Fragmentation dynamics and Compressibility of the EOS

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November 2025, FAZIA days

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6. Fragmentation dynamic
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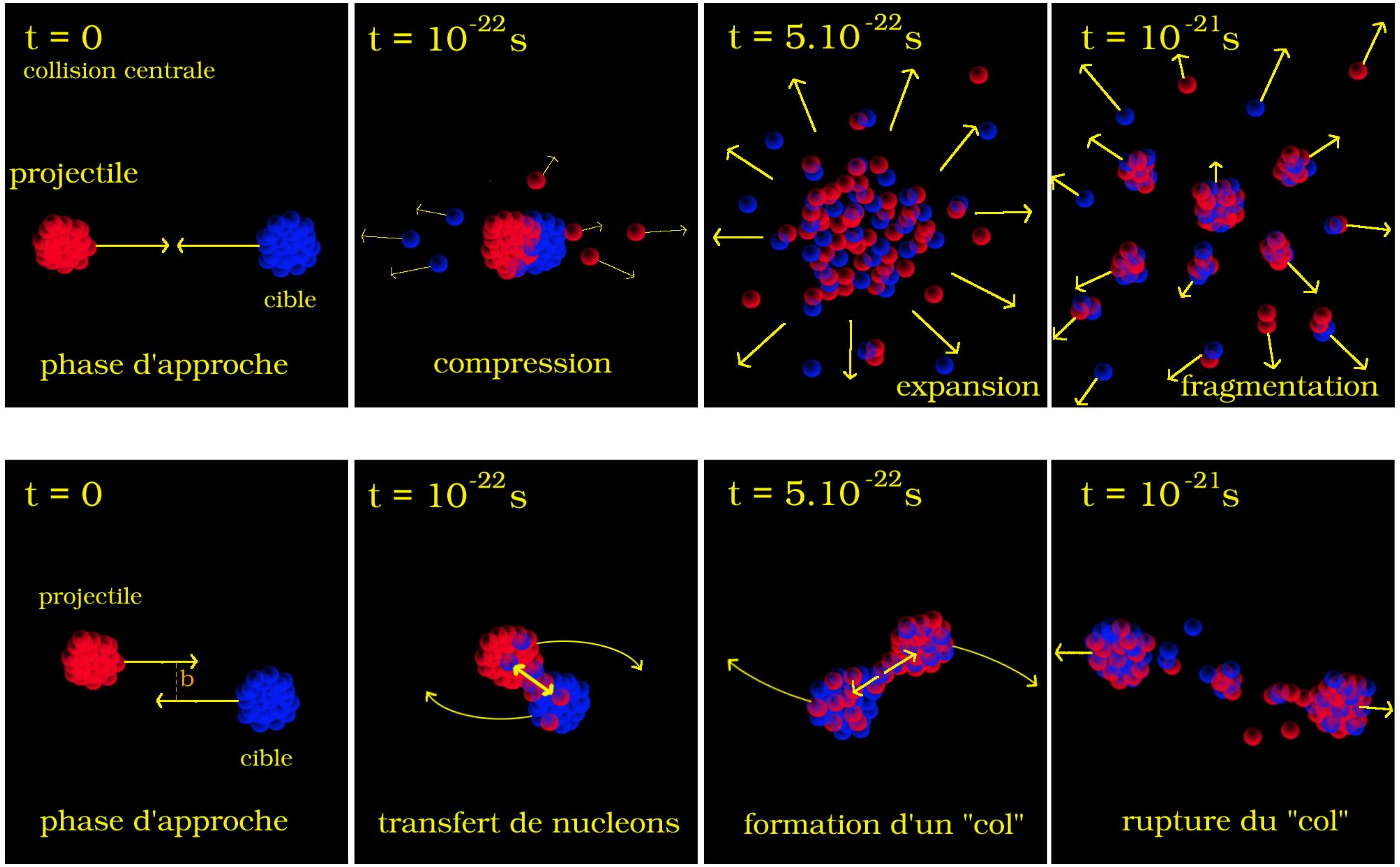
1. Introduction



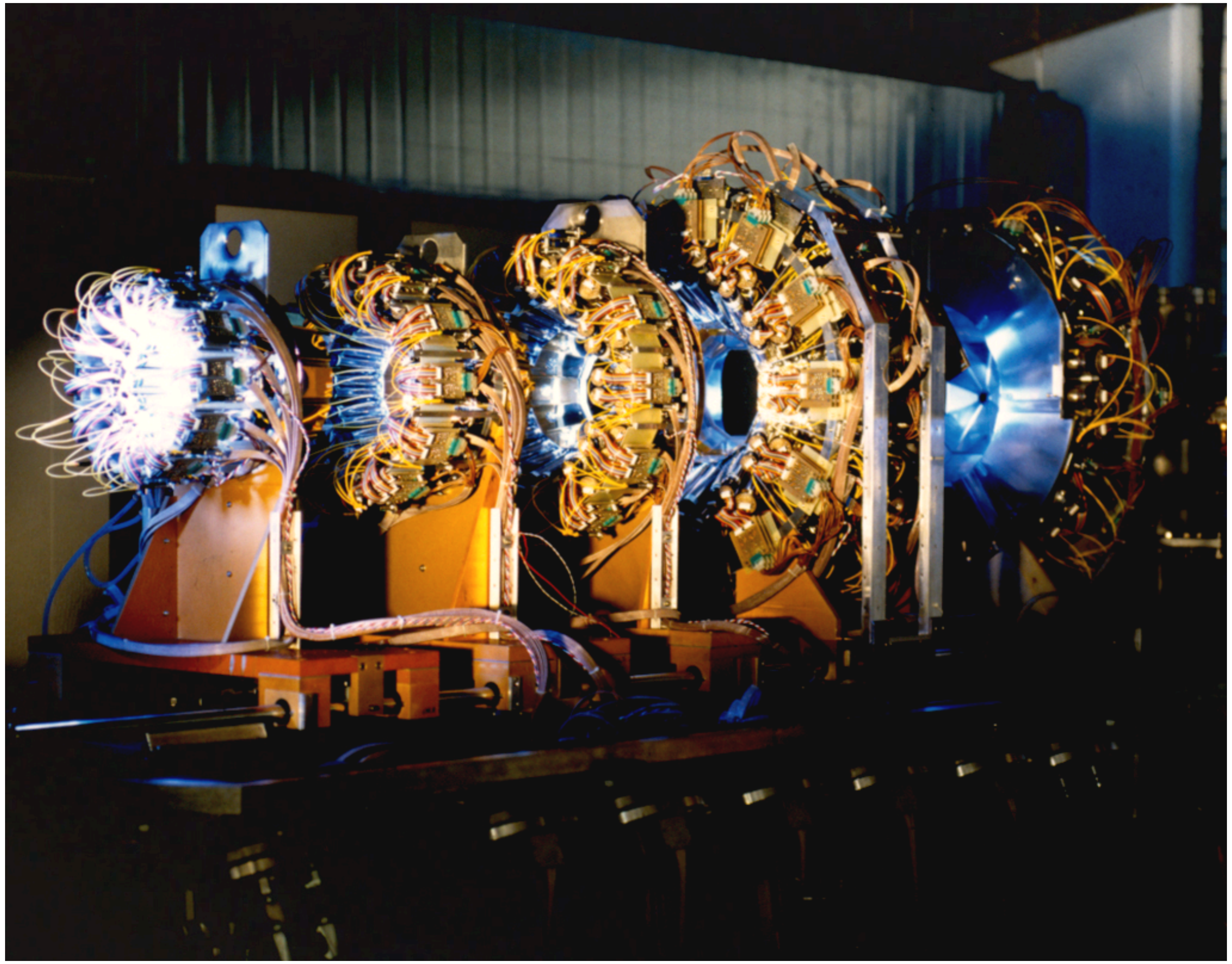
- Heavy ion collisions (HIC) around Fermi energy offer the possibility of compressing nuclear matter in order to explore densities far from saturation density ($\rho \sim 0.1 - 2\rho_0$).

- We can also heat the nuclear matter ($T \sim 1 - 10$ MeV) and perform nucleon exchange to measure isospin transfer ($N/Z \sim 1 - 1.5$).

- How can we do it? We use the INDRA device...

Experimental Setup

INDRA Detector



- INDRA is a light charged particle detector covering a wide solid angle of detection.
- We can collect the charge (Z), the energy (E), the mass (A for $Z < 4$) and the angles (ϕ, θ) to rebuild the linear momentum $(\vec{p})$, the multiplicity (M), …
- We can then proceed to construct global variables to characterize the equation of state (EOS)…

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Equation of State

$$E(\delta, \rho) = E_{iso}(\rho) + E_{vec}(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

With:

$$E_{iso}(\rho) = E_{sat} + \frac{K_{sat}}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3)$$

$$E_{vec}(\rho) = E_{sym} + L_{sym}\left(\frac{\rho - \rho_0}{3\rho_0}\right) + \cdots$$

$$\cdots + \frac{K_{sym}}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3)$$

- The equation of state (EOS) relates pressure, energy density and other thermodynamic properties of nuclear matter. The equation is Taylor expanded around ρ_0 and made of two parts: isoscalar and isovector.
- The first term (isoscalar) depends on density ρ. The second term (isovector) has isospin dependencies δ (isospin asymetry) and in density ρ.
- E_{sat} the saturation energy and K_{sat} is the curvature of the isoscalar energy component. E_{sym} is the symmetry energy, L_{sym} the slope, K_{sym} the curvature of the isovector part.
- Constraining these parameters is the objective of my thesis. This can be achieved using a Machine Learning approach...

