# Feynman-Diagram Gauge and the Unitary Gauge

Based on: Higgs production in association with a Z boson at TeV-scale lepton colliders, arXiv:260401686

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# Outlines

- Motivation
- The problem in the unitary gauge
- Basic idea of the FD gauge
- Analysis setup and diagram groups
- Energy dependence
- Summary

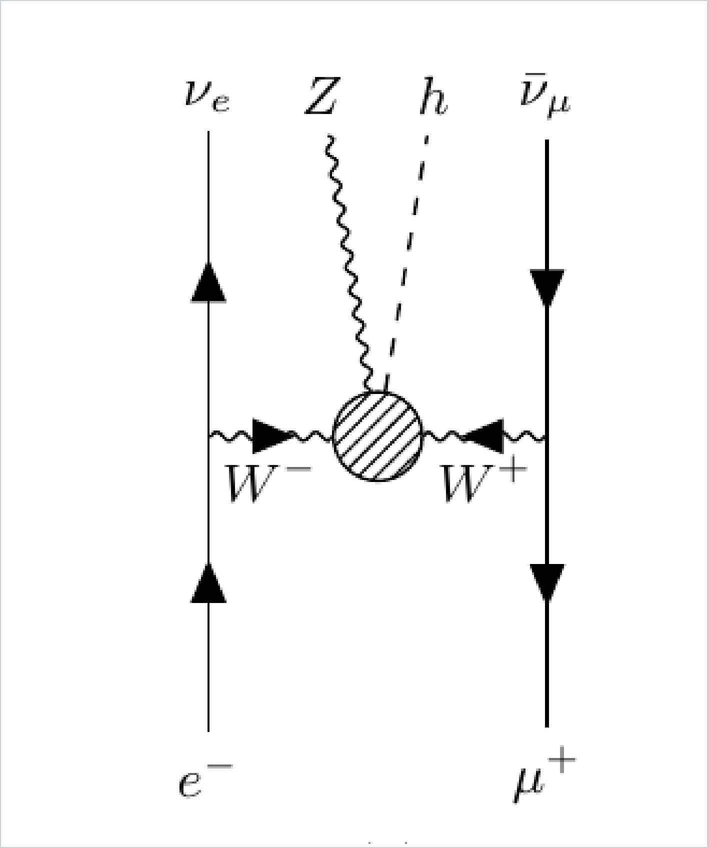
# Motivation and target process

$$\ell^-\ell^+ \to \nu\bar{\nu}Zh$$

- At multi-TeV energies, processes with t-channel weak boson exchange is important.

- **The initial leptons emit W bosons. The W bosons produce Zh → Vector Boson Scattering (VBS)**

- The longitudinally-polarized ($\lambda = 0$) Z boson is dominantly produced in high energies.




Representative VBS topology

**Main question: how can we calculate ~~and interpret~~ the longitudinal contribution at high energy?**

Source: arXiv:2604.01686v2, Fig. 6(a)

# Why unitary gauge becomes difficult

### Massive vector propagator

$$G_{\mathrm{U}}^{\mu\nu}(q) = \frac{i}{q^2 - m^2 + i\epsilon}\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m^2}\right)$$

### Longitudinal polarization

$$\epsilon_L^\mu \sim \frac{q^\mu}{m} \sim \frac{E}{m}$$

1. Individual diagram contributions can grow strongly with energy.
2. The large terms cancel only after all relevant amplitudes are summed.
3. The final answer is correct, but numerical integration and diagram-by-diagram interpretation become difficult.

# FD gauge: vector and Goldstone together

### Light-cone gauge vector

$$n^\mu = (\mathrm{sgn}(q^0), -\vec{q}/|\vec{q}|) \, . \tag{3}$$

### Five-component propagator

$$G_{MN}(q) = \frac{i}{q^2 - m^2 + i\epsilon} \begin{pmatrix} -g_{\mu\nu} + \frac{q_\mu n_\nu + n_\mu q_\nu}{n \cdot q} & i \frac{m\, n_\mu}{n \cdot q} \\ -i \frac{m\, n_\nu}{n \cdot q} & 1 \end{pmatrix} \tag{4}$$

M,N = 0,…,4: four vector components + one Goldstone component

## How to read Eqs. (3) and (4)

- Choose a light-cone vector n opposite to the boson three-momentum.
- Combine the four-vector Vμ and the Goldstone boson πV.

