Astrophysics of compact objects

Part II: Global models and microphysics input

Micaela Oertel

micaela.oertel@astro.unistra.fr

Observatoire astronomique de Strasbourg
CNRS / Université de Strasbourg

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Outline

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Neutron star formation via accretion induced collapse

If the WD accretion has a low enough rate it can indeed collapse and form a neutron star not leading to a thermonuclear explosion.

Review article
`https://iopscience.iop.org/article/10.1088/1674-4527/20/9/135`

Produces a priori faint optical transient, not yet observed

Collapse or explosion ? Depends on competition between electron capture and nuclear fusion reactions in a massive WD close to Chandrasekhar mass

Plan

Some timescales

Object	evolution	reactions
Neutron star	100y	$\tau_{strong,em} \ll \tau_e$; $\tau_{weak} \ll \tau_e$
Proto-neutron star	few minutes	$\tau_{strong,em} \ll \tau_e$; $\tau_{weak} \approx \tau_e$
Supernova	few 100 ms	$\tau_{strong,em} \ll \tau_e$; $\tau_{weak} \approx \tau_e$
Binary merger inspiral	few minutes	$\tau_{strong,em} \ll \tau_e$; $\tau_{weak} \ll \tau_e$
Binary merger post-merger	few 10 ms	$\tau_{strong,em} \ll \tau_e$; $\tau_{weak} \approx \tau_e$

- Thermal equilibrium for baryons, charged leptons $\longrightarrow$ equation of state
- Timescale for weak reactions strongly dependent on temperature (and density) $\longrightarrow$ chemical equilibrium not always achieved
- Hydrodynamical timescale $\sim$ size of the object/sound speed $\sim 10^{-3}s$

Plan

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Plan

1. Introduction
2. Global modeling
   - Modeling mature neutron stars
   - Modeling CCSN, PNS and binary mergers
3. Constructing an equation of state
   - The Free Fermi gas
   - The nuclear interaction comes into play
   - Sub-saturation matter
   - Beyond NSE
   - Supra-saturation matter
4. Neutrino matter interactions

Physical ingredients

First step, building equilibrium models :

- assume that nuclear matter can be treated as a perfect fluid ;
- assume chemical equilibrium ;
- neutrinos freely leave the system ;
- give the gravitational law (self-gravitating body) ;
- write the hydrostatic equilibrium ;
- give a law for the pressure as a function of nuclear matter density (temperature ?) ⇒equation of state (EOS).

The EOS specifies the nuclear matter properties and, in particular the strength of the strong interaction between particles, which is to equilibrate gravity.

Gravitational law

Due to the intense gravitational field ($\Xi \sim 0.2$), a newtonian description is not adequate and we have to use general relativity, i.e. solve Einstein equations :

$$R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi G}{c^4} T^{\alpha\beta}$$

The matter part is entirely specified by the energy-momentum tensor $T^{\alpha\beta}$ (of a perfect fluid) :

$$T^{\alpha\beta} = \left(\varepsilon + \frac{p}{c^2}\right) u^\alpha u^\beta + p\, g^{\alpha\beta}.$$

To describe the gravitational field in GR, one needs the metric $g_{\alpha\beta}$. In the static and spherically symmetric case it can be written with Schwarzschild coordinates as :

$$\mathrm{d}s^2 = g_{\alpha\beta} \mathrm{d}x^\alpha \mathrm{d}x^\beta = -N(r)^2 c^2 \mathrm{d}t^2 + A(r)^2 \mathrm{d}r^2 + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)$$

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Tolman-Oppenheimer-Volkov system

Taking

- Einstein equations in the static and spherically symmetric case
- Define $m(r)$ and from $A = \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$ and $\Phi(r)$ from $N = \exp\left(\Phi/c^2\right)$,

Einstein and hydrostatic equations reduce to :

$$\begin{aligned}
\frac{\mathrm{d}m}{\mathrm{d}r} &= 4\pi r^2 \varepsilon \\
\frac{\mathrm{d}\Phi}{\mathrm{d}r} &= \left(1 - \frac{2Gm}{rc^2}\right)^{-1} \left(\frac{Gm}{r^2} + 4\pi G \frac{p}{c^2} r\right) \\
\frac{\mathrm{d}p}{\mathrm{d}r} &= -\left(\varepsilon + \frac{p}{c^2}\right) \frac{\mathrm{d}\Phi}{\mathrm{d}r}
\end{aligned}$$

In the newtonian limit $m(r)$ describes the enclosed mass and $\Phi(r)$ the gravitational potential.
This system of partial differential equations is called the Tolman-Oppenheimer-Volkov (TOV) system

Solving the system : EOS

In order to integrate the TOV system, one must first specify an EOS :

- soon after their birth in supernovae, neutron stars cool down below their Fermi temperature $\rightarrow$ temperature effects can in general be neglected
- strong, electromagnetic, and weak reactions are at equilibrium (this includes in particular β-equilibrium for $n \leftrightarrow p$)
- cold catalyzed matter at the endpoint of thermonuclear evolution.

$\Rightarrow$all state variables are functions of only one parameter, chosen conveniently to be *e.g.* the baryon number density n_B.

Details from the microphysics (nuclear/particle physics) point of view on the construction of the EOS will be discussed later.

Solving the system : boundary conditions

In addition to the EOS suitable boundary conditions have to be specified in order to integrate the TOV system :

- a value of the central density (to vary the resulting mass)
- regularity conditions at $r = 0$
  - $m(r = 0) = 0$
  - $\Phi(0) = \Phi_0$

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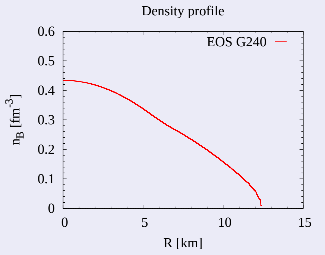
The surface of the star is defined by the condition of vanishing pressure $p(r = R) = 0$.
The system can thus be integrated until the surface, where it is matched with the vacuum spherical static solution (Schwarzschild metric).



Global quantities

In GR, the following global quantities can be defined :

- the gravitational mass can be defined for an isolated system, and here as
$$M_g = \int_0^R 4\pi r^2 \varepsilon(r)\mathrm{d}r = m(R).$$
- the baryon mass is given by the number of baryons contained in the star
$$M_b = m_b \int_0^R 4\pi r^2 A(r) n(r)\mathrm{d}r.$$
- the gravitational redshift is the frequency relative redshift undergone by a signal emitted at the surface of the star and measured by a distant observer
$$z = \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} - 1 = (1 - 2\Xi)^{-1/2} - 1.$$

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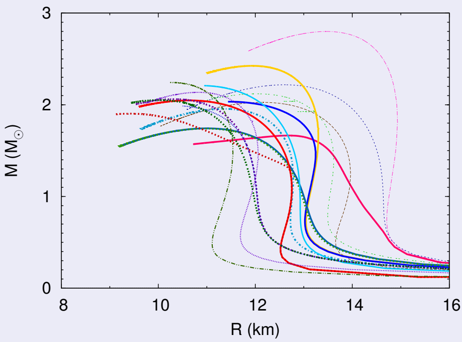
Solving TOV equations

Mass-radius relation

- Solving TOV system $\rightarrow M = m(R)$ and $R = r(p = 0)$
- Maximum mass is a GR effect, value given by the EOS

More details $\rightarrow$ exercices.

Different EOS models (taken from CompOSE)



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Rotating models : more complications

Next step : take into account rotation (still assume stationarity) ; two possibilities

- Analytically perturb spherical models (valid for low rotation frequencies)
- Numerically compute full models in axisymmetry.

Assumption of circularity (no meridional convective currents), ⇒four gravitational potentials, depending on (r, θ). Two more differences with spherical models :

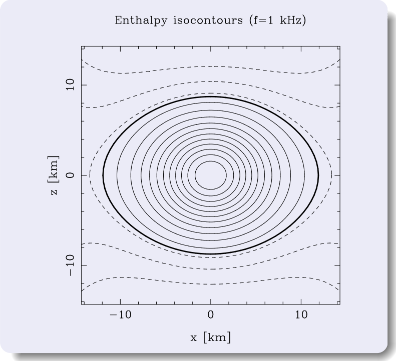
- need to specify the rotation law as $\Omega = f(r \sin\theta)$, $f =$ const being rigid rotation
- the gravitational potentials must be integrated up to spatial infinity, where space-time is asymptotically flat.