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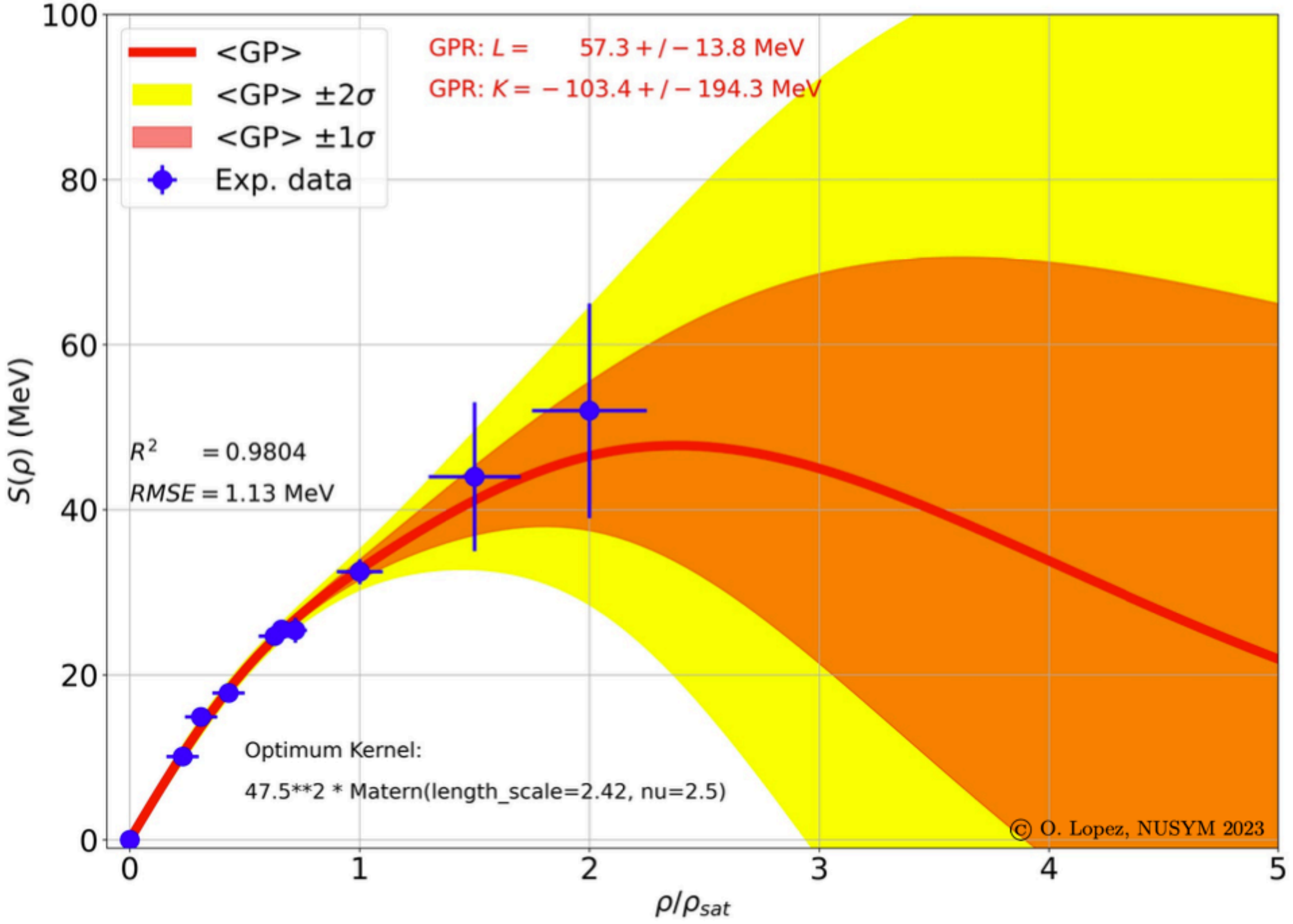
2. Motivations



- For example, we can cite the work of O. Lopez. This is a gaussian emulation symmetry energy from the experimental data in the literature. (O. Lopez, NUSYM 2023)

- Symmetry energy $S(\rho)$, which is the energy cost per nucleon to go from symmetric matter (N=Z) of a nuclear system to a given baryonic density ρ to a system consisting purely of neutrons (N=A).

- The more the density range is explored, the better the constraint on the parameters L_{sym} and K_{sym}.

- The aim of my thesis is to use machine learning and Bayesian inference techniques on INDRA datasets in order to improve and constrain equation of state (EOS) parameters.

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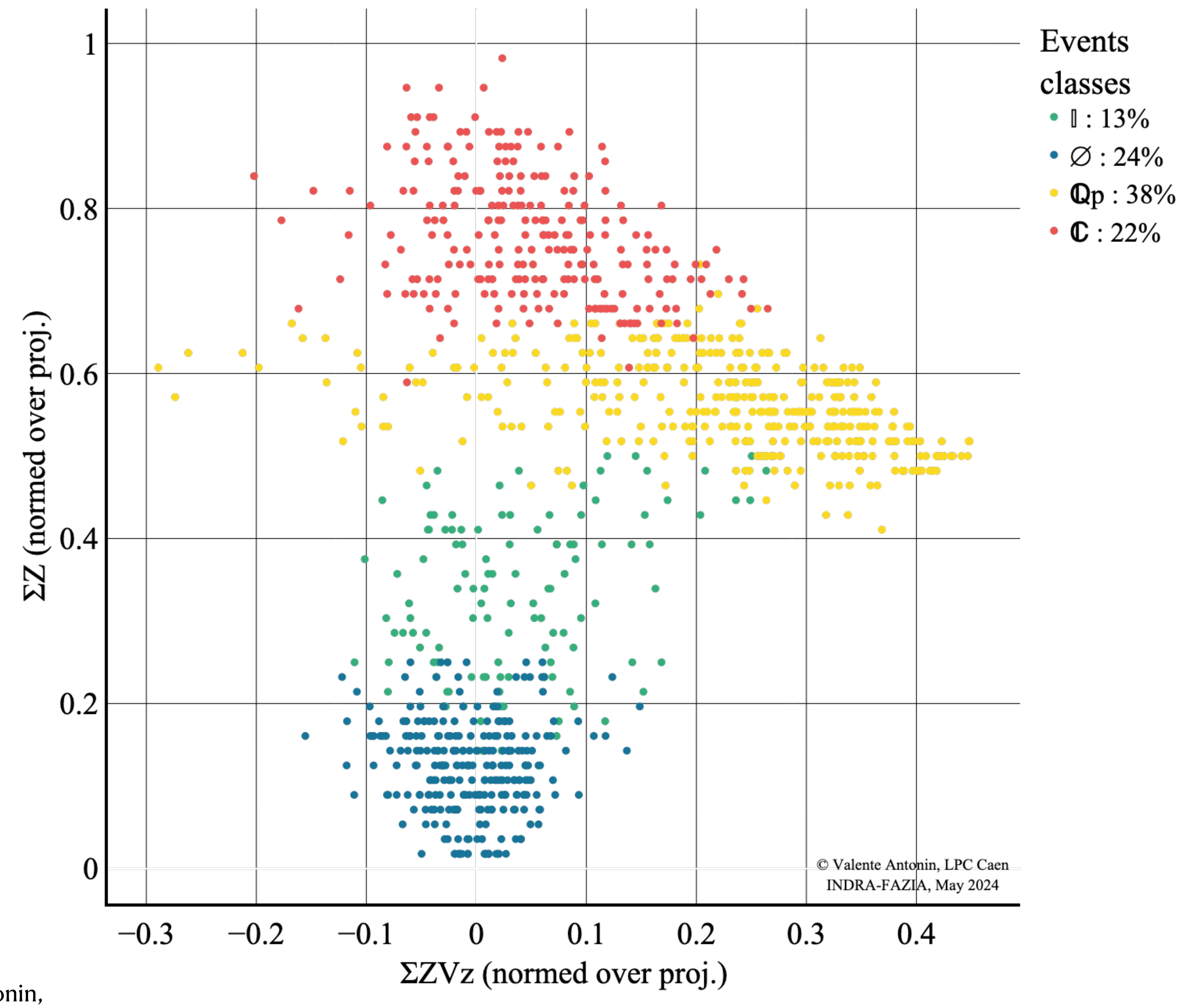
Complete and central selection

Completeness selection

$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 52 Mev/Nucleon (INDRA, Run n°7)

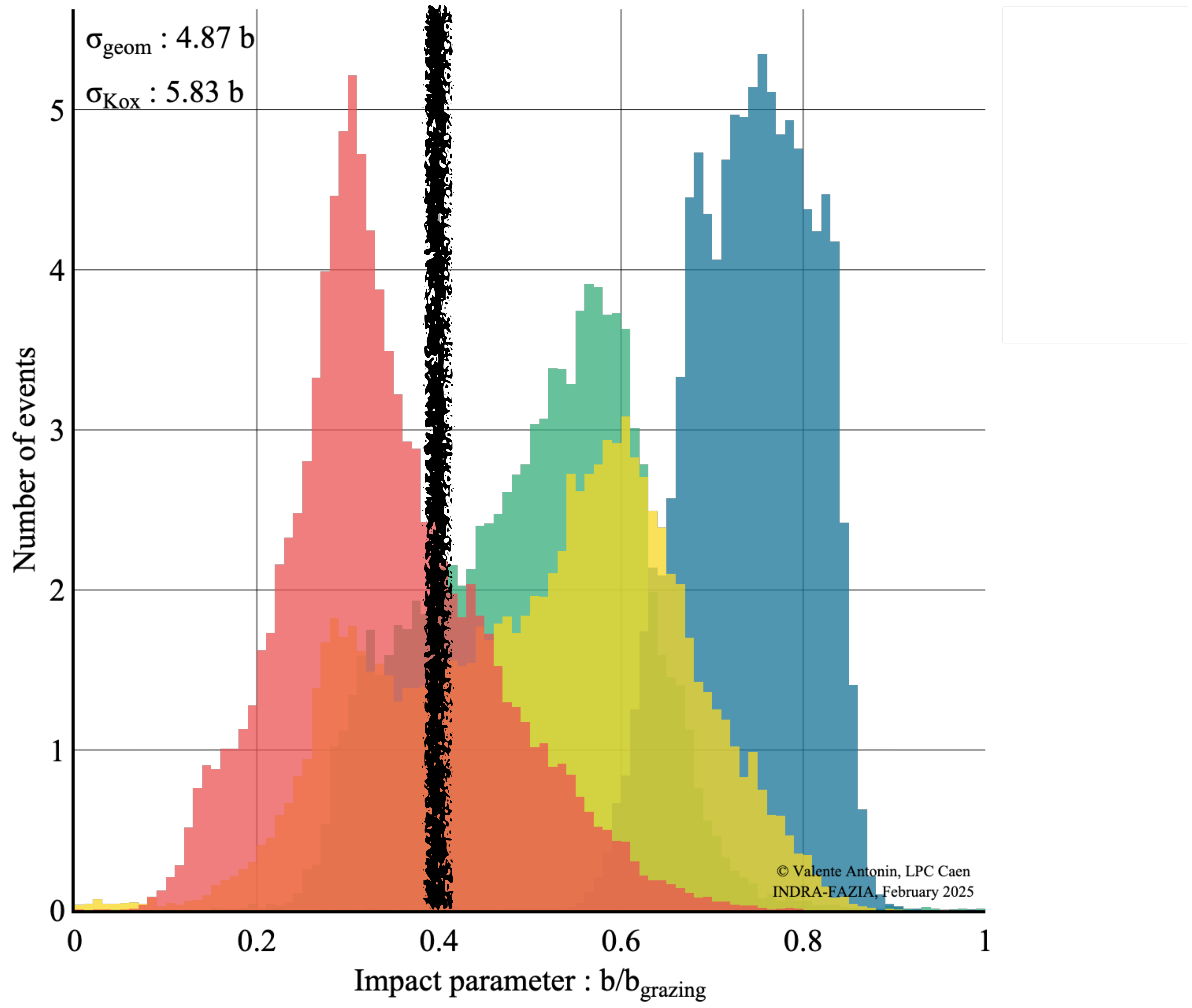
Scatter plot of the events on (ΣZ, ΣZVz)



Centrality selection

$^{128}Xe_{54}$ on $^{119}Sn_{50}$ at 50 Mev/Nucleon (INDRA, Run n°7)

Overlay histogram of the reconstructed impact parameters (ELIE)



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3. Bayesian inference of the maximal density ρ

We use an uniform distribution of the density ρ.

