Exploring QCD through phenomenology: small-x and heavy flavors

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Personal tastes

Phenomenological studies with perturbative QCD:

- Factorization theorems
- Effective field theories (NRQCD, SCET, CGC)

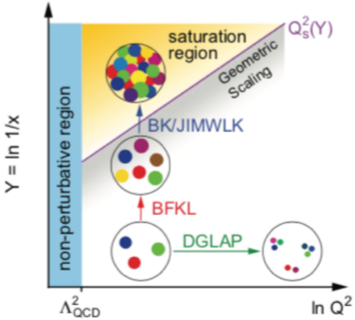
Expertise, research activities:

- Small-x physics
- Problem-solving using "simple" calculations or models
- ~~Precision physics, N^3LO calculations, etc...~~

Contents

Small-x physics and heavy flavors

Proton structure



Very small x: CGC
Dominated by gluons.
Numerically involved.

Small x: k_t factorization
Dominated by gluons?
Technically simpler than the CGC
Public code KaTie.
Hameren: Comput.Phys.Commun. 224 (2018) 371

Heavy flavors: interesting observable, test for the small-x formalism.

Open questions and work in progress

- How the different formalisms relate?
- Improvements
  CGC: higher-order calculations, next-to-eikonal corrections.
  k_t factorization: discussed in the following.
- Search for clear signals of gluon saturation.

High-energy or k_t-factorization

Seminal papers: Collins, Ellis: *Nucl. Phys. B 360 (1991) 3-30*; Catani, Ciafaloni, Hautmann: *Phys. Lett. B242 (1990) 97, Nucl. Phys. B 366(1991) 135, Nucl.Phys. b427 (1994) 475*

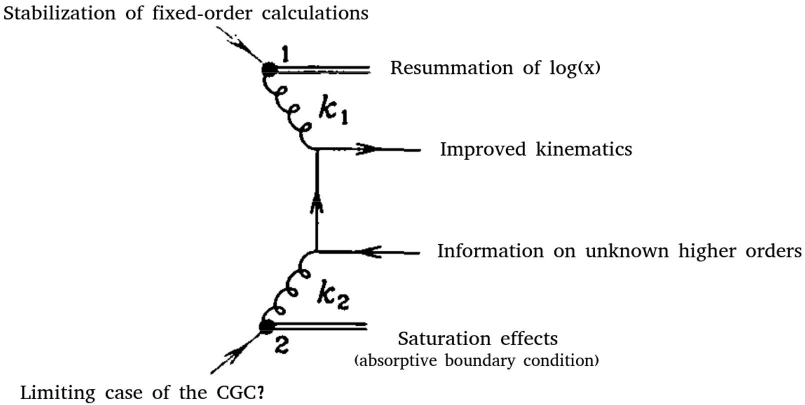
$$\frac{d\sigma}{dx_1 dx_2 d^2p_t} = \int^{k_{max}^2} d^2k_{1t} d^2k_{2t} F_i(x_1, k_{1t}^2; \mu^2) F_j(x_2, k_{2t}^2; \mu^2) \hat{\sigma}_{ij}(\hat{s}, k_{1t}^2, k_{2t}^2, p_t^2)$$

$\hat{\sigma}$ is the off-shell cross section. The unintegrated PDFs, $F(x, k_t^2; \mu^2)$, are related to collinear PDFs by

$$f_i(x, \mu^2) = \int^{\mu^2} F_i(x, k_t^2; \mu^2) dk_t^2$$

$F_i(x, k_t^2; \mu^2)$ obey the BFKL equation: Resums large $\log(1/x)$.

Advantages of k_t factorization



Implementation (since 2000)

- Many UPDFs:
  Some include non-linear or CCFM evolution.
  Most popular: Kimber-Martin-Ryskin-Watt (KMRW).
- KMRW UPDFs
  Built from collinear PDFs (include both quark and gluon densities).
  No BFKL evolution: focuses on P_t rather than x dependence.

Unsatisfactory features:

- ⋆ Calculations including only $gg \to Q\overline{Q}$ (formalism initially developed for asymptotic energies).
- ⋆ Inconsistent use of the factorization scheme.
- ⋆ No publication found specifying the value for k_{max}.
- ⋆ Several issues related to the KMRW UPDFs.

