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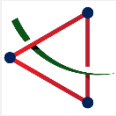
A Deep Learning Framework for Amplitude Generation of General EMRIs

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A Deep Learning Framework for Amplitude Generation of General EMRIs

I. EMRI Basic

II. Method

III. Result

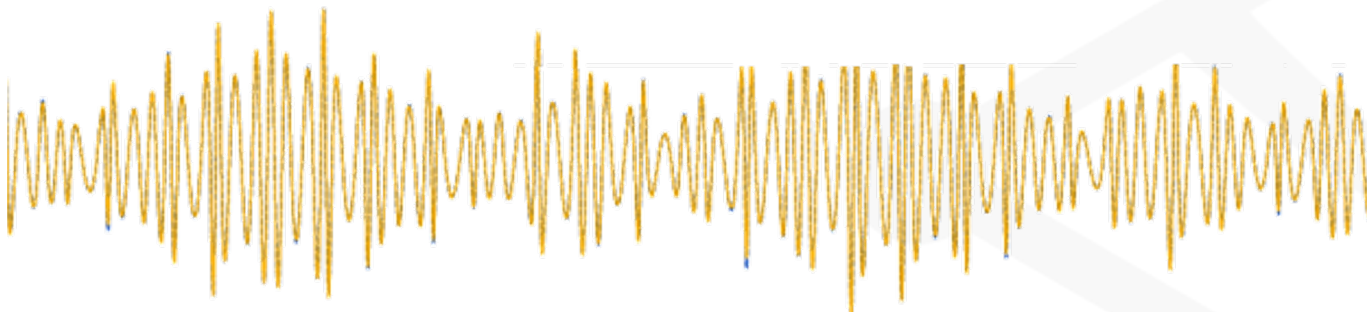
IV. Summary



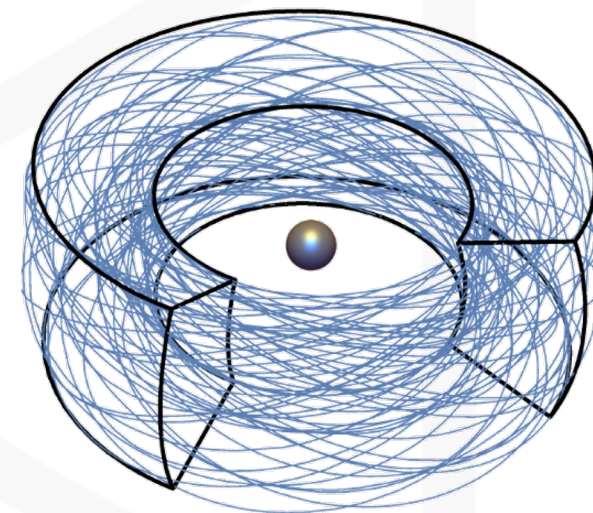
I. EMRI Basic

□ EMRI and its waveform

- Massive black hole ($M = 10^5 \sim 10^7 M_{\odot}$)+Stellar mass compact object ($m = 1 \sim 10^2 M_{\odot}$)
- Accuracy for parameter estimation: $1/\varepsilon$ for intrinsic parameter
- Fruitful scientific objectives[2,3]:
Nature of Gravity, Nature of BH, Astrophysics
- **Very high requirement on accuracy (1PA) and speed (ms)**



Credit: [4]



Credit: [5]

- [1] Sasaki et al. LRR 6, 6 (2003)
- [2] Fan et al. PRD 102, 063016 (2020)
- [3] Zi et al. PRD 104, 064008 (2021)
- [4] Isoyama et al. PRL 128, 231101 (2022)
- [5] Barack et al. RPP 82, 016904. (2019)



I. EMRI Basic

□ Current status

- FastEMRIWaveform: a framework for general EMRI waveform generation[2,3]
- FEW v1: 0PA Schwarzschild
- FEW v2: 0PA Kerr Equatorial