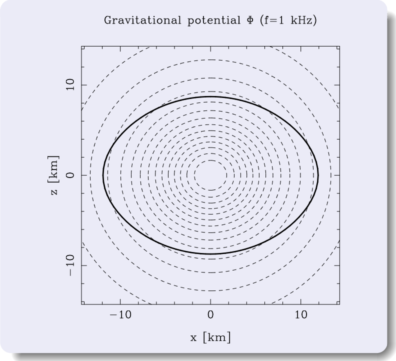
With $H = \log\left(\frac{\varepsilon + p}{n_B\, m_B}\right)$ the pseudo-enthalpy, the fluid equilibrium reads (γ is the fluid Lorentz factor) :

$$H + \Phi - \log\gamma = \text{const.}$$

Rotating models : numerical integration

System of four coupled non-linear Poisson-like equations, with non-compact sources ⇒only numerical solutions





Note : the bold line is the $H = 0$ iso-contour, representing the star's surface.

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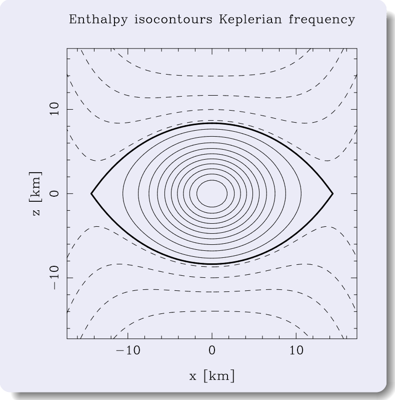
Rotating models : Kepler limit

In the case of rigid rotation, the angular frequency is physically limited by the mass shedding limit.



Also called Kepler limit, here $\Omega_K \simeq 1100 Hz$.

For each central density (baryon mass) one can define an absolute maximal rotational frequency.

Maximum frequency depends on EOS.

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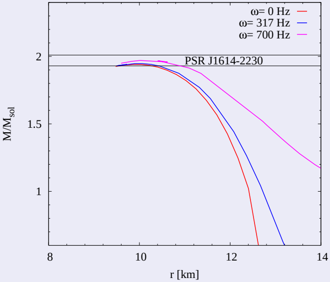
Effects of rotation

For a given number of baryons, the effect of rotation is to :

- increase the radius of the star,
- decrease its central density,
- increase the gravitational mass.



The maximal mass associated to a given EOS is even more increased from the existence of supermassive sequences : rotating solutions that cannot exist in spherical symmetry.

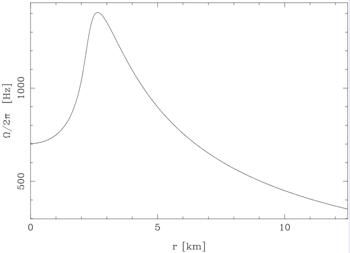
Differential rotation

Rotation law not constant in many cases (CCSN, post-merger, etc)
$\rightarrow\ \Omega = f(r\sin\theta)$

$$\partial_i(H + \Phi - \log\gamma) = F\partial_i\Omega$$

- global quantities depend on rotation law
- viscous effects rigidify rotation

Typical profile for a BNS merger remnant



The maximal mass associated to a given EOS is even more increased from the existence of hypermassive sequences : rotating solutions that cannot exist in spherical symmetry nor with rigid rotation

Tidally deformed neutron stars

Tidal Love numbers : constant of proportionality between external tidal field applied to the body and the resulting multipole moment of its mass distribution

- assume weak tidal field ;
- assume field slowly varying with time ;

For a quadrupolar field $\longrightarrow$

$$Q_{ij} = -\frac{2}{3} R^5 k_2 \mathcal{E}_{ij}$$

The tidal Love number k_2 depends the star's inner structure ;

It can be computed from a perturbation of the spherical equilibrium (TOV) solution ;

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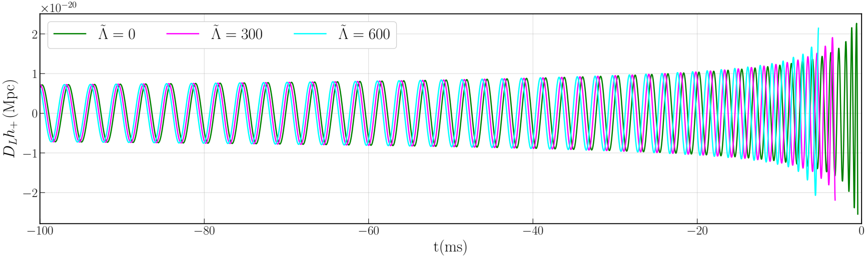
Tidally deformed neutron stars

Binary inspiral

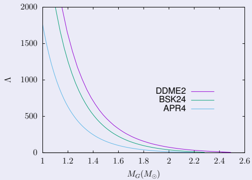


[Chatziioannou 2020]

Tidal deformation during late inspiral of binary coalescence influences the phase of GW emission

Lowest order changes sensitive to a combination of deformabilities $\Lambda_i \propto (k_2)_i (R_i/M_i)^5$ of both stars

$$\tilde\Lambda = \frac{16}{3}\frac{(M_1 + 12 M_2) M_1^4 \Lambda_1^4 + (M_2 + 12 M_1) M_2^4 \Lambda_2}{(M_1 + M_2)^5}$$



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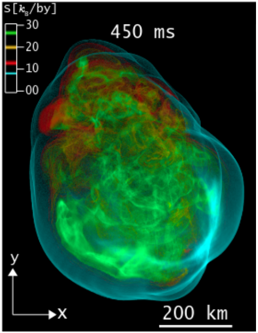
Modeling of dynamical processes

Timescales in CCSN, binary mergers too short to assume stationarity

$$\nabla_\alpha T^{\alpha\beta} = \sigma^\beta[f_\nu]$$
$$\nabla_\alpha J^\alpha_B = 0$$
$$\nabla_\alpha J^\alpha_{L_e} = S[f_\nu]$$



[Bollig+ 2021]

$+$ transport equation for neutrinos

$$p^0\frac{\partial f_\nu}{\partial t} + p^i\frac{\partial f_\nu}{\partial x^i} - \Gamma^i_{ab}p^a p^b\frac{\partial f_\nu}{\partial p^i} = (-u_a p^a)\mathcal{B}[f_\nu]$$

$+$ equation for gravity (Einstein equations)

$\rightarrow$ numerical simulations

- Simulations too time consuming without approximations : newtonian + corrections vs GR, 1D hydro vs multi-D, quasi-stationarity, approximate treatment of transport, ...

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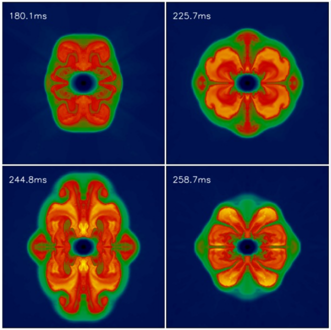
Global modeling vs internal structure

Global models based on numerical simulations

Many ingredients : gravity law, (multiD) hydrodynamics, magnetic field, rotation . . .

Microphysics information needed to close the system of equations

- Neutrino-matter interactions
- An equation of state



[Buras+ 2003]

Thermodynamic conditions very different for the different astrophysical objects

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Chemical equilibrium

- Thermal and mechanical equilibrium in general very quickly acheived and temperature T and pressure p are well defined variables, except for neutrinos (and photons)
  - these particles have to be treated by transport equations coupled to hydrodynamics
- Chemical equilibrium ? Only if

reaction timescale $\ll$ timescale of the system's hydrodynamic evolution

Examples :

- Weak reactions such as $p + e \to n + \nu_e$ too slow to reach equilibrium in parts of CCSN and BNS merger matter
- Complete equilibrium under nuclear species ("nuclear statistical equilibrium") not reached at low temperatures and densities $\to$nuclear reaction network

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How to construct an equation of state ?

