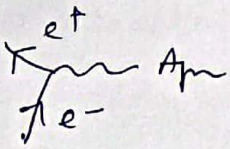
①

fermion – Gauge boson vertex in SM

Two type of interactions other than QED.

$e\,\bar{e}\gamma^\mu e\, A_\mu$



$ie\gamma^\mu$ (a) This appears after redefinition $\{W_\mu^3, B_\mu\} \to \{A_\mu, Z_\mu\}$

$$\mathcal{L} \to \bar{L} i \not{\partial} L$$

$$= (\bar{\nu}_e \;\; \bar{e}_L)\, i\gamma^\mu \left(\partial_\mu - ig W_\mu^i \frac{\sigma^i}{2} - ig' \frac{Y}{2} B_\mu\right) \begin{pmatrix} \nu \\ e \end{pmatrix}_L$$

$$W_\mu^i \frac{\sigma^i}{2} = \begin{pmatrix} \cancel{W_\mu^1 - iW_\mu^2}\;\; W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}$$

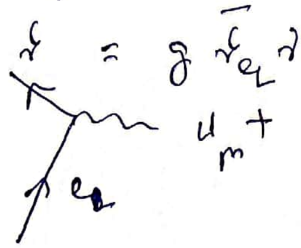
$$= \begin{pmatrix} W_\mu^3 & W_\mu^+ \\ W_\mu^- & -W_\mu^3 \end{pmatrix}$$

(b)

$\longrightarrow g\,\bar{\nu}_{e_L} \gamma^\mu W_\mu^+ e_L^- + h.c$

$= g\,\bar{\nu}_{e_L}\gamma^\mu W_\mu^+ P_L e^-$

Charged current interaction



$= g\, j^{\mu +} W_\mu^- + h.c$ where $j^{\mu +} = \bar{\nu}_{e_L}\gamma^\mu e_L$

$j^{\mu -} = \bar{e}_L \gamma^\mu \nu_{e_L}$

(c) $\bar{L} i \not\partial L + \bar{e}_R i \not\partial e_R$

W_μ^3 interacts with $\bar{\nu}_{e_L} - \nu_{e_L}$ component

both W_μ^3 and B_μ interacts with $\bar{e}_L - e_L$ &

$\bar{e}_R - e_R$.

Redefine W_μ^3 & B_μ linear combination as

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}} = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}} = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$$

$\bar{f} \gamma^\mu Z_\mu f$ where $f = e_L$

$f = e_R$

$g \sin\theta_W = e_0 = g' \cos\theta_W$

from #

and

$\bar{\nu}_L \gamma^\mu Z_\mu \nu_L$ where $f = \nu_{e_L}$, the vertex

factor is $\boxed{\frac{e_0}{\sin\theta_W \cos\theta_W} \times \frac{1}{2}}$.

②

Z_μ interaction with e_L & e_R

$$\frac{e_0}{\sin\theta_W \cos\theta_W} \left\{ \left[-\tfrac{1}{2} + \sin^2\theta_W \right] \left(\bar{e}_L \gamma^\mu e_L \right) + \sin^2\theta_W \left(\bar{e}_R \gamma^\mu e_R \right) \right\} Z_\mu$$

One can write this as

$$\frac{e_0}{\sin\theta_W \cos\theta_W} \bar{f} \gamma^\mu \frac{1}{2} \left(c_V^f - c_A^f \gamma_5 \right) f Z_\mu$$

$$= \left(\frac{e_0}{S_W C_W} \right) Z_\mu \, J^\mu{}_{NC}$$

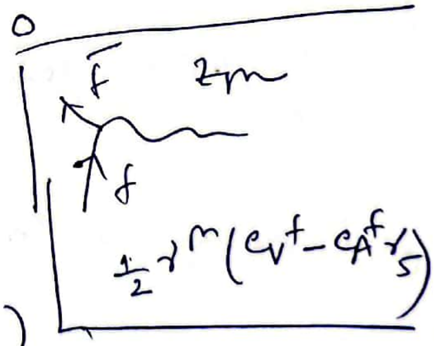
where

$$c_V^f = T_3^f - 2 \sin^2\theta_W \, q^f$$

$$c_A^f = T_3^f$$

for example for ν, $q^f = 0$

$c_A^f = \tfrac{1}{2}$, $c_V^f = \tfrac{1}{2}$

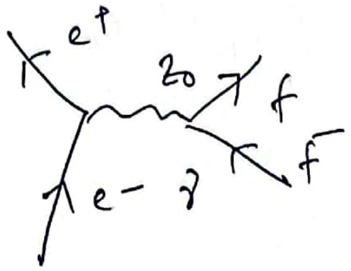


for e_L = $c_V^f = (-\tfrac{1}{2}, + 2\sin^2\theta_W)$

$c_A^f = -\tfrac{1}{2}$ & so on.

Both charge current interaction & neutral current interaction violates parity.

One of the experimental manifestation is assymmetry in $e^+e^- \to Z_0 \to f\bar{f}$ $(\mu^+\mu^-)$ processes.



The QED part $\bar{f}\gamma^\mu f A_\mu$

$$= \bar{f_L}\gamma^\mu f_L A_\mu + \bar{f_R}\gamma^\mu f_R A_\mu$$

$\longrightarrow$ parity symmetric

$\longrightarrow$ But since Z_μ coupling with $\bar{\mu}_L \mu_L$ & $\bar{\mu}_R \mu_R$ different

This leads to $A_{FB} = \dfrac{N_F - N_B}{N_F + N_B} \neq 0$

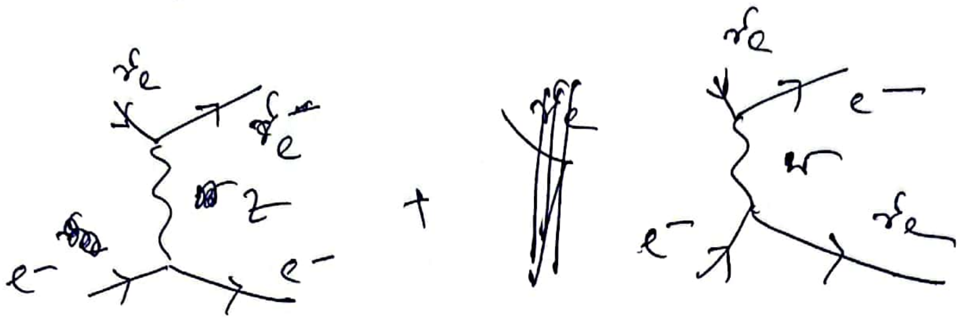
$N_F \to$ Number of μ hitting in the forward hemisphere (w.r.t e^- beam $\theta < 90°$).

$N_B \to$ Backward " $(\theta > 90°)$ w.r.t e^- beam direction

$$\left\{ \frac{d\sigma}{d\cos\theta} \propto \underline{(1+\cos^2\theta) + \frac{8}{3}\alpha \underline{\underline{\cos\theta}}} \right\}$$ $\to$ This makes the $A_{FB} \neq 0$

(3) Example of few of the other processes where both $W^{\pm}$ & Z contribute

$$\nu_e + e^- \longrightarrow \nu_e + e^-$$



$$M = |M_Z + M_W|^2$$

$\longrightarrow$ This kind of processes are extremely important for detection of neutrinos. [you don't see neutrino, but see charged particle track]

SSB - 1

Spontaneous symmetry breaking

\# No reason why a symmetry of a Q.M system will be a symmetry of ground state.

Example - Ferromagnet

ferromagnet ⇒ rotational symmetry

With external fields / lowering of temp all the spin allign in same direction. rotational invariance lost.

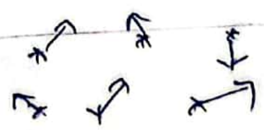
In particle physics

SSB → essential ingredient that postulates goldstone excitations ⇒ Can be related to gauge boson mass term, This is called Higgs mechanism.

The Higgs mechanism in other sense describes the short range weak interaction by a gauge theory without spoiling gauge invariance,

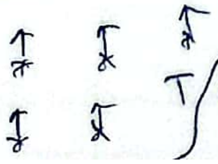
→ $W^{\pm}, Z$ discovery at 1983 is the first experimental evidence of SSB in electroweak theory.

→ * Ferromagnet *



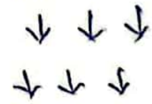
$T > T_c$ ⇒ spin rotationally invariant

⇒ average magnetization becomes 0.



$T < T_c$ ⇒ $\vec{M}_{average}$ (average magnetization non - zero)

or



Since, the direction of spin is arbitrary, there are infinite possible ground states.

✓ The system chooses one among the infinite possible non-invariant ground states.

