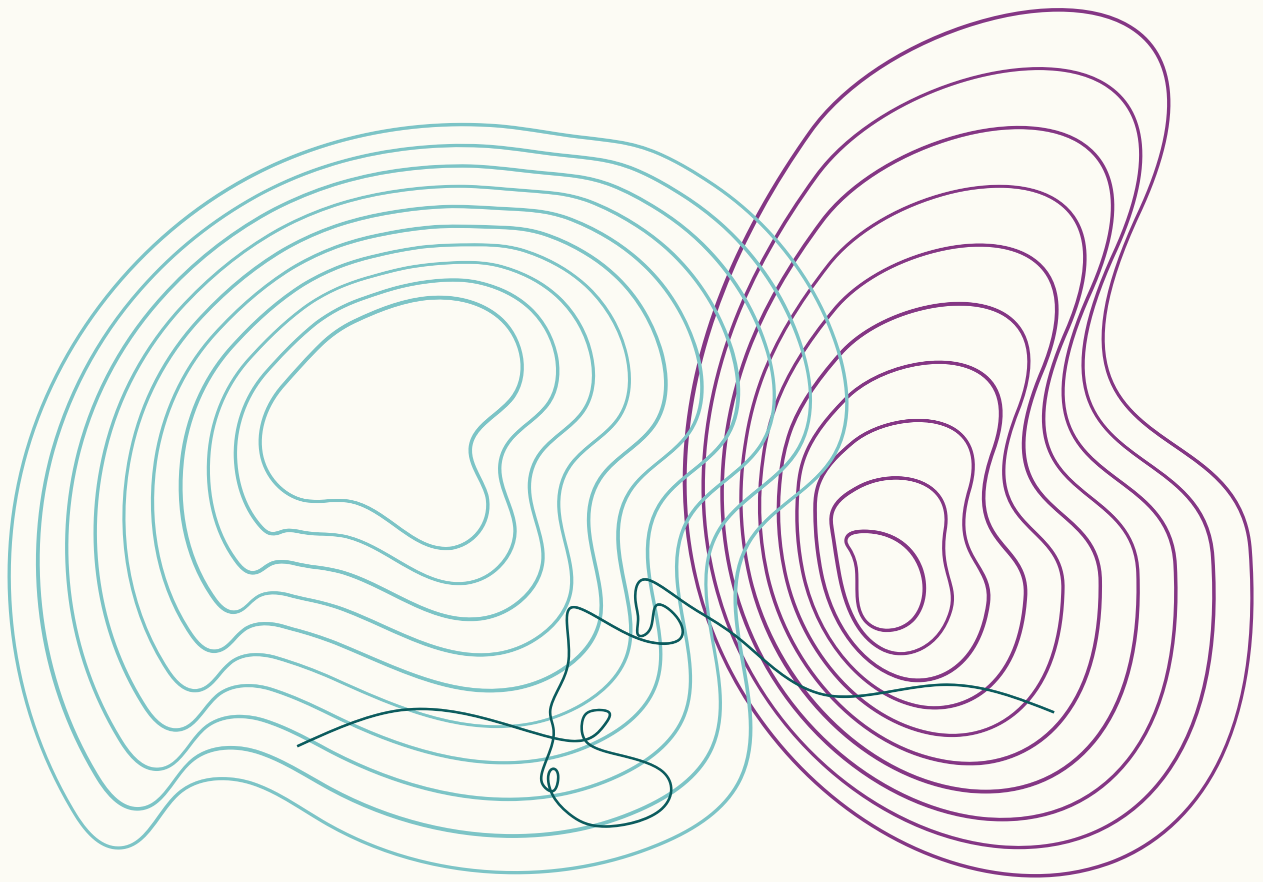
A SUPERPOSITION QUANTUM UNIVERSE AND ITS PERTURBATIONS

… an alternative description for quantum cosmology?

FRIF Day, IAP

17 Nov 2025



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[arXiv:2508.06231] – *work in collaboration with Kratika Mazde and Patrick Peter*

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LEVERHULME TRUST

INTRO: QUANTUM COSMOLOGY

- **Minisuperspace models**: Wheeler-de-Witt quantisation on the reduced phase space of general relativity to obtain a quantum description of the universe

→ approximation to a full theory of quantum gravity

- **Problem of time** [Isham, ('93)]
  - GR is a fully constrained system → no external time parameter
  - Evolution happens w.r.t. an internal degree of freedom serving as a clock (here: perfect fluid)

[Małkiewicz, Peter, ('19)]
[Gielen, Menéndez-Pidal, ('20, `21)]
[de Cabo Martin, Małkiewicz, Peter, ('22)]
[Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]
....

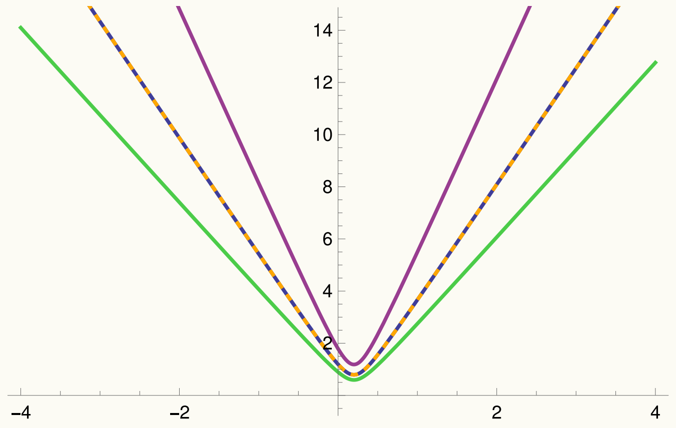
- **Ambiguities**
  - Quantisation: choice of clock degree of freedom, canonical variables, quantisation scheme
  - Extraction of an effective evolution of the scale factor
  - (Semiclassical) state of the universe

→ **different phenomenology** (e.g. singularity resolution)

IN A NUTSHELL

Minisuperspace quantisation of FLRW spacetime → bounce

Quantum **trajectories** to obtain quantum corrected evolution of scale factor →



Universe in a **superposition** $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$ → Influence on **perturbations**?