An equation of state is a relation between two or more functions describing the thermodynamic state of matter, e.g. temperature, density, pressure, energy, i.e. it describes the state of matter under a given set of physical conditions. It assumes thermal equilibrium.

Examples :

- Stellar interior : hot and dilute gas $\longrightarrow$ ideal Maxwell-Boltzmann gas
- White dwarfs : degenerate electron gas $\longrightarrow$ ideal Fermi gas
- Neutron stars : cold strongly interacting matter

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Reminder of thermodynamic identities

There are different thermodynamic potentials depending on the temperature (T)/the entropy (S), the volume (V)/the pressure (p), the particle number (N)/the chemical potential (μ), or the corresponding densities (s, n)

- the energy density $\varepsilon(s, n_i)$
- the free energy density $f(T, n_i) = \varepsilon - Ts$
- the grand canonical potential density $\omega(T, \mu_i) = \varepsilon - Ts - \sum_i \mu_i n_i$
- the conjugate variables are related via derivatives, e.g. $n_i = -\frac{\partial \omega}{\partial \mu_i}$

There is a chemical potential associated with each conserved quantity (charge, baryon number, lepton number) and the individual chemical potentials are linear combinations of μ_q, μ_B, μ_l, e.g. $\mu_{proton} = \mu_B + \mu_q$.

At zero temperature the entropy vanishes.

The ideal gas EoS

- R. Boyle (1662) and E. Mariotte (1676) : at constant temperature and keeping the amount of substance constant $pV = const$
- J. Charles (1787) and J. L. Gay-Lussac (1802) : at constant pressure and keeping the amount of substance constant $V/T = const$
- A. Avogadro (1811) : at constant temperature and pressure $V/n = const$
- Combination gives the ideal gas law,

$$pV = Nk_B T$$

Valid for a dilute gas at high temperature.

NS matter is a strongly interacting system at high density → ideal gas law not applicable to neutron star matter

J.L. Gay-Lussac [wikipedia]

R. Boyle [wikipedia]

A. Avogadro [wikipedia]

J. Charles [wikipedia]

Equation of state for an ideal Boltzmann gas

Hydrodynamics equations need EoS as closure relation, mostly in the form of pressure as function of density and temperature.

- individual number densities $n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_{\rm MB} = \frac{g_i}{2\pi^2} e^{\frac{\mu_i - m_i}{T}} \int p^2 e^{-\frac{p^2}{2m_i}} dp$

$$n_i = g_i \frac{(m_i T)^{3/2}}{(2\pi)^{3/2}} \exp(\frac{\mu_i - m_i}{T})$$

- energy density and pressure $\varepsilon = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} E_i\, f_{\rm MB}$

$$\varepsilon = \sum_i m_i n_i + \frac{3}{2}\frac{T}{(2\pi)^{3/2}} \sum_i g_i (m_i T)^{3/2} e^{\frac{\mu_i - m_i}{T}} = \sum_i m_i n_i + \frac{3}{2} T \sum_i n_i$$

$$p = \frac{T}{(2\pi)^{3/2}} \sum_i g_i (m_i T)^{3/2} e^{\frac{\mu_i - m_i}{T}} = T \sum_i n_i$$

This corresponds to the ideal gas law $P = \frac{\rho}{\mu} T$ used in stellar evolution calculations($\mu^{-1} = \sum Y_i$, with Y_i particle fractions)

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An equation of state for neutron stars

Start with homogeneous neutron star matter :

- $f(T, n_B, n_q, n_l)$ would be a pertinent (and convenient) equation of state

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An equation of state for neutron stars

Start with homogeneous neutron star matter :

- $f(T, n_B, n_q, n_l)$ would be a pertinent (and convenient) equation of state
- soon after their birth in supernovae ($\sim$ minutes), neutron stars are sufficiently cool for temperature effects on the EoS to be neglected in general
- strong, electromagnetic, and weak reactions are at equilibrium (this includes in particular β-equilibrium)
- global charge neutrality should be fulfilled ($n_q = 0$)

$\Rightarrow$all state variables are functions of only one parameter, chosen to be *e.g.* the baryon number density n_B.

More precisely ...

Homogeneous (bulk) matter in the core of neutron stars

- should be charge neutral : $\sum_i q_i n_i = 0$. For matter composed of neutrons, protons, and electrons this means just $n_p = n_e$.
- should be in β-equilibrium. This means the reactions

  $$p + e^- \to n + \nu_e \quad \text{and} \quad n \to p + e^- + \bar{\nu}_e$$

  should be in equilibrium. In bulk matter this can be achieved by the following condition on the chemical potentials :

  $$\mu_p + \mu_e = \mu_n + \mu_{\nu_e} \ .$$

  Remind that the chemical potential corresponds to the energy needed to add one particle to the Fermi sea.
- Since neutrinos can freely leave the (cold) neutron star, the β-equilibrium condition reduces to $\mu_p + \mu_e = \mu_n$.

Attention : in the literature, the chemical potentials of the nucleons are sometimes defined without the particle mass. If this is the case, the masses should be added to the above relation explicitly.

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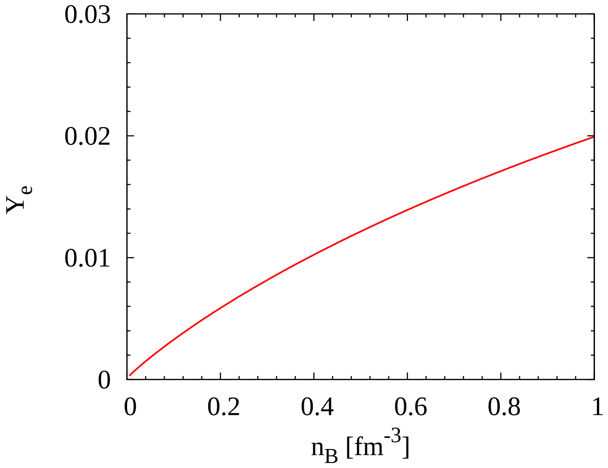
The free Fermi gas

The most simple equation of state would be a free Fermi gas of n, p, e.

- $n_i = \frac{p_F^3}{3\pi^2}$ with $p_F^i = \sqrt{\mu_i^2 - m_i^2}$ giving $\mu_i = \sqrt{(n_i\, 3\pi^2)^{2/3} + m_i^2}$
- charge neutrality ($n_p = n_e$) and the β-equilibrium condition then allow to determine n_e as a function of $n_B = n_n + n_p \Rightarrow n_i = n_i(n_B)$

The free Fermi gas



$$Y_e = n_e/n_B$$

→ charge neutral matter in β-equilibrium becomes neutron rich

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The free Fermi gas

The most simple equation of state would be a free Fermi gas of n, p, e.

- $n_i = \frac{p_F^3}{3\pi^2}$ with $p_F^i = \sqrt{\mu_i^2 - m_i^2}$ giving $\mu_i = \sqrt{(n_i\, 3\pi^2)^{2/3} + m_i^2}$
- charge neutrality ($n_p = n_e$) and the β-equilibrium condition then allow to determine n_e as a function of $n_B = n_n + n_p \Rightarrow n_i = n_i(n_B)$
- energy density

$$\varepsilon = \sum_{i=n,p,e} 2 \int_0^{P_F^i} \frac{d^3p}{(2\pi)^3} \sqrt{p^2 + m_i^2}$$

$$= \sum_{i=n,p} (m_i n_i + \frac{3}{5} \frac{(3\pi^2)^{2/3}}{2m_i} n_i^{5/3}) + \frac{3}{4} (3\pi^2)^{1/3} n_e^{4/3}$$

(Here I have used the non-rel. approximation for nucleons and neglected m_e)

- pressure $p = -\varepsilon + \sum_{i=n,p,e} \mu_i n_i$ (note that $\sum_{i=n,p,e} \mu_i n_i = \mu_B n_B$)

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c) Calculer l'énergie du gaz de positrons et celle du gaz d'électrons en équilibre dans l'enceinte, sachant que

$$\int_0^\infty \frac{x^3\,dx}{e^x+1} = \frac{7\pi^4}{120}.$$

Comparer avec l'énergie du gaz de photons dans l'enceinte.