**5-component field:** $V^M = (V^\mu, \pi_V)$

- The propagator is a 5 × 5 matrix. Its mixed entries connect vector and Goldstone components.

# FD gauge: reduced longitudinal polarization

### Five-component polarization

$$\epsilon^M(q,\pm) = (\epsilon^\mu(q,\pm),\, 0)\ , \qquad (5)$$
$$\epsilon^M(q,0) = (\tilde{\epsilon}^\mu(q,0),\, i)\ , \qquad (6)$$

### Reduced longitudinal vector

$$\tilde{\epsilon}^\mu(q,0) = \epsilon^\mu(q,0) - \frac{q^\mu}{Q} = -\mathrm{sgn}(q^2)\frac{Q\, n^\mu}{n\cdot q}\ , \qquad (7)$$

$$Q = \sqrt{|q^2|}$$

| Representation | High-energy size |
| --- | --- |
| **Ordinary vector εL** | E / m → large |
| **Reduced vector ε̃L** | m / E → small |
| **Goldstone component** | Carries the leading longitudinal behavior |

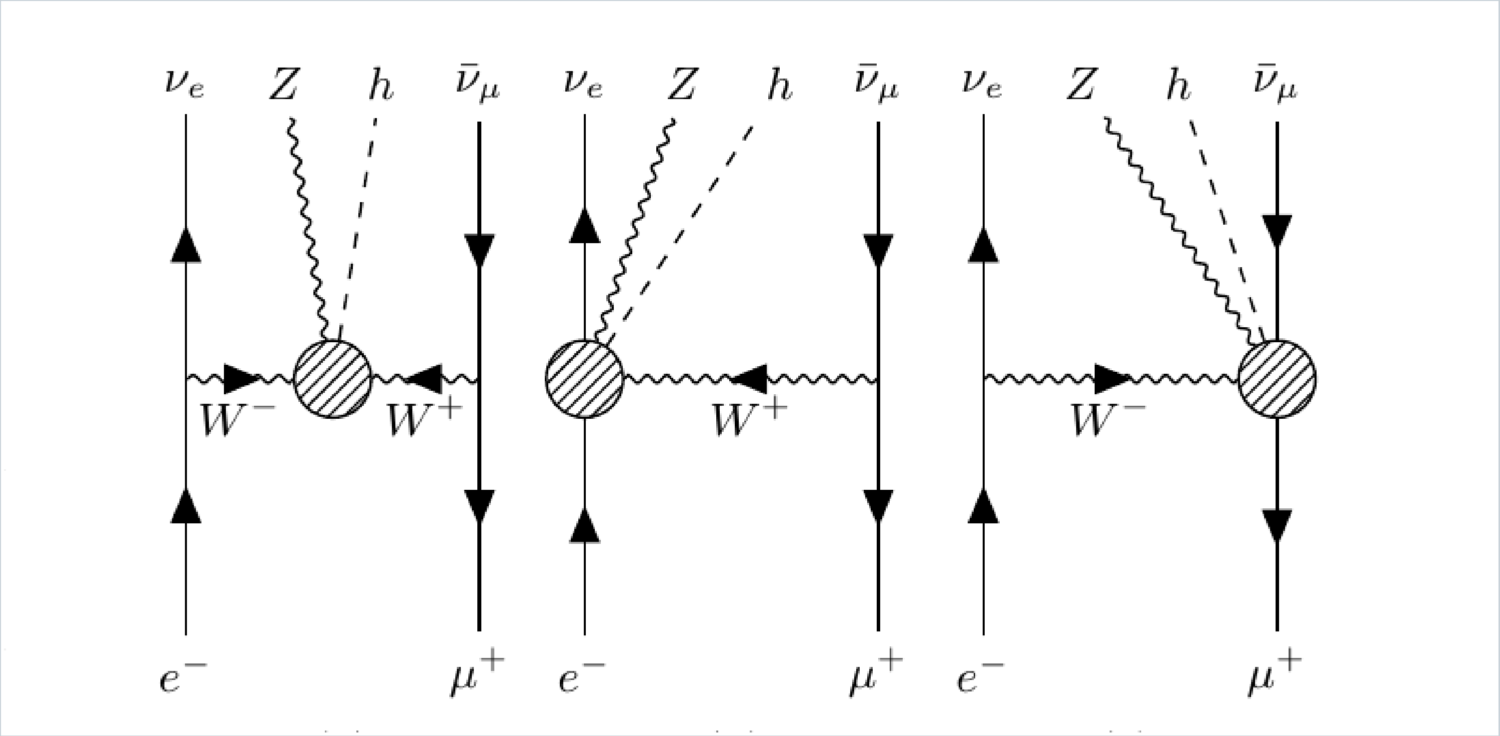
**Result: partial contributions no longer need artificial high-energy cancellations.**
**Caveat: individual diagrams are still gauge dependent.**