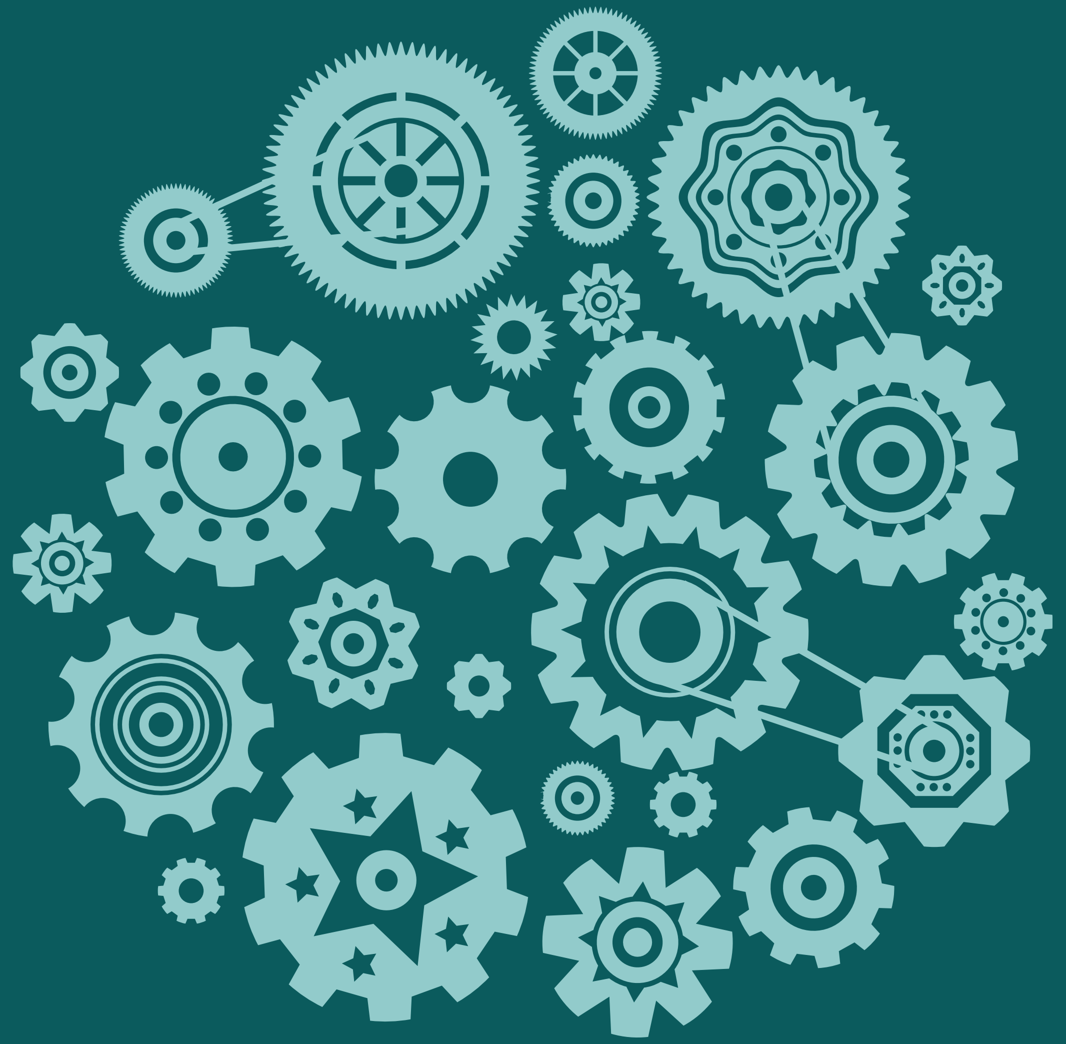
Interaction between multiple background states and their perturbations leads to non Gaussianities in perturbations

[Bergeron, Małkiewicz, Peter, ('24)]
[Bergeron, Małkiewicz, Peter, ('25)]

Trajectories give drastically different background evolution that alters the dynamics of perturbations

SETUP

Quantising the universe



THE SYSTEM

- Quantise phase space of FLRW spacetime $\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$

lapse

$\kappa = 8\pi G$

$$\mathcal{H}_{\mathrm{ADM}} \to \mathcal{H}_{\mathrm{FLRW}} = -\frac{\kappa N}{12\mathcal{V}_0 a} p_a^2$$

momentum conjugate to scale factor

scale factor

spatial section volume

- Perfect fluid as matter clock fixes the lapse $N = -a^{3w}$

equation of state parameter

[Schutz, ('70) ('71)]

$$\mathcal{H}_{\mathrm{fluid}} = \gamma(w)\frac{N}{a^{3w}}\, p_\phi^{1+w}$$

- Canonical transformation to convenient variables
  - $(a, p_a) \to (q, p)$ with $p \propto a^{\frac{3}{2}(1-w)} H$, $q \propto a^{\frac{3}{2}(1-w)}$