VI.26. Ordres de grandeur concernant les étoiles à neutrons. Lorsqu'une étoile a épuisé son combustible nucléaire, elle s'effondre sous l'effet des forces gravitationnelles. Si sa masse est supérieure à la limite de Chandrasekhar (complément VI.B, § III.4), la pression du gaz dégénéré d'électrons qu'elle contient n'est pas suffisante pour arrêter le processus. L'effondrement se poursuit alors jusqu'à ce que l'étoile devienne un trou noir. Mais il peut arriver que l'échauffement qui accompagne l'effondrement conduise à une explosion, donnant lieu à une supernova; l'étoile éjecte alors de la matière, et sa masse résiduelle peut descendre au-dessous de la limite de Chandrasekhar. Dans ce cas l'astre, trop comprimé pour pouvoir se stabiliser en une naine blanche, se transforme en une «étoile à neutrons».

1) La différence de masse $m_n - m_p$ entre le neutron et le proton est telle que $(m_n - m_p)c^2 \simeq 1{,}3$ MeV.

a) Calculer (cf. § III.2 du complément VI.B) la masse volumique ρ qu'il faut atteindre pour que l'énergie de Fermi des électrons dépasse cette valeur.

b) On admet que les protons et les neutrons de l'astre sont pratiquement au repos (comment pourrait-on s'assurer que cette hypothèse est vérifiée?). Lorsque la masse volumique atteint l'ordre de grandeur calculé en *a*, la réaction

$$e^- + p \longrightarrow n + \nu$$

devient possible, les neutrinos s'échappant de l'étoile. Montrer qu'en outre la désintégration du neutron

$$n \longrightarrow p + e^- + \bar{\nu}$$

est bloquée par le principe de Pauli (appliqué aux électrons).

2) Le processus qui vient d'être esquissé aboutit à la formation d'un gaz de neutrons dégénéré; c'est essentiellement la pression quantique de ce gaz qui stabilise l'astre.

a) Montrer que les ordres de grandeur correspondants s'obtiennent à partir des formules du complément VI.B (§ III.4) en remplaçant la masse m de l'électron par celle du neutron m_n.

b) En déduire que la masse limite M_0 reste pratiquement la même, mais que le rayon d'une étoile à neutrons est environ 1 000 fois plus petit que celui d'une naine blanche de même masse. Quel est l'ordre de grandeur de la masse volumique d'une étoile à neutrons?

VI.27. Ordres de grandeur concernant le rayonnement fossile (complément VI.G).

1) Montrer que le nombre total de photons d'un corps noir est, comme l'entropie, proportionnel à T^3. Évaluer le nombre de photons par unité de volume dans le rayonnement cosmique à 3 K (avec les notations de la note 36 du chapitre VI, on donne $\Gamma(3)\zeta(3) = 2 \times 1{,}202\ldots$).

2) Pour $T = 3$ K, quelle est la longueur d'onde correspondant au maximum de la distribution de Planck?

From a book of statistical mechanics : *The process which has been discussed leads to the formation of a degenerate gas of neutrons. It is mainly the quantum pressure of this gas which is stabilising the star.*

Free gas a good EoS ?

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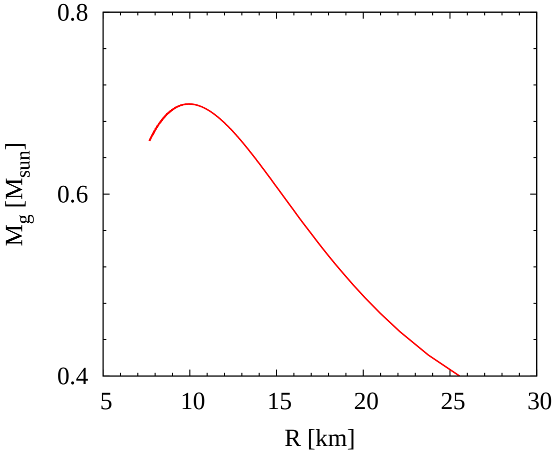
The free Fermi gas

But life is not as simple !



Historical note : A neutron Fermi gas EoS led Oppenheimer and Volkov (Oppenheimer& Volkov, Phys. Rev. 55 (1939) 374) to predict maximum NS masses of $\sim 0.7 M_\odot$

But : Maximum neutron star mass $< 1 M_\odot$ in contradiction with observations !

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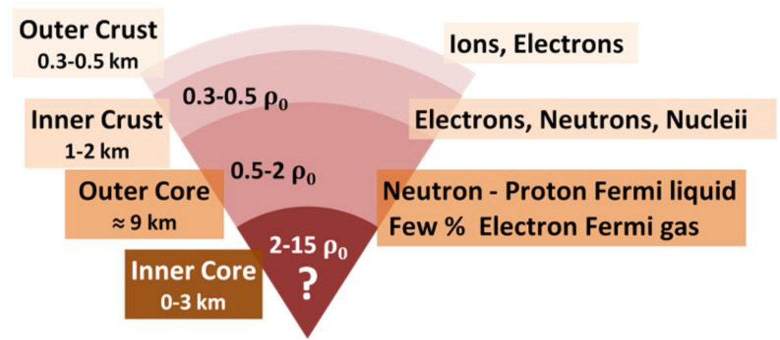
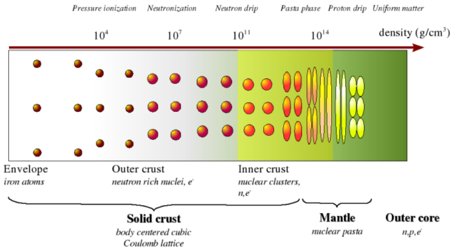
Standard picture of the inner structure



- crust formed of nuclei, neutron gas in inner crust
- transition to the core characterised by transition to homogeneous matter
- composition close to the center almost unknown (hyperons, kaon/pion condensate, quark matter . . . ?)

Neutron star matter not accessible in terrestrial laboratories (density, asymmetry) nor to ab-inito calculations

The neutron star crust



- The outer crust is the part of the crust below neutron drip density, composed of a lattice of nuclei and an electron gas. It extends up to densities of roughly $10^{11}\mathrm{g/cm}^{-3}$.
- The inner crust is the part above neutron drip, extending up to $\sim \rho_0/3 \sim 10^{14}\mathrm{g/cm}^{-3}$. It is characterised by the presence of neutrons outside the nuclei.
- Some models predict so-called "pasta" phases in which the nuclei become strongly deformed at the transition from the inner crust to homegeneous core matter.

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The outer crust

The main ideas of the description of the outer crust are given by Baym, Pethick, Sutherland (1971) :

- The total energy density is given by
  $$\varepsilon_{tot} = n_N E(A,Z) + \varepsilon_e + \varepsilon_L$$
- Electrons can be treated as ideal Fermi gas
- Lattice energy can be estimated in Wigner-Seitz approximation (body centered cubic structure seems favored)
- $E(A,Z)$ not experimentally known for the neutron rich nuclei eventually appearing the denser layers.

$\rho_{\max}$ [g/cm^3]	Element	Z	N
8.02×10^6	^{56}Fe	26	30
2.71×10^8	^{62}Ni	28	34
1.33×10^9	^{64}Ni	28	36
1.50×10^9	^{66}Ni	28	38
3.09×10^9	^{86}Kr	36	50
1.06×10^{10}	^{84}Se	34	50
2.79×10^{10}	^{82}Ge	32	50
6.07×10^{10}	^{80}Zn	30	50
8.46×10^{10}	^{82}Zn	30	52
9.67×10^{10}	^{128}Pd	46	82
1.47×10^{11}	^{126}Ru	44	82
2.11×10^{11}	^{124}Mo	42	82
2.89×10^{11}	^{122}Zr	40	82
3.97×10^{11}	^{120}Sr	38	82
4.27×10^{11}	^{118}Kr	36	82

[Rüster, Hempel, Schaffner-Bielich 2005]

The inner crust

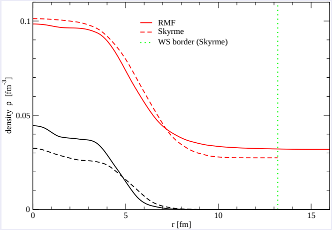
With increasing density, the value of Z/A decreases and neutrons become less and less bound. At some point the "neutron drip line" is reached, i.e. neutrons drip out of nuclei. This point is reached at a density of roughly $\rho_{ND} \sim 4 \times 10^{11} \mathrm{g/cm}^{-3}$.

