The Higss Mass as fundamental parameter of the MSSM
- or m_hMSSM -

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Based on: **Rima El-Kosseifi, Gilbert Moultaka, JLK, Dirk Zerwas** **(arXiv:2202.06919)**

Zoom meeting 24/09/25

Outline

- Motivation
- Concept
- illustrations
  - -One-loop
  - -Exact one-loop
  - -Exact one-loop + (dominant) two-loop
- Discussion

Motivation

MSSM Predicts a light Higgs Boson (<140GeV)

- Interesting pheno at TeV scales

3 neutral Higgs bosons: h, A, H
1 charged Higgs boson: $H^{\pm}$
+ many supersymmetric particles

Restrict to R-Parity+CP-conserving models

- Main issue : in MSSM, physical m_h dependence on SUSY (soft-breaking) parameters is highly non-trivial, specially from large radiative correction contributions
- (depends practically on all MSSM parameters)

Generic 'low scale' model:

ㄱㅅ (p)MSSM (minimal supersymmetric standard model) has many (~22-24) parameters :

-Squarks, sleptons $\tilde{q}_{L,R}, \tilde{l}_{L,R}$ mass terms (3 generations)

- Gaugino mass terms : M_1, M_2, M_3 *for* $U(1) \otimes SU(2) \otimes SU(3)$

-Trilinear couplings : $A_t, A_b\, A_\tau$

-Higgs sector : $\mu, \tan\beta, m^2_{H_u}, m^2_{H_d}$

ㄱㅅOr more constrained high scale models
mSUGRA
AMSB
...

Goal : replace one MSSM parameter with presently best measured mass: m_h input

- Conceptually similar to m(Z) input choice in electroweak +BSM physics in early '90s (LEP 1 era)
- SUSY pheno & constraint studies : more efficient inversion algorithm (compared to general scan)

Basic concept

Large sensitivity + non-trivial $m_h(A_t)$: → from $m_h(A_t)$ to $A_t(m_h)$

- **Higgs mass matrix (diagrammatic 'fixed order' approach) : invert mh dependence on A_t**

$$M_s^2(p^2) = \begin{pmatrix} \overline{m}_{11}^2 - \Pi_{11}(p^2) + \frac{t_1}{v_1} & \overline{m}_{12}^2 - \Pi_{12}(p^2) \\ \overline{m}_{12}^2 - \Pi_{12}(p^2) & \overline{m}_{22}^2 - \Pi_{22}(p^2) + \frac{t_2}{v_2} \end{pmatrix}$$

← Tree-level →

← Loops: Π_{ij}, t_i

$$\overline{m}_{11}^2 = \overline{m}_Z^2 \cos^2\beta + \overline{m}_A^2 \sin^2\beta,$$
$$\overline{m}_{22}^2 = \overline{m}_Z^2 \sin^2\beta + \overline{m}_A^2 \cos^2\beta,$$
$$\overline{m}_{12}^2 = -\frac{1}{2}(\overline{m}_Z^2 + \overline{m}_A^2)\sin 2\beta.$$

Standard calculation: electroweak symmetry breaking (EWSB) determines iteratively:

- **Higgs mass parameter mu**
- **Pseudoscalar running mass**

$$\overline{m}_A^2(M_{EWSB}) = \frac{1}{\cos 2\beta}(\hat{m}_{H_u}^2 - \hat{m}_{H_d}^2) - \overline{m}_Z^2,$$
$$\mu^2(M_{EWSB}) = \frac{1}{2}\left((\hat{m}_{H_u}^2 \tan\beta - \hat{m}_{H_d}^2 \cot\beta)\tan 2\beta - \overline{m}_Z^2\right).$$

Add the determination of A_t from pole mass m_h :