FEW v1

FEW v2

This work

Background Spacetime	Orbital Configuration	Adiabatic	Post-1-adiabatic			
		1SF (Dissipative)	1SF (Conservative)	2SF (Dissipative)	Spin Effects (Conservative)	Spin Effects (Dissipative)
Schwarzschild	Circular	✓✓✓	✓✓✓	✓✓✓	✓✓✓	✓✓✓
	Eccentric	✓✓✓	✓✓✓	X	✓✓, ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	X	✓, ✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✓✓	X	✓, ✓✓*	✓✓*
	Generic	✓✓✓	✓	X	✓	✓*
	Resonances	✓✓✓	✓	X	X	X
✓✓✓ Evolving Waveform		✓✓ Driven Inspiral		✓ Snapshot Calculation		*(Anti-)Aligned Spin Only

[1] LISA waveform WG. LRR 28, 1 (2025)

[2] Chua et al. PRL 126, 051102 (2021)

[3] Chapman-Bird et al. PRD 112.104023 (2025)

Credit: [1]



II. Method

□ The process for adiabatic waveform modelling

- 0PA waveform: only include the dissipative part of 1SF
- The effect is equivalent to the flux of the radiation: $\dot{c} = (\dot{E}, \dot{L}, \dot{Q})$
- Waveform as a superposition of multiple modes

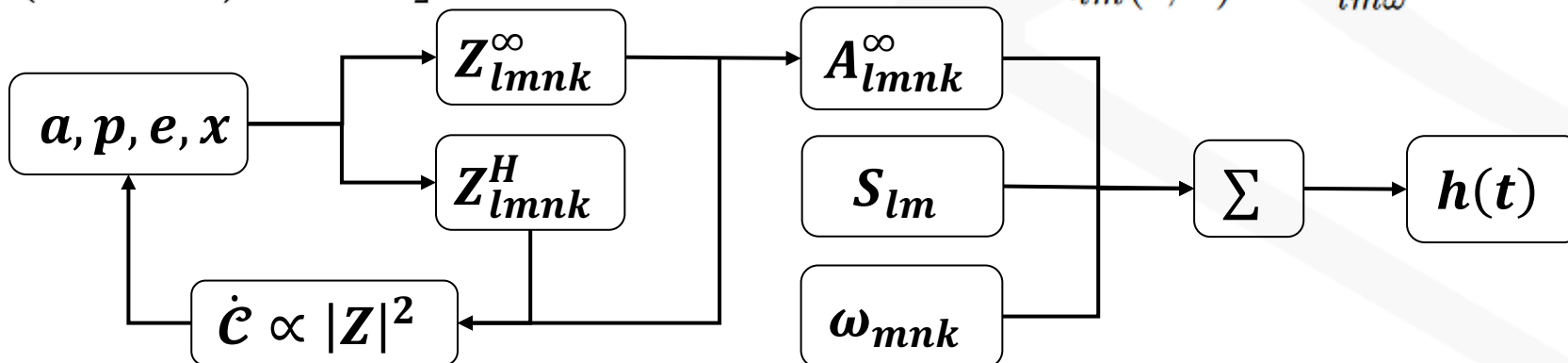
$$h(t) = \sum_{lmnk} s_{lm}(\theta; a\omega_{mnk}) A_{lmnk}^{\infty}(p, e, x) e^{-i\omega_{mnk}t},$$

- $A_{lmnk}^{\infty} \propto Z_{lmnk}^{\infty}/\omega_{mnk}^2$ obtained by radial Teukolsky equation:

$$\left[\Delta^{-s} \frac{d}{dr} \left(\Delta^{s+1} \frac{dR}{dr} \right) + V(r) \right] R_{lm\omega}(r) = \mathcal{T}_{lm\omega}(r)$$

$$R_{lm}(r, \omega) \rightarrow Z_{lm\omega}^{\infty} r^3 e^{i\omega r_*}, \quad r \rightarrow \infty$$

$$R_{lm}(r, \omega) \rightarrow Z_{lm\omega}^H \Delta e^{-i(\omega - m\Omega_H)r_*}, \quad r \rightarrow r_+$$

