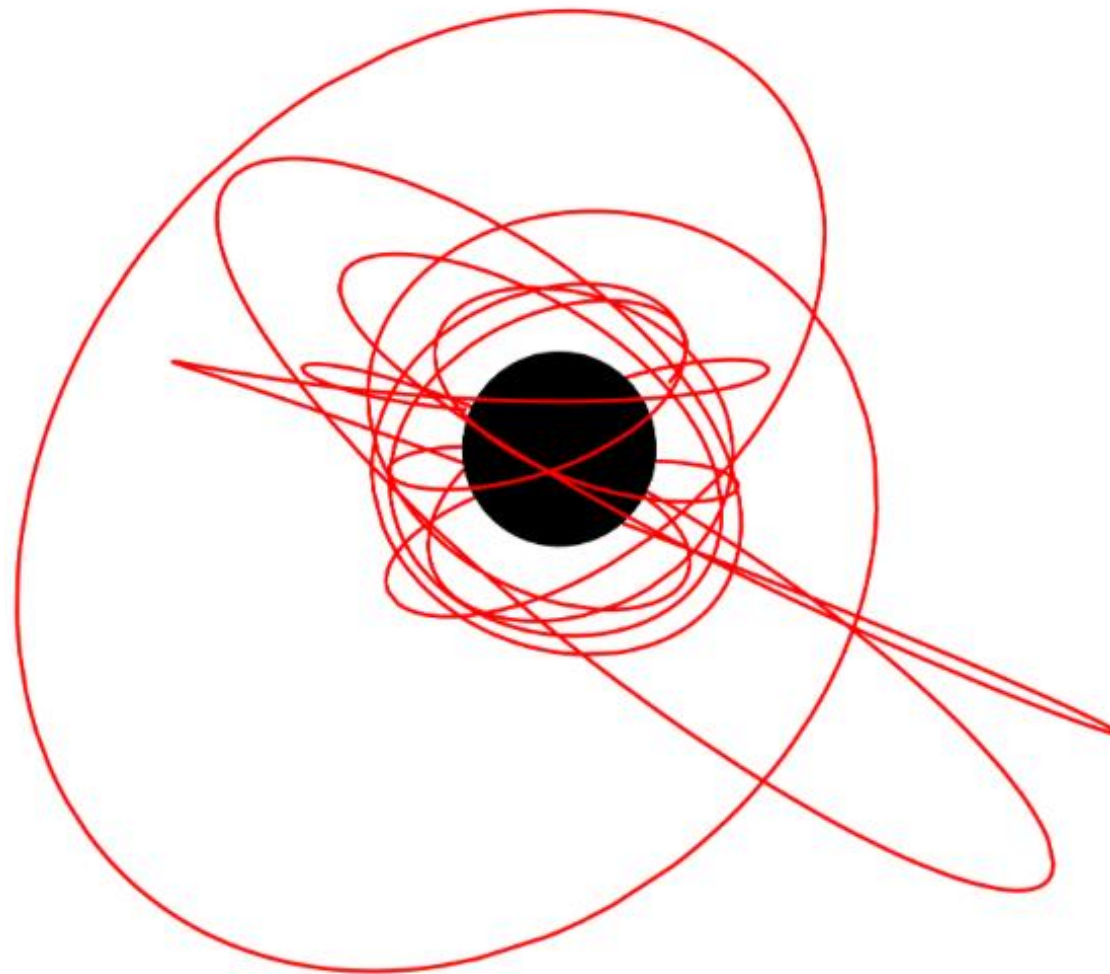


Towards EMRI Search via f-function

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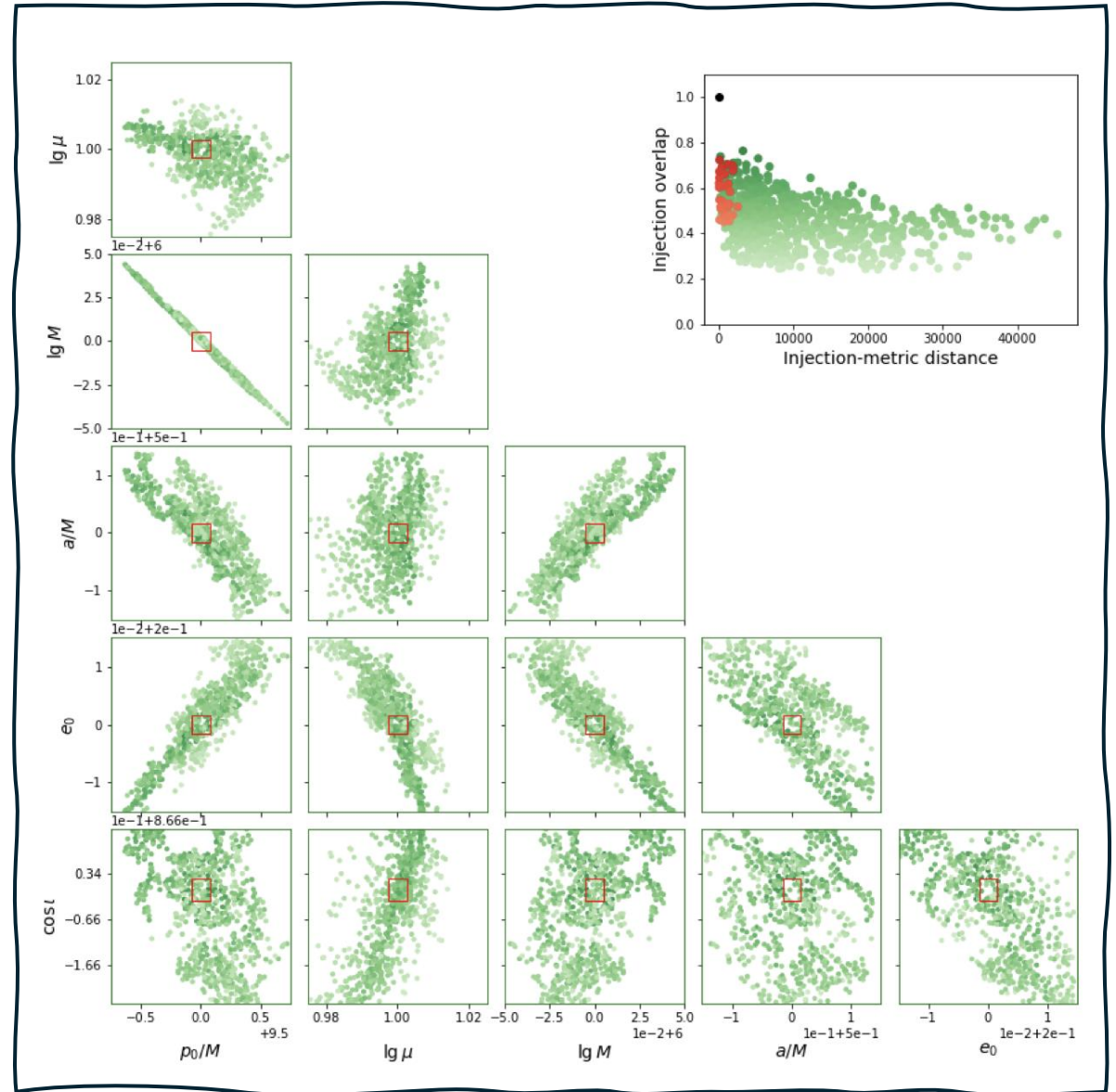