Source: arXiv:2604.01686v2, Eqs. (5)–(7)

# Analysis setup and topology groups

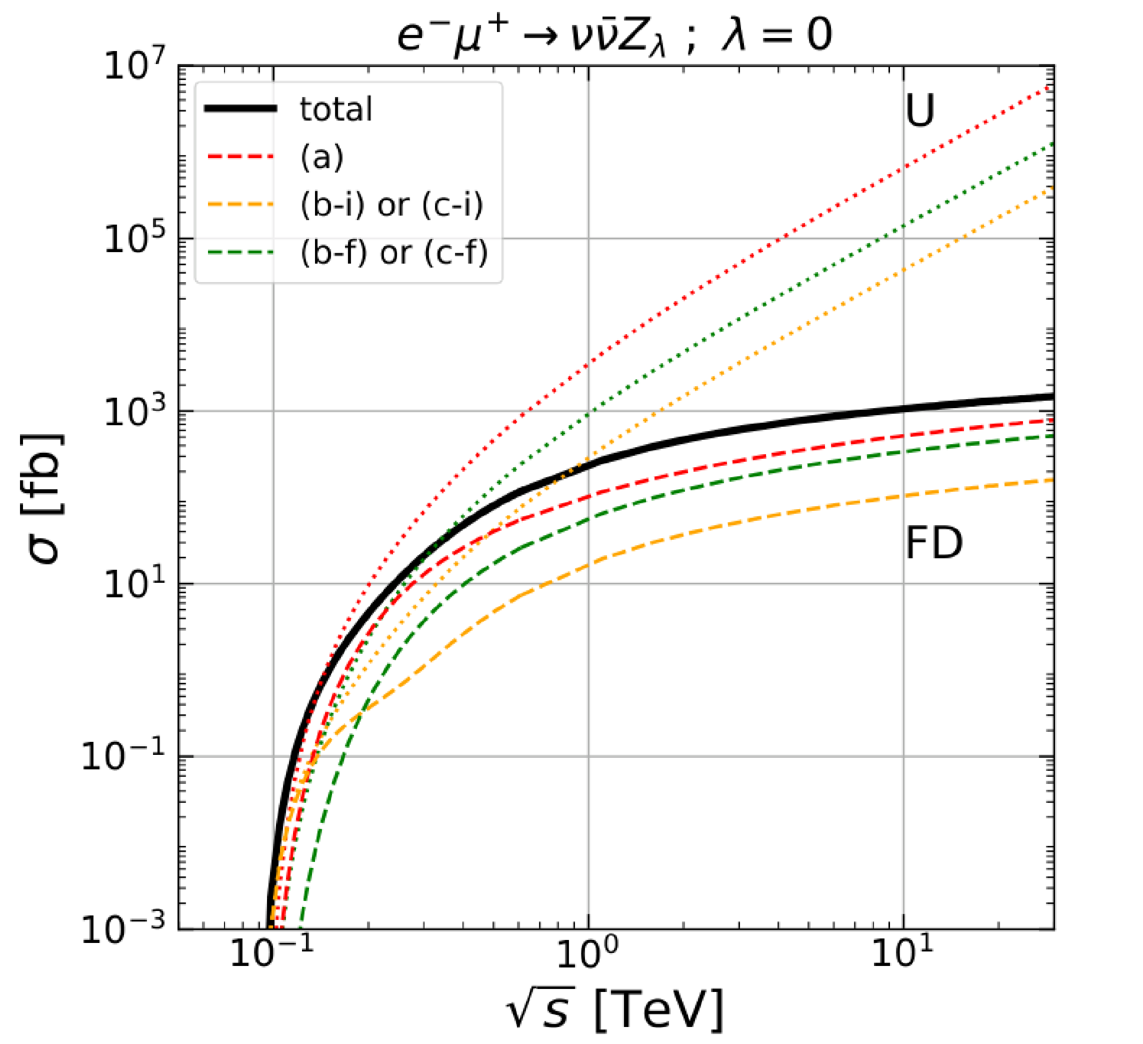
$$e^-\mu^+ \to \nu_e \bar{\nu}_\mu Z h$$