Can only be implemented post-EWSB

- **Includes all (known) radiative corrections**

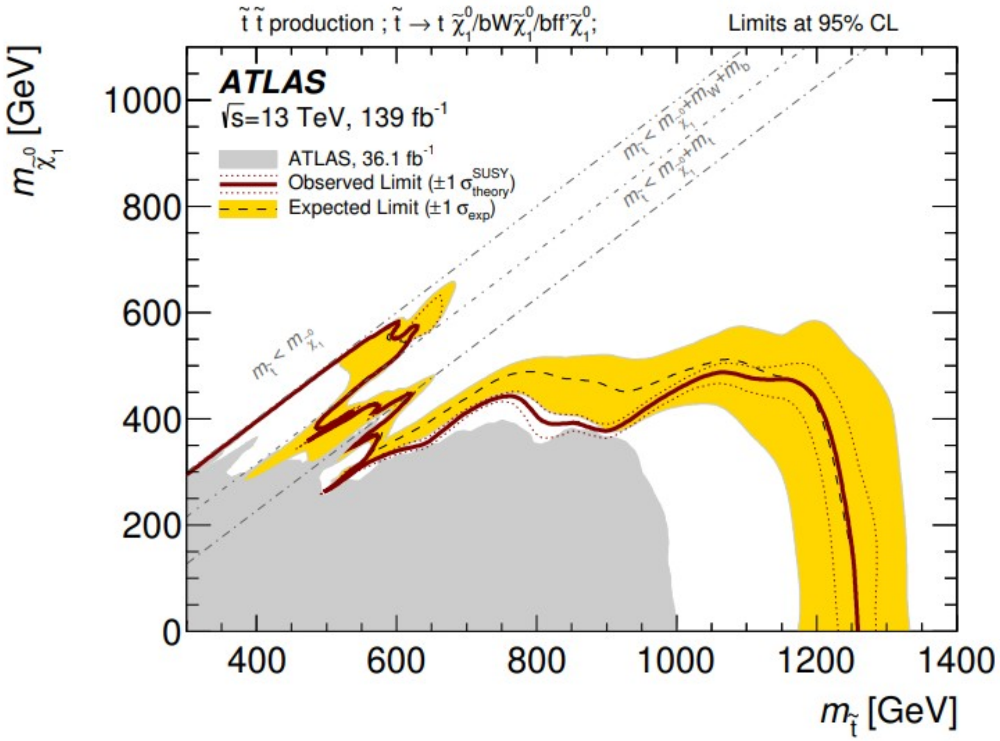
Important remark : Standard spectrum calculation already iterative :

- **RGE: high<-> low scale<-> Z scale**
- **EWSB +threshold radiative corrections**

The Stop Cliff

A test/benchmark point:

- **Heavy squarks and sleptons**
- **Light LSP (Bino)**



EW	2.0 TeV
$m^2_{H_d}$	3.65740418 TeV^2
$m^2_{H_u}$	-0.213361994 TeV^2
sign(μ)	+
A_t	3.610 TeV
$m_{\tilde{t}_R}$	1.27 TeV
$m_{\tilde{q}3_L}$	3 TeV
M_1	300 GeV
M_2	2 TeV
M_3	3 TeV
$A_b,\ A_\tau$	0 GeV
$\tan\beta$	10
$m_{\tilde{e}_L} = m_{\tilde{\mu}_L} = m_{\tilde{\tau}_L} = m_{\tilde{e}_R} = m_{\tilde{\mu}_R} = m_{\tilde{\tau}_R}$	2 TeV
$m_{\tilde{q}1_L} = m_{\tilde{q}2_L} = m_{\tilde{u}_R} = m_{\tilde{c}_R} = m_{\tilde{d}_R} = m_{\tilde{s}_R} = m_{\tilde{b}_R}$	3 TeV
m_h	125.012 GeV
$m_{\tilde{t}_1}$	1306 GeV
$m_{\tilde{\chi}_1^0}$	294 GeV

Stop sector:

- **Lightest stop at detection mass limit**
- **At=3610 GeV**

Higgs mass:

- **Experimental error ~ 0.15GeV**
- **Typical theoretical error: ~2GeV**
  (unkown higher orders, different renormalization schemes, ...)