The EMRI Challenge

Strong presence of non-local degeneracies

Cause: similar initial frequencies & their time derivatives

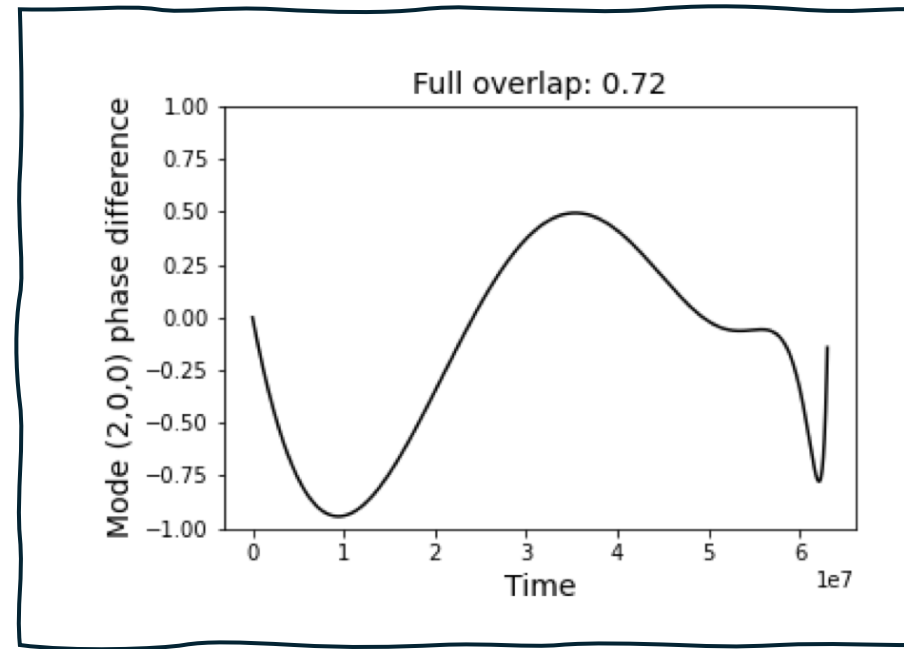
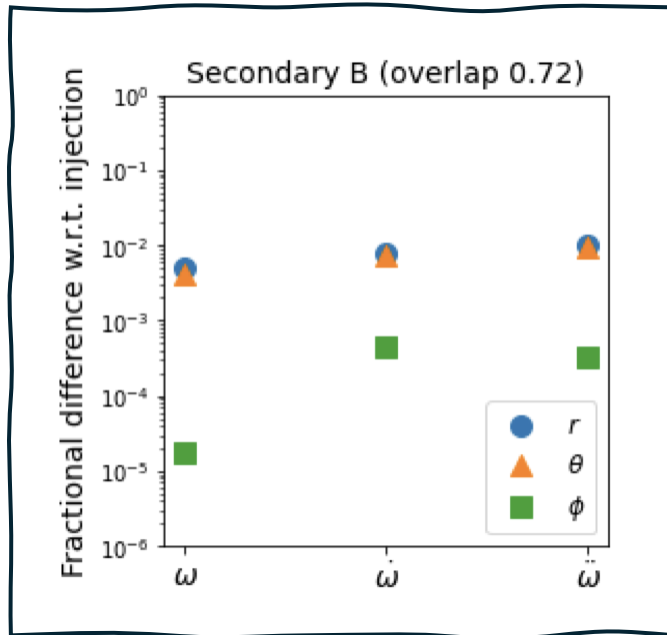
→ phase-aligned





The EMRI Challenge

Key finding: dominant mode of secondaries are phase-aligned with primary, other modes are not





***f*-function**

Suppression through mismatch between *recovered* and *optimal* per mode SNR

$$[\mathbf{X}(\theta)]_{\text{m}} = X_{\text{m}}(\theta) = \frac{\langle x | h_{\text{m}}(\theta) \rangle}{\rho_{\text{m}}(\theta)}$$

$$[\boldsymbol{\rho}(\theta)]_{\text{m}} = \rho_{\text{m}}(\theta) = \sqrt{\langle h_{\text{m}}(\theta) | h_{\text{m}}(\theta) \rangle}$$

Chi-square like statistic: $\chi^2(\theta) = |\mathbf{X}(\theta) - \boldsymbol{\rho}(\theta)|^2$



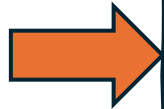
***f*-function**

Suppression through mismatch between *recovered* and *optimal* per mode SNR

$$[\mathbf{X}(\theta)]_{\text{mm}} = X_{\text{mm}}(\theta) = \frac{\langle x | h_{\text{mm}}(\theta) \rangle}{\rho_{\text{mm}}(\theta)}$$

$$[\boldsymbol{\rho}(\theta)]_{\text{mm}} = \rho_{\text{mm}}(\theta) = \sqrt{\langle h_{\text{mm}}(\theta) | h_{\text{mm}}(\theta) \rangle}$$

Chi-square like statistic: $\chi^2(\theta) = |\mathbf{X}(\theta) - \boldsymbol{\rho}(\theta)|^2$


$$f(\theta) = X(\theta) \exp\left(-\frac{1}{2}\beta(\theta)\chi^2(\theta)\right)$$