Factorization schemes and HF production

Fixed-flavor-number scheme

Proton made of gluon+light quarks

$\sigma = \mathrm{PDF}^{FFNS}_{\text{order}} \otimes \hat{\sigma}^{FFNS}_{\text{order}}$

Dominant process: $gg \to Q\bar{Q}$

Variable-flavor-number scheme

gluon+light quarks+heavy quarks

$\sigma = \mathrm{PDF}^{VFNS}_{\text{order}} \otimes \hat{\sigma}^{VFNS}_{\text{order}}$

Dominant processes: $gg \to Q\bar{Q}$ and $gQ \to gQ$

- Older calculations used $\hat{\sigma}^{FFNS}_{\text{LO}}$ with $\mathrm{PDF}^{VFNS}_{\text{LO, NLO}}$.
- Expect better results with a VFNS at LHC energies (where you can reach $P_t \gtrsim 5m_Q$).

Improvements

1. Change the FFNS for a VFNS.
   - Expect better results for, e.g., p_t-dependent observables.
2. Modified KMRW approach.
   - Main issue: does not obey exactly the condition

$$f_i(x,\mu^2) = \int^{\mu^2} F_i(x,k_t^2;\mu^2)dk_t^2$$

   - *Golec-Biernat and Staśto, Phys. Lett. B 781, 633-638 (2018); Guiot: Phys.Rev.D 101 (2020) 5; Nefedov and Saleev, Phys. Rev. D 102, 114018 (2020); Guiot: Phys.Rev.D 107 (2023) 1*
3. Detailed study of the upper limit of integration k_{max}
   - Why do calculations based on $gg \to Q\overline{Q}$ "work"? (underestimation by a factor of 4 expected)
   - Inconsistency between the normalization condition and

$$\boldsymbol{\sigma} = \int^{k_{max}^2 \sim s} dk_t^2 F_i \otimes F_j \otimes \hat{\boldsymbol{\sigma}}$$

k_t factorization in a Yukawa theory

Inspired by "Basics of factorization in a scalar Yukawa field theory", *Aslan, Gamberg, Gonzalez-Hernandez, Rainaldi, Rogers*.

$$\mathscr{L} = \sum_j \frac{i}{2}\left[\overline{\psi}_j \gamma^\mu \partial_\mu \psi_j - (\partial_\mu \overline{\psi}_j)\gamma^\mu \psi_j\right] - M_j \overline{\psi}_j \psi_j + \frac{1}{2}(\partial\phi)^2 - \frac{m_s^2}{2}\phi^2$$
$$- \lambda\left(\overline{\psi}_1 \psi_2 \phi + \overline{\psi}_2 \psi_1 \phi\right)$$

The structure functions can be computed exactly = w/o factorization (but still in perturbation theory).

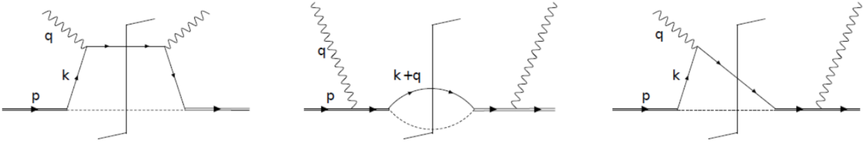
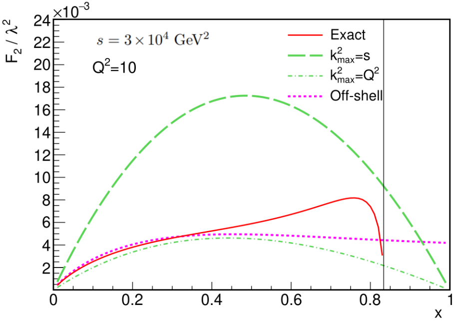


Figure 1. Order $\mathcal{O}(\lambda^2)$ diagrams for the DIS on a quark target in the Yukawa theory.

k_t factorization in a Yukawa theory

Guiot, Hameren: JHEP 04 (2024) 085

The KMRW UPDFs are also computed in the Yukawa theory:
$\Rightarrow$ Behave as $1/k_t^2$ at large k_t.



$k^2_{max} \sim s$ overshoots the exact result.