Basic Concept : Approximate one-loop

Starts from a well-known approximation to m_h **: dominant (heavy) stop contributions** (Carena et al ‘96, Haber, Hempfling ‘97,...)**:**

$$m_h^2 = \overline{m}_h^2 + \frac{3 g_2^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12 M_S^4}\right]$$

$$M_S^2 = \sqrt{(m_{\tilde{q}3_L}^2 + (\frac{1}{2} - \frac{2}{3}s_W^2)m_Z^2 \cos 2\beta + m_t^2)} \cdot \sqrt{(m_{\tilde{t}_R}^2 + \frac{2}{3}s_W^2 m_Z^2 \cos 2\beta + m_t^2)}$$

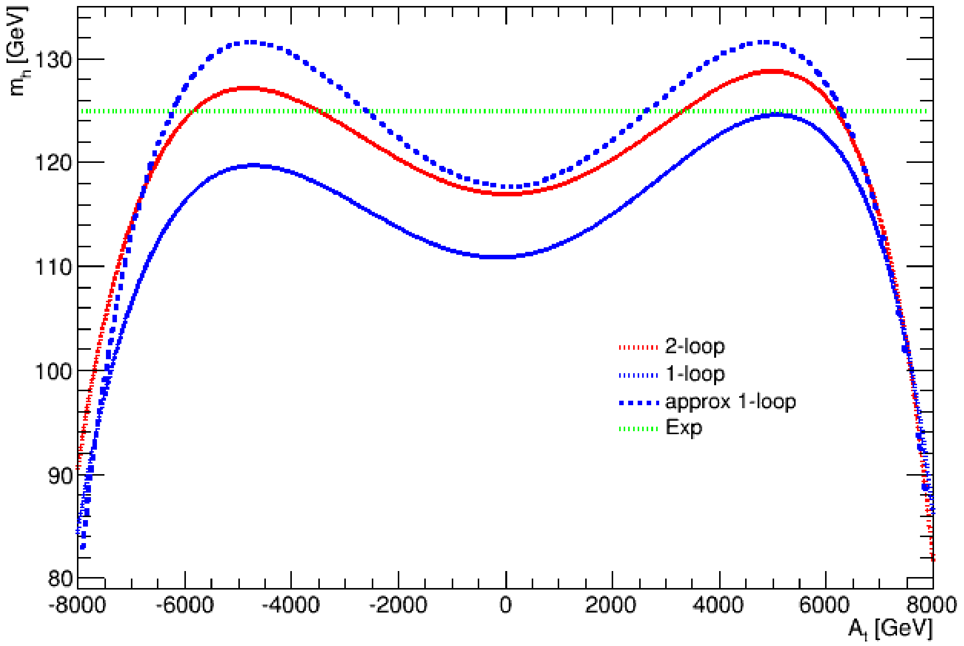
$$X_t = A_t - \mu \cot \beta$$

Approximate 1-loop:

- **Involves up to A_t^4**
- **Chosen benchmark has sizeable subdominant contributions**

Full 1-loop (Pierce et al ‘96)
+ dominant 2-loop contributions (Degrassi et al ‘01-’02)**:**

- **Structure preserved**
- **Non-negligeable contribution from 2-loop**



		Stop cliff
s1	A_t [TeV]	-5.44
s2	A_t [TeV]	-3.61
s3	A_t [TeV]	2.87
s4	A_t [TeV]	6.36

Inversion with approximate 1-loop:

- **Invertible analytically**
- **4 solutions for At**
- **But: true A_t is off by 30%**

Proof of Concept – full one-loop

-Start from full Higgs mass matrix eigenvalue equation:

$$m_{h,H}^4 - m_{h,H}^2((M_s^2)_{11} + (M_s^2)_{22}) + (M_s^2)_{11}(M_s^2)_{22} - ((M_s^2)_{12})^2 = 0,$$

