Stochastic Loop Quantum Cosmology

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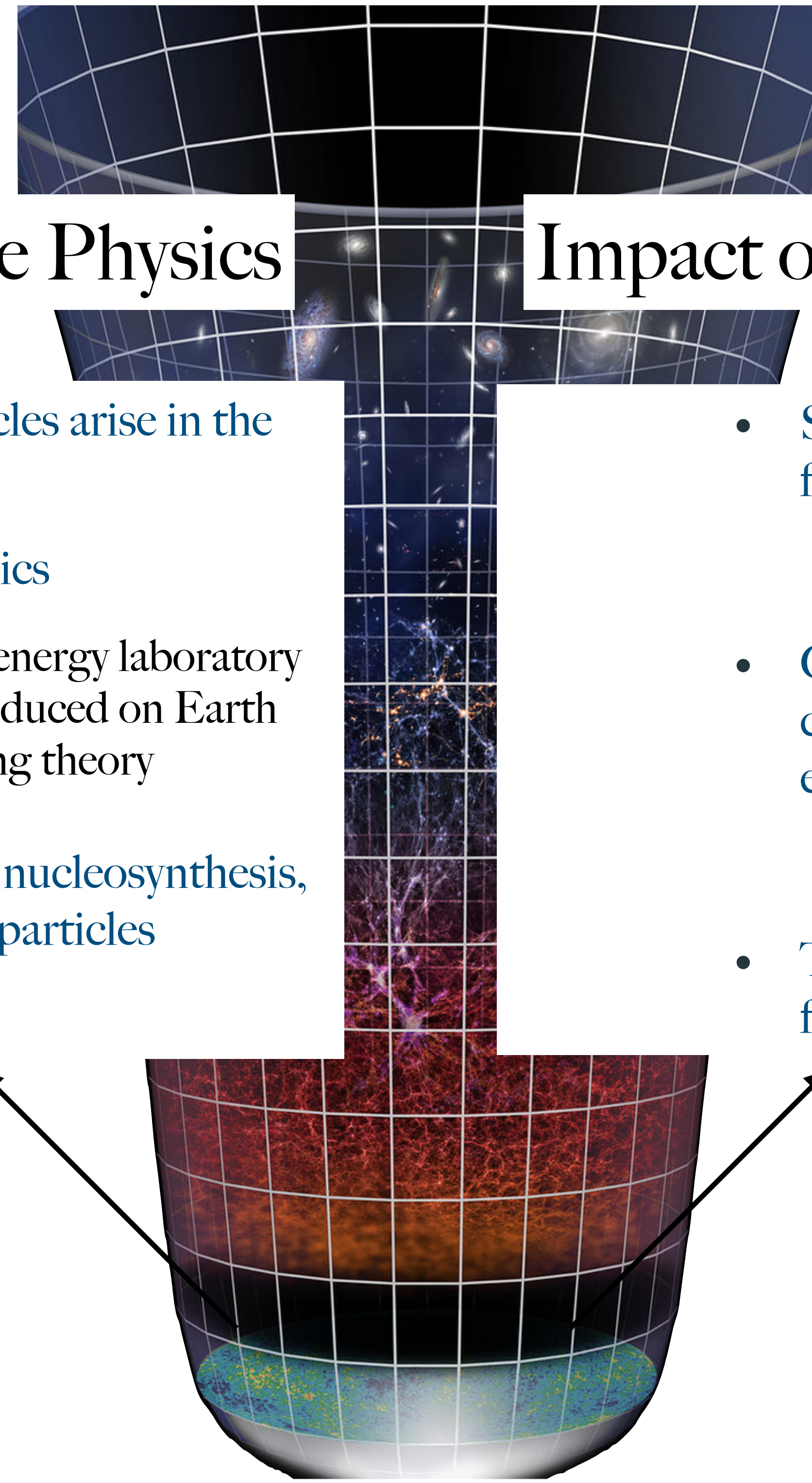
Why is it important to understand the Early Universe?



Impact on Particle Physics

- Fundamental forces and particles arise in the primordial universe
- Insights into high energy physics
  - Primordial universe as a natural high-energy laboratory
  - Study conditions that cannot be reproduced on Earth
  - Test high-energy theories such as string theory
- Mass of neutrinos, primordial nucleosynthesis, axions and other hypothetical particles

Impact on the Large Scale Structure

- Structure of the universe originated from initial density fluctuations
- Consistent initial conditions are crucial for accurate models of cosmic evolution
- Tracing the history of the universe from the Big Bang to the present day

?