$k^2_{max} \sim Q^2$ is also justified by theoretical considerations.

Parton-Branching UPDFs: rapidly decreasing when $k_t > Q$:
$k^2_{max} \to \infty$ is fine.

Implementation of the VFNS *Guiot, Hameren: Phys.Rev.D 104 (2021)*

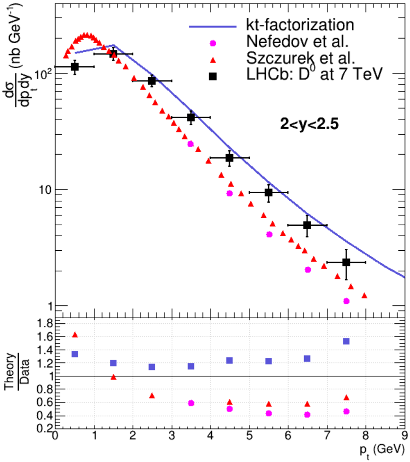
- All partonic channels included, in particular the dominant $Qg \to Qg$ contribution.
- Modified KMRW UPDFs.
- (Off-shell) partonic cross sections obtained with KaTie ($k_{max} \sim Q$).
- Peterson's fragmentation functions:

$$D_{Q\to H_Q}(z) \propto f(Q \to H_Q)\frac{1}{z\left(1-\frac{1}{z}-\frac{\varepsilon_i}{1-z}\right)^2}$$

Fixed scale
Cacciari's fragmentation fractions.
Standard value $\varepsilon_c = 0.05$ and $\varepsilon_b = 0.01$.

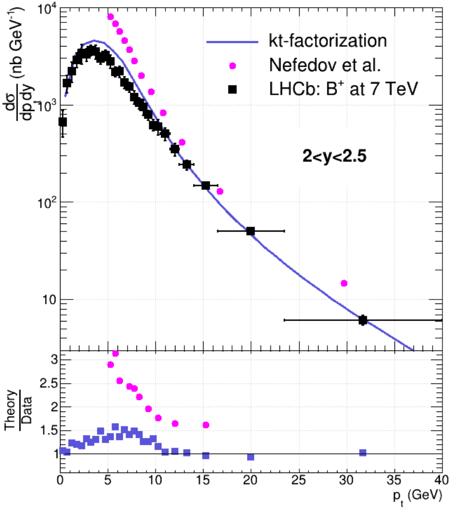
Results: D mesons

Improved agreement between theory and data.



Nefedov et al.: *Phys. Rev. D 91 (2015) 5, 054009*
Szczurek et al.: *Phys. Rev. D 87 (2013) 094022*

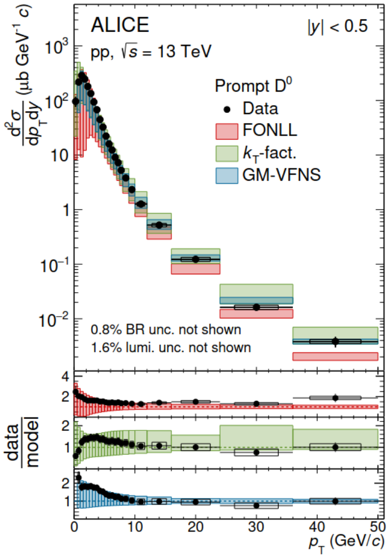
Results: B mesons



Nefedov et al.: *Int. J. Mod. Phys. A 30 (2015) 04n05, 1550023*

Fewer theoretical results available for B mesons.

Prediction for D-meson production at 13 TeV



More plots in: Alice Collaboration, *JHEP 12 (2023) 086.*

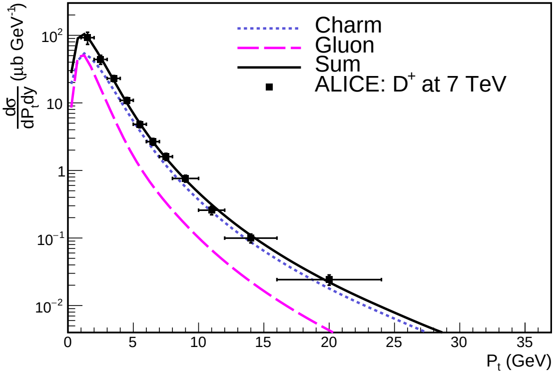
Complete implementation: scale-dependent FF