    Hubble rate
  - $p_\phi \to p_\tau = -\gamma p_\phi^{1+w}$

- Total Hamiltonian after deparametrisation: $\mathcal{H} = \mathcal{H}_{\mathrm{FLRW}} + \mathcal{H}_{\mathrm{fluid}} \propto p^2 + p_\tau$

matter clock

QUANTISATION $\mathcal{H} \to \hat{\mathcal{H}}$

- Quantisation based on the Weyl-Heisenberg group $x \in \mathbb{R}$ $p \in \mathbb{R}$
  - $U(q,p)\psi(x) = e^{\mathrm{i}p(x-q/2)}\psi(x-q)$

- Here: $x \geq 0$, $p \in \mathbb{R}$ $\longrightarrow$ use affine group $U(q,p)\psi(x) = \frac{e^{\mathrm{i}px}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$

- Quantisation map $\hat{A}_f = \mathcal{N}\int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\,|q,p\rangle\, f(p,q)\,\langle q,p|$ where $|q,p\rangle = U(q,p)|\psi_0\rangle$

  $f(p,q)$: phase space function; $|q,p\rangle$: coherent state; $|\psi_0\rangle$: fiducial state

- Quantisation of FLRW Hamiltonian $\mathcal{H} \propto p^2 + p_\tau$ leads to a repulsive potential

$$\hat{A}_{p^2}\,\psi = -\partial_x^2\,\psi + \frac{K}{x^2}\psi \longrightarrow \hat{\mathcal{H}} \propto \hat{p}^2 + \frac{K}{\hat{q}^2} + \hat{p}_\tau$$

$\hat{p}_\tau$: time evolution; $\frac{K}{\hat{q}^2}$: introduces a bounce

with

$$\hat{p}\,\psi = -\mathrm{i}\partial_x\psi$$
$$\hat{q}\,\psi = x\,\psi$$
$$\hat{p}_\tau\psi = -\mathrm{i}\partial_\tau\psi$$

[Klauder, ('99)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

SEMICASSICAL STATES

- Use semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$
  - Satisfies the Schrödinger equation $\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \frac{K}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2} \qquad p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

  - Specific choice of fiducial state $\langle x|q(\tau), p(\tau)\rangle = \frac{1}{\sqrt{q(\tau)}}\exp\left(\mathrm{i}\frac{p(\tau)}{2q(\tau)}x^2\right)\Phi_n\left(\frac{x}{q(\tau)}\right)$

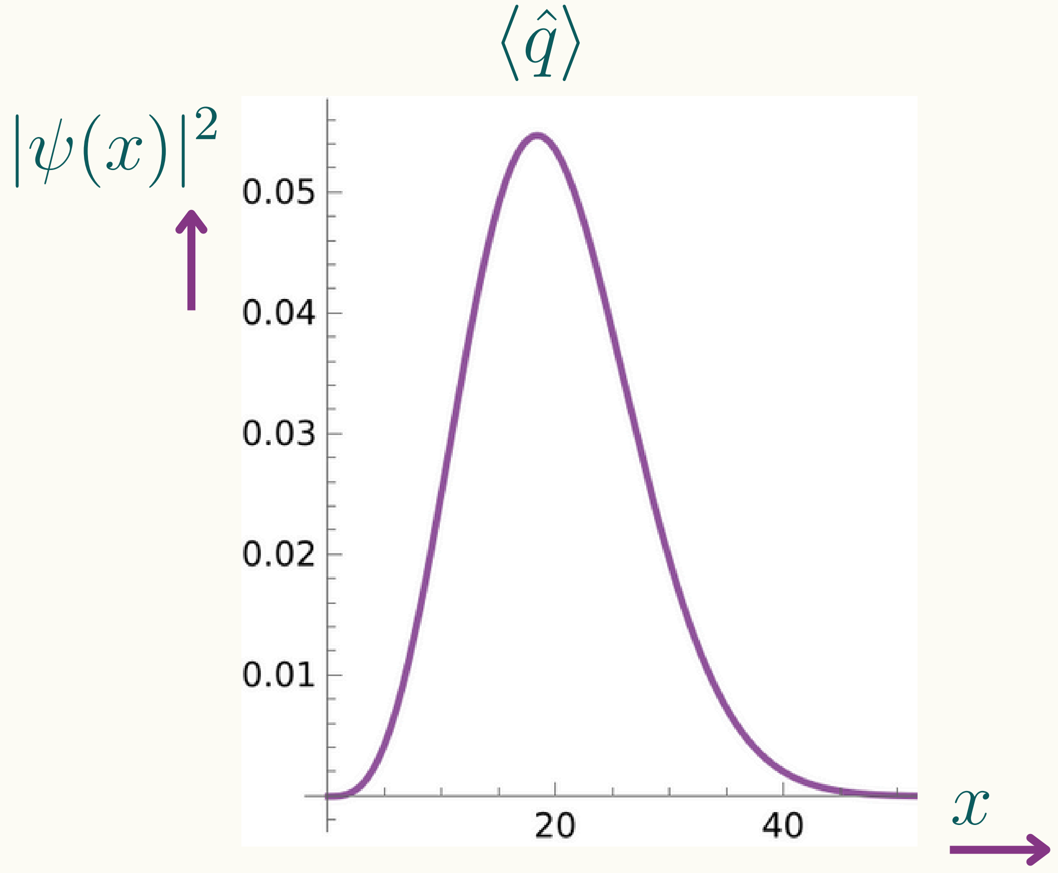
[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