→ T near T_c, free energy density ✓

$$u(\vec{M}) = (\partial_i \vec{M})(\partial_i \vec{M}) + V(\vec{M}) \quad ✓$$

$$i = 1, 2, 3$$

$$V(\vec{M}) = \alpha_1 (T - T_c)(\vec{M}\cdot\vec{M}) + \alpha_2 (\vec{M}\cdot\vec{M})^2 \quad ✓$$

$$\alpha_1, \alpha_2 > 0$$

condition of extremum

$$\frac{\partial V(\vec{M})}{\partial M_i} = 0 = \vec{M} \cdot \left(\alpha_1 (T - T_c) + 2\alpha_2 (\vec{M}\cdot\vec{M})\right) = 0 \quad ✓$$

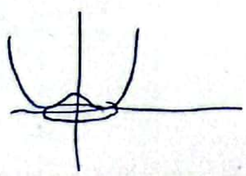
→ If $T > T_c \Rightarrow \vec{M} = 0$ ✓

→ $T < T_c \Rightarrow |\vec{M}| = \sqrt{\dfrac{\alpha_1 (T_c - T)}{2\alpha_2}}$ ✓

$$M = (M_x, M_y)$$

→ Infinite absolute minima having all the same $|\vec{M}|$, but different direction. ✓

⇒ system has degenerate ground states, that are not rotationally invariant. ✓



→ 3/ Discrete symmetry spontaneous broken

✓ $\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4$

$\mathcal{L}$ is invariant under $\varphi \to -\varphi$, a discrete symm of the model. ✓

Energy $E = \int d^3x \left[\frac{1}{2}(\partial_0 \varphi)^2 + \frac{1}{2}(\partial_i \varphi)^2 + \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 \right]$ ✓

The energy ✓ is bounded from below if $\lambda > 0$

→ The ground state of the model corresponds ✓ to the field configuration with minimal energy.

→ E is ✓ minima if $\partial_0 \varphi = 0$ & $\partial_i \varphi = 0$

i.e only solution is φ = constant.

i.e ground state field configuration is ‖ ~~ap~~ independent of x & t. ✓

→ for $m^2 > 0$ minima at $\varphi = 0$ ✓

this state is invariant under $\varphi \to -\varphi$

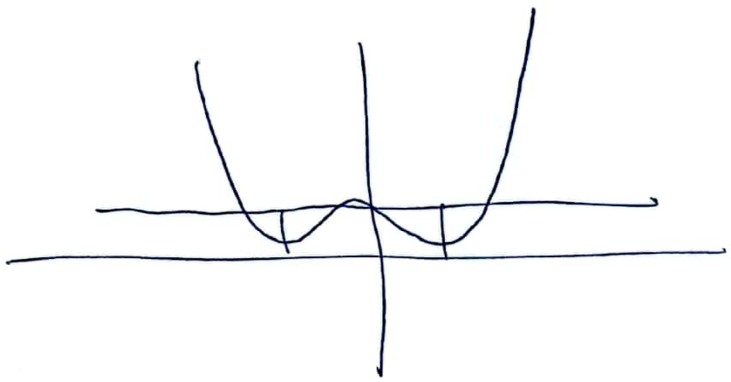
trivial ground state doesn't break the symm of the model. ✓

→ $m^2 < 0$, $m^2 = -\mu^2$, two options, a) to expand near $\varphi = 0$ ~~&~~ or to expand near $\varphi = \pm\sqrt{\frac{\mu^2}{\lambda}}$

→ $\varphi = 0$ is a saddle point. The modes are ✓ tachyonic. ~~[illegible]~~ ✓

→ $\varphi = \sqrt{\frac{\mu^2}{\lambda}}$ is the ~~a~~ ground state. ✓

Remedy is to expand the perturbation theory near minima.



$$V(\varphi) = -\frac{m^2}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4$$

$$\frac{\partial V}{\partial \varphi} = -m^2\varphi + \frac{\lambda}{3!}\varphi^3 = 0$$

$$\Rightarrow \varphi = \pm\sqrt{\frac{m^2 3!}{\lambda}}$$

Two minima at $\varphi = \sqrt{\frac{3!\, m^2}{\lambda}}$ and

$$\varphi = -\sqrt{\frac{3!\, m^2}{\lambda}}$$

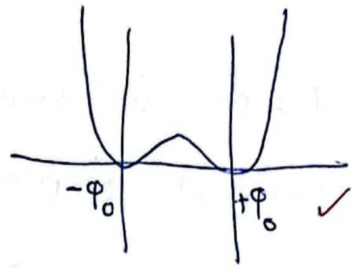
→ Expand $\alpha = \sqrt{\frac{m^2 3!}{\lambda}} - \phi$

S1

Expand φ around $\varphi_0 \neq 0$

Let us chose $+\varphi_0$

$$\varphi = \varphi_0 + x$$



$$\Rightarrow x = \varphi - \varphi_0$$

as $\varphi \to \varphi_0$,

$x \to 0$

One can consider x as small fluctuation around $\varphi = \varphi_0$

$$V = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$

$$= -\frac{\mu^2}{2}(\varphi_0 + x)^2 + \frac{\lambda}{4}(\varphi_0 + x)^4$$

$$= -\frac{\mu^2}{2}\left(\varphi_0^2 + 2\varphi_0 x + x^2\right) + \frac{\lambda}{4}\left(\varphi_0^2 + 2\varphi_0 x + x^2\right) \times \left(\varphi_0^2 + 2\varphi_0 x + x^2\right)$$

$$= -\frac{\mu^2}{2}\varphi_0^2 + \frac{\lambda}{4}\varphi_0^4 - \mu^2\varphi_0 x + \lambda\varphi_0^3 x$$

$$-\frac{\mu^2}{2}x^2 + \frac{\lambda}{2}\varphi_0^2 x^2$$

$$+\frac{\lambda}{4} \times 4\varphi_0^2 x^2$$

$$+\frac{\lambda}{4}x^4 + \lambda\varphi_0 x^3$$

$$= A - \cancel{\mu^2\varphi_0 x} + \lambda \times \frac{\mu^2}{\lambda}\cancel{\varphi_0 x} - \cancel{\frac{\mu^2}{2}x^2} + \cancel{\frac{\mu^2}{2}x^2}$$

$$+ \mu^2 x^2 + \frac{\lambda}{4}x^4$$

$$+ \lambda\sqrt{\frac{\mu^2}{\lambda}}\,x^3$$

$$x = \varphi - \sqrt{\frac{3!\mu^2}{\lambda}}$$

Ex-2 Re-write $V(\varphi)$ in terms of $V(x)$.

→ Identify the mass & x^3, x^4 terms. Show that no Z_2 symmetry left. Which term breaks the Z_2 sym?

SSB ⟶ choosing one of the minima breaks the $\varphi \to -\varphi$ symmetry. The sym is no more there in coreln funn

→ Remember as $x \to \infty$, $\varphi \to 0$, this corresponds to crossing the boundary. $x \to \infty$, $\varphi \to -\sqrt{\frac{\mu^2 3!}{\lambda}}$

→ In order to transfer the field from one ground state $\varphi_0 = \sqrt{\frac{\mu^2}{\lambda}}$ to $-\sqrt{\frac{\mu^2}{\lambda}}$ we need to add an energy.

Ex-3
→ Ground state energy at $\varphi = \varphi_0$

$$V(\varphi_0) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$$

$$= +\frac{\mu^2}{2} \times \frac{\mu^2}{\lambda} + \frac{\lambda}{4}\frac{\mu^4}{\lambda^2}$$

$$= \frac{\mu^4}{2\lambda} + \frac{\mu^4}{4\lambda} = \frac{3\mu^4}{4\lambda}$$

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SSB implies that the ground stat is not invariant under the symmetry. Neither the Lagrangian for perturbation have the original symmetry.

Continuous symmetry

U(1) symmetry

Complex scalar field $\phi = \phi_1 + i\phi_2$

Let $U = \frac{\lambda}{4!}\left(\phi^*\phi - a^2\right)^2$

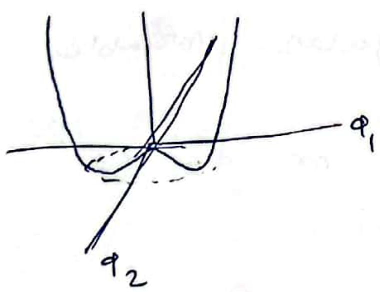
upon expansion U will give the form

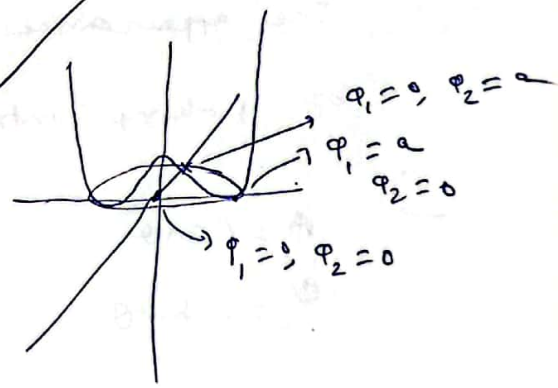
$a|\phi|^2 + b|\phi|^4 + c$.

