



Rapid waveform generation strategies for EMRI search and parameter inference

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A meandering path...

- The FastEMRIWaveforms (**FEW**) framework (briefly)
- The high-eccentricity limit: challenges and solutions
- Eliminating the radial mode decomposition: radial phase-space
- Redefining waveform generation completely: full phase-space summation

EMRI waveform basics

- At a large distance from the source,

$$h(t) \equiv h_+(t) - ih_\times(t) = \frac{\mu}{d_L} \sum_{lmkn} {}_{-2}Y_{lm}(\theta, \phi) A_{lmkn}(t) e^{-i\Phi_{mkn}(t)}$$

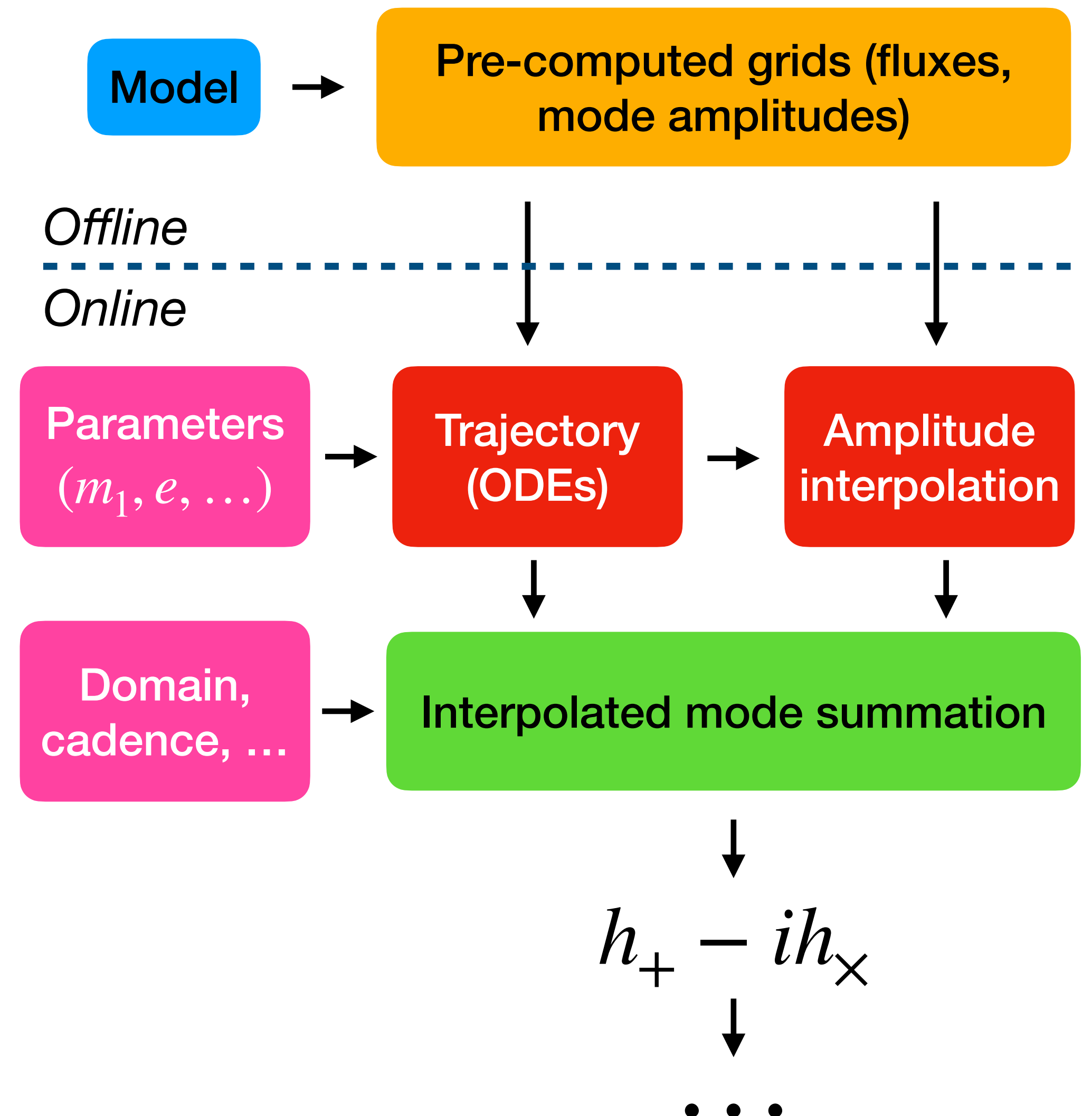
$(\Phi_{mkn} = m\Phi_\phi + k\Phi_\theta + n\Phi_r)$



- Waveform a sum of harmonics of three fundamental frequencies
- Amplitudes A_{lmkn} and phases $\Phi_{\phi,\theta,r}$ smoothly evolve during inspiral

The FastEMRIWaveforms (FEW) framework

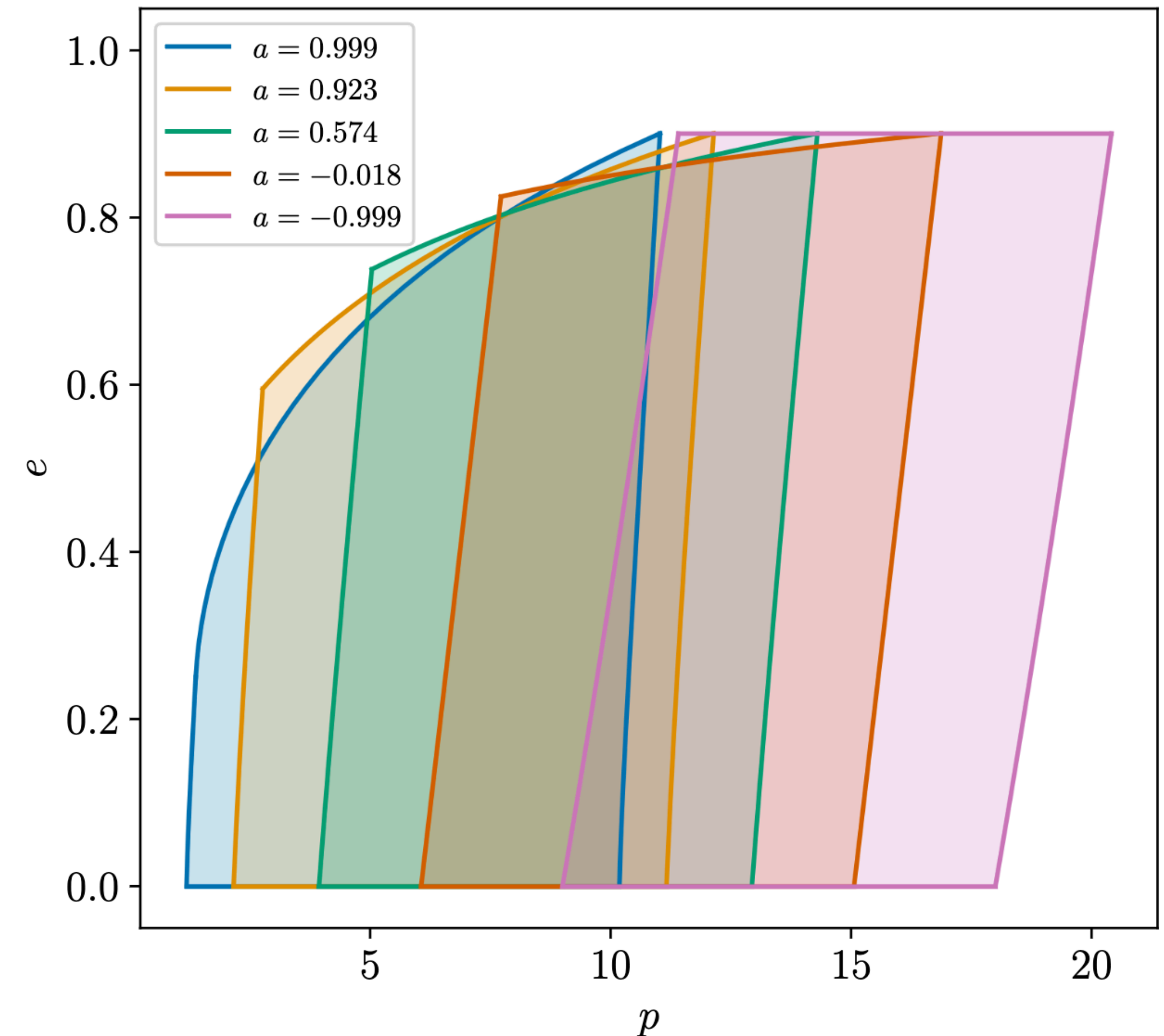
- Efficient waveform generation scheme built upon the multiscale expansion
- Pre-compute ingredients (**offline**), then interpolate for fast waveforms (**online**)
- For long-duration (LISA) signals, leverage graphics processing units (GPUs) heavily
- Waveform generation in < 1 s



Kerr Eccentric Equatorial in FEW

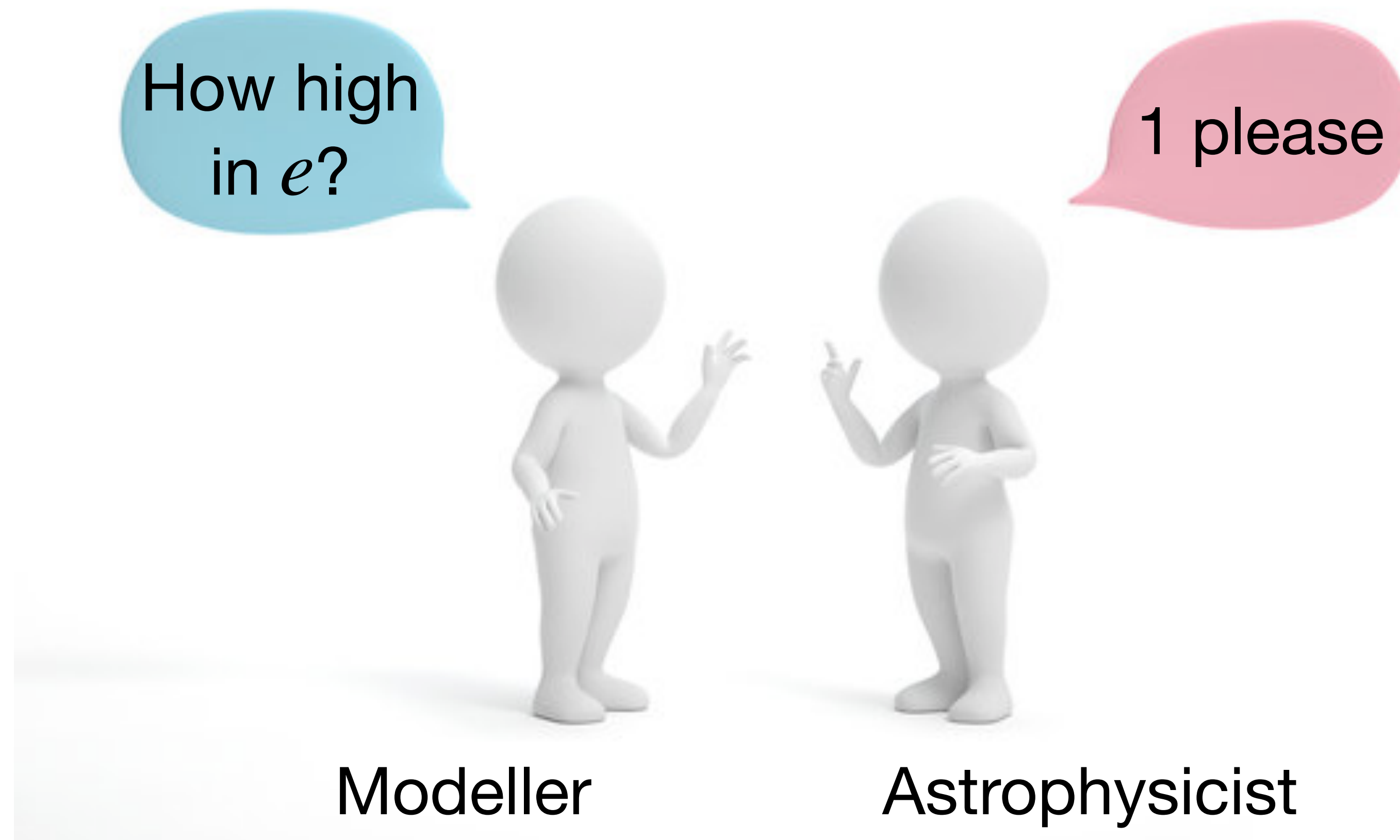
- Most physically complete adiabatic model currently in FEW:
 - $|a| \leq 0.999$; $e \lesssim 0.85$ (**tapered**)
- Main limitation of the model is completeness at high eccentricity:
 - Cut-off of grids at $e \leq 0.9$
 - Incomplete n -mode information at high eccentricities

