

```

In[3]:= (* plus and minus spinor solutions *)
4  U["+",p_]:=({
5      {sqrt[plus[p]]},
6      {sqrt[minus[p]]*Exp[I*Phi[p]]},
7      {0},
8      {0}
9  });
10 U["-",p_]:=({
11     {0},
12     {0},
13     {sqrt[minus[p]]*Exp[-I*Phi[p]]},
14     {-sqrt[plus[p]]}
15 });

```

```

In[5]:= (* Weyl matrices *)
19 WeylGamma[0]=({
20     {0, 0, 1, 0},
21     {0, 0, 0, 1},
22     {1, 0, 0, 0},
23     {0, 1, 0, 0}
24 });
25 WeylGamma[1]=({
26     {0, 0, 0, -1},
27     {0, 0, -1, 0},
28     {0, 1, 0, 0},
29     {1, 0, 0, 0}
30 });
31 WeylGamma[2]=({
32     {0, 0, 0, I},
33     {0, 0, -I, 0},
34     {0, -I, 0, 0},
35     {I, 0, 0, 0}
36 });
37 WeylGamma[3]=({
38     {0, 0, -1, 0},
39     {0, 0, 0, 1},
40     {1, 0, 0, 0},
41     {0, -1, 0, 0}
42 });

```

```

In[9]:= (* Phase in terms of Momenta Component *)
46 SubsPhase = {
47     Exp[I*Phi[p_]]>=>(dot[p,1]+I*dot[p,2])/sqrt[plus[p]]/sqrt[minus[p]],
48     Exp[-I*Phi[p_]]>=>(dot[p,1]-I*dot[p,2])/sqrt[plus[p]]/sqrt[minus[p]]
49 };

```

```
In[10]:= ImposeReal = {
53   Conjugate[sqrt[plus[p_]]]:>sqrt[plus[p]],
54   Conjugate[sqrt[minus[p_]]]:>sqrt[minus[p]],
55   Conjugate[Phi[p_]]:>Phi[p]
56 };
```

```
In[11]:= (* Ubar spinors from U spinor *)
60 Ubar[h_,p_] := ConjugateTranspose[U[h,p]] /. WeylGamma[0] /. ImposeReal;
```

```
In[125]:= (* spinor products *)
64 spa[a_,b_] := Ubar["-",a].U["+",b];
65 spb[a_,b_] := Ubar["+",a].U["-",b];
66 (* slashed momentum *)
67 slashedmom[p_] := WeylGamma[0]*dot[p,0] - WeylGamma[1]*dot[p,1] - WeylGamma[2]*dot[p,2] - We
```

```
In[111]:= ToPcomp[x_] := {
71   plus[x] -> dot[x,0] + dot[x,3],
72   minus[x] -> dot[x,0] - dot[x,3]
73 };
```

```
In[116]:= (* <ij> + <ji> = 0 *)
77 spa[p1,p2] + spa[p2,p1]
78 (* [ij] + [ji] = 0 *)
79 spb[p1,p2] + spb[p2,p1]
```

```
Out[116]=
{{0}}
```

```
Out[117]=
{{0}}
```

```
In[124]:= (* [ij]^* - <ji> = 0 *)
83 (spb[p1,p2] /. Phi[x_] -> -Phi[x]) - spa[p2,p1]
```

```
Out[124]=
{{0}}
```

```
In[131]:= (* slashed[p] = |i>[i] + |i><i| *)
87 slashedmom[p];
88 Outer[Times, Flatten[U["-",p] /. ToPcomp[p]], Flatten[Ubar["-",p] /. ToPcomp[p]] + Out
89 % - % // MatrixForm
```

```
Out[133]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[139]:= (* Schouten identity *)
93 spa[p1,p2]*spa[p3,p4] + spa[p1,p3]*spa[p4,p2] + spa[p1,p4]*spa[p2,p3] /. SubsPhase /. To
```

```
Out[139]=
{{0}}
```

```

96 (* |p1>[p2| + |p2><p1| *)
97 tmpA = (Outer[Times,Flatten[U["-",p2] /. ToPcomp[p2]],Flatten[Ubar["-",p1] /. ToPcomp[p1]]
98 +Outer[Times,Flatten[U["+",p1] /. ToPcomp[p1]],Flatten[Ubar["+",p2] /. ToPcomp[p1]]