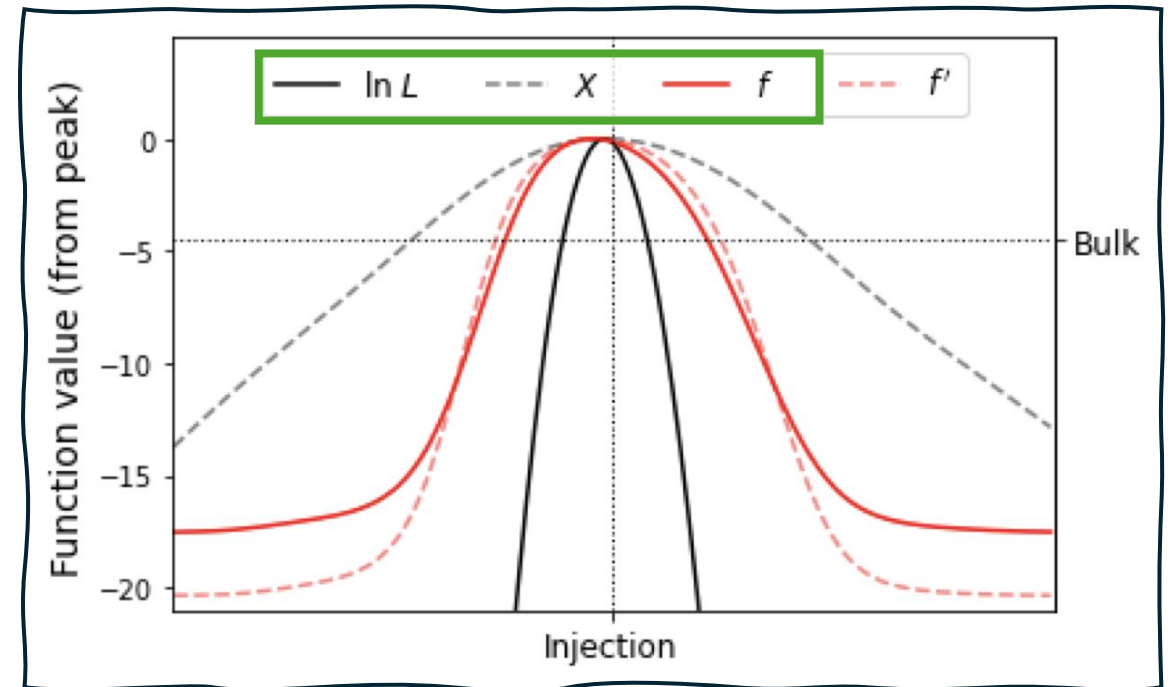
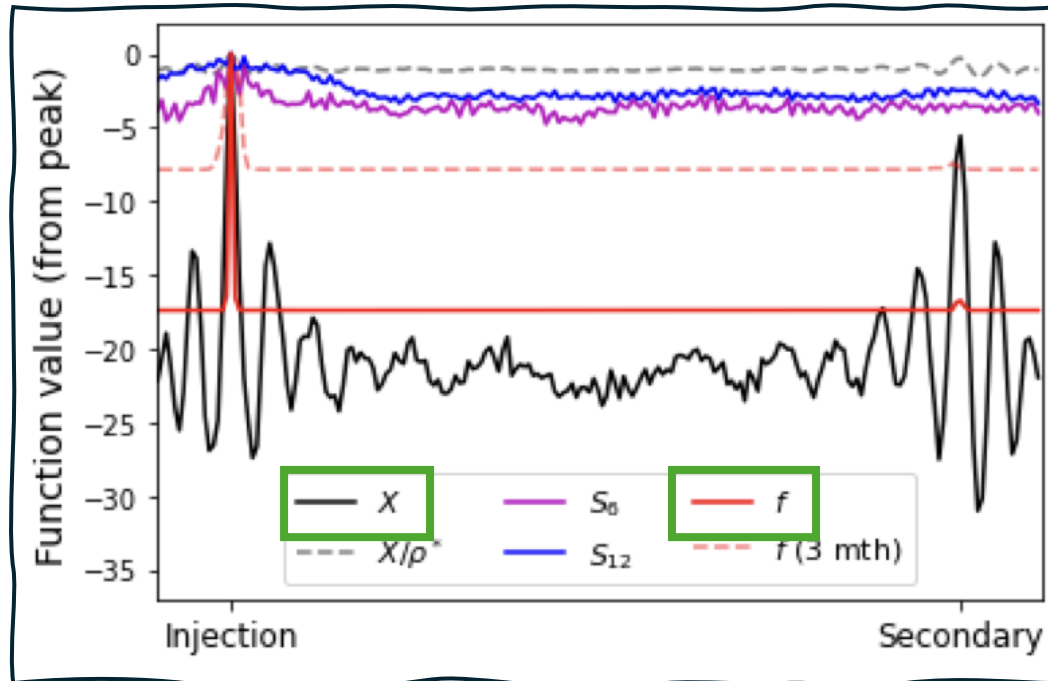
$\beta(\theta)$ = calibration factor

Valid for uncorrelated modes with small residual



f -function

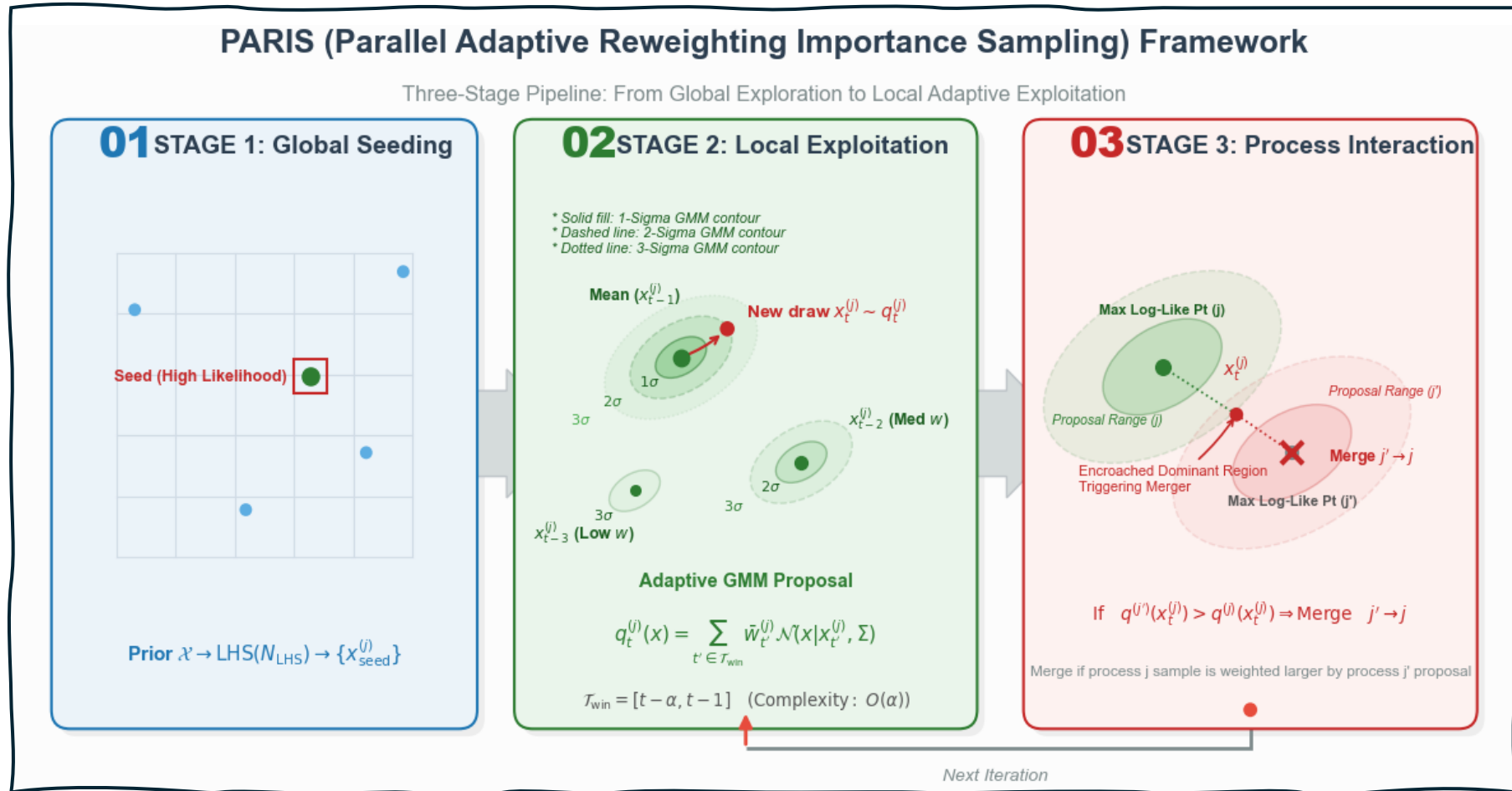
Suppression of secondaries (and noise)