The history of the Universe

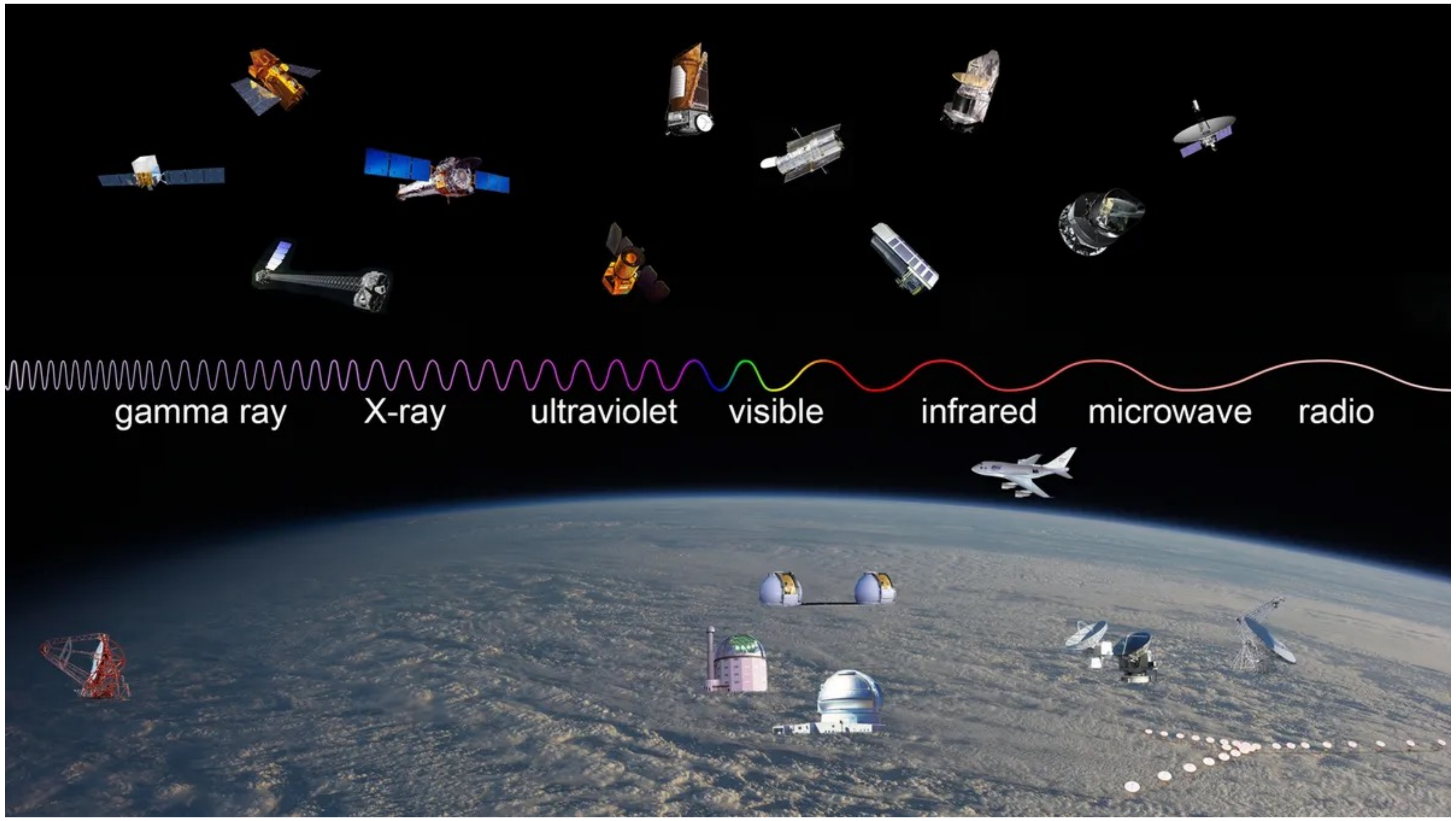
- Quantum fluctuations (inflation or bounce)→ classical perturbations in the density of all species at early times
- Fluctuations grow over time through a process of gravitational instability to form all of the structure we see today
  - We see these fluctuations at $t \sim 380{,}000$ years as fluctuations in the temperature of the cosmic microwave background (CMB) radiation with $\Delta T/T \sim 10^{-5}$
  - and "today" with galaxy surveys with $\delta \gg 1$
  - Probing a wide range of energy densities, temperatures, ...
- Growth is a competition between gravity and expansion
  - Depends on the laws of gravity (general relativity)
  - Depends upon the expansion of the Universe (metric)
  - Depends on constituents and their properties

Probe metric, particle content and both epochs of accelerated expansion ... with high precision!

