on disk model, the density is highly variable.
- $10^{-21} \lesssim \epsilon \lesssim 10^{-9}$.
- GWs get weaker further from LSO.
- Aero forces dominate at high e.
- $f_{\rm d} \sim 1/v^2$, prolonged exposure near apoapsis.



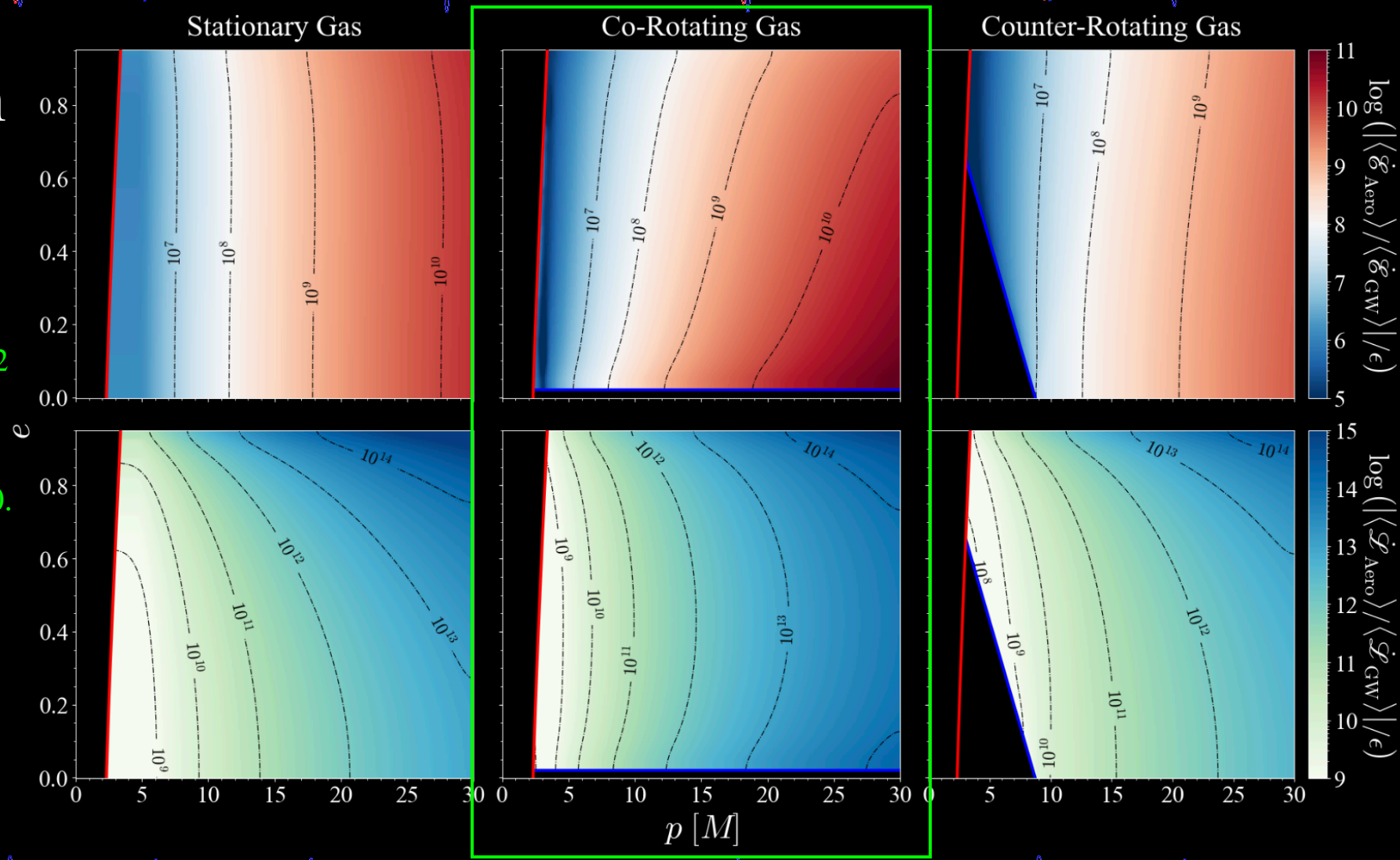
Fluxes without Spin

Case 2: Prograde EMRI, Co-rotating disk

- Diverges at $e = 0$.
- This makes sense, as $f_{\rm d} \sim 1/v^2$



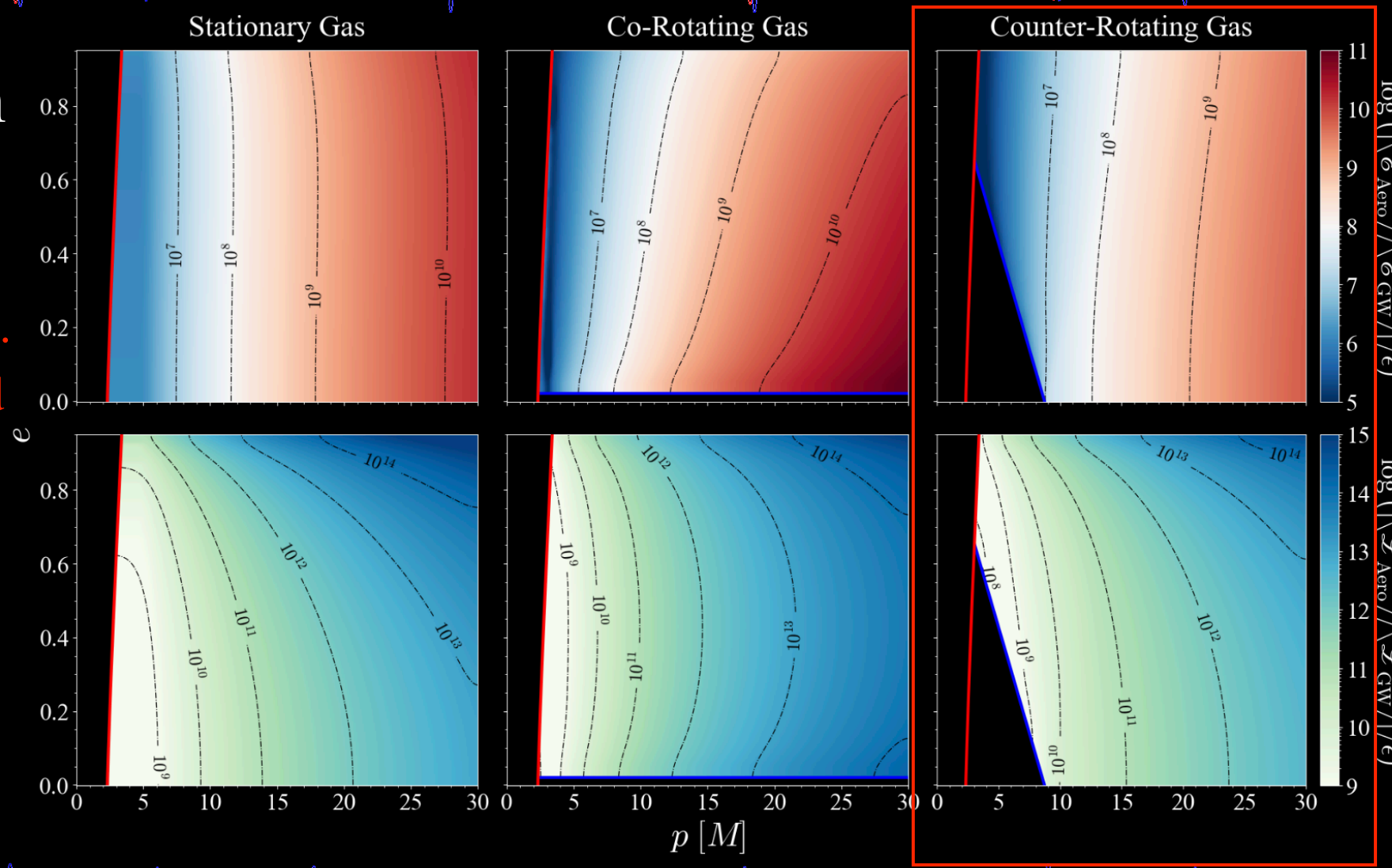
$e = 0$, $v_{\rm rel} = 0$.



Fluxes without Spin

Case 3: Prograde EMRI, Counter-rotating disk.

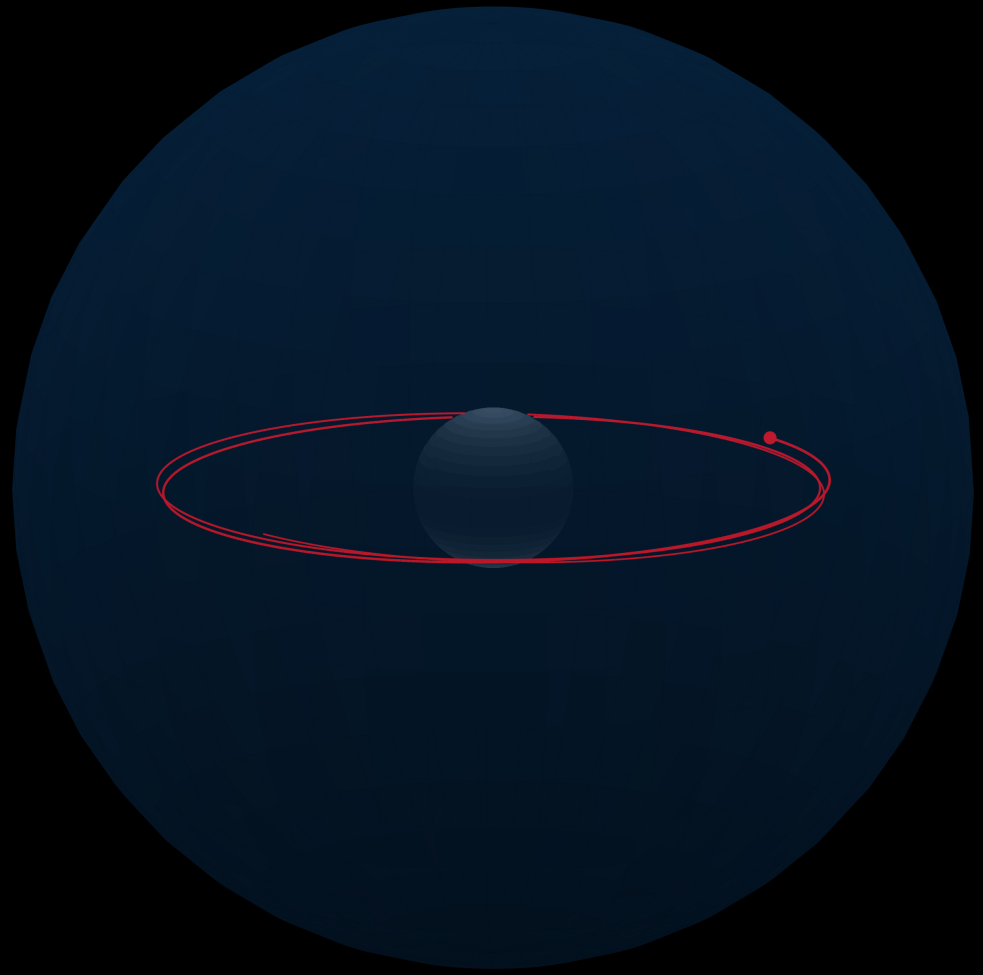
- Low e fits our expectations.
- Why is there a large voided region?
- Why is the flux smaller for high e?