Goal: Implement f-function



PARIS

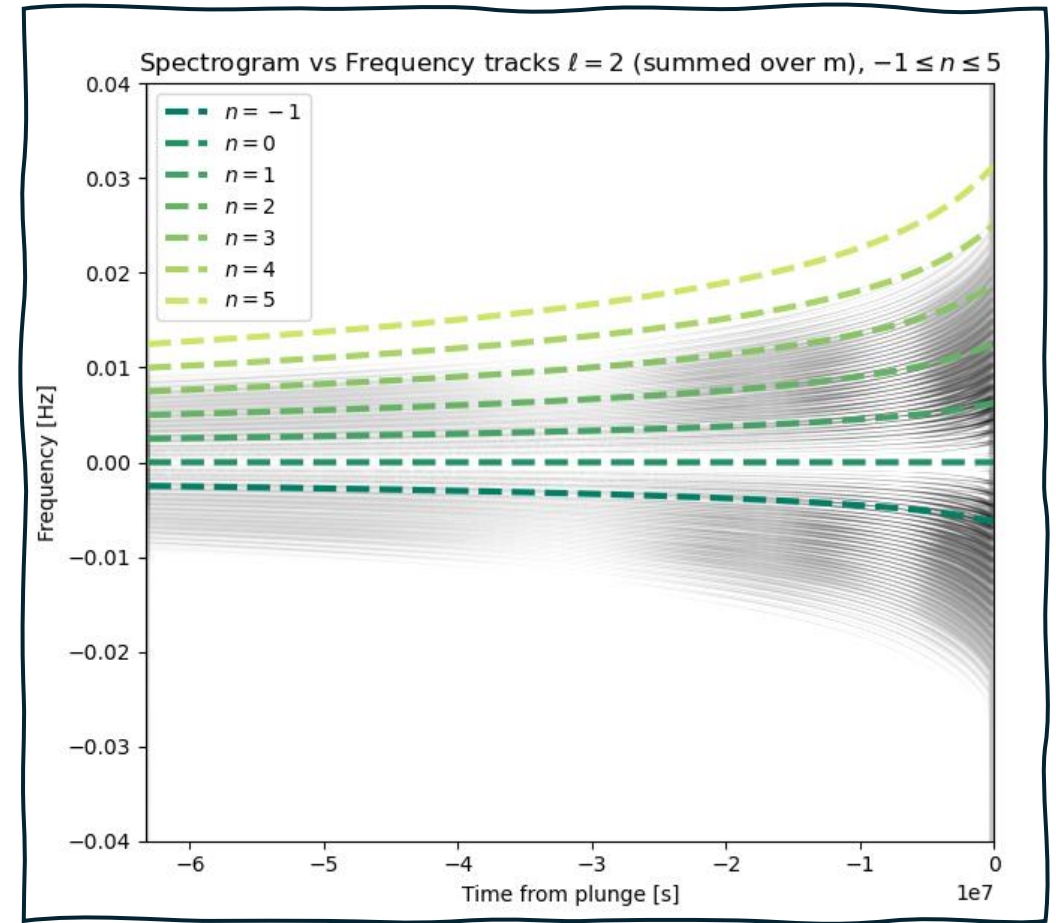




Mode groups

Select **orthogonal** mode groups: $h_n(\theta) = \sum_m h_{mn}^{\ell=2}(\theta)$

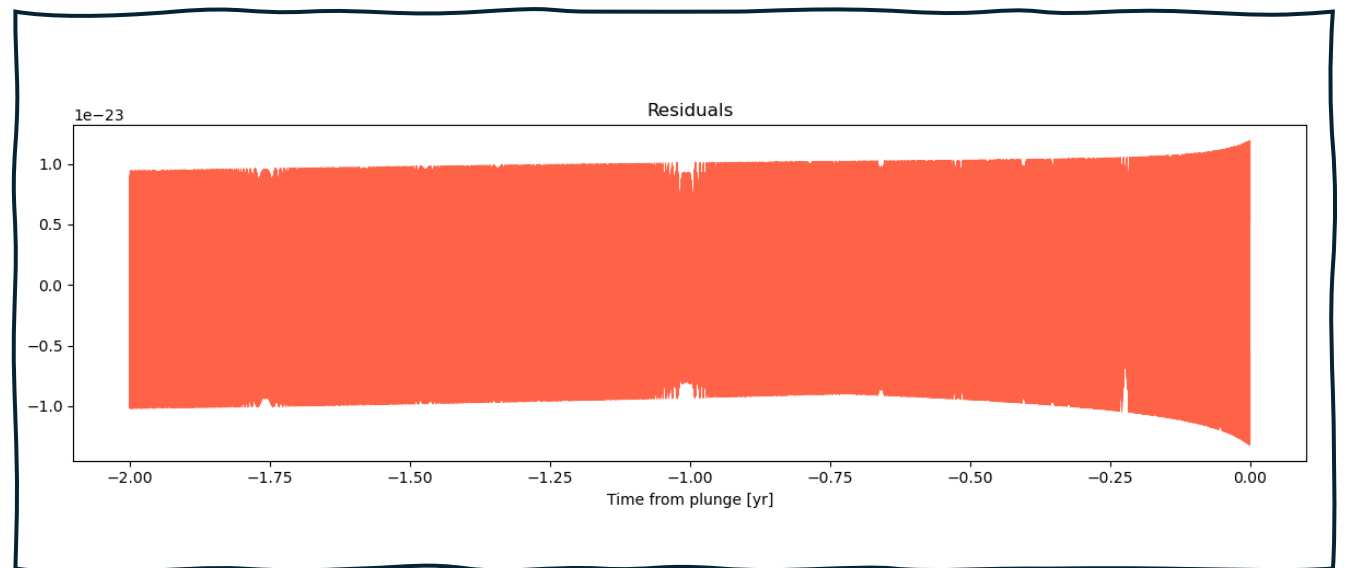
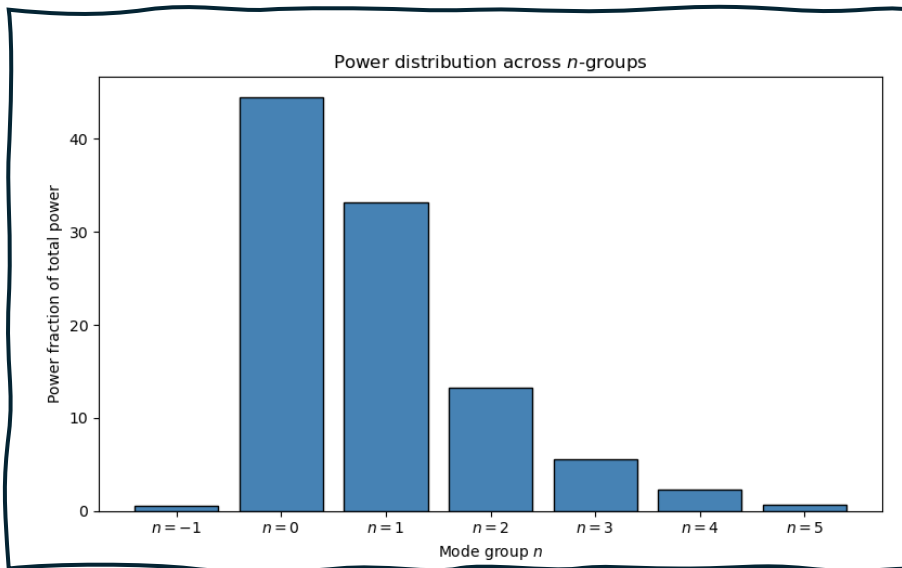
each n occupies a distinct frequency track