- Different initial flavors remove s-channel annihilation
- Longitudinal Z: λ = 0
- MadGraph with FD-gauge implementation
- Tree-level Standard Model




| (a) VBS | (b) $e^-W^+$ scattering | (c) $W^-\mu^+$ scattering |
|---|---|---|

# Figure 10: energy dependence and gauge cancellation




## What the curves show

| | |
|---|---|
| **Black solid** | Physical total; identical in both gauges |
| **Dotted** | **Unitary-gauge groups grow approximately as $(\sqrt{s})^4$** |
| **Dashed** | **FD-gauge groups remain near the physical scale** |

**The physical hierarchy is visible in FD gauge: Zh emission is about two orders of magnitude smaller.**

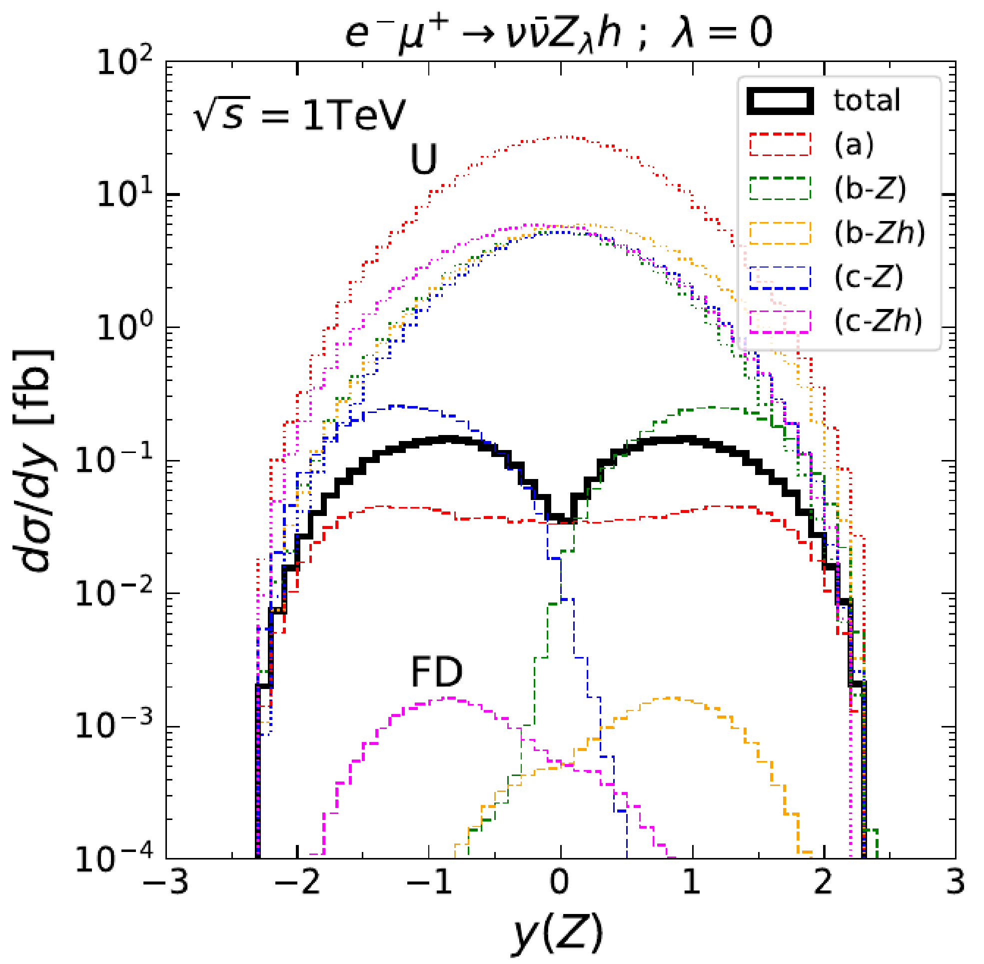
# Summary: same physics, clearer intermediate picture