Cosmological Agenda in a Nutshell

Electromagnetic spectrum

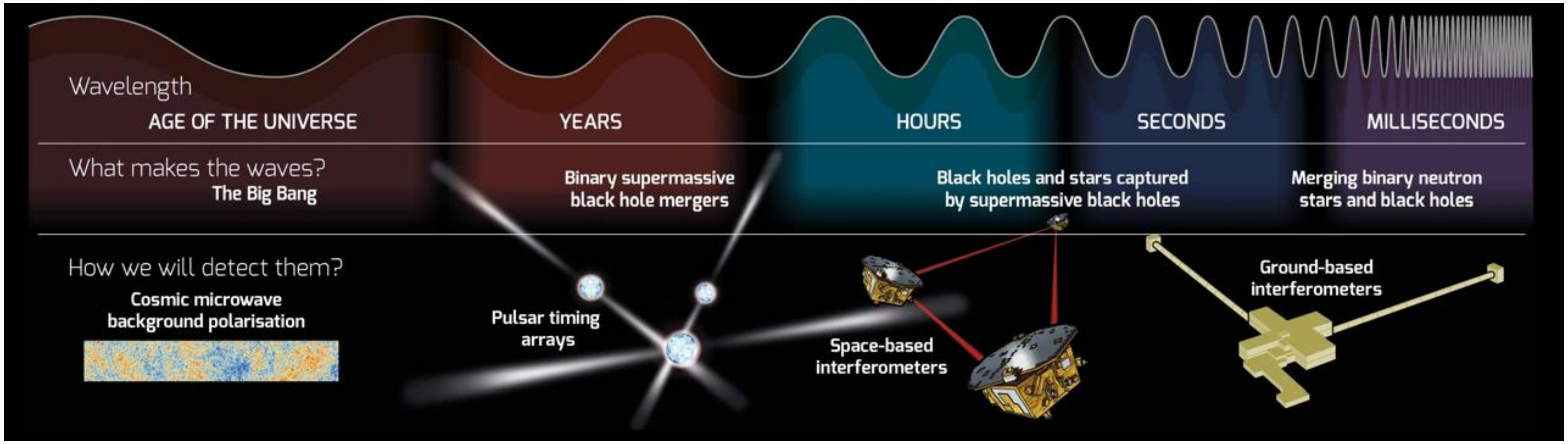
- Fermi, Swift, Integral
- Chandra, XMM-Newton
- Solar Orbiter, JUNO
- HUBBLE, VLT, LSST
- JWST, SOFIA
- ALMA, SPT
- COBE, WMAP, Planck
- SKA, EHT