-Identify explicit dependence on A_t -in h-stop-stop couplings:

$$g_{s_2t_1t_1} = c_t^2\, g_{s_2\tilde{t}_L\tilde{t}_L} + 2c_t s_t\, g_{s_2\tilde{t}_L\tilde{t}_R} + s_t^2\, g_{s_2\tilde{t}_R\tilde{t}_R}$$

$$g_{s_2t_2t_2} = s_t^2\, g_{s_2\tilde{t}_L\tilde{t}_L} - 2c_t s_t\, g_{s_2\tilde{t}_L\tilde{t}_R} + c_t^2\, g_{s_2\tilde{t}_R\tilde{t}_R}$$

$$g_{s_2t_1t_2} = s_t c_t\,(g_{s_2\tilde{t}_R\tilde{t}_R} - g_{s_2\tilde{t}_L\tilde{t}_L}) + (c_t^2 - s_t^2)\, g_{s_2\tilde{t}_L\tilde{t}_R}$$

$$g_{s_2\tilde{t}_L\tilde{t}_R} = \frac{y_t}{\sqrt{2}} A_t;$$

-in the one-loop tadpole (log treated as 'constant') :

$$A_0(m_{\tilde{t}_i}) = m_{\tilde{t}_i}^2 \left(1 - \ln\left(\frac{m_{\tilde{t}_i}^2}{Q^2}\right)\right)$$

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left(M^2 \mp \sqrt{a_s A_t^2 + b_s A_t + c_s}\right)$$

$$M^2 = m_{\tilde{q}3_L}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + \frac{1}{2} m_Z^2 \cos 2\beta,$$

$$a_s = 4m_t^2,$$

$$b_s = -8m_t^2 \mu \cot\beta,$$

$$c_s = \left(m_{\tilde{q}3_L}^2 - m_{\tilde{t}_R}^2 + \left(\frac{1}{2} - \frac{4}{3}s_W^2\right) m_Z^2 \cos 2\beta\right)^2 + 4m_t^2 \mu^2 \cot^2\beta.$$

Rewrite tadpoles + self-energies (0 : no, or log dependence):

$$\frac{t_1}{v_1} = t_1^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_1^{(0)}$$

$$\frac{t_2}{v_2} = t_2^{(1s)} A_t \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_2^{(1)} A_t + t_2^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_2^{(0)}$$

$$\Pi_{11} = \pi_{11}^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + \pi_{11}^{(0)}$$

$$\Pi_{12} = \pi_{12}^{(1)} A_t + \pi_{12}^{(0)}$$

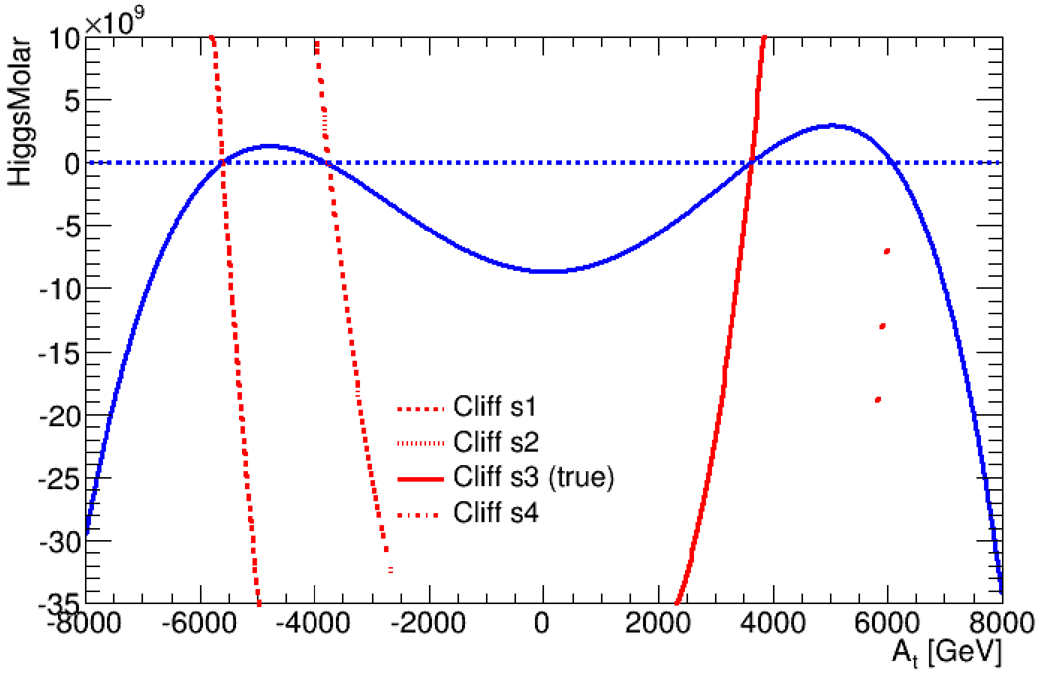
$$\Pi_{22} = \pi_{22}^{(2)} A_t^2 + \pi_{22}^{(1)} A_t + \pi_{22}^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + \pi_{22}^{(0)}$$

