Universal clustering in some nonequilibrium systems

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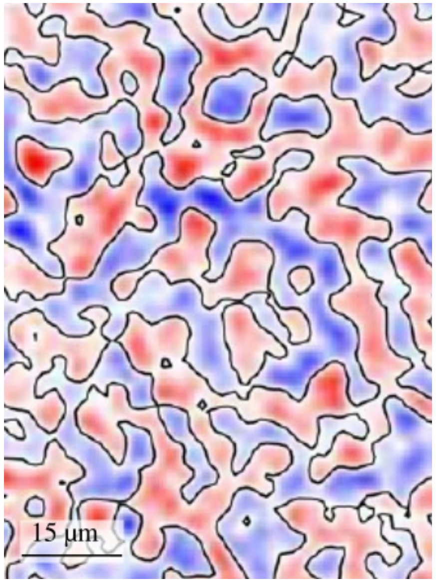
- Part 1: Liquid crystals
- Part 2: Growing interfaces

Clusters & universality

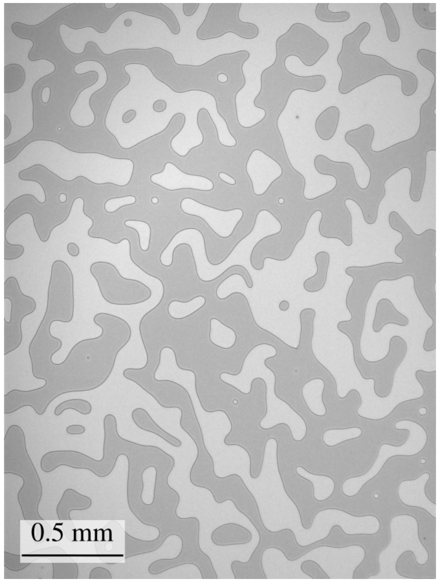
- Focus on 2D macroscopic systems with two local states

Collective motion of cells



Andersen et al., Nat. Phys. (2025)

Liquid crystals



Almeida, PRL (2023)

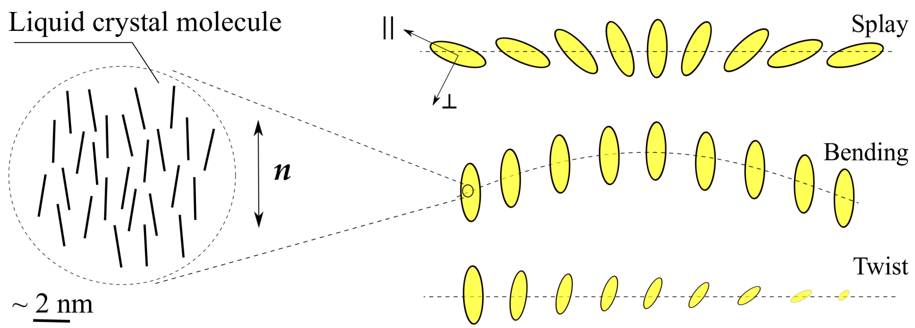
Part 1

Liquid crystals: (a) Experiment

In collaboration with K. A. Takeuchi (U. Tokyo, Japan)

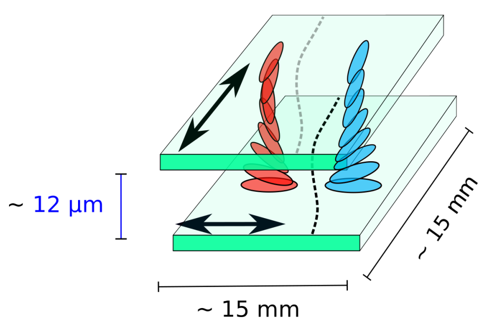
Nematic phase

- Fluid with orientational order (it stores elastic energy)