→ The theory has a continuous group of symmetries $\equiv SO(2)$

$$\left.\begin{aligned} \phi_1 &\to \phi_1 \cos\theta + \phi_2 \sin\theta \\ \phi_2 &\to -\phi_1 \sin\theta + \phi_2 \cos\theta \end{aligned}\right\} \text{keeps } U \text{ invariant.} \to A$$

i.e the minima lies on a circle





infinite number of degenerate ground states.

choosing one vacuum will break $SO(2)$ symmetry spontaneously.

let us choose $\left.\begin{aligned}\langle\phi_1\rangle &= a \\ \langle\phi_2\rangle &= 0\end{aligned}\right\}$

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We choose $\varphi' = \varphi_1 + i\varphi_2 - a = \varphi_1' + i\varphi_2'$

Show that $U = \frac{\lambda}{4!}\left(\varphi_1'^2 + \varphi_2'^2 + 2a\varphi_1'\right)^2$

→ The mass term for $\varphi_1' \Rightarrow \left.\frac{\partial^2 u}{\partial \varphi_1'^2}\right|_{\varphi_1' = 0} =$

$$u = \frac{\lambda}{4!}\left(\varphi_1'^4 + \varphi_2'^4 + 4a^2\varphi_1'^2 + 2\varphi_1'^2\varphi_2'^2 + 4\varphi_1'^3 a + 4a\varphi_1'\varphi_2'^2\right)$$

$\cancel{+ 4a\varphi_1}$

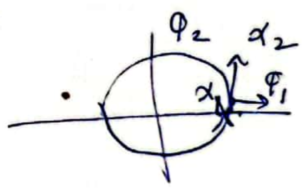
The mass term for $\varphi_1' \neq 0$

- No $\varphi_2'^2$ term in u.

⇒ $\varphi_2'(x)$ field is a massless boson, this is called Goldstone mode. φ_2' corresponds to flat direction upto $\varphi_2'^2$ term

→ The appearance of G.M can be seen by looking into angular variables.

→ $\left.\begin{array}{l} \varphi_1 = \rho\cos\theta \\ \varphi_2 = \rho\sin\theta \end{array}\right\}$ or $\varphi = \rho e^{i\theta}$



→ so the transformation Eq A becomes $\rho \to \rho$
$\theta \to \theta + \theta_0$

→ $\mathcal{L} = \frac{1}{2}(\partial_\mu\rho)^2 + \frac{1}{2}\rho^2(\partial_\mu\theta)^2 - V(\rho)$

$\hookrightarrow (\partial_\mu\varphi)^+(\partial^\mu\varphi) = \partial_\mu(\rho e^{i\theta})$

$(\partial^\mu\varphi) = \partial_\mu\rho\, e^{i\theta} + \rho i(\partial_\mu\theta)e^{i\theta}$

$(\partial_\mu\varphi)^+ = (\partial_\mu\rho)e^{-i\theta} - i\rho(\partial_\mu\theta)e^{-i\theta}$

$$(\partial_\mu \varphi)^* (\partial^\mu \varphi) = \left((\partial_\mu \rho) e^{-i\theta} - i\rho (\partial_\mu \theta) e^{-i\theta}\right)$$
$$\left((\partial_\mu \rho) e^{i\theta} + i\rho (\partial_\mu \theta) e^{i\theta}\right)$$

$$= (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta)^2$$

and $V(\rho) = \frac{\lambda}{4!}(\rho^2 - a^2)^2$

$$\therefore \mathcal{L} = (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta)^2 - \frac{\lambda}{4!}(\rho^2 - a^2)^2$$

θ direction is the flat direction. as the potential does not change as $\theta \to \theta + \theta_0$, $\therefore$ θ ~~direction~~ field is the Goldstone mode.

$\rightarrow$ Expand $\varphi(x) = a + \ldots$

$\rho' = \rho - a$

$\theta' = \theta$

$\frac{\partial^2 V}{\partial \rho'^2} < 0$ tachyonic $\quad \frac{\partial^2 V}{\partial \rho'^2} > 0$

$$\rightarrow \mathcal{L} = (\partial_\mu \rho')^2 + (\rho' + a)^2 (\partial_\mu \theta')^2 - \frac{\lambda}{4!}\left((\rho' + a)^2 - a^2\right)^2$$

$\rightarrow$ θ' meson is massless. only has kinetic term $(\partial_\mu \theta')^2$

$\rightarrow$ potential $+\frac{\lambda}{4!}(\rho'^2 + 2\rho' a)^2$

$$= +\frac{\lambda}{4!}\left(\rho'^4 + 4\rho'^2 a^2 + 4\rho'^3 a\right)$$

$\rightarrow$ Mass of $\rho' \longrightarrow \frac{\lambda}{3!} a^2$

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The reason this is happening, bcz,

there is a circle passing through the vacuum,

along which potential is constant.

2nd derivative of potential gives mass term.

If potential is flat, mass term will be zero.

→ As we are choosing a vacuum and

doing perturbative expansion ~~along that~~,

no. term can appear involving

the variable that measures

displacement along the curve - θ variable.

$\therefore$ θ' is goldstone boson.

$\frac{u}{24}$

SSB – 2

From yesterday's example,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho')^2 + \frac{1}{2}(\rho'+a)^2(\partial_\mu \theta')^2 - U(\rho'+a)$$

upon expanding $\rho = \rho' - a$ in $\frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2(\partial_\mu \theta)^2 - U(\rho)$

$$\rightarrow \left.\begin{aligned} \rho' &= \rho - a \\ \theta' &= \theta \end{aligned}\right\}$$

→ θ meson is massless, as θ field enters in $\mathcal{L}$ only through $(\partial_\mu \theta')$.

Continuous symmetry spontaneously broken

→ Goldston Boson's in general case: → Massless modes

Set of n real fields $\vec{\phi}$, such that

$\vec{\phi} \rightarrow e^{it_a \alpha^a} \vec{\phi}$ is a symmetry transformation of Lagrangian.

Let $T_a = it_a$

T_a are the group generators. for a

α_a are arbitrary real parameters.

$\delta\vec{\phi}_a = T_a \delta\alpha^a \vec{\phi}$ the change in ϕ

$[T_a, T_b] = C_{abc} T_c$, C_{abc} are the structure constant of the group.

→ Symmetry implies $U(\phi) = U(e^{T_a \alpha_a} \phi)$

i.e Potential $U(\phi)$ is same as $U(\phi')$

Let us consider that H is a smaller subgroup, that leaves $\langle\phi\rangle$ invariant.

→ H can span from $\mathbb{1}$ to G, i.e the bigger group itself.

→ Let us consider G generators are T^a where $a = 1 \dots N$

H generators are T^λ where $\lambda = 1 \dots M$

There are $N - M$ generators that don't keep potential minima $\langle\phi\rangle$ invariant.

→ $T_\lambda \delta\alpha^\lambda \langle\phi\rangle = 0$ for $\lambda = 1 \dots M$

$T_a \delta\alpha^\lambda \langle\phi\rangle \neq 0$ for $\lambda = M+1 \dots N$

→ Passing through $\langle\phi\rangle$ an $N - M$ dimensional surface of constant U $\left[\frac{\partial^2 U}{\partial\phi_i \partial\phi_j} = 0\right]$

(since $U(\phi) = U(\phi + T_a \alpha^a \phi)$)

that corresponds to massless spinless mesons, one for each spontaneously broken generators.

Higgs mechanism for U(1)

$$\mathcal{L} = (\mathcal{D}_\mu \varphi)^* (\mathcal{D}^\mu \varphi) + \mu^2 \varphi^* \varphi - \lambda (\varphi^* \varphi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{D}_\mu \varphi = (\partial_\mu - i g A_\mu)\varphi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$\mathcal{L}$ invariant under $\varphi(x) \to \varphi'(x) = e^{-i\alpha(x)} \varphi(x)$

$$A_\mu(x) \to A_\mu'(x) = A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x)$$

$\rightarrow$ Minimization of the potential

$$V(\varphi) = -\mu^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2$$

gives $\langle 0|\varphi|0\rangle$, the vacuum expectation value of φ w.r.t ground state is $|\varphi| = \frac{v}{\sqrt{2}}$

where $v = \sqrt{\frac{\mu^2}{\lambda}}$

$\rightarrow$ $\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$ implies $\left.\begin{array}{l}\langle 0|\varphi_1|0\rangle = v \\ \langle 0|\varphi_2|0\rangle = 0\end{array}\right\}$ choice

Now expanding $\varphi_1' = \varphi_1 - v$

$\varphi_2' = \varphi_2$ ✓

→ φ_2' is the Goldstone

→ for local gauge symmetry,

$$|D_\mu \varphi|^2 = |(\partial_\mu - i g A_\mu)\varphi|^2$$

$$= \frac{1}{2}(\partial_\mu \varphi_1' + g A_\mu \varphi_2')^2 + \frac{1}{2}(\partial_\mu \varphi_2' - g A_\mu \varphi_1')^2$$

$$- g v A^\mu (\partial_\mu \varphi_2' + g A_\mu \varphi_1') + \frac{g^2 v^2}{2} A^\mu A_\mu$$

→ verify ✓ → ③

→ the mass term comes from $|ig A_\mu \varphi|^2$ term

$$|ig A_\mu \varphi|^2 \longrightarrow |ig A_\mu \langle\varphi\rangle|^2 = g^2 A_\mu A^\mu \frac{v^2}{2}$$ ✓

→ The mass term for A_μ is generated from $|D_\mu \varphi|^2$ upon expansion around the minima. ✓

→ Unitary gauge:

all the terms in 3 except $A^\mu \partial_\mu \varphi_2'$ are $\varphi_1'^2$, $(\partial_\mu \varphi_1')^2$ or $A_\mu A^\mu \varphi_2'^2$, $\partial_\mu \varphi_1' A_\mu \varphi_2'$ etc.

