DM 2: Comparing DM models to cosmological constraints

There are many variations of dark matter – spanning wave-like dark matter, primordial black holes, and thermal particle dark matter – that could have implications for how dark matter is distributed on specific scales. A common situation is that modes that are "inside the horizon" (roughly speaking, with wavelengths smaller than the inverse Hubble scale H^{-1}) at a particular epoch are affected, while modes that are "outside the horizon" are not.

(a) Assuming a homogeneous universe, estimate the total dark matter mass enclosed inside the radius cH^{-1} when the universe had a temperature of (i) 1 MeV (corresponding to BBN), (ii) 1 keV, (iii) 1 eV (corresponding to matter-radiation equality), and (iv) 10^{-3} eV (close to the present day). This gives a mass scale associated with the modes with wavelength comparable to that radius, and a sense of the largest dark matter structures that could be significantly modified by new physics at that epoch. Give your answers in units of the mass of the Sun.

Only order-of-magnitude estimates are needed for this problem. You may assume a spatially flat universe, and the Planck measurement of the dark matter density (and you may assume all these epochs are after the initial production of the DM abundance). In each epoch, to obtain H, you may make the approximation that the energy content of the universe is 100% radiation during radiation domination, 100% matter during matter domination, etc. You may use the approximation that $H \sim T^2/m_{\text{Planck}}$ during radiation domination (this follows directly from $\rho \propto T^4$, Newton's constant $G \propto 1/m_{\text{Planck}}^2$, and $H^2 = 3\pi G\rho/3$).

(b) For each of the cases above, how light would the DM have to be for its Compton wavelength to be larger than the (physical) horizon scale at that epoch? Again, only order-of-magnitude estimates are needed.

(c) For primordial black holes in the mass range from $10^{17} - 10^{23}$ grams (where as we will discuss, it is observationally allowed for primordial black holes to make up all the dark matter), if all the mass within a horizon / Hubble radius collapsed to form the primordial black hole, roughly what would the universe's temperature be when these modes entered the horizon?

(d) There could be a self-interaction between dark matter particles, which could become important in the dense cores of dark matter halos at late times. Consider a galaxy cluster with typical DM velocity dispersion ~ 1000 km/s. At the virial radius where the dark matter density is roughly 200 times the cosmological value, what must the DM-DM scattering cross section σ be such that the scattering time for a DM particle is less than 10^{10} years? (Your answer will depend on the DM mass.)

For a 100 GeV particle DM candidate, compare this cross section to a Standard Model cross section of your choice (e.g. the Thomson cross section or the cross section for particles scattering through the weak interaction). What do you notice? What if the DM mass is instead 10^{-6} eV?

(e) (extension problem) Suppose instead the dark matter is a particle possessing a non-gravitational interaction, via a new light $U(1)$ gauge boson ϕ (much lighter than the dark matter) which couples both to the dark matter (with coupling g_D) and to electrons (with coupling g_V). At temperatures much less than the mass of the ϕ, but much greater than the electron mass, the dark matter scatters off the thermal bath of relativistic electrons with a scattering cross section of order,

$$\langle \sigma v_{\mathrm{rel}} \rangle \sim \alpha_D \alpha_V T^2 / m_\phi^4 .$$

Kinetic decoupling occurs when this interaction is no longer fast enough to keep the dark matter at the same temperature as the electrons. Suppose that keeping the dark matter heated to the same temperature as the electrons requires m_{DM}/T scatterings to occur in a Hubble time.

Estimate the temperature of the universe when the dark matter kinetically decouples as a function of m_ϕ, α_D and α_V. Give the numerical value of the decoupling temperature when the dark matter mass is 1 TeV and $\alpha_D = 1/30$, for the two cases where (i) $\alpha_V = 1/30$, $m_\phi = m_Z$, and (ii) $\alpha_V = 10^{-6} \times 1/30$, $m_\phi = 30$ MeV (the first case corresponds to the standard WIMP calculation, the second is an example of dark matter in a hidden sector with a light force carrier). Following your approach in (a), what masses would you expect to be contained within the horizon at these temperatures?

You may neglect $\mathcal{O}(1)$ factors in your calculation, and you may assume kinetic decoupling occurs during radiation domination and while the electrons and positrons are relativistic.

Note that this calculation only holds when $T \gtrsim m_e$, which implies radiation domination – once the electrons become non-relativistic, unless the dark matter has a large coupling to the neutrinos, the rate becomes very small due to a lack of targets and decoupling is usually immediate.