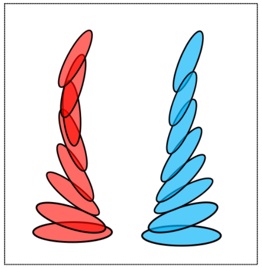


- Twisted nematics

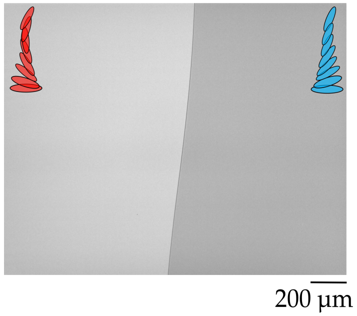
Quasi-2D cell



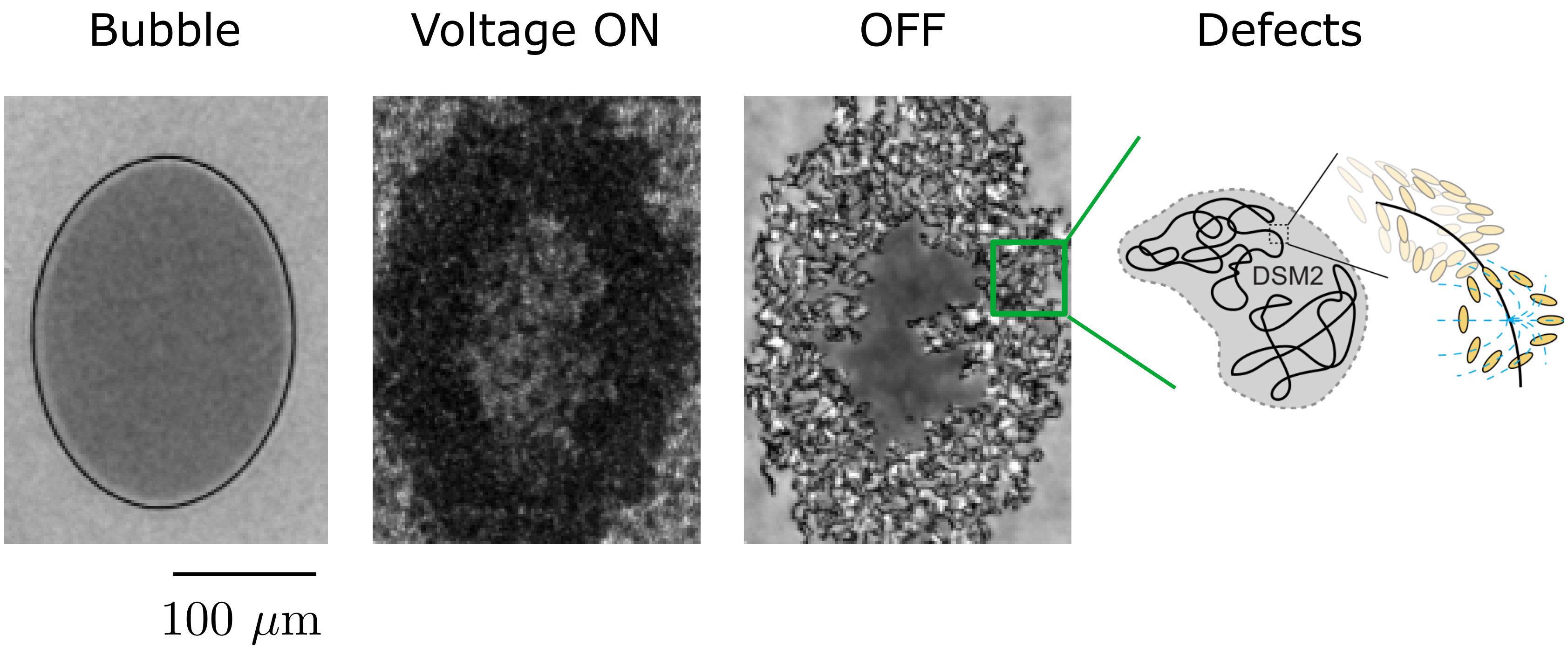
Two states



Optical microscopy

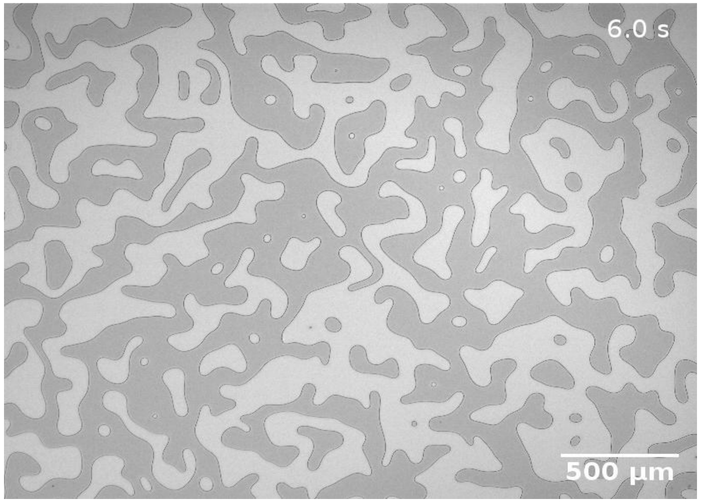


Disordered phase at high voltages



Nonequilibrium, nematic-disordered phase

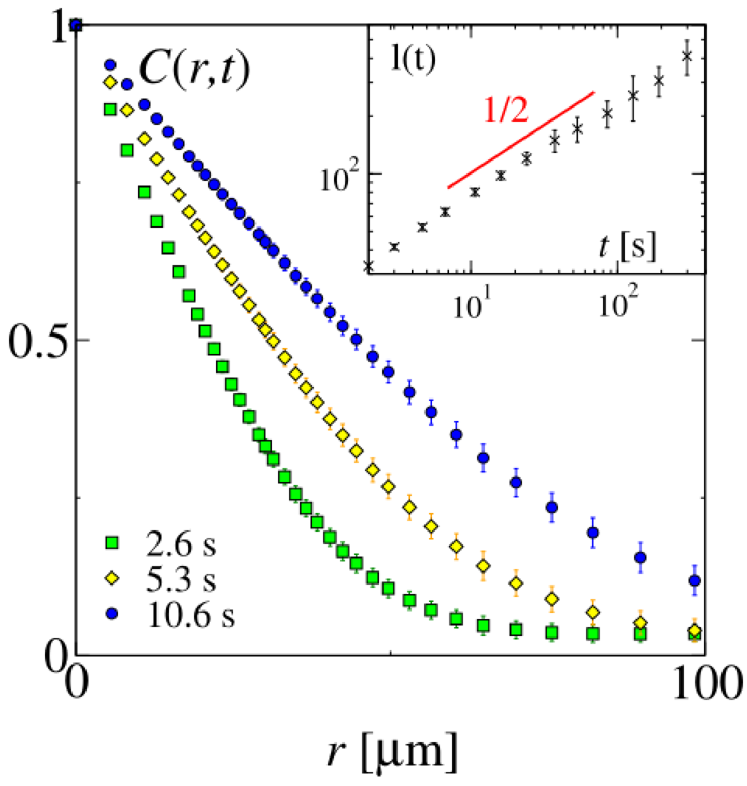
Ordering after voltage removal

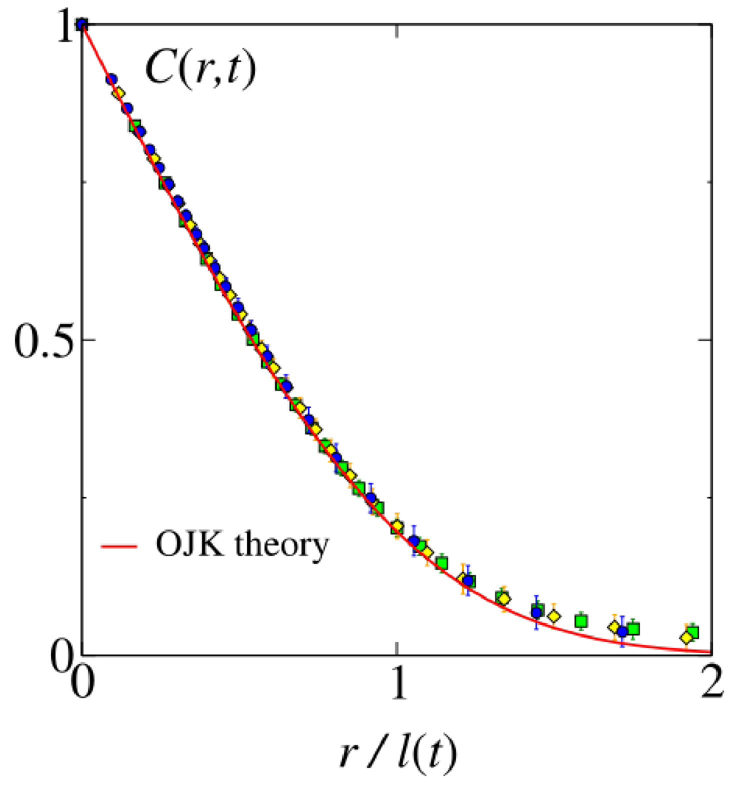


$$s(r,t) = \begin{cases} +1 & \text{pixel} \\ -1 & \text{pixel} \end{cases}$$

Dynamic scaling

- Correlator $C(r,t) = \langle s(r',t)s(r'+r,t)\rangle \simeq f[r/l(t)]$
- Growth law $C(l,t) := 0.2 \quad \rightarrow \quad l(t) \sim t^{1/z}, \quad z = 2$





(b) Percolation

Critical percolation & coarsening

PRL **98,** 145701 (2007) PHYSICAL REVIEW LETTERS week ending 6 APRIL 2007

Exact Results for Curvature-Driven Coarsening in Two Dimensions

Jeferson J. Arenzon,[1] Alan J. Bray,[2] Leticia F. Cugliandolo,[3] and Alberto Sicilia[3]
[1]*Instituto de Física, Universidade Federal do Rio Grande do Sul, CP 15051, 91501-970 Porto Alegre RS, Brazil*
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(Received 16 August 2006; published 3 April 2007)

PRL **109,** 195702 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012

Fate of 2D Kinetic Ferromagnets and Critical Percolation Crossing Probabilities

J. Olejarz, P. L. Krapivsky, and S. Redner
Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 02215, USA
(Received 15 August 2012; published 7 November 2012)

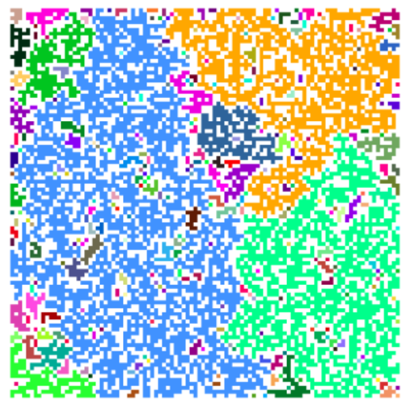
2D percolation (a brief reminder)

- Take a lattice Λ and occupy its sites with prob. p

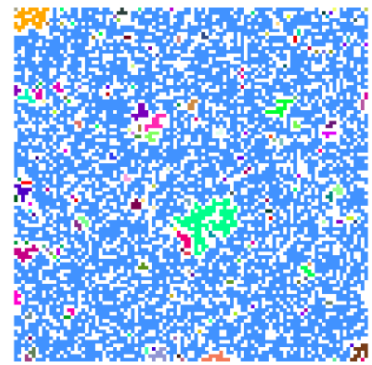
Define a cluster as a connected region of neighboring occupied sites