Remaining term → a bilinear in $\varphi_2' - A_\mu$

is $A^\mu \partial_\mu \varphi_2'$ ⟶ in k-space this leads

to $\tilde{A}^\mu(k)\, \tilde{\varphi}_2'(k) (ik_\mu)$

→ a momentum dependent mixing term

between $\tilde{A}_\mu(k)$ $\tilde{\varphi}_2'(k)$

→ One can remove $\tilde{\varphi}_2'$ / φ_2' by suitably choosing the gauge.

→ Go to polar co-ordinate.

$$\varphi(x) = \frac{1}{\sqrt{2}} \left(v + \eta(x)\right) \exp\left(i \frac{\zeta(x)}{v}\right)$$

$$= \frac{1}{\sqrt{2}} \left[v + \eta(x) + i\zeta(x) + \dots\right]$$

{ Remember yesterday's $\varphi(x) = \rho\, e^{i\alpha}$ }

if $\eta(x)$, $\zeta(x)$ are small,

then $\varphi_1'(x) = \eta(x)$ & $\varphi_2'(x) = \zeta(x)$.

→ Fix the gauge

$$\varphi^u(x) = \exp\left[-\frac{i\zeta(x)}{v}\right] \varphi(x)$$

$$= \frac{1}{\sqrt{2}} \left(v + \eta(x)\right)$$

$$\boxed{A_\mu \to A_\mu^\varphi - \frac{1}{g} \frac{\partial_\mu \theta}{v}}$$

i.e the phase away the d term or $\zeta(x)$ ⇒ goldstone mode.

define

$$B_\mu(x) = A_\mu(x) - \frac{1}{qv} \partial_\mu \zeta(x)$$

$\rightarrow$ Proof $\quad D_\mu \varphi = (\partial_\mu - ig A_\mu)\varphi$

$$= \left\{\partial_\mu - ig\cdot\left(B_\mu(x) + \frac{1}{qv}\partial_\mu \zeta(x)\right)\right\} e^{i\zeta/v}\varphi^u$$

$$= e^{i\zeta/v}\left(\partial_\mu \varphi^u + \frac{i}{v}\partial_\mu\zeta \right.$$

$$\left. - ig B_\mu \varphi^u \right)$$

$$= \frac{\exp[-i\zeta/v]}{\sqrt{2}} \left\{\partial_\mu \eta(x) - ig B_\mu (v+\eta)\right\}$$

$$|D_\mu \varphi|^2 = \frac{1}{2}\left|\partial_\mu \eta - ig B_\mu (v+\eta)\right|^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$= \partial_\mu B_\nu - \partial_\nu B_\mu$$

The Lagrangian

$$\mathcal{L} = |D_\mu \varphi|^2 - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - V(\varphi)$$

$$= \frac{1}{2}\left|\partial_\mu \eta - ig B_\mu (v+\eta)\right|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+ \frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4$$

$$\left\{ V(\varphi) = -\mu^2|\varphi|^2 + \lambda|\varphi|^4 \right\}$$

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free part

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \eta)^2 - \mu^2\eta^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + \frac{1}{2} g^2 v^2 B_\mu B^\mu$$ ✓

$$\mathcal{L}_1 = \frac{1}{2} g^2 B_\mu B^\mu \, 2v\eta + \frac{1}{2} g B_\mu B^\mu \eta^2 - \lambda v^2 \eta^3 - \frac{1}{4}\lambda \eta^4$$ ✓

→ $\mathcal{L}_0$ has a massive vector boson of mass $M = gv$, a scalar with mass ✓ $\sqrt{2}\mu$. The field $\xi(x)$ completely disappeared from $\mathcal{L}$.

✓

→ Counting of d.o.f → $A_\mu \to 2$ d.o.f
$\phi_1, \phi_2 \to 2$ d.o.f

now → $B_\mu \to 3$ d.o.f (three polarization)
one scalar $\eta(x) \to 1$ d.o.f

→ ✓ $\xi(x)$ ~~gives mass~~ is eaten up by A_μ to give mass to B_μ.

✓ → Higgs mechanism in Abelian case.

SSB-4

Standard Model - Non-abelian extension

$G \longrightarrow H$ (G breaks to subgroup H)

$n_G - n_H$ number of broken generators.

They coupled with $n_G - n_H$ number of massless vectors bosons.

~~for large k, divergence, as 2nd term in~~

· Standard Model:

$SU(3) \times SU(2)_L \times U(1)_Y$

→ $SU(3)$ unbroken

→ $SU(2) \times U(1)_Y \to U(1)_{em}$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \to U \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{where } U = \exp[i \tfrac{\sigma^a}{2} \alpha^a(x)]$$

→ $SU(3)$ unbroken

$\phi^+ \to 2$ d.o.f

$\phi^0 \to 2$ d.o.f

→ $$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$$

→ Individually $$\sigma_1 \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\sigma_2 \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} -iv \\ 0 \end{pmatrix}$$

$$\sigma_3 \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} - v \\ v \end{pmatrix}$$

→ But one can find out what linear combination of $\alpha_i \sigma_i + \frac{y}{2} \mathbb{I}$ will be unbroken

$$\left[\frac{\alpha_1}{2} \sigma_1 + \frac{\alpha_2}{2} \sigma_2 + \frac{\alpha_3}{2} \sigma_3 + \frac{y}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \left[\begin{pmatrix} \alpha_3/2 & \frac{\alpha_1}{2} - \frac{i\alpha_2}{2} \\ \frac{\alpha_1 + i\alpha_2}{2} & -\frac{\alpha_3}{2} \end{pmatrix} + \frac{y}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Q3

$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ complex scalar doublet ✓

under $SU(2)$ $\quad \phi \to V\phi$ ✓

$U(1)_Y$ $\quad Y_H = +1, \quad Y_L = -1$ ✓

$Y_R = -2$

$\phi \to e^{i\alpha Y_H/2}\phi = e^{i\alpha/2}\phi$ ✓

$U(1)_{em}$ charge $q_{em} = T_3 + Y/2$ ✓

$\phi^+ \to \frac{1}{2} + \frac{1}{2} = 1$ ✓

$\phi^0 \to -\frac{1}{2} + 1/2 = 0$ ✓

Kinetic term $\quad (D_\mu\phi)^\dagger (D^\mu\phi) \to (D_\mu\phi)^\dagger V^\dagger V (D^\mu\phi)$ ✓

$= (D_\mu\phi)^\dagger (D^\mu\phi)$

$D_\mu\phi = \left(\partial_\mu - ig W_\mu^i \frac{\tau_i}{2} - ig' \frac{Y_H}{2} B_\mu(x) \mathbb{1}\right)$ ✓

$V(\phi^\dagger\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$ ✓

Minima at $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ ✓

$SU(2)_L \times U(1)_Y$ both do not leave the configuration invariant.

$\phi_0 \xrightarrow{SU(2)} V\phi_0 \neq \phi_0$ ✓

$\xrightarrow{U(1)_Y} e^{i\alpha/2}\phi_0 \neq \phi_0$.

$$\begin{pmatrix} \frac{\alpha_3 + Y/2}{2} & \frac{\alpha_1 - i\alpha_2}{2} \\ \frac{\alpha_1 + i\alpha_2}{2} & \frac{-\alpha_3}{2} + Y/2 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(\alpha_1 - i\alpha_2)v}{2} \\ -\frac{\alpha_3}{2} v + Y/_2 v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\frac{\alpha_3}{2} = Y/_2$$

$$(\alpha_1 - i\alpha_2)v = 0$$

$$\Rightarrow \alpha_1 = 0, \quad \alpha_2 = 0$$

$\therefore$ $\left(\frac{\sigma_3}{2} + \frac{Y}{2}\right)$ Remains unbroken

$$\downarrow$$

$$T_3 \rightarrow \begin{pmatrix} Y_2 \\ -Y_2 \end{pmatrix}$$

$\therefore$ $T_3 + Y/2 = Q$ leaves ϕ_0 invariant.

This corresponds to the generator of $U(1)_{em}$

$$e^{iQ\alpha(x)} = e^{i\left(\frac{\sigma_3}{2} + Y/2\right)\alpha(x)}$$

$$= e^{i\alpha(x)/2} \begin{pmatrix} e^{i\alpha(x)/2} & \\ & e^{-i\alpha(x)/2} \end{pmatrix}$$

$$e^{iQ\alpha(x)} \phi_0 = \begin{pmatrix} e^{i\alpha(x)} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v \end{pmatrix} = \phi_0$$

A linear combination of $SU(2) \times U(1)_Y$ generators keep φ_0 invariant.

$$\mathcal{L}_{Higgs} = |\partial_\mu \varphi|^2 - V(\varphi^\dagger \varphi)$$

$$V(\varphi^\dagger \varphi) = -\mu^2 |\varphi^\dagger \varphi| + \lambda (\varphi^\dagger \varphi)^2$$

minimization gives $\varphi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix} \frac{1}{\sqrt{2}}$ with $\frac{v^2}{2} = \frac{\mu^2}{2\lambda}$

The masses of the gauge bosons are now generated by SSB.