Chapman-Bird+ 2025



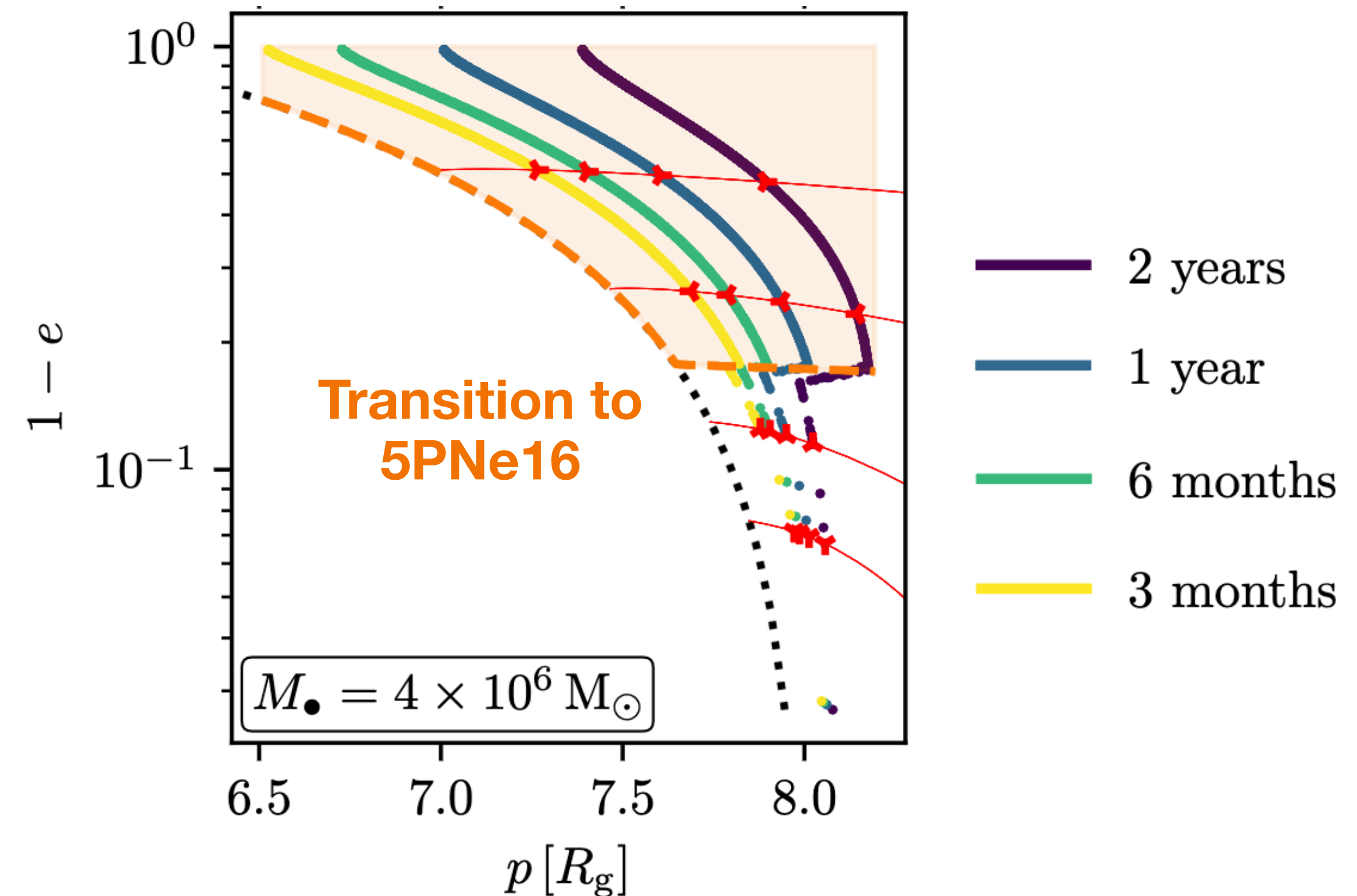
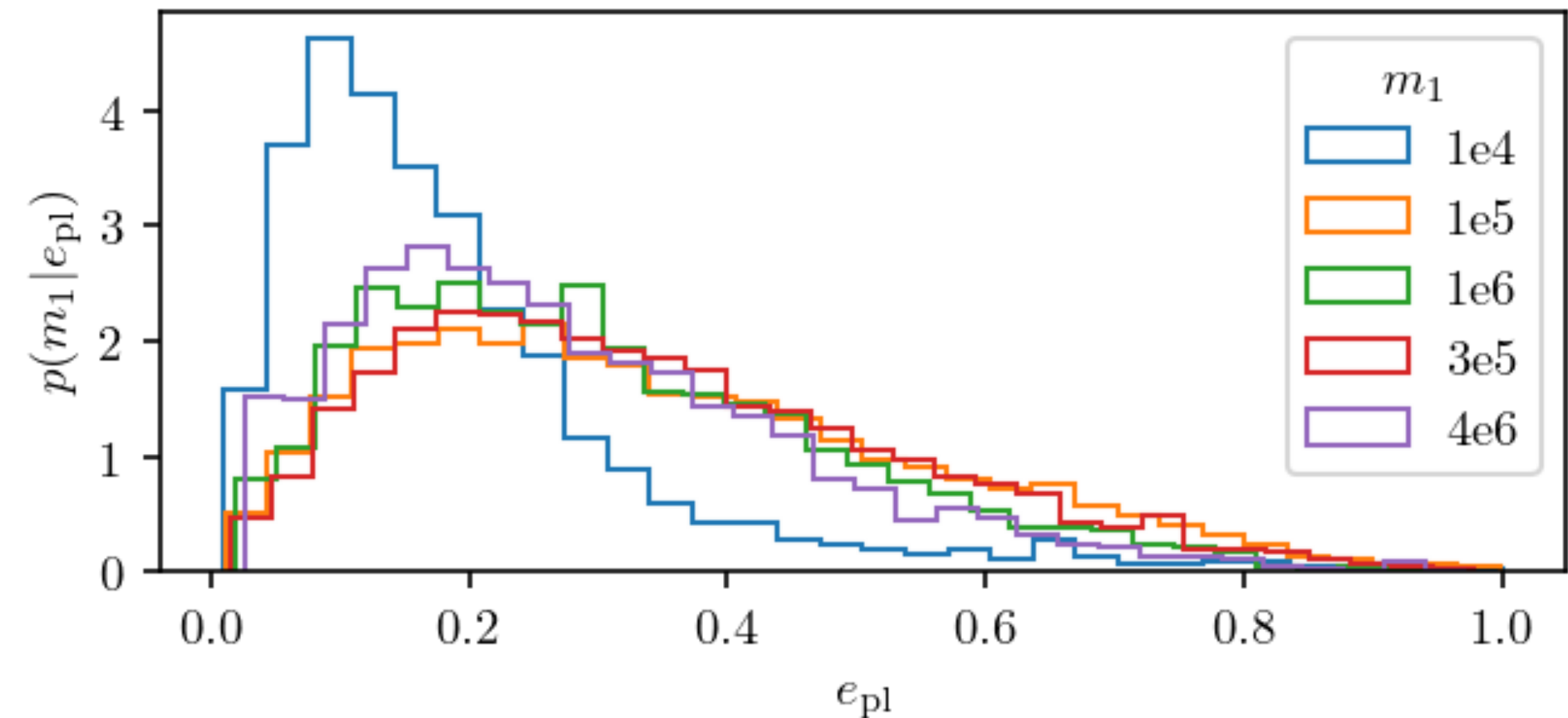
Other challenges

Parameter space boundaries



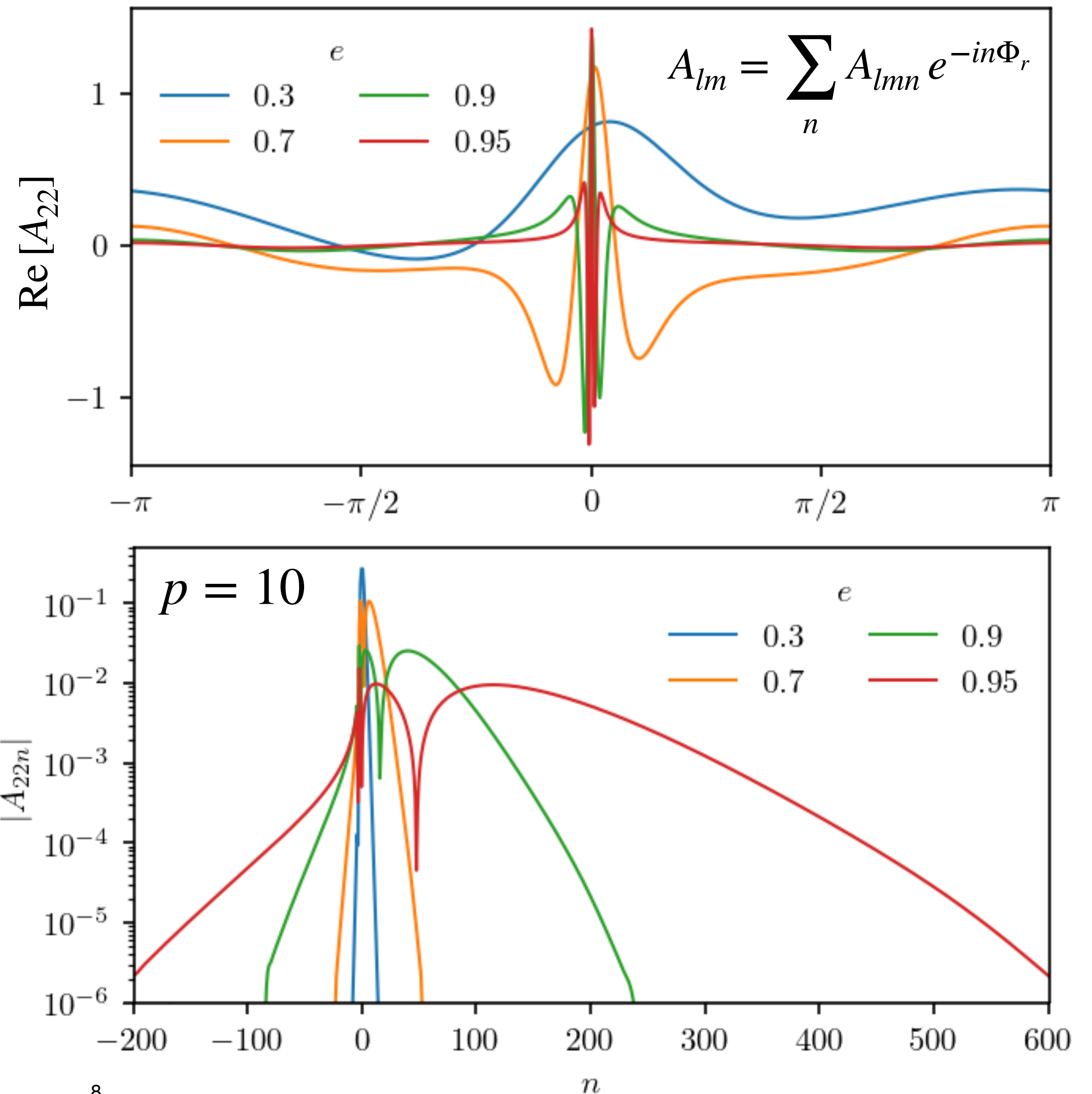
Expected eccentricity distribution for (some) EMRIs

- Astrophysical simulations by Mancieri et al. (2025) of loss-cone dynamics
- They predict a long tail of EMRIs with **significant eccentricity at plunge** — these rapidly exit model domain!
- Small- e expanded PN a poor substitute
 - We **must** extend models to higher e



The problem with high eccentricities

- As $e \rightarrow 1$, increasingly sharp burst structure in $A_{lm}(\Phi_r)$
 - Mode spectrum **diverges**
- Bad news for fast waveforms
 - Storage limitations
 - Linear cost scaling