$p < p_{\mathrm{c}}$

$p \approx p_{\mathrm{c}}$



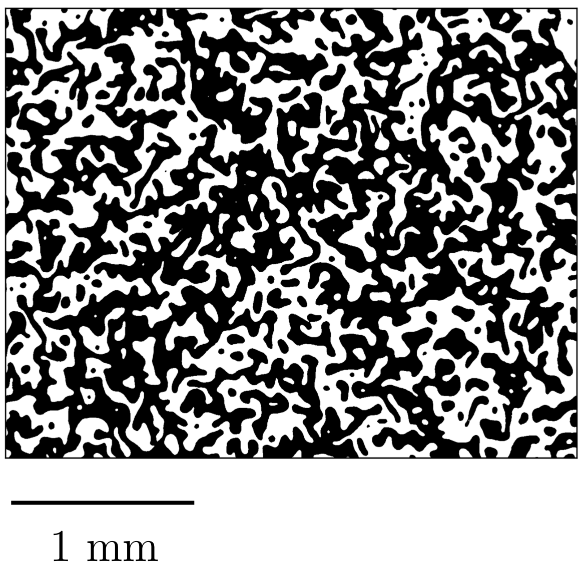
$p > p_{\mathrm{c}}$



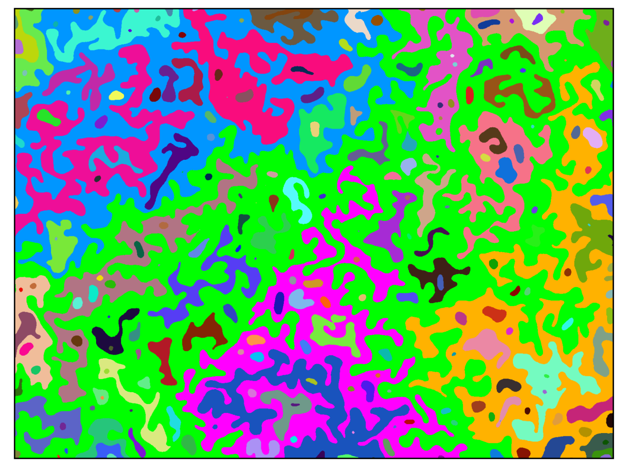
$$\lim_{L \to \infty} P(p, L) = \begin{cases} 0, & p \leq p_{\mathrm{c}}(\Lambda) \\ 1, & p > p_{\mathrm{c}} \end{cases}$$

$P(p)$: Prob. of there being a percolating (infinite) cluster

Labeling



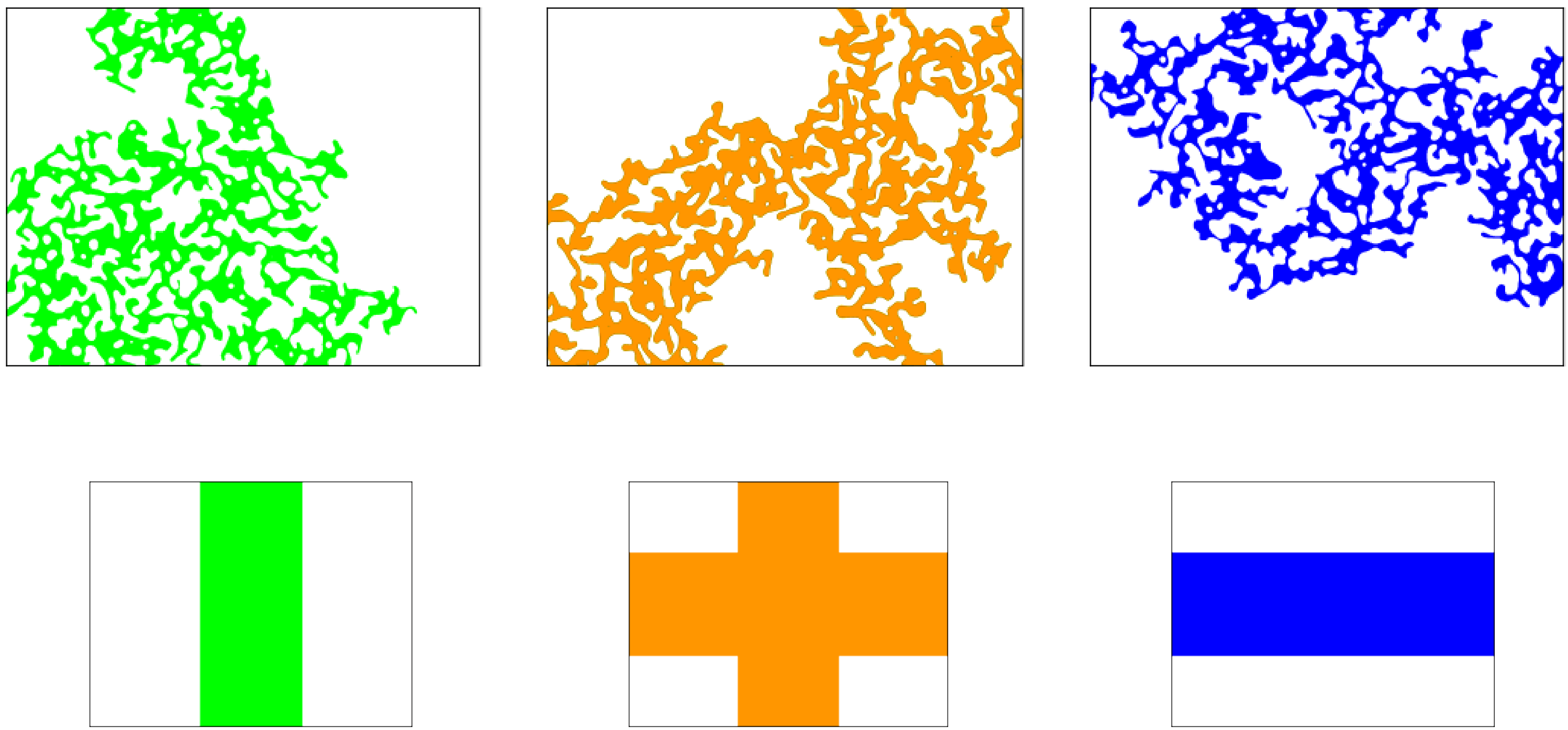
before



after

Types of crossing a rectangle

- Different samples at $t = 2.8$ s. Only largest cluster is shown



$\overline{h}v$ $\quad$ hv $\quad$ $h\overline{v}$

Crossing probabilities

Cardy, J. Phys. A: Math. Gen. **25** L201 (1992)

Smirnov, C. R. A. Sci. Paris, t. **333**, p. 239 (2001)