$\rightarrow$ $|\partial_\mu \varphi|^2 = \left| \left(\partial_\mu - i g W_\mu^i \frac{\sigma_i}{2} - i g' \frac{Y_H}{2} B_\mu \mathbb{1} \right) \phi \right|^2$

$\longrightarrow$ evaluate at φ_0

$$\frac{1}{8} \begin{pmatrix} 0 & v \end{pmatrix} \left[g W_\mu^i \tau^i + g' B_\mu \mathbb{1} \right] \left[g W_\mu^i \tau^i + g' B_\mu \mathbb{1} \right] \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{v^2}{8} \left[\left(g^2 W_\mu^{1\,2} + g^2 W_\mu^{2\,2} \right) + \left(g' B_\mu - g W_\mu^3 \right)^2 \right]$$

$$= \frac{v^2}{4} \left[g^2 W_\mu^+ W^{\mu -} + \frac{1}{2} (g^2 + g'^2) Z_\mu Z^\mu \right]$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$\cos\theta_W =$

$\sin\theta_W =$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

$$M_W^2 = \frac{g^2 v^2}{4} \quad \& \quad M_Z^2 = \frac{v^2 (g^2 + g'^2)}{4}$$

Extra

SM Higgs potential

$$\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4\end{pmatrix} = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix}$$

$Y_\varphi = +1$ $\qquad Q = t_3 + Y/2$

$$Q(\varphi^+) = \frac{1}{2} + \frac{1}{2} = 1$$

$$Q(\varphi^0) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\begin{pmatrix}\varphi^{++} \\ \varphi^+\end{pmatrix} \qquad 2 = \frac{1}{2} + Y/2$$

$$\Rightarrow Y/2 = 2 - 1/2$$

$$= \frac{4-1}{2} = 3/2$$

$$Y = 3$$

$$1 = -1/2 + 3/2 = 1$$

$$\begin{pmatrix}\varphi^0 \\ \varphi^-\end{pmatrix} \qquad 0 = 1/2 + Y/2 \qquad Y = -1$$

$$1 = -1/2 + Y/2 \Rightarrow Y/2 = 3/2$$

$$-1$$

$$|\varphi|_0 = \begin{pmatrix}0 \\ v/\sqrt{2}\end{pmatrix} \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$

or

$$\sigma_1 |\varphi|_0 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}0 \\ v/\sqrt{2}\end{pmatrix} \neq 0$$

$$\sigma_2 |\varphi_0| \neq 0$$

$$\sigma_3 |\varphi_0| \neq 0 \,, \qquad \left(\frac{\sigma_3}{2} - \frac{Y}{2}\right)\begin{pmatrix}0 \\ v/\sqrt{2}\end{pmatrix} \neq 0$$

$$Q_{em} \longrightarrow \text{But } \underline{\left(\frac{\sigma_3}{2} + Y/2\right)}\begin{pmatrix}0 \\ v/\sqrt{2}\end{pmatrix} = 0$$

AM

$\sigma_1, \sigma_2, \sigma_3$ generators break SU(2) symmetry

$$\phi_0 \xrightarrow[SU(2)]{} V\phi_0 \neq \phi_0 \rightarrow (1)$$

$$\phi_0 \xrightarrow[U(1)_Y]{} e^{iY\theta/2}\phi_0 \neq \phi_0 \rightarrow (2)$$

(1) Can be understood as $\sigma_1 \phi_0 \neq 0$

$\sigma_2 \phi_0 \neq 0$

$\sigma_3 \phi_0 \neq 0$

But a linear combination $\frac{\sigma_3}{2} + Y/2$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\left(\frac{\sigma_3}{2} + \frac{Y}{2}\right)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This is q_{em}.

$\therefore$ ~~as~~ the $U(1)_{em}$ is unbroken.

photon is massless.

$$A_\mu = \frac{g W_\mu^3 + g' B_\mu}{\sqrt{g^2 + g'^2}}$$ remains massless

bcz of unbroken $U(1)$ em.

$$\rightarrow \frac{M_Z^2}{M_W^2} = \frac{g^2 + g'^2}{g^2} = \frac{1}{\cos^2\theta_W}$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Masses of the leptons

The Higgs Mechanism also helps to give masses to the leptons (and quarks)

The gauge invariant combination which is the yukawa Lagrangian is

$$\mathcal{L}_{yuk} = -\lambda_e \left[\bar{L}\phi e_R + \bar{e}_R \phi^\dagger L \right]$$

$[L] \to 3/2$
$[e_R] \to 3/2$
$[\phi] \to 1$

{ L has $Y_L = -1$
e_R " $Y_R = -2$ }

$\therefore$ λ_e is dimensionless

$\longrightarrow$ $\bar{L}\phi \xrightarrow{V} (\bar{L}V^\dagger)(V\phi) = \bar{L}\phi$ under $SU(2)$

$e_R \xrightarrow{V} e_R$ [As e_R is $SU(2)$ singlet]

$\longrightarrow$ $\bar{L}\phi e_R$ is $SU(2)$ invariant.

$\longrightarrow$ This is also invariant under $U(1)_Y$.

$L \xrightarrow{\alpha} e^{iY_L/2\,\alpha(x)} L = e^{-i/2\,\alpha(x)} L$

$e_R \xrightarrow{\alpha} e^{iY_R/2\,\alpha(x)} e_R = e^{-i\alpha(x)} e_R$

$$\bar{L}\varphi e_R \longrightarrow e^{i/2\,\alpha(x)}\, e^{+i\alpha(x)/2}\, e^{-i\alpha(x)} (\bar{L}\varphi e_R)$$

$$= \bar{L}\varphi e_R$$

↳ $U(1)_Y$ invariant

The 's

→ Let us expand the term around minima

$$\varphi = \varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$\mathcal{L}_{Yuk} = -\frac{\lambda_e}{\sqrt{2}} v(\bar{e}_L e_R + \bar{e}_R e_L)$$

$$-\frac{\lambda_e}{\sqrt{2}} h \bar{e} e$$

↓ Dirac mass term for the electron

→ $m_e = \dfrac{\lambda_e v}{\sqrt{2}}$, i.e $\lambda_e = m_e \sqrt{2}/v$

→ Similarly, we can have mass terms for muons & tau

$$m_\mu = \lambda_\mu v/\sqrt{2} \quad \& \quad m_\tau = \frac{\lambda_\tau v}{\sqrt{2}}$$

The mass hierarchy of the leptons is due to the hierarchy of the yukawa couplings.

→ Interaction of h (Higgs field) with fermions are proportional to the mass.

Quark Mass and Generation Mixing

The yukawa Lagrangian

$$\mathcal{L} = -\lambda_d \left(\bar{q}_L \phi\, d_R\right) + h.c.$$

q_L $(Y_L = 1/3)$

$Y_R^u = 4/3$

$Y_R^d = -2/3$

→ $SU(3)$ is contracted between $\bar{q}_L^a d_R^a$

→ for $SU(2)$ $\bar{q}_L \phi\, d_R \to \bar{q}_L U^\dagger U \phi\, d_R$

$= \bar{q}_L \phi\, d_R$

→ $U(1)_Y$ conserved $-1/3 + 1 - 2/3$

$= 0$

$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$

$u \to 2/3$

$d \to -1/3$

$q = T_3 + Y/2$

Down quark mass can be obtained from The

$\lambda_d \bar{q}_L \phi d_R = \bar{d}_L d_R v \lambda_d$

→ how up quark obtain mass?

→ $\mathcal{L}_{yuk} = -\lambda_u \bar{q}_L \tilde{\phi} u_R + h.c$

$\tilde{\phi} = i\sigma_2 \phi^*$

→ A different gauge invariant term generate the masses of up type of quark.

→ Ex: Prove that $\bar{q}_L \tilde{\phi} u_R$ is SU(2) invariant.

use: $V^T \sigma_2 = \sigma_2 V^\dagger$

→ evaluating $\mathcal{L}_{yuk}$ around ϕ_0

$$-\frac{\lambda_u v}{\sqrt{2}} \left(\bar{u}_L u_R + \bar{u}_R u_L\right)$$

$$= -\frac{\lambda_u v}{\sqrt{2}} \bar{u} u.$$

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The most general gauge invariant and renormalizable yukawa couplings

$$\mathcal{L}_{yuk} = -\lambda_d^{ij}\left[\bar{q}_L^{\,i}\varphi\, d_R^{\,j}\right] - \lambda_d^{ij*}\left[\varphi^{\dagger}\bar{d}_R^{\,j} q_L^{\,i}\right]$$

$$-\lambda_u^{ij}\left[\bar{q}_L^{\,i}\, i\sigma_2\varphi^{*}\, u_R^{\,j}\right]$$

$$-\lambda_u^{ij*}\left[\bar{u}_R^{\,j}\,\varphi^{T}(i\sigma_2)\, q_L^{\,i}\right]$$

→ How we generate masses of the quarks?

→ Why for leptons, we do not have λ^{ij}.

→ λ_d^{ij} and λ_u^{ij} can in general be arbitrary complex numbers.

→ λ_d & λ_u might not be symmetric or hermitian

→ The off diagonal elements of λ_d & λ_u mix between generations.