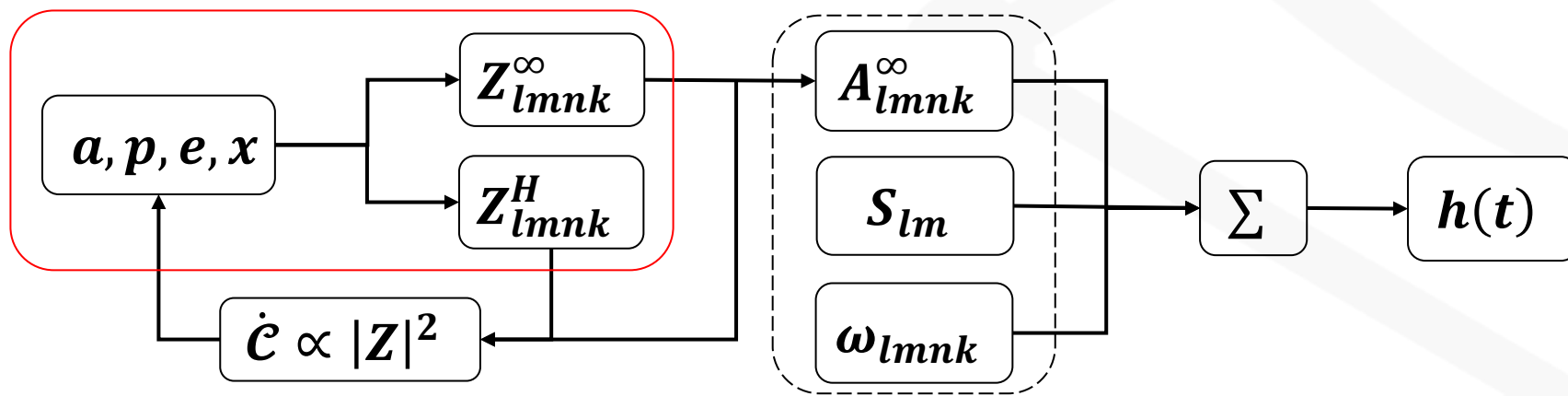


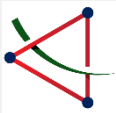


II. Method

□ Two major challenges

- Slow for solving the amplitudes + Too many modes
- Amplitudes: Offline calculation + Real time interpolation
- $(a, p, e, x_I) \Rightarrow \{Z_{lmnk}\}$: $40 \times 100 \times 40 \times 40 = 6,400,000$ points
 $a \in [0, 1]$; $p \in [p_{lso}, 50]$; $e \in [0, 1]$; $x_I \in [-1, 1]$
- Number of modes: 133,614 modes with only a few hundreds (which?) contribute
 $l \in [2, 10]$; $m \in [0, l]$; $n \in [-50, 50]$; $k \in [-10, 10]$





II. Method

□ Generation of amplitude in FEW

- **FEW v1: Schwarzschild with 3,843 modes, $l \in [2, 10]$; $n \in [-30, 30]$**

Use reduced order modeling on $1,650 = 50 \times 33$ samples

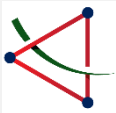
$$p \in [p_{lso} + 0.03, p_{lso} + 41.6] ; e \in [0, 0.8]$$

- **FEW v2: Equatorial Kerr with 6,993 modes, $l \in [2, 10]$; $n \in [-55, 55]$**

Direct interpolation on $615,810 = 129 \times 65 \times 65 + 65 \times 33 \times 33$ samples for fluxes

$$45,474 = 33 \times 33 \times 33 + 17 \times 17 \times 33 \text{ samples for amplitude}$$

