

EMRI models for data analysis

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EMRI Search and Inference within the LISA Global Fit - Part II

13 July 2026



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First principles: because that's how we roll

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What else do we need EMRI models to be? **IMRI models**

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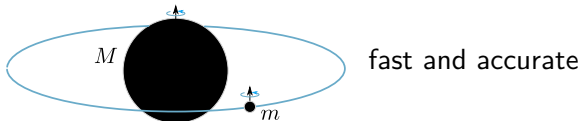
Extensive: coverage of full parameter space

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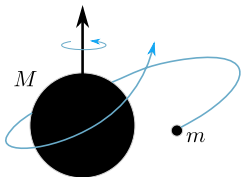
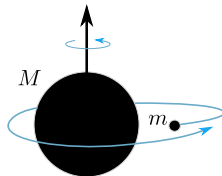
What else do we need EMRI models to be? **IMRI models**

What else do we need EMRI models to be? **Beyond vacuum GR**

EMRI models: what are they now?

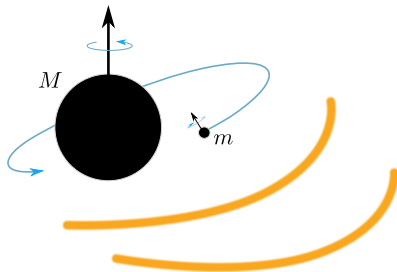


fast, less accurate



fast, even less accurate

Gravitational self-force theory



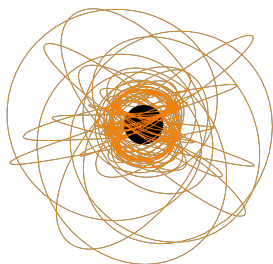
- $\varepsilon = 1/q = m/M \ll 1$
- small body perturbs spacetime:

$$g_{\alpha\beta} = g_{\alpha\beta}^{\text{Kerr}} + \varepsilon h_{\alpha\beta}^{(1)} + \varepsilon^2 h_{\alpha\beta}^{(2)} + \dots$$

- perturbation affects m 's motion:

$$\frac{D^2 z^\alpha}{d\tau^2} = \varepsilon f_{(1)}^\alpha + \varepsilon^2 f_{(2)}^\alpha + \dots$$

Zeroth order: test mass in Kerr



- constants $P_A = (M, a, E, L_z, Q)$:

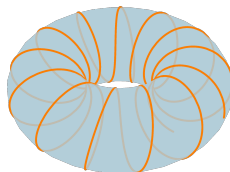
- ① energy E
- ② angular momentum L_z
- ③ Carter constant Q , related to orbital inclination

(often use $p, e, \theta_{\text{inc}}$ instead)

- phases $\Phi_A = (\Phi_r, \Phi_\theta, \Phi_\phi)$ with frequencies $\Omega_A(P_B)$

- simple ODEs:

$$\frac{d\Phi_A}{dt} = \Omega_A(P_B)$$
$$\frac{dP_A}{dt} = 0$$



- choose “nice” variables to describe the perturbed orbital evolution:

$$\begin{aligned}\frac{d\Phi_A}{dt} &= \Omega_A^{(0)}(P_B) + \varepsilon \Omega_A^{(1)}(P_B) + \mathcal{O}(\varepsilon^2) \\ \frac{dP_A}{dt} &= \varepsilon \left[F_A^{(0)}(P_B) + \varepsilon F_A^{(1)}(P_B) + \mathcal{O}(\varepsilon^2) \right]\end{aligned}$$

- treat metric as function on orbital phase space, expand at fixed phase-space coordinates:

$$h_{\alpha\beta} = \sum_{j,m,n,k} \varepsilon^j h_{\alpha\beta}^{(j,m,n,k)}(P_B, x^i) e^{-i(m\Phi_\phi + n\Phi_r + k\Phi_\theta)}$$

- solve EFE for $h_{\alpha\beta}^{(j,m,n,k)}$ and compute $\Omega_A^{(j)}(P_B)$ and $F_A^{(j)}(P_B)$ on grid of P_A values.
- secondary spin enters $\Omega_A^{(1)}$ and $F_A^{(1)}$ and adds a precession phase

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Adiabatic order (0PA)

determined by dissipative modes of $h_{\alpha\beta}^{(1)}$ (1SF)

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Multiscale expansion [Hinderer, Flanagan, AP, van de Meent, Warburton]

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- solve EFE for $h_{\alpha\beta}^{(0)}$ and grid of P_A values.