Barattini, Dib, Guiot: JHEP 05 (2025) 115

- ⋆ Contributions: $D_{Q\to H_Q}(z;\mu)$ and $D_{g\to H_Q}(z;\mu)$.
- ⋆ Evolution with QCDNUM.
- ⋆ Initial conditions: $D_{g\to H_Q}(z;\mu_0)=0$ and $D_{Q\to H_Q}(z;\mu_0)$ with Peterson parametrization.
- ⋆ Applied to D and B mesons, charmed baryons and B_c meson.

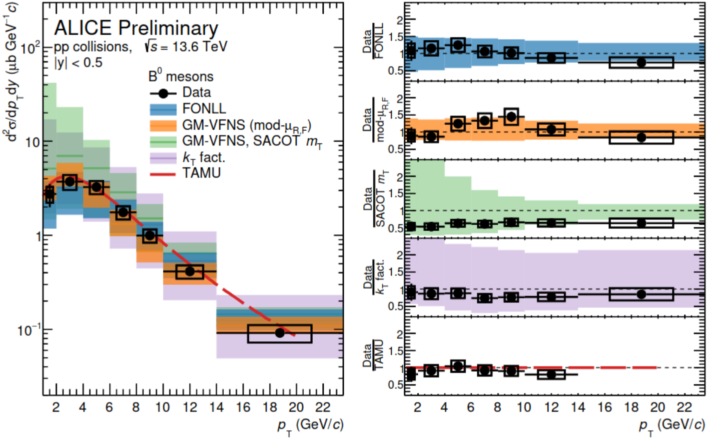
$$\frac{d\sigma(pp\to H_Q+X)}{dYd^2P_t}(Y,P_t)=\sum_j\int_{z_{\mathsf{min}}}^1\frac{dz}{z^2}D_{j\to H_Q}(z;\mu_f)$$
$$\times\frac{d\sigma(pp\to j+X)}{dyd^2p_t}\left(y,\frac{P_t}{z};\mu_f\right)$$

Results: D^+ meson



$D_{g \to D}$ improves the agreement between theory and data at small P_t.

Prediction for B-meson production at 13.6 TeV



Color Glass Condensate

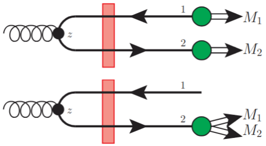
Phenomenology

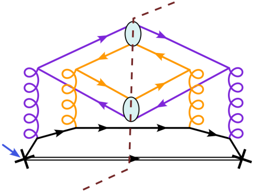
CGC limit $\Rightarrow k_t$ and TMD factorization

Improved k_t factorization, or ITMD

Double quarkonia production

- ⋆ Ongoing work with Marat Siddikov and Franco Barattini
- ⋆ Focus on the fragmentation contributions.
- ⋆ Calculations for the P_t-integrated cross section.





Collinear factorization

$$\sigma^{ep\to j+X} = \sum_i f_{i/p}(x;\mu^2) \otimes \hat{\sigma}^{ei\to ej+X}\left(x, \frac{Q^2}{\mu^2}\right) + \text{remainder}$$

PDFs are by-products of the factorization proof.

How to use it:

Choose a scheme and scales.

Compute the infrared-safe cross section $\hat{\sigma}$ (subtraction of double counting).

Requires Feynman rules for PDFs.

DGLAP Eq. = renormalization group equation for PDFs.

Factorization and double counting

Regularized PDFs: $f_{a/b}^{(1)}(\xi;\mu) \propto \mu^{2\varepsilon}\left(\frac{1}{\varepsilon_{IR}} - \frac{1}{\varepsilon_{UV}}\right) P_{ab}(\xi)$

Renormalized PDFs: $f_{a/b}^{(1)}(\xi;\mu) \propto \mu^{2\varepsilon}\left(\frac{1}{\varepsilon_{IR}}\right) P_{ab}(\xi)$