Mode groups

Select orthogonal **mode groups**: $h_n(\theta) = \sum_m h_{mn}^{\ell=2}(\theta)$



~80% of the total power are in the selected mode groups



Search Pipeline

Stage 1 Global search

Proof of concept:

f-function

time & phase shift approx. max.
shorter templates (e.g., 3 mths)

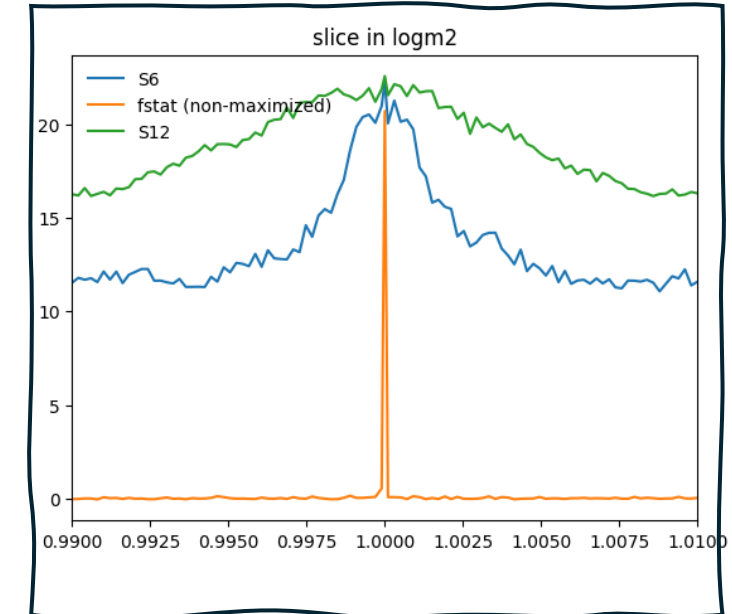
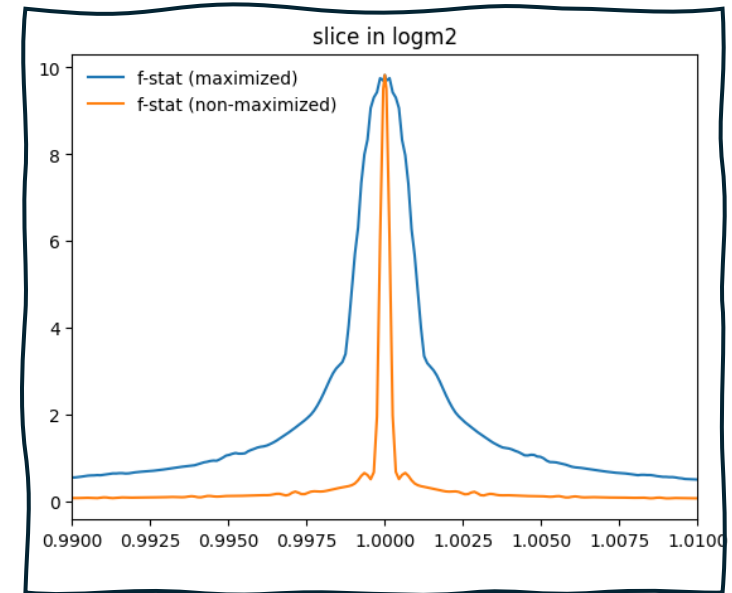
Testing:

Semi-coherent detection SNR
iteratively lower N segments

$N=\{12,6\}$

$$S_N := \frac{\langle x|h \rangle_N}{\sqrt{\langle h|h \rangle_N}} = \frac{\langle x|h \rangle_N}{\rho} = \langle x|\hat{h} \rangle_N.$$

noiseless



noisy



Search Pipeline

Stage 1

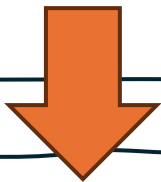
Global search: approximate maximization templates

Proof of concept: f-function **with** time & phase shift approximate maximization for shorter duration templates (3 mths)

Testing:

Semi-coherent detection SNR with iteratively lower N segments $N=\{12,6\}$

$$S_N := \frac{\langle x|h \rangle_N}{\sqrt{\langle h|h \rangle_N}} = \frac{\langle x|h \rangle_N}{\rho} = \langle x|\hat{h} \rangle_N.$$



Stage 2

Local refinement: f-function **without** time & phase shift maximization for full duration template (1 year)



Setup

1-year SNR ~ 14 (noiseless) / ~ 25 (noisy) Kerr Eccentric Equatorial (FEW) waveform

Parameter	Injected	Priors
$\log_{10} m_1 (M_{\odot})$	6.0	[5.6, 6.4]
$\log_{10} m_2 (M_{\odot})$	1.0	[0.8, 1.3]
a	0.7	[0.3, 0.99]
p_0	9	[8.0, 11.0]
e_0	0.4	[0.2, 0.5]