First post-adiabatic order (1PA)

determined by

- dissipative modes of $h_{\alpha\beta}^{(2)}$ (2SF)
- all of $h_{\alpha\beta}^{(1)}$ (1SF)

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- all of $h_{\alpha\beta}^{(1)}$ (1SF)

- secondary spin enters $\Omega_A^{(1)}$ and $F_A^{(1)}$ and adds a precession phase

$$h_+ - ih_\times = \sum_{j,l,m,n,k} \varepsilon^j h_{lmnk}^{(j)}(P_B) e^{-i(m\Phi_\phi + n\Phi_r + k\Phi_\theta)} {}_{-2}Y_{lm}(\theta, \phi)$$

- waveform's time dependence dictated by orbital evolution:

$$\begin{aligned} \frac{d\Phi_A}{dt} &= \Omega_A^{(0)}(P_B) + \varepsilon \Omega_A^{(1)}(P_B) + \mathcal{O}(\varepsilon^2) \\ \frac{dP_A}{dt} &= \varepsilon \left[F_A^{(0)}(P_B) + \varepsilon F_A^{(1)}(P_B) + \mathcal{O}(\varepsilon^2) \right] \end{aligned}$$

- offline-online split:
 - Pre-compute $h_{lmnk}^{(j)}(P_B)$, $\Omega_A^{(j)}(P_B)$, $F_A^{(j)}(P_B)$ offline
 - Rapidly generate waveforms online

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- offline-online split:
 - Pre-compute $h_{lmnk}^{(j)}(P_B)$, $\Omega_A^{(j)}(P_B)$, $F_A^{(j)}(P_B)$ offline
 - **Rapidly generate waveforms online**
 $\Rightarrow$ FastEMRIWaveforms (FEW)
[Chapman-Bird, Katz, Nasipak, &co]

Goal: 1PA waveforms over full parameter space

Background Spacetime	Orbital Configuration	Adiabatic	Post-1-adiabatic			
		1SF (Dissipative)	1SF (Conservative)	2SF (Dissipative)	Spin Effects (Conservative)	Spin Effects (Dissipative)
Schwarzschild	Circular	✓✓✓	✓✓✓	✓✓✓	✓✓✓	✓✓✓
	Eccentric	✓✓✓	✓✓✓	✗	✓✓, ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	✗	✓, ✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✓✓	✗	✓, ✓✓*	✓✓*
	Generic	✓✓✓	✓	✗	✓	✓*
	Resonances	✓✓✓	✓	✗	✗	✗
✓✓✓ Evolving Waveform		✓✓ Driven Inspiral	✓ Snapshot Calculation		*(Anti-)Aligned Spin Only	

[table from LISA WavWG White Paper]