This simulated distribution is then filtered and gives us the filtered *prior* for our analysis:

$$P_{prior}(\theta) \xrightarrow{\text{filtering}} P^{\star}_{prior}(\theta)$$

This eliminates events that are impossible to detect and to obtain a better quality *prior* for analysis.

We use the most central events

$b/b_{max} < 0.4$ **and "complete" events**

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Discrete Bayes Theorem

Bayes' theorem is then used for this *prior*.

By applying a numerical version of this theorem for a **sampling** j **of** D **(data)** and i **of** θ **(model parameter)** :

$$P_{posterior}(\theta_i|D_j) = \frac{P^{\star}_{prior}(D_j|\theta_i)P^{\star}_{prior}(\theta_i)}{P(D_j)}$$

where :

- $P(D)$, is the experimental distribution of a global observable D
- $P^{\star}_{prior}(D\,|\,\theta)$, is the simulated filtered distribution of an observable D given a parameter value θ.

The result is a density map $P_{posterior}(\theta_i\,|\,D_j)$.

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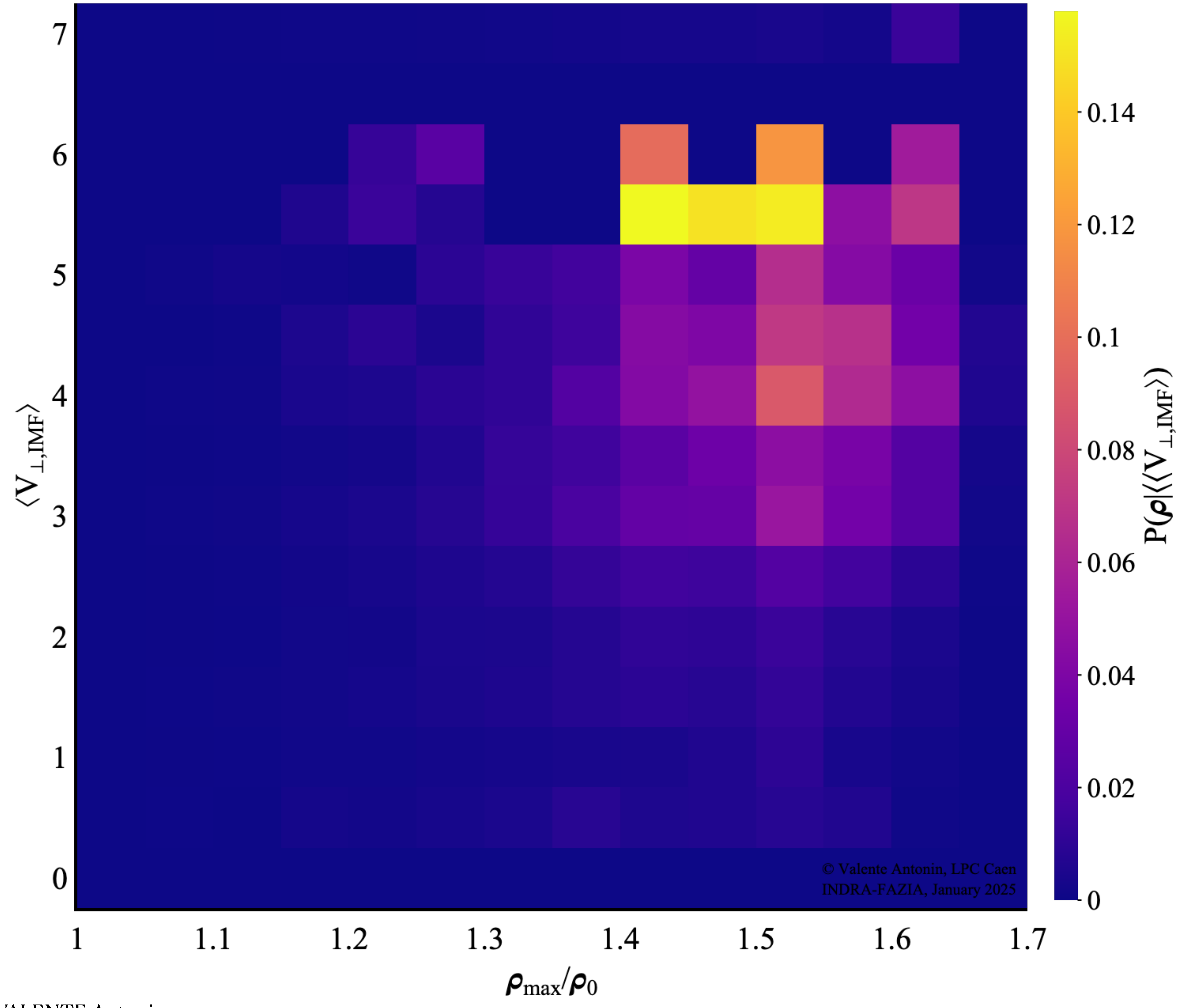
Bayesian inference for the density ρ_{max}

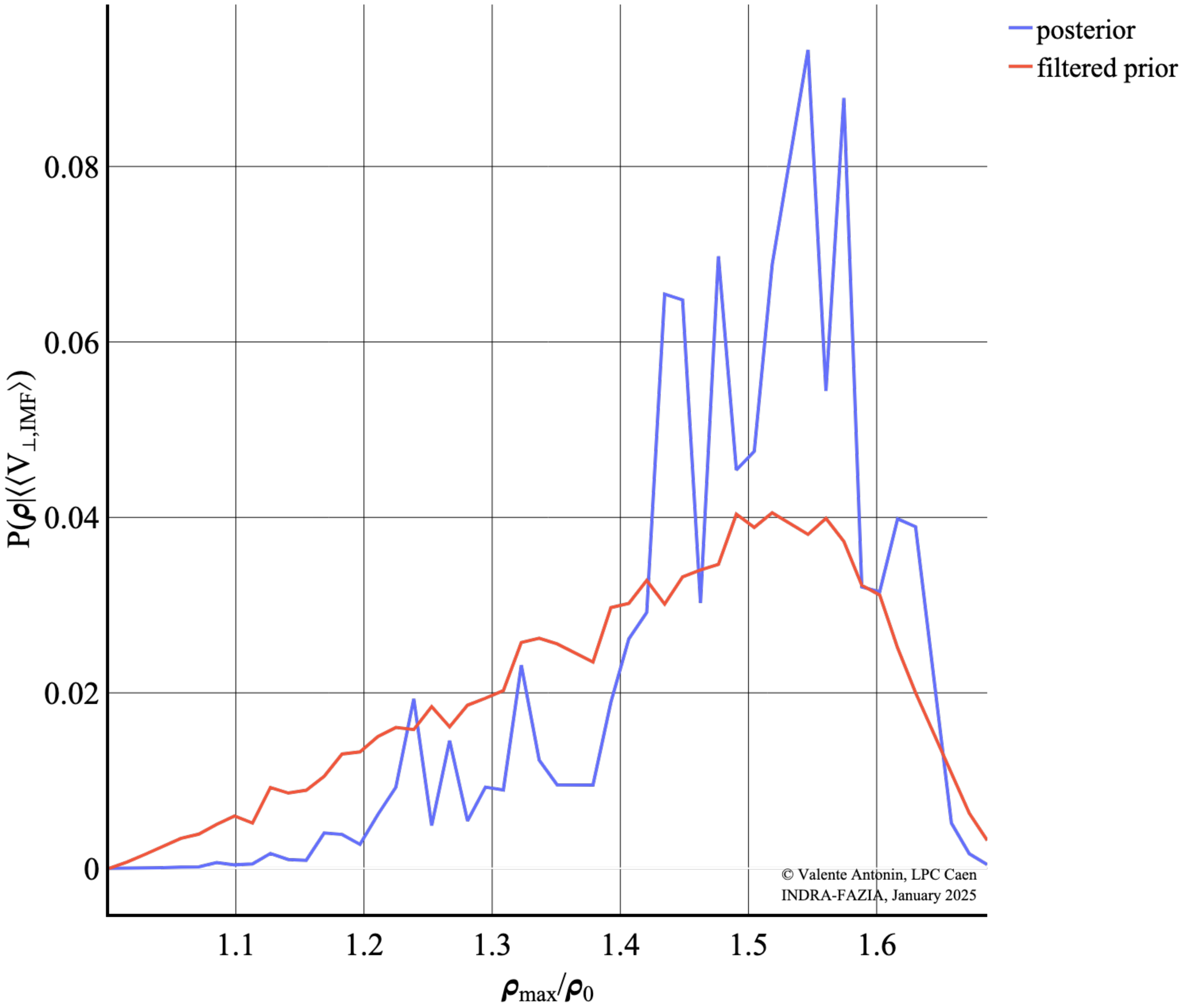
$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 74 Mev/Nucleon (INDRA, Run n°8)

Résultats de l'inférence bayésienne de la densité sachant la $\langle V_{\perp,IMF} \rangle$



$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 74 Mev/Nucleon (INDRA, Run n°8)

Projection marginale de l'inférence bayésienne de la densité



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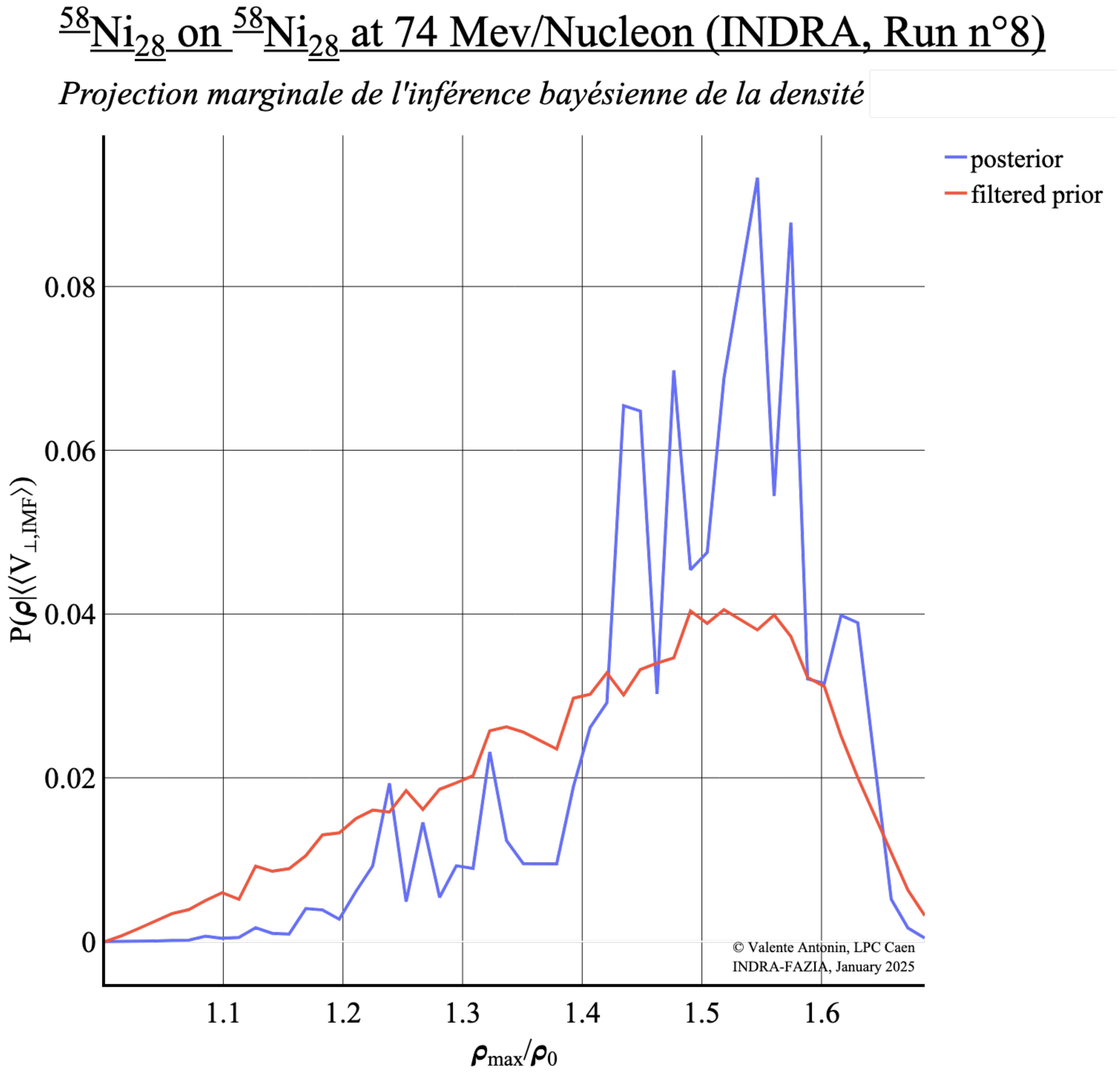