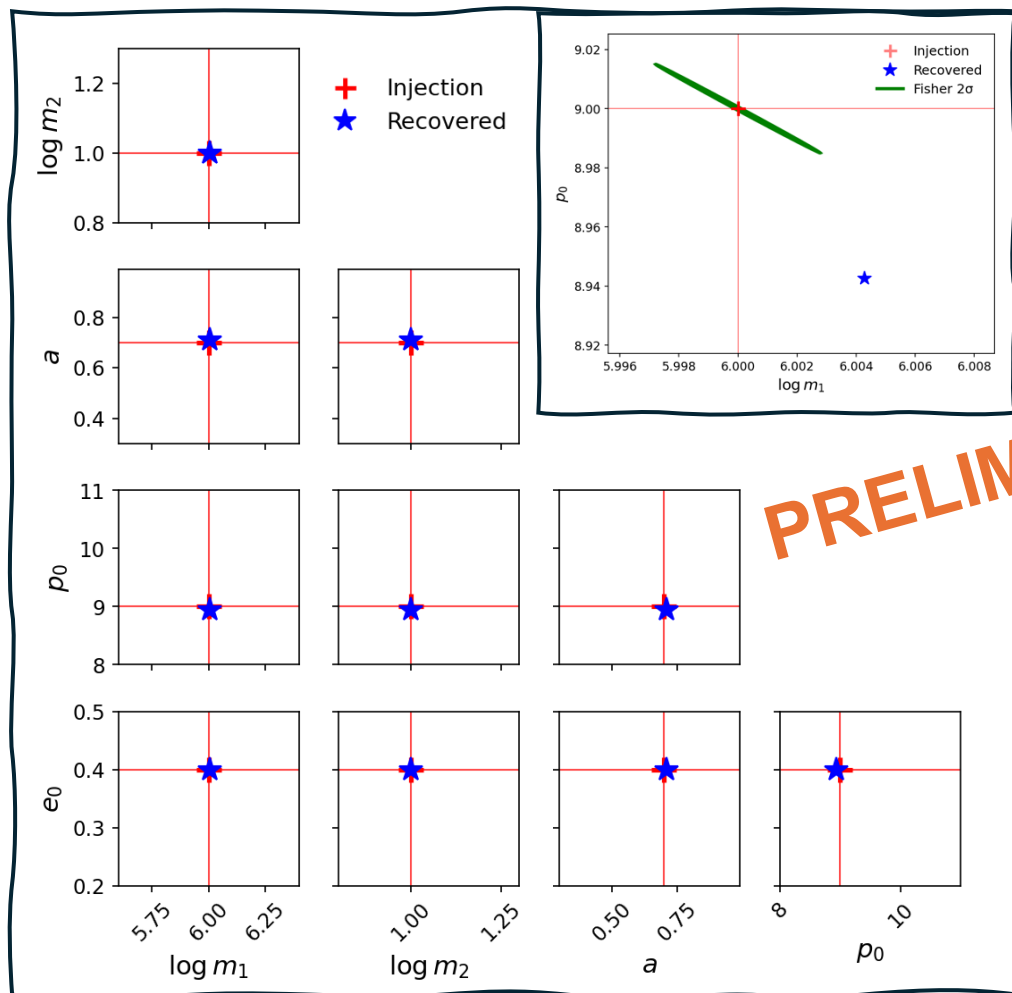
Stage	Step	Calls
1	LHS Seeding	100.000
	Run	~ 100.000 (Excessive)
2	LHS Seeding	100.000
	Run	~ 100.000 (Excessive)
Total		~ 500.000

< 1-3 days



1st stage (Noiseless)

3-mth SNR~6 signal

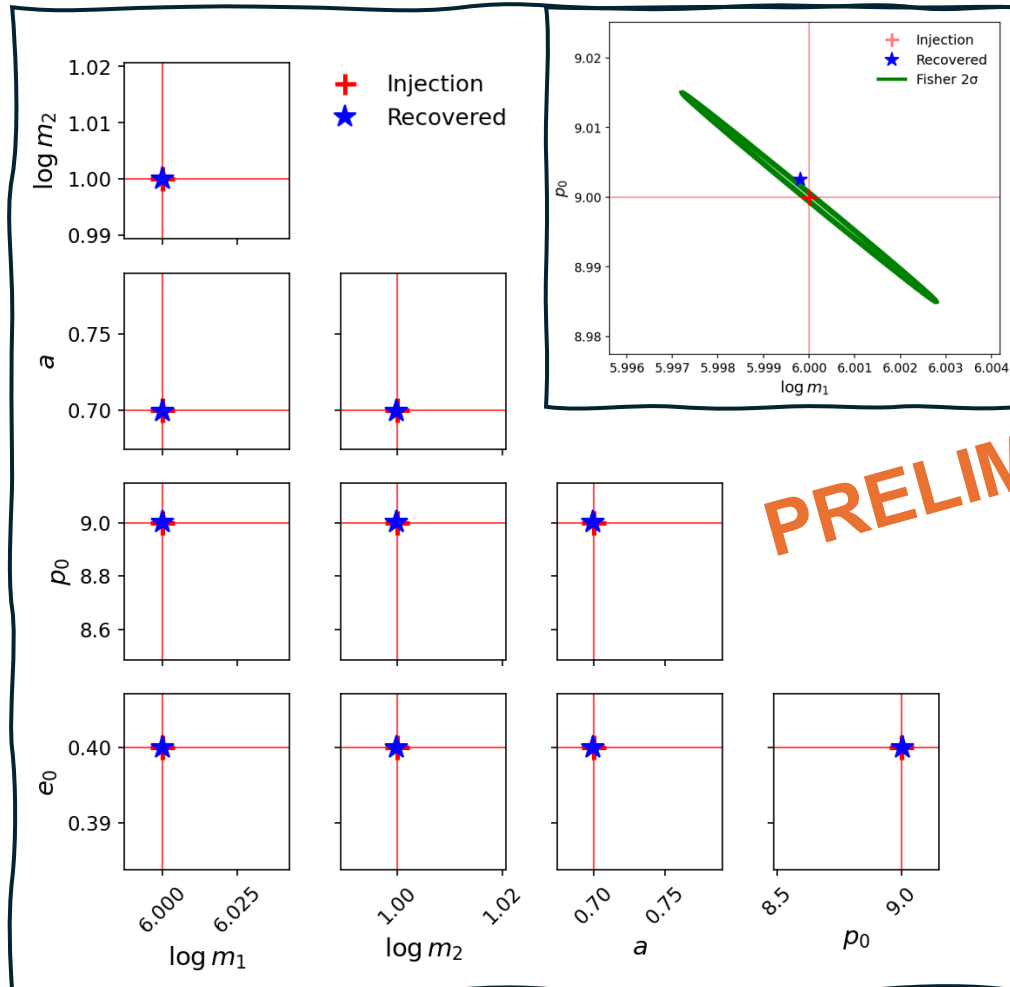


Parameter	Priors	Injected	Recovered
$\log_{10} m_1 (M_\odot)$	[5.6, 6.4]	6.0	6.00428558
$\log_{10} m_2 (M_\odot)$	[0.8, 1.3]	1.0	1.00036002
a	[0.3, 0.99]	0.7	0.7101336
p_0	[8.0, 11.0]	9	8.94251488
e_0	[0.2, 0.5]	0.4	0.3992983



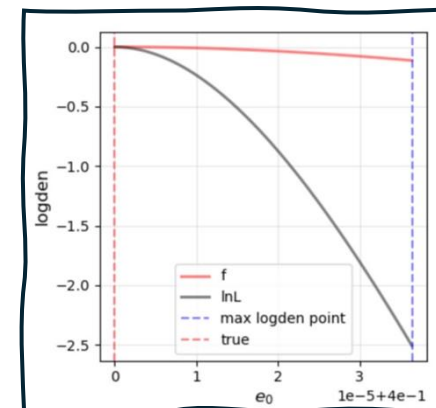
2nd stage (Noiseless)

1-year SNR~14 signal



PRELIMINARY

Parameter	Priors	Injected	Recovered
$\log_{10} m_1 (M_\odot)$	[5.98699, 6.04277]	6.0	5.9998059
$\log_{10} m_2 (M_\odot)$	[0.98935, 1.02067]	1.0	0.9999736
a	[0.67449, 0.78962]	0.7	0.69955029
p_0	[8.48613, 9.14781]	9	9.00257535
e_0	[0.38373, 0.40714]	0.4	0.40003642