| Point | Unitary gauge | FD gauge |
|---|---|---|
| **Physical total** | Correct and gauge invariant | The same correct result |
| **Partial terms** | **Very large; strong cancellations** | **Well behaved at high energy** |
| **Interpretation** | Topology is obscured | Topology and kinematics are clearly related |

**FD gauge is useful for numerical stability ~~and physical interpretation~~.**
**Individual diagrams are still gauge dependent; only the complete result is observable.**

Thank you.

# Figure 11: rapidity and topology




## Rapidity convention

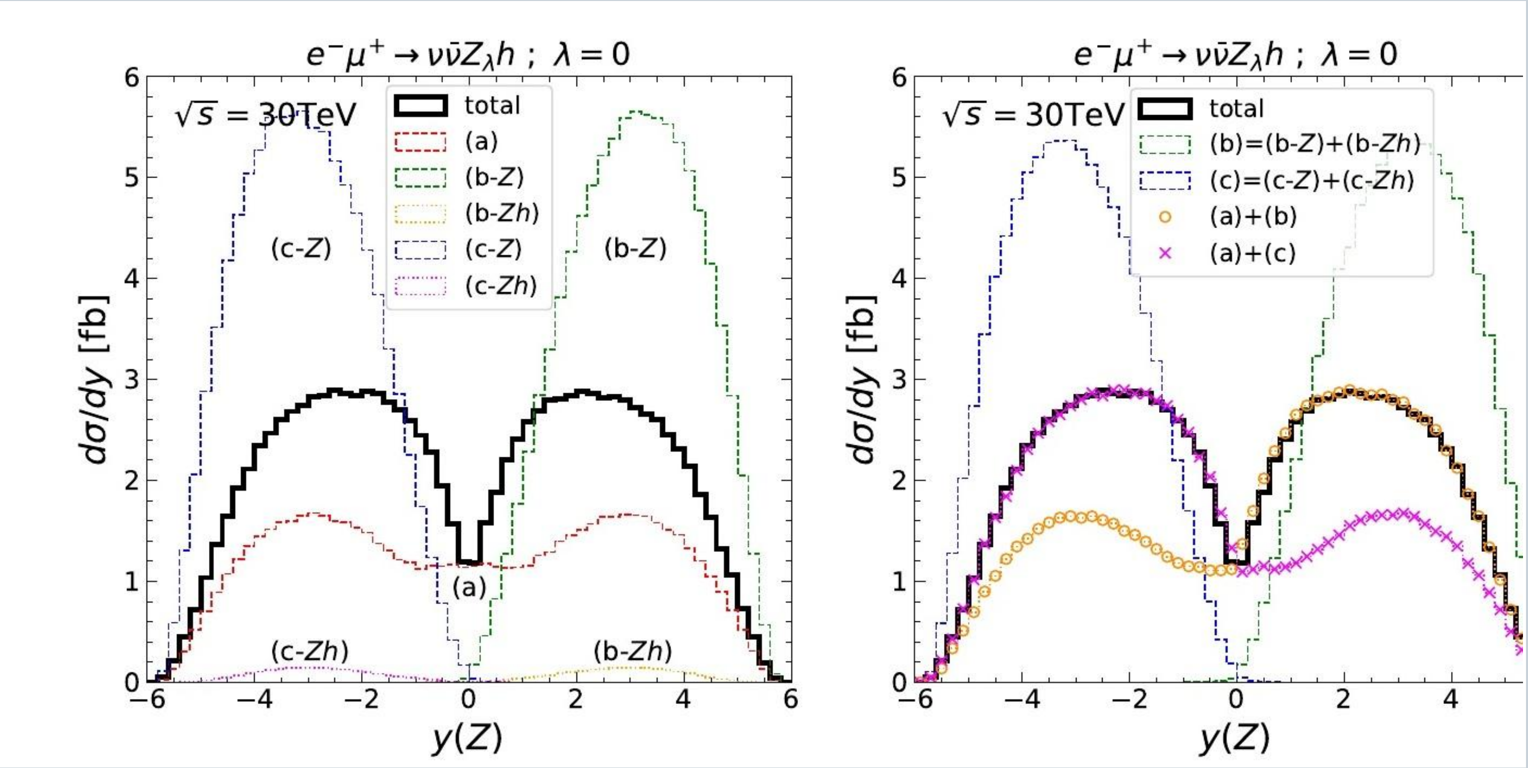
y > 0: electron direction | y < 0: muon direction