Evaluate at the minimum $\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\mathcal{L}_{yuk} = -\lambda_d^{ij} v \bar{d}_L^i d_R^j + h.c$$
$$- \lambda_u^{ij} v \bar{u}_L^i u_R^j + h.c$$

for $i \neq j \rightarrow$ mixing term between different generations. u_L^i & u_R^j are not the mass eigenstates.

→ i) On the propagator the effect will be $(\delta^{ij} + M^{ij})\bar{\chi}^i \chi^j$
↓ analogue of $\not{p}$

ii) As $M^{ij} \bar{\chi}^i \chi^j$ not diagonal, we need to go to a basis where we have a diagonal mass term.

→ Diagonalize $\lambda_d^{ij} v = M^{d\,ij}$ and
$\lambda_u^{ij} v = M^{u\,ij}$

as

$$(W_d^\dagger)^{ij} d_R^j = \tilde{d}_R^i \,, \quad V_d^\dagger d_L = \tilde{d}_L$$

$$W_u^\dagger u_R = \tilde{u}_R \,, \quad V_u^\dagger u_L = \tilde{u}_L$$

→ The Yukawa mass term

$$\frac{v}{\sqrt{2}} \left[\bar{d}_L \lambda_d d_R + h.c \right]$$

$$= \left[\bar{d}_L V_d D_d W_d^\dagger d_R + h.c \right] v/\sqrt{2}$$

$$= \frac{v}{\sqrt{2}} \left(\bar{\tilde{d}}_L D_d \tilde{d}_R + h.c \right]$$

$$= \frac{v}{\sqrt{2}} \left[\bar{\tilde{d}}_L^i D_d^{ii} \tilde{d}_R^i + h.c \right]$$

$$= \frac{v}{\sqrt{2}} \bar{\tilde{\mathcal{D}}} D_d \tilde{D}$$

where

$$\tilde{D} = \begin{pmatrix} \tilde{d}_L^1 + \tilde{d}_R^1 & & \\ & \tilde{d}_L^2 + \tilde{d}_R^2 & \\ & & \tilde{d}_L^3 + \tilde{d}_R^3 \end{pmatrix},$$

→ Similarly

$$\frac{v}{\sqrt{2}} \left(\bar{u}_L \lambda_u u_R + h.c \right)$$

$$= \frac{v}{\sqrt{2}} \Big(\underbrace{\bar{\tilde{u}}_L}_{1\times 3} \underbrace{D_u}_{3\times 3 \text{ diagonal matrix}} \underbrace{\tilde{u}_R}_{3\times 1} + h.c \Big)$$

CS CamScanner

4

Statement : Given an arbitrary complex matrix M, it can be diagonalised by a biunitary diagonalization

↳ two unitary matrix.

→ It is possible to find two unitary matrices V & W such that

$$M = V D W^{\dagger}$$

↳ Real positive diagonal matrix.

↳ Complex matrix.

D → semi-definite.

→ Apply this to the matrices λ_d and λ_u

$$\lambda_d = V_d D_d W_d^{\dagger}$$

$$\lambda_u = V_u D_u W_u^{\dagger}$$

→ We re-define our fields d_L^i, d_R^i, u_L^i & u_R^i

Kinetic term

$$\bar{u}_L i \not\partial u_L^i + \bar{d}_L i \not\partial d_L^i + \bar{u}_R i \not\partial u_R^i + \bar{d}_R i \not\partial d_R^i$$

$= \mathcal{L}$

→ This comes from

$$\bar{q}_L \not\partial q_L + \bar{u}_R \not\partial u_R + \bar{d}_R \not\partial d_R$$

$$= i\left(\bar{\tilde{u}}_L^i \not\partial \tilde{u}_L^i + \bar{\tilde{u}}_R^i \not\partial \tilde{u}_R^i + \bar{\tilde{d}}_L^i \not\partial \tilde{d}_L^i + \bar{\tilde{d}}_R^i \not\partial \tilde{d}_R^i \right)$$

→ The kinetic term has the same form in the mass and flavor basis.

What happens with other interaction terms?

quarks interact with other quarks, gauge bosons via $SU(3)_c$, $SU(2)_L$ & $U(1)_Y$

$SU(3)_c$ → triplet under $SU(3)$

$$\begin{pmatrix} u^1 \\ u^2 \\ u^3 \end{pmatrix}$$

$SU(2)_L$ → doublet $\begin{pmatrix} u \\ d \end{pmatrix}_L$

$U(1)_Y$ → has non-zero hypercharge.

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Hence we have

$$M_d^i \left(\bar{\tilde{d}}_L^i \, \tilde{d}_R^i + \bar{\tilde{d}}_R^i \, \tilde{d}_L^i \right)$$
$$+ M_u^i \left(\bar{\tilde{u}}_L^i \, \tilde{u}_R^i + \bar{\tilde{u}}_R^i \, \tilde{u}_L^i \right).$$

The mass matrix

$$M_d^i = (D_d)^{ii} \frac{v}{\sqrt{2}}$$

$$M_u^i = (D_u)^{ii} \frac{v}{\sqrt{2}}$$

By using the freedom of redefining the fields, λ_d & λ_u have transformed into a diagonal term.

$$W_d^\dagger d_R = \tilde{d}_R, \quad V_d^\dagger d_L = \tilde{d}_L$$
$$W_u^\dagger u_R = \tilde{u}_R, \quad V_u^\dagger u_L = \tilde{u}_L$$

Let us look at the effect of redefinition on other terms of Lagrangian

Interaction terms in the Lagrangian,

SU(3)$_c$:   Coupling is via

(f : family) $\sum_f \bar{q}^f_L \not{D} q^f_L + \bar{d}^f_L \not{D} d^f_L + \bar{u}^f_R \not{D} u^f_R$

$= 1,2,3$ ↳ has a gluon component

$\longrightarrow \bar{u}^f_L \gamma^\mu (G^\alpha_\mu T_\alpha) u^f_L + \bar{u}^f_R \gamma^\mu (G^\alpha_\mu T^\alpha) u^f_R$

$+ \bar{d}^f_L \gamma^\mu (G^\alpha_\mu T_\alpha) d^f_L + \bar{d}^f_R \gamma^\mu (G^\alpha_\mu T_\alpha) . d^f_R$

$\bar{u}^f_L \gamma^\mu (G^\alpha_\mu T_\alpha) u^f_L$ is actually

$\bar{u}^{f,a}_L \gamma^\mu (G^\alpha_\mu T_\alpha)_{ab} u^{f,b}_L$

where a, b are 1,2,3 color indices.

$\longrightarrow$ under field redefinition

$\bar{\tilde{u}}^f_L \gamma^\mu (G^\alpha_\mu T_\alpha) \tilde{u}^f_L$ as $V_u^\dagger V_u = \mathbb{1}$

$\longrightarrow$ Color : interaction with gluon remains invariant under basis redefinition.

$U(1)_{em}$:

interaction with photon

$$ie_0\left[\tfrac{2}{3}\left(\bar{u}_L^f \gamma^\mu u_L^f\right)A_\mu + \tfrac{2}{3}\left(\bar{u}_R^f \gamma^\mu u_R^f\right)A_\mu\right]$$

$$+ ie_0\left[\left(-\tfrac{1}{3}\right)\left[\left(\bar{d}_L^f \gamma^\mu d_L^f\right)A_\mu + \left(\bar{d}_R^f \gamma^\mu d_R^f\right)A_\mu\right]\right]$$

$$= ie_0\,\tfrac{2}{3}\left(\bar{\tilde{u}}_L^f \gamma^\mu \tilde{u}_L + \bar{\tilde{u}}_R^f \gamma^\mu \tilde{u}_R^f\right)A_\mu$$

$$- ie_0\,\tfrac{1}{3}\left(\bar{\tilde{d}}_L^f \gamma^\mu \tilde{d}_L^f + \bar{\tilde{d}}_R^f \gamma^\mu \tilde{d}_R^f\right)A_\mu$$

$$= ie_0\,\tfrac{2}{3}\left(\bar{\tilde{u}}^f \gamma^\mu \tilde{u}^f\right)A_\mu - ie_0\,\tfrac{1}{3}\left(\bar{\tilde{d}}^f \gamma^\mu \tilde{d}^f\right)A_\mu$$

$\longrightarrow$ Remains unchanged under field redefinition.

Remember

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$$

$$Z_\mu = \frac{g W_\mu^3 - B_\mu g'}{\sqrt{g^2 + g'^2}}$$

$$W_\mu^\mp = \frac{W_\mu^1 \pm W_\mu^2}{\sqrt{2}}$$

$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W$

$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W$

$\frac{\sin\theta_W}{\cos\theta_W} = \frac{g'}{g}$

$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$

$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

$g\sin\theta_W = e_0 = g'\cos\theta_W$

Z_μ :

$$k_1\, \bar{u}_L^f \gamma^\mu Z_\mu u_L^f + k_2 (\bar{u}_R^f \gamma^\mu Z_\mu u_R^f)$$

$$+ k_3\, \bar{d}_R^f \gamma^\mu Z_\mu d_R^f + k_4\, \bar{d}_L^f \gamma^\mu Z_\mu d_L^f .$$

Again $L-L$ & $R-R$ & same for every generation

$\longrightarrow$ will be unchanged under the field redefinition

* $W_\mu^\pm$: $W_\mu^\pm$ couples d_L & u_L.

for n-generation the coupling to $W_\mu^\pm$ is

$$\left\{ (\bar{u}_L^i \gamma^\mu d_L^i) W_\mu^+ + (\bar{d}_L^i \gamma^\mu \bar{u}_L^i) W_\mu^- \right\}$$

$\hookrightarrow$ Charged current interaction as this can be written as

$$(J_\mu^- W^{\mu +} + J_\mu^+ W^{\mu -})$$

$$\hookrightarrow (\bar{u}_L^i \gamma^\mu d_L^i) \quad (\bar{d}_L^i \gamma^\mu u_L^i).$$

→ This term is diagonal in flavor basis.