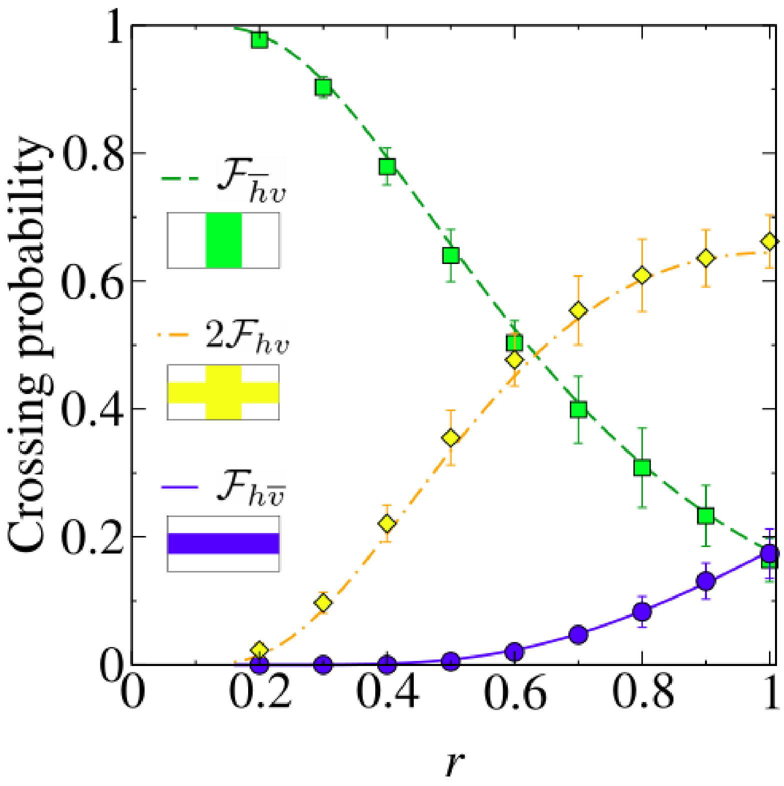
- Vertical crossing $\mathcal{F}_{\overline{h}v}(r) = \dfrac{\eta(r)}{\Gamma(1/3)\Gamma(2/3)}\ {}_3F_2(1,1,4/3;5/3,2,\eta)$
- Horizontal crossing $\mathcal{F}_{h\overline{v}}(1/r) = \mathcal{F}_{\overline{h}v}(r)$
- Dual-spanning $2\mathcal{F}_{hv}(r) = 1 - \mathcal{F}_{\overline{h}v}(r) - \mathcal{F}_{h\overline{v}}(r)$

$\eta := \left(\dfrac{1-k}{1+k}\right)^2, \quad r := \dfrac{2K(k^2)}{K(1-k^2)}, \quad K(\cdot):$ Complete elliptic int. of 1st kind

rectangle aspect ratio $r := \text{height} \ / \ \text{width}$

Crossing probabilities

Almeida, Phys. Rev. Lett. **131,** 268101 (2023)

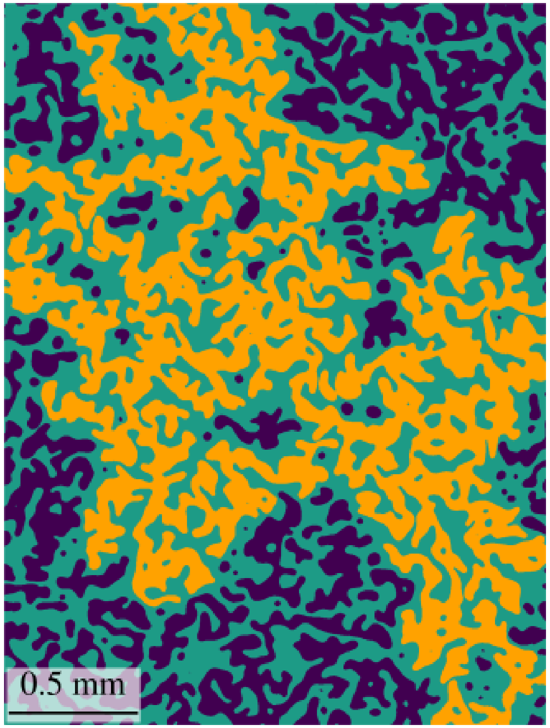


(c) Geometry

In collaboration with J. J. Arenzon (UFRGS, Brazil)

Large clusters look fractal

Almeida & Arenzon, Phys. Rev. Lett. **134**, 178101 (2025)



2 s

8 s

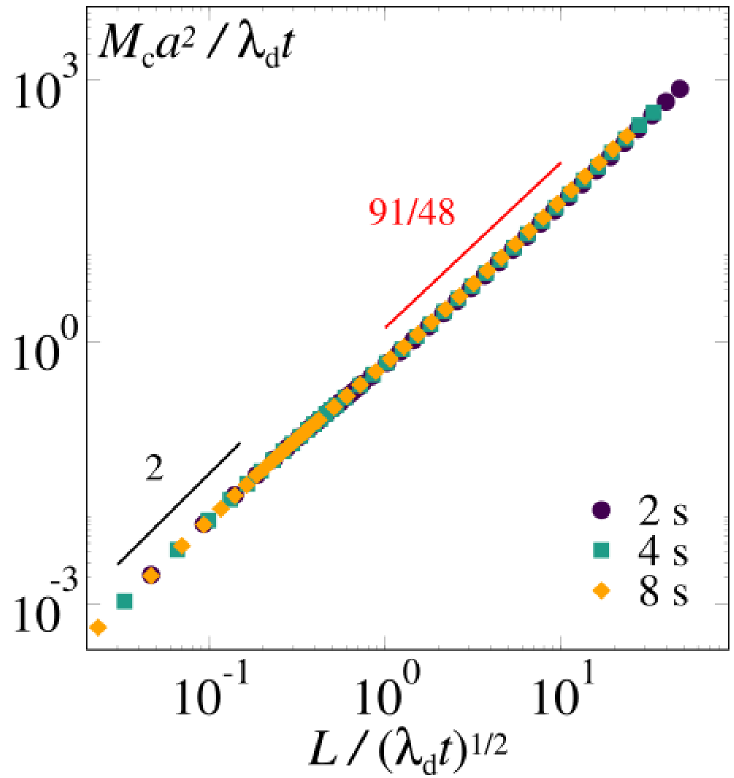
Fractal dimension (d_F)

$$A(L,t) := M_c(L,t)a^2 \sim L^{d_F}$$

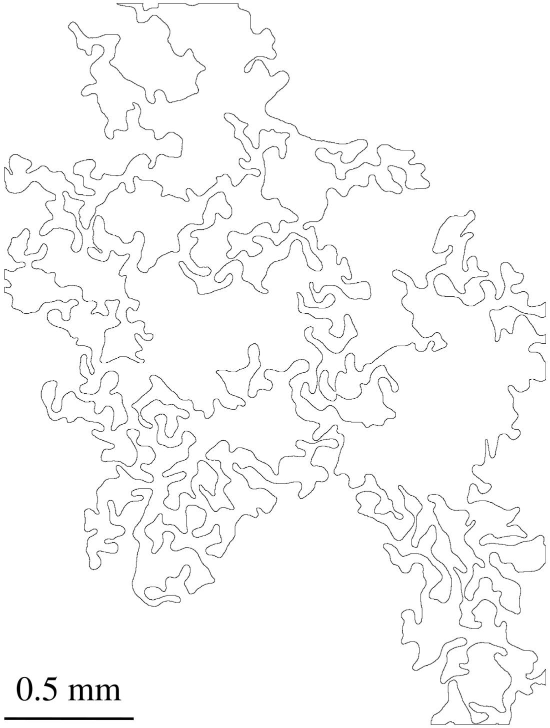
$$d_F = \frac{91}{48}$$ Critical percolation

M_c, a Mass of a cluster, pixel size

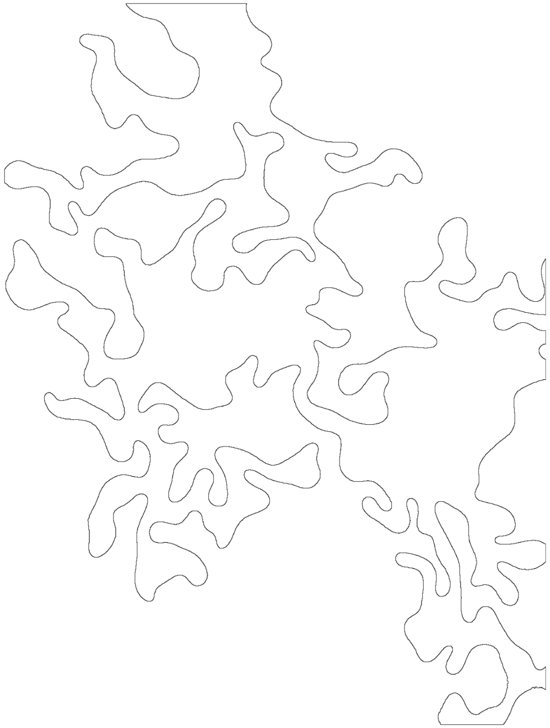
L Size of a square cut



How winding are the boundaries?