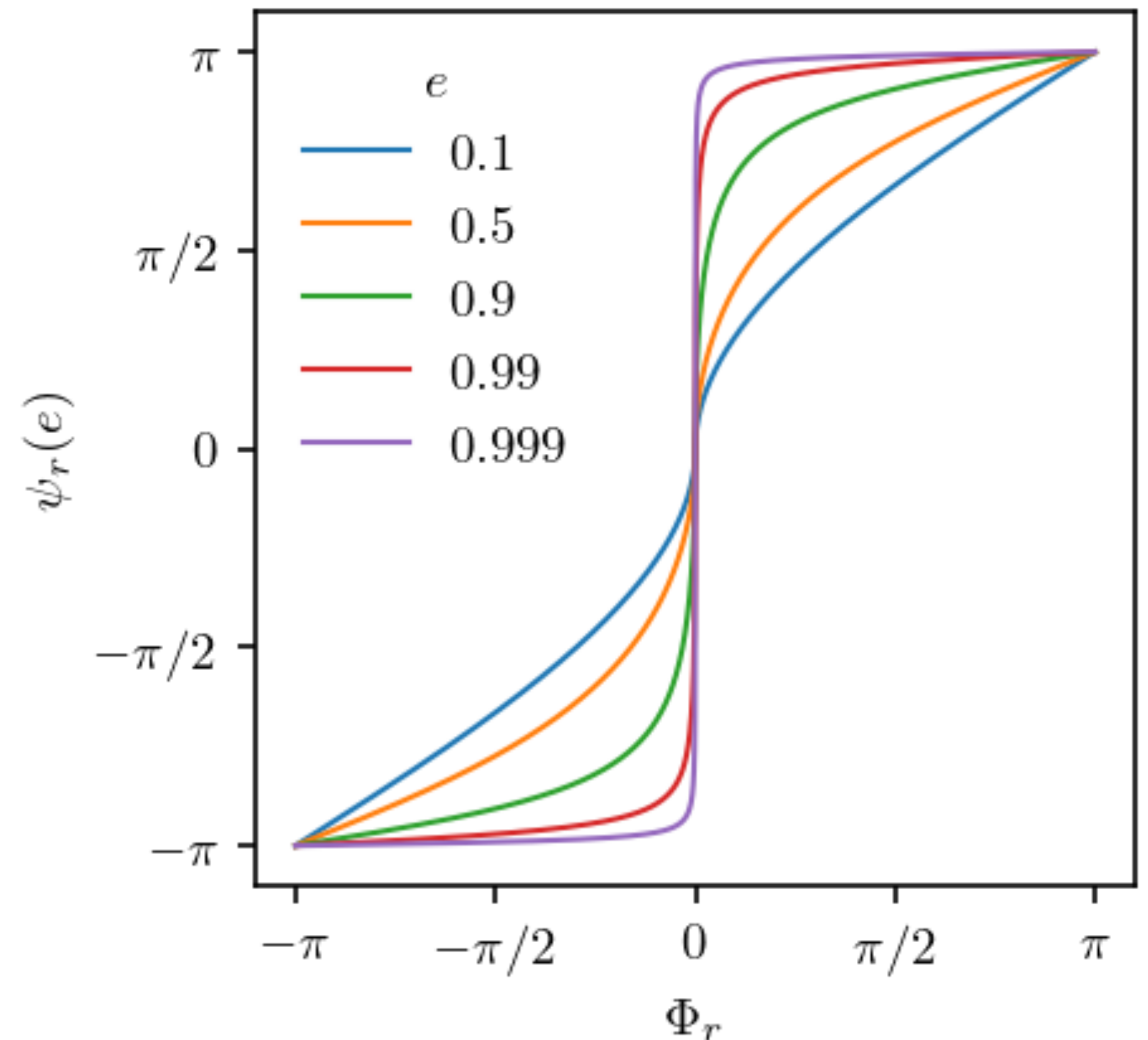


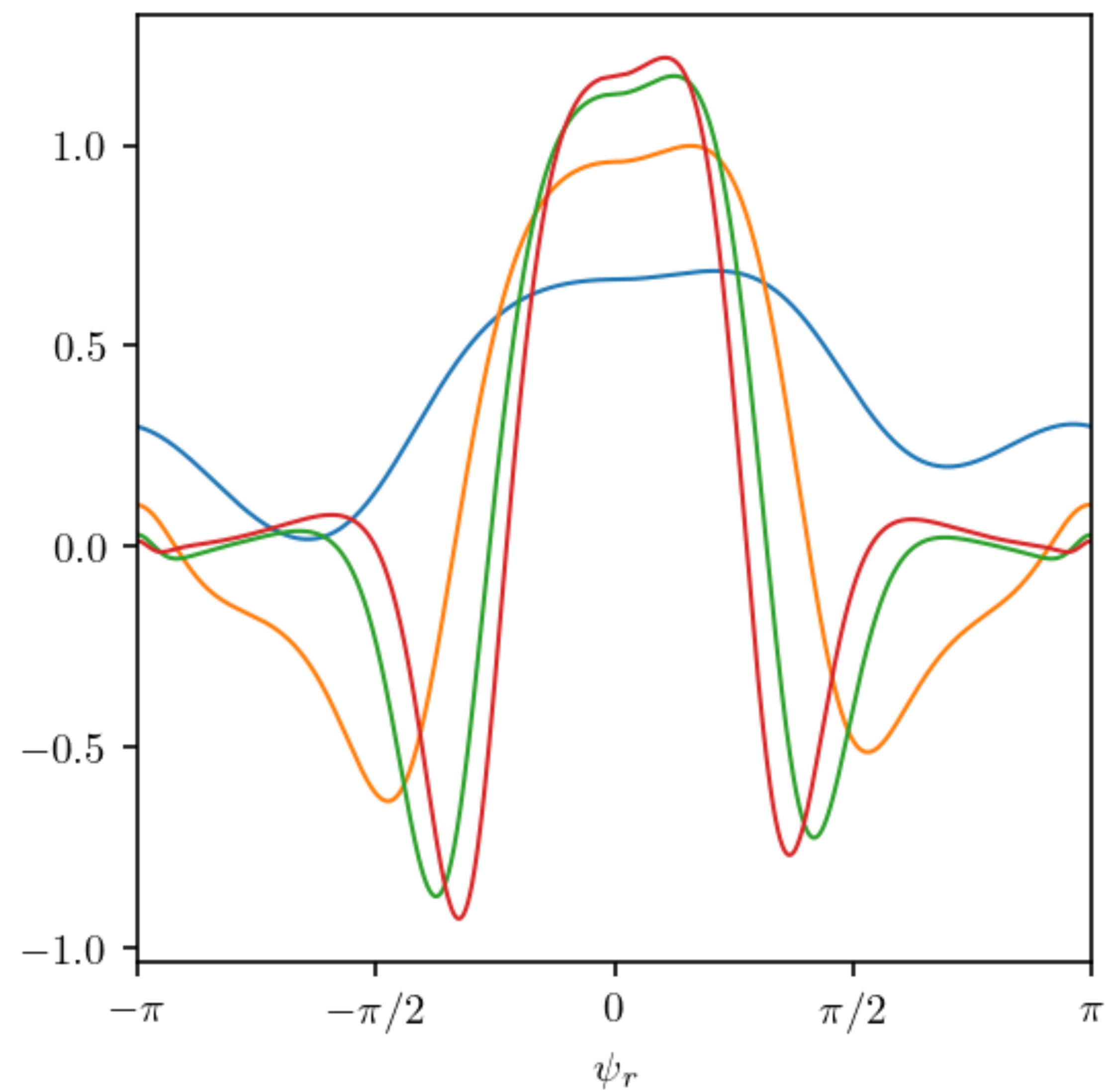
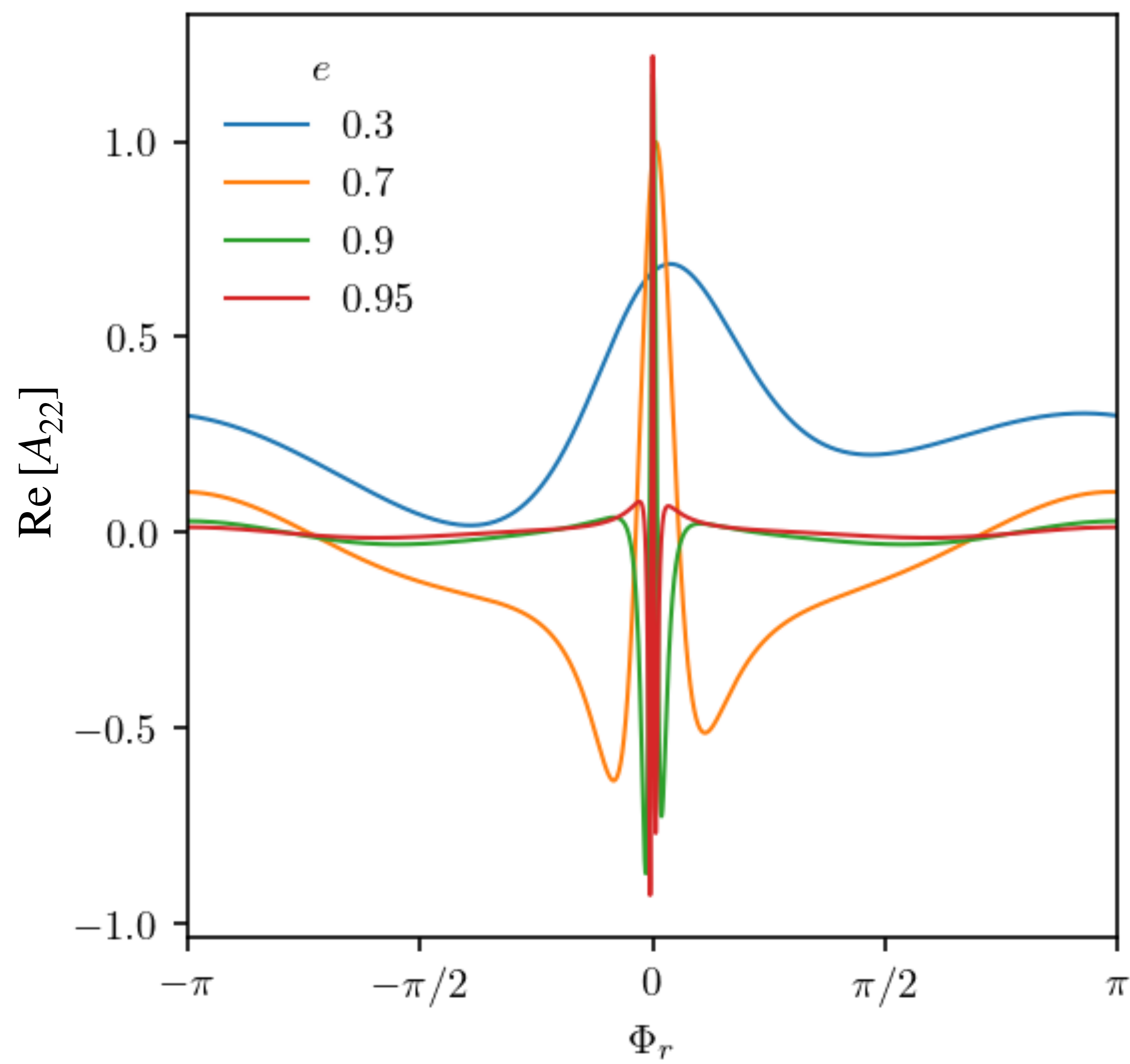
Defining a new radial phase variable

- Curing the poor Fourier convergence requires a change of phase variable
- Draw inspiration from Darwin angle parameterisation of orbits:

$$\sin \frac{\Phi_r}{2} \sim \frac{1}{1 + e \cos \psi_r}$$

- Has desirable properties for our problem:
 - “Spreads out” periapsis passage
 - Still well-behaved at lower e





Modifying the mode summation

- At this stage, two options for how to proceed:
 1. **Define new Fourier modes for ψ_r** : proceed as normal
 2. **Continue to work in terms of ψ_r** : define a new summation
- Why choose the second option?

Radial phase summation

- Are Fourier coefficients the best choice for fast waveform generation?

$$h = \frac{\mu}{d_L} \sum_{lmn'} -2 Y_{lm}(\theta, \phi) A_{lmn'} e^{-i(m\Phi_\phi + n'\psi_r)}$$

- Fourier series gives function over all ψ_r , but we only need the waveform at a **specific value of ψ_r** for each time-step
- i.e. Fourier series a **global basis**, but a **local basis** may be better suited

Radial phase summation

- Instead, define phase grid $\psi_r^{(g)}$ and pre-compute the partial mode-sum:

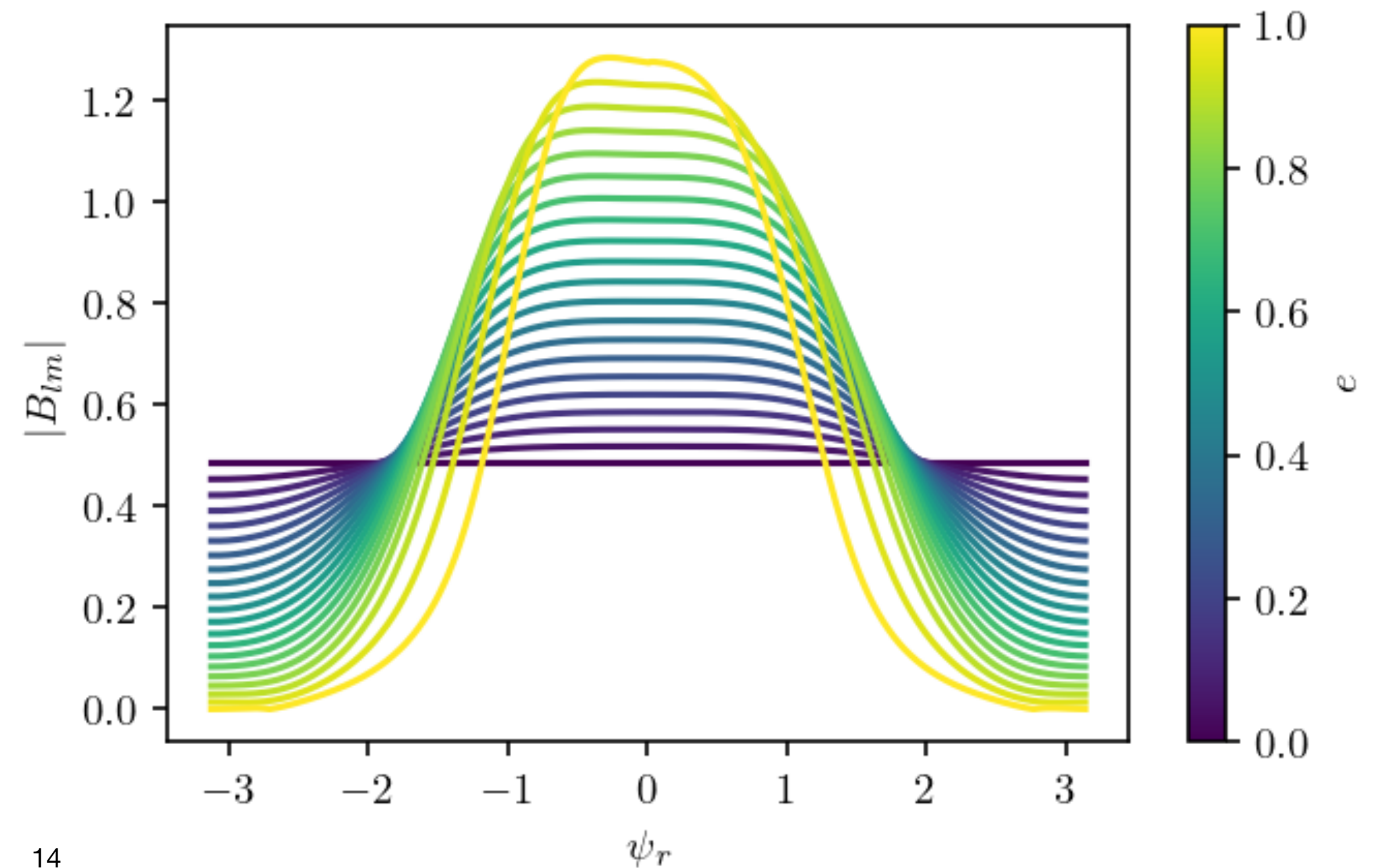
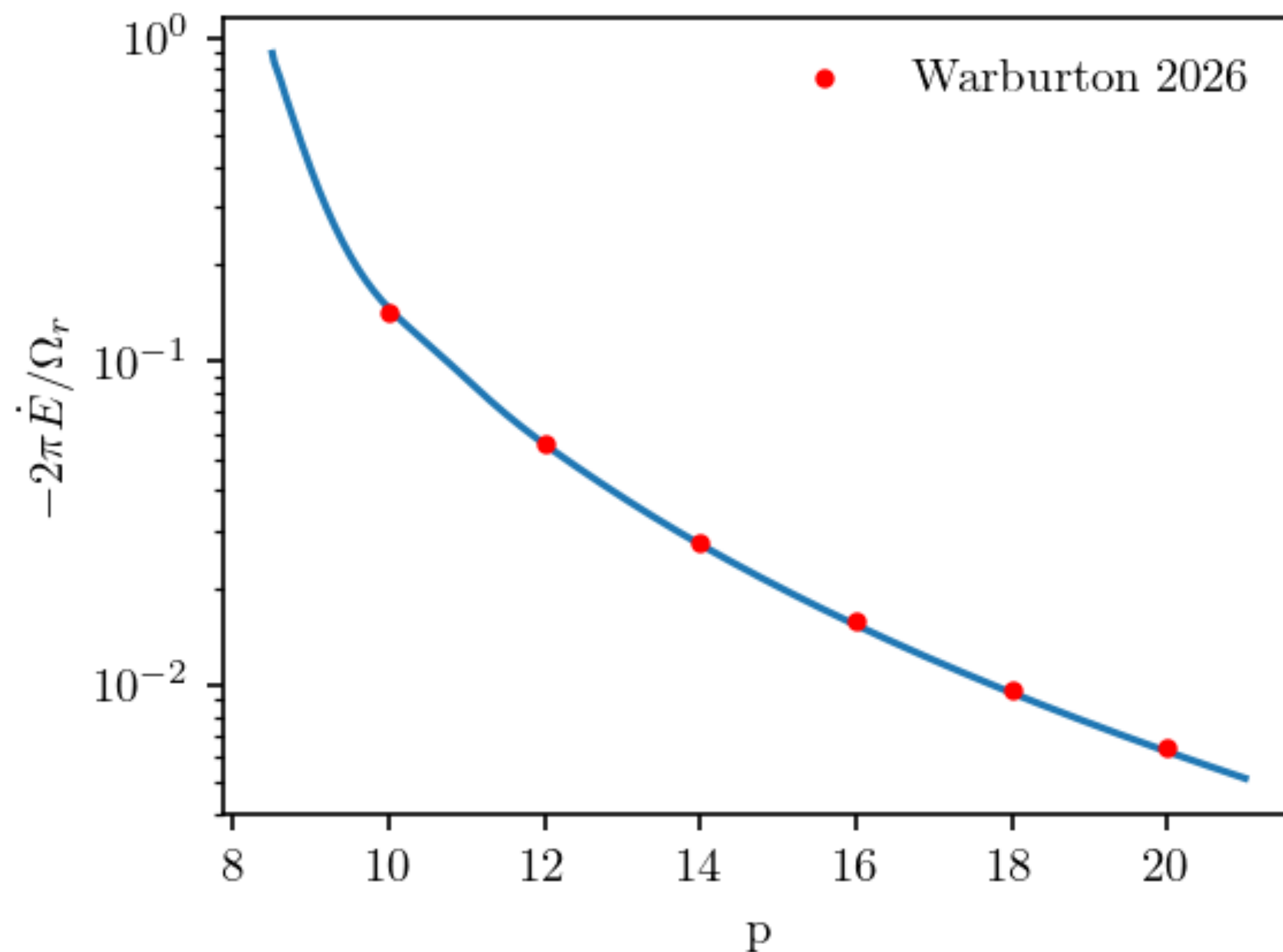
$$B_{lm}(\psi_r^{(g)}) = \sum_{n'} A_{lmn'} e^{-i\psi_r^{(g)}}$$