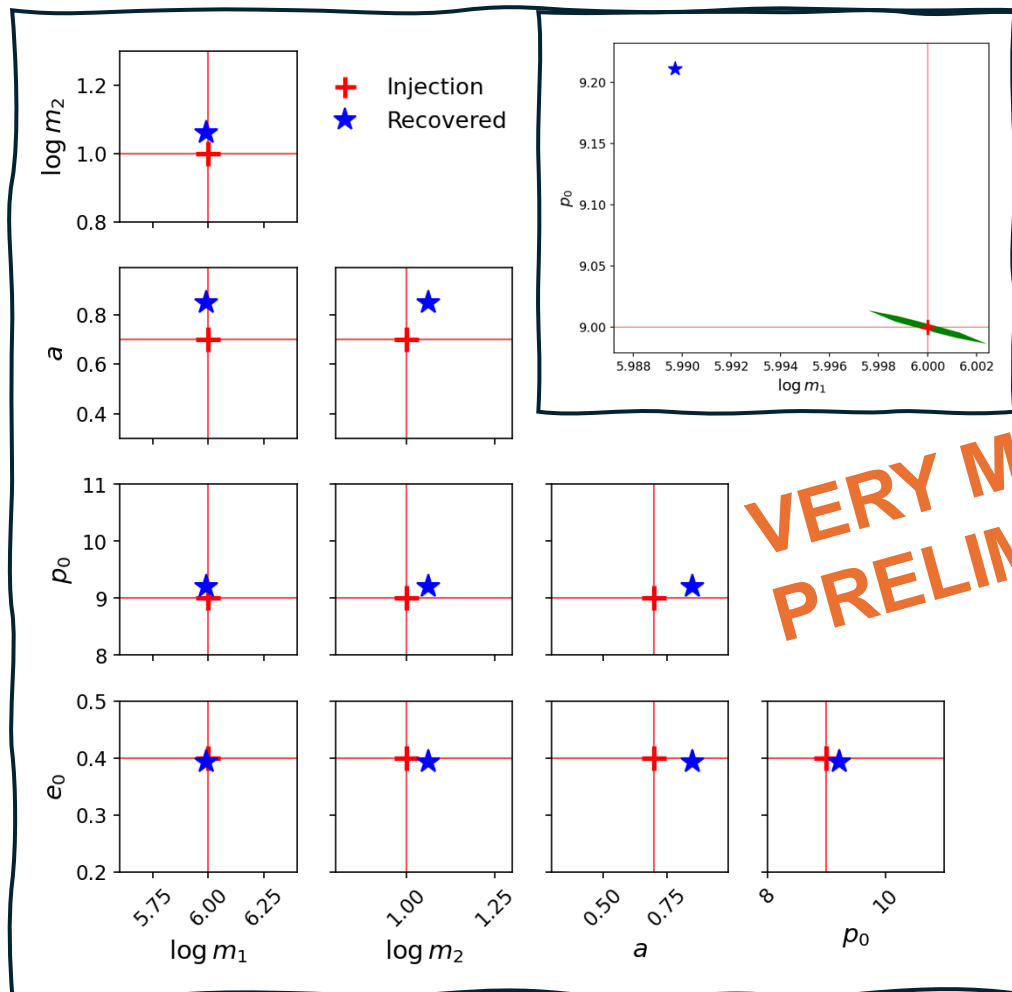




1st stage (Noisy)

1-year SNR~25 signal

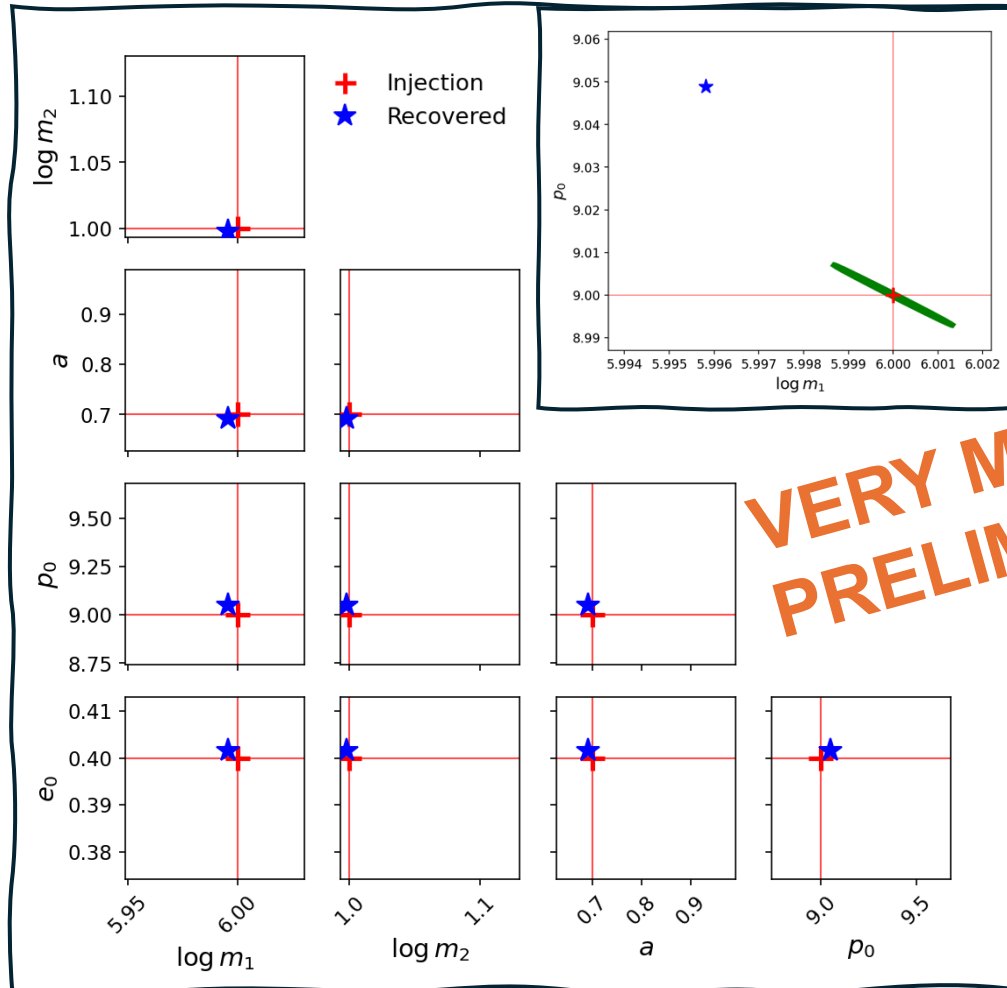
Parameter	Priors	Injected	Recovered
$\log_{10} m_1 (M_\odot)$	[5.6, 6.4]	6.0	5.9897139
$\log_{10} m_2 (M_\odot)$	[0.8, 1.3]	1.0	1.06187119
a	[0.3, 0.99]	0.7	0.84993711
p_0	[8.0, 11.0]	9	9.21099004
e_0	[0.2, 0.5]	0.4	0.39363801





2nd stage (Noisy)

