Lecture 1 Notes (VSOP 2026): Essentials of Cosmology for Dark Matter

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I. THE EXPANSION OF SPACETIME

A. The FLRW metric

General relativity provides the framework for modern cosmology. The metric of spacetime determines how light and particles propagate, while the matter and energy content determine the metric and its evolution through Einstein's equations.

On large scales, we approximate the Universe as spatially isotropic and homogeneous, evolving in time. We typically assume the matter content can be well approximated as a perfect fluid, completely characterized by its pressure P and energy density ρ.

Under these assumptions, Einstein's equations of general relativity admit the Friedmann–Lemaître–Robertson–Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]. \tag{1}$$

Here the parameter k describes the *spatial* curvature of the cosmos (not spacetime curvature): $k = 1, -1, 0$ corresponds to closed, open, or flat spatial sections respectively. Einstein's equations then imply the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{2}$$

It is typical to define the Hubble parameter as:

$$H \equiv \frac{1}{a}\frac{da}{dt} = \frac{\dot{a}}{a}. \tag{3}$$

B. Comoving coordinates, observers, and lengths/volumes

"Comoving observers" are observers at fixed (r, θ, ϕ) coordinates, and we refer to this coordinate system as "comoving coordinates". Since the FLRW metric has no preferred spatial direction, particles initially at rest in comoving coordinates remain there, in the absence of external forces. To a first approximation, we can often treat galaxies, galaxy clusters, and even particles of non-relativistic matter as being at rest in comoving coordinates. Their physical separation changes with the scale factor $a(t)$ – in this sense they are moving together with the expansion of the universe, hence the term "co-moving" – but their comoving coordinate separation remains constant.

It is often then convenient to speak of a comoving distance between points or test particles at fixed comoving spatial coordinates, and express this distance in terms of the physical distance between the particles at the

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present time; essentially, the physical distance is the comoving distance rescaled by a/a_0, where a_0 is the scale factor at the present time. Earlier in the universe's history, this comoving distance (we may also use the terms "comoving separation" or "comoving length scale") would be the same, but the physical distance between the particles/points would be smaller.

Similarly, a "comoving volume" is a fixed region of r–θ–ϕ space. Its physical volume changes with $a(t)$, scaling as $a(t)^3$.

C. Redshifting of light and temperature

Light, and other particles traveling at relativistic speeds relative to the comoving coordinate system, of course cannot be treated as at-rest. As the universe expands, light loses energy as it propagates. Concretely, if a photon is emitted at time $t_{\rm em}$ and received at $t_{\rm rec}$, then

$$\frac{E_{\rm em}}{E_{\rm rec}} = \frac{a(t_{\rm rec})}{a(t_{\rm em})}. \tag{4}$$

Thus, in an expanding universe with $a(t_{\rm rec}) > a(t_{\rm em})$, the received energy is smaller and the light is redshifted. We typically define the *redshift factor*:

$$1 + z \equiv \frac{a_0}{a(t)} \tag{5}$$

The present day thus corresponds to $z = 0$, higher redshifts correspond to earlier times.

It will also often be useful to treat the temperature of the universe as a measure of its age. The universe is filled with a bath of photons; at earlier times the bath contained other relativistic particles, coupled tightly to the photons. Because the photons' energy redshifts as $1 + z$ with the expansion of the universe, the temperature of the bath redshifts in the same way. There are some exceptions to this rule – for example, when the electrons and positrons became non-relativistic, they largely annihilated into photons, and this heated up the photon bath – but unless there are many extra particle degrees of freedom in the early universe that we do not know about, we do not expect this effect to be large (and it is largely non-existent at temperatures much below ~ 1 MeV). So as a first approximation, we can estimate the temperature of the photon bath of the universe at some redshift $(1 + z)$ as just $(1 + z)T_0$, where $T_0 \approx 2.7$ K $\approx 2 \times 10^{-4}$ eV is the photon bath temperature today.

This does not hold true for the temperature of *non*-relativistic particles. We can show (using the geodesic equation of general relativity) that the 3-momentum $\vec{p}$ of a free non-relativistic particle redshifts as $(1 + z)$, and since the kinetic energy of such a particle is $|\vec{p}|^2/(2m)$, this means its temperature redshifts as $(1 + z)^2$. So non-relativistic particles become colder relative to the photon bath over time. The exception is if the particles have interactions (with the photon bath or with other species) that allow them to gain energy. In the Standard Model, the matter temperature tracks the temperature of the photon bath until $z \sim 200$, because before that point the Compton scattering between photons and charged electrons equalizes their temperatures.