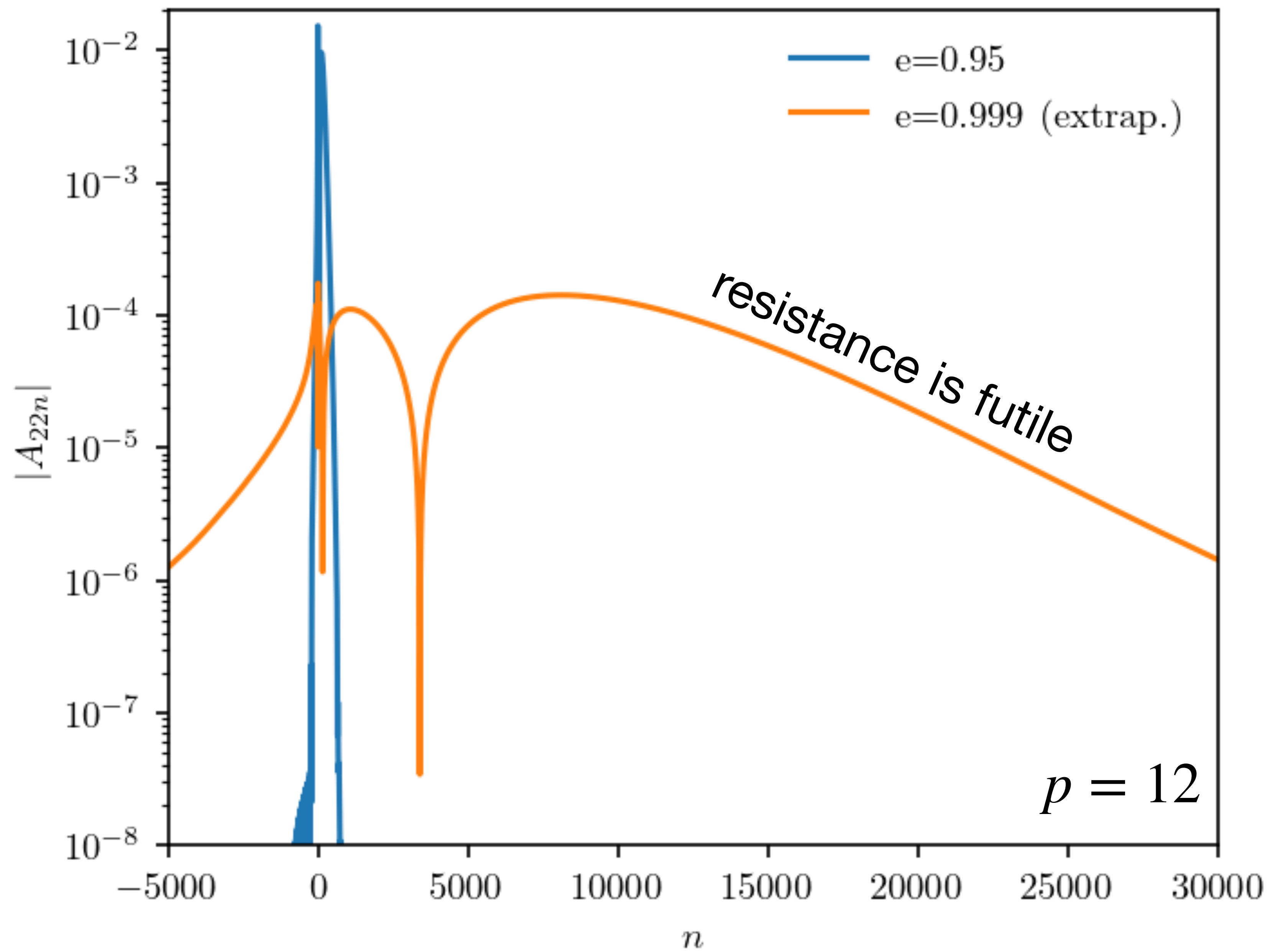
$$h(t) = \frac{\mu}{d_L} \sum_{lm} {}_{-2}Y_{lm}(\theta, \phi) B_{lm}(t, \psi_r(t)) e^{-im\Phi_\phi(t)}$$

- We are now free to construct $B_{lm}(t, \psi_r)$ via bivariate spline interpolation

Test case: High-eccentricity Schwarzschild

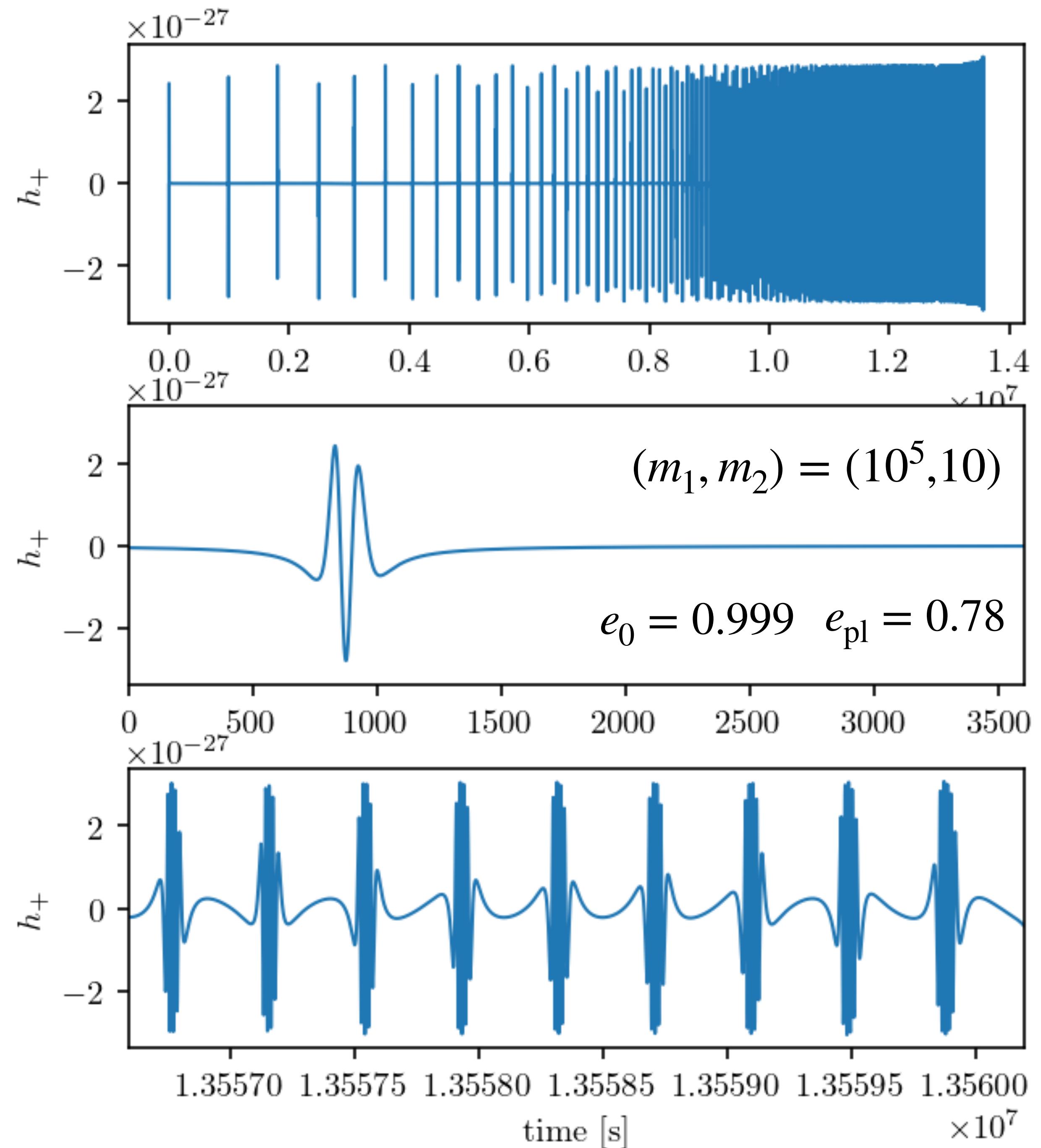
- Extrapolate grids for $e \in (0.95, 1]$ as a test case — $(l, m) = (2, 2)$ only
- $e = 1$ flux agrees with parabolic results of Warburton (2026) to $\sim 1\%$