Note: use different representation

$$V(q,p)\psi(x) = \frac{1}{\sqrt{q}}\exp\left(\mathrm{i}\frac{p}{2q}x^2\right)\psi\left(\frac{x}{q}\right)$$

SEMICLASSICAL SCALE FACTOR

- Operator related to the scale factor $q \propto a^{\frac{3}{2}(1-w)}$ $\quad q \to \hat{q}$
- In order to connect to GR: need classical scale factor on spacetime
- Common: semiclassical scale factor from an operator expectation value in a highly peaked state semiclassical state = the most likely state for the system $\langle\psi|\hat{q}|\psi\rangle \propto a^{\frac{3}{2}(1-w)}$
- Contrary to lab experiments (or cosmological perturbations) one cannot repeat the experiment many times: no statistical distribution



- Here: use Bohmian trajectories [de Broglie ('27)] [Bohm ('52)]
  - Predictions of the trajectory approach are equivalent to the Copenhagen / orthodox version of quantum mechanics: give the same probabilistic outcomes
  - Applications in e.g. theoretical chemistry [Sanz ('18)] [Gindensperger ('00)]

THE TRAJECTORY APPROACH

In a nutshell: The quantum system follows a trajectory, but our knowledge of system properties at any given moment is limited

[Holland ('93)]

- Physical system = wave + point particle moving under guidance of the wave

$$\psi(x,t) = R(x,t)e^{\mathrm{i}S(x,t)} \longrightarrow x(t)$$

trajectory

- Wave function obeys the Schrödinger equation

$$\mathrm{i}\partial_t\psi = H\psi$$

- Particle motion is obtained calculated from the phase (initial condition dependent)

$$\dot{x} = \frac{1}{m}\nabla S(x,t)$$

  - Different initial conditions $x(t_0)$ give an ensemble of particles associated to the same wave
  - Arbitrary initial condition, but not in regions where $\psi(x,t) = 0$

- Probability of the particle to lie in an interval $x + \mathrm{d}x$ is determined by the wave function amplitude $R^2(x,t)\mathrm{d}x$

EXAMPLE: DOUBLE SLIT EXPERIMENT

- Consider the double slit experiment as an example
  - Two wave packets emerge from the slits: Gaussian in y-direction, plane waves in x-direction

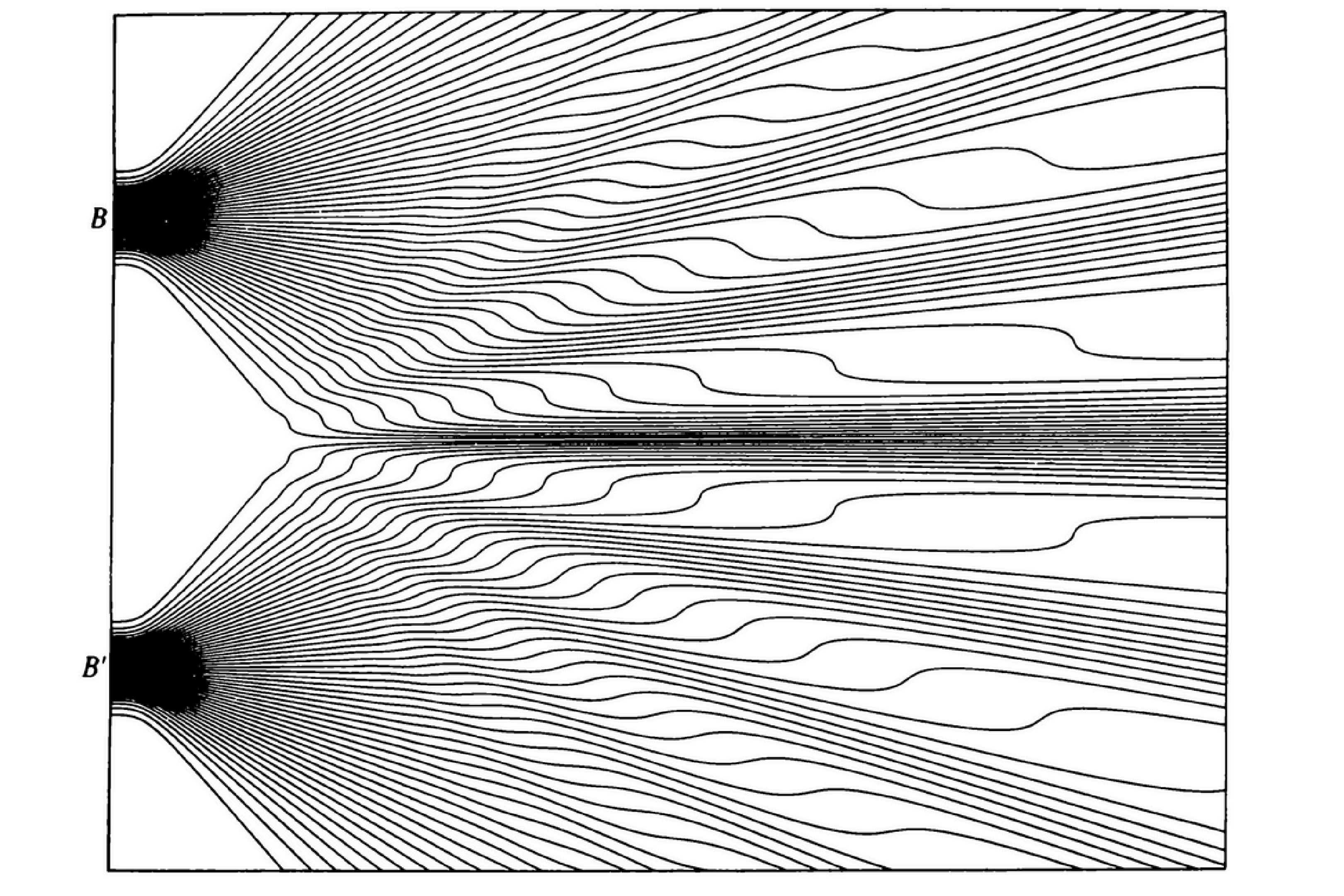
$$\psi_B(x, y, t_0) \quad \text{and} \quad \psi_{B'}(x, y, t_0)$$