Fluxes without Spin

Case 3: Prograde EMRI, Counter-rotating disk.

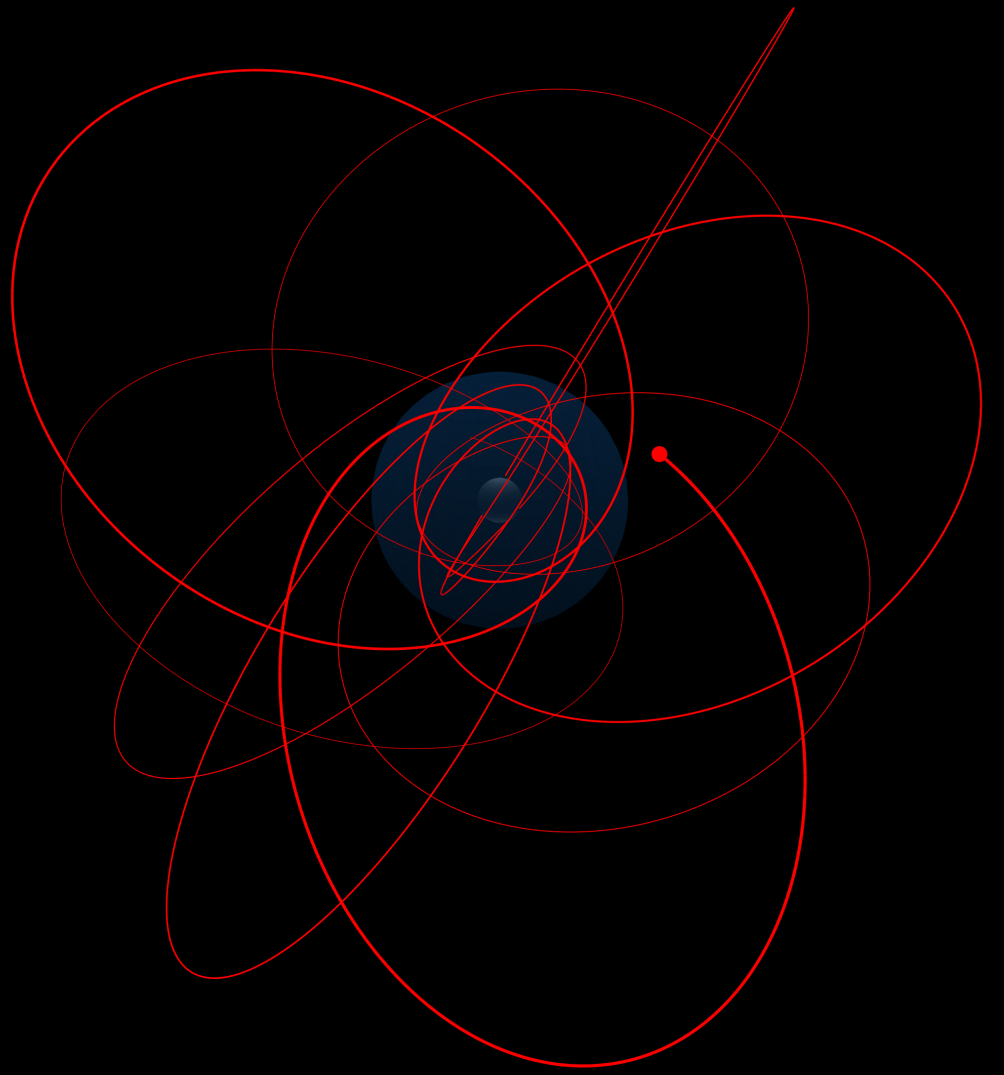
- Low *e* fits our expectations.
- Why is there a large voided region?
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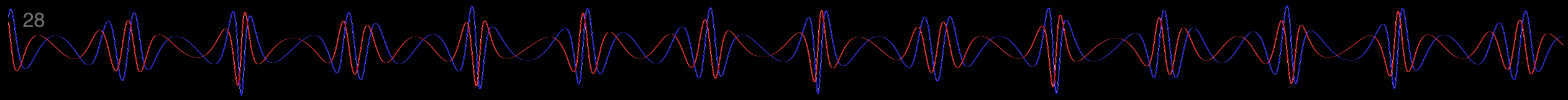


Fluxes without Spin

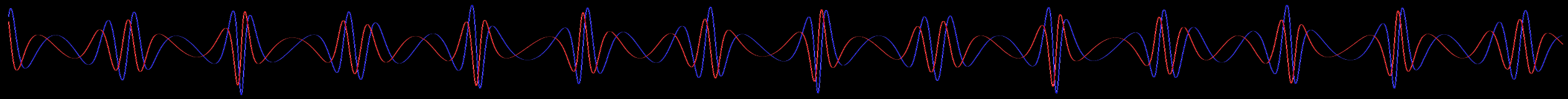
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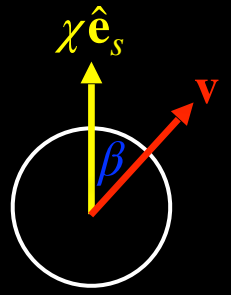




Fluxes with Spin

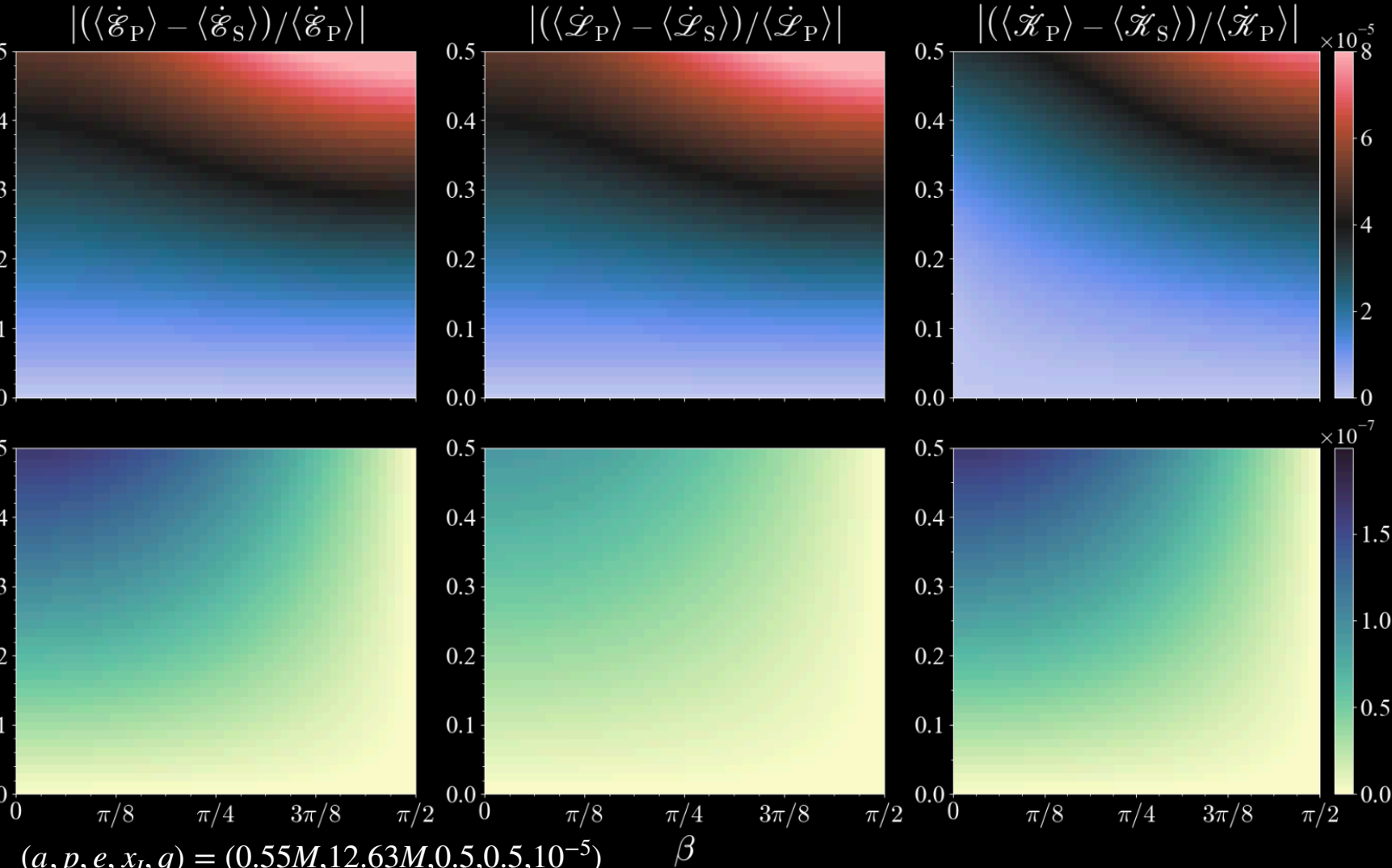


Fluxes with Spin



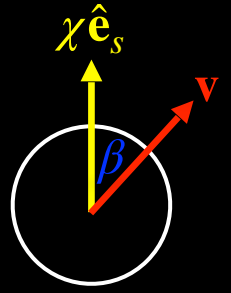
Top panel: Relative difference of aerodynamical fluxes for EMRI with secondary with/without spin.

Bottom panel: Relative difference of GW fluxes for EMRI with secondary with/without spin.