→ Now let us do basis redefinition.

→ $\hookrightarrow \left[\bar{\tilde{u}}_L (V_u^\dagger V_d) \gamma^\mu \tilde{d}_L \right] W_\mu^+$

$+ \left[\bar{\tilde{d}}_L \left(V_d^\dagger V_u\right) \gamma^\mu \tilde{u}_L \right] W_\mu^-$

Not diagonal in mass eigenstate.

→ $\left[\bar{\tilde{u}}_L^i V_{ij} \gamma^\mu \tilde{d}_L^j \right] W_\mu^+$

$+ \text{h.c}$

$V = V_u^\dagger V_d$ is an unitary 3×3 matrix.

↳ This is called CKM Matrix.

→ for n generation, we have $n\times n$ unitary matrix

V has n^2 independent entries (complex)

<u>Counting</u>

$$\left.\begin{array}{l} (n-1)^2 \text{ real parameters} \\ \dfrac{(n-1)(n-2)}{2} \text{ phases} \end{array}\right\}$$

$$V \sim \begin{pmatrix} 0.97 & 0.22 & 0.0037 \\ 0.22 & 0.97 & 0.04 \\ 0.008 & 0.04 & \sim 0.99 \end{pmatrix}$$

In general V can be parametrised as

$$\begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} c_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

θ_{12}, θ_{23}, θ_{13} are mixing angles

$\delta \longrightarrow$ phase.

→ ~~for leptonic~~

→ Since we can measure only V_{CKM}, we consider $\tilde{u}_L = u_L$ &

$$\tilde{d}_L = V_{CKM}\, d_L$$

i.e mixing is only in down type of quark.

→ for 2 flavor, only one angle θ_c & no-phase

→ $n=2$, $V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$

θ_c = cabibo angle.

→ $\tilde{d}_L = \cos\theta_c\, d_L + \sin\theta_c\, s_L$

$\tilde{s}_L = -\sin\theta_c\, d_L + \cos\theta_c\, s_L$

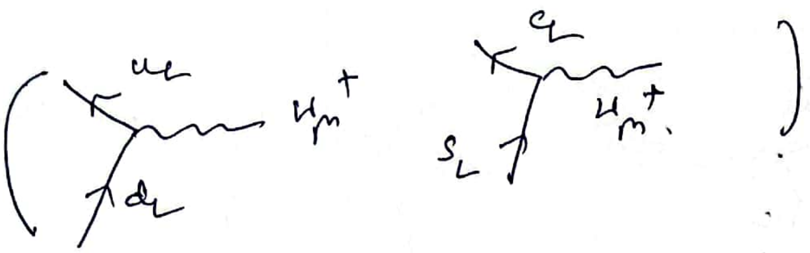
C.C interaction

$$\frac{e_0}{\sqrt{2} S_W}\left[\bar{\tilde{u}}_L \gamma^\mu \tilde{d}_L \cos\theta_c + \bar{\tilde{u}}_L \gamma^\mu \tilde{s}_L \sin\theta_c\right] W^+_\mu$$

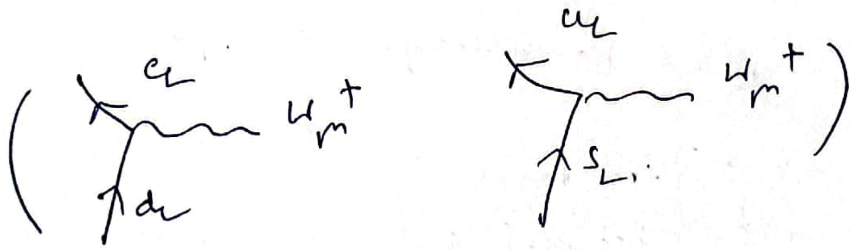
$$+ \frac{e_0}{\sqrt{2} S_W}\left[\bar{\tilde{c}}_L \gamma^\mu \tilde{d}_L (-\sin\theta_c) + \bar{\tilde{c}}_L \gamma^\mu \tilde{s}_L \cos\theta_c\right] W^+_\mu$$

mixing in mass basis.

$$\begin{pmatrix} \tilde{u}_L & \tilde{c}_L \\ \tilde{d}_L & \tilde{s}_L \end{pmatrix}$$



$\propto \cos\theta_c$

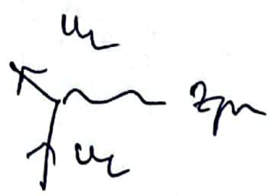


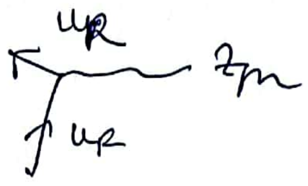
$\propto \sin\theta_c$

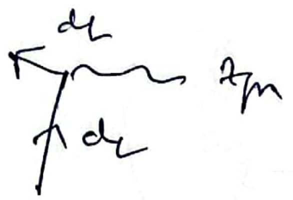
Experimentally $\cos\theta_c \sim 0.97$

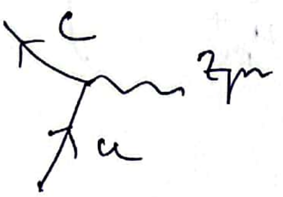
$\longrightarrow$ experimentally what has been observed is there is mixing in first two generations (d, s) with very little mixing with the 3rd generation (b)

[This mixing doesn't occur in neutral current, as interaction is diagonal in flavor & mass basis. There is no flavor changing neutral current]









Z_μ prohibited

$\bar{b}\gamma^\mu s \, Z_\mu$

Rare Kaon decays

$K_L \to \mu^+\mu^-$,

$K^+ \to \pi^+ \nu\bar{\nu}$

$K_L \to \pi^0 \nu\bar{\nu}$

$K_L \to \pi^0 e^+ e^-$

$B^0 \to \mu^+\mu^-$

$b \to s$
$s \to d$
] tree level not possible in SM

] via loops

(1)

Neutrino Mass

What has been observed

$\nu_e \longrightarrow \nu_\mu \longrightarrow \nu_\tau$ and vice-versa.

$$P(\nu_e - \nu_\mu) \sim \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \sin^2 2\theta$$

To have non-zero probability, we need both

$\theta \neq 0$ & $\Delta m^2 \neq 0$, $\Delta m_{ij}^2 = m_i^2 - m_j^2$

Extend 2 generation to 3 generation $(\nu_e, \nu_\mu, \nu_\tau)$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \cancel{s_{13}e^{-i\delta}}\ s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{+i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{+i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}$$

---

from neutrino oscillation

$\Delta m_{21}^2 = (7.05 - 8.14) \times 10^{-5}\ eV^2 \rightarrow \nu_\mu \rightarrow \nu_e + \nu$

$|\Delta m_{31}^2| = (2.41 - 2.60) \times 10^{-3}\ eV^2 \rightarrow \nu_\mu - \nu_\tau$

$\sin^2\theta_{12} = 0.273 - 0.375$, $\sin^2\theta_{23} = 0.445 - 0.559$

$\theta_{12} \sim 34.5^\circ$, $\theta_{23} \sim 47.7^\circ$ & $\theta_{13} \sim 8.41^\circ$.

CamScanner

$\nu \to$ Dirac particle or Majorana

$\bar{\nu}\nu$

$(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$

$\downarrow$ No ν_R in SM

$\nu^T C^{-1} \nu$

$\downarrow$

Not gauge invariant

Q: ν_1 or ν_3 lightest mass eigenstate?