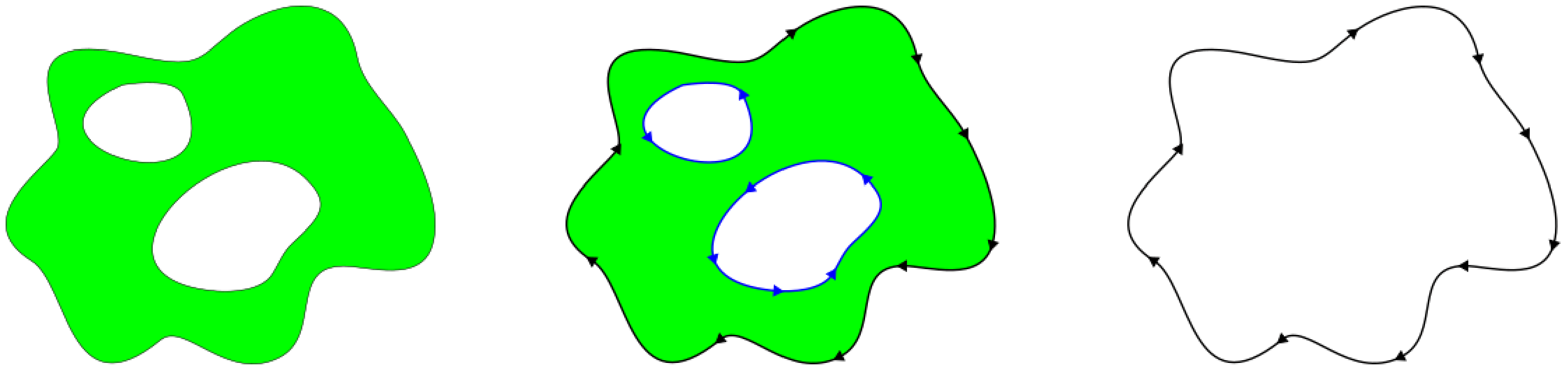


2 s

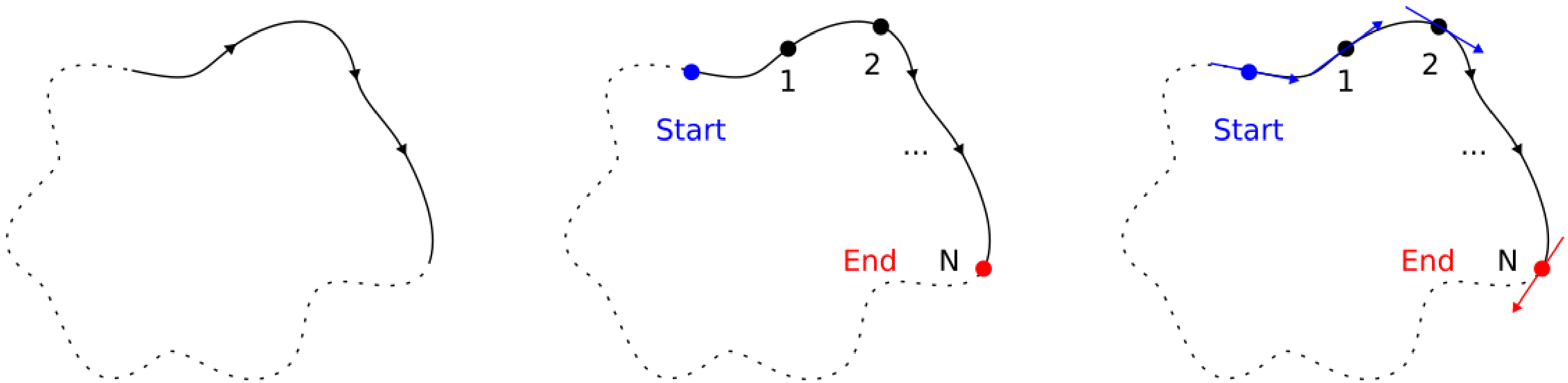


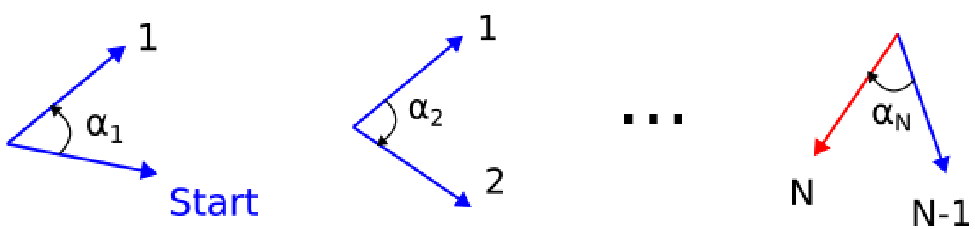
8 s

Oriented path



Winding angle





$$\theta(l) := \sum_{i=1}^{N} \alpha_i$$

α_i : Local turning angle

l : Curvilinear length of the path

Winding angle variance

Duplantier & Saleur PRL **60**, 2343 (1988)

Wieland & Wilson PRE **68**, 056101 (2003)

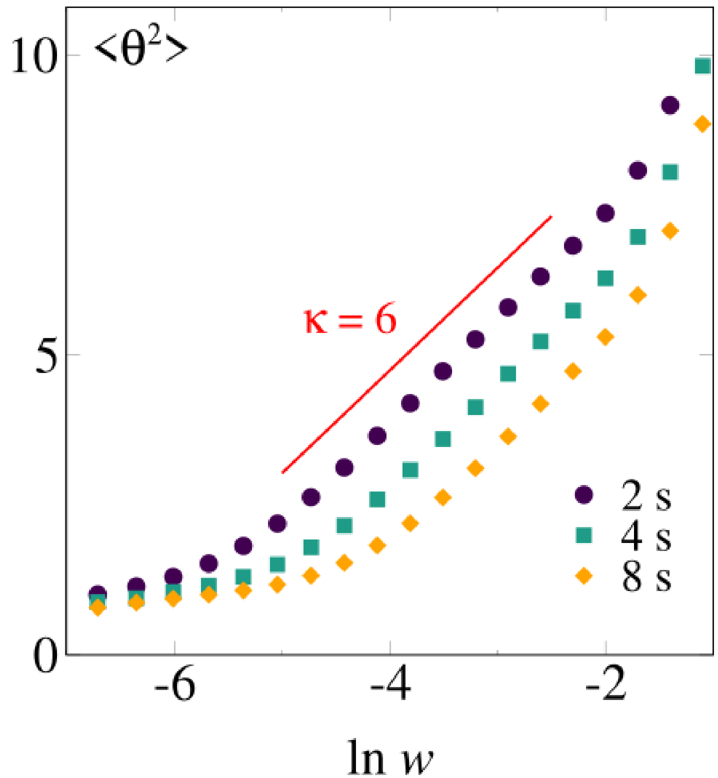
- For curves with conformal invariance (empirical law)

$$\langle \theta^2(l) \rangle_c = \text{cst.} + \frac{4k}{8+k} \ln(l); \qquad k \in \mathbb{R}, k \leq 8$$

l : length along the path

$k = 6$ for percolation hulls

Boundaries have conformal invariance



p : total length of the loop

$w = l/p$: Normal distance

Conclusions (part 1)