$(a, p, e, x_I, q) = (0.55M, 12.63M, 0.5, 0.5, 10^{-5})$

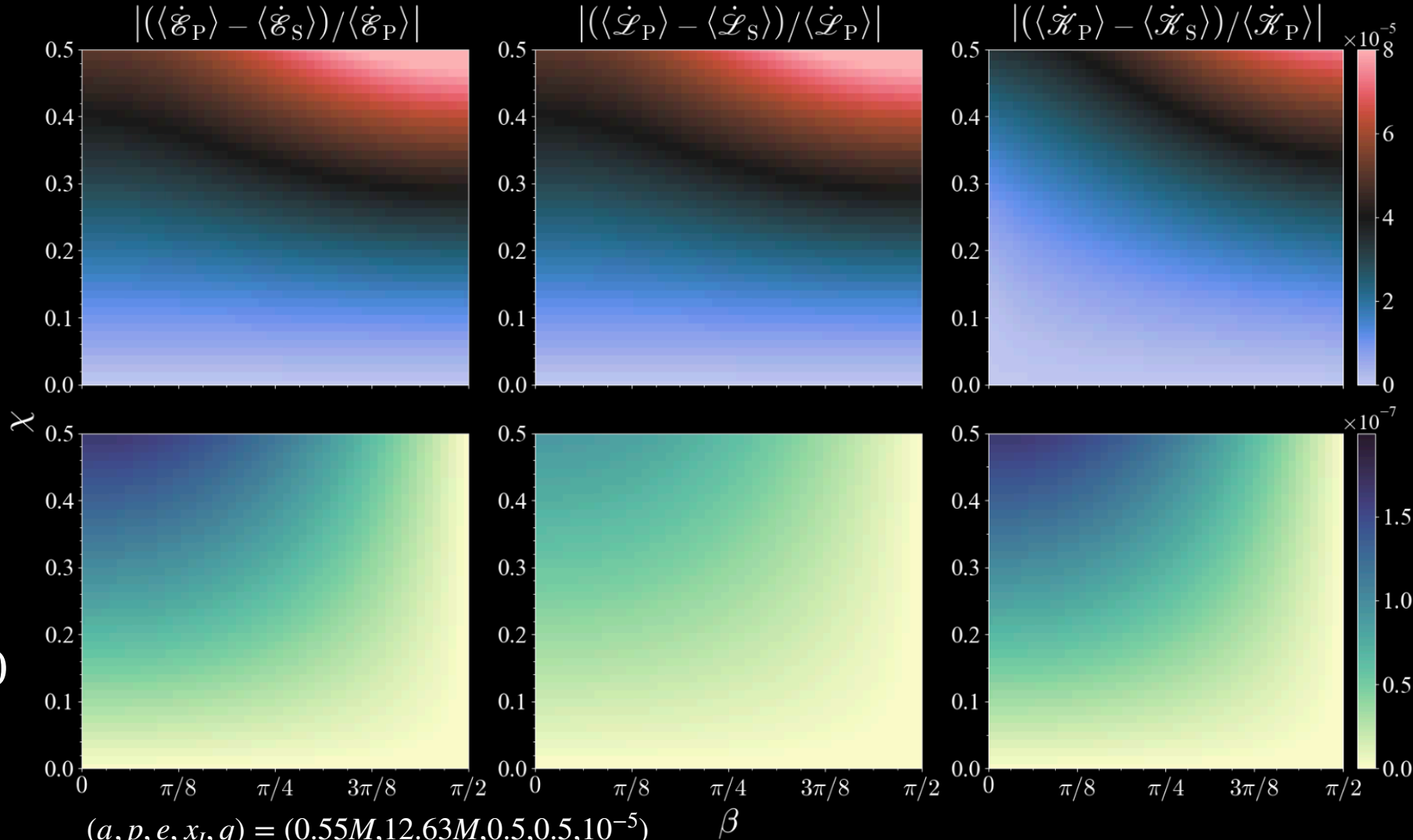
Fluxes with Spin



Top panel: Relative difference of aerodynamical fluxes for EMRI with secondary with/without spin.

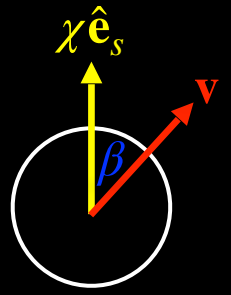
Bottom panel: Relative difference of GW fluxes for EMRI with secondary with/without spin.

- $\Delta\dot{\mathscr{C}}/\mathscr{C}$ minimum at $\beta = 0$ for aerodynamic fluxes.
- Converse effect for GWs.

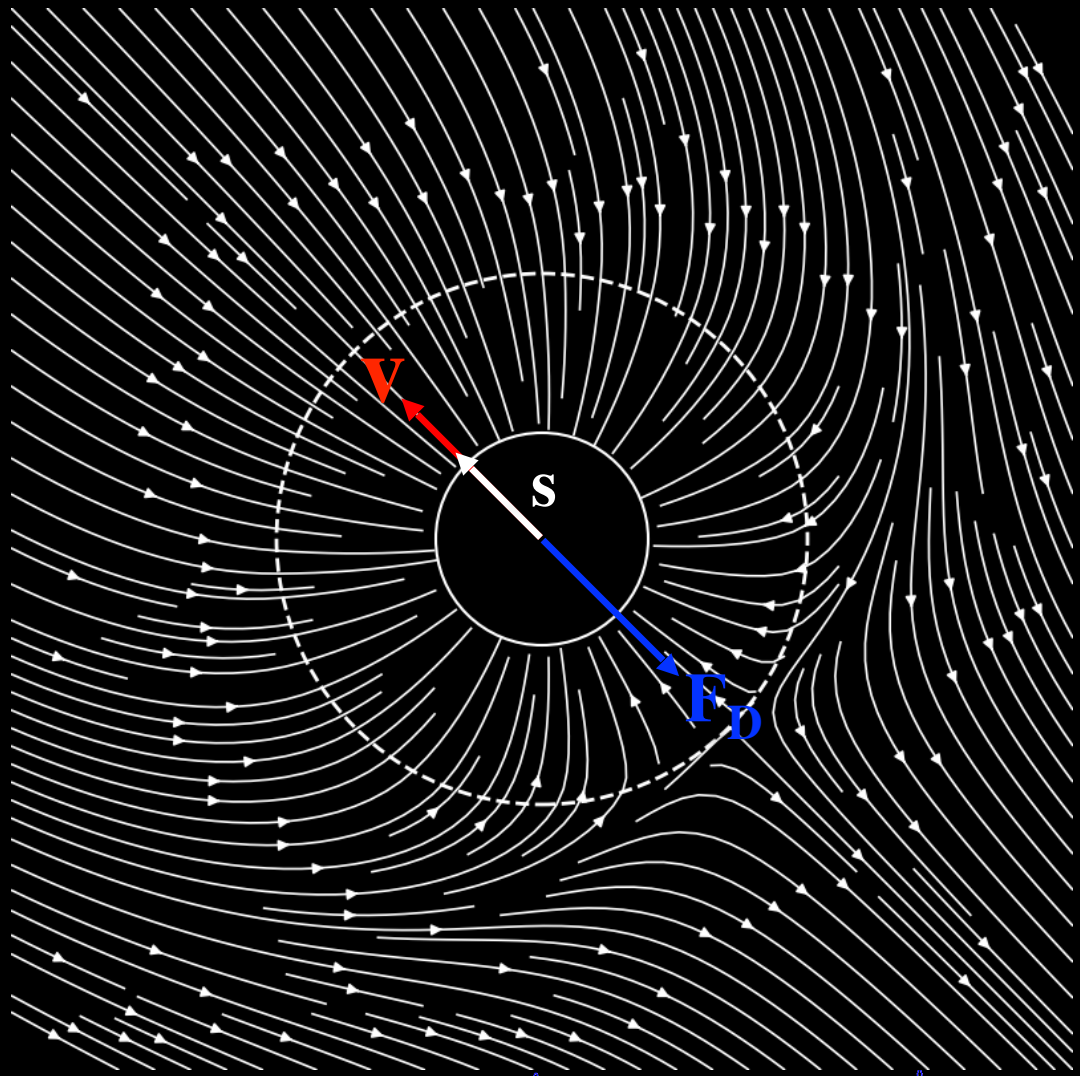


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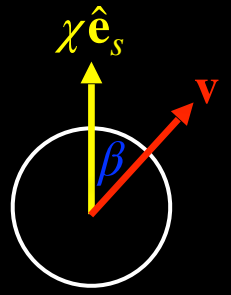
Fluxes with Spin



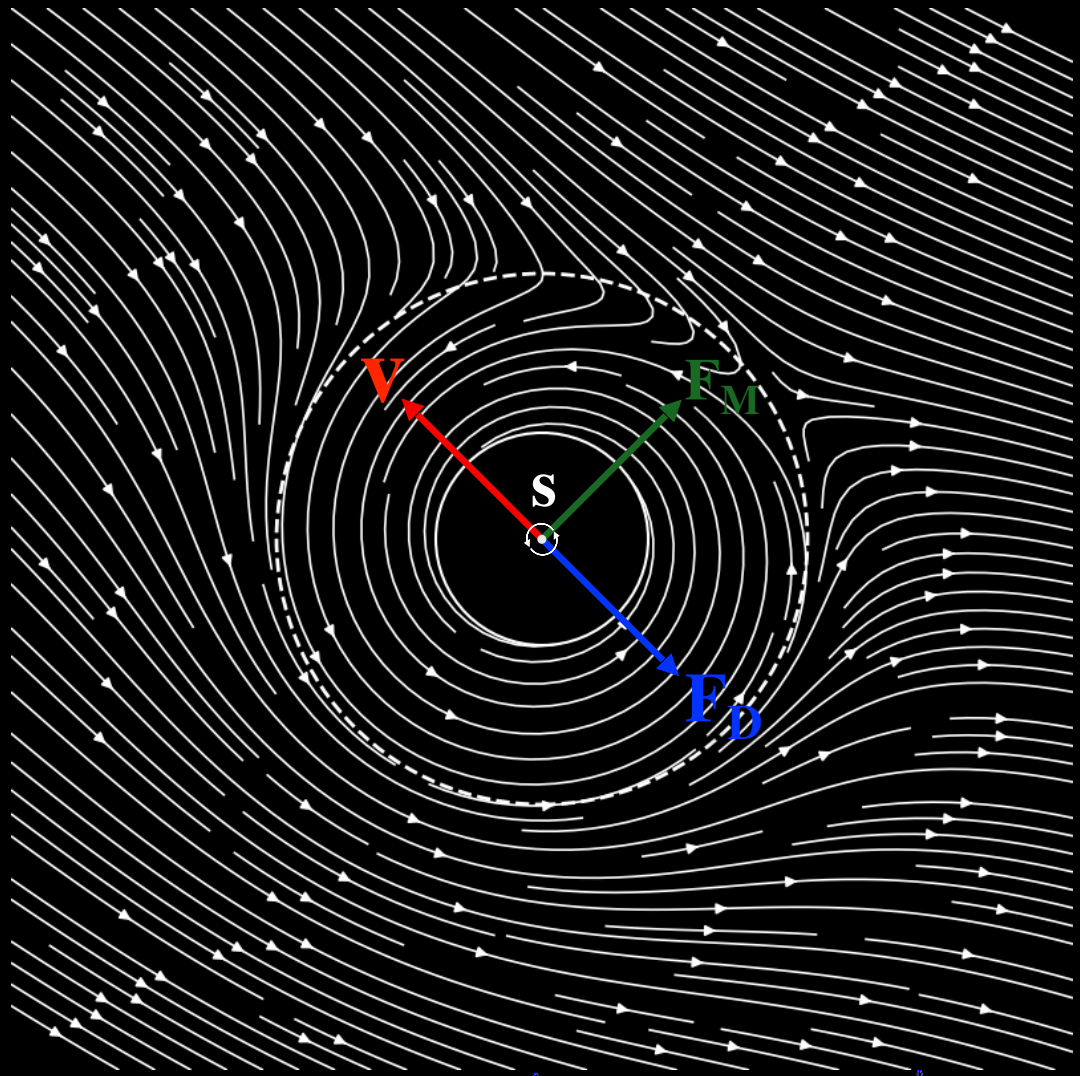
- For $\beta = 0$, spin-aligns with velocity direction.
- Axisymmetry is preserved.
- No Magnus/Lift forces.