Use factorization at the partonic level:

$$\sigma^{(1)} = f^{(0)} \otimes \hat{\sigma}^{(1)} + f^{(1)} \otimes \hat{\sigma}^{(0)}$$

$$\hat{\sigma}^{(1)} = \sigma^{(1)} - f^{(1)} \otimes \hat{\sigma}^{(0)}$$

IR divergence of $\sigma^{(1)}$ canceled by $f^{(1)}$

Improved TMD factorization *Kotko et al.: JHEP 2015 (2015) 9*

- **Goal:** improvement of the k_t factorization at small k_t.
- The TMD factorization requires several gluon densities (observable dependent).
- Initial proposition:

$$\sigma = \text{TMD PDFs} \otimes \hat{\sigma}^{\text{off-shell}}$$

Cannot work: $\hat{\sigma}^{\text{off-shell}}$ has more IR divergences than $\hat{\sigma}^{\text{on-shell}}$

But: phenomenological applications use small-x PDFs from CGC, not TMD PDFs.

Ambiguity: small-x PDFs $\equiv$ TMD PDFs?

Ongoing and future work

- Study of the equivalence between small-x and TMD PDFs.

- Improved k_t factorization:
  - Derivation of k_t factorization starting from TMD factorization.
  - CGC $\rightarrow$ ITMD factorization.

- Phenomenological applications with observables sensitive to small P_t.
  - Comparison of k_t and ITMD factorization.

Contents

Quarkonia production and scale evolution

- ⋆ All non-perturbative functions have a scale evolution: PDFs, FFs...
- ⋆ Exception: Quarkonia production. Hardly justifiable
- ⋆ Fragmentation function formalism by Kang, Ma, Qiu, Sterman:

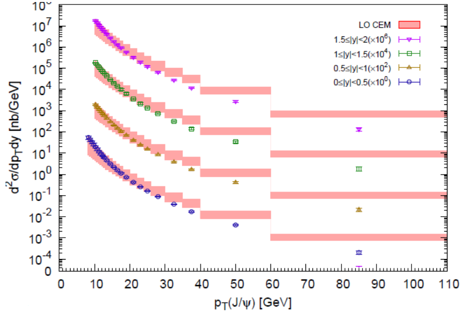
$$d\sigma_{\text{NLP}} = \sum_{\kappa} D_{[Q\bar{Q}(\kappa)]\to H}(z;\mu) \otimes d\hat{\sigma}_{[Q\bar{Q}(\kappa)]}$$

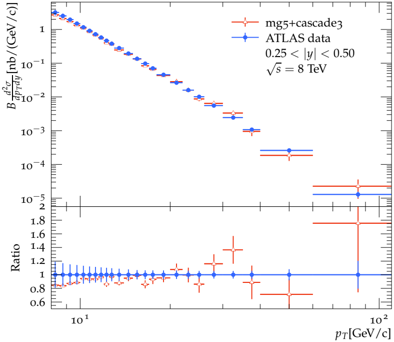
LDME sets not always compatible: but some are extracted at small P_t, other at large P_t.

Apply the FF idea to the color evaporation model (CEM).

J/ψ production with a scale-dependent CEM

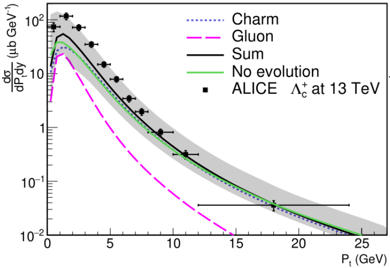
- A 30-year-old issue with the CEM at NLO: solved by evolution.
- k_t factorization $\Rightarrow$ description of low P_t data.
- Madgraph5 + parton shower + CEM

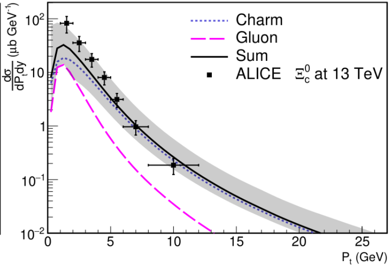




No free parameters: a pure pQCD mechanism

Heavy-baryon Enhancement





Can be explained by, e.g., the coalescence model.

Search for a perturbative mechanism: motivated by claims that the effect cannot be explained by fragmentation.

Mechanism inspired by *Braaten, Jia, Mehen, Phys.Rev.D 66 (2002) 034003*: analogy with quarkonia.

$$d\sigma_{\Lambda_c} = D_{[cud]\to\Lambda_c} \otimes d\hat{\sigma}_{[cud]}$$

Start with a CEM-inspired model.