Almeida & Arenzon PRL **134**, 178101 (2025)

- Exp. confirmation: System self-tunes to critical percolation
- Large clusters are fractal, but morphed into regular structures
- Boundaries have conformal invariance

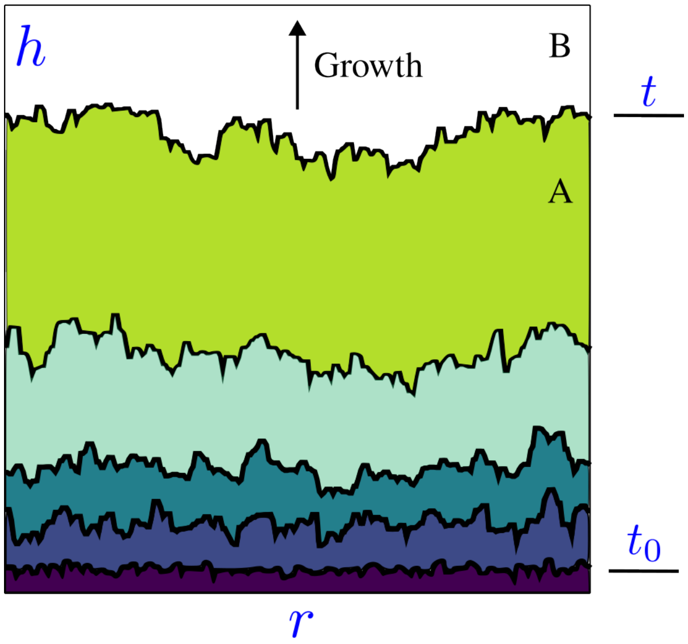
Part 2

Growing interfaces

Theoretical work in collaboration with L. Cugliandolo (LPTHE, Paris)

Concept in 1D

- Phase A grows at the expense of phase B. Local interactions



- Growth equation (symmetries): $\partial_t h(r,t) = v_0 + \mathcal{F}\{(\nabla h)\} + \eta(r,t)$

Growth equation

Kardar, Parisi, Zhang PRL **56**, 889 (1986)

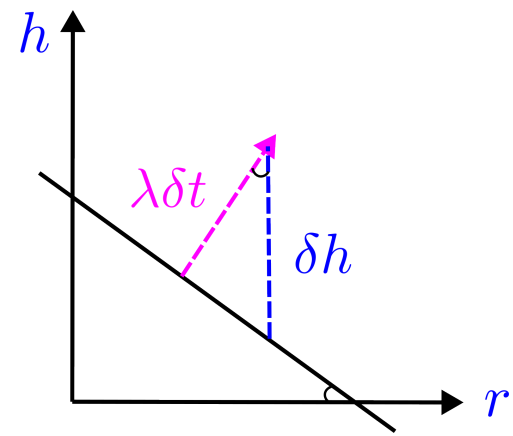
$$\partial_t h(r,t) = v_0 + \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(r,t)$$

v_0 : growth velocity

$\nabla^2 h$: elastic contribution

$(\nabla h)^2$: lateral growth

$\eta(r,t)$: Gaussian noise



$$\delta h \approx \lambda \delta t \left[1 + \frac{1}{2}(\nabla h)^2 \right]$$

$$\overline{\eta(r,t)} = 0$$

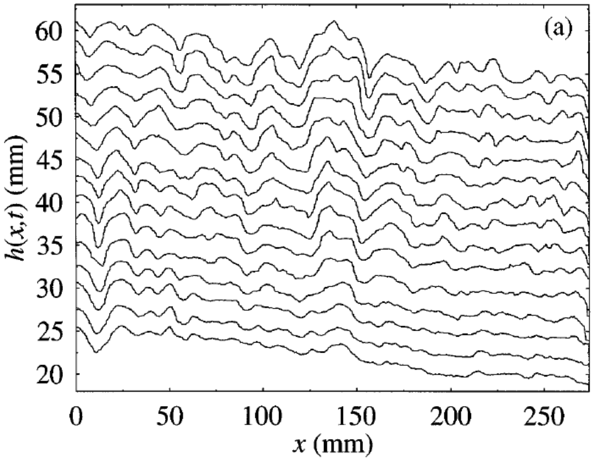
$$\overline{\eta(r,t)\eta(r',t')} = 2D\delta^d(r-r')\delta(t-t')$$

ν, λ, D : parameters

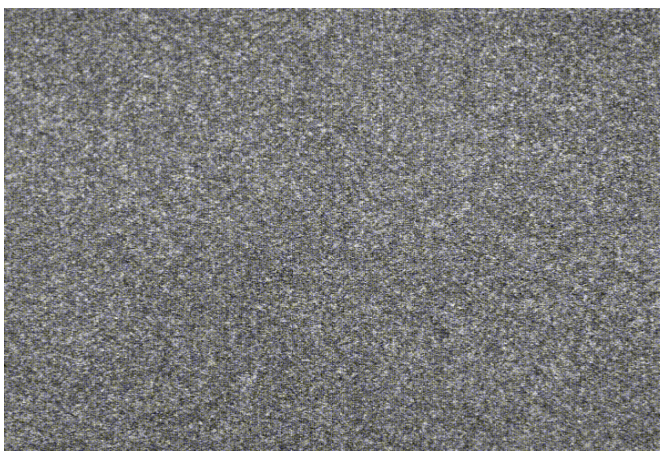
KPZ growth in 1D (experimental examples)

Flame fronts in papers

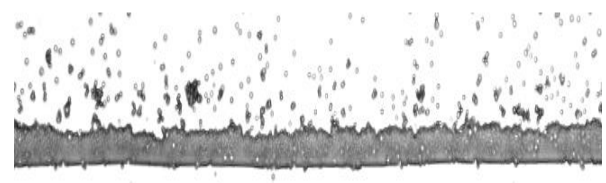
Maunuksela et al. PRL **79**, 1515 (1997)



Liquid crystals



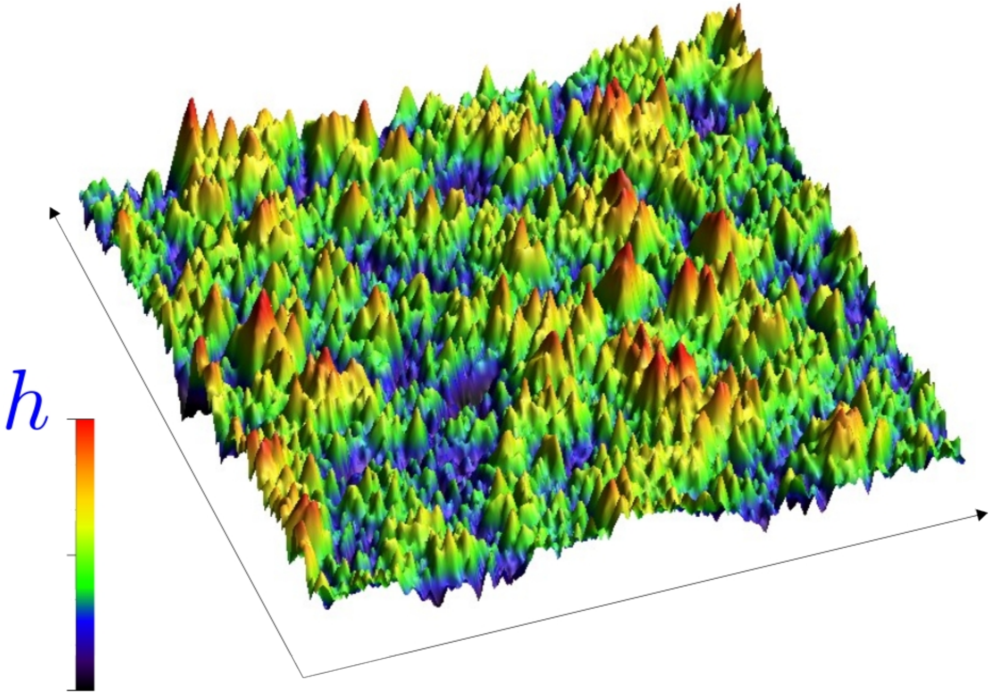
Colloids in evaporating drops



Takeuchi et al. PRL **104**, 230601 (2010)