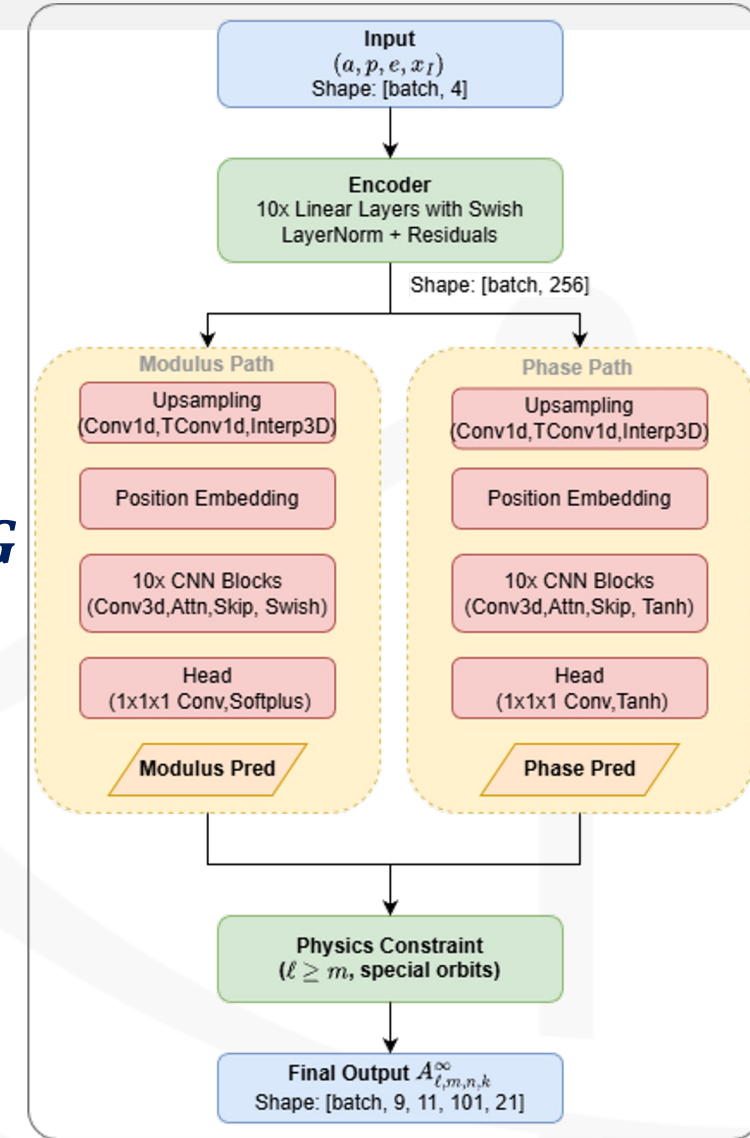
- **Very hard to extend to general Kerr orbits (storage & computation cost)**



II. Method

□ CNN for amplitude generation and selection

- The amplitudes as a 3rd-order tensor for each l
- Encoder + Decoder: Modulus part + Phase part
- Weighted loss function: $L = w_{\text{rel}}L_{\text{rel}} + w_{\text{cross}}L_{\text{cross}} + w_{\phi}L_{\phi}$.
- Transfer learning: $SC \rightarrow SE \rightarrow KEC \rightarrow KEE \rightarrow KIC \rightarrow KG$
- Predict which modes are dominate (99% in Power)



Orbital Type	Notation	Constraints
Schwarzschild Circular	SC	$a = 0, e = 0, x_I = 1$
Schwarzschild Eccentric	SE	$a = 0, e > 0, x_I = 1$
Kerr Equatorial Circular	KEC	$a \neq 0, e = 0, x_I = 1$
Kerr Equatorial Eccentric	KEE	$a \neq 0, e > 0, x_I = 1$
Kerr Inclined Circular	KIC	$a \neq 0, e = 0, x_I < 1$
Kerr Generic	KG	$a \neq 0, e > 0, x_I < 1$



