DM3: Thermal Freezeout for Asymmetric Dark Matter

Suppose that the dark matter is a Dirac fermion, and there is an asymmetry in the dark matter sector analogous to that in the baryon sector, so the abundance of dark matter and anti-dark-matter is different. In this case, either the depletion of the less abundant component (as for the baryons) *or* the expansion of the universe (as for conventional WIMP dark matter) can cut off the annihilation processes that deplete the dark matter density. This idea is appealing because of the apparent coincidence between the amounts of dark and baryonic matter in the universe – if their abundances are set by entirely different processes, why are they only different by a factor of five or so?

In this problem we will work out the freeze-out of annihilations for this scenario, and see how it differs from the symmetric case discussed in lecture.

(a) Let us denote the DM by χ^+ and the anti-DM by χ^-. Without loss of generality, we will assume χ^+ is more abundant than χ^-. Let the average $\chi^+\chi^-$ annihilation cross section be given by $\langle\sigma v\rangle$. The coupled Boltzmann equations for this system are:

$$\frac{dn^\pm}{dt} + 3Hn^\pm = -\langle\sigma v\rangle(n^+ n^- - n^+_{\rm eq} n^-_{\rm eq}).$$

Here n describes the number density of the two species and the subscript "eq" refers to an equilibrium value. (We are assuming here that the annihilation reaction is the only number-changing reaction for either DM or anti-DM.)

Describe the meaning of each of the terms in these equations, and justify why these are the correct equations (qualitatively, in words), including any factors of 2.

(b) Let us define $Y^\pm = n^\pm/s$, where s is the entropy density of the universe. You may assume that entropy is conserved, so $s \propto a^{-3}$. Let $\eta = Y^+ - Y^-$. Show that η is conserved by the Boltzmann equations from (a), and explain the physical reason for this behavior.

(c) Define the fractional asymmetry $r = n^-/n^+$. Note by definition $0 \leq r \leq 1$. What is the annihilation rate in terms of the overall DM density $n = n^+ + n^-$ and r? Comment on the limits $r = 1$ and $r = 0$.

(d) Assume the universe is radiation dominated so $H(T) \propto T^2$ and $T \propto 1/a$ (i.e. you may assume no significant entropy injection over the time period of greatest interest). Define $r_{\rm eq} = n^-_{\rm eq}/n^+_{\rm eq}$, and $x = m/T$. Show that the Boltzmann equations yield a dynamical equation for r of the form:

$$\frac{dr}{dx} \propto -\eta\langle\sigma v\rangle x^{-2}\left[r - r_{\rm eq}\left(\frac{1-r}{1-r_{\rm eq}}\right)^2\right] \propto -\eta\langle\sigma v\rangle x^{-2}\left[r - \frac{Y_{\rm eq}^{+}Y_{\rm eq}^{-}}{\eta^2}\left(1-r\right)^2\right].$$

(e) As the DM freezes out, the $Y_{\rm eq}$ terms in the above equation will be exponentially suppressed (by the usual Boltzmann suppression). In the limit where this term approaches zero, solve the resulting differential equation for the late-time value of r. How does it depend on the annihilation cross section? Comment.