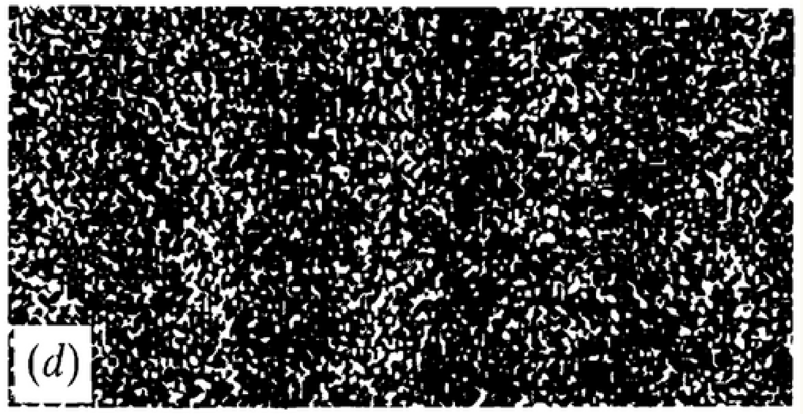
  - Total wave function is a superposition

$$\psi = \mathcal{N}(\psi_B(x, y, t) + \psi_{B'}(x, y, t))$$

- Obtain interference pattern from $R^2 = |\psi|^2$
- Numerically calculate trajectories
  - Can reconstruct the path of an electron that hit the screen (up to measurement uncertainty)

[Image: Philippidis et al., ('82)]





[Image: Tonomura et al., ('89)]

BOUNCING TRAJECTORIES

- Continuous ensemble of trajectories can be obtained from the wave function:

trajectory gives the scale factor:

$$x(\tau) = \sqrt{\frac{12\mathcal{V}_0}{\kappa}\frac{2\,a^{\frac{3}{2}(1-w)}}{3(1-w)}}$$

evolution governed by the wave function:

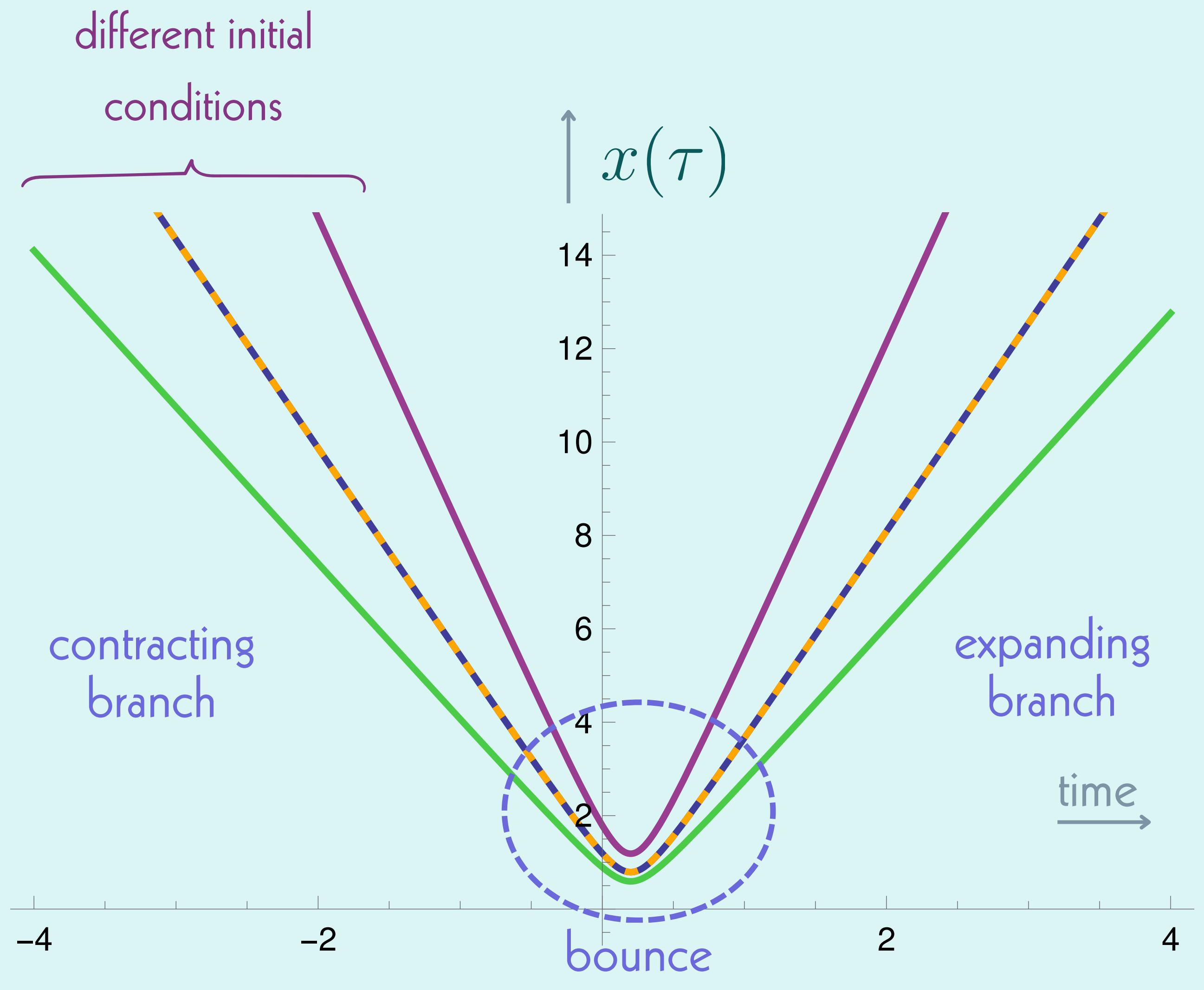
$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln\frac{\psi}{\psi^\star}$$