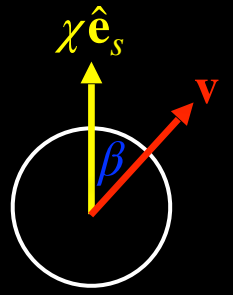
Fluxes with Spin

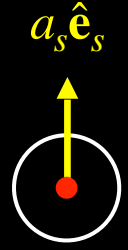


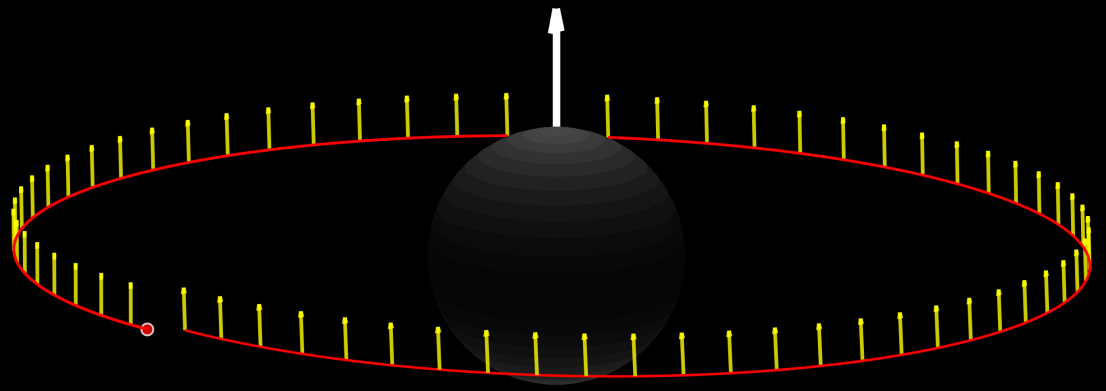
- For $\beta = \pi/2$, breaking of axisymmetry is maximised.
- Stronger Magnus and lift forces.
- Maximises the flux deviation.



Fluxes with Spin

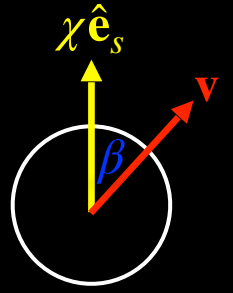




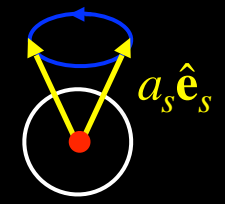


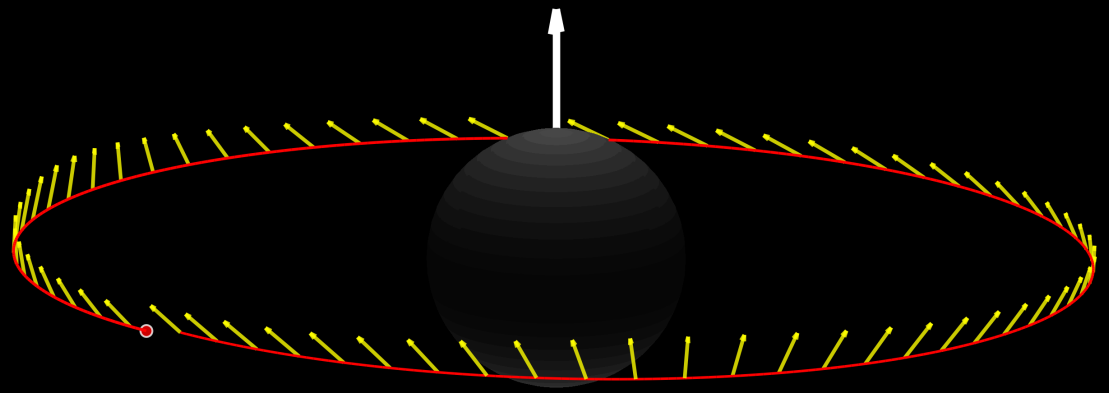
- For equatorial orbits, $\beta = \pi/2$ corresponds to aligned spin.
- $J_{\rm orb}$ is aligned
- GW emission is “isotropic”.

Fluxes with Spin



- $\beta \neq \pi/2$ corresponds to misaligned spins.
- Spin-precession occurs.
- GW radiation becomes “anisotropic”.





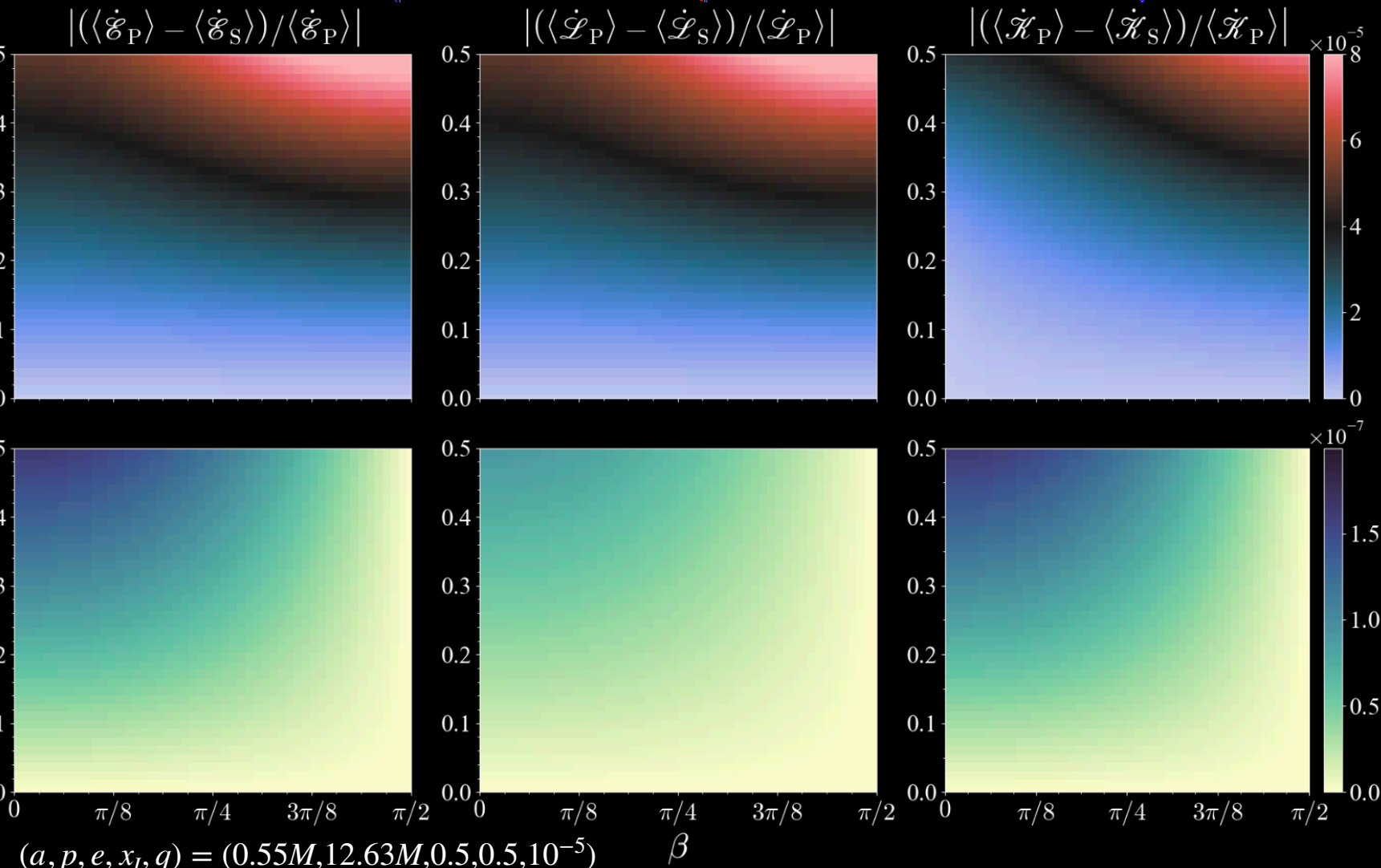
Fluxes with Spin

Top panel: Relative difference of aerodynamical fluxes for EMRI with secondary with/without spin.

Bottom panel: Relative difference of GW fluxes for EMRI with secondary with/without spin.

Interesting features:

- $\Delta\dot{\mathscr{C}}/\mathscr{C}$ for the aero case is $\sim 10^2\times$ higher.



$(a, p, e, x_I, q) = (0.55M, 12.63M, 0.5, 0.5, 10^{-5})$

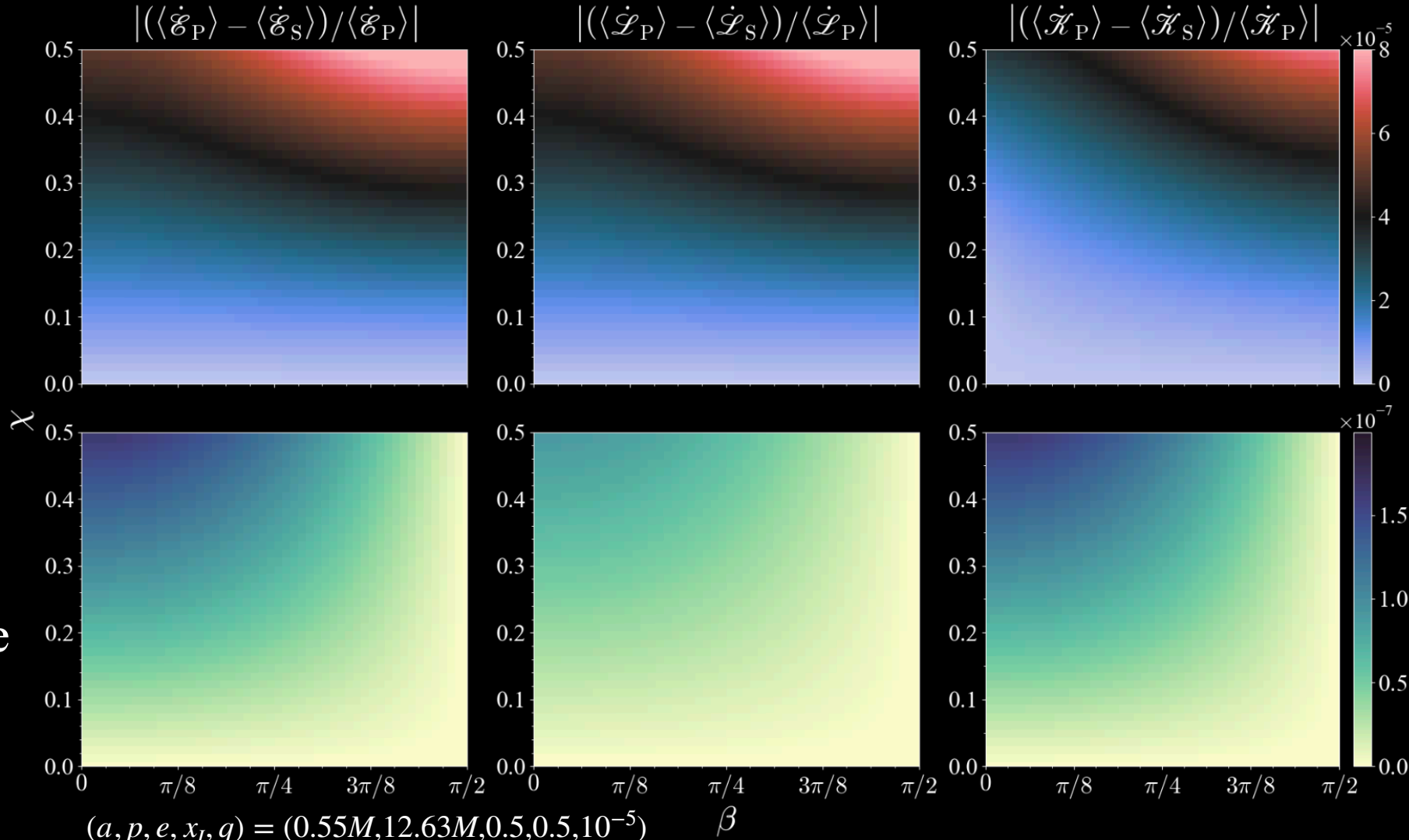
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Top panel: Relative difference of aerodynamical fluxes for EMRI with secondary with/without spin.