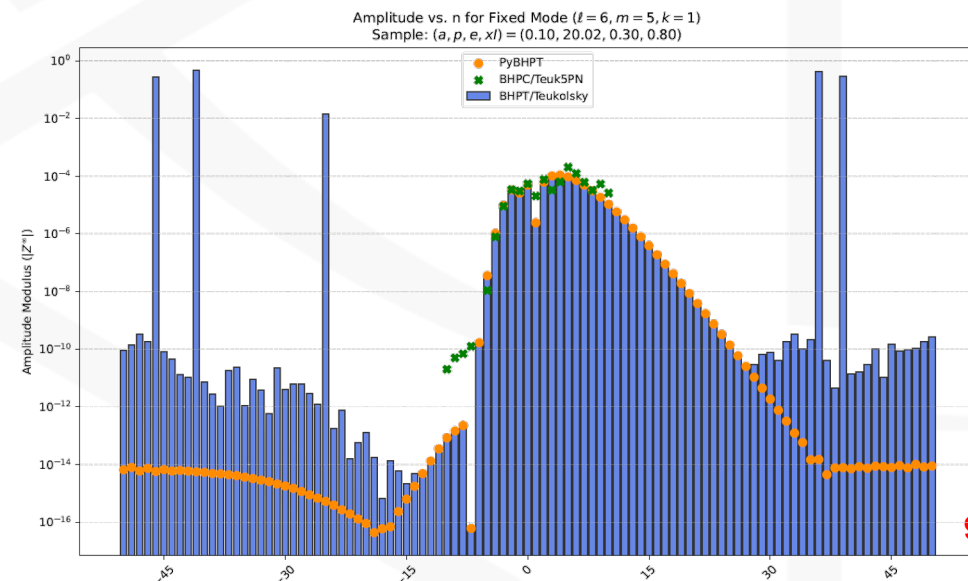
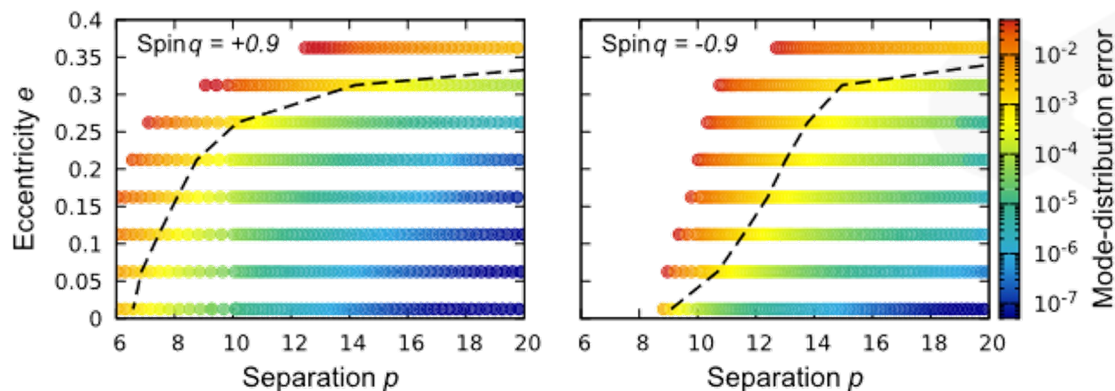
II. Method

□ Amplitude generation

- **pybhpt**: based on Python, fast
- **BHPT/Teukolsky**: based on Mathematica, slow
- **BHPC[2]**: Analytic PN-Teukolsky Amplitude, fast
- Chose to use **BHPC**, but:
 - Only for $p \geq 10$ and $e \leq 0.3$
 - Limited modes: $2 \leq \ell \leq 12$, $|m| \leq \ell$, $|m+k| \leq 12$, $|n| \leq 10$
- Only for validation of this framework

Credit: [1]

(ℓ, m, n, k)	BHPTToolkit [ms]	pybhpt [ms]
(2,2,0,0)	6.44×10^4	45
(2, 2, 0, 5)	2.31×10^5	63
(2,2,10,0)	1.96×10^5	48
(2,2,50,0)	Aborted	389
(4, 4, 0, 0)	4.93×10^4	53
(4, 4, 0, 5)	1.79×10^5	47
(4,4,10,0)	2.94×10^5	54