with

$$|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau),p(\tau)\rangle$$

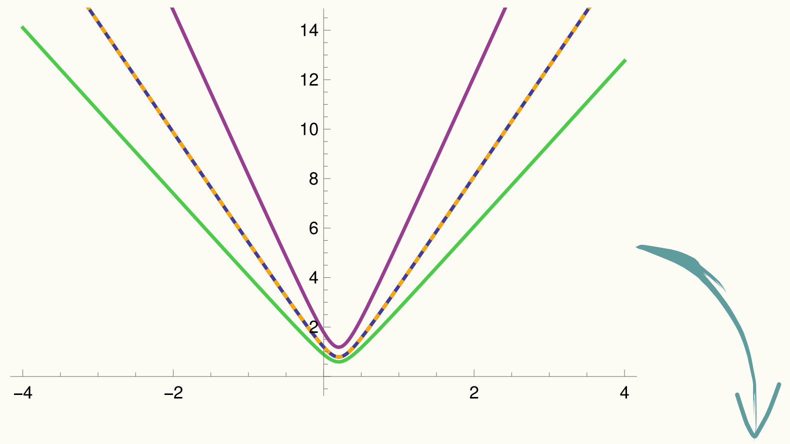
$$\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$$

  - quantum uncertainty in the initial conditions
  - assign a concrete value to the effective scale factor at all times
  - classical dynamics away from bounce

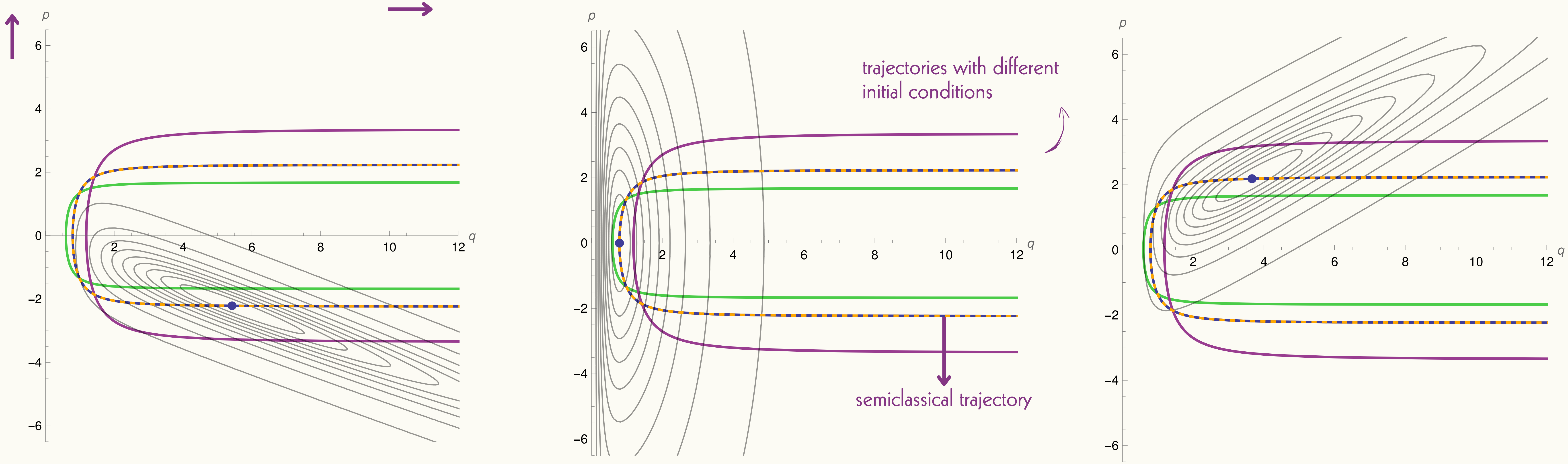


BOUNCING SINGLE STATE TRAJECTORIES

- State and trajectories follow dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\text{sem}} = p^2 + \frac{K}{q^2}$



