1. **Neutron stars and the EOS of a free Fermi gas of neutrons**

— Calculate the pressure p and the energy density ε of a free Fermi gas of neutrons as a function of the baryon number density n_B.

— Solve the TOV system

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2 \varepsilon \\ \frac{d\Phi}{dr} &= \left(1 - \frac{2Gm}{rc^2}\right)^{-1} \left(\frac{Gm}{r^2} + 4\pi G \frac{p}{c^2} r\right) \\ \frac{dp}{dr} &= -\left(\varepsilon + \frac{p}{c^2}\right) \frac{d\Phi}{dr}\end{aligned}$$

with this equation of state and give the gravitational mass as a function of the radius. Does (as often stated) the degeneracy pressure of the neutrons give the counterpart to gravitation ? What else could play a role ?

— Option : introduce charge neutrality and neutrinoless β-equilibrium to repeat the calculation with a free gas of n, p, e. Does it help ?

— Option : Solve the newtonian limit of the TOV system

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2 \rho_B \\ \frac{dp}{dr} &= -\rho_B G \frac{m}{r^2}\end{aligned}$$

and give again the mass as a function of radius. ρ_B is here a mass density, you can take it as n_B multiplied by a constant "baryon mass" (e.g. the neutron mass). Remarks ?

— Option : if you are interested, you can download a nuclear EoS from the Compose data base, e.g. `https://compose.obspm.fr/eos/134`, and solve again the TOV system. You only need to download the `eos.thermo` and the `eos.nb` file. The latter contains the grid points in baryon number density n_B in fm^{-3} and the former in column 2 the index for the grid point in n_B and in column 4 the pressure divided by n_B. The energy density ε is contained in column 10 in the form $\varepsilon/(m_n n_b) - 1$ (dimensionless). Or you can use the analytic fits from `http://www.ioffe.ru/astro/NSG/BSk/index.html` and solve again the TOV system to obtain mass and radius of a spherical star.

Some of the nuclear EoS tables have been fitted to piecewise polytropes : instead of a table with a large number of points, we use a parametrized analytical expression of a polytrope with parameters κ and Γ, using for the pressure

$$P = \kappa_i \rho^{\Gamma_i} \ . \tag{1}$$

Since only one polytrope cannot accuretly reproduce the tabulated EoS of an elaborate nuclear model, we divide the table in 7 parts, each of those are associated to a set of parameters κ and Γ. We present in Table the parameters of each polytrope associated to the Relativistic Mean field nucleonic EoS DD2.

As the fit of the table is done for the mass density ρ, all parameters aforementionned are defined for the fit of $\rho(P)$ in cgs units (ρ in g/cm^3 and P in dyne/cm^2) ; note that $\rho = n_b * m_b$ with m_b the baryonic mass in appropriate units. As the energy density is needed to solve the TOV equations, it can be calculated from the first law of thermodynamics applied to polytropes, which renders the following expression :

$$\epsilon(\rho) = (1 + a_i)\rho + \frac{\kappa_i}{\Gamma_i - 1}\rho^{\Gamma_i}, \tag{2}$$

Solve again the TOV equations for DD2, try to vary the central pressure and find what is the maximum mass (in solar mass units) that this EoS can reach and its associated radius (in km). Conclude whether or not this EoS is acceptable with regards to mass measurement.
You might also use the Compose EoS directly within the `Lorene` software, `https://lorene.obspm.fr` to obtain rotating neutron star models.

Polytrope	1	2	3	4	5	6	7
Γ	1.6369	1.3114	0.6260	1.2833	2.3253	3.4041	2.6026
$log(\kappa)$	12.4878	14.7438	22.5525	14.4024	9.34901×10^{-2}	-15.4188	-3.49896502
$log(\rho_t)$		6.9309	11.3929	12.3993	13.7322	14.3792	14.8719
a	0.	-0.00014575	0.01872659	0.00541335	0.01962209	0.03210719	-0.08520517