On the marginal projection in density of :

$$P_{posterior}(\theta_i|D_j)$$

We can have :

$$P_{posterior}(\theta_i)$$

This distribution can then be defined by the mean and the width:

$$\langle \rho_{max}/\rho_0 \rangle \pm \sigma_{\rho_{max}}$$

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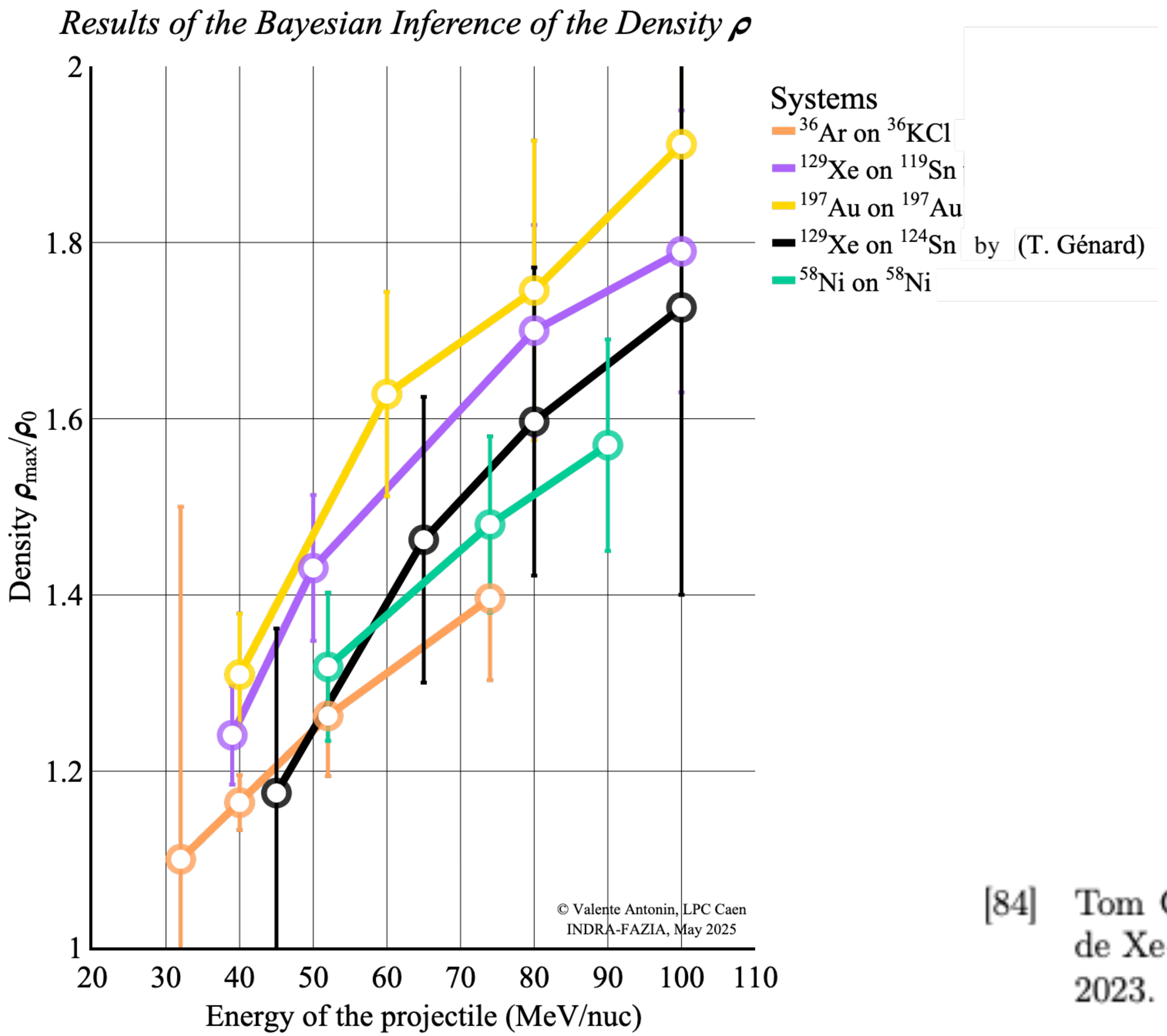
4. Systematics Results

<u>Bayesian analysis of INDRA data</u>



The most sensitive observable most in this work is the average transverse velocity of the IMFs $\langle V_{\perp,IMF,\theta>60^\circ} \rangle$ which was originally suggested by Dominique DURAND in a previous work using ELIE.

[84] Tom GENARD. “Dynamique de production des clusters dans les collisions de Xe+ Sn entre 32 et 150 A MeV”. Thèse de doct. Normandie Université, 2023.