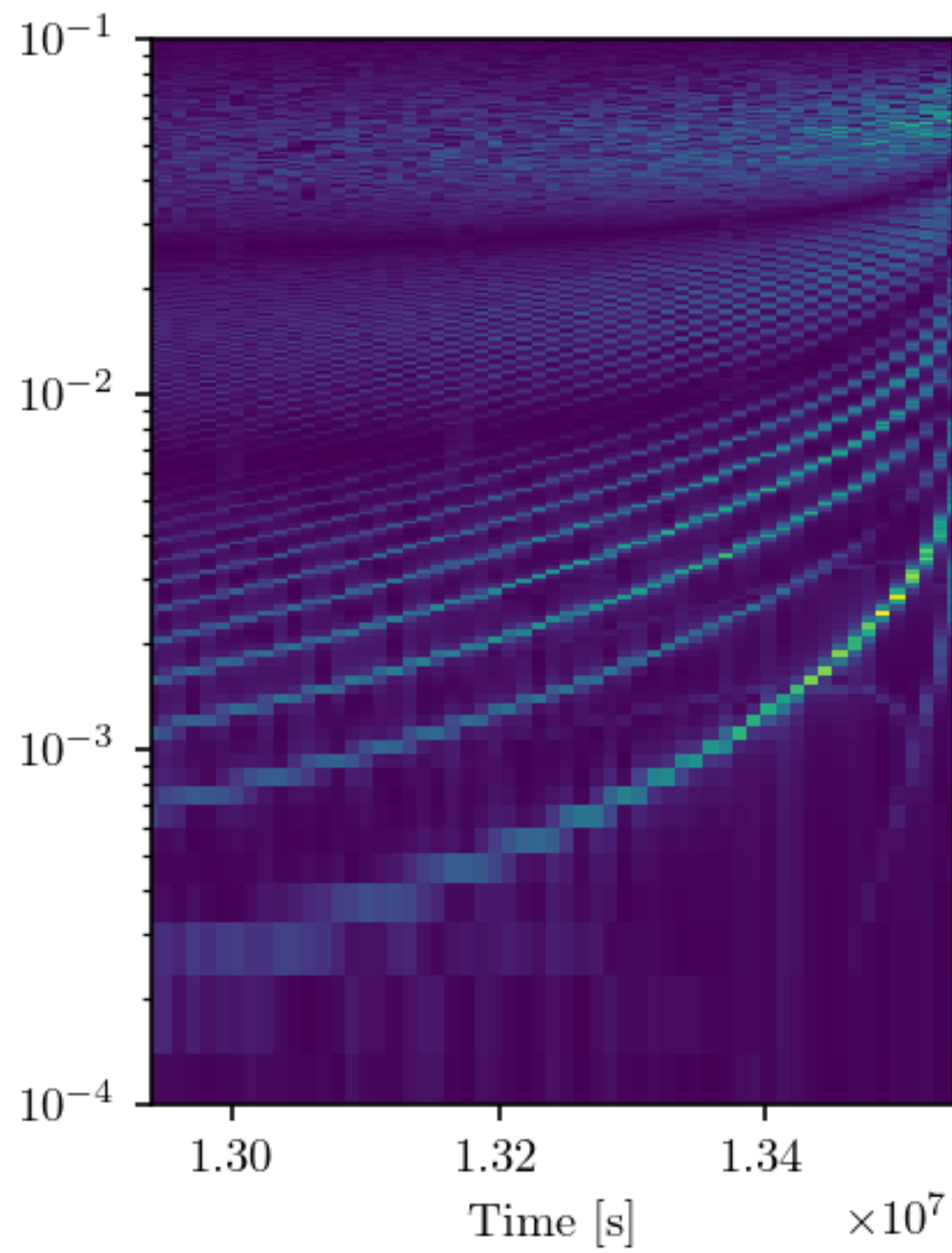
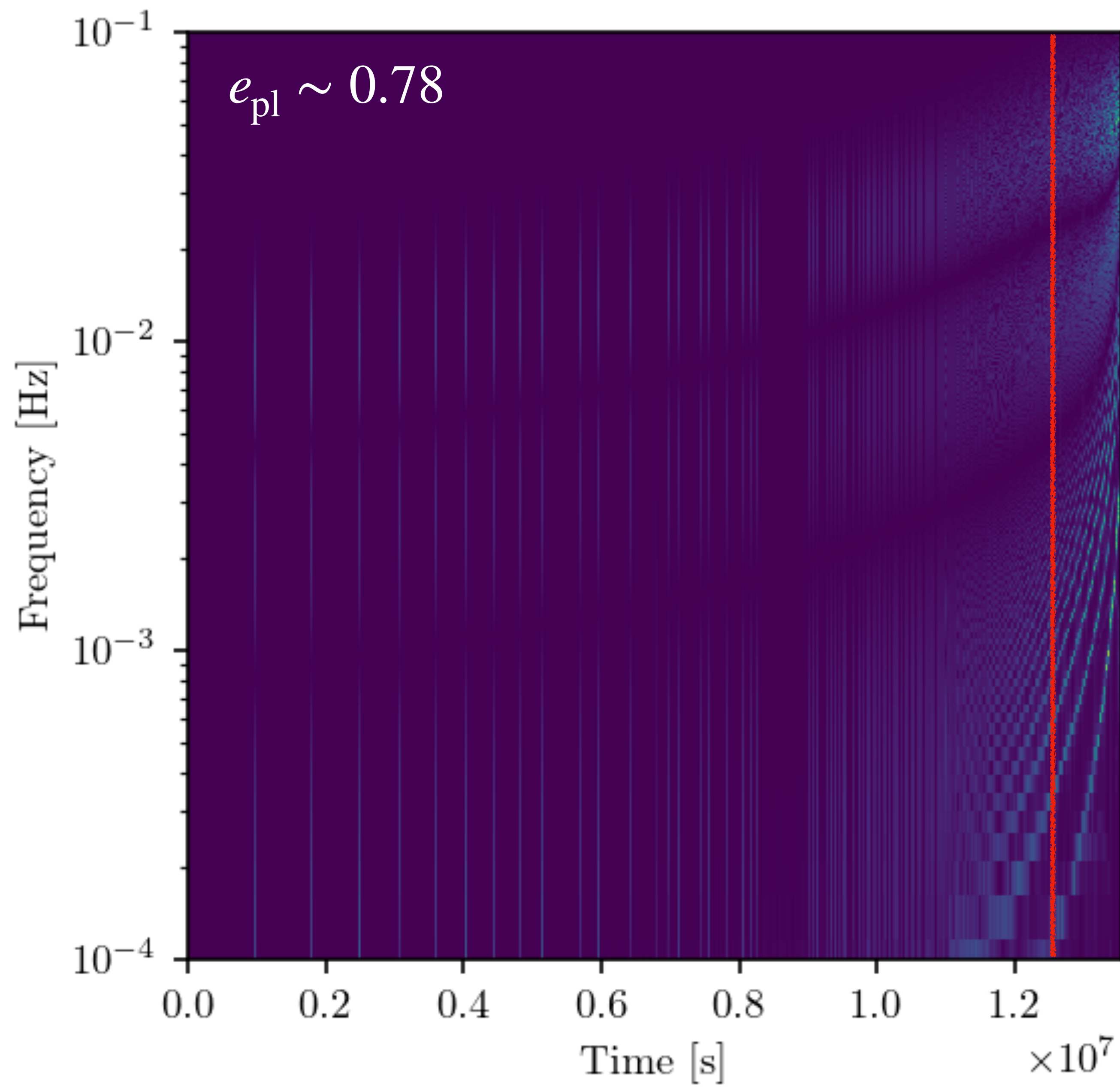




Fast high-eccentricity (2,2) waveforms

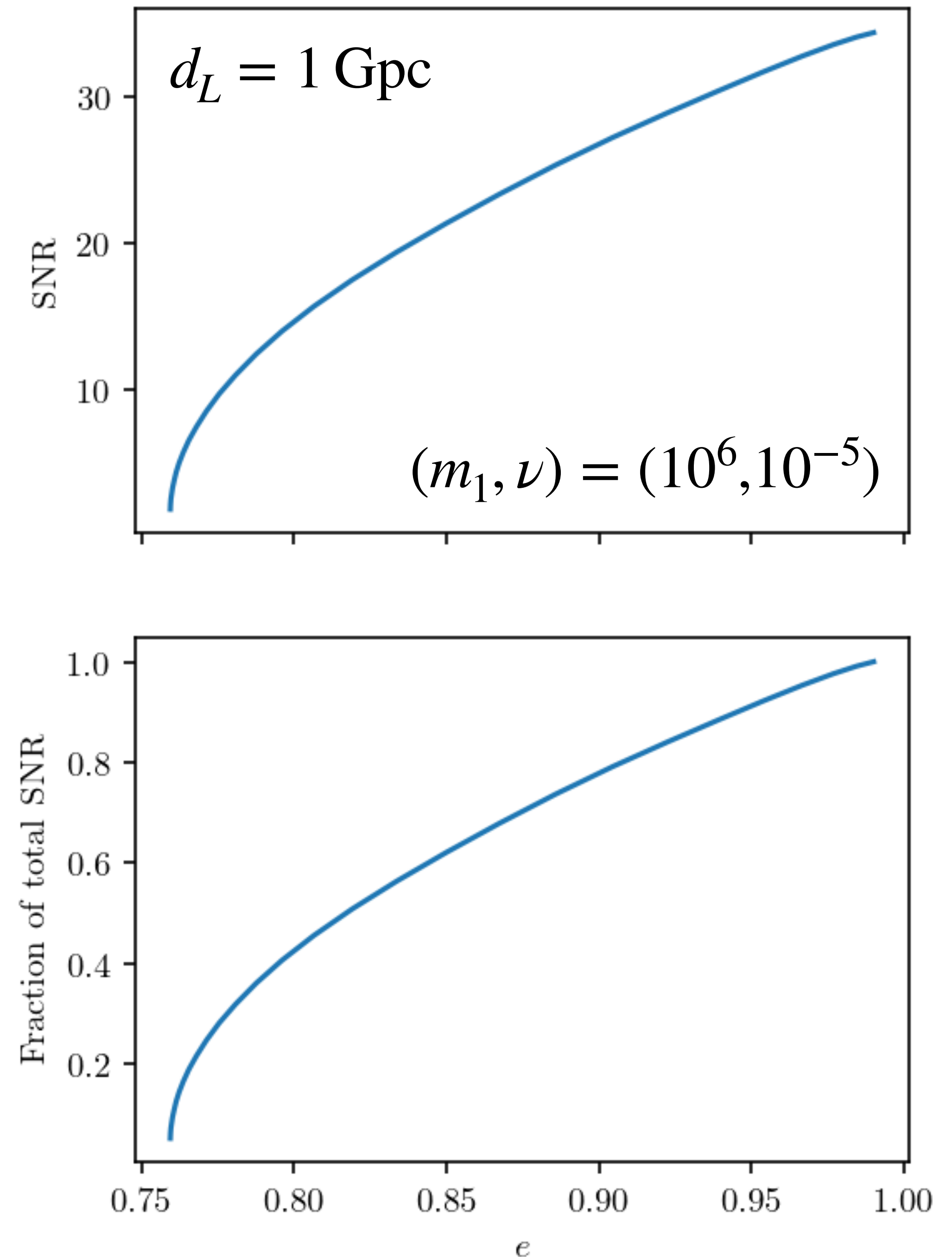
- With extrapolated data, construct waveform with modified summation
 - 10^7 point (2,2) waveform sum in 500 (5) ms on CPU (GPU)
- No clear transition point between “bursts” and “continuous waveform”





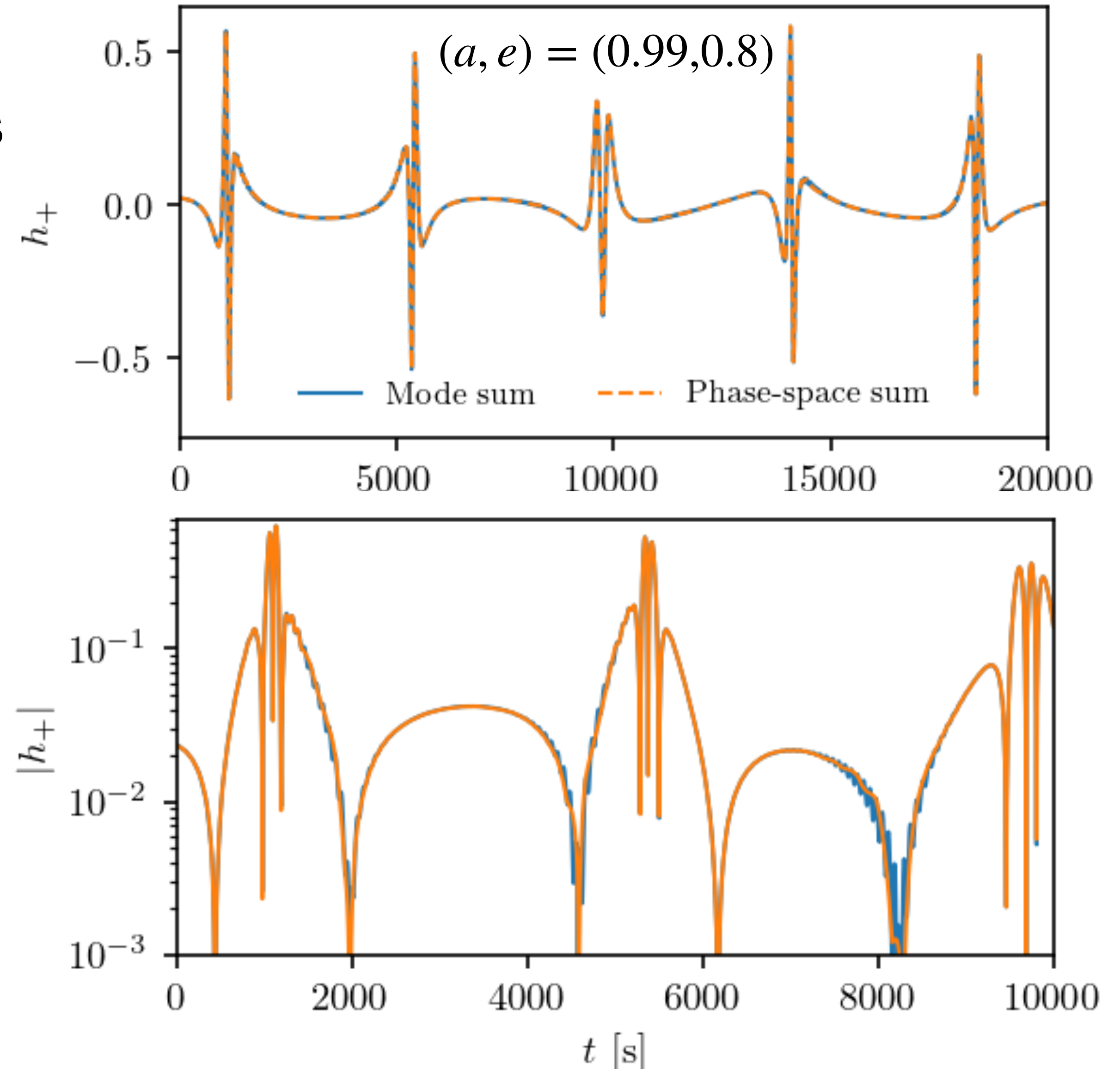
Detectability

- Can we ignore the high-eccentricity part?
 - Spot-check: we probably shouldn't!
- Bear in mind this is still $(l, m) = (2, 2)$ in Schwarzschild!
- More comprehensive detectability study to be done (by next week...)