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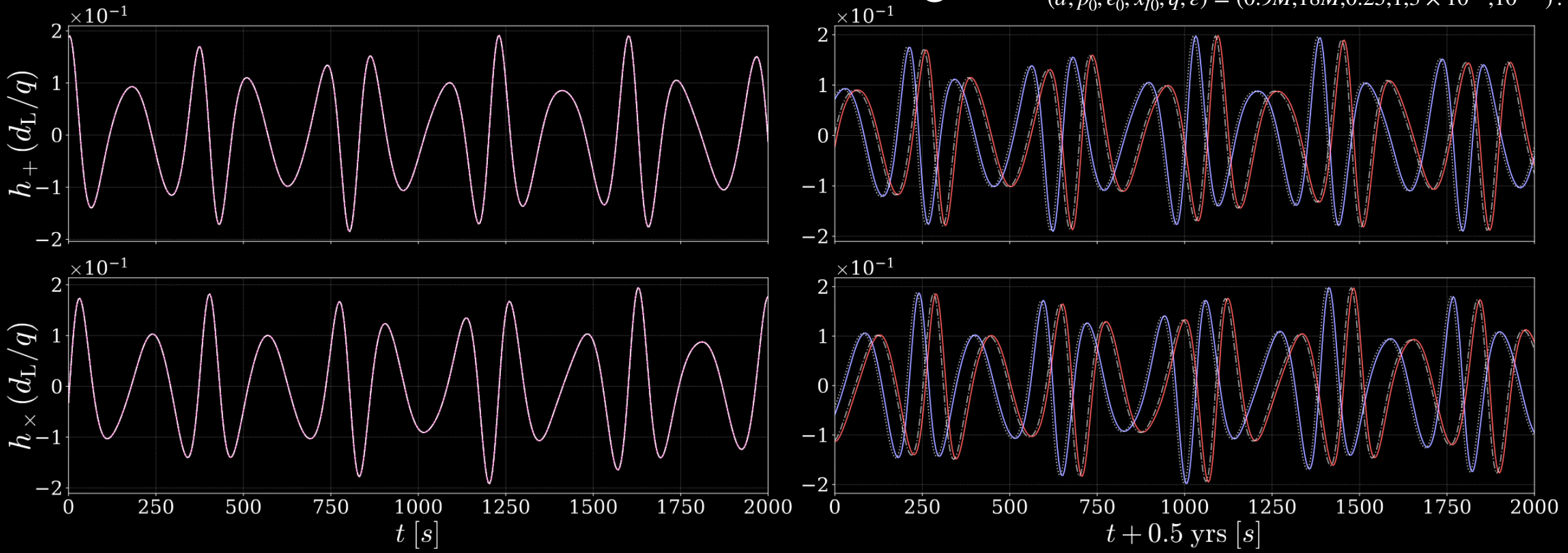
- $\Delta\dot{\mathscr{C}}/\mathscr{C}$ for the aero case is $\sim 10^2\times$ higher.
- Spin-aero effects may be more discernible.



$(a, p, e, x_I, q) = (0.55M, 12.63M, 0.5, 0.5, 10^{-5})$

Observables: GW Signals

$(a, p_0, e_0, x_{I0}, q, \varepsilon) = (0.9M, 18M, 0.25, 1, 3\times10^{-5}, 10^{-12})$.

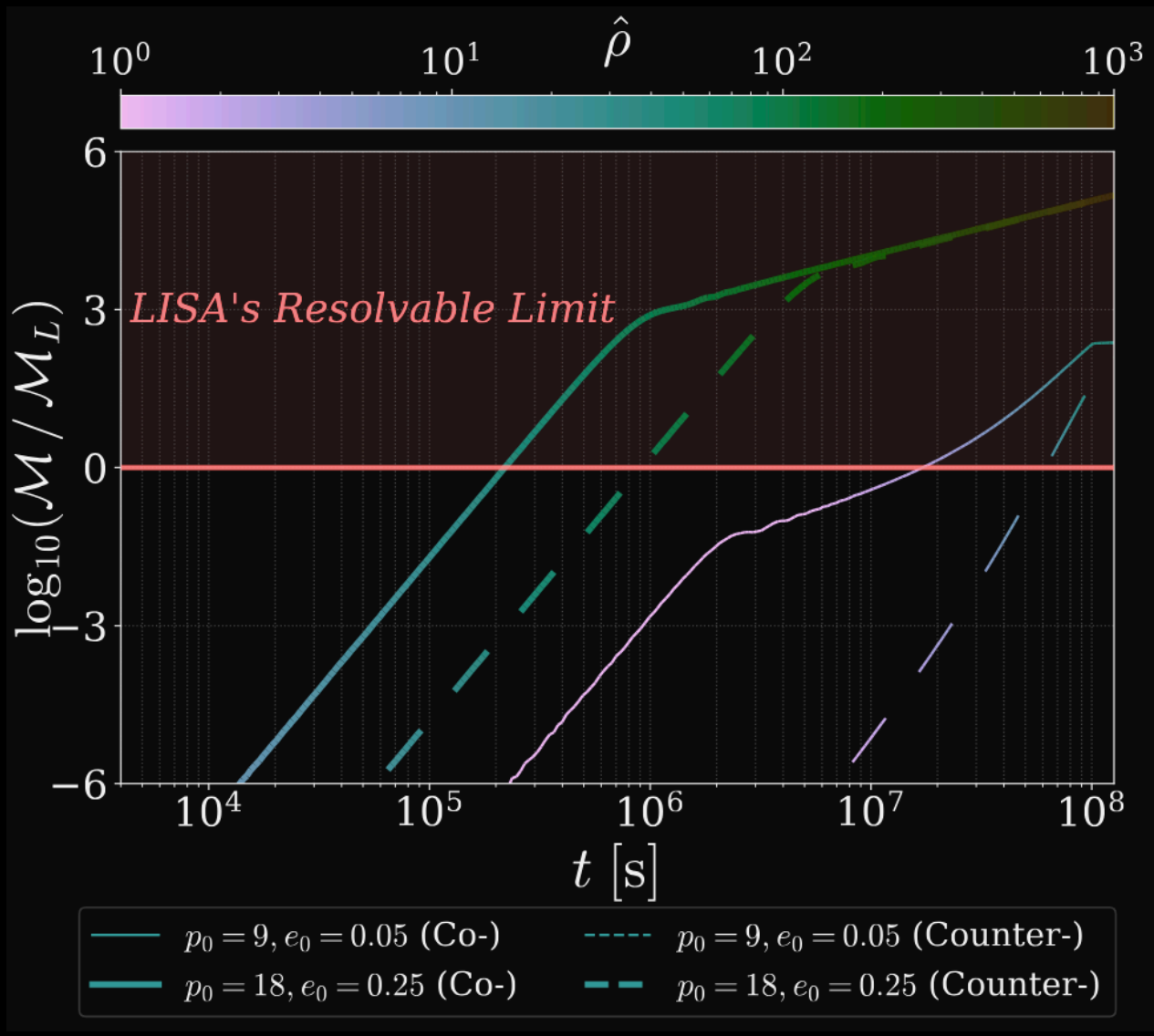


Observables: GW Signals

- Lindblom criteria

$$\rho \geq \sqrt{\frac{D}{2\mathscr{M}_L}}.$$

- Mismatch with and without spins are smaller.
- EEs dominate far from LSO.
- Co-rotating case produce larger mismatches.

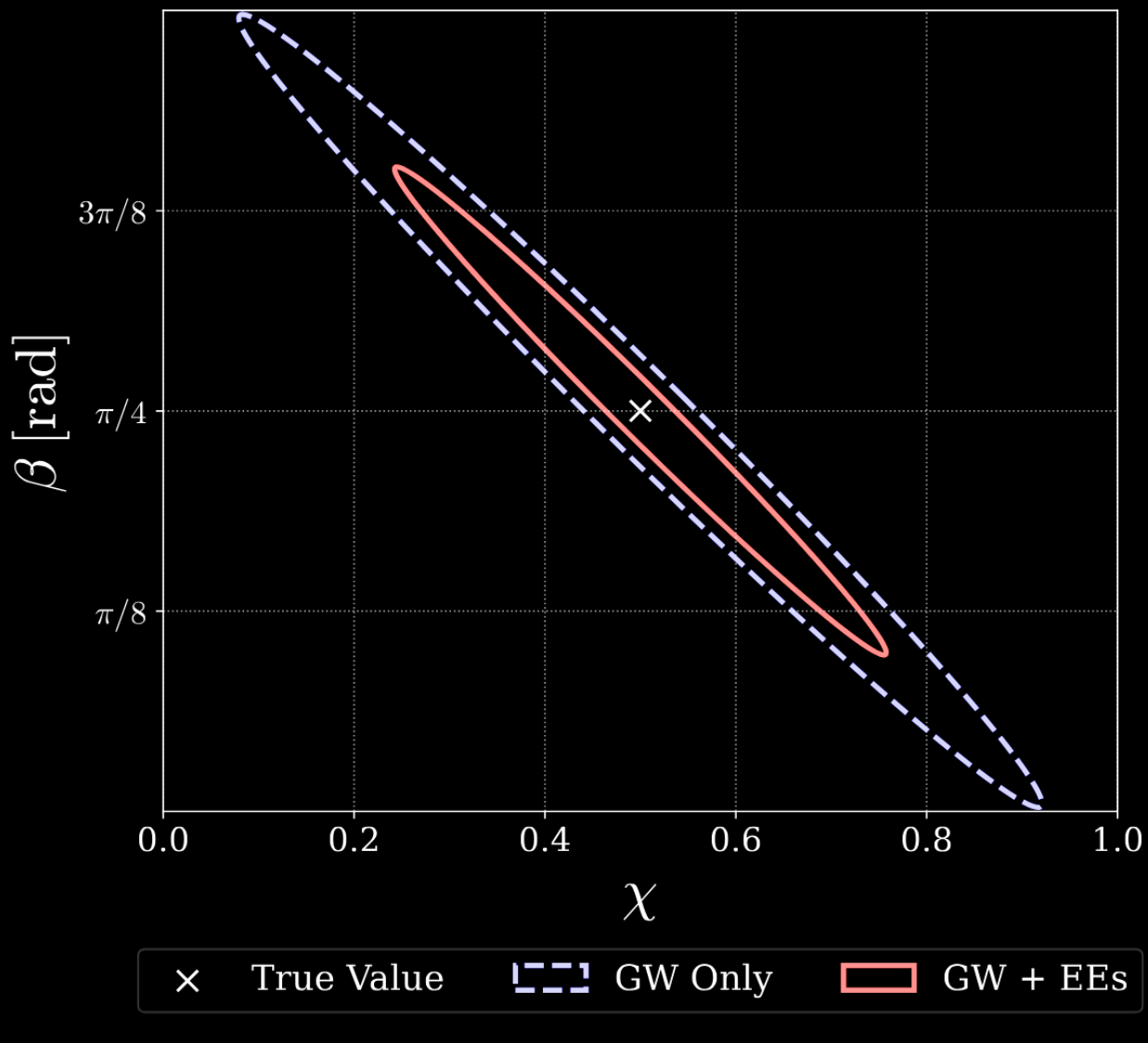


Constraints: Fisher Analysis

- Gaussian priors with $\sigma_\beta = \pi/2$ and $\sigma_\chi = 1$.