- Within the liquid drop approach, for the total energy density that of the "neutron gas" has to be included

$$\varepsilon_{tot} = n_N E(A,Z) + \varepsilon_e + \varepsilon_L + \varepsilon_n$$

- Qualitatively, the results are almost the same in more sophisticated Thomas-Fermi and quantum calculations employing in general the Wigner-Seitz approximation
- "Pasta" phases with very deformed nuclei in form of rods, slabs, ...are not found with all nuclear interaction models (depends on the ratio of surface and bulk energy)
- The neutrons in the inner crust are probably in a superfluid state

Density distribution within a WS cell



[Gögelein & Müther 2007]

Models for bulk nuclear matter

- Ab-initio calculations up to $A \approx 12$ (not adequate for the description of nuclear matter !)
- for nuclear matter there are two types of models
  1. phenomenological models with effective interactions
     - ⋆ liquid drop
     - ⋆ mean field
  2. "ab initio" microscopic calculations starting from the basic two-body interaction
     - ⋆ Brueckner-Hartree-Fock (BHF)
     - ⋆ Self-consistent Green's function
     - ⋆ Variational techniques
     - ⋆ Many-body perturbation therory with RG evolved forces
     - ⋆ ...

Mean field models for bulk matter

Within mean field models, the starting point is a phenomenological "effective" interaction (here in a non-relativistic form) :

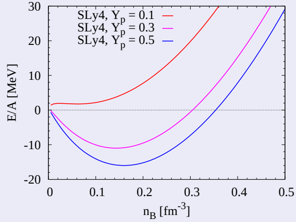
$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j=1}^{A} V_{ij}^{eff}$$

- Take "effective" interaction, not the NN-interaction, in order to take correlations into account
- Modern interpretation in terms of Kohn-Sham energy density functional theory
- Parameter of the interaction fitted to nuclear data $\longrightarrow$ good description of the nuclear chart up to very heavy nuclei
- In practice calculations reduce to free gas equations with effective masses and effective chemical potentials

The non-relativistic Skyrme mean field model

- Starting point is a functional for the energy density $\varepsilon(n_n, n_p)$
- Zero range interaction
- Nucleons treated with non-relativistic kinematics

- Equation of state can be written in the form of a free Fermi gas with effective masses and chemical potentials + an interaction potential dependent on the densities
- Many versions exist with interaction parameters adjusted to nuclear data



- How to proceed to obtain the pressure ?
- How to proceed to get the EoS for neutron star matter ?

A Skyrme EoS

1. Pressure

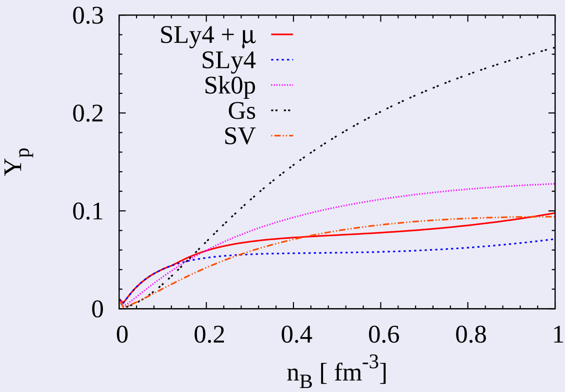
- Thermodynamic relations
  - $\frac{\partial \varepsilon}{\partial n_i} = \mu_i$ and
  - $P = -\varepsilon + \sum_i \mu_i n_i$

2. Neutron star EoS

- Add leptons, i.e. electrons and possibly muons (as free Fermi gas)
- Charge neutrality $n_q = 0$
- β-equilibrium $\mu_n = \mu_p + \mu_e$ (and muons ?)

A Skyrme EoS

The proton fraction in neutron star matter for different parametrisations



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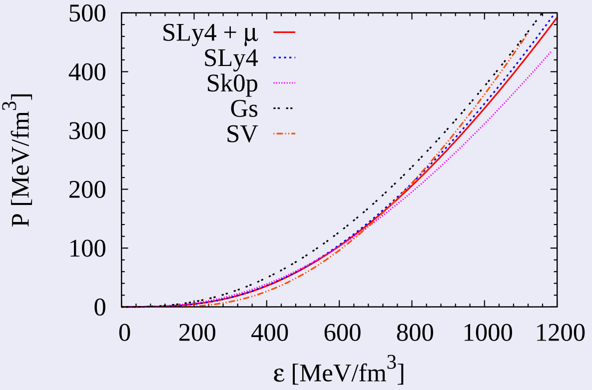
A Skyrme EoS



SLy4 + μ
SLy4
Sk0p
Gs
SV
P [MeV/fm3]
ε [MeV/fm3]
0
100
200
300
400
500
0
200
400
600
800
1000
1200

Covariant density functionals

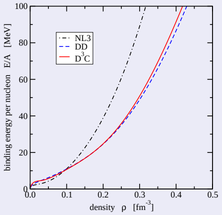
- Relativistic (special relativity) treatment of the nucleons
- Interaction described via effective (!) meson exchange :

$$\mathcal{L}_{RMF} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma + \ldots\right)\psi$$

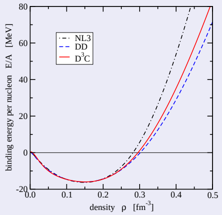
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 + \ldots$$

- In the original version (Walecka model) only σ, ω
- In order to reproduce data, many refinements : isovector channel (ρ, δ), nonlinear meson-interactions, density dependent couplings, . . .

Neutron matter



symmetric nuclear matter



[Typel 2005]

(Dirac)-Brueckner-Hartree-Fock calculations

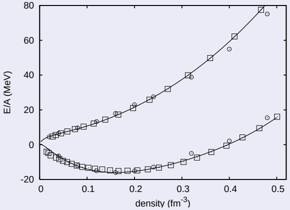
- Starting point is the bare NN-interaction V
- Construction of the Brueckner G-matrix :

$$G(\omega)_{ij,kl} = V_{ij,kl} + \sum_{a,b} V_{ij,ab} \frac{Q_{ab}}{\omega - e_a - e_b} G(\omega)_{ab,kl}$$

- The Pauli operator Q prevents the baryons (they are fermions !) to be scattered to states below their respective Fermi momenta
- The single-particle energies are determined self-consistently

$$e_i(\vec{k}) = M_i + \frac{\vec{k}^2}{2M_i} + \sum_j \langle ij|G(\omega)_{ij,ij}|ij\rangle$$

- Three-body forces necessary to reproduce empirical behaviour of $E/A(n_B)$.