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Schwarzschild	Circular	✓✓✓	✓✓✓	✓✓✓	✓✓✓	✓✓✓
	Eccentric	✓✓✓	✓✓✓	X	✓✓, ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	X	✓, ✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✓✓	X	✓, ✓✓*	✓✓*
	Generic	✓✓✓	✓	X	✓	✓*
	Resonances	✓✓✓	✓	X	X	X
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	Eccentric	✓✓✓	✗✗✗	✗	✓✓, ✓✓✓*	✓, ✓✓*
Kerr	Circular	✓✓✓	✓✓	✗	✓, ✓✓*	✓✓✓*
	Eccentric Equatorial	✓✓✓	✗✗✗	✗	✓, ✓✓*	✓✓*
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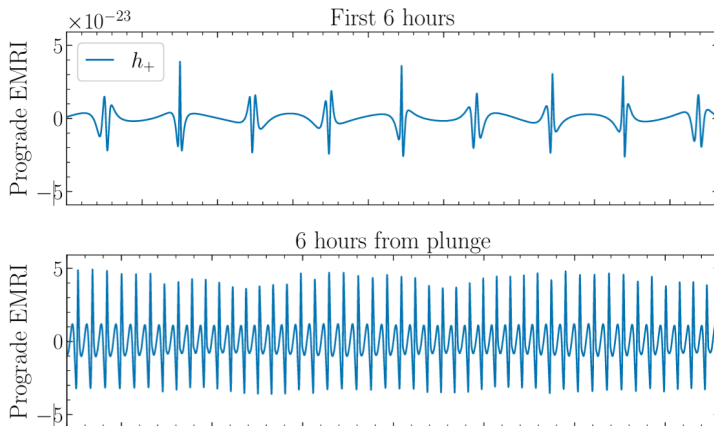
DDPC & Consortium timeline

- 2025  Mojito Light data set – 0PA eccentric equatorial Kerr, nonspinning secondary
- 2027  Mojito Heavy data set – 1PA quasicircular, slowly spinning primary, spinning secondary
- 2027  Key Project: Mojito Light followup studies – environments and beyond GR
- 2029  Long Island Iced Tea – 0PA generic Kerr, best 1PA available, IMRIs
- 2031  Aunt Roberta – “fully realistic”

State of the art in extensiveness: KerrEccEq0PA

[Chapman-Bird et al.]

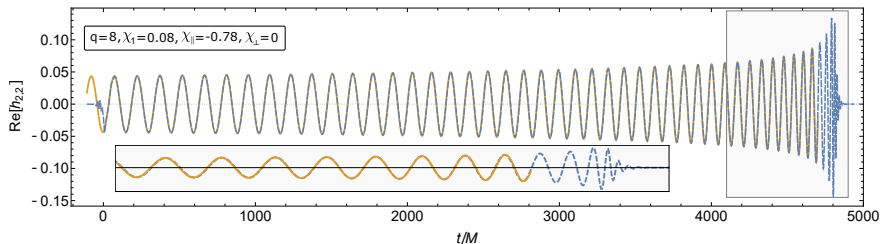
- leading order (0PA): eccentric equatorial orbits in Kerr
- $|a/M| \leq 0.998$, $0 \leq e_0 \lesssim 0.85$



State of the art in accuracy: Circ1PAT1R

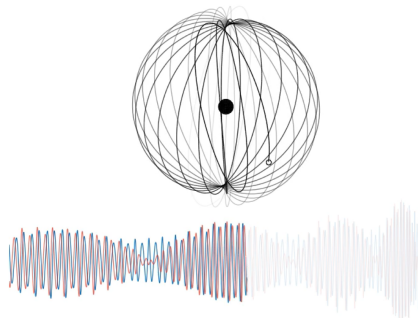
[Honet, Mathews, AP, & M2SF]

Comparison with NR waveform from SXS collaboration



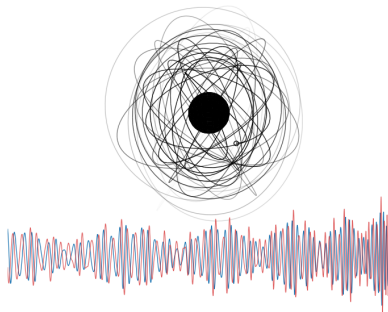
- 1PA quasicircular, nonprecessing orbits
- rapidly spinning secondary, slowly spinning primary

Spherical orbits in Kerr:
fully functioning



[Image credit: Nasipak]