  [Piovano, et al. (2021).]

- Spin-induced EEs produce larger dephasing.
- Improving χ and β constraints.
- Here $\epsilon = 10^{-13}$.



Summary and Conclusion

What we did:

- Developed a simple model to incorporate spin-induced EEs
- Calculated the corresponding inspiral and GW signals.
- Performed FM analysis to constrain χ and β .

Our Findings:

- Spinning EE fluxes depend more strongly on the geometry in the LIF.
- This produces a larger spin-induced dephasing.
- Our FM analysis reveals that EEs improve constrains on χ and β .

Summary and Conclusion

Some caveats:

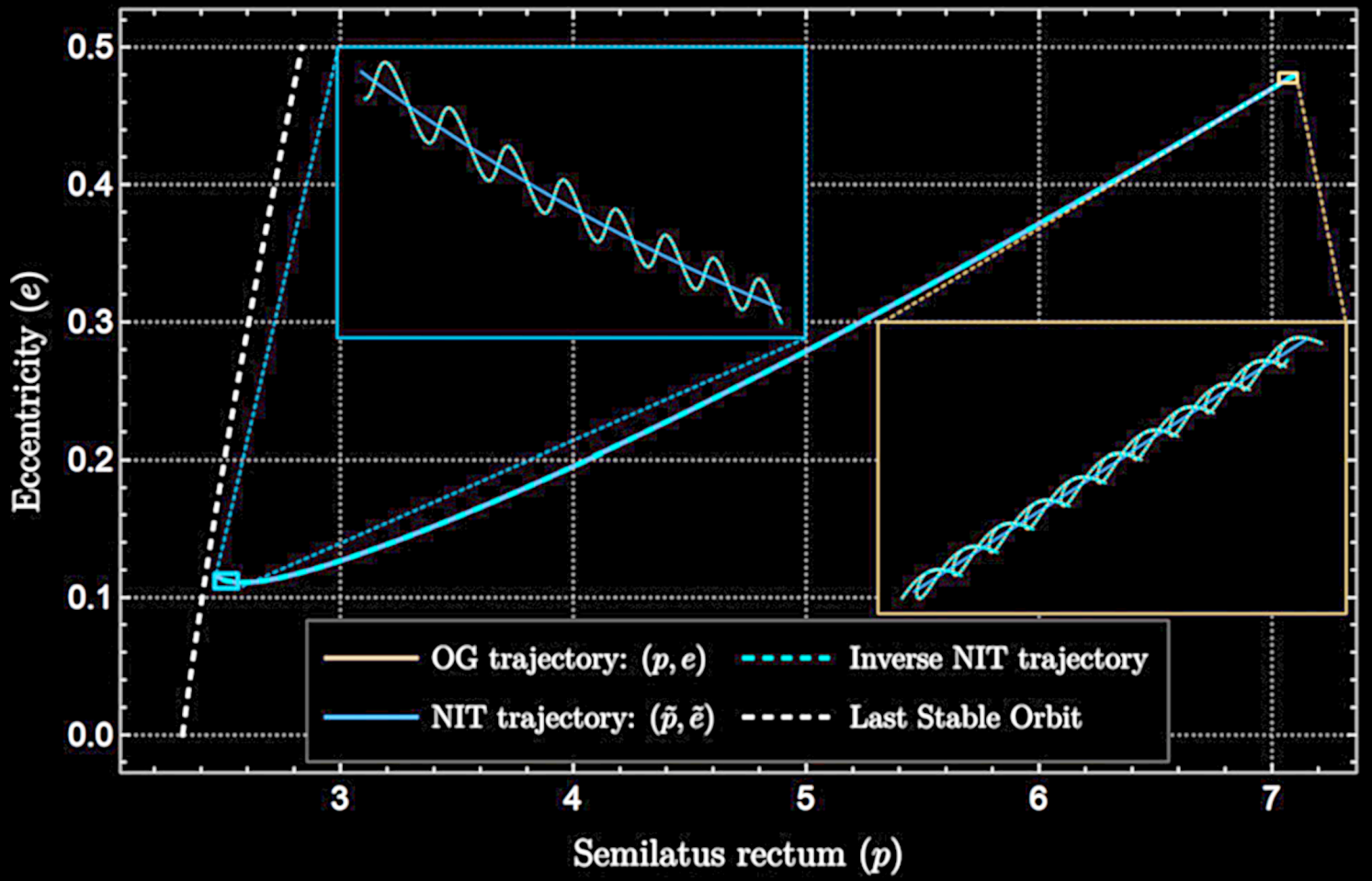
- Only includes “local” environmental interactions.
- Neglects resonances. [Duque, et al. (2025). Hegade, et al. (2025)].
- Only Fisher analysis.
- Neglects self-torque effects. [Matthews, et al. (2026)].
- Doesn’t consider Hoyle-Lyttleton effect (upcoming project).

Future Work

Some interesting questions

- Can the inclusion of PA terms improve spin constraints?

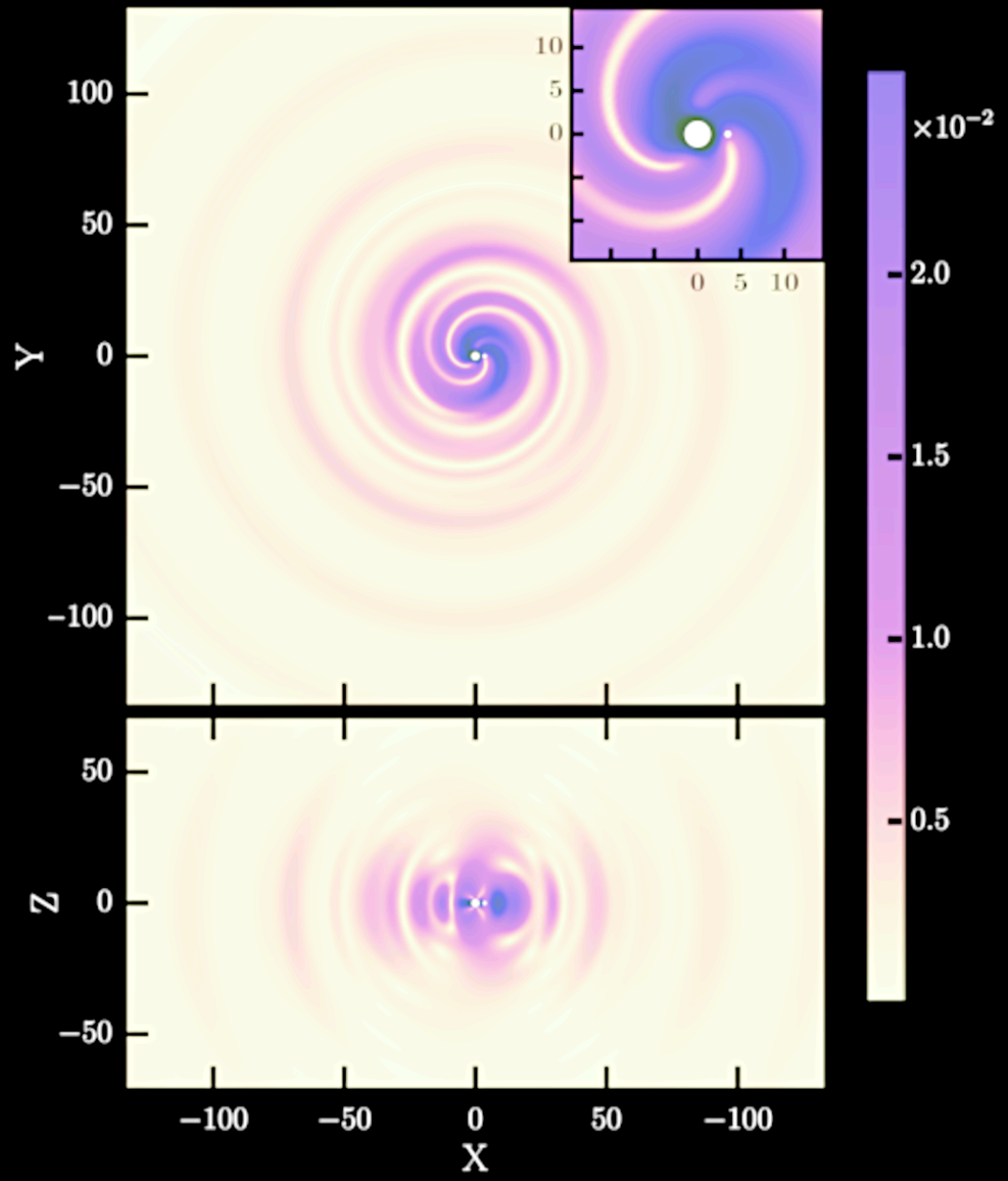
[Lynch, et al. (2022).]



Future Work

Some interesting questions

- Can the inclusion of PA terms improve spin constraints?
- Perturbations to the environments?



[Dyson, et al. (2025).]