[Baldo & Burgio 2011]

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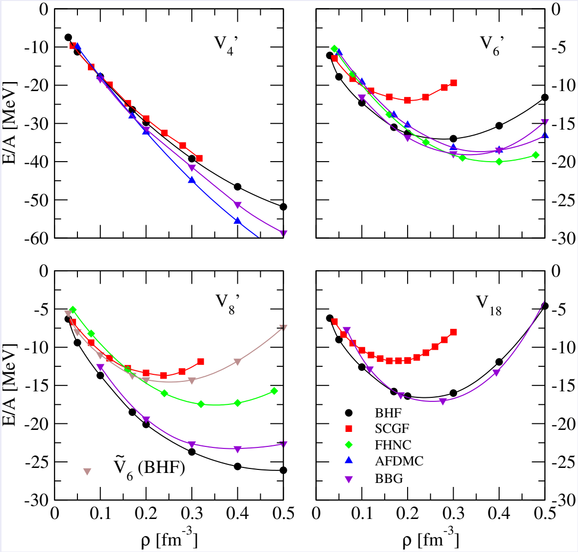
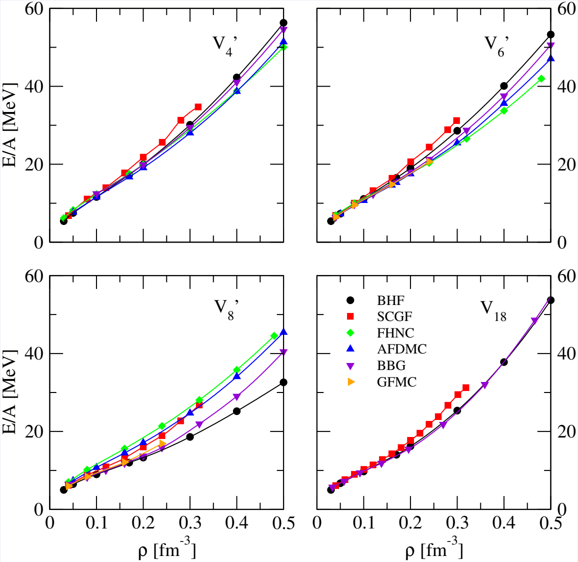
Energy per baryon for symmetric matter, different approaches



[Baldo et al. 2012]

Energy per baryon for neutron matter, different approaches



[Baldo et al. 2012]

Conditions in CCSN and BNS mergers

The equation of state (EoS) thermodynamically relates different quantities to close the system of hydrodynamic equations.
The number of parameters depends on equilibrium conditions :

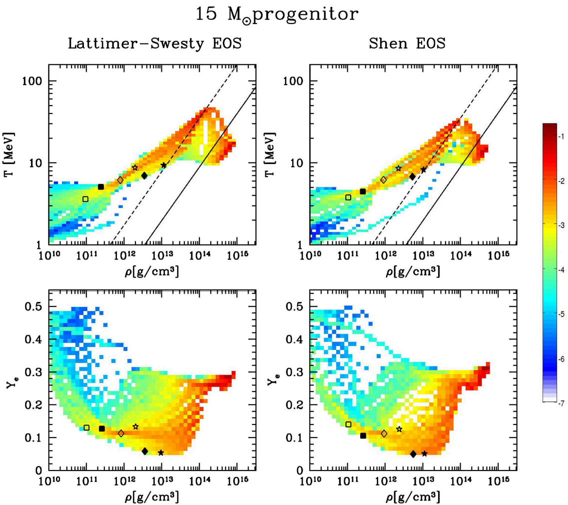
- For a cold and charge-neutral neutron star in β-equilibrium :
  EoS is $P(n_B)$ (or equivalent)

- For core collapse and neutron star mergers :
  - charge neutrality always fulfilled, i.e.

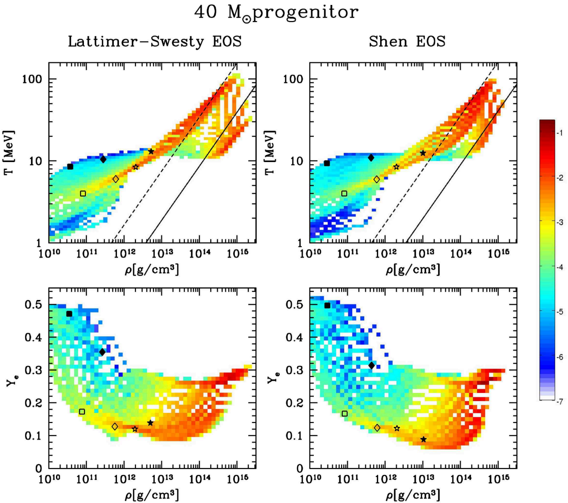
$$Y_e = \sum_{hadrons} n_{q,h}/n_B \equiv Y_q$$

  - hydrodynamical timescale $\sim 10^{-6}$ s $\longrightarrow$ β-equilibrium not always achieved

What about the temperature ?


15 $M_\odot$progenitor
Lattimer−Swesty EOS
Shen EOS
T [MeV]
ρ[g/cm³]
Y_e
100
10
1
0.5
0.4
0.3
0.2
0.1
0
10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15}
-1
-2
-3
-4
-5
-6
-7

[A. Perego]



[A. Perego]

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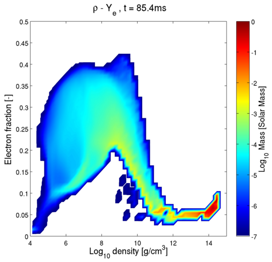

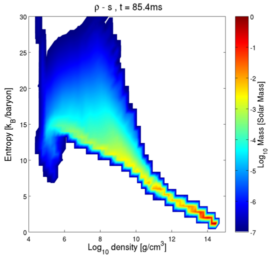

[A. Perego]

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Temperature effects in a Fermi gas

- Recall : Fermi-Dirac distribution function $f_{FD}(p) = \frac{1}{\exp((E(p)-\mu)/T)+1}$ becomes a step function in the degenerate limit, $T \ll \mu$

Temperature corrections :

- In the non-relativistic case (nucleons) :

$$\varepsilon = mn + a_1 \frac{n^{5/3}}{m} + a_2 T n + a_3 T^2 m n^{1/3} + \cdots$$

Numerical estimate for $m = 1$ GeV, $n = 0.1\ \mathrm{fm}^{-3}$ (T in MeV) :

$\varepsilon[\mathrm{MeV/fm}^3] = 100 + a_1 0.86 + a_2 0.1T + a_3 T^2 0.011604 + \cdots$

- In the ultra-relativistic case (electrons) :

$$\varepsilon = a_1 n^{4/3} + a_2 n^{2/3} T^2 + a_3 T^4 + \cdots$$

Numerical estimate for $n = 0.1\ \mathrm{fm}^{-3}$ (T in MeV) : $\varepsilon[\mathrm{MeV/fm}^3] = a_1 9.3 + a_2\ T^2\ 0.001 + a_3\ T^4\ 8 \times 10^{-8} + \cdots$

$\rightarrow$ for core collapse and NS merger matter temperature effects not negligible

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Conditions in CCSN and BNS mergers

The equation of state (EoS) thermodynamically relates different quantities to close the system of hydrodynamic equations.
The number of parameters depends on equilibrium conditions :

- For a cold and charge-neutral neutron star in β-equilibrium :
  EoS is $P(n_B)$ (or equivalent)
- For core collapse and neutron star mergers :
  - charge neutrality always fulfilled
  - β-equilibrium not always achieved
  - temperature effects not negligible !

  $\rightarrow$EoS is $P(n_B, T, Y_e)$ (or equivalent)

Very large ranges to be covered :

$$n_B = 10^{-8}\mathrm{fm}^{-3} \cdots 1\mathrm{fm}^{-3}$$
$$T = 0.2\mathrm{MeV} \cdots 150 \text{ MeV}$$
$$Y_e = 0.05 \cdots 0.5$$

We need an EoS for hot dense asymmetric matter...

- Three parts of the EoS :
  - Hadrons
  - Charged leptons, free Fermi gas coupled to hadrons only via charge neutrality
    Neutrinos are not in thermal equilibrium, can thus not be treated via EoS
  - Photons, free (massless) Bose gas with

$$p = \frac{\pi^2}{15}\frac{T^4}{3} \qquad \varepsilon = \frac{\pi^2}{15}T^4$$

In the following we will concentrate on the hadronic part.

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The hadronic EoS

Composition of hadronic matter changes dramatically depending on baryon number density, charge fraction (asymmetry), and temperature.