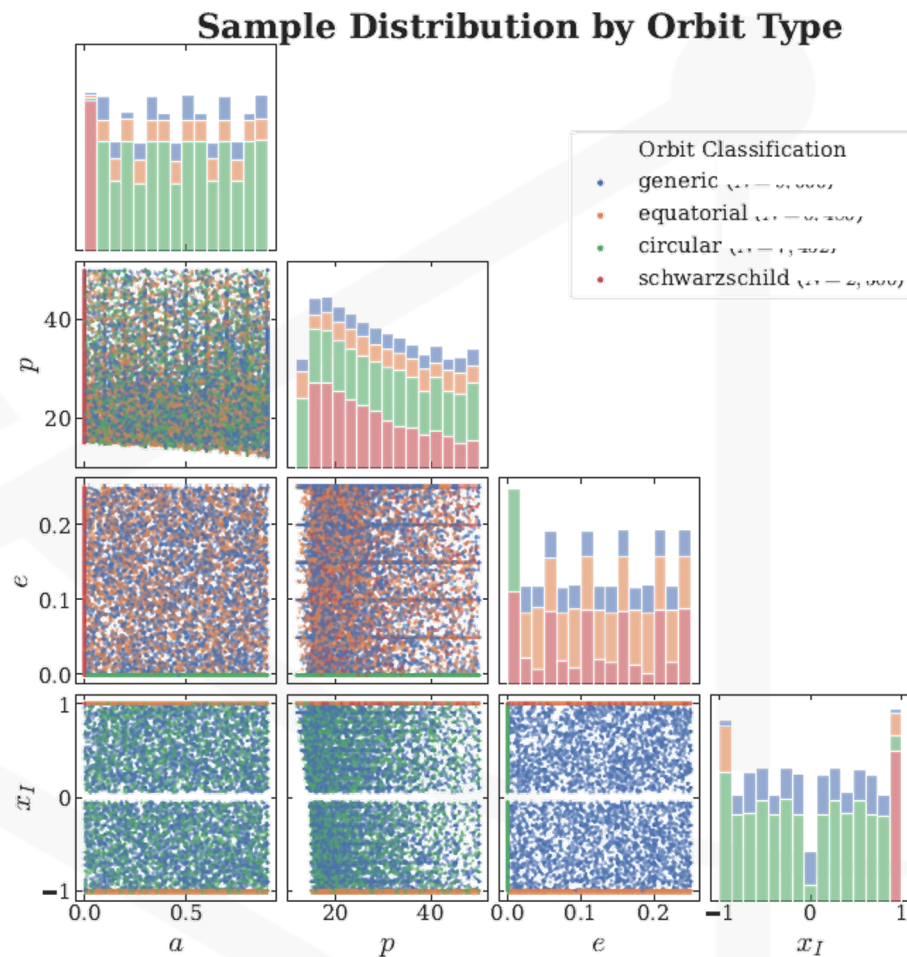


III. Result

□ Dataset

- The BHPC sample: grid based sample + equal number random sample
- 1/10 of the random samples for validation
- $a \in [0, 0.9], e \in [0, 0.25], x \in [-1, 1]$
- $p \in [\max(p_{LSO}, 10), 50], u = 1/\sqrt{p - 0.9p_{LSO}(a, e, x)}$
- Uniform sample on a, e, x, u , 12940+11646=24586

Orbit type	a	x	e	p	number
SC	0	1	0	200	200
SE	0	1	{0.05, 0.1, ..., 0.25}	200	1000
KEC	{0.1, 0.2, ..., 0.9}	± 1	0	50	900
KEE	{0.1, 0.2, ..., 0.9}	± 1	{0.05, 0.1, ..., 0.25}	40	3600
KIC	{0.1, 0.2, ..., 0.9}	{-0.9, -0.8, ..., 0.9}	0	20	3240
KG	{0.1, 0.3, ..., 0.9}	{ $\pm 0.9, \pm 0.6, \pm 0.3, \pm 0.1$ }	{0.05, 0.1, ..., 0.25}	20	4000