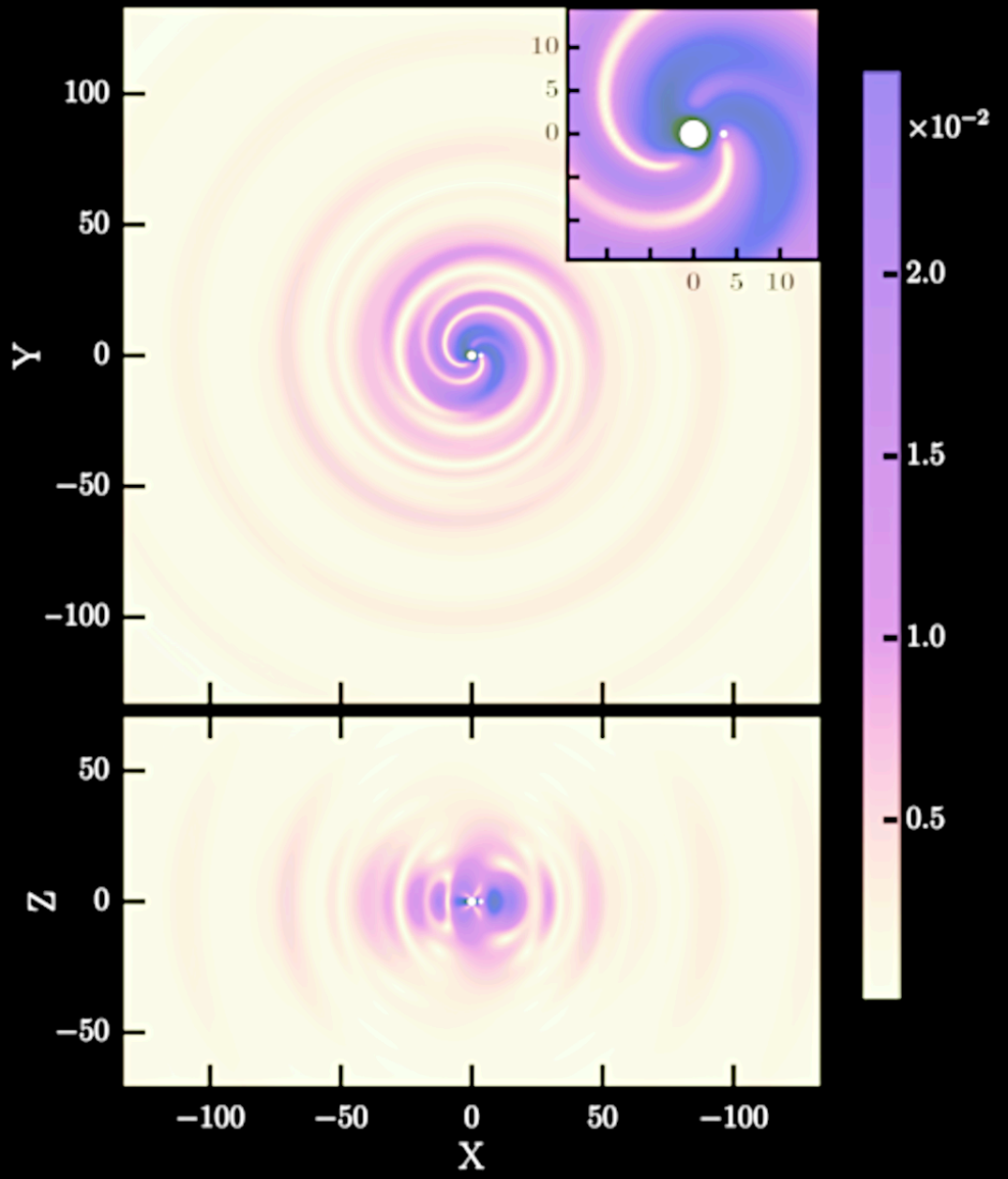
Future Work

Some interesting questions

- Can the inclusion of PA terms improve spin constraints?
- Perturbations to the environments?
- Relevant for IMRIs?

$$f^{\mu} = q\left[f^{\mu}_{(1,0,0)} + s f^{\mu}_{(1,1,0)} + \epsilon\left(f^{\mu}_{(1,0,1)} + s f^{\mu}_{(1,1,1)}\right)\right] + \mathcal{O}(q^2, s^2, \epsilon^2),$$

- Do EEs dominate?



[Dyson, et al. (2025).]

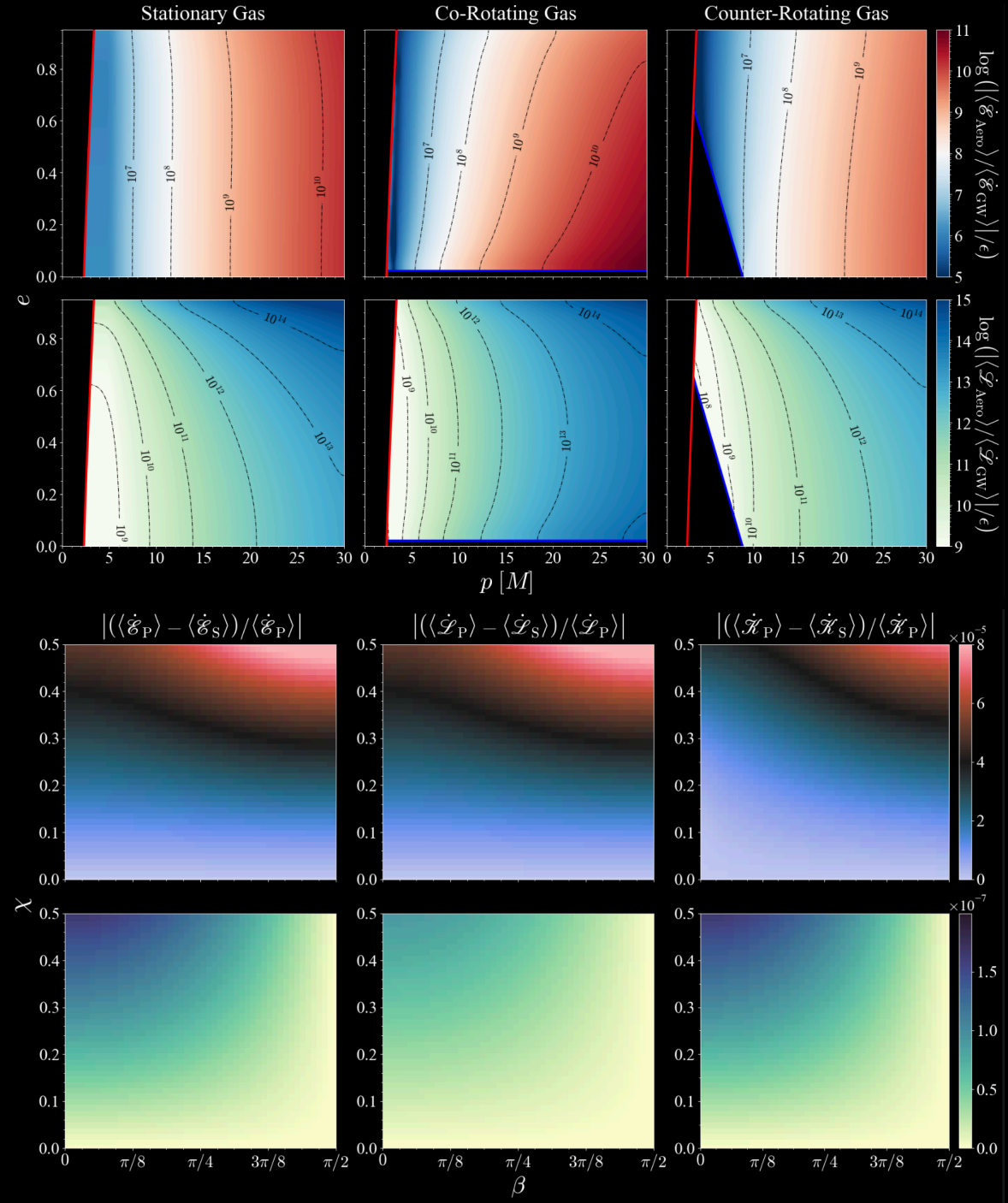
Future Work

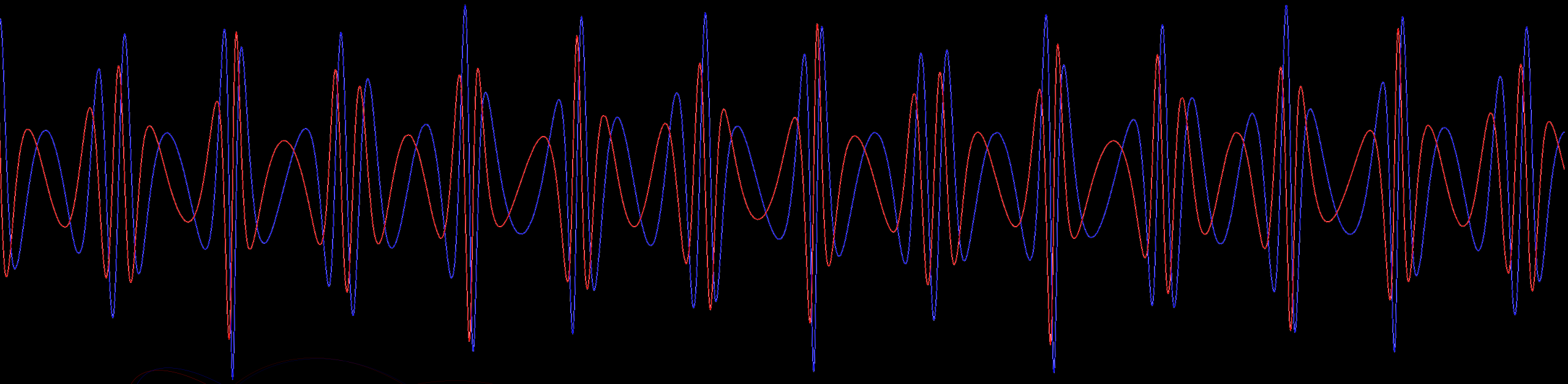
Some interesting questions

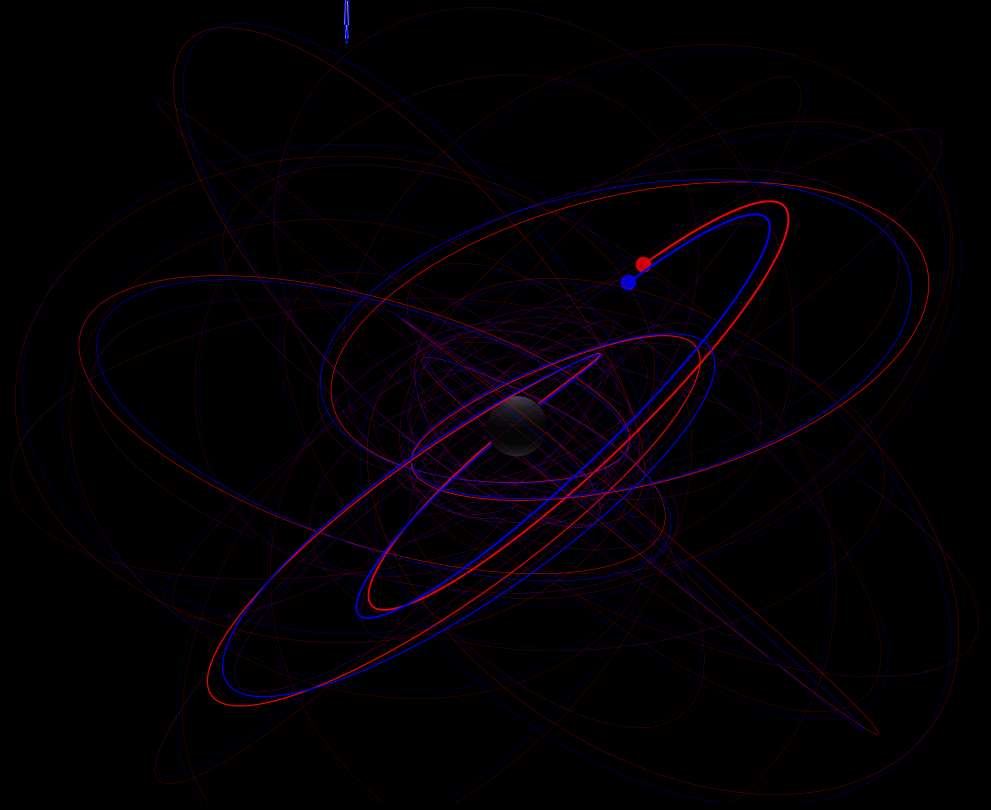
- Can the inclusion of PA terms improve spin constraints?
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$$f^{\mu} = q\left[f^{\mu}_{(1,0,0)} + s f^{\mu}_{(1,1,0)} + \epsilon\left(f^{\mu}_{(1,0,1)} + s f^{\mu}_{(1,1,1)}\right)\right] + \mathcal{O}(q^2, s^2, \epsilon^2),$$

- Do EEs dominate?
- Eventually incorporate into FEW?