1-year SNR~25 signal



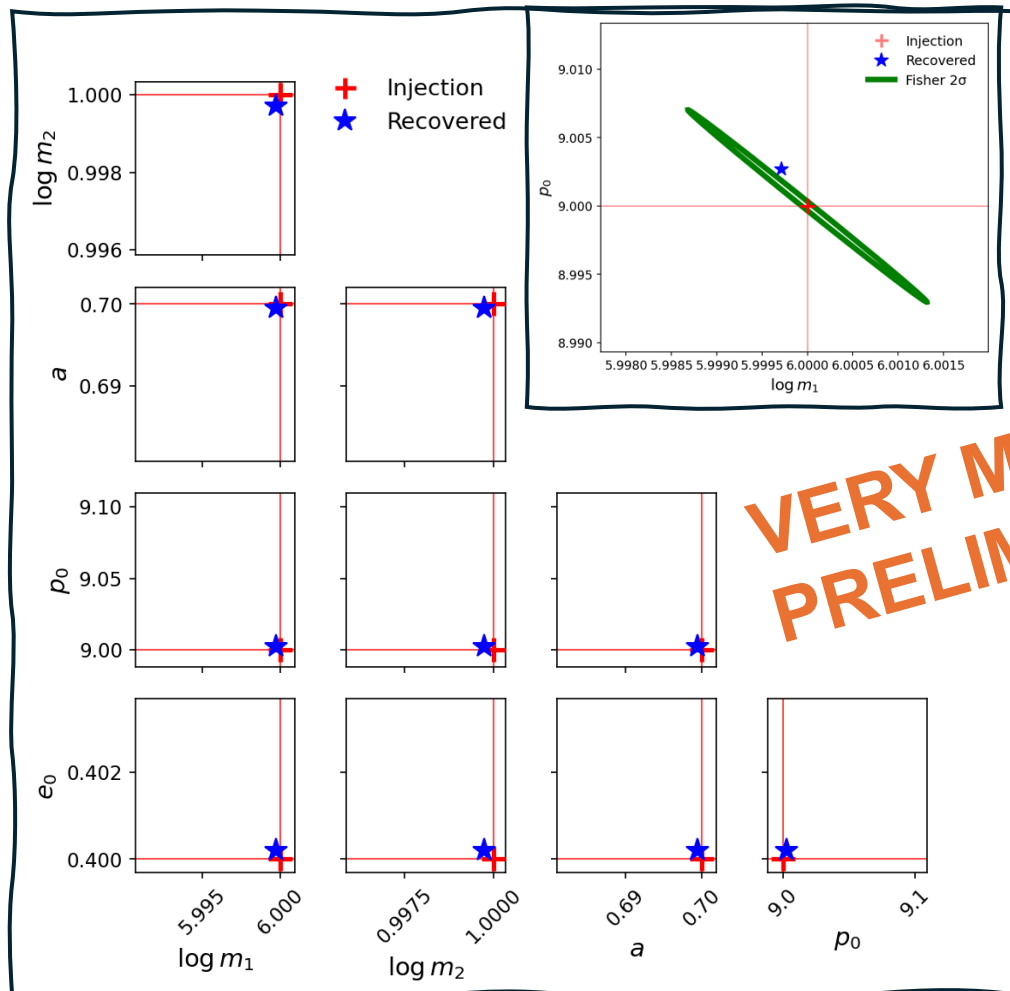
VERY MUCH
PRELIMINARY

Parameter	Priors	Injected	Recovered
$\log_{10} m_1 (M_{\odot})$	[5.94889, 6.03054]	6.0	5.99582573
$\log_{10} m_2 (M_{\odot})$	[0.99336, 1.13038]	1.0	0.99810132
a	[0.62652, 0.99000]	0.7	0.69144752
p_0	[8.74104, 9.68094]	9	9.04888015
e_0	[0.37427, 0.41300]	0.4	0.40169474

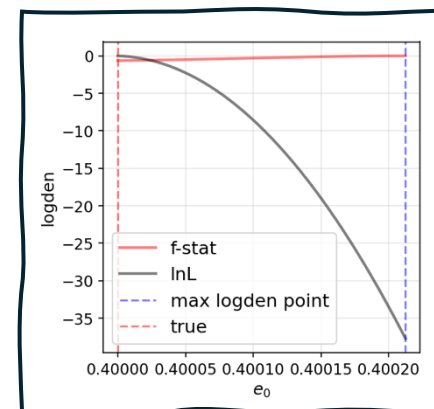


3rd stage (Noisy)

1-year SNR~25 signal



Parameter	Priors	Injected	Recovered
$\log_{10} m_1 (M_{\odot})$	[5.99072, 6.00093]	6.0	5.99971119
$\log_{10} m_2 (M_{\odot})$	[0.99586, 1.00034]	1.0	0.99972285
a	[0.68086, 0.70203]	0.7	0.69952266
p_0	[8.98819, 9.10957]	9	9.00270586
e_0	[0.39969, 0.40370]	0.4	0.40021255





Conclusion

- Detection of single EMRI: $\text{SNR} < 20$ (noiseless) and $\text{SNR} > 20$ (noisy)
- Statistic exploits mode asymmetry between secondaries & primary
 - Might need to adapt to the upcoming phase-domain waveforms for computational gains?



Future Directions

- Need (more) stress tests to determine which is more advantageous between f-function & PARIS
- Testing validity with extrinsic parameters; lower & higher SNRs
- Adaptive (region-dependent) mode basis
 - Dominant modes shift across the space
- Trying on multiple EMRIs, LISA MDC

Thank you

Questions? Feedback (please)?

References

1. M. L. Katz et al., *FastEMRIWaveforms: New tools for millihertz gravitational-wave data analysis*, Phys. Rev. D **104**, 064047 (2021).
2. A. J. K. Chua & C. J. Cutler, *Non-local parameter degeneracy in the intrinsic space of gravitational-wave signals from extreme-mass-ratio inspirals*, Phys. Rev. D **106**, 124046 (2022).
3. A. J. K. Chua, *A one-stop function for gravitational-wave detection, identification and inference*, Phys. Rev. D **106**, 104051 (2022).
4. M. Liu & A. J. K. Chua, *Parallel adaptive reweighting importance sampling for Bayesian astrophysics*, arXiv:2603.21249 (2026).



Inner Product

- Noise-weighted inner product

$$\langle a|b \rangle = 4Re \left(\Delta f \sum_{f>0} \frac{\overline{A(f)}B(f)}{S_n(f)} \right)$$

- Stage 1 time & phase maximization

$$\langle a|b \rangle = \max_{\tau} \left| IFFT \left(\frac{\overline{A(f)}B(f)}{S_n(f)} \right) \right|$$

* Phase maximization = taking the absolute value



f-function

- Suppression strength:

Fractional optimal SNR at each parameter point

$$\alpha(\theta) = \frac{\rho_M}{\rho}$$

M refers to dominant mode

Measures mode dominance

Calibration factor

$$\beta(\theta) = \frac{2\ln(\alpha(\theta)\rho(\theta))}{(1 - \alpha(\theta)^2)\rho(\theta)^2}$$

Controls suppression strength