Different regimes :

- Very low densities and temperatures :
  - dilute gas of non-interacting nuclei
    $\rightarrow$ nuclear statistical equilibrium (NSE)
- Intermediate densities and low temperatures :
  - gas of interacting nuclei surrounded by free nucleons
    $\rightarrow$ approaches beyond NSE
- High densities and temperatures :
  - nuclei dissolve
    $\rightarrow$ strongly interacting (homogeneous) hadronic matter
  - potentially transition to the quark gluon plasma

Plan

Nuclear statistical equilibrium (NSE)

Basic assumption : mixture of nucleons (n, p) and nuclei (X) in chemical equilibrium

- chemical equilibrium expressed via equality of chemical potentials for a nucleus with Z protons and N neutrons :

$$ {}^A_Z X_N : Zp + (A-Z)n \qquad \mu_X = Z\mu_p + (A-Z)\mu_n $$

- simplest model (called NSE in the literature) assumes in addition non-interacting independent particles
  - partition function factorises : $Z = \Pi_i Z_i$
  - particles form an ideal gas (Maxwell-Boltzmann or Fermi/Bose)

Attention : Approximation not really valid below $T \sim 0.5$ MeV and low densities : nuclear reaction network necessary, determining abundances from individual reaction rates (mainly for stellar evolution, not here)

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Simplest model

- Maxwell-Boltzmann statistics with $f_{\rm MB} = e^{-E_i/T}\ e^{\mu_i/T}$
- non-relativistic kinematics : $E_i = m_i + \frac{p_i^2}{2m_i}$
- energy density and pressure $\varepsilon = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} E_i\ f_{\rm MB}$

$$\varepsilon = \sum_i m_i n_i + \frac{3}{2}\frac{T}{(2\pi)^{3/2}}\sum_i g_i (m_i T)^{3/2} = \sum_i m_i n_i + \frac{3}{2} T \sum_i n_i$$

$$p = \frac{T}{(2\pi)^{3/2}}\sum_i g_i (m_i T)^{3/2} \exp(\frac{\mu_i - m_i}{T}) = T \sum_i n_i$$

- $g_i = (2J_i + 1)$ are the degeneracy factors
- $\exp((\mu_i - m_i)/T)$ are often called fugacities
- consider only nuclear ground states
- masses and spins of individual nuclei from data tables (e.g. NuBase 2012 evaluation with 3350 nuclides) or mass formulae

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A SIMPLE EXAMPLE

- take a mixture of neutrons (n), protons (p) and deuterons ($d = ^2$ H)

$$n + p \Leftrightarrow d \quad \Rightarrow \quad \mu_p + \mu_n = \mu_d$$

- $m_d = m_n + m_p - B_d$, deuteron binding energy $B_d = 2.225$ MeV

- individual number densities

$$n_i = g_i \frac{(m_i T)^{3/2}}{(2\pi)^{3/2}} \exp(\frac{\mu_i - m_i}{T})$$

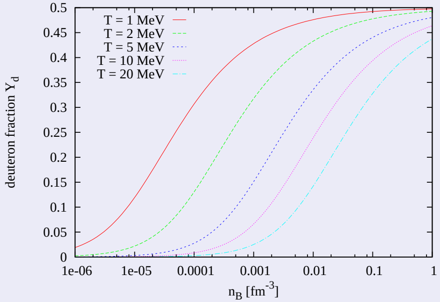
- deuteron fraction $Y_d = \frac{n_d}{n_B}$

- pressure and energy density

$$p = T \sum_{n,p,d} n_i$$

$$\varepsilon = \sum_{n,p,d} (m_i + \frac{3}{2} T n_i)$$

DEUTERON FRACTION FOR SYMMETRIC MATTER ($n_n = n_p$)



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Possible improvements ?

- At (very) low densities the dilute non-interacting ideal gas is a good approximation with some refinements :

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- At (very) low densities the dilute non-interacting ideal gas is a good approximation with some refinements :
  - at finite temperature excited states j of nuclei will be populated

    $$\rightarrow \quad g_i(T) = g_i(T=0) + \sum_j (2J_j + 1)\exp(-\frac{E_j}{T})$$

    in general semi-empirical formulae used
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    in general semi-empirical formulae used
  - Coulomb corrections
- At higher densities ($n_B \sim 10^{-4}\mathrm{fm}^{-3}$) medium effects become important, the (strong) interaction of clusters and with the surrounding nucleons cannot be neglected

  In particular : Pauli exclusion principle leads to the dissolution of clusters !

  $\rightarrow$ different approaches beyond NSE

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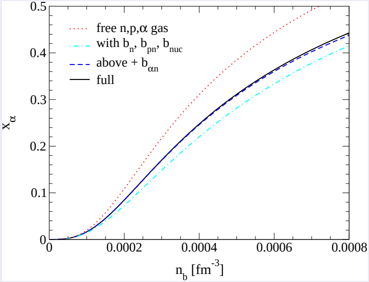
Different approaches beyond NSE

1. Virial expansion

- Idea : as far as fugacities $z_i = \exp((\mu_i - m_i)/T) \ll 1$, the (grand canonical) partition function can be expanded in terms of z_i
- bound states (clusters) and scattering states (phase shifts) can be included
- limited to $n_i \ll (m_i T/(2\pi))^{3/2}$
  ($n_i \ll 2 \times 10^{-4} T_{\mathrm{MeV}}^{3/2}$ for nucleons)

α mass fraction for symmetric matter ($n_n = n_p$)(Horowitz & Schwenk, NPA 2006)



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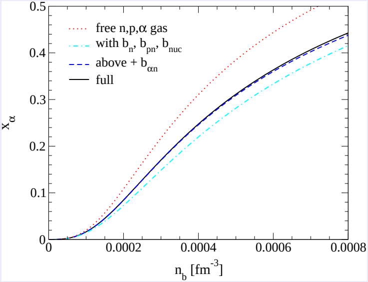

2. Quantum statistical

- Idea : solve self-consistently for in-medium propagators and T-matrix
- medium dependent shift of binding energies (Pauli principle) and phase shifts
- dissolution of clusters at high densities (Mott effect)

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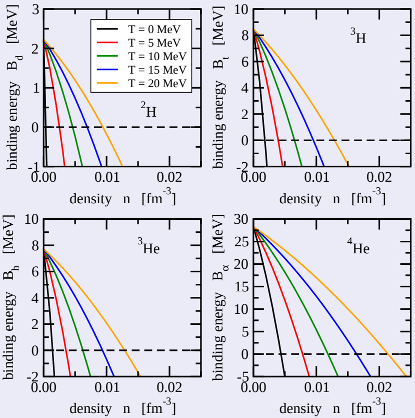
Different approaches beyond NSE

3. Generalised energy density functional approach

- Idea : include light clusters explicitely in the EDF with medium dependent binding energies
- heavy clusters ?

in-medium cluster binding energies(Typel et al. PRC 2010)



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Different approaches beyond NSE

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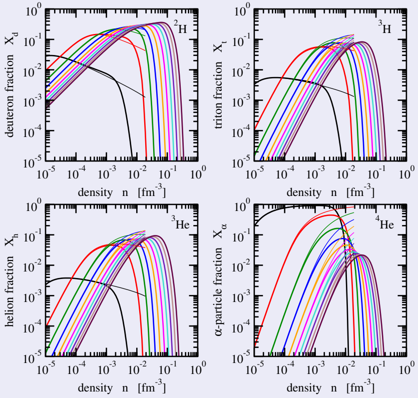
comparison with NSE (Typel et al. PRC 2010)



Different approaches beyond NSE

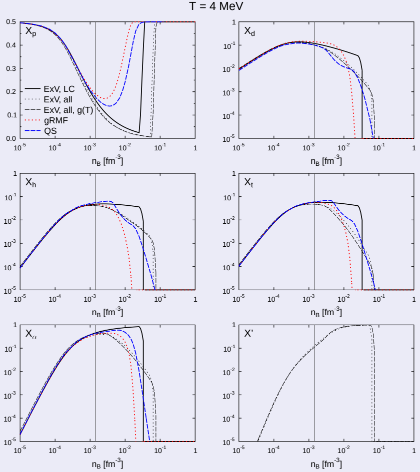
3. Generalised energy density functional approach

- Idea : include light clusters explicitely in the EDF with medium dependent binding energies
- heavy clusters ?