Yunker et al. PRL **110**, 035501 (2013)

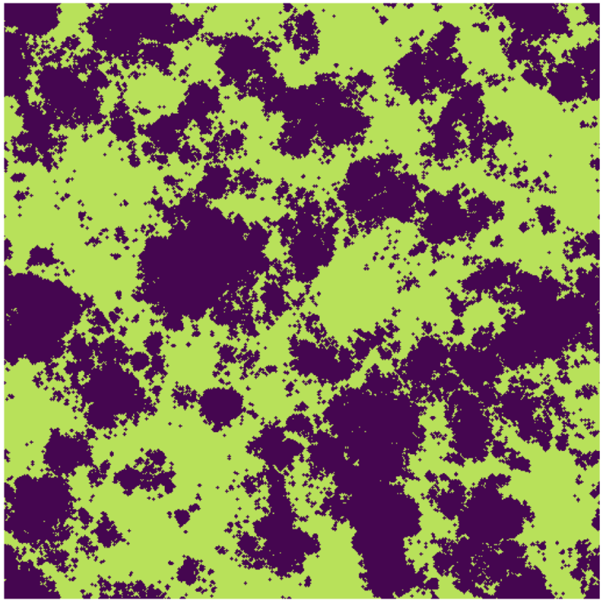
KPZ growth in 2D



Clusters

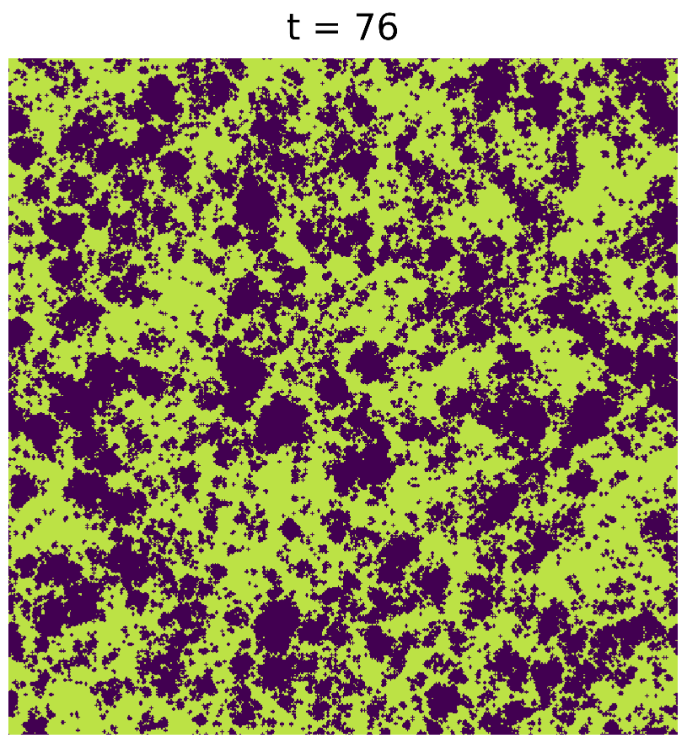
Almeida & Cugliandolo (To be published)

- Projection of connected regions above/below the mean height



- Key observable: $s(r,t) = \text{sign}[h(r,t) - \langle h(r,t) \rangle]$ $\quad s(r,t) = \begin{cases} +1 \\ -1 \end{cases}$

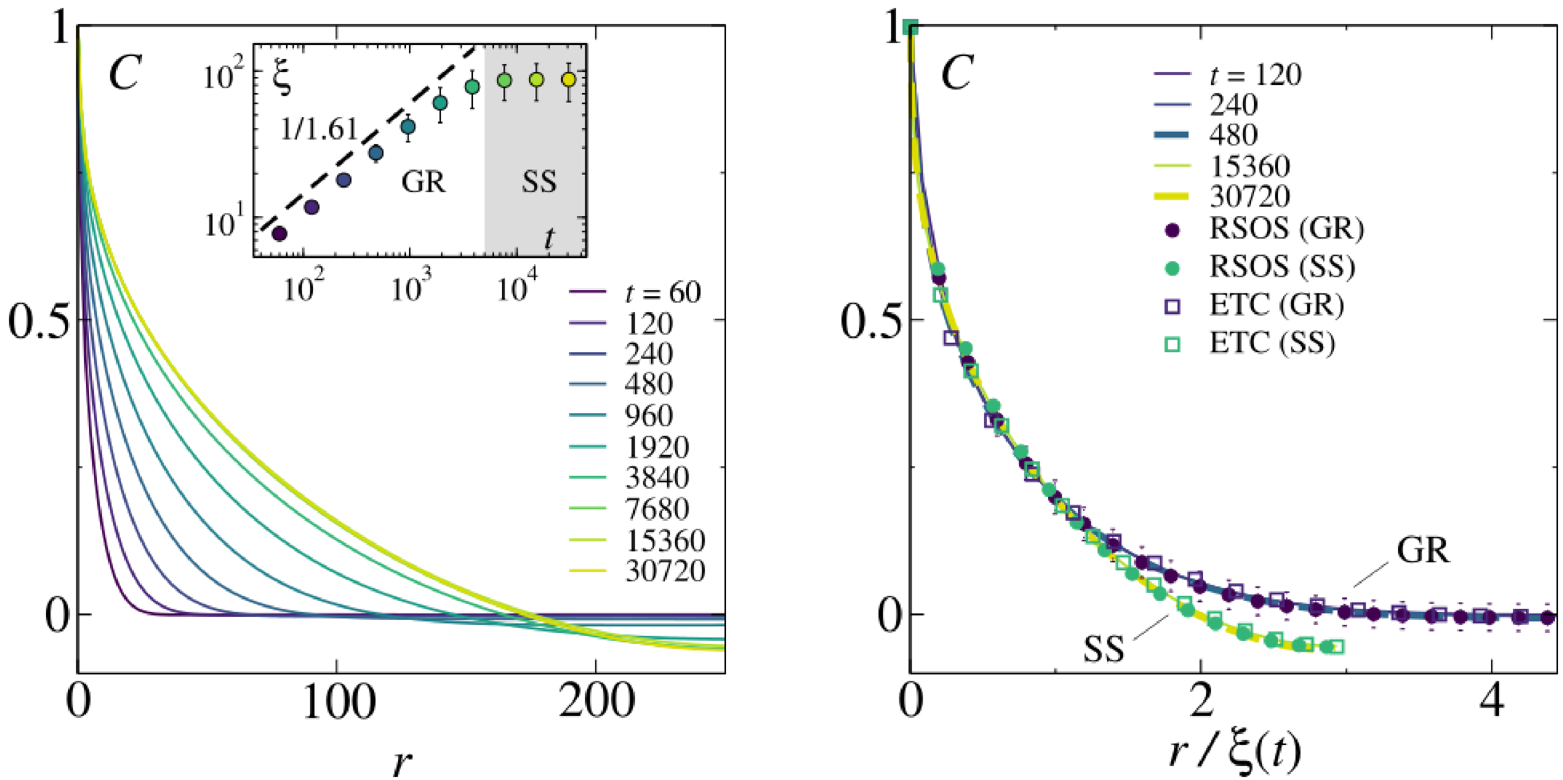
Time evolution (KPZ model)