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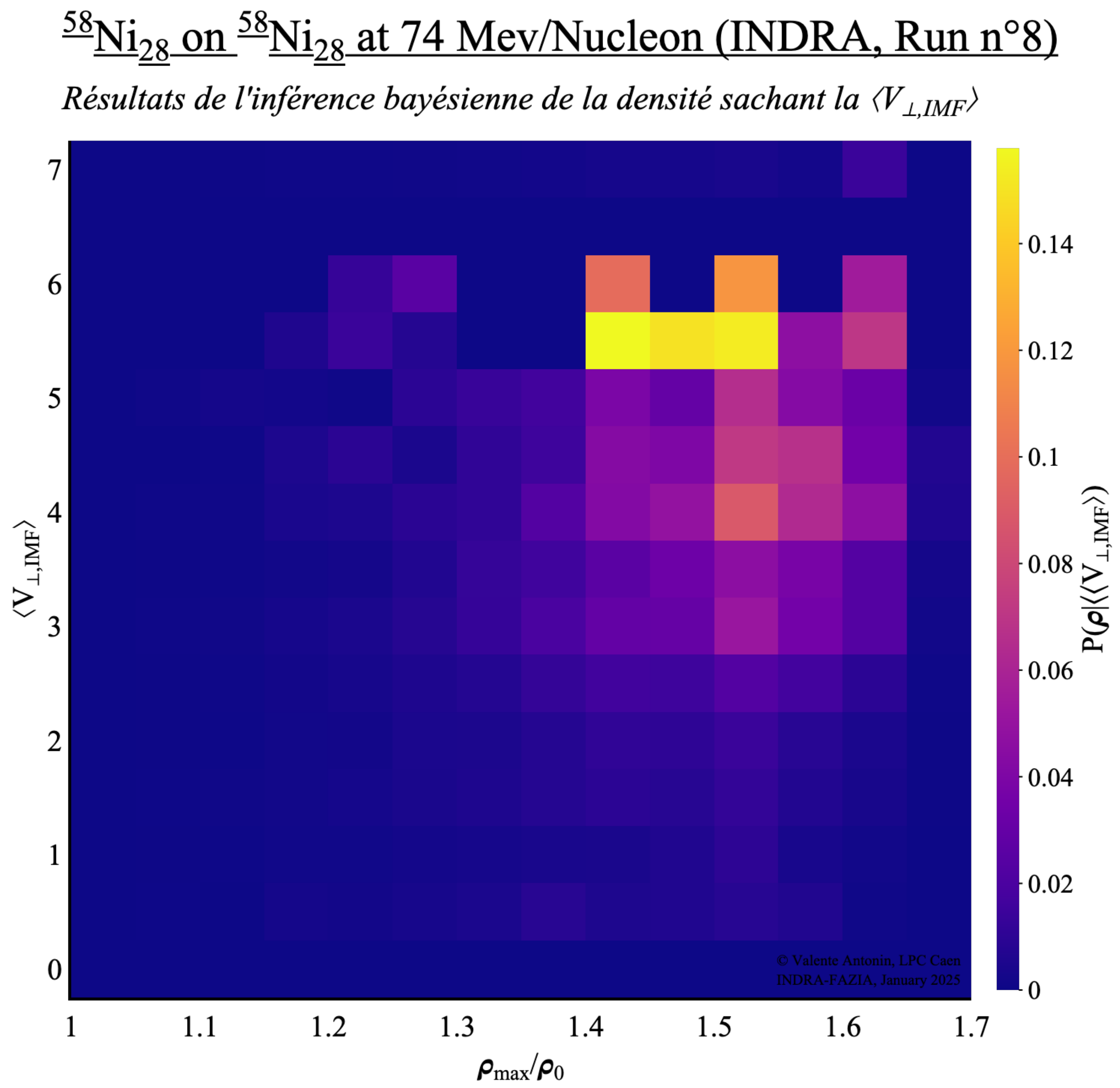
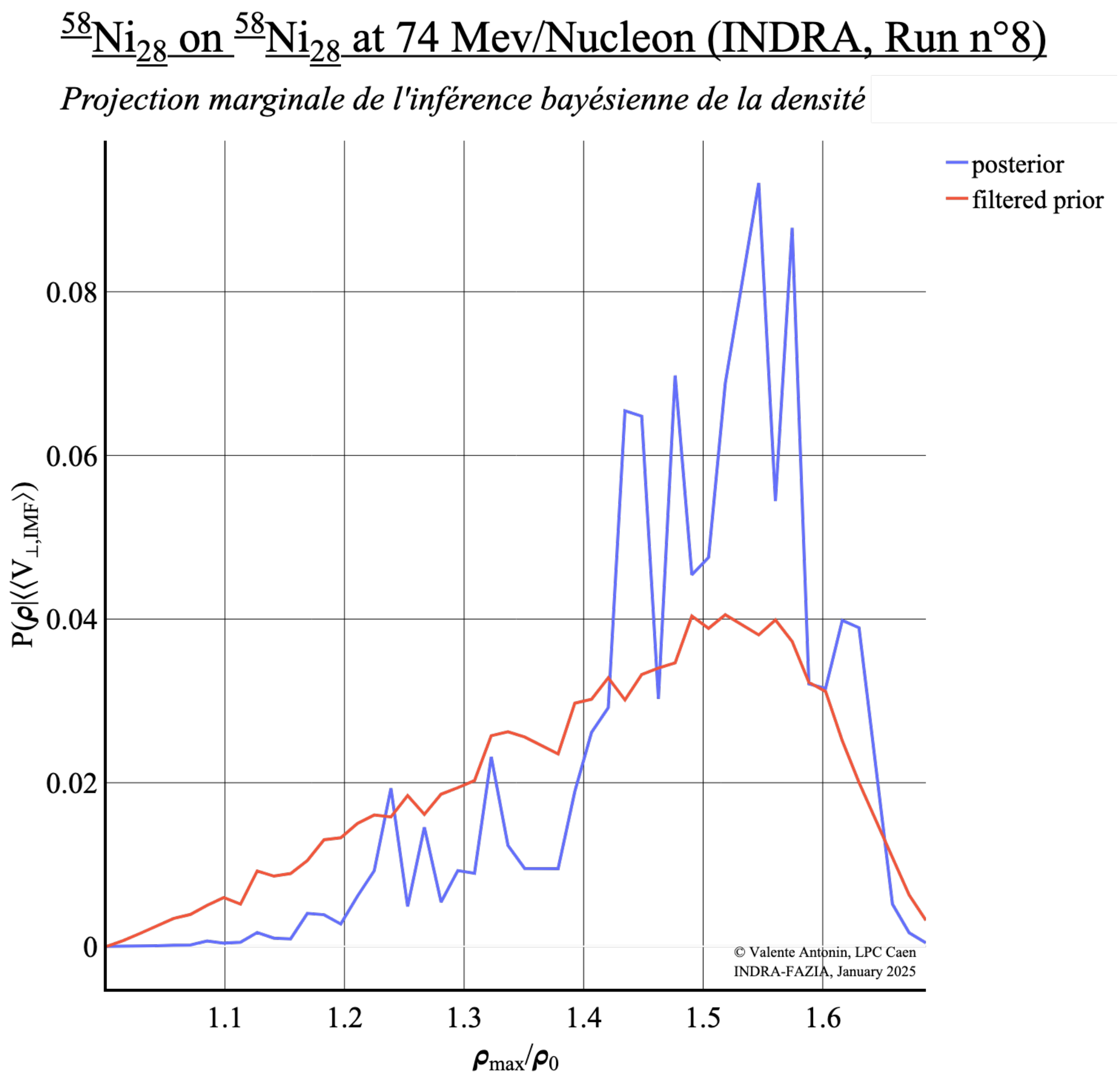
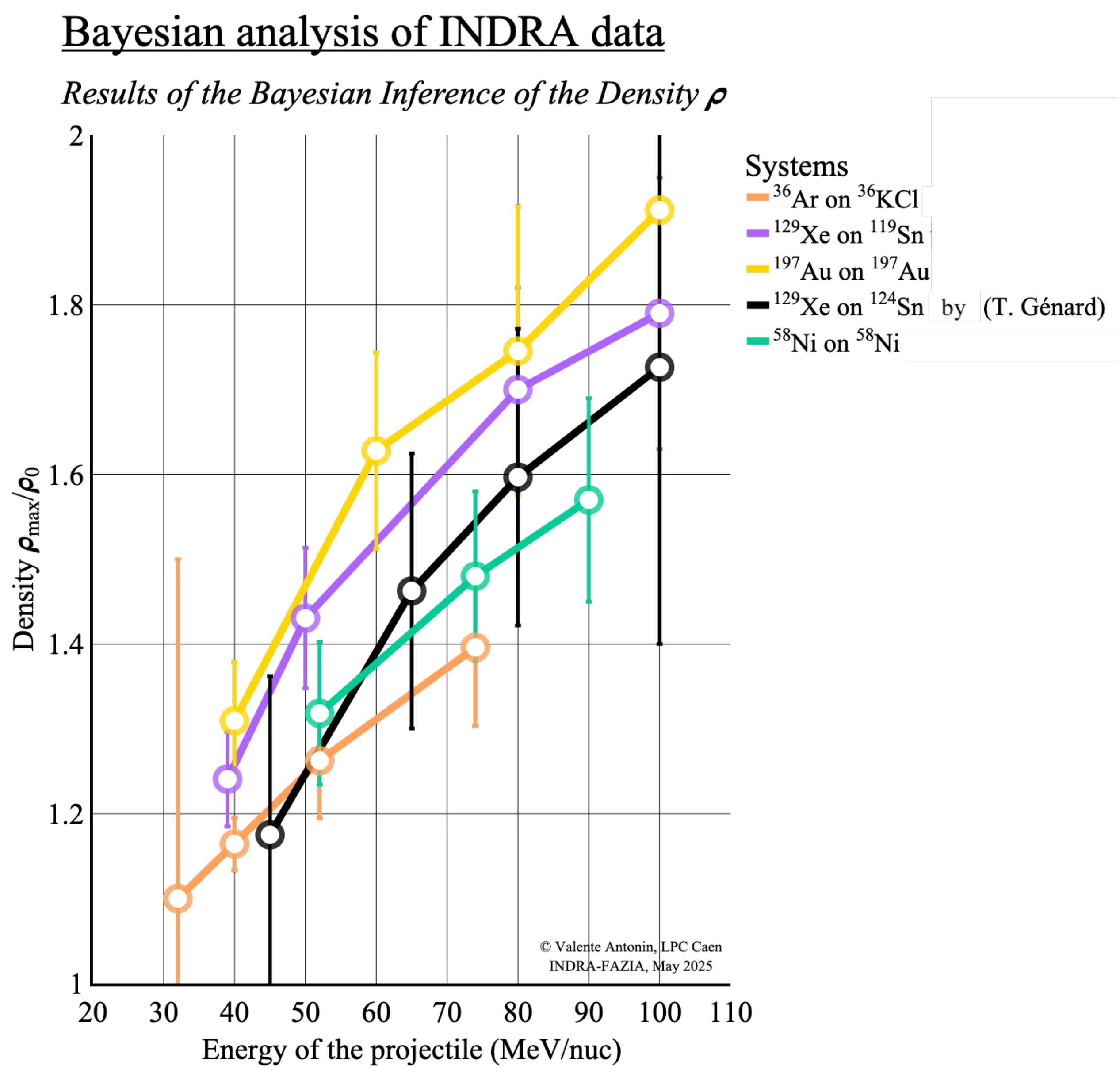
Summary for ρ_{max}







- Maximum density-sensitive selection of global observables ρ_{max}
- **Replicating and improving Tom's PhD Thesis results** (T. Génard 2023 PhD Thesis) with Bayesian inference.
- A conservative **estimate of the uncertainty is obtained at** $\sim 10\,\%$ **level based on experimental information**.
- ρ_{max} **reach** $1.8\rho_0$ **at** 100 **MeV/nucleon**.
- The mass hierarchy is well ordered : Ar + KCl < Ni+Ni < Xe +Sn < Au + Au

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5. Contribution for the compression energy

Hypothesis : $$E_{comp} = \gamma \left(E_{cm} - E_{th}\right)$$

From the EOS : $$E_{comp}(\rho) = K_\infty \frac{(\rho - \rho_0)^2}{18\rho_0}$$

Results : $$\frac{\rho}{\rho_0} = 1 + \sqrt{\frac{\gamma(E_{cm} - E_{th})}{18K_\infty}}$$

as a linear form : $$\left(\frac{\rho}{\rho_0} - 1\right)^2 \frac{1}{18} = \frac{\gamma}{K_\infty}\left(E_{cm} - E_{th}\right)$$

- We use a linear ansatz for the compression. We approximate the compression energy as a fraction of the energy available in the center of mass. γ **is the compression factor and** E_{th} **is a energy threshold.**
- We can plug this formula into the expression of the EOS compression energy. We then deduce a functional of ρ and the energy available in the center of mass E_{cm}.
- We then reformulate this functional to get a linear form and fit the (γ, E_{th}) for a given systematic with (ρ, E_{cm})...

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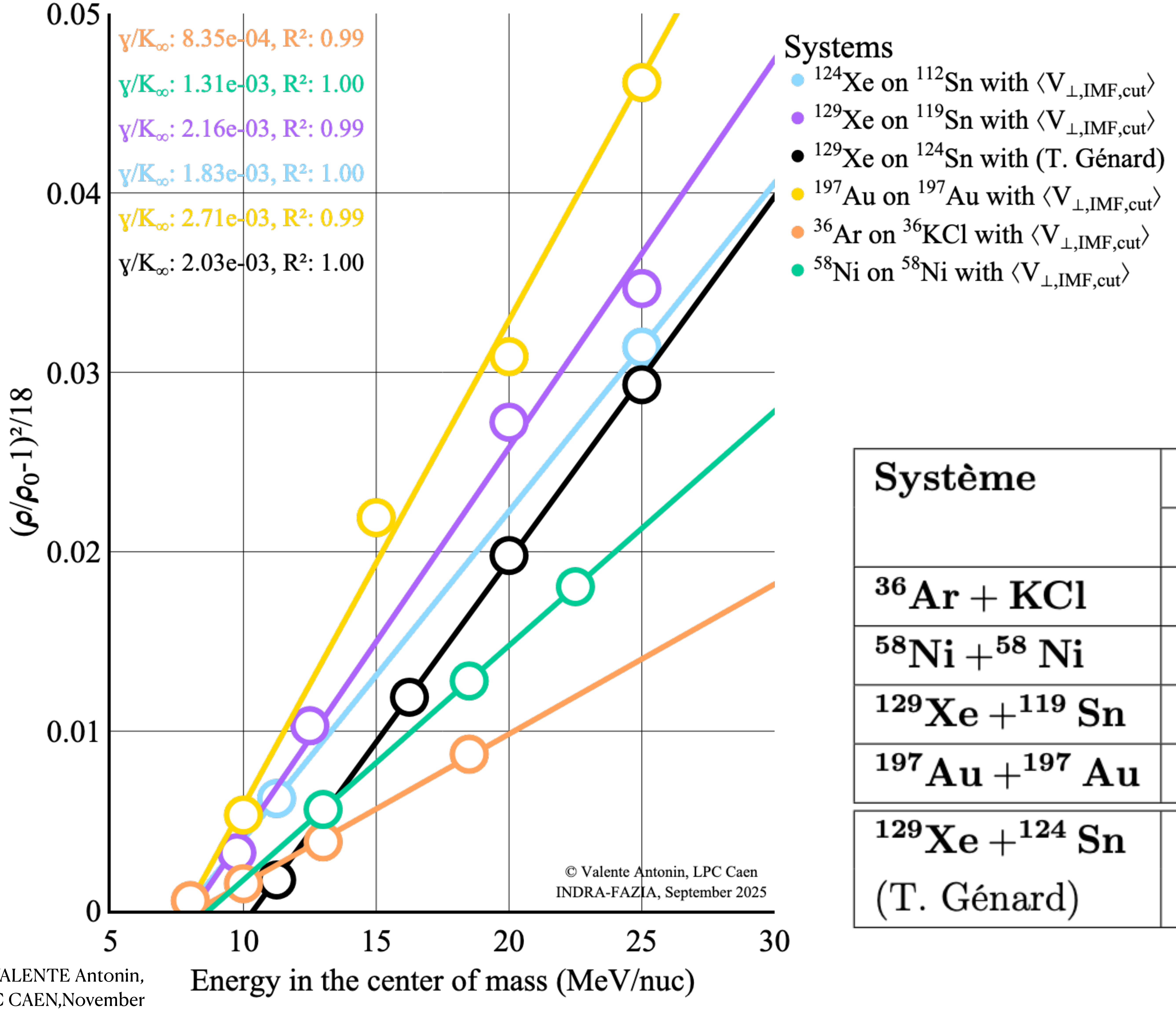
Bayesian analysis of INDRA data

Fits of the Density functionnal ρ with E_{cm}



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- With this linear form one can deduce the γ and E_{th}.
- The linear behavior is confirmed with $R^2 \sim 1.0$
- We can now compile those value in a table.