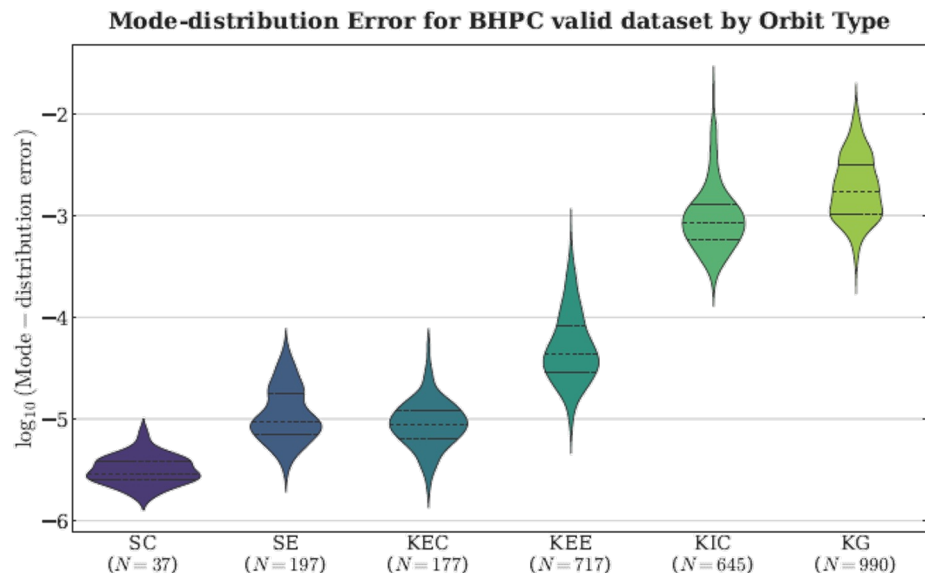




III. Result

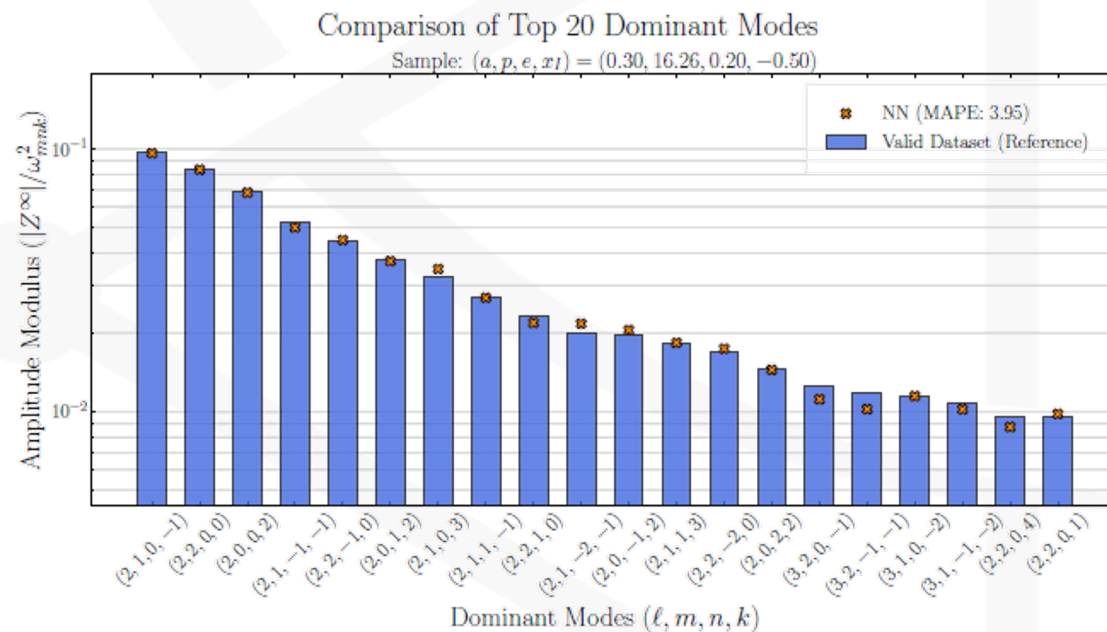
□ Training result

- 7 days on 4x NVIDIA A100
- Mismatch less than 10^{-4} for special orbit
- Mismatch less than 10^{-2} for most general orbit
- Efficient prediction of the dominate modes
- About 1.26ms for each orbit parameter



$$\mathcal{M}_{\text{amp}} = 1 - \frac{\Re \left(\sum_i \mathcal{A}_i^{\text{pred}} \cdot (\mathcal{A}_i^{\text{true}})^* \right)}{|\mathcal{A}^{\text{pred}}| \cdot |\mathcal{A}^{\text{true}}|}$$

Orbit Type	Recall Score (%)
Schwarzschild Circular (SC)	100.0
Schwarzschild Eccentric (SE)	99.7
Kerr Equatorial Circular (KEC)	99.7
Kerr Equatorial Eccentric (KEE)	99.0
Kerr Inclined Circular (KIC)	94.0
Kerr Generic (KG)	83.9