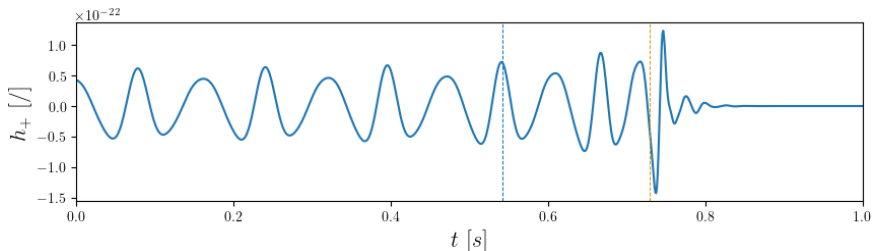
Fully generic::
grid up to $e = 0.2$



[Image credit: Nasipak]

Inspiral-merger-ringdown in FEW

Quasicircular, nonspinning binaries [Honet, Küchler, AP, & M2SF]

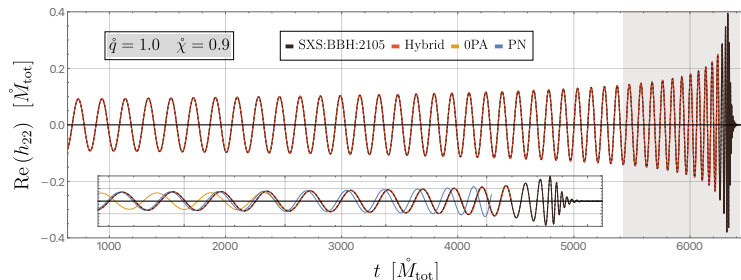


- 1PA inspiral, next-to-leading transition, leading-order plunge
- extension to eccentric underway [Lhost, Compere, Barack, et al.]
- see also:
 - 1PA-NR hybrid [Iglesias et al.]
 - surrogate [Islam et al.]

Work in progress: hybridization with PN

[Honet, AP, Compere; Trestini, Nasipak, AP; Mathews et al.]

- two motivations:
 - 2SF data is very hard to get $\Rightarrow$ use PN to approximate it
 - 2PA terms contribute a large amount at large separations $\Rightarrow$ critical for IMRIs $\Rightarrow$ use PN to approximate them
- example: WaSABI-C quasicircular model for (anti)aligned spins

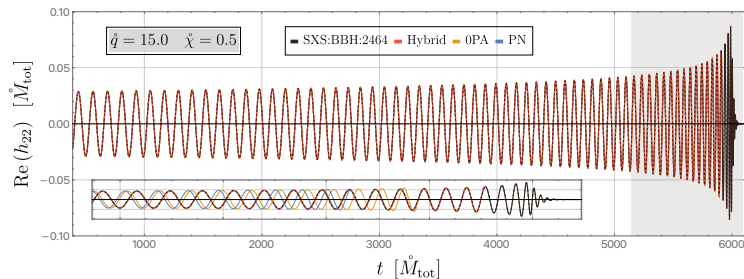


- two eccentric models implemented in FEW. See talk by Mathews

Work in progress: hybridization with PN

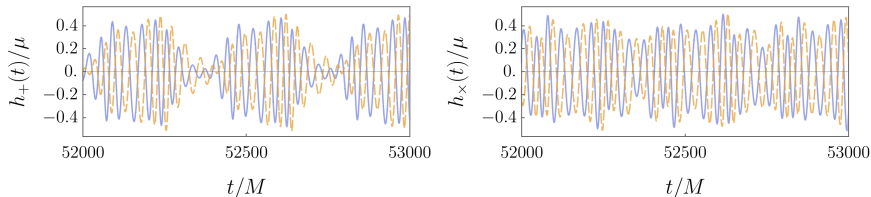
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1PA secondary spin effects [Prague groups; Drummond & Hughes et al.]