Credits: NASA

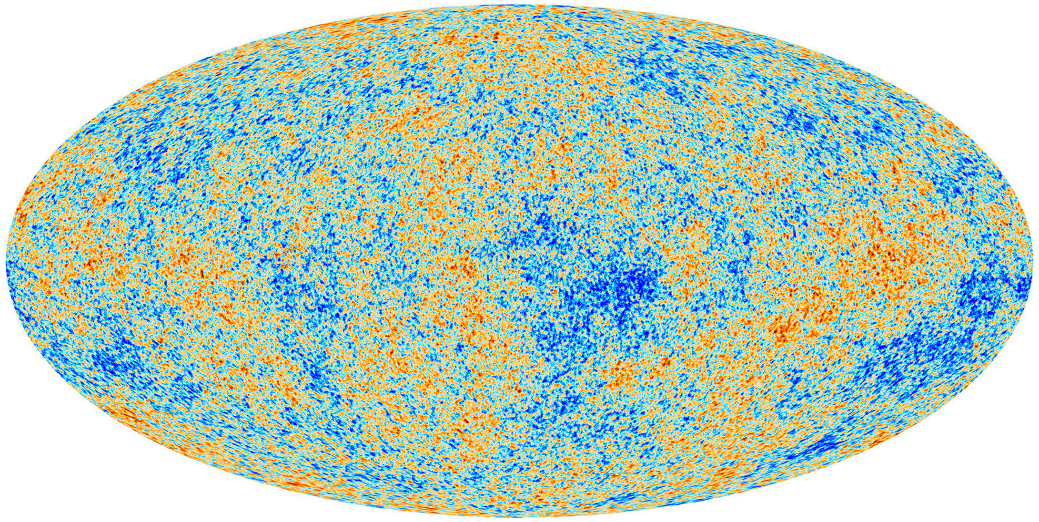
Gravitational wave spectrum

- CMB S4
- LIGO
- Virgo
- KAGRA
- ET
- LISA
- DECIGO



Credits: Ben Gilliland

The most accurate Gaussian distribution



ESA Planck Collaboration

The distribution of temperature fluctuations is consistent with a Gaussian distribution!

$$\Delta T/T \sim 10^{-5}$$

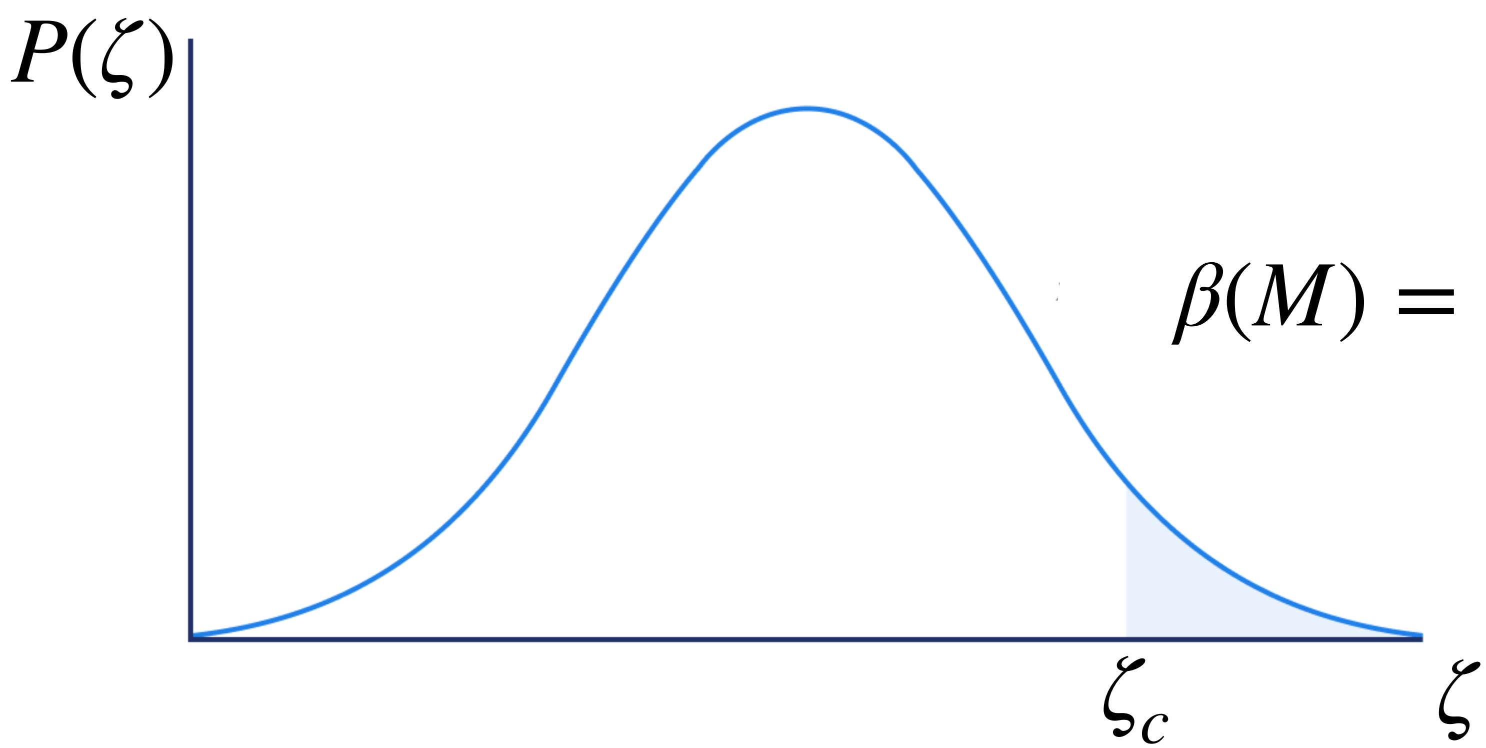
- The detection of gravitational waves by LIGO in 2015 has opened up a new era in cosmology, investigating physics beyond observations of cosmic background radiation;
- NANOGrav in 2020 has reported evidence of stochastic gravitational waves possibly coming from primordial black holes, and the future LISA mission promises more advances;
- Current predictions do not take into account the non-linear effects of general relativity in the Primordial Universe;