$\longrightarrow$ SMEFT $\quad \hat{O} = LLHH \dfrac{y^2}{\Lambda} \to \Delta L = 2$ unit

$$\nu^T C^{-1} \nu \left(\frac{y^2 v^2}{\Lambda}\right) \Rightarrow m_\nu \sim \frac{y^2 v^2}{\Lambda}$$

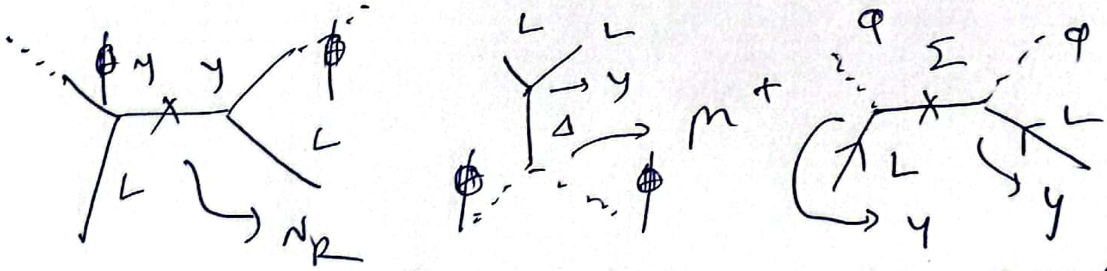
Actually

$$m_\nu = m_D^T M_R^{-1} m_D$$

---

open LLHH operator at tree level

Λ



$$\frac{1}{\not{p} - M_{N_R}} \sim \frac{\not{p} + M_{N_R}}{p^2 - M_{N_R}^2} \quad \text{for } p \ll M_{N_R}$$

$$\sim \frac{1}{M_{N_R}} Y^T Y v^2$$

2 N.M

$$\frac{1}{p^2 - M_\Delta^2}\, y\, m \sim \frac{y\, m\, v^2}{M_\Delta^2}$$

$M_\nu \sim y\, v_\Delta$; $\Delta \to (\Delta^{++}, \Delta^{+}, \Delta^{0})$

↳ $(3, +2)$ → Type-II seesaw

→ Type-I + Type-II can be embedded into a bigger symmetry group

$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

$B-L \to$ g.B

B-L

Bm

$SU(3)_C$ ↓ As SM; $SU(2)_L$ ↓ As SM; $SU(2)_R$ ↓ additional 3 gauge boson $W_R^{\mu i}$

→ ~~symmetry breaking~~

particle contents

$$L_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \& \quad L_R = \begin{pmatrix} N \\ e^- \end{pmatrix}_R$$

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \& \quad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$\Delta_L = (3, 1, +2)$

$\Delta_R = (1, 3, +2)$

$\phi = (2, 2, 0)$

$$\mathcal{L}_Y = \ell_L^T c^{-1} \Delta_L \ell_L + \ell_R^T c^{-1} \Delta_R$$

Spontaneous violation of parity:

$L_L \leftrightarrow L_R$

$q_L \leftrightarrow q_R$

$\Delta_L \leftrightarrow \Delta_R$

$W_L^i \leftrightarrow W_R^i$

$C: \{W_L, q_L, \ell_L\} \leftrightarrow \{-W_R^{\dagger}, q_R^c, \ell_R^c\}$ parity P

↳ charged conjugation.

A consequence of this symmetry is

$\bar{L}_L \phi L_L + \bar{L}_R \phi L_R$ should be

invariant under $L_L \leftrightarrow L_R$

$W^{\mu i}_L \leftrightarrow W^{\mu i}_R$

$\Rightarrow \quad g_L = g_R = g \rightarrow SU(2)$ coupling of SM.

---

symmetry breaking

$$\Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix}, \quad \Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix}$$

$$\left.\begin{array}{ll} P : & \Delta_L \leftrightarrow \Delta_R \\ & \phi \leftrightarrow \phi^\dagger \end{array}\right\} \quad \phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}$$

$$\Delta_L \longrightarrow U \Delta_L U^\dagger, \quad \Delta_R \longrightarrow V \Delta_R V^\dagger$$

$$\phi \longrightarrow U \phi V^\dagger$$

$\langle \Delta_R^0 \rangle = v_R$, $\rightarrow$ breaks $SU(2)_R \otimes U(1)_{B-L}$ symmetry

$\downarrow$

$U(1)_Y$.

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\downarrow$$

$$SU(3)_c \otimes SU(2)_L \times U(1)_Y$$

~~In this process at~~

$$\langle \phi \rangle = \mathrm{diag}\left(v_1 \, , \, -v_2 \, e^{-i\alpha}\right)$$

$$\langle \Delta_L^0 \rangle = v_L$$

→ breaks $SU(2)_L \otimes U(1)_Y$ symmetry down to $U(1)_{em}$

Neutrino mass generation

$$\mathcal{L}_Y = -\frac{y_L}{2} L_L^T C^{-1} \Delta_L L_L - \frac{y_R}{2} L_R^T C^{-1} \Delta_R L_R + \bar{L}_L \left(y_1 \phi + y_2 \phi^c\right) l_R + h.c.$$

$$\phi^c = \left(i\sigma_2 \, \phi^* \, i\sigma_2\right)$$

~~In the potential we have~~

$\cancel{\mathrm{Tr}\left[\Delta_L^\dagger \phi \, \Delta_L\right]}$

& $\mathrm{Tr}\left[\Delta_R^\dagger \phi\right]$

→ $\bar{L}_L \phi \, l_R$ → generates terms such as

$$\left(\bar{\nu}_{e_L} \;\; \bar{e}_L\right)\begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & -\phi_2^{0*} \end{pmatrix}\begin{pmatrix} N_R \\ e_R \end{pmatrix}$$

N.M

ν mass (Dirac) $\bar{\nu}_L \, N_R \langle \phi_1^0 \rangle y_1$

$+ \bar{\nu}_L \, N_R \langle \phi_2^0 \rangle (-y_2)$

$\longrightarrow \quad L_L^T c^{-1} \Delta_L L_L \longrightarrow \nu_L^T v_{\Delta_L} \nu_L c^{-1} \dfrac{y_L}{2} = M_L^{\nu} \nu_L^T c^{-1} \nu_L$

$\longrightarrow \quad L_R^T c^{-1} \Delta_R L_R \longrightarrow N_R^T v_{\Delta_R} N_R \dfrac{y_R}{2}$

$= M_{N_R} N_R^T c^{-1} N_R$

$$\begin{pmatrix} \bar{\nu}_L & \bar{N}_R^c \end{pmatrix} \begin{pmatrix} M_L & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix}$$

In the limit $M_R \gg m_D$

$m_\nu \sim M_L - m_D^T M_R^{-1} m_D$

$\hookrightarrow y_\nu v_\Delta \qquad \hookrightarrow$ Type - I seesaw.

$\hookrightarrow$ Type - II

4 N.M

Assignment

Consider

$$\bar{q}_L \left(y_1^q \phi + y_2^q \phi^c \right) q_R + h.c$$

$\longrightarrow$ Check the quark masses.

$$M_e = v \left(y_1 \langle \phi_2^0 \rangle + y_2 \langle \phi_1^0 \rangle \right)$$

Among the Gauge bosons

$$\begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} = \begin{pmatrix} 1 & \zeta^* \\ -\zeta^* & 1 \end{pmatrix} \begin{pmatrix} W^- \\ W_R^- \end{pmatrix}$$

$$Tr\left[(\partial_\mu \Delta_L)^\dagger (\partial^\mu \Delta_L) \right] + Tr\left[(\partial_\mu \Delta_R)^\dagger (\partial^\mu \Delta_R) \right] + Tr\left[(\partial_\mu \phi)^\dagger (\partial^\mu \phi) \right]$$

$\to g_L T_3^i W_\mu^{iL}$

$\to g_R T_3^j W_\mu^{iR}$

$\to g_L T_3^i W_\mu^{iL} + g_R T_3^j W_\mu^{jR}$

$\hookrightarrow$ Introduces mixing between W_L & W_R

$$\sim \left. \begin{aligned} M_{W_R}^2 &\sim g^2 v_R^2 \\ M_{W_L}^2 &\sim \frac{g^2 v^2}{2} \end{aligned} \right\}$$

(5) For neutral gauge boson

W_L^3 & W_R^3 & B_μ will mix

$$\begin{pmatrix} Z' \\ Z \\ A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{\theta_W} & -S_{\theta_W} \\ 0 & S_{\theta_W} & C_{\theta_W} \end{pmatrix} \begin{pmatrix} C_{\theta_R} & 0 & -S_{\theta_R} \\ 0 & 1 & 0 \\ S_{\theta_R} & 0 & C_{\theta_R} \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B_\mu \end{pmatrix}$$

$$\boxed{M_{Z'}^2 \simeq \frac{2 C_W^2}{C_W} M_{W_R}^2}$$

$$\boxed{M_Z^2 \simeq \frac{M_W^2}{C_W^2}}$$

A_μ massless.

How many gauge bosons get mass?

$SU(3)_C \otimes SU(2)_L \otimes \underbrace{SU(2)_R \otimes U(1)_{B-L}}$

$\downarrow U(1)_Y$

$2^2 - 1 + 1 = N_G$

$1 = n_H$

3 gauge bosons massive $W_R^\pm, Z'_\mu$

$3 + 1 = n_G$

$\downarrow$

$1 = n_H$

$\downarrow$

$\underline{SU(2)_L \otimes U(1)_Y} \longrightarrow$ 3 gauge bosons massive $\hookrightarrow W^\pm, Z$

All together

8 Gluons + 1 photon massless

Massive

$W^\pm, Z, W_R^\pm, Z'$

✓

(5) N.M C.C + N.C

$\rightarrow$ $\bar{\nu}_L^e \gamma^\mu W_\mu^- e_L\, g + \bar{\nu}_R \gamma^\mu W_\mu^R e_R\, g$

$\rightarrow$ N.C. $\bar{\nu}_L^e \gamma^\mu Z_\mu \nu_L^e\, () + \bar{\nu}_R \gamma^\mu Z_\mu \nu_R\, ()$

Parity restoration at high scale

interaction terms are parity symmetric.