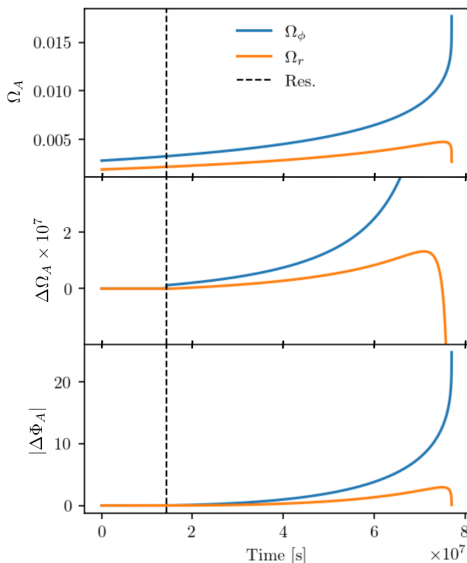
- complete prescription available [Mathews & AP '25, Skoupy '26]
- no complete data sets
—but new fast code has emerged [Witzany & Claude]
- see talk by G. Piovano

Work in progress: orbital resonances [Chapman-Bird et al.]

frequencies become
momentarily
commensurate

Ω_A jump slightly across
the resonance

leads to a significant
cumulative change in
phase



[Image credit: C. Chapman-Bird]

Work in progress: high eccentricity & new waveform construction [Chapman-Bird, Nasipak, AP, Roy]

A project born at NUS 1 year ago...

- problem: too many radial modes! Max. n grows with e .
- why do we have modes? Frequency domain for geodesic orbits:

$$h_{\alpha\beta}^{(1)} = \sum_{m,n,k} h_{\alpha\beta}^{(1,m,n,k)}(P_B, r, \theta, \phi) e^{-i(m\Omega_\phi^{(0)} + n\Omega_r^{(0)} + k\Omega_\theta^{(0)})t}$$

- multiscale expansion replaces this with “phase domain” for evolving orbits

$$h_{\alpha\beta} = \sum_{j,m,n,k} \varepsilon^j h_{\alpha\beta}^{(j,m,n,k)}(P_B, r, \theta, \phi) e^{-i(m\Phi_\phi + n\Phi_r + k\Phi_\theta)}$$

- why stop here?

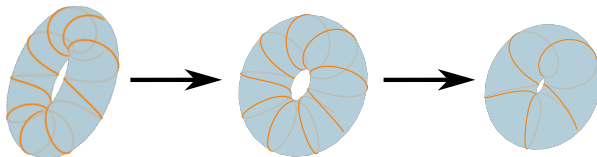
Work in progress: high eccentricity continued

- $\sum_{j,m,n,k} \varepsilon^j h_{\alpha\beta}^{(j,m,n,k)}(P_B, r, \theta, \phi) e^{-i(m\Phi_\phi + n\Phi_r + k\Phi_\theta)}$ is the Fourier representation of a function on a 3-torus
- for high e , a Fourier series is a *bad* representation
 $\Rightarrow$ work directly with interpolants on each torus of constant P_B :

$$h = \sum_{j,l,m} \varepsilon^j h_{lm}^{(j)}(\Phi_A, P_A) {}_{-2}Y_{lm}(\theta, \phi)$$

- (note: solving the multiscale EFE yields a function on the orbital phase space, not a solution for a particular source trajectory)