Système	γ				E_{th} (MeV/A)		
	$\frac{\gamma}{K_\infty}$ (MeV^{-1})	γ	$u(\gamma)$	$\frac{u(\gamma)}{\gamma}$	E_{th}	$u(E_{th})$	$\frac{u(E_{th})}{E_{th}}$
^{36}**Ar + KCl**	$8,35 \times 10^{-4}$	0,20	0,03	15 %	8,21	5,3	65 %
^{58}**Ni +** ^{58} **Ni**	$1,31 \times 10^{-3}$	0,31	0,06	19 %	8,02	10,3	128 %
^{129}**Xe +** ^{119} **Sn**	$2,16 \times 10^{-3}$	0,52	0,07	13 %	8,07	2,8	35 %
^{197}**Au +** ^{197} **Au**	$2,71 \times 10^{-3}$	0,65	0,15	23 %	7,84	3,07	39 %
^{129}**Xe +** ^{124} **Sn** (T. Génard)	$2,03 \times 10^{-3}$	0,70	0,17	24 %	-	-	-

Results for the compression energy

- We also did this study with the data from Borderie et al radial energy study and we use their energy data to do this estimation of γ and E_{th}. Our results are compatible with Borderie and are suggesting :

[38] B. Borderie et M. F. Rivet. "Nuclear Multifragmentation and Phase Transitions in Hot Nuclei". In : *Progress in Particle and Nuclear Physics* 61.2 (2008), p. 551-601. DOI : 10.1016/j.ppnp.2008.05.001.

Système	Our study			Borderie et al. [38]		
	γ	$u(\gamma)$	$\frac{u(\gamma)}{\gamma}$	γ	$u(\gamma)$	$\frac{u(\gamma)}{\gamma}$
$^{36}Ar + KCl$	0,20	0,03	15%	-	-	-
$^{58}Ni + ^{58}Ni$	0,31	0,06	19%	0,33	0,28	86%
$^{129}Xe + ^{119}Sn$	0,52	0,07	13%	0,38	0,30	78%
$^{197}Au + ^{197}Au$	0,65	0,15	23%	0,43	0,09	21%
$^{36}Ar + ^{45}Sc$	-	-	-	0,47	0,05	11%
$^{84}Kr + ^{197}Au$	-	-	-	0,38	0,10	26%

$$\langle \gamma \rangle = 0.4 - 0.5$$

$$E_{th} = 8.00 \pm 1.89\ MeV$$

Système	Our study			Borderie et al. [38]		
	E_{th} (MeV/A)	$u(E_{th})$	$\frac{u(E_{th})}{E_{th}}$	E_{th} (MeV/A)	$u(E_{th})$	$\frac{u(E_{th})}{E_{th}}$
$^{36}Ar + KCl$	8,21	5,30	65%	-	-	-
$^{58}Ni + ^{58}Ni$	8,02	10,3	128%	5.40	4.58	85%
$^{129}Xe + ^{119}Sn$	8,07	2,8	35%	6.61	3.20	48%
$^{197}Au + ^{197}Au$	7,84	3,07	39%	5.27	2.81	53%
$^{36}Ar + ^{45}Sc$	-	-	-	3.27	1.82	56%
$^{84}Kr + ^{197}Au$	-	-	-	5.83	2.47	42%

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6. Fragmentation Dynamics

$$V = \frac{4}{3}\pi r^3 = \frac{A_T}{\rho} \qquad \Delta t_{\text{comp}} = \frac{r_0 - r_i}{v_{comp}}$$

$$\Delta t_{\text{comp}} = \left(\frac{3A_T}{4\pi\rho_{sat}}\right)^{1/3}\left(1-\left(\frac{\rho_{sat}}{\rho_{max}}\right)^{1/3}\right)\sqrt{\frac{m}{2\gamma\left(E_{cm}-E_{th}\right)}}$$

Δt_{comp} decompression/compression :

$$\rho_{sat} \rightarrow \rho_{max}$$

Δt_{chim} chemical freeze out :

$$\rho_{sat} \rightarrow \frac{\rho_{sat}}{3}$$

Δt_{cin} cinematic freeze out :

$$\rho_{sat} \rightarrow \frac{\rho_{sat}}{20}$$

- We consider the evolution of the system from the maximum density to low density. During this stage, the system expands from a smaller radius (high density) to a larger radius (lower density). This expansion defines the characteristic **freeze-out time** of the process.
- The freeze-out time corresponds to the duration required for the system to evolve from the compressed state to the dilute state. It is estimated by comparing the change in radius to a characteristic expansion velocity.
- The expansion velocity is derived from the energy available in the center-of-mass frame. We assume that only a fraction of this energy contributes to the compression–decompression dynamics. This is expressed through a **compression factor** and an **energy threshold**, which set how much energy effectively drives the expansion.
- This approach links the expansion timescale directly to the available collision energy and provides a simple way to compare different reaction systems.

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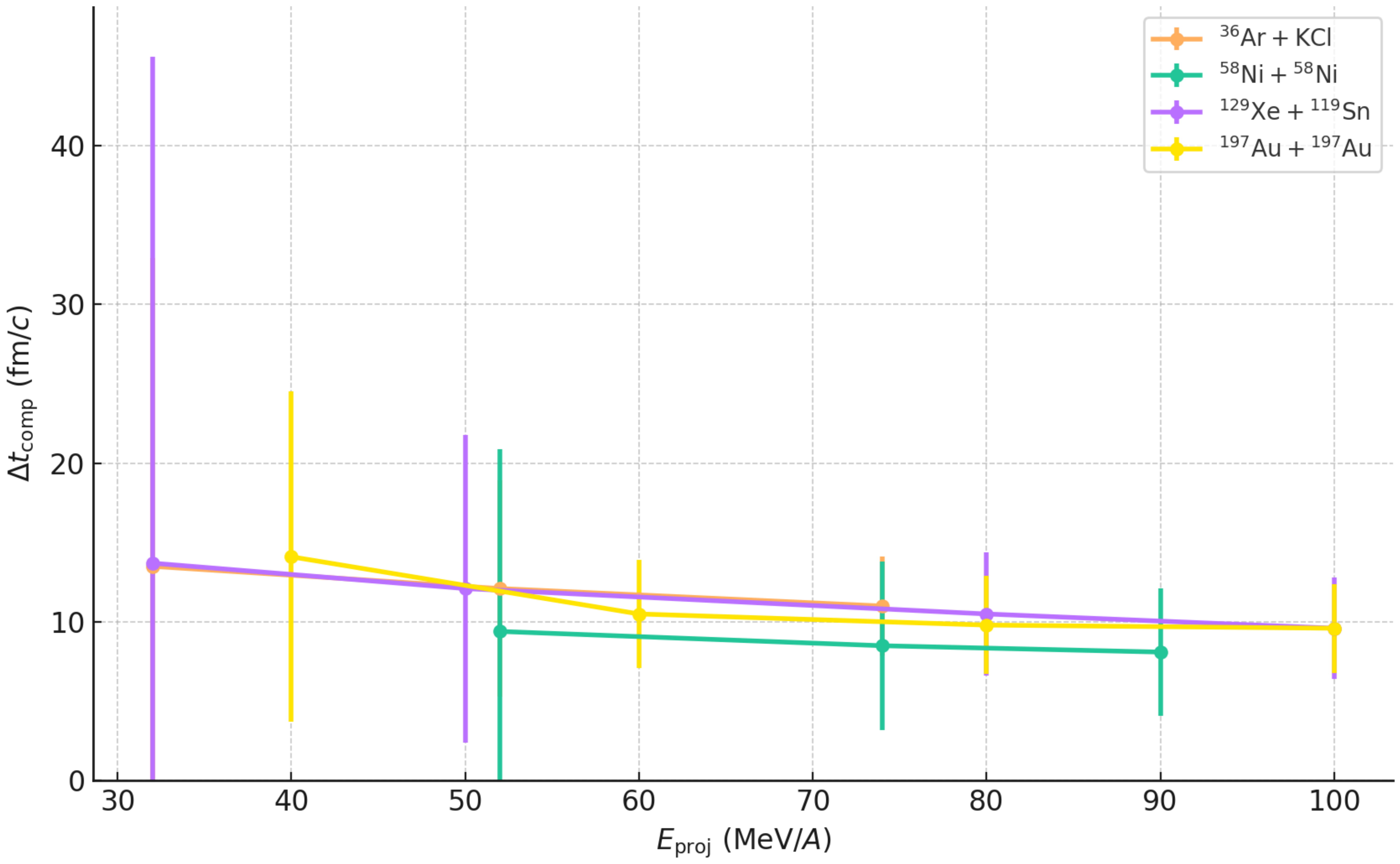

We describe the early evolution of the system as it expands from its maximum density to the saturation density. At high density, the system has a smaller radius; as it decompresses, the radius increases. The characteristic freeze-out time for this stage corresponds to the duration required for this expansion.

The expansion velocity is estimated from the fraction of the center-of-mass energy that effectively drives the collective motion, regulated by a compression factor and an energy threshold.

This phase therefore establishes the initial dynamical timescale of the system after the collision.