Proof of Concept: full one-loop

= equivalent rewriting of one-loop contributions, but defines new function:

$$\mathrm{HiggsMolar}(A_t) = C_3 A_t^3 + C_2 A_t^2 + C_1 A_t + C_0 + (R_2 A_t^2 + R_1 A_t + R_0)\sqrt{a_s A_t^2 + b_s A_t + c_s} = 0$$

m_h^2 enters C_2, C_1, C_0, R_1, R_0



$$C_0[A_t] = c_s(\pi_{11}^{(s)} - t_1^{(s)})(\pi_{22}^{(s)} - t_2^{(s)}) + (m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2)(m_h^2 + \pi_{22}^{(0)} - t_2^{(0)} - \overline{m}_{22}^2) - (\pi_{12}^{(0)} - \overline{m}_{12}^2)^2,$$

$$C_1[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(b_s(\pi_{22}^{(s)} - t_2^{(s)}) - c_s t_2^{(1s)}) + (\pi_{22}^{(1)} - t_2^{(1)})(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2) + 2\pi_{12}^{(1)}(\overline{m}_{12}^2 - \pi_{12}^{(0)}),$$

$$C_2[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(a_s(\pi_{22}^{(s)} - t_2^{(s)}) - b_s t_2^{(1s)}) + \pi_{22}^{(2)}(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2) - (\pi_{12}^{(1)})^2,$$

$$C_3[A_t] = a_s(t_1^{(s)} - \pi_{11}^{(s)})t_2^{(1s)},$$

$$R_0[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(m_h^2 + \pi_{22}^{(0)} - t_2^{(0)} - \overline{m}_{22}^2) + (\pi_{22}^{(s)} - t_2^{(s)})(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2),$$

$$R_1[A_t] = (t_1^{(s)} - \pi_{11}^{(s)})(t_2^{(1)} - \pi_{22}^{(1)}) + (t_1^{(0)} - \pi_{11}^{(0)} + \overline{m}_{11}^2 - m_h^2)t_2^{(1s)},$$

$$R_2[A_t] = \pi_{22}^{(2)}(\pi_{11}^{(s)} - t_1^{(s)}),$$

HiggsMolar:

- **Includes exact 1-loop contributions**
- **Similar form as the approximate 1-loop**
- **Zeros correspond to m_h=125GeV**
- **Four solutions**
- **A_t^3 yet 4 solutions:**

At this stage machine learning could help, how ?

C_k

Proof of Concept : 1-loop + 2-loop

Solve for A_t ?