$p \propto a^{\frac{3}{2}(1-w)} H$ $\qquad$ $q \propto a^{\frac{3}{2}(1-w)}$



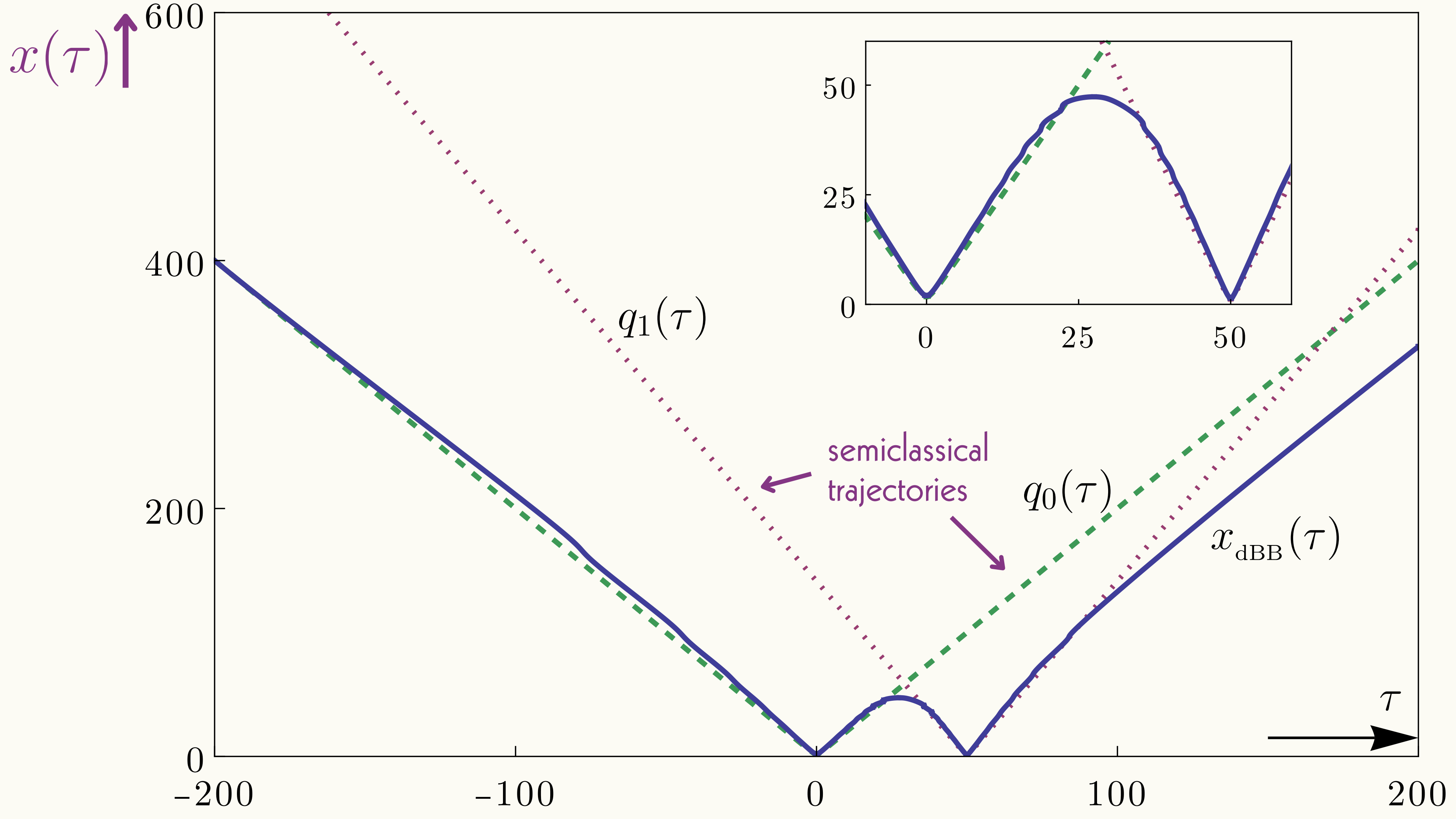
SUPERPOSITION UNIVERSE

Bouncing biverse

UNIVERSE IN A SUPERPOSITION

$$\Psi = \mathcal{N} \sum_n \alpha_n \psi_n$$

- Biverse: $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$
- Recall that trajectories are calculated from: $\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\mathrm{i}\partial_x \ln \frac{\Psi}{\Psi^*}$ with $x \propto a^{\frac{3}{2}(1-w)}$

