DM4: Direct detection of dark matter

The baseline calculation for dark matter direct detection is quite simple (i.e. non-relativistic classical $2 \to 2$ elastic scattering), so we will work through it on this problem set.

Suppose we have a Standard Model target of mass m_T at rest in the laboratory frame, and an incident dark matter particle of mass m_{DM} with velocity vector $\vec{v}$. You may assume the dark matter particle is non-relativistic, as the typical velocity of dark matter particles at our location in the Milky Way is around $10^{-3}c$, and the escape velocity from the Milky Way is only a factor of two or so higher. You may treat both particles as classical point particles, ignoring quantum-mechanical effects. (Accounting for the finite size of the nucleus introduces a form factor in the spectrum – ask me if you want more information about this.) You may assume that the scattering is elastic (e.g. we are not exciting internal states in either the target or the dark matter, although if this assumption is violated it can lead to interesting signals).

(a) Prove that the recoil energy of the target particle is given by:

$$E_R = \frac{2\mu^2 v^2 \cos^2\theta}{m_T}, \quad \mu = \frac{m_T m_{\text{DM}}}{m_T + m_{\text{DM}}}, \tag{1}$$

where the scattering angle θ is defined as the angle between the initial velocity vector $\vec{v}$ and the momentum of the target particle after the scattering.

(b) If the target is an atomic nucleus, its typical mass will be 10-100 GeV (depending on the element). Suppose we take $m_T = 10$ GeV. Estimate the scale of E_R (and how it scales with m_{DM}) for the two cases of (i) $m_{\text{DM}} \gg m_T$, and (ii) $m_{\text{DM}} \ll m_T$ (for a concrete number you can consider $m_{\text{DM}} = 100$ MeV), taking $v_{\text{rel}} = 10^{-3}c$.

(c) Suppose we fix the recoil energy E_R. What is the minimum velocity for DM particles that can contribute to the scattering signal at this recoil energy ($v_{\min}(E_R)$), as a function of m_T, E_R and μ?

For a concrete example, suppose a candidate DM event is detected in a silicon detector with recoil energy 12.3 keV (you may round all atomic masses to the nearest integer), and interpreted as the scattering of a 10 GeV WIMP. What is the smallest possible speed of the

DM particle in question, in the lab frame?

(d) Because higher recoil energies correspond to higher minimum DM velocities, they sample a smaller fraction of the velocity distribution of dark matter, and consequently we expect the number of scattering events to fall monotonically as a function of recoil energy. Quantitatively, the differential event rate $\frac{dR}{dE_R} \propto \int_{v_{\min}(E_R)} \frac{dv}{v} f(v)$, where $f(v)$ describes the velocity distribution of DM particles and v_0 is a characteristic velocity scale. In the "standard halo model", $f(v) = \frac{4}{\sqrt{\pi}} \frac{v^2}{v_0^3} e^{-v^2/v_0^2}$ (note: for simplicity, we are here ignoring the difference between the lab frame and the Galactic frame in which the DM velocity distribution is approximately isotropic, in reality this will introduce corrections).

Determine how the spectral shape dR/dE_R depends on E_R in the case of the standard halo model.