Context and State-of-the-art

Quantum fluctuations in the scalar field induce perturbations in space (called curvature perturbations)

$$\zeta = \frac{\delta\varphi_{\text{quan}}}{\delta\varphi_{\text{clas}}}$$

When $\zeta \sim 1$ it means $\delta\varphi_{\text{quan}} \sim \delta\varphi_{\text{clas}}$, i.e, stochastic effects are important!



$$\beta(M) = \int_{\zeta_c}^{\infty} P(\zeta) d\zeta$$

Context and State-of-the-art

Primordial black holes formed in the radiation era from a dimensionless density perturbation, ζ, which exceeds the threshold value $\zeta > \zeta_c \sim 1$

Standard abundance calculations assume a Gaussian PDF, $P(\zeta)$, possibly with some non-Gaussian modification, but are not suitable for describing the tail of the distribution

Stochastic formalism → non-perturbative approach to calculating large or rare variations in the primordial density field (V.Vennin and A. Starobinsky, arXiv:1506.04732)

$$\varphi = \varphi_{\text{long}} + \varphi_{\text{short}} = \int_0^{k_\sigma} d^3k\, \varphi_k\, e^{ikx} + \int_{k_\sigma}^{\infty} d^3k\, \varphi_k\, e^{ikx}$$

k_σ = coarse graining scale

A power-law expansion/collapse universe

Set up: Homogeneous scalar field (I. Heard and D. Wands, arXiv: gr-qc/0206085)

$$\rho = V(\varphi) + \frac{1}{2}\dot{\varphi}^2 , \quad \text{and} \quad P = -V(\varphi) + \frac{1}{2}\dot{\varphi}^2 \quad \text{where} \quad V(\varphi) = V_0 e^{-\kappa\lambda\varphi}$$

Reducing dynamics to a one-dimensional problem

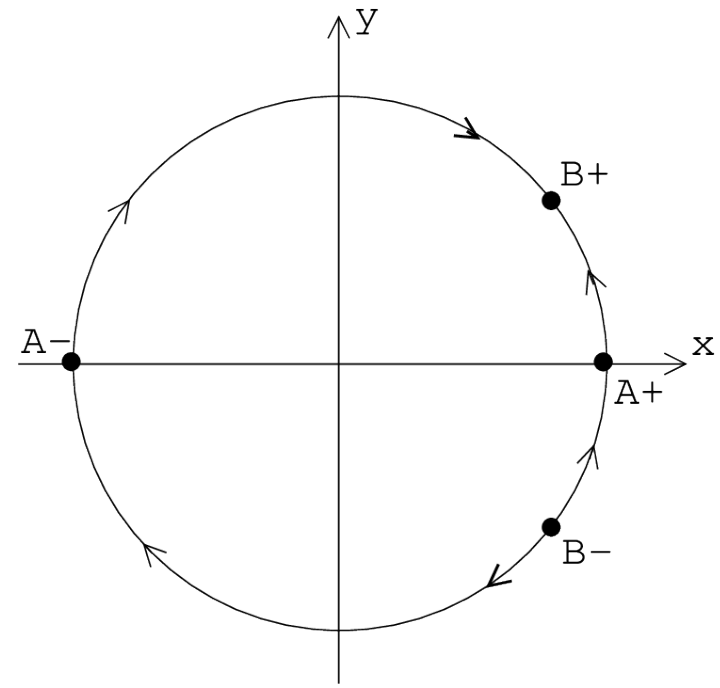
$$x = \frac{\kappa\dot{\varphi}}{\sqrt{6}H} , \quad y = \frac{\kappa\sqrt{\pm V}}{\sqrt{3}H}$$

Critical points

$$(A_\pm)\ x_{A_\pm} = \pm 1 , \quad y_A = 0$$

$$(B)\ x_B = \frac{\lambda}{\sqrt{6}} , \quad y_B = \sqrt{1 - \frac{\lambda^2}{6}}$$