Conclusion

- The relations between different small-x formalisms are not yet fully understood, but this is a work in progress.
- Future exp like the EIC have an important part of their physics program dedicated to small-x.
- Play an important role in our understanding of the QGP.
- A more factorization-oriented point of view may help improve calculations and modeling
  - VFNS implementation of the k_t factorization.
  - Work by Sterman et al. on quarkonia.
  - CEM at NLO.
  - Heavy baryon production?

Backup slides

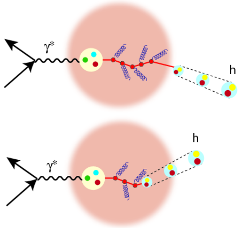
SIDIS on nuclei

Semi-inclusive DIS: $l + A \rightarrow l' + h + X$.

Comparison of in-vacuum and in-medium observables provide information on hadronization and related mechanisms.

A central quantity is the production length Lp.

$Lp =$ length/time necessary for the parton to turn into a color singlet dipole (prehadron).



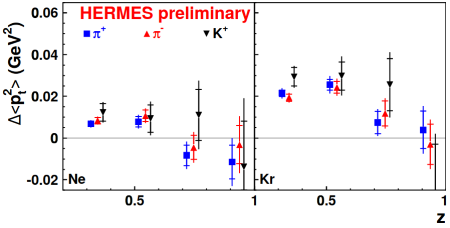
Induced energy loss (IEL) and nuclear absorption depend on Lp.

Observables

p_t **broadening:**

$$\Delta P_t^2 = \langle P_t^2 \rangle_A - \langle P_t^2 \rangle_D$$

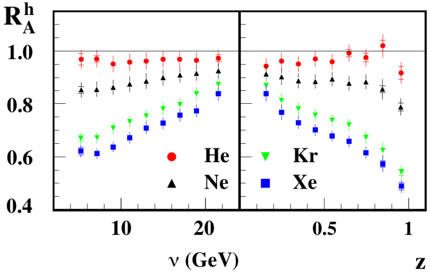
$$\langle P_t^2 \rangle_D \simeq 0.3 \text{ GeV}^2$$



Multiplicity ratio

$$R_A^h(z,Q^2,\nu) = \frac{\frac{1}{N_A^e}\frac{dN_A^h}{d\nu dz dQ^2}}{\frac{1}{N_D^e}\frac{dN_D^h}{d\nu dz dQ^2}}$$

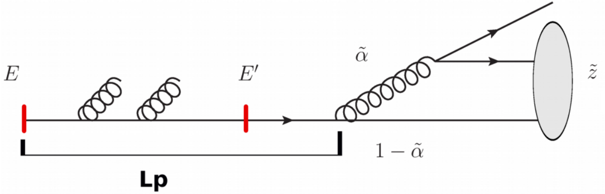
$$R_A^h(z,Q^2,\nu) \simeq \frac{D_A^h(z,Q^2,\nu)}{D^h(z,Q^2,\nu)}$$



DIS variables: Q^2, ν, x. Hadronic variables: P_t, $z = \frac{E_h}{\nu}$.

Our model *B.G and B.Kopeliovich, Phys.Rev.C 102 (2020) 4, 045201*

Improved Berger's model (for the leading quark)



$Lp \equiv t_r = \frac{(1-\tilde{z})zE}{p_t^2}$, with t_r the gluon radition time and $\tilde{z} = \frac{z}{1-\Delta E/E}$.

$(1-\tilde{z})E$ expected from energy conservation and boost.

ΔE includes both vacuum and IEL.

Initial virtuality Q^2. *The hypothesis of nerly on-shell parton overestimates the amount of IEL.*

$$\frac{\partial D}{\partial Lp}(z, E, Q^2, Lp) \propto \frac{1-\tilde{z}}{4zE}$$

Pt-broadening

$$\Delta \left\langle p_t^2 \right\rangle (s, z_1, b, Lp) = 2C(s) \int_{z_1}^{z_1+Lp} \rho(l,b) dl$$

$$\Delta \left\langle p_t^2 \right\rangle_h (z, Q^2, E) = \tilde{z}^2 \frac{2C}{A} \int d^2 b \int dz_1 \rho(z_1, b) \int_{L_{min}}^{L_{max}} dLp \frac{1}{N_L} \frac{\partial D}{\partial Lp} T_A(b, z_1, Lp)$$

- Absolute value and shape $(1-z)$ of Lp confirmed by the exp.
- Generally, IEL based models cannot reproduce this obs. : Use $Lp \to \infty$.