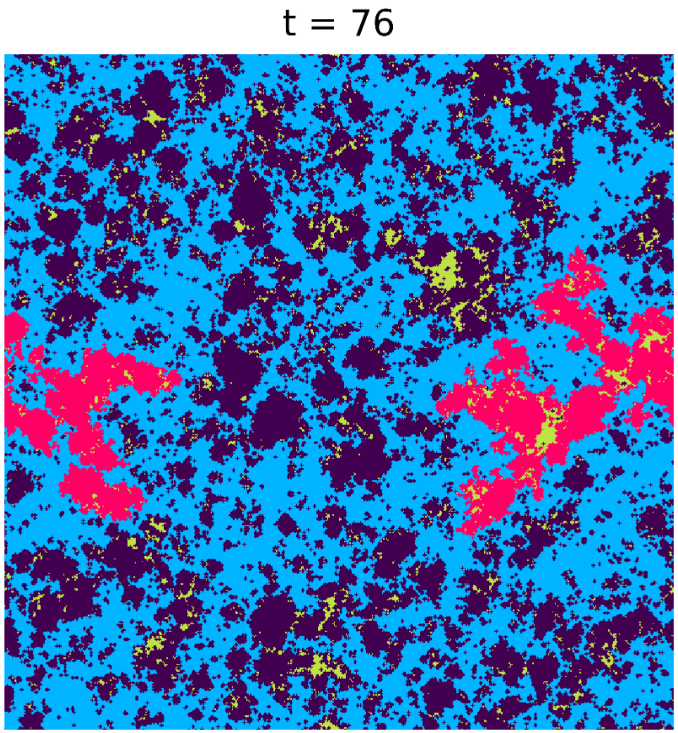
$$s(r,t) = \begin{cases} +1 & \blacksquare \quad \text{pixel} \\ -1 & \square \quad \text{pixel} \end{cases}$$

Dynamic scaling & universality

- Correlator $C(r,t) = \langle s(r',t)s(r'+r,t)\rangle \simeq f[r/\xi(t)]$
- Growth law $C(\xi,t) := 0.2 \quad \rightarrow \quad \xi(t) \sim t^{1/z}, \quad z \approx 1.61$



Configurations w/ largest clusters detected



$$s(r,t) = \begin{cases} +1 \\ -1 \end{cases}$$

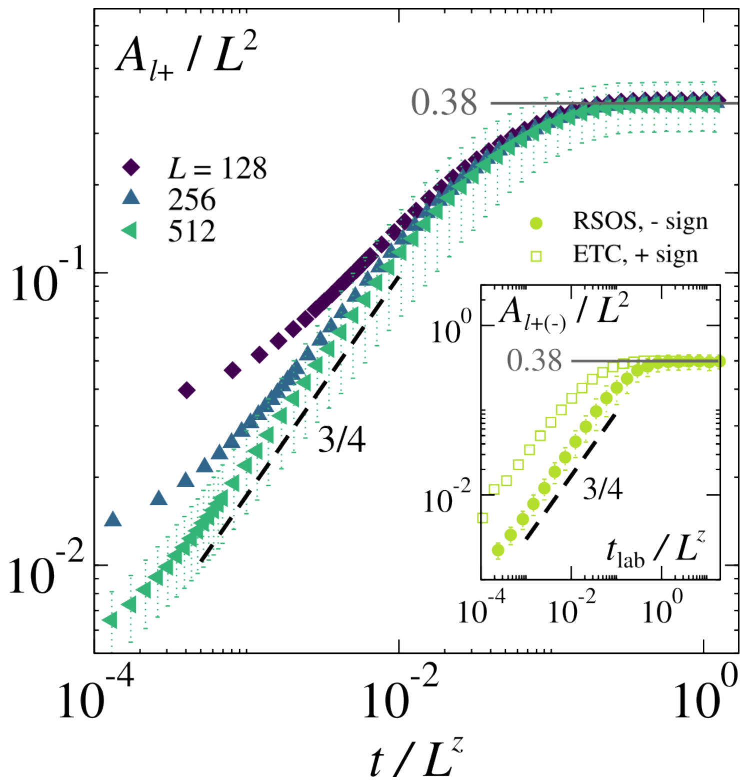
$+1$	pixel		Largest cluster
-1	pixel		Largest cluster

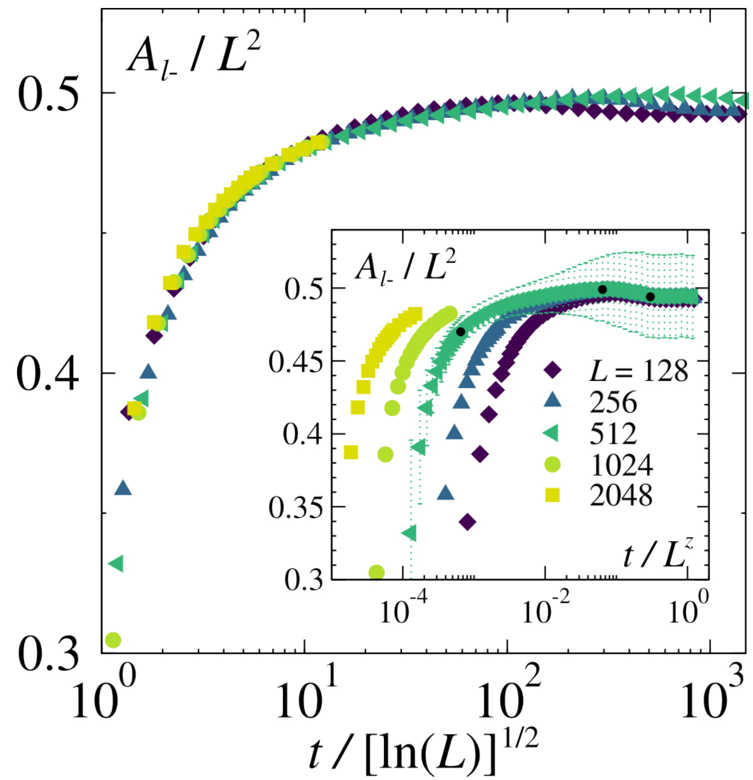
A new lengthscale in KPZ dynamics

- Area of the largest cluster (ensemble-averaged) $A_{l+(-)}(L,t)$

Normal growth: $l_+(t) \sim t^{1/z}$

Faster growth: $l_-(t) \sim e^{(t/t_0)^2}$

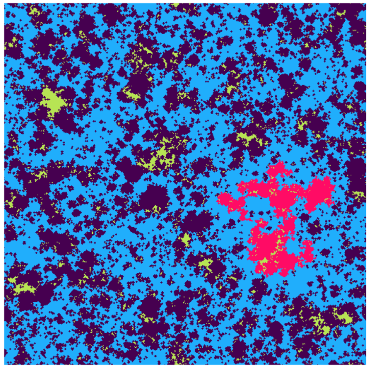




Conclusions (part 2)

Almeida & Cugliandolo (To be published)

- We uncovered new (universal) aspects of growing interfaces
- Asymmetry: Giant cluster detected for the first time
- Symmetry recovered if KPZ nonlinearity is off (not shown)



Thanks!

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