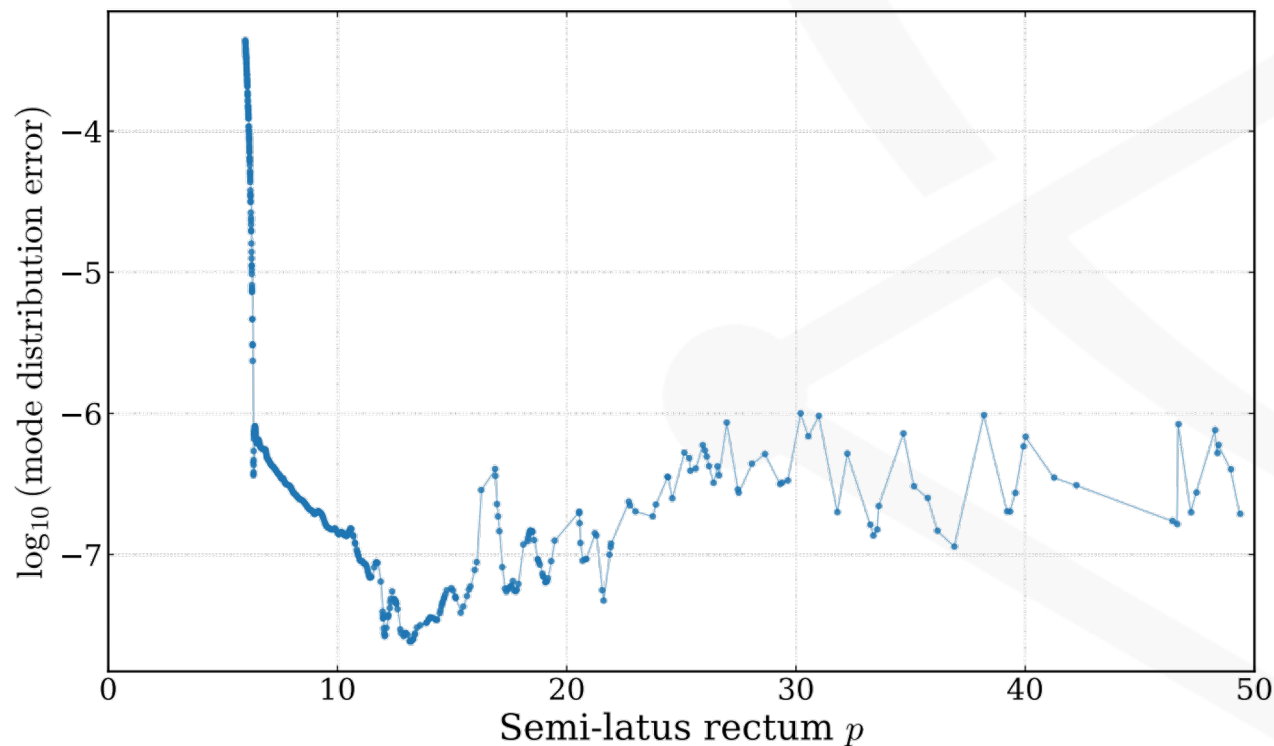


III. Result

□ Preliminary result with pybhpt

- Schwarzschild circular orbit, $p \in [6 + 0.001, 50]$
- Trained with 800 samples (doubled)
- $\mathcal{M}_{amp} < 10^{-6}$ except for the region very close to 6 (p_{lso})

Schwarzschild Circular Mode-Distribution Error

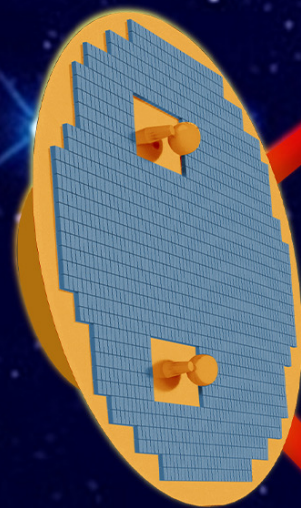




IV. Summary

□ 0PA waveform generation for general EMRIs

- Use transfer learning to train a CNN
- Could predict the dominate modes and their amplitude
- Very accurate for special orbits
- Need to be improved for the most general case
- **Future work:**
 - Better dataset (pybhpt)
 - Larger parameter space
 - Amplitude at horizon
 - Waveform generation
 - Fluxes and 1PA effect



Thanks!