```

```

In[232]:= (* (1/2)*<p1 \gamma_\mu p2>*\gamma^\mu *)
102 (1/2)*Flatten[{
103   Ubar["-",p1] . WeylGamma[0] . U["-",p2] /. sqrt[x_]^2:->x /. SubsPhase /. ToPcomp[p1]
104   Ubar["-",p1] . WeylGamma[1] . U["-",p2] /. SubsPhase,
105   (Ubar["-",p1] . WeylGamma[2] . U["-",p2] /. SubsPhase // Expand) /. sqrt[x_]^2:->x,
106   (Ubar["-",p1] . WeylGamma[3] . U["-",p2] /. SubsPhase // Expand) /. sqrt[x_]^2:->x /.
107   tmpB = %[[1]]*WeylGamma[0] - %[[2]]*WeylGamma[1] - %[[3]]*WeylGamma[2] - %[[4]]*WeylGamma[3]

```

```

In[234]:= tmpA-tmpB /. ToPcomp[p1] /. ToPcomp[p2] /. Exp[x_->Exp[Expand[x]] /. {
111   Exp[a_+b_->tmp[Exp[a]]*tmp[Exp[b]]
112 } // SubsPhase /. tmp->Identity /. ToPcomp[p1] /. ToPcomp[p2] // FullSimplify

```

```

Out[234]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

```

```

In[240]:= (* <p1 \gamma^\mu p2> *)
116 tmpA = Flatten[{
117   Ubar["-",p1] . WeylGamma[0] . U["-",p2] /. sqrt[x_]^2:->x /. SubsPhase /. ToPcomp[p1]
118   Ubar["-",p1] . WeylGamma[1] . U["-",p2] /. SubsPhase,
119   (Ubar["-",p1] . WeylGamma[2] . U["-",p2] /. SubsPhase // Expand) /. sqrt[x_]^2:->x,
120   (Ubar["-",p1] . WeylGamma[3] . U["-",p2] /. SubsPhase // Expand) /. sqrt[x_]^2:->x /.
121   (* [p2 \gamma^\mu p1> *)
122   tmpB = Flatten[{
123     Ubar["+",p2] . WeylGamma[0] . U["+",p1] /. sqrt[x_]^2:->x /. SubsPhase /. ToPcomp[p1]
124     Ubar["+",p2] . WeylGamma[1] . U["+",p1] /. SubsPhase,
125     (Ubar["+",p2] . WeylGamma[2] . U["+",p1] /. SubsPhase // Expand) /. sqrt[x_]^2:->x,
126     (Ubar["+",p2] . WeylGamma[3] . U["+",p1] /. SubsPhase // Expand) /. sqrt[x_]^2:->x /.

```

```

In[242]:= tmpA - tmpB

```

```

Out[242]= {0, 0, 0, 0}

```

```

In[17]:= (* <i \gamma^\mu i> = 2*p_i^\mu *)
133 Ubar["-",p] . WeylGamma[0] . U["-",p] /. sqrt[x_]^2:->x /. ToPcomp[p]
134 Ubar["-",p] . WeylGamma[1] . U["-",p] /. SubsPhase
135 (Ubar["-",p] . WeylGamma[2] . U["-",p] /. SubsPhase // Expand) /. sqrt[x_]^2:->x
136 (Ubar["-",p] . WeylGamma[3] . U["-",p] /. SubsPhase // Expand) /. sqrt[x_]^2:->x /. ToPcomp[p]

```

```

Out[17]= {{2 dot [p, 0]}}

```

```

Out[18]= {{2 dot [p, 1]}}

```

```

Out[19]= {{2 dot [p, 2]}}

```

```

Out[20]= {{2 dot [p, 3]}}

```