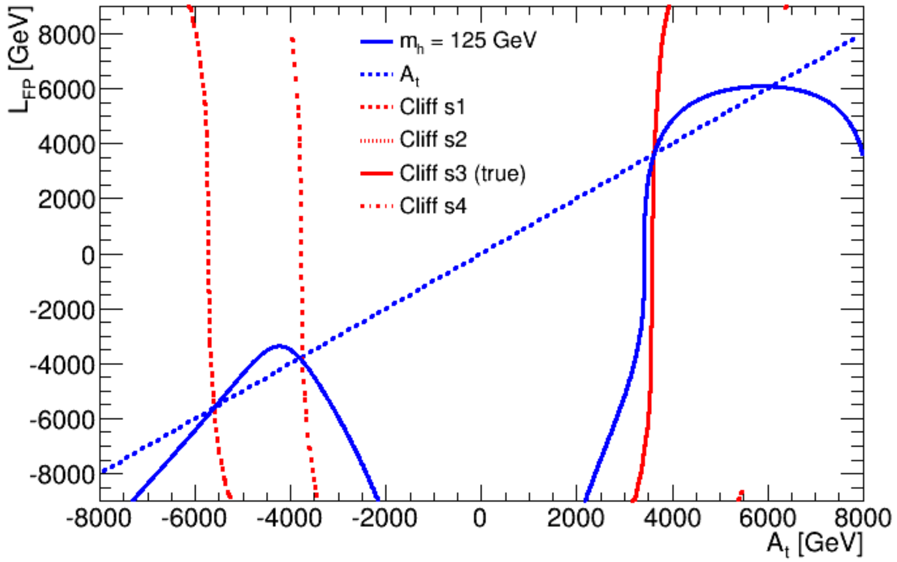
- **Not possible analytically in general...**

→ Transform Molar to a Fixed Point (FP) equation:

$$\mathrm{C_{FP}}(A_t) = -\frac{1}{C_3}[C_2A_t^2 + C_1A_t + C_0 + (R_2A_t^2 + R_1A_t + R_0)\sqrt{a_sA_t^2 + b_sA_t + c_s}],$$

$$A_t = \sqrt[3]{\mathrm{C_{FP}}(A_t)}.$$

$$\mathrm{L_{FP}}(A_t) \equiv \sqrt[3]{\mathrm{C_{FP}}(A_t)},$$



C_{FP} and LFP:

- **Convergence needs |LFP'| < 1 (against repulsive FPs etc)**
- **Tricky redefinition of parameter dependence can enforce convergence (but not always p[illegible])**

$$\mathrm{L_{FP}}_\tau(A_t) = \frac{1}{\tau}(\mathrm{L_{FP}}(A_t) - A_t) + A_t.$$

2-loop (and higher orders):

- **Enter as self-energy + tadpoles in the mass matrix**
- **A_t -dependence 'screened' by an extra loop factor ->included as 'constant' (but varies in FP iterations)**

Remnants, log(A_t) and 2-loop At dependencies:

- **Accounted exactly in FP + EWSB iterations**

Proof of Concept – Full Algorithm

Full Algorithm:

1. Complication: need to 'stabilize' top yukawa (threshold corrections at Q~Mz)
2. Use approximate 1-loop inversion as first guess
3. EWSB: add fixed point iteration on At
4. Adapt tau (convergence parameter) locally

$$L_{FP}'_{\tau}(A_t) = 1 + \frac{L_{FP}'(A_t) - 1}{\tau},$$

stop cliff	s1	s2	s3	s4
A_t [GeV]	-5617.3	-3796.1	3609.7	6082.5
m_h [GeV]	125.012	125.012	125.012	125.012

Algorithm is quite efficient:

- 0.1 permil precision reached on At
- m_h excellent (too good for practical purposes)
- A_t precision better than requested (effect of iterations)
- Everything accounted, but quite long and tricky FP algorithm : here ML could really help !

EWSB complication :

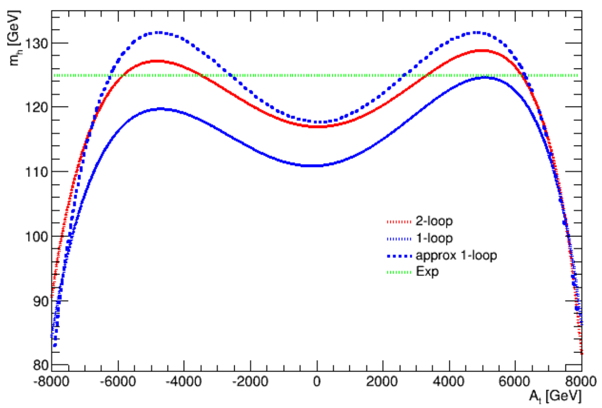
- Not uniquely defined (see SLHA) :
  -either m_{Hu}, m_{Hd}, sign(μ)
  -or $m_A(Q)$, μ
- -or pole m_A, μ