| | |
|---|---|
| **(a) VBS** | approximately symmetric |
| **(b) e side** | mainly at positive y |
| **(c) μ side** | mainly at negative y |

**Unitary-gauge partial curves are much larger and do not explain the physical shape.**

Source: arXiv:2604.01686v2, Fig. 11

# Figure 12: physical interference in FD gauge




## At √s = 30 TeV

- Initial- and final-state Z emission interfere constructively within (b) or (c).
- **These groups interfere destructively with VBS.**
- (a)+(b) works in the forward region; (a)+(c) works in the backward region.

**This is physical interference between well-behaved amplitudes.**
**It is different from the artificial high-energy cancellation seen in the unitary gauge.**

Source: arXiv:2604.01686v2, Fig. 12

# Is the unitary gauge wrong?

**No.**

**The complete physical result is correct.**

## What is the problem?

- The propagator and longitudinal polarization contain factors that grow with energy.
- Individual contributions become much larger than the physical total.
- Large gauge cancellations make numerical calculation and interpretation difficult.

| | | | |
|---|---|---|---|
| **Final result** | Correct and gauge invariant | **Intermediate picture** | **Correct but inefficient** |

Related discussion: arXiv:2604.01686v2, Secs. II and IV

# Is an individual FD-gauge diagram physical?

**No.**

**It is still gauge dependent.**

## Then why is it useful?

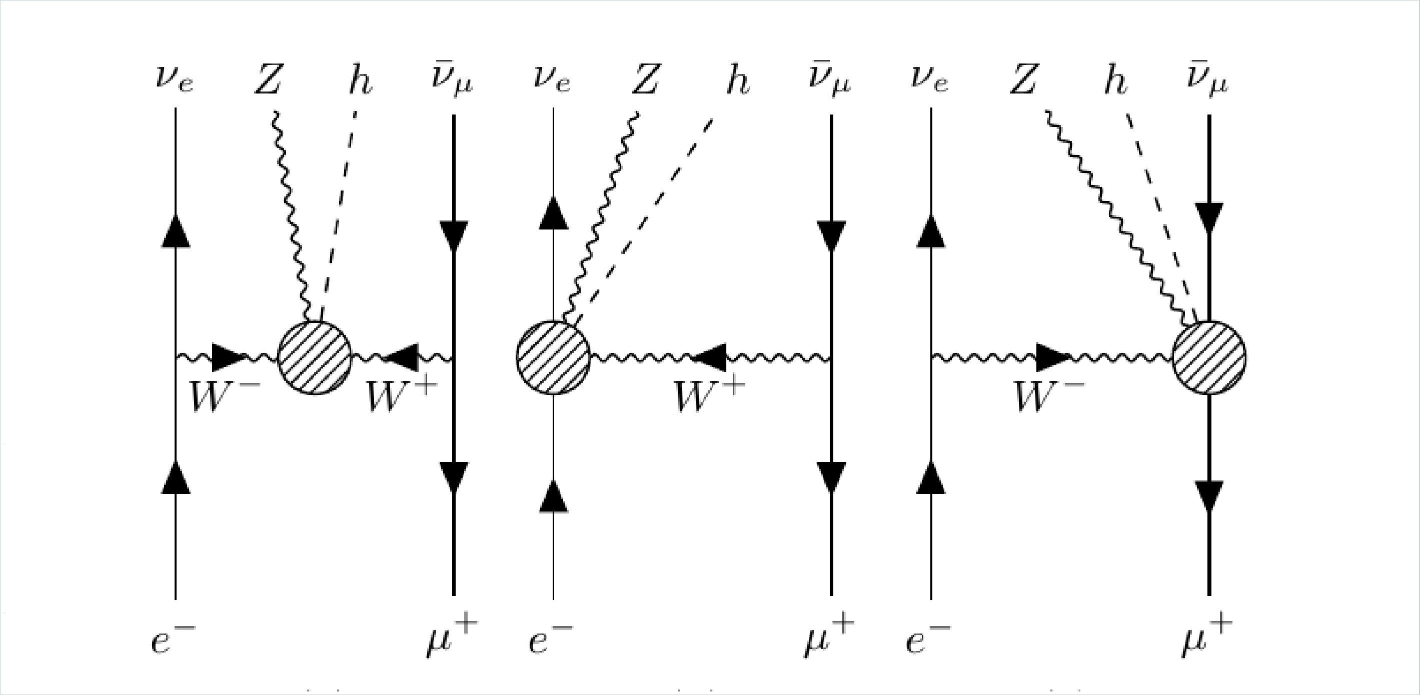
- FD-gauge contributions stay well behaved at high energy.
- Their rapidity shapes often follow the corresponding diagram topology.
- They provide a useful interpretation, but they are not observables.

| | | | |