Multiplicity ratio

$$R_A^h(z, Q^2, \nu) \simeq \frac{D^A(z,Q^2,\nu)}{D^h(z,Q^2,\nu)}$$

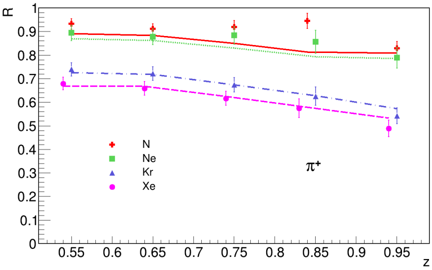
$$D^A(z, Q^2, E) \propto \frac{1}{A} \int d^2b \int dz_l \, \rho(b, z_l) \int_{Lp_{min}}^{Lp_{max}} dLp \frac{\partial D}{\partial Lp}(\tilde{z}) Tr(z, Q^2, E, b, z_l + Lp, \infty).$$

- IEL taken into account by $\tilde{z}$.
- Two free parameters for the initial dipole size at Lp (effectively, just one).

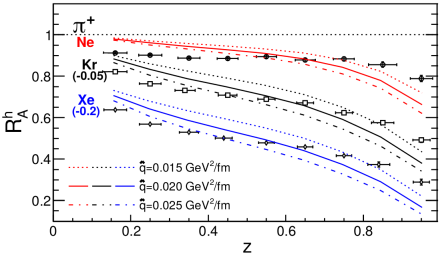
Multiplicity ratio

$z \to 1 \Rightarrow Lp \to 0$: No IEL, nucl. absorption maximum.

IEL contribution at $z \simeq 0.5$: about 7-25 %.



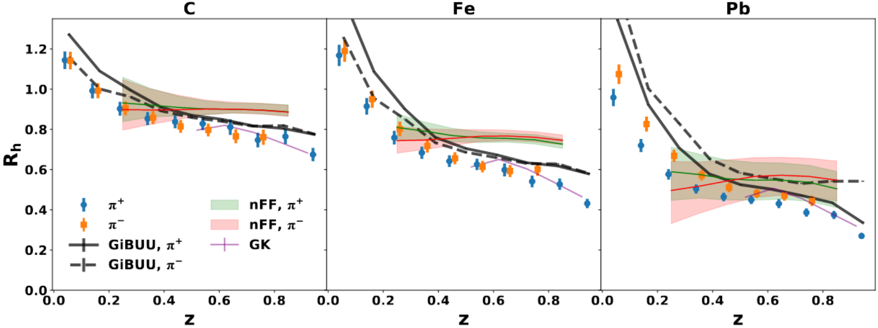
However, also possible to reproduce app. the data just with induced energy loss *(i.e. Chang, Deng, Wang; Phys.Rev.C 89 (2014) 3)*



Fail to reproduce the z-dependence of the pt-broadening.

CLAS multiplicity ratio

Predictions for CLAS data shown in *CLAS Collaboration, Phys.Rev.C 105 (2022) 1, 015201.*



The pt-broadening puzzle

- Work with Esteban Molina (Master student) and Hayk Hakobyan (Prof., CLAS).
- Strong assumption revisited: What if $\langle k_t^2 \rangle_D \neq \langle k_t^2 \rangle_A$?
$$\Delta P_t^2 = z_h^2 \left(\langle k_t^2 \rangle_A - \langle k_t^2 \rangle_D + 2C\rho L p \right)$$
- $\langle k_t^2 \rangle$ extracted in analysis using TMD PFDs $f(x, k_t; \mu) = \frac{e^{-k_t^2/\langle k_t^2 \rangle}}{\langle k_t^2 \rangle} f(x, \mu)$.
- Note that $\langle k_t^2 \rangle_D \sim 5 - 10 \times \Delta P_t^2$.