PROBLEM SET #1

1. Using the expressions for $u_\pm(p)$ and $\bar{u}_\pm(p)$ given in the lecture, prove the following identities involving spinor products / strings:

$$\langle ij \rangle = -\langle ji \rangle$$

$$[ij] = -[ji]$$

$$\not{p}_i = |i\rangle[i| + |i]\langle i|$$

$$\langle ij\rangle\langle kl\rangle + \langle ik\rangle\langle lj\rangle + \langle il\rangle\langle jk\rangle = 0 \quad \text{(Schouten identity)}$$

$$\frac{1}{2}\langle i\gamma_\mu j]\gamma^\mu = |i\rangle[j| + |j]\langle i| \quad \text{(Fierz identity)}$$

$$\langle i\gamma^\mu j] = [j\gamma^\mu i\rangle$$

$$\langle i\gamma^\mu i] = 2p_i^\mu$$

Hint: to evaluate $|i\rangle[j|$ or $|j]\langle i|$, use the following outer product rule

$$\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} c & d \end{pmatrix} = \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix}$$

(2) Massless vector boson polarisation can be written in terms of spinor products / strings as follows

$$\varepsilon_{+}^{*\mu}(p_i, r) = \frac{1}{\sqrt{2}} \frac{\langle r \gamma^\mu i]}{\langle r i \rangle}$$

$$\varepsilon_{-}^{*\mu}(p_i, r) = -\frac{1}{\sqrt{2}} \frac{[r \gamma^\mu i\rangle}{[r i]}$$

where r is an arbitrary massless reference vector.

(a) <u>Show that</u> :

$$\varepsilon_{\pm}^{*\mu}(p, r) \cdot p_\mu = 0$$

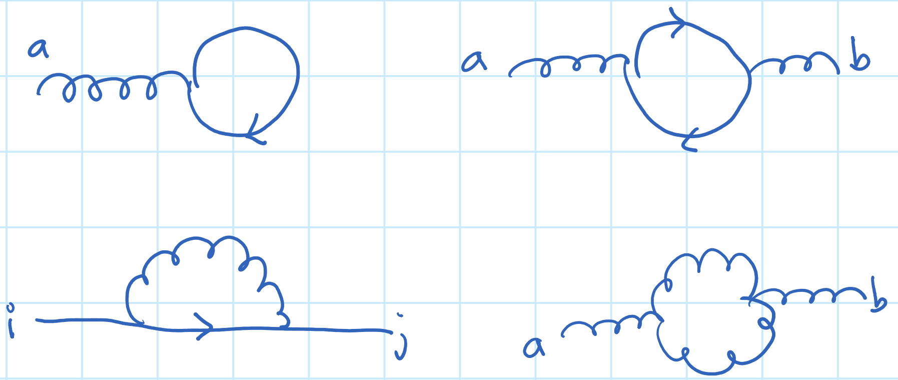
$$\varepsilon_{\pm}^{*\mu}(p, r) \cdot r_\mu = 0$$

(b) <u>Evaluate</u> scalar product involving 2 polarisation vectors

$$\varepsilon_{\lambda_1}^{*}(k, r) \cdot \varepsilon_{\lambda_2}^{*}(k', r')$$

where $(\lambda_1, \lambda_2) = (+,+), (+,-), (-,+), (-,-)$

(3) Calculate the colour factors for the following Feynman diagrams



4. We have considered the following process in the lecture

$$u(p_1) + \bar{u}(p_2) \to g(p_3) + g(p_4)$$

The colour decomposition is given by:

$$M = t^a t^b A_{ab} + t^b t^a A_{ba} \qquad (*)$$

where we need to focus on the A_{ab} computation only. From the expressions of A_{ab} given in the lecture notes show that

$$A_{ab}^{+-++} = 0$$

$$A_{ab}^{+-+-} = 2 i g_s^2 \frac{\langle 14\rangle^3 \langle 24\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

$$A_{ab}^{+--+} = 2 i g_s^2 \frac{\langle 13\rangle^3 \langle 23\rangle}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

obtain the A_{ab} for $-+-+$ and $+--+$ using results for $+-+-$ and $+--+$

Obtain helicity amplitudes for A_{ba} from those of A_{ab}

From the colour decomposition in ($*$), show that the colour summed squared amplitude is given by

$$\sum_{col} |M|^2 = \frac{(N_c^2-1)^2}{4N_c} \left\{ |A_{ab}|^2 + |A_{ba}|^2 \right\}$$

$$- \frac{1}{4N_c}(N_c^2-1) \left\{ A_{ab}^* A_{ba} + A_{ab} A_{ba}^* \right\}$$

Compute the $|A_{ab}|^2$, $|A_{ba}|^2$, $A_{ab}^* A_{ba}$, $A_{ab} A_{ba}^*$ from helicity amplitudes, express them in terms of Mandelstam invariants s, t, u, wher

$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$

$$t = (p_1-p_4)^2 = (p_2-p_3)^2$$

$$u = (p_1-p_3)^2 = (p_2-p_4)^2$$

Hint: use the following reference vector:

$$r_3 = p_4$$

$$r_4 = p_3$$

use results from Problem 3 for products of polarisation vectors