Radial phase sum: Kerr eccentric equatorial

- High spin, high eccentricity waveforms most expensive in model domain
- 1100 $(l, m, n) : l < 6, n \in [-55, 55]$
- Over four years of observation:
 - **Regular mode sum:** ~ 600 ms
 - **Phase-space sum:** ~ 15 ms
 - **Speed-up:** $\sim 40 \times$



Full phase-space waveform generation

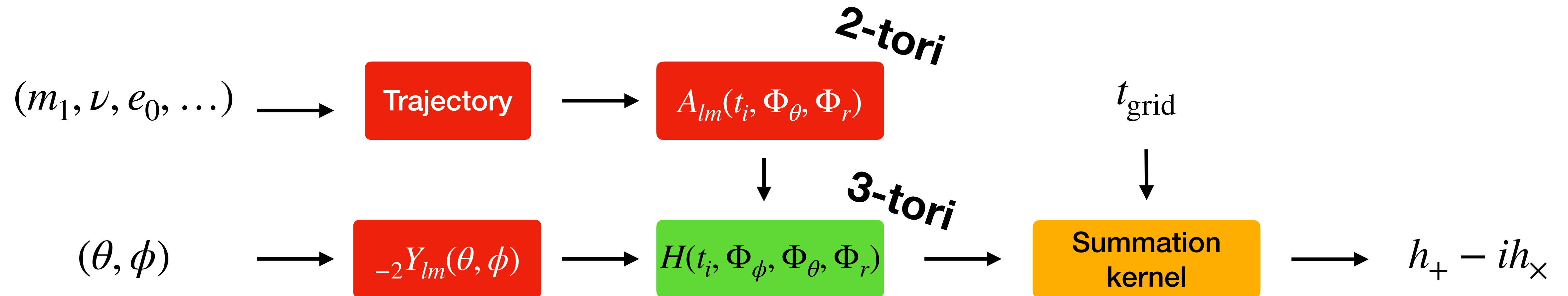
- This idea could in principle be taken further: no mode-sum at all!
- Waveform is a slowly evolving function on the 3-torus:

$$h(t) = \frac{\mu}{d_L} H(t, \Phi_\phi(t), \Phi_\theta(t), \Phi_r(t))$$

- H is a 4-D interpolant over sparsely sampled tori
 - Efficiency gains will be largest when higher-mode content is significant

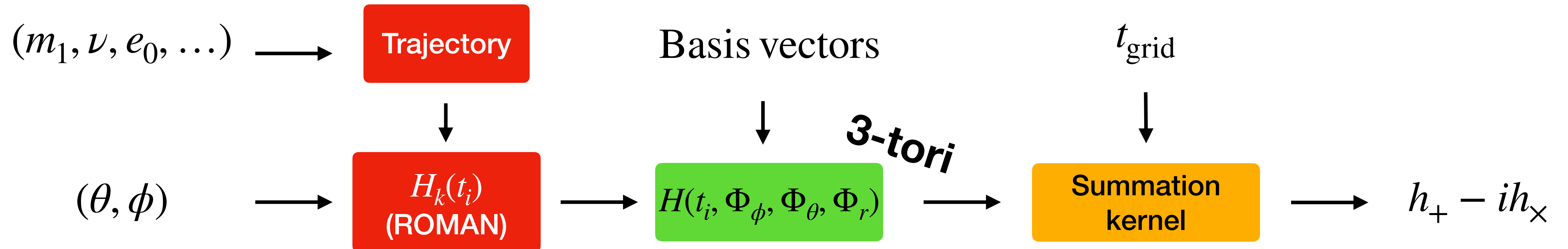
Fiducial phase-space waveform build

- Offline, retain (l, m) decomposition for compactness
- Construct 3-torus online given user-specified viewing angle

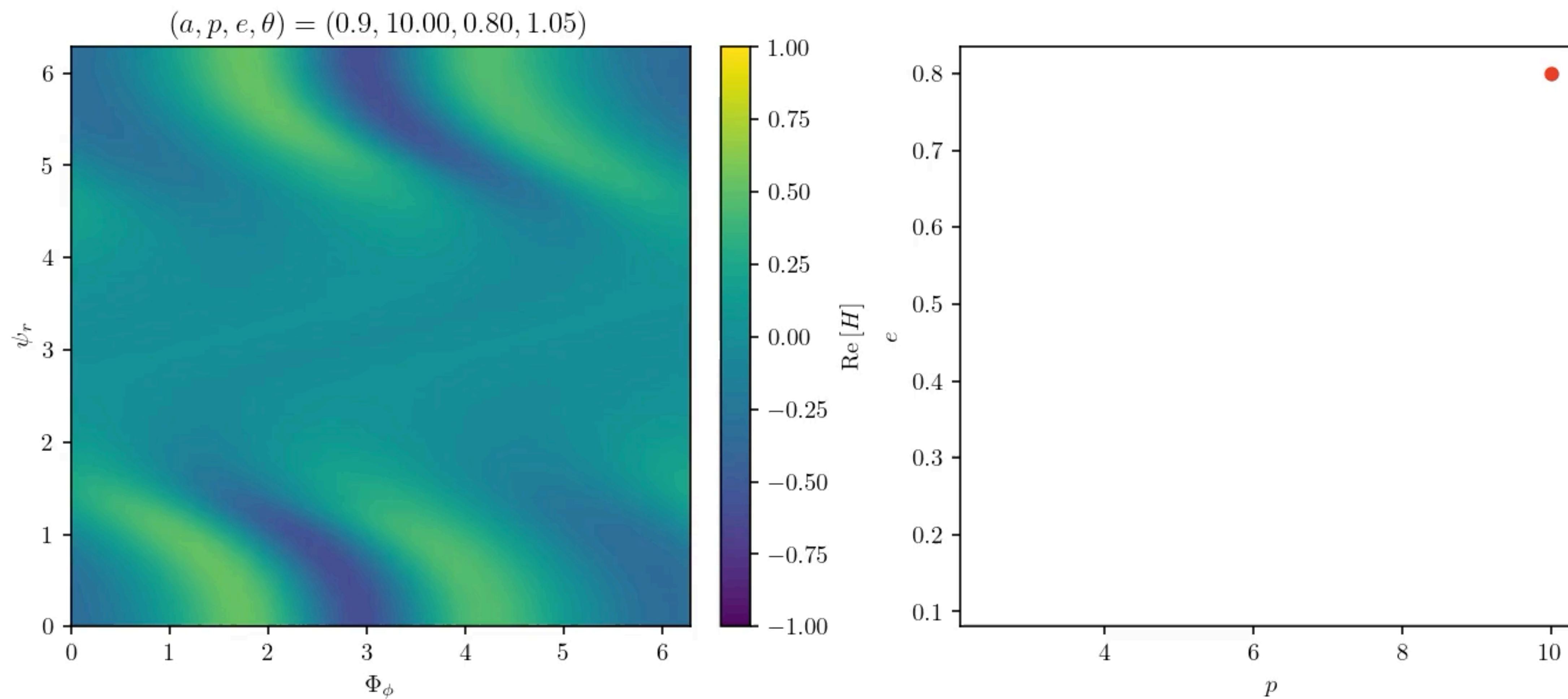


Compact phase-space waveform build

- Amplitude storage a fundamental FEW limitation: need compactification
- ROMAN-style approach: compress 3-tori to reduced basis and interpolate coefficients in $(a, p, e, x, \theta, \phi)$ parameter space

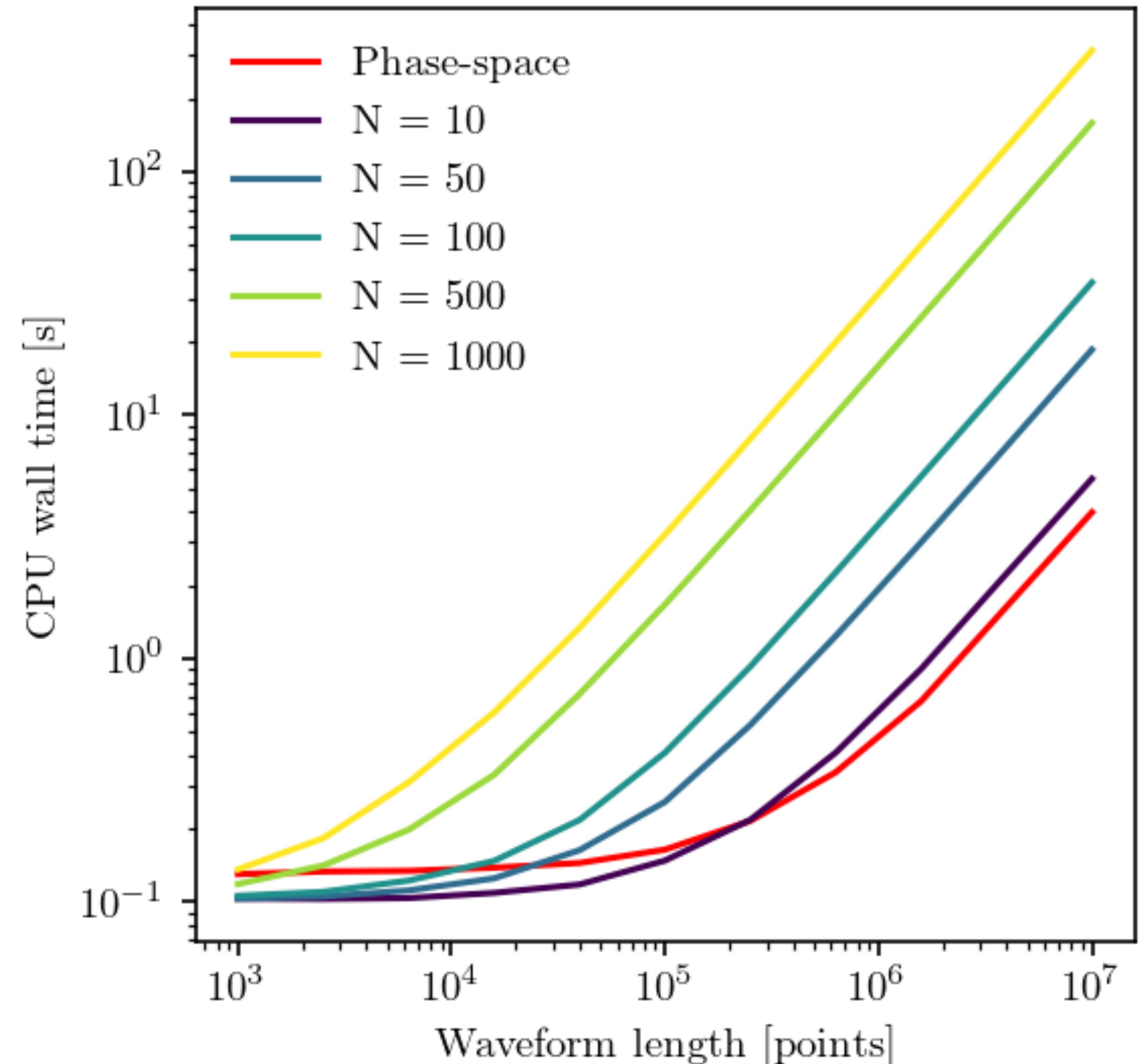


Phase-space sum: Kerr equatorial

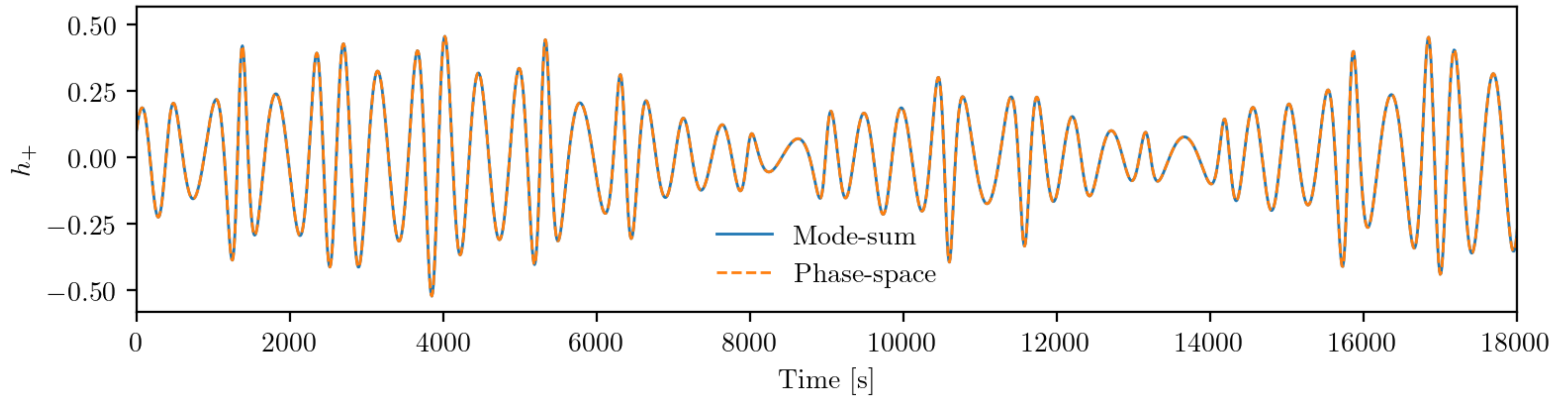


Phase-space sum: Kerr equatorial

- Year-long waveform in ~ 1 s (for $\delta t = 10$ s) on CPUs
- Up to 2 orders of magnitude speed-up in best case
- Laptop EMRI data analysis?
- Expected GPU sum cost < 10 ms (once I code it up...)