Time evolution

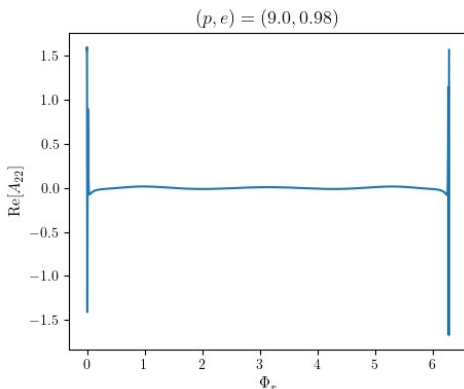


Work in progress: high eccentricity continued

- Example:
in Schwarzschild, write

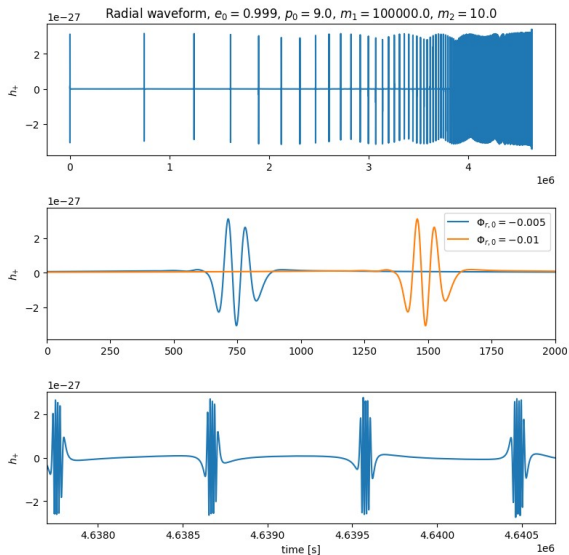
$$h_{lm}^{(1)} = A_{lm}(\Phi_r, p, e) e^{-im\Phi_\phi}$$

- using a Fourier series
would be mad



[Credit: Nasipak]

Work in progress: high eccentricity continued

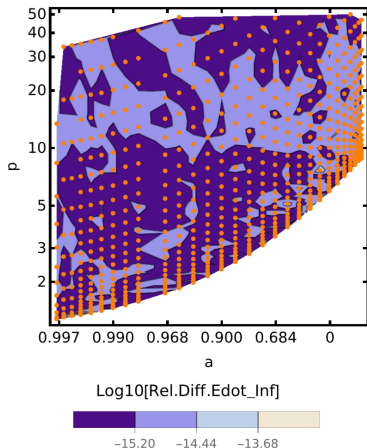


[Credit: Chapman-Bird]

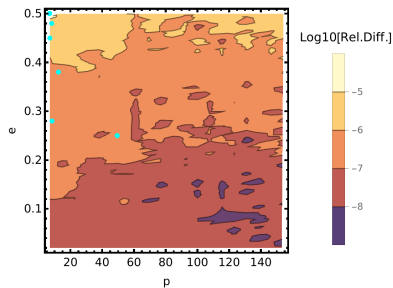
LISA Consortium: common/verified data sets

Community-generated, cross-verified data sets for all model ingredients

0PA flux for circular orbits in Kerr



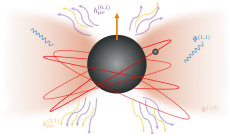
1SF redshift for eccentric orbits
in Schwarzschild



[Credit: Akcay]

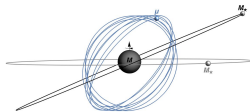
LISA Consortium: beyond-vacuum-GR models

EMRIs in boson clouds



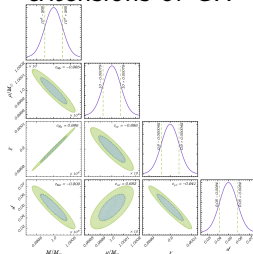
[Image: Li et al.]

Tidal resonances



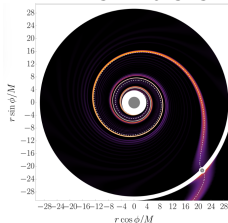
[Image: Bonga]

EMRIs in EFT extensions of GR



[Image: Maselli et al.]

EMRIs in disks



[Image: Dyson & D'Orazio]

EMRIs in dark matter



[Image: Duque et al.]

Common goal:
waveform models

Summary

- EMRI waveforms: fast, accurate, first principles
—but far from complete

2026

- 0PA: equatorial inspirals into Kerr primary
- 1PA: quasicircular inspirals into slowly spinning primary
- resonances, secondary spin, 1PA Kerr: commencing data generation

2027

- 0PA: good spherical, “bad” generic, highly eccentric
- 1PA: PN hybrid eccentric, quasicircular Kerr (?!)
- IMRIs: inspiral-merger-ringdown & PN hybridization
- many beyond-vacuum-GR models