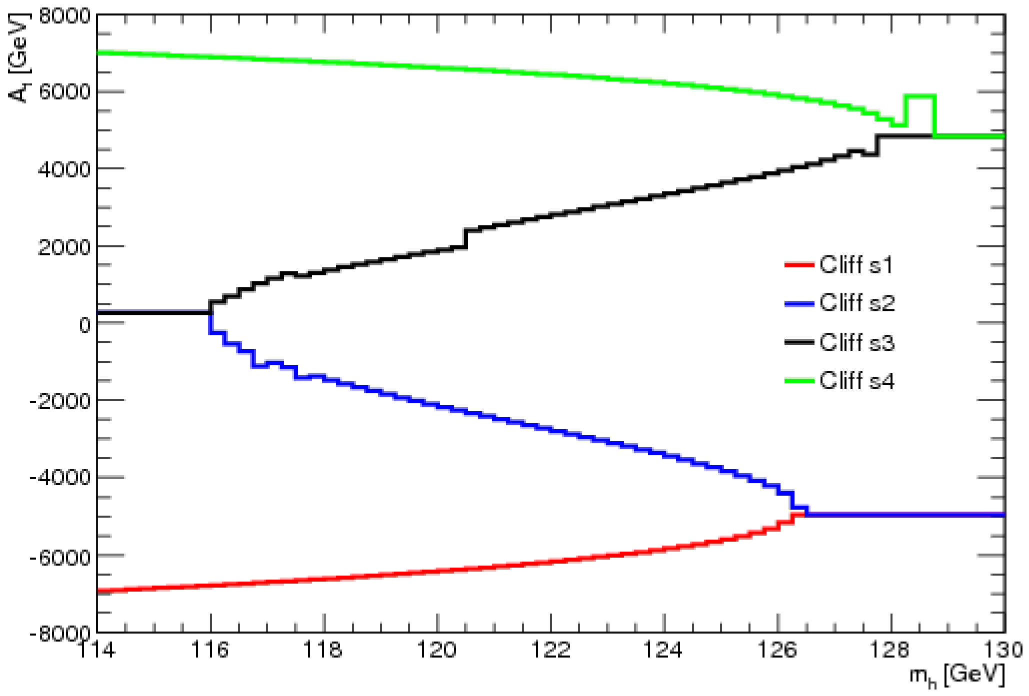
EWSB	stop cliff	s1	s2	s3	s4
$m^2_{H_d}$, $m^2_{H_u}$, sign(μ)	A_t [GeV]	-5617.8	-3795.0	3610.5	6085.9
	m_h [GeV]	125.012	125.012	125.012	125.012
$m^2_A(Q)$, μ	A_t [GeV]	-5606.9	-3795.1	3610.7	6090.1
	m_h [GeV]	125.012	125.012	125.012	125.012
m_A, μ	A_t [GeV]	-5607.2	-3794.7	3610.7	6089.9
	m_h [GeV]	125.012	125.012	125.012	125.012

It works:

- Similar precision achieved in all cases

Beyond the benchmark point





Proof of complete Inversion : more than 1 point:

- **Stepping through m_h**
- **Specifying s1, s2, s3, s4**

  But needs a stepper function applied regularly to identify the local minima and maxima in m_h
- **Close to extremal FP is complemented by a standard Bisection algorithm**

It works (better than expected):

- **Regions well separated**
- **continuous**
- **3 points (of 256) did not converge**

- **But hope that ML could improve this stage**

Inversion works: from $m_h(A_t)$ to $A_t(m_h)$!

Conclusions

Proof of concept:
- m_h as fundamental parameter of the MSSM: worked out
- Formally correct to all orders (at least for diagrammatic calculations)
- Stop cliff benchmark: works
- 1d scan: works

Extensions:
- Improve (better optimize) the algorithm
- Complete Algorithm for all configurations
- ML could really help to speed up, both at several stages,
  Or more generically with a globally different algorithm ?