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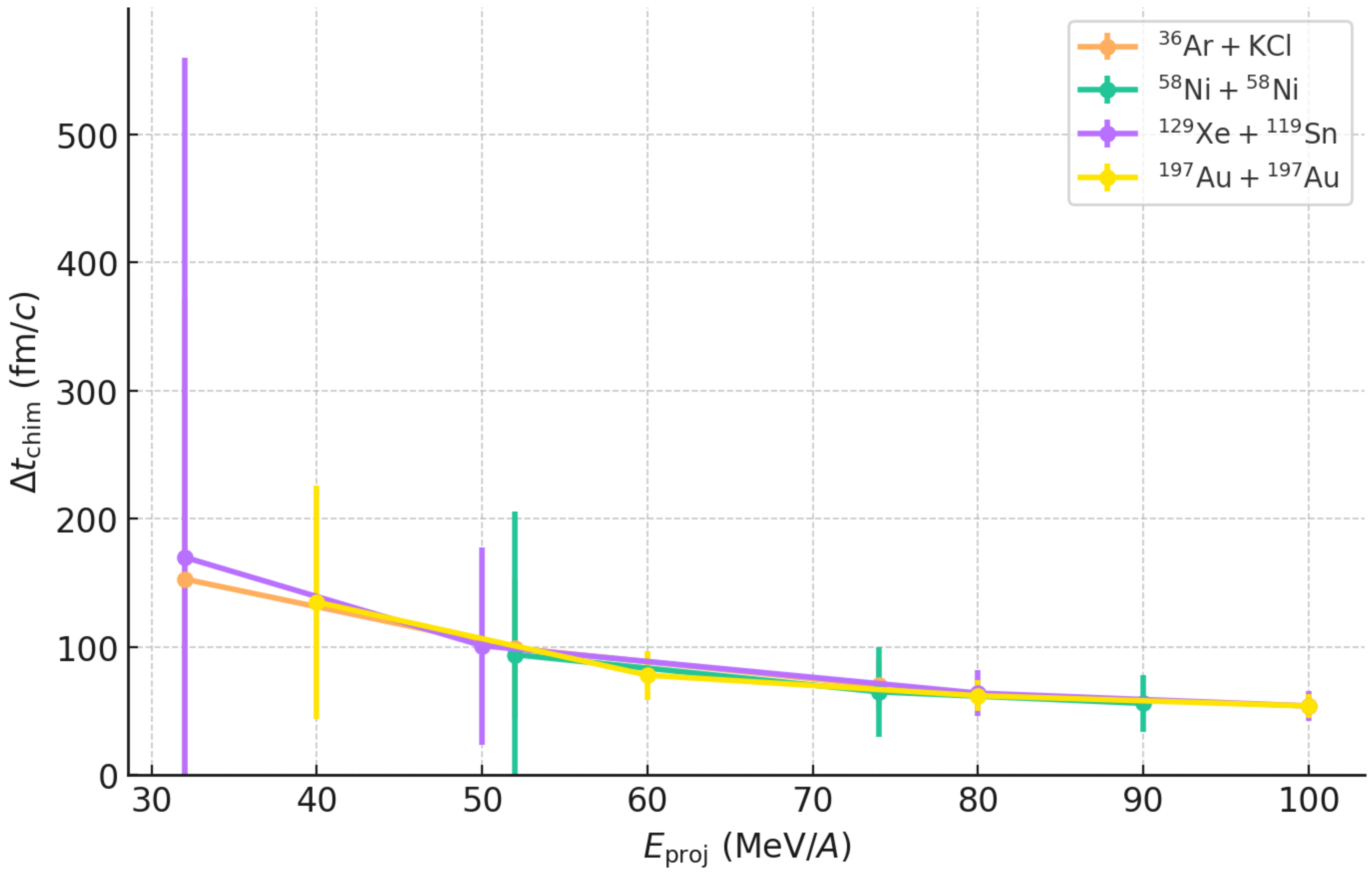

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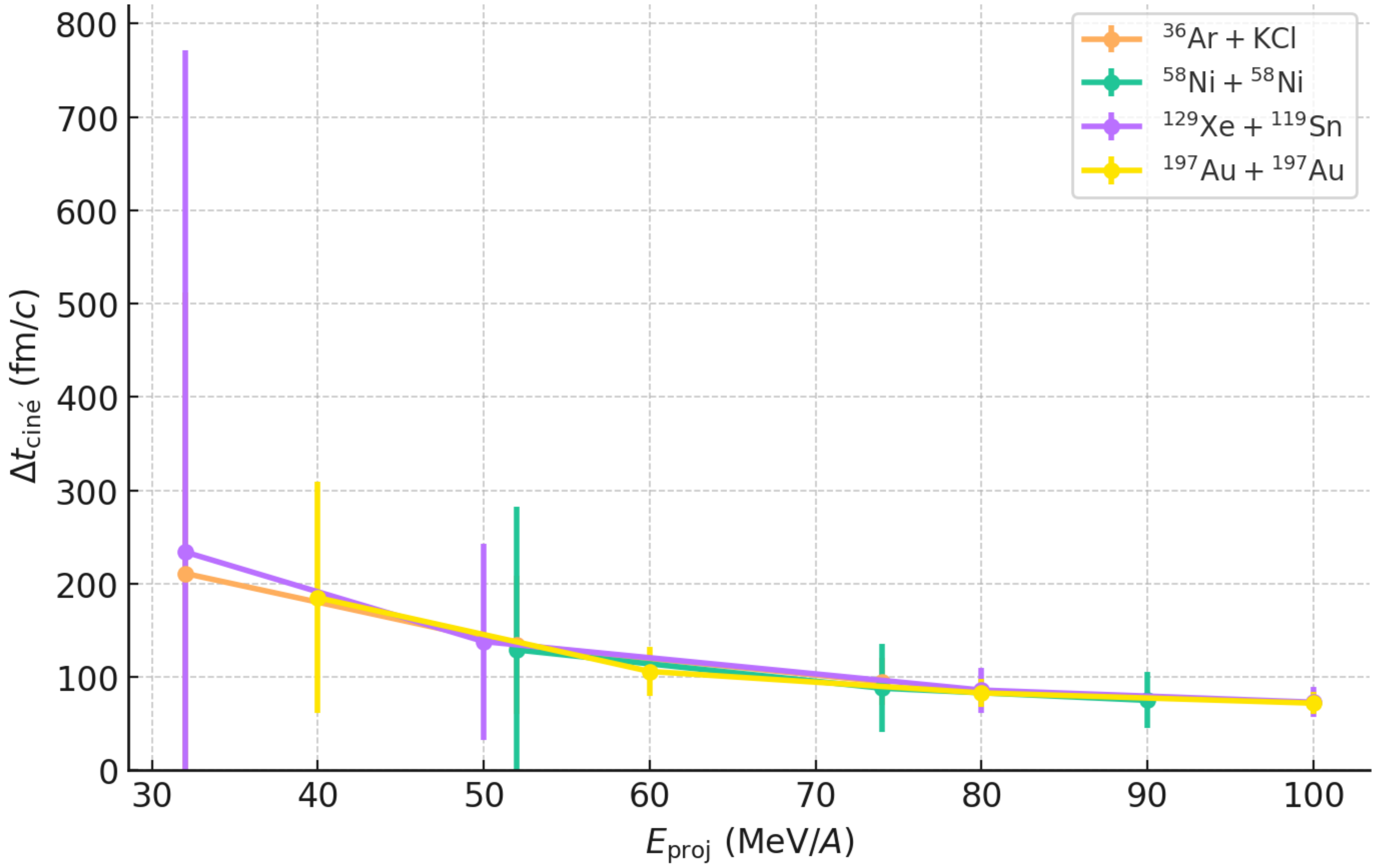

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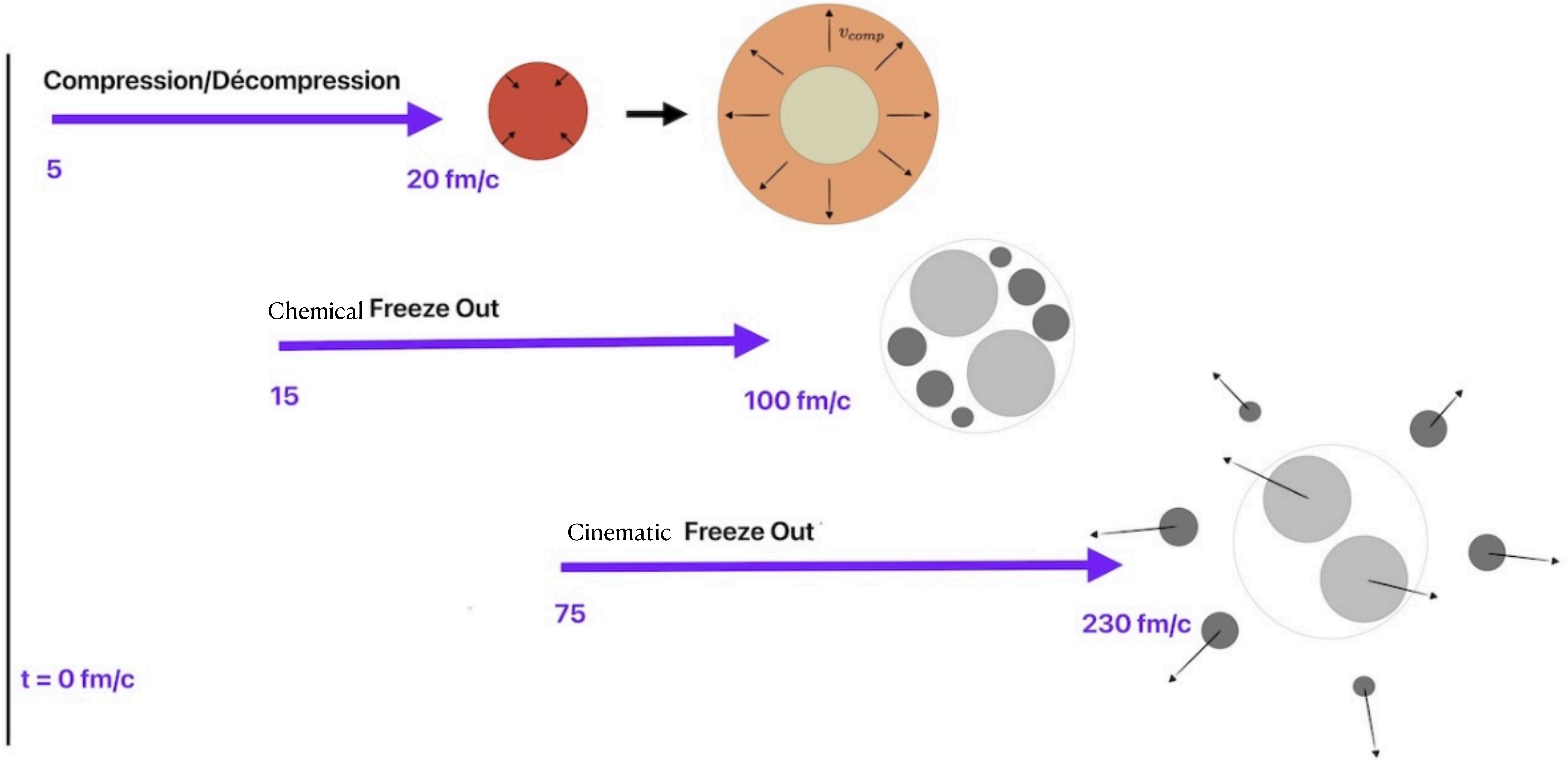

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7. Compressibility

$$P = \rho^2 \frac{\partial E}{\partial \rho}$$

- With the density, the compression, the threshold energy and the energy in the center of mass one can deduce a pressure.
- The pressure is derived from the derivative of the energy which is obtained from the compression energy.
- We also take in account the Fermi pressure.