II. THE COSMIC ENERGY BUDGET

It is convenient to characterize a perfect fluid by its equation-of-state parameter w, which relates energy density ρ to pressure P:

$$P = w\rho. \tag{6}$$

The standard cosmological model describes our universe as a combination of three such fluids:

$$\text{matter:} \quad w = 0, \tag{7}$$

$$\text{radiation:} \quad w = \frac{1}{3}, \tag{8}$$

$$\text{dark energy:} \quad w = -1. \tag{9}$$

The continuity equation obeyed by the stress-energy tensor (if you have not done general relativity, just think of this as a generalization of energy conservation) tells us how the energy density varies as the universe expands, as a function of the fluid's pressure:

$$\rho \propto a^{-3(1+w)}. \tag{10}$$

Substituting in the appropriate values of w gives:

$$\rho_{\rm m} \propto a^{-3}, \quad \rho_{\rm r} \propto a^{-4}, \quad \rho_\Lambda \propto a^0. \tag{11}$$

We see that matter just dilutes as we would expect; the total number (and mass) of particles in a comoving volume remains constant, but the volume of that region expands by a factor of a^3, so the energy density goes down by the same factor (here "matter" implies "non-relativistic" so almost all the energy is stored in mass). The extra factor of a^{-1} for radiation (relative to matter) reflects the redshifting of the energy per photon. Dark energy behaves in a qualitatively different way; as the universe expands, the density of dark energy stays constant, so doubling the volume of a comoving region also doubles the amount of energy contained therein. Often dark energy is attributed to an intrinsic "vacuum energy" (although trying to actually calculate this vacuum energy currently gives numbers many orders of magnitude too large to match observations, suggesting that we are missing something critical about how to do the calculation).

One caveat here – recent observational results from the DESI collaboration suggest that the component we usually identify as dark energy may actually have an evolving value of w [1]. The detection is at $2.8 - 4.2\sigma$ depending on exactly what datasets are used, so it is not yet highly statistically significant, but definitely worth keeping an eye on! For purposes of these lectures we will assume $w = -1$ for dark energy.

We can thus write the total energy density as:

$$\rho(t) = \rho_{\rm m,0} \left(\frac{a(t)}{a_0}\right)^{-3} + \rho_{\rm r,0} \left(\frac{a(t)}{a_0}\right)^{-4} + \rho_{\Lambda,0}, \tag{12}$$

where a_0 is the scale factor today. It is convenient to normalize the energy densities in terms of the *critical density*

$$\rho_c \equiv \frac{3H^2}{8\pi G}. \tag{13}$$

Then if we define the present-day density fractions as

$$\Omega_x \equiv \frac{\rho_{x,0}}{\rho_{c,0}}, \qquad \Omega_k \equiv -\frac{k}{a_0^2 H_0^2}, \tag{14}$$

we can write

$$\frac{H^2}{H_0^2} = \Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda. \tag{15}$$

The current best-fit values measured by the *Planck* experiment and large-scale structure data (or inferred from the temperature in the case of the photon bath) are [2]:

$$\Omega_r \simeq 9 \times 10^{-5}, \qquad \Omega_m \simeq 0.3111 \pm 0.0056, \qquad \Omega_\Lambda \simeq 0.6889 \pm 0.0056, \qquad \Omega_k \simeq 0.001 \pm 0.002, \tag{16}$$

with Ω_m being split between baryonic matter ($\sim 15.8\%$) and cold dark matter ($\sim 84.2\%$).

At early times, when the $1+z$ factor is very large, we see that the radiation term will dominate the energy density. As the universe expands, eventually the matter term will come to dominate, and finally the dark energy term (you can show that given these coefficients there is no intermediate period where the curvature term dominates). In the very early universe there could have been other components that once dominated the universe but then decayed away – in particular, we believe that at very early times there was another epoch dominated by a fluid with $w \approx -1$, known as *inflation*.

[1] M. Abdul Karim *et al.* (DESI), "DESI DR2 results. II. Measurements of baryon acoustic oscillations and cosmological constraints," Phys. Rev. D **112**, 083515 (2025), arXiv:2503.14738 [astro-ph.CO].

[2] N. Aghanim *et al.* (Planck), "Planck 2018 results. VI. Cosmological parameters," Astron. Astrophys. **641**, A6 (2020), [Erratum: Astron.Astrophys. 652, C4 (2021)], arXiv:1807.06209 [astro-ph.CO].