|---|---|---|---|
| **Partial diagram or group** | **Gauge dependent** | **Complete cross section** | **Observable** |

# Why use e⁻μ⁺ instead of e⁻e⁺?

**It isolates the t-channel W-exchange structure.**

- Different initial flavors remove s-channel annihilation diagrams.
- Only the t-channel W-exchange topologies remain.
- The comparison of groups (a), (b), and (c) becomes simpler.




| | | |
|---|---|---|
| **(a) VBS** | **(b) electron side** | **(c) muon side** |

# Why focus on a longitudinal Z boson?

**The high-energy gauge-cancellation problem is most visible for $\lambda = 0$.**

| Polarization | High-energy behavior | Why it matters |
|---|---|---|
| **Longitudinal: $\lambda = 0$** | **$\varepsilon_L \sim E / m$ in the ordinary vector representation** | **Large cancellations and a direct link to the Goldstone mode** |
| **Transverse: $\lambda = \pm 1$** | **No $E / m$ enhancement** | The gauge-cancellation problem is less severe |

Related discussion: arXiv:2604.01686v2, Secs. II and IV

# What does the Goldstone component do?

### Five-component longitudinal state

$$\epsilon^M(q,\pm) = (\epsilon^\mu(q,\pm), 0) \ , \tag{5}$$
$$\epsilon^M(q,0) = (\tilde{\epsilon}^\mu(q,0), i) \ , \tag{6}$$

### Reduced vector part

$$\tilde{\epsilon}^\mu(q,0) = \epsilon^\mu(q,0) - \frac{q^\mu}{Q} = -\mathrm{sgn}(q^2)\frac{Q\, n^\mu}{n\cdot q} \ , \tag{7}$$

$$\varepsilon^M(q,0) = (\,\tilde{\varepsilon}\mu(q,0), i\,)$$

## High-energy interpretation

- The reduced vector component behaves as m/E and becomes small.
- **The Goldstone component carries the leading longitudinal behavior.**
- This is consistent with the Goldstone-boson equivalence theorem.