Thank you for Listening!

Comments and questions are fully welcomed!

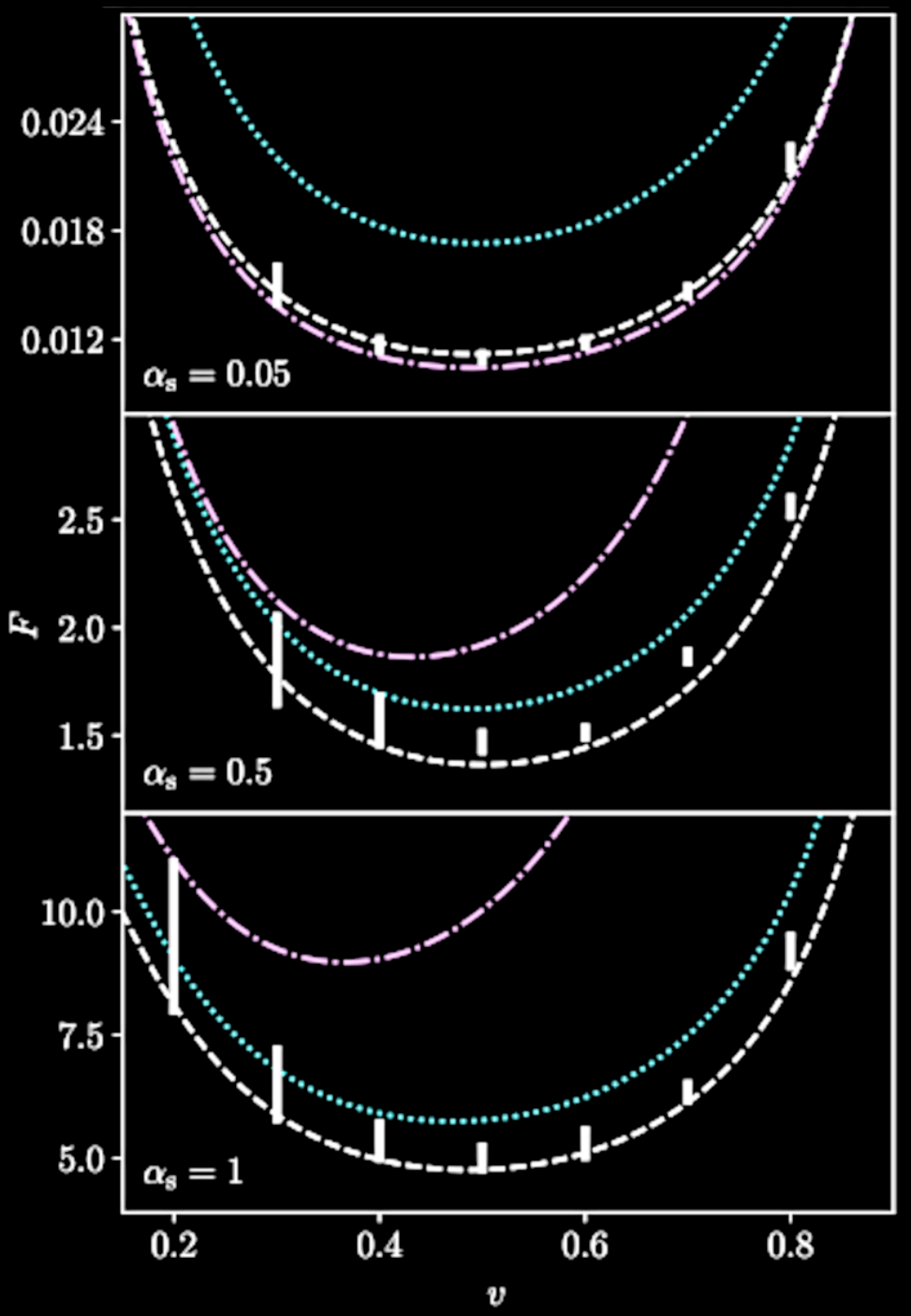
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Beijing Institute
of Mathematical
Sciences and Applications

NUS
National University
of Singapore

Drag Force [Traykova, (2023).]

$$F_{\bar{x}} \approx -\frac{4\pi\rho M^2}{v^2}\gamma^2(1+v^2)^2\Bigg\{\overbrace{\ln\left(\frac{\Lambda}{\beta}\right)-1+\chi(v)}^{\mathcal{D}(\bar{\alpha}_s\gg 1,\Lambda\gg\beta)} + \underbrace{\frac{v^4}{(1+v^2)^2}\left(\frac{b_{\rm cr}(v)}{2M}\right)^2}_{\mathcal{A}(\bar{\alpha}_s\gg 1)\equiv\mathcal{A}_{\rm particle}}\Bigg\},$$

$$\left(\frac{b_{\rm cr}}{M}\right)^2 \coloneqq \frac{-1+8v^4+\sqrt{1+8v^2}+4v^2(5+2\sqrt{1+8v^2})}{2v^4}$$



Magnus and Lift Forces [Dyson, et al. (2024).]

$$\delta F_D^{(P)} = a^2(1.11 - 3.26v + 0.74a - 1.07\sqrt{v}a - 0.59\cos\beta + 1.38\sqrt{v}\cos\beta),$$

$$F_M^{(P)} = a\sin\beta(2.55 - 31.81\sqrt{v} + 69.81v - 59.15v^{3/2} - 0.12a\sqrt{v}\cos\beta),$$

$$F_L^{(P)} = a^2\sin 2\beta(0.82 - 2.60\sqrt{v} + 5.06v - 3.79v^{3/2} + 0.28a + 1.28va - 0.26\sqrt{v}a\cos\beta + 0.11a\cos\beta - 1.22\sqrt{v}a - 0.02\cos\beta + 0.04\sqrt{v}\cos\beta + 0.23\sqrt{v}a^2),$$

$$\delta F_D^{(W)} = a^2(-16.45 - 30.84\sqrt{v} + 316.40v - 296.10v^{3/2} + 5.44\cos^2\beta - 23.92\sqrt{v}\cos^2\beta + 12.25\sqrt{v}a^2 - 9.16a^2),$$

$$F_M^{(W)} = a\sin\beta(36.45 - 241.9\sqrt{v} + 415.8v - 248.6v^{3/2}),$$

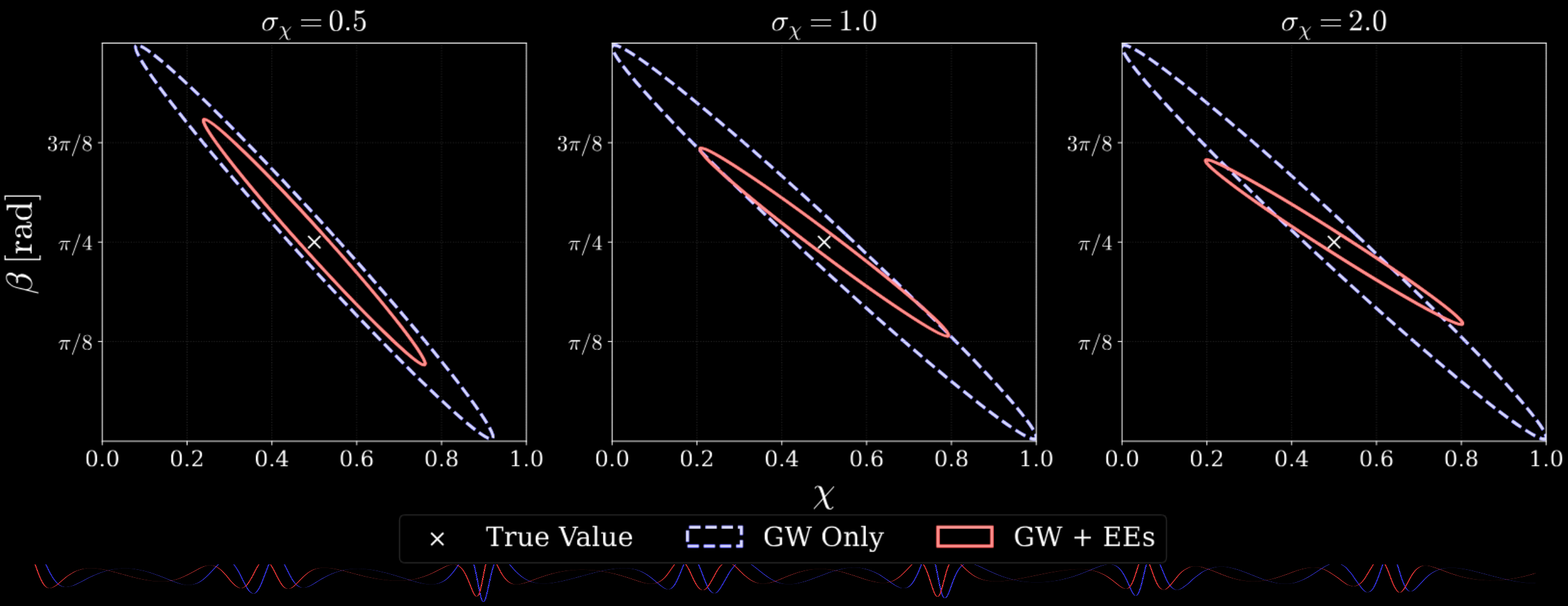
$$F_L^{(W)} = a^2\sin 2\beta(-1.023463 + 58.2938v - 165.9078v^{3/2} + 150.6706v^2).$$

Deriving the fluxes

$$\frac{\mathrm{d}\mathscr{E}}{\mathrm{d}\lambda} = -\xi^{(t)}_{\mu;\nu}u^\nu u^\mu - \xi^{(t)}_\mu\left(\frac{Du^\mu}{\mathrm{d}\lambda}\right) + \frac{1}{2}\xi^{(t)}_{\mu;\nu;\rho}u^\rho s^{\mu\nu} + \frac{1}{2}\xi^{(t)}_{\mu;\nu}\left(\frac{Ds^{\mu\nu}}{\mathrm{d}\lambda}\right).$$

- Term 1: Vanishes by Killing equation.
- Term 2: $-\xi^{(t)}_\mu\left(-\frac{1}{2}R^\mu{}_{\nu\rho\sigma}u^\nu s^{\rho\sigma} + f^\mu\right) = \frac{1}{2}R_{\mu\nu\rho\sigma}\xi^{(t)\mu}u^\nu s^{\rho\sigma} - \xi^{(t)}_\mu f^\mu$
- Term 3: $-\frac{1}{2}R_{\sigma\rho\mu\nu}\xi^\sigma u^\rho s^{\mu\nu}$
- Term 4: $\frac{Ds^{\mu\nu}}{d\tau} = \left(\frac{1}{2u^\lambda p_\lambda}s^{\mu\rho}R_{\rho\alpha\beta\gamma}u^\alpha s^{\beta\gamma}\right)u^\nu - \left(\frac{1}{2u^\lambda p_\lambda}s^{\nu\rho}R_{\rho\alpha\beta\gamma}u^\alpha s^{\beta\gamma}\right)u^\mu \sim \mathcal{O}(s^2).$

Dependence on Priors



Bondi-Hoyle-Lyttleton Accretion