The solution (B) exists for $\pm(6 - \lambda^2) > 0$



Phase-space for flat positive potentials, $\lambda^2 < 6$. Friedmann constraint $x^2 + y^2 = 1$. Arrows indicate evolution in cosmic time, t

Quantum fluctuations in a collapsing FRW cosmology behave like inflationary perturbations (A. Starobinsky, 1979, D. Wands, arXiv: gr-qc/9809062)

$$\delta\varphi_k = \frac{i}{a}\sqrt{\frac{1}{4\pi k}}\frac{\Gamma(|\nu|)2^{|\nu|}}{|k\eta|^{|\nu|-1/2}} \quad \text{where} \quad \nu = \frac{3}{2} + \frac{\lambda^2}{2-\lambda^2} \quad \xrightarrow[|k\eta| \to 0]{} \quad \mathcal{P}_{\delta\varphi} = \left[\frac{\Gamma(|\nu|)2^{|\nu|}}{(\nu - 1/2)2^{3/2}\Gamma(3/2)}\right]^2 \left(\frac{H}{2\pi}\right)^2 |k\eta|^{3-2|\nu|}$$

Stochastic formalism in a power-law expansion/collapse universe

Quantum fluctuations gives a finite lifetime for $w = 0$ (T. Miranda, E. Frion and D. Wands, arXiv: 1910.10000)

$$\langle 0 | \hat{\xi}_x(N_\star)\hat{\xi}_x(N) | 0 \rangle = g(\nu)\frac{(|\nu|-\nu)^2}{\sigma^{2|\nu|-3}}\kappa^2 H_\star^2 \exp\left[-\frac{3-2\nu}{\nu-1/2}(N_\star - N)\right]$$

No noise for $\nu > 0$: adiabatic perturbations, includes power-law inflation ($\nu = 3/2$) and ekpyrosis ($\nu = 1/2$)

Langevin equation

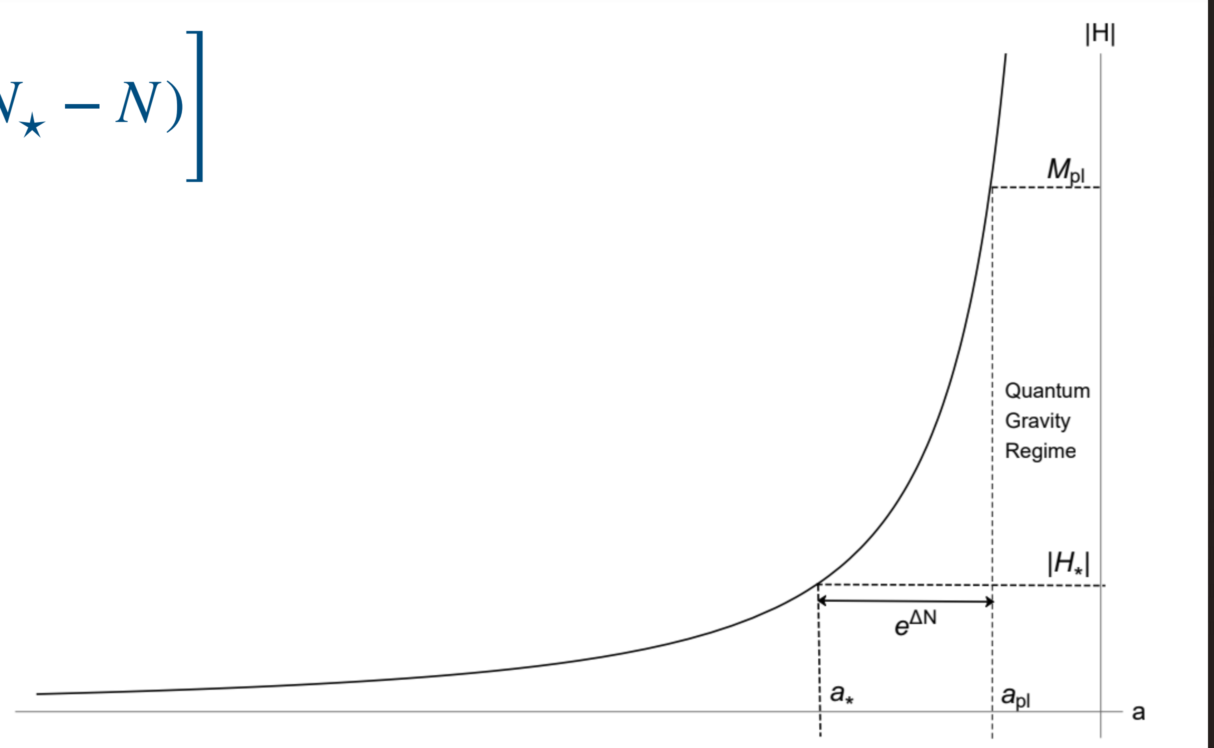
$$\bar{x}' = m(\bar{x} - x_B) + \hat{\xi}_x \quad \text{with} \quad m = \frac{\lambda^2 - 6}{2}$$