$$P_{\text{tot}}(\rho) = P_{\text{comp}}(\rho) + P_{\text{F}}(\rho)$$

$$P_{\text{tot}}(\rho, \gamma, E_{\text{th}}, E_{\text{cm}}) = \frac{K_\infty}{9} \times \frac{\rho^2}{\rho_0^2} \times \sqrt{\frac{18\rho_0^2}{K_\infty} \times \gamma \times (E_{\text{cm}} - E_{\text{th}})} + 0.367\,\rho^{5/3}$$

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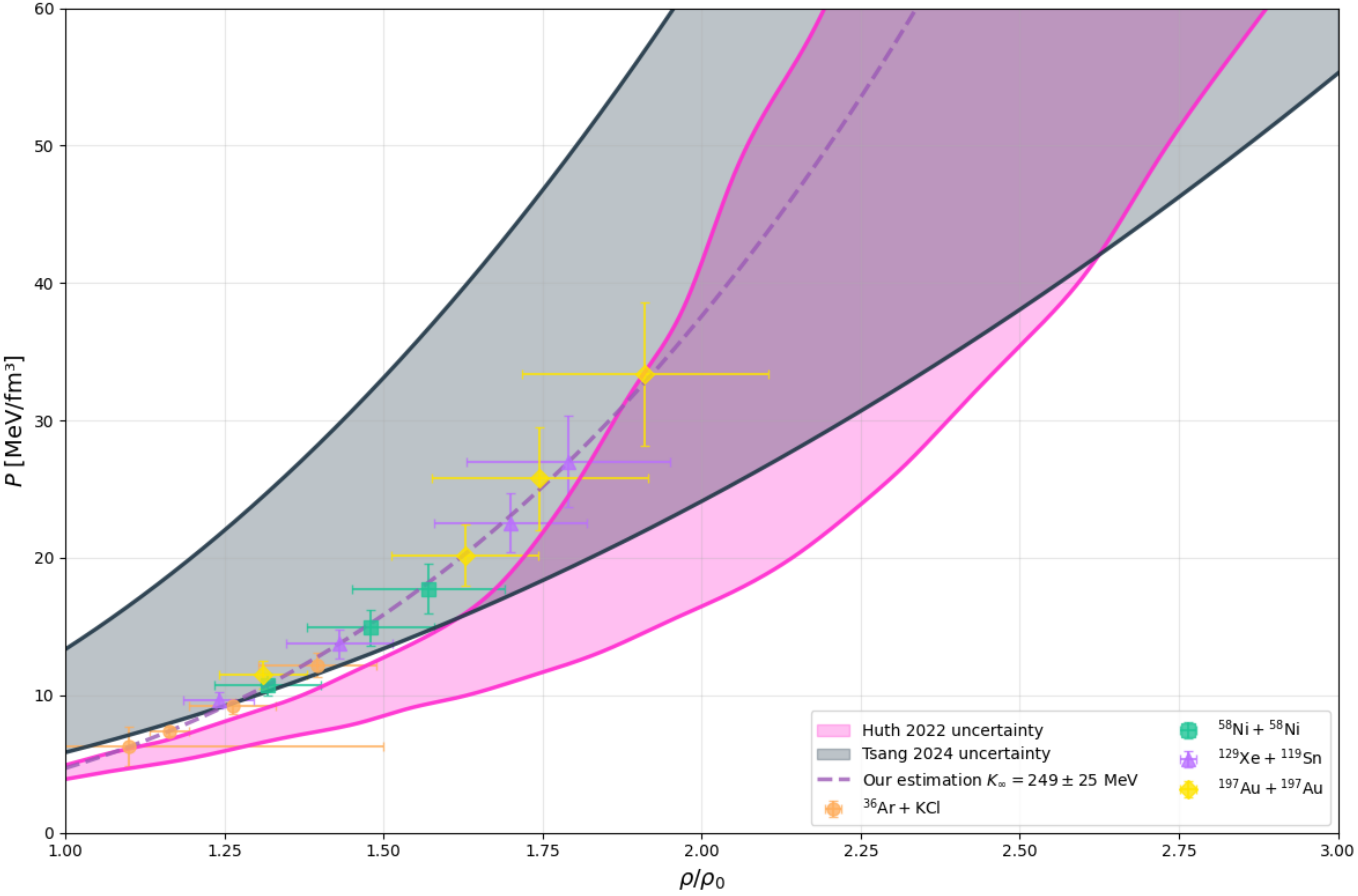
Pressure vs max. density



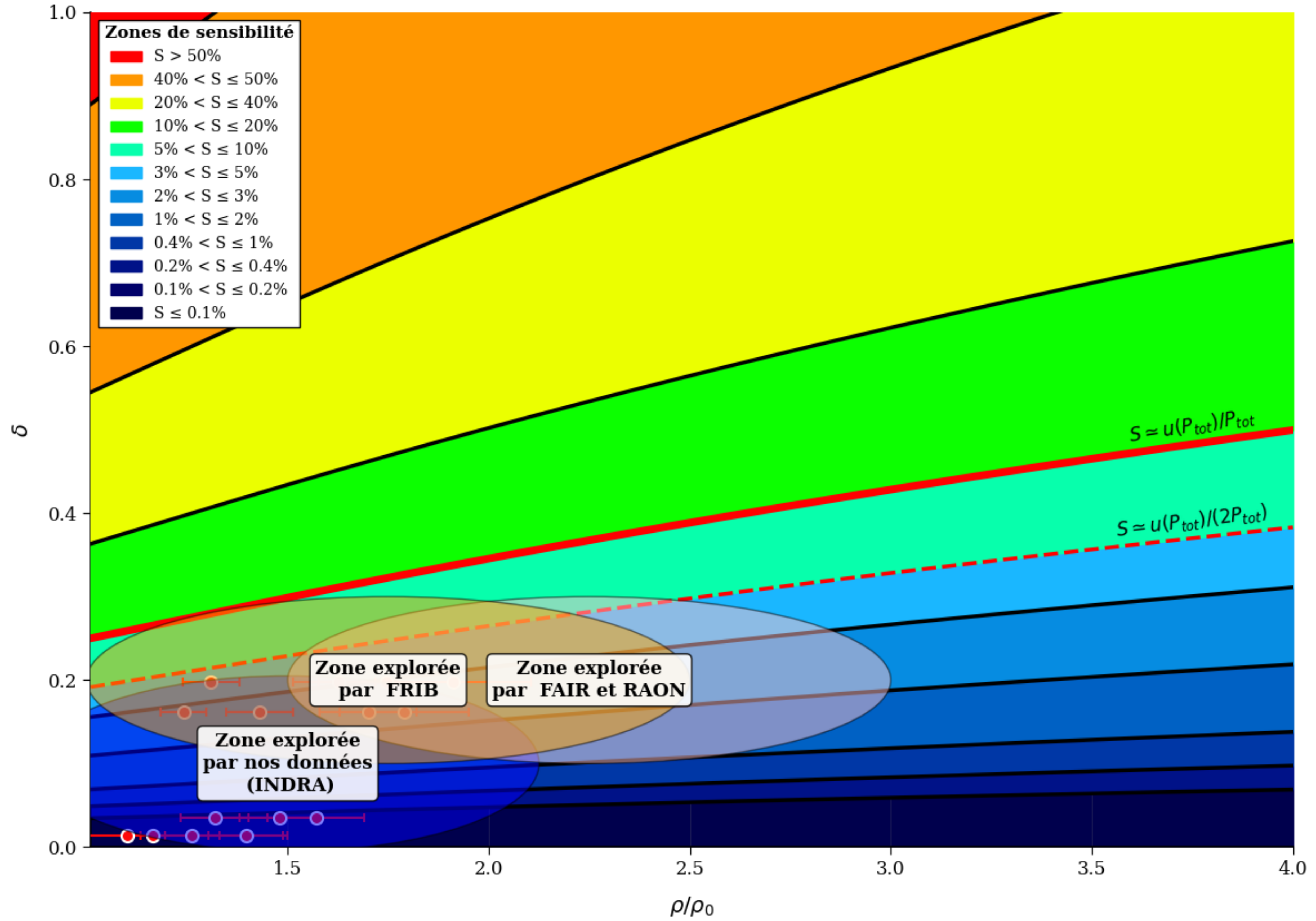
- With this calculation we can produce a pressure systematic with each system studied.
- We have a unique pressure functionnal $P(\rho)$
- We compare our results with the most recent estimation of pressure from Huth et al. who deduce the pressure with bayesian inference on astrophysical data and nuclear collision data. We also show the work of Tsang et al. who also produced an estimation.
- We are compatible with both estimation !
- With our estimation we deduce a $K_\infty = 249 \pm 25\ MeV$ which is the range of usual value in the literature.

[18] Sabrina Huth et al. "Constraining neutron-star matter with microscopic and macroscopic collisions". In : *Nature* 606.7913 (2022), p. 276-280.

[19] Chun Yuen Tsang et al. "Determination of the equation of state from nuclear experiments and neutron star observations". In : *Nature Astronomy* 8.3 (2024), p. 328-336.

9. Sensibility to the symmetry pressure

Système	δ^2	P_{sym} (MeV/fm^3)	P_{tot} (MeV/fm^3)	$S = P_{\text{sym}}/P_{\text{tot}}$ (%)
^{36}Ar + KCl (74 MeV)	0,000177	0,001	12,20	0,008
^{58}Ni + ^{58}Ni (90 MeV)	0,00119	0,007	17,75	0,039
^{129}Xe + ^{119}Sn (100 MeV)	0,0260	0,148	27,02	0,548
^{197}Au + ^{197}Au (100 MeV)	0,0392	0,223	33,40	0,668



$$P_{\text{sym}} = \rho^2 \frac{\partial E_{\text{sym}}}{\partial \rho}\delta^2 \simeq \rho_0 x^2 \delta^2 \left(\frac{L_{\text{sym}}}{3} + \frac{K_{\text{sym}}}{9}(x-1)\right)$$

- In the collision energy range considered (32–100 MeV/nucleon), the total pressure is dominated by the isoscalar component.
- The isovector (symmetry) contribution remains very small because the systems studied have low isospin asymmetry.
- As a result, the sensitivity to the symmetry-energy parameters stays below experimental resolution.
- The symmetry pressure contribution is insufficient to constrain L_{sym} or K_{sym}.
- Higher isospin asymmetry is required to adress this effect.
- Future facilities (FRIB, FAIR, RAON) and improved resolution (e.g., INDRA + FAZIA) will be needed to probe the isovector part of the EOS more effectively.

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To conclude...

- **Bayesian and ML** approaches to better **characterise the nuclear EOS**.
- Selection of central events with a dedicated **Neural Network**.
- Evaluation of the **maximum density** using **bayesian** and **ML** selection on HIPSE/ELIE models
- Finding the **mass/energy dependence** of the **maximum density**.
- Study the **dynamics of the fragmentation** to evaluate the **timescales**.
- Study of the **compressibility** of the EOS through $P(\rho)$ and **comparison** with lab and **astro** data.

Thanks for your listenning