4. Phenomenolgical excluded volume

- Idea : mimic medium effects (Pauli principle) by excluding the volume occupied by a cluster for all other clusters
- cluster dissolution not well described since medium modifications of cluster properties not included

α comparison of light cluster abundances for symmetric matter (Hempel et al. PRC 2011)



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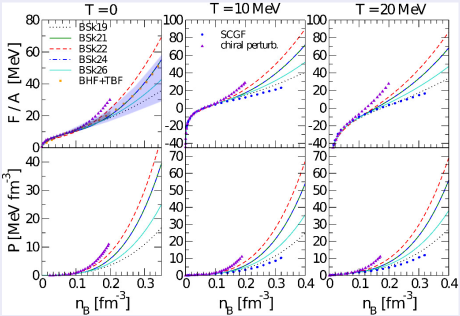
Choice of the interaction for bulk matter

Ab-initio calculations at finite temperature very demanding
→ only few results for restricted n_B, T, Y_q ranges
→ mainly phenomenological models used
Question : Can we use the same (phenomenological) effective interactions in the whole (T, n_B, Y_q) range ?

- asymmetry (Y_q dependence) via constraints on neutron and cold neutron star matter
- temperature effects on the interaction small, enter only via the kinetic energy terms

→ Basic assumption : the same (effective) interaction can be used throughout the entire EoS range

EoS of neutron matter in a Skyrme mean field compared with microscopic calculations (A. Fantina)



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Neutrino matter interactions

- Different types of interactions with matter (nucleons, nuclei and charged leptons, photons)
  - scattering (neutral current)
  - absorption/creation processes (charged current)
  - pair creation (neutral current)

Typical reactions

$$
\begin{aligned}
p + e^-(+N) &\leftrightarrow n + \nu_e(+N) \\
n + e^+(+N) &\leftrightarrow p + \bar{\nu}_e(+N) \\
(A, Z) + e^- &\leftrightarrow (A, Z-1) + \nu_e \\
N + N &\rightarrow \nu + \bar{\nu} + N + N \\
\nu + A &\rightarrow \nu + A \\
\nu + N &\rightarrow \nu + N
\end{aligned}
$$

- Will not treat them all here ...

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Thermally averaged reaction rates

Consider a reaction

$$1(p_1) + 2(p_2) \to 1'(p_1') + 2'(p_2')$$

with the transition rate $r(1 + 2 \to 1' + 2')$ (final spin-summed, initial spin averaged)

$$r dn(p_1) dn(p_2) [d^3 p_1'][d^3 p_2']$$

In a stellar plasma, particles have a thermal momentum distribution and rate has to be averaged :

$$\Pi_i \left(\int d^3 p_i \right) f_1(p_1) f_2(p_2) (1 \pm f_1'(p_1'))(1 \pm f_2'(p_2')) r(1 + 2 \to 1' + 2')$$

$\to$ Fermi's golden rule with thermodynamic distribution functions f_i

- $1 - f_i$: final state Pauli blocking for fermions
- $1 + f_i$: final state Bose enhancement for bosons

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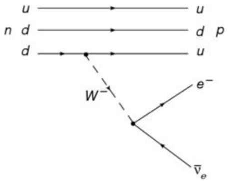
Neutrino-nucleon charged current reactions

Basic charged current weak interaction [Fermi 1934,...] :

$$G_F V_{ij} \bar{q}_i \gamma_\mu (1-\gamma_5) q_j \, \bar{\psi}_{l_1} \gamma^\mu (1-\gamma_5) \psi_{l_2}$$

Attention : interaction with quarks not nucleons !



- Governs the following reactions (not all of them are equally relevant)

Electron/positron capture

$$p + e^- \leftrightarrow n + \nu_e$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

Neutron/proton decay

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$
$$p \leftrightarrow n + e^+ + \nu_e$$

- Main problem : in medium nuclear response (matrix element + phase space)

General form (here : $p + e^- \rightarrow n + \nu_e$)

$$\frac{\partial f_\nu}{\partial t} \propto (1 - f_\nu) \int d^3 p_e \, f_e \, \int d^3 p_n d^3 p_p f_p (1 - f_n) |\mathcal{M}|^2$$

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Basic approximation

Elastic approximation [Bruenn 1985]

- Nuclear matrix element

$$\langle N|\bar{q}\gamma_\mu q|N\rangle \rightarrow g_V \bar{N}\gamma_\mu N \text{ (vector)}$$

and

$$\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle \rightarrow g_A \bar{N}\gamma_\mu\gamma_5 N \text{ (axial)}$$

- $g_V = 1$ and $g_A = 1.26$ from free neutron decay
- Neglect momentum transfer to the nucleons
- Non (special-)relativistic kinematics
- No nuclear interaction :
  $\longrightarrow$ free nucleon masses and single particle energies
  $\longrightarrow$ energy transferred to the nuclear medium is $E_e - E_\nu = m_n - m_p$

$\longrightarrow$ simple analytic expressions for opacities widely used in simulations

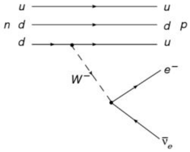
- We are concerned with a hot and dense asymetric ($n_n \neq n_p$) matter
  $\longrightarrow$ different corrections considered (but large uncertainties)

Charged current reactions on nuclei

Basic charged current weak interaction [Fermi 1934,...] :

$$G_F V_{ij} \bar{q}_i \gamma_\mu (1-\gamma_5) q_j \, \bar{\psi}_{l_1} \gamma^\mu (1-\gamma_5) \psi_{l_2}$$

Attention : interaction with quarks not nucleons !



- Most important are nuclear β decay and electron capture $(Z, A) \to (Z+1, A) + e^- + \bar{\nu}_e$, and $(Z, A) + e^- \to (Z-1, A) + \nu_e$
- Main problem :

Nuclear matrix elements for the transitions

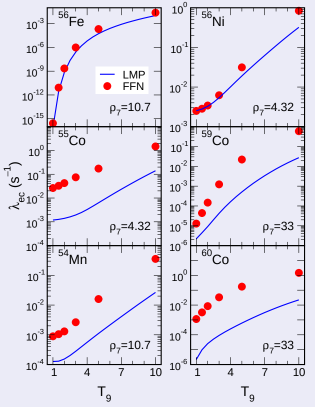
Fermi : $\langle f|\bar{u}\gamma_\mu d|i\rangle$ (vector) and Gamow-Teller : $\langle f|\bar{u}\gamma_\mu\gamma_5 d|i\rangle$ (axial)

- For core-collapse conditions : only allowed ($l = 0$) transitions important
- β-decay only relevant for $\rho \lesssim 10^{10}\mathrm{g/cm}^3$ (electron Pauli blocking)

Electron capture rates

- Continuous electron capture
  $\rightarrow$ reduction of Y_e and heavy (neutron rich) nuclei more abundant
- Fuller (1982) : GT transitions on nuclei are suppressed for $Z < 40$ and $N > 40$
  $\rightarrow$ electron capture on free protons dominant
- This is not true !
  Thermal excitations, mixing of states
- New (shell-model) rates considerably different

  [Langanke & Martínez-Pinedo 2000, Juodagalvis et al. 2009]
- Calculations using different approaches for nuclear interaction available [Paar et al. 2009]



[Langanke & Martínez-Pinedo 2000]

- Same principle for scattering reactions : main difficulty hadronic in-medium matrix element and phase space

Some useful software

- If you want to know more and test for example rotating neutron stars with your favorite EoS, there are two publicly available codes :
  - The RNS code written by Nick Stergioulas.

    See https://www.gravity.phys.uwm.edu/rns/
  - The LORENE library developped at Meudon mainly by E. Gourgoulhon, P. Grandclément, J.-A. Marck, J. Novak. K. Taniguchi.

    See https://www.lorene.obspm.fr
- ROXAS, a tool based on LORENE to describe evolution (oscillations) of isolated NS, https://zenodo.org/records/14849547
- https://www.Stellarcollapse.org is a website aimed at providing resources supporting research in stellar collapse, core-collapse supernovae, neutron stars, and gamma-ray bursts
- https://einsteintoolkit.org/ is a platform for different open source codes for relativistic astrophysics
- Tables of realistic EoS for neutron stars and core collapse are available on different web sites, e.g. CompOSE, https://compose.obspm.fr

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