Variance split into classical/quantum parts

$$\sigma_x^2(N) = \sigma_x^2(N_\star)e^{2m(N-N_\star)} + \int_{N_\star}^{N} dS\, e^{2m(N-S)}\Xi_{x,x}(S)$$

Backreaction condition

If $\sigma^2_{x,qu} = 1 \implies$ does quantum noise change the dynamics?



For the pressureless collapse to get a sensible deviation from the fixed point we start in, the initial scale must be set at low energy

For slightly red spectrum ($\nu = -3/2 - \epsilon$, $n_s < 1$): classical perturbations grow faster

Why LQC?

Loop Quantum Cosmology replaces the classical Friedmann equation by (E. Wilson-Ewing, arXiv: 1211.6269)

$$H^2 = \frac{\kappa^2}{3}\rho\,(1-z) \quad \text{where} \quad z = \frac{\rho}{\rho_c}$$

Introduces a maximum energy density, forces a bounce at $z = 1$

Scalar-field potential in LQC model

$$V(\varphi) = \left(2 - \frac{\lambda^2}{3}\right)\frac{V_0 e^{-\kappa\lambda\varphi}}{\left(1 + \frac{V_0}{2\rho_c}e^{-\kappa\lambda\varphi}\right)^2}$$

with $\lambda(s) = \lambda_0 \frac{1-s}{1+s}$ and $s = \frac{V_0}{2\rho_c}e^{-\kappa\lambda_0\varphi}$

LQC dynamical variables

$$X = x\sqrt{1-z} \qquad Y = y\sqrt{1-z}$$

Even in LQC these satisfy $X^2 + Y^2 = 1$

Critical points

A) Classical regime ($z = 0$)

Scaling & kinetic/potential points reappear + a new one:

$$X_c(1) = \frac{\lambda_0}{\sqrt{6}} \qquad X_c(2) = \pm 1$$

$$X_c(3) = 0 \quad \text{with} \quad \lambda = z = 0$$

B) Quantum regime ($z \neq 0$)

A continuous line of fixed points:

$$X_c = 0 \quad Y_c = \pm 1$$
$$\lambda_c = 0 \quad z_c \in [0,1]$$

Near the bounce $z \to 1$

$(1-z)^{-3/2} \to \infty \Rightarrow$ fast quantum-dominated behaviour!

Behaviour of Z''/Z near the bounce

For the background scalar field in LQC, the Mukhanov-Sasaki variable Z is

$$Z = \frac{\sqrt{6}}{\kappa} a X$$

If X is slowly varying (nearly constant at the fixed point)

$$\frac{Z''}{Z} = a^2 H^2 \left[2 - \frac{3X^2(1-2z)}{1-z} \right]$$

which shows

- A non-standard effective potential in the Mukhanov–Sasaki equation
- Sensitivity to the equation-of-state w around the bounce, since $w = 2X^2 - 1$
- Possible suppression or enhancement of quantum diffusion depending on the background path

What we've learned so far

- In classical contracting universes, collapse models with an exponential potential show that the fixed point is classically unstable for $\nu < 0$
- Stochastic fluctuations introduce quantum diffusion, but in most cases the variance stays tiny on super-Hubble scales as long as $|H| \ll M_{Pl}$
- The only exceptional case is pressureless collapse ($\nu = -3/2$), where the lifetime of the classical solution becomes sensitive to noise
- LQC modifies the effective potential felt by scalar perturbations
- These results provide the baseline scenario for asking:

Does the LQC bounce amplify, suppress, or qualitatively change the impact of quantum diffusion?

Muito obrigada!