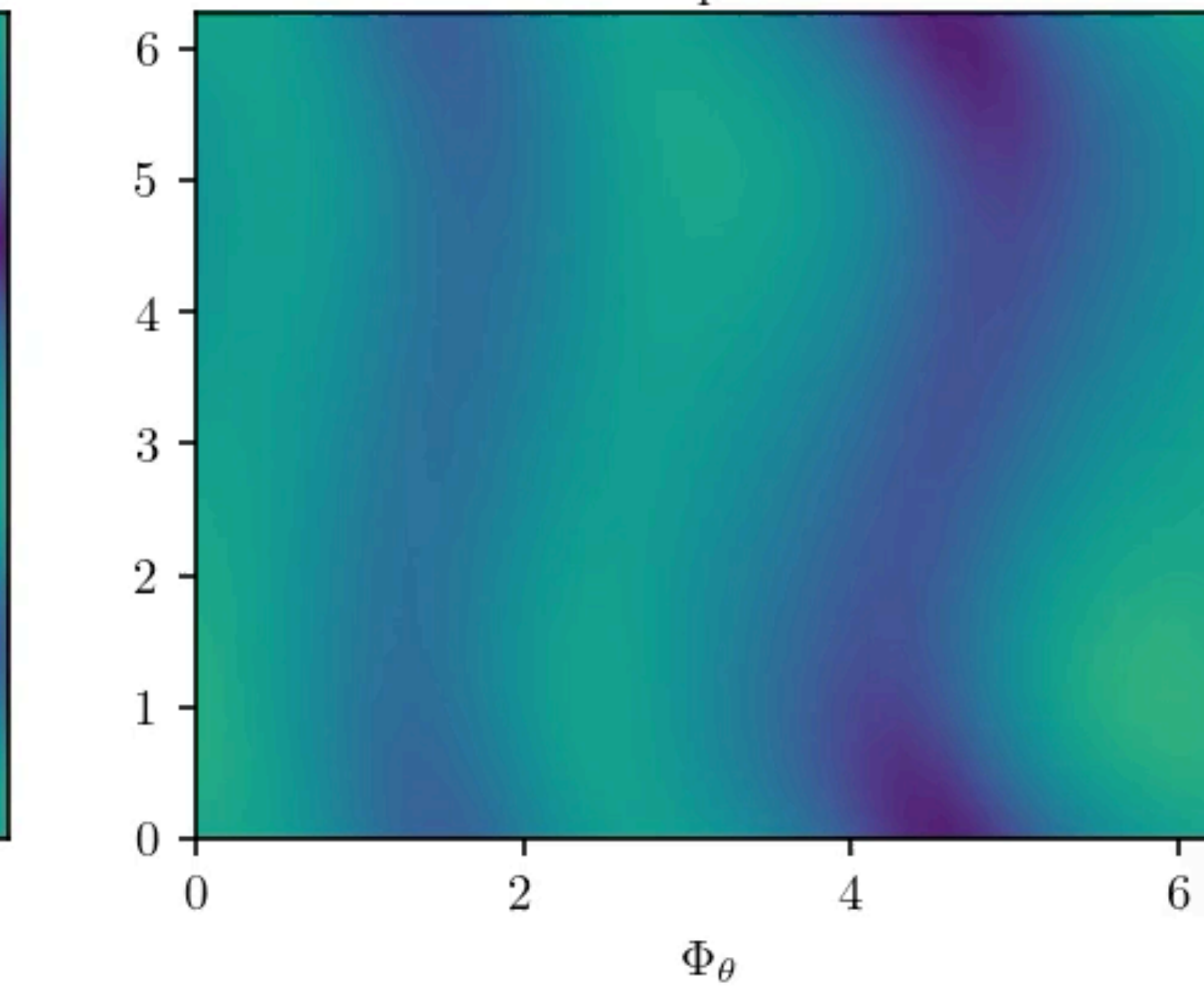
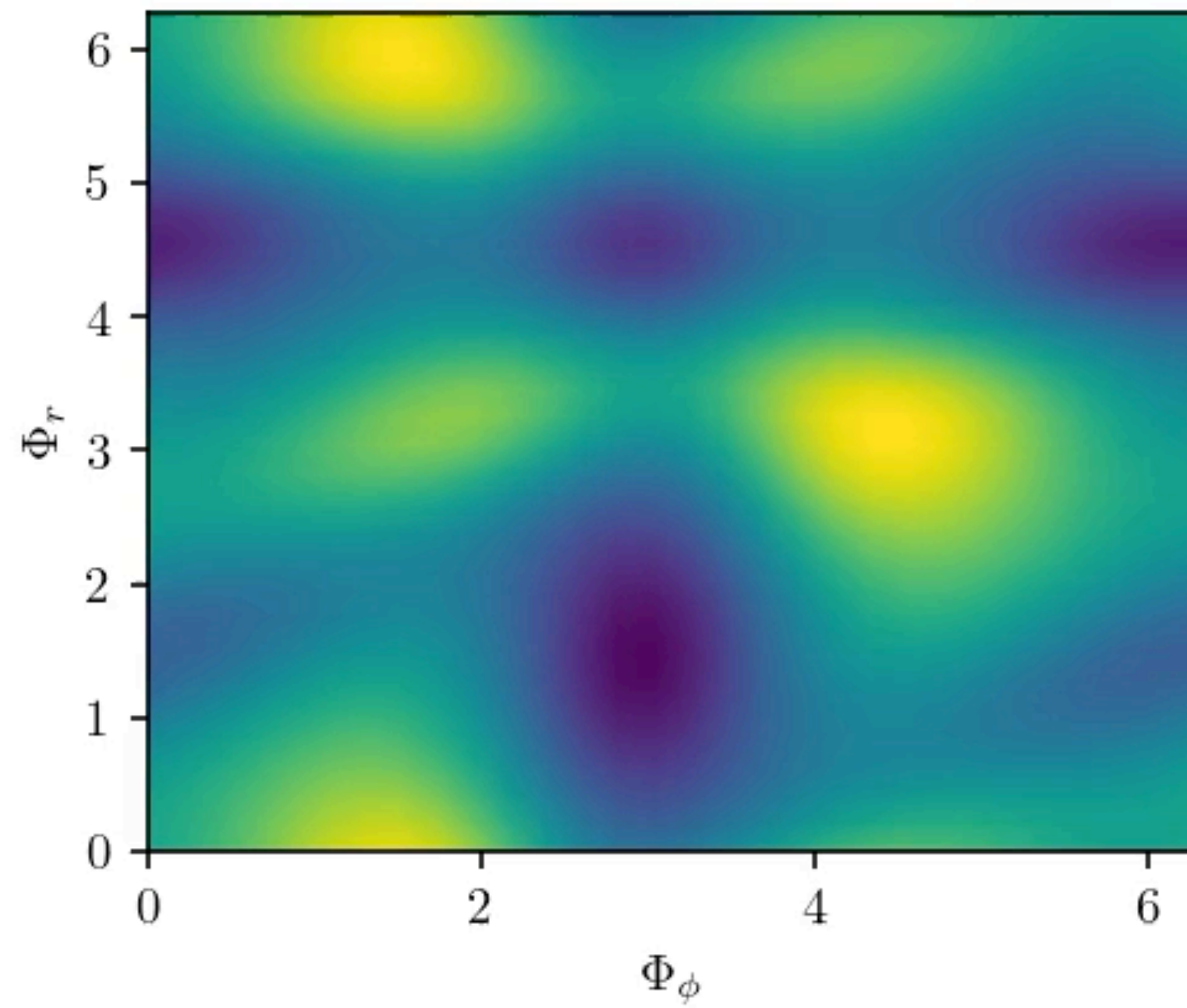
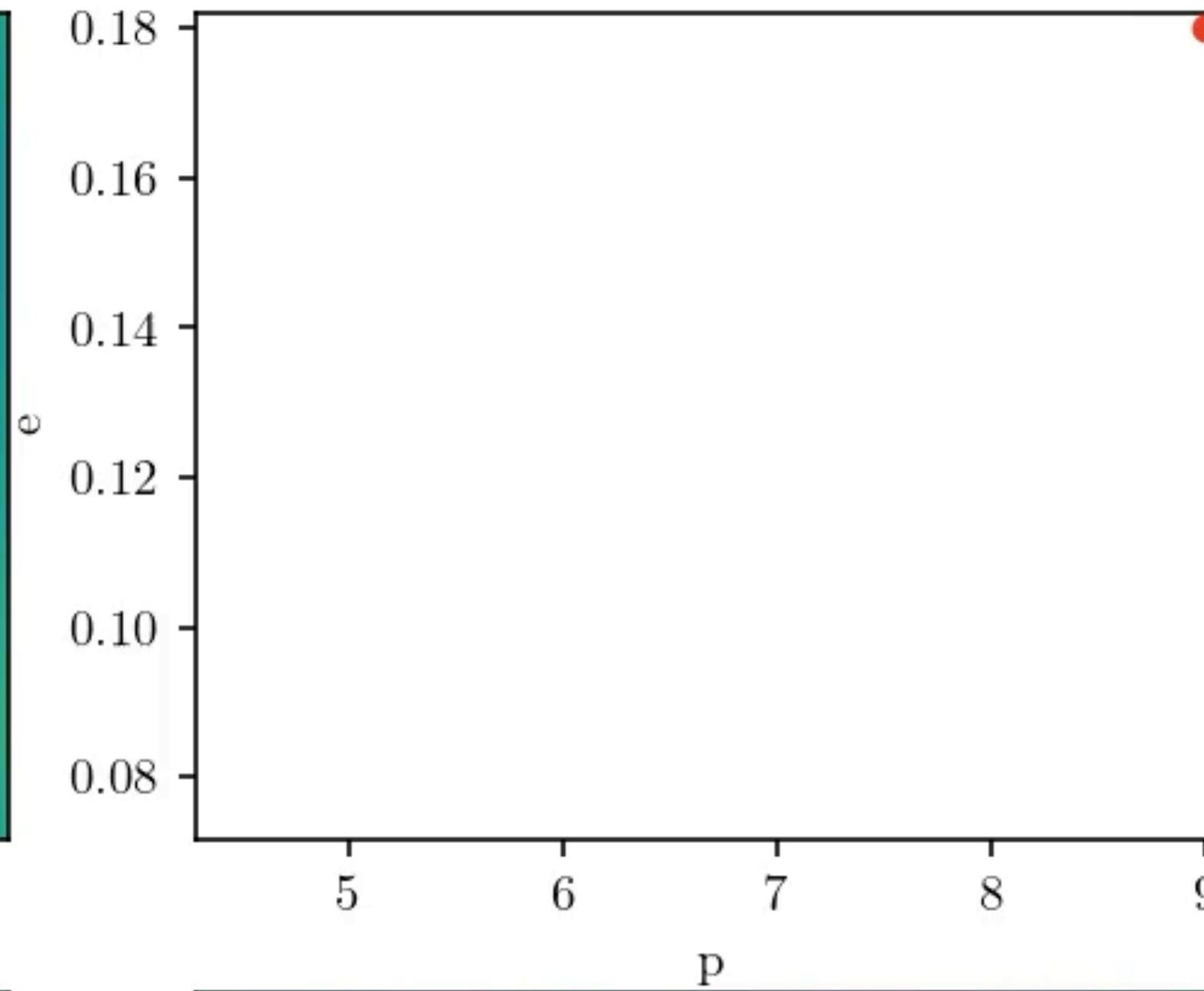
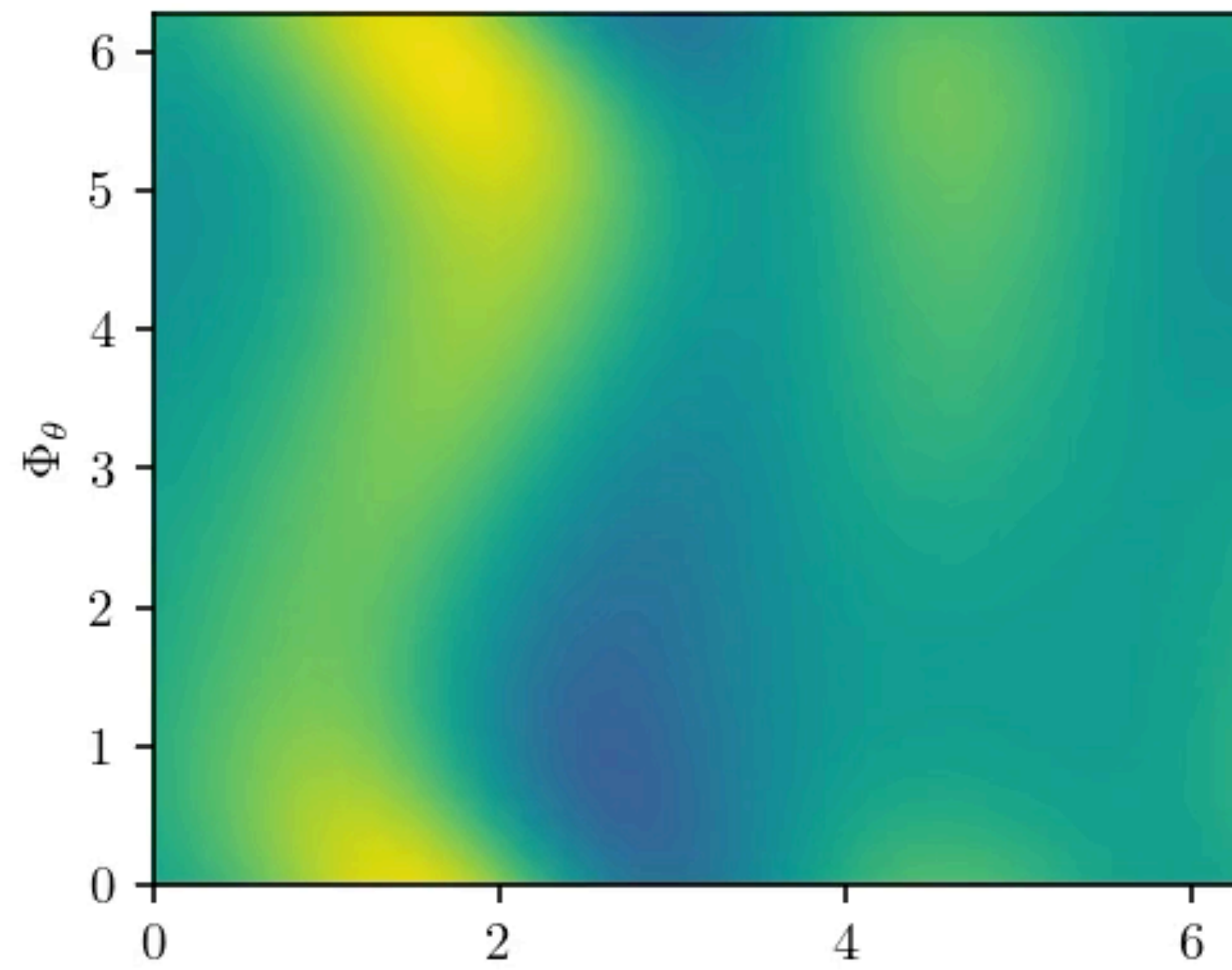