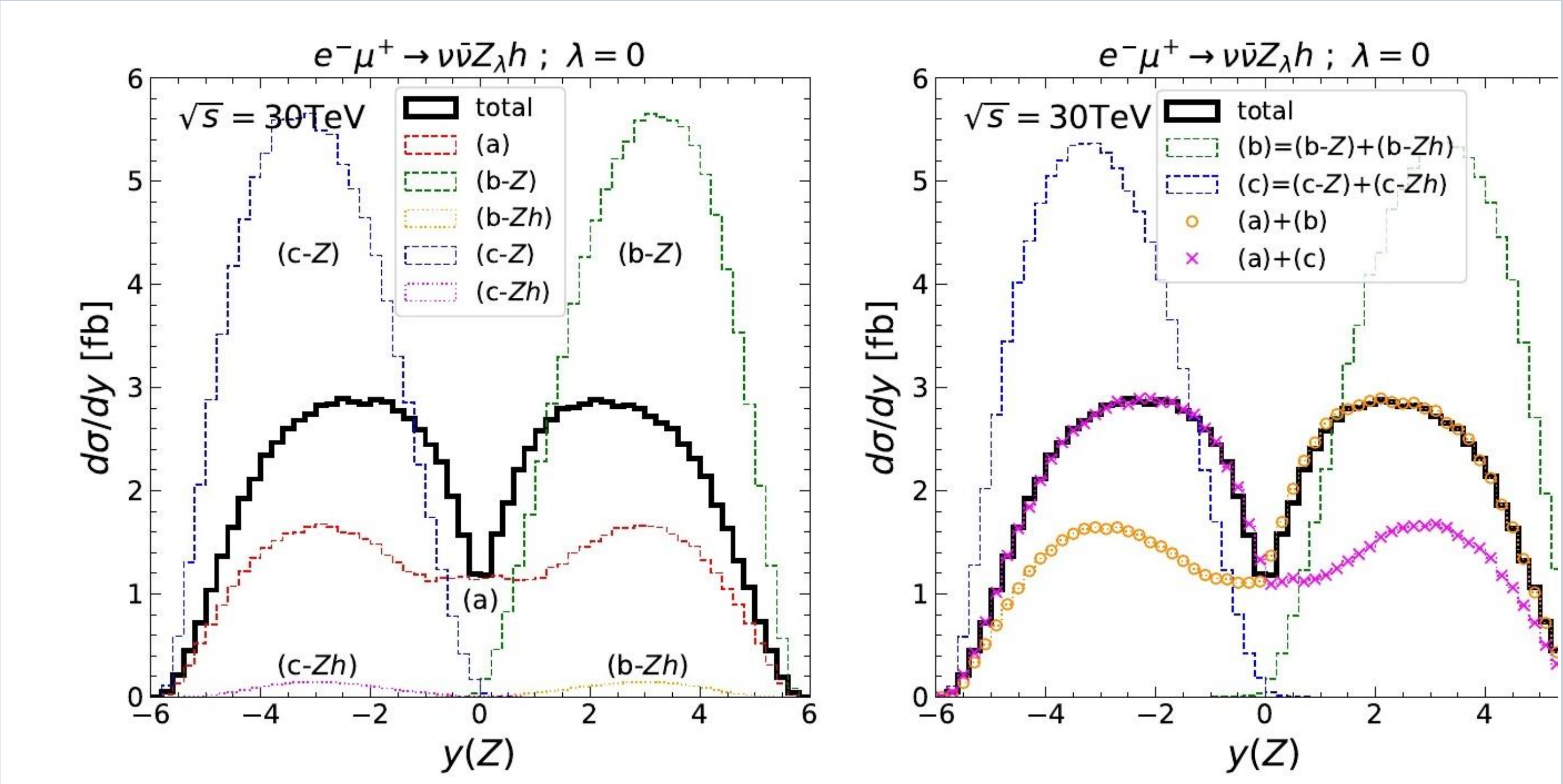
**Leading behavior: vector → Goldstone**

# Does FD gauge remove all interference?

## No.

**Physical interference remains.**

- Constructive interference remains within the electron-side or muon-side group.
- **Destructive interference remains between these groups and VBS.**




| | | | |
|---|---|---|---|
| Artificial gauge cancellation | Strongly reduced | Physical interference | Remains |

Source: arXiv:2604.01686v2, Fig. 12