Phase-space sum: Generic Kerr



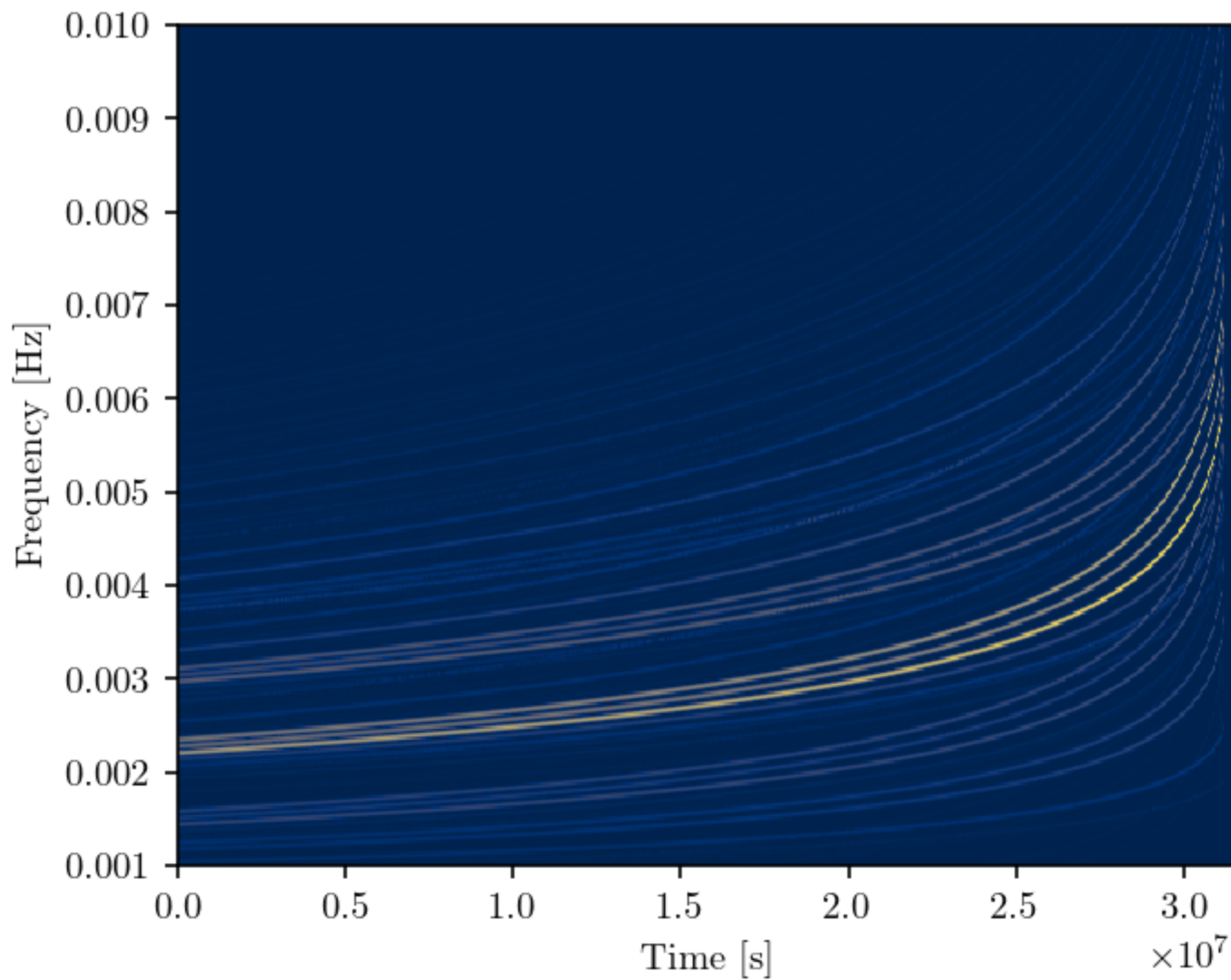
- Readily extensible to generic: test with on-the-fly amplitudes (thanks to Zach!)
- Polar phase can be sampled coarsely (much simpler than radial)
- Generic is only $\sim 4 \times$ the cost of equatorial by construction*

t=0.000, p=9.000000, e=0.180000, x=0.400000

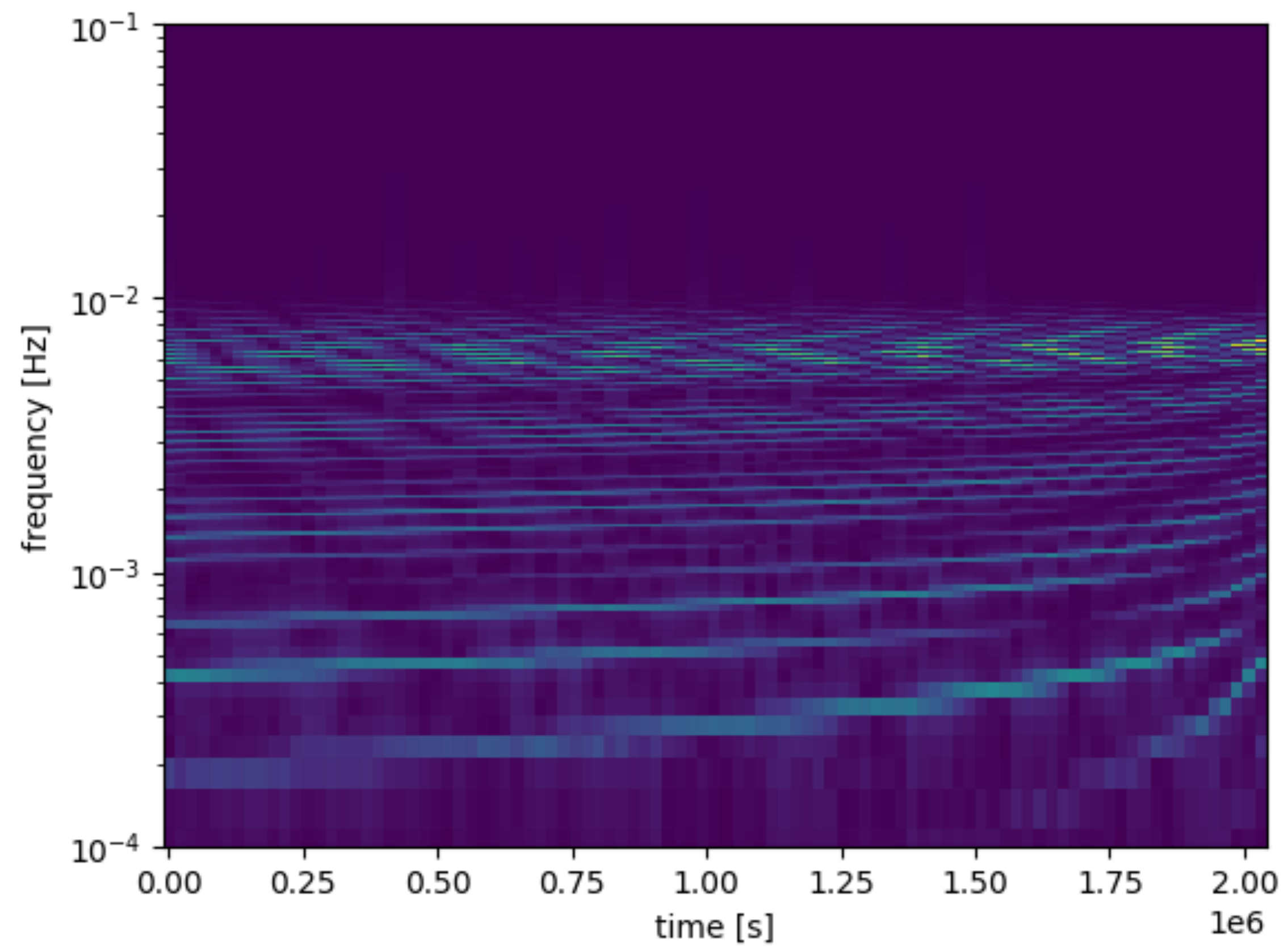


Conclusions

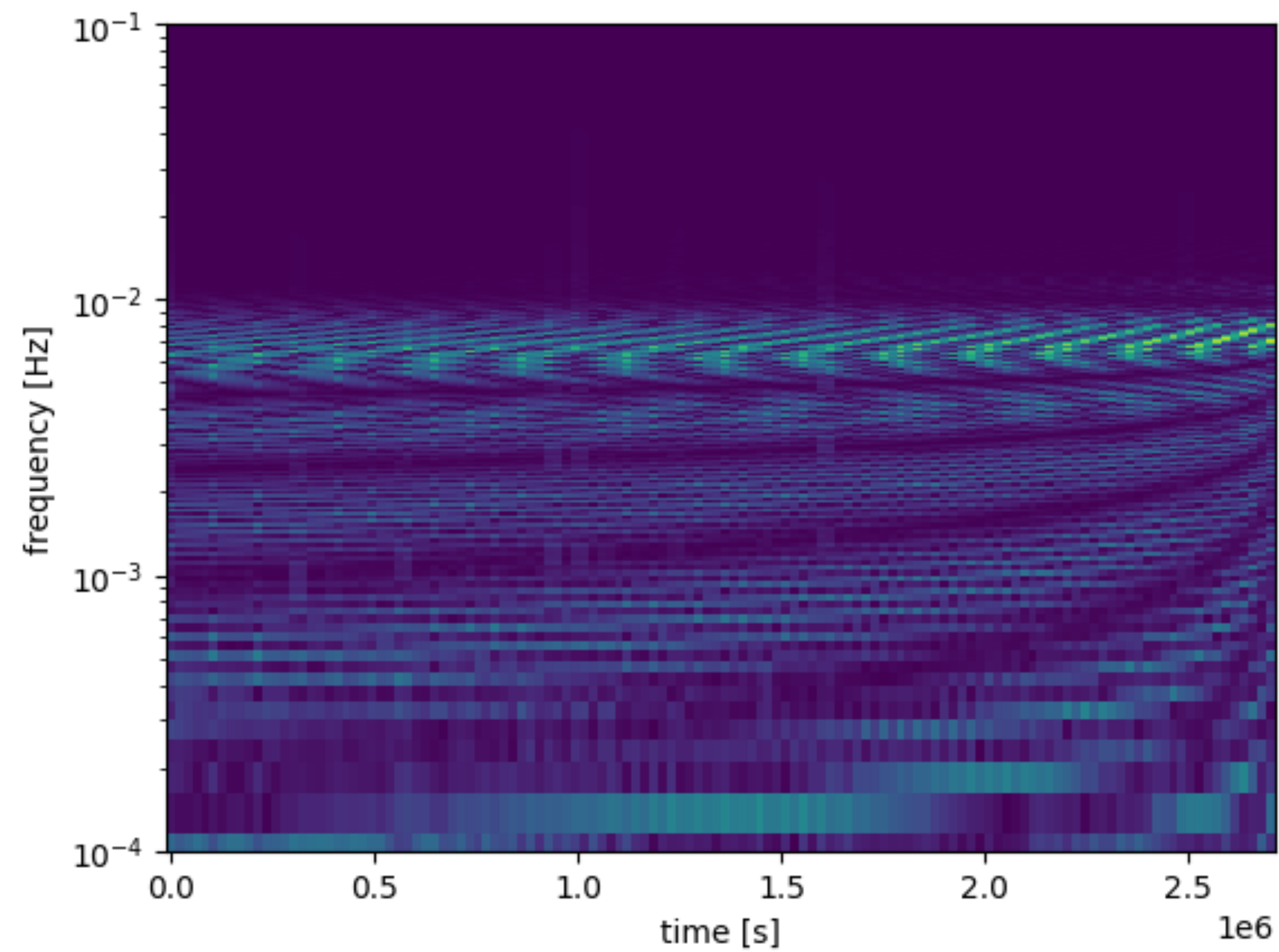
- FastEMRIWaveforms struggles to cover high eccentricity / generic parameter space due to the sharp increase in waveform mode content
 - Alleviate poor eccentricity scaling with change of phase variable
- Constructing EMRI waveforms directly on the 3-torus has significant potential
 - Significant computational savings without compromising accuracy
 - More readily compressible via reduced bases than Fourier coefficients
- Lots of machinery to set up for efficient waveforms — watch this space



$$e_{\text{pl}} \sim 0.7$$



$$e_{\text{pl}} \sim 0.9$$



t=0.000, p=9.000000, e=0.180000, x=0.400000