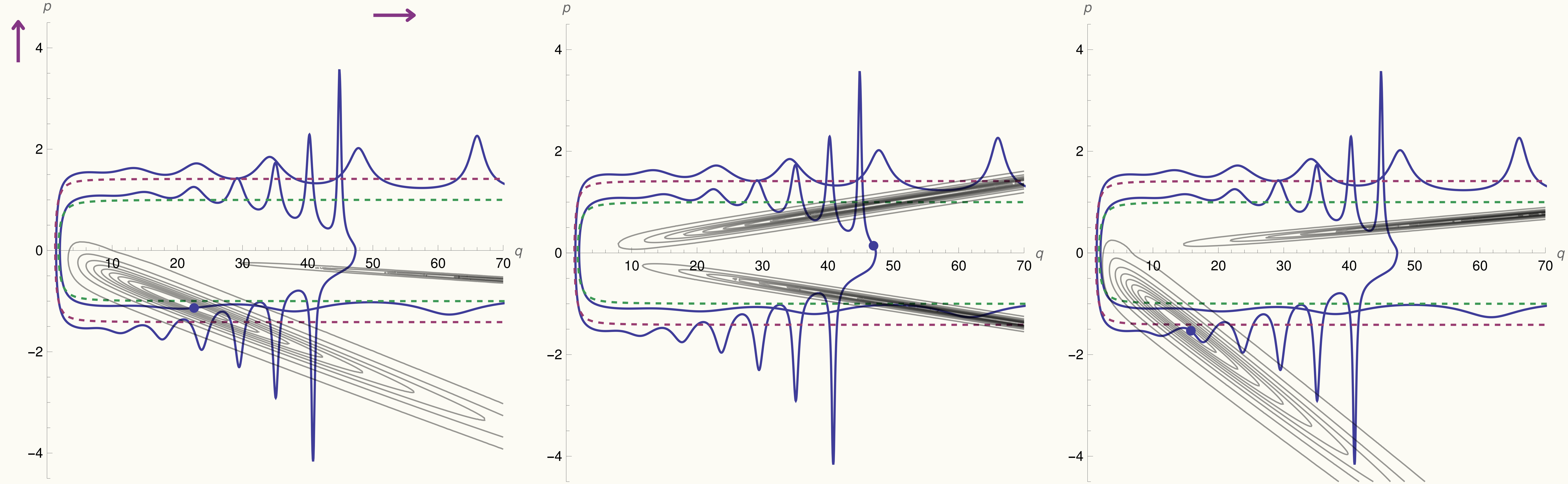


- Start on semiclassical trajectory
- Parameters:
  - r ○ ratio of late time momenta of semiclassical solutions
  - $\Delta\tau$ ○ difference in bounce times
  - $\rho,\ \delta$ ○ contribution of second wave function

UNIVERSE IN A SUPERPOSITION

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

$p \propto a^{\frac{3}{2}(1-w)} H$ $\quad q \propto a^{\frac{3}{2}(1-w)}$

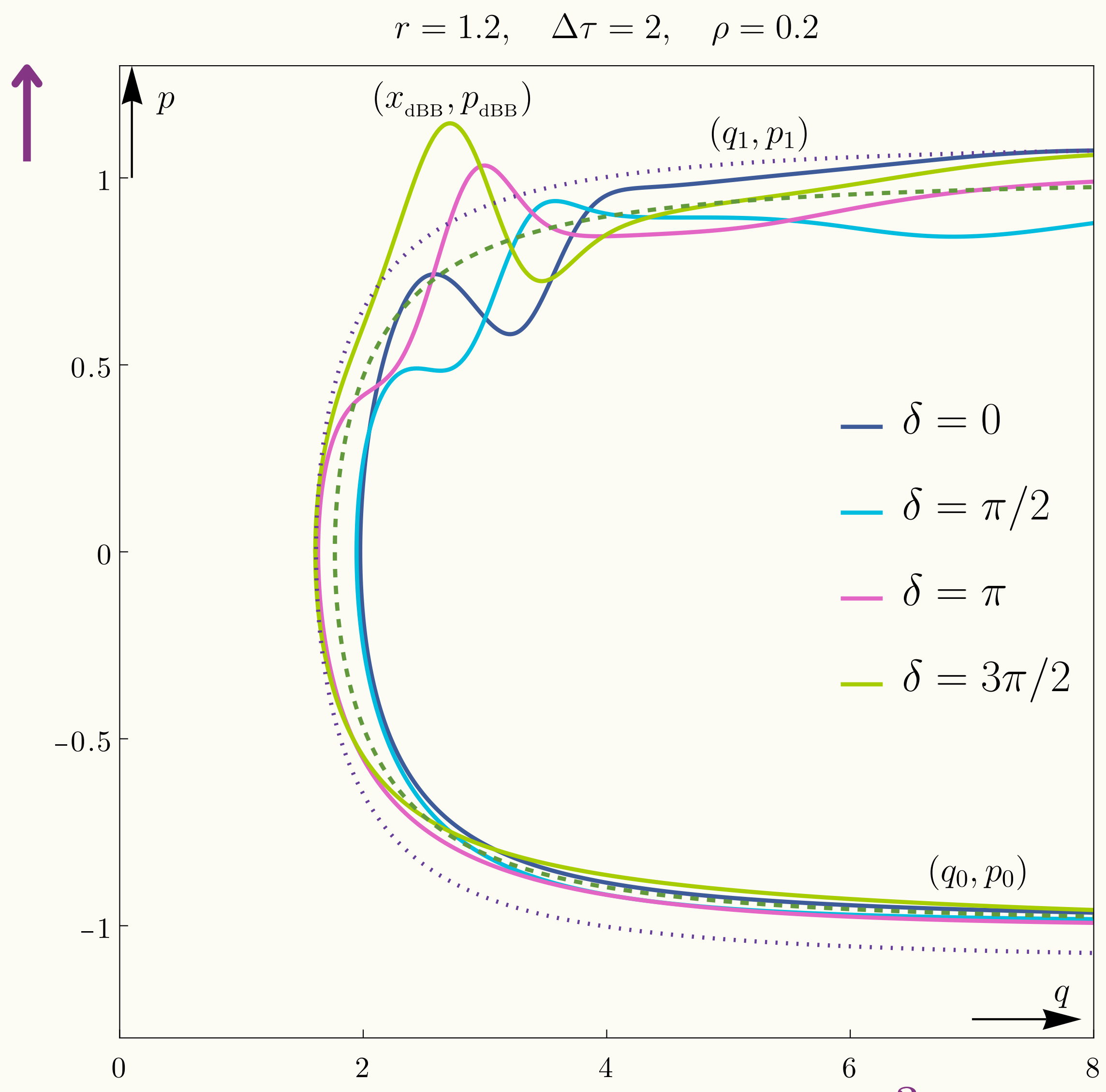


- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

ANOTHER EXAMPLE & LATE TIME LIMIT

- Phase space portraits for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

$p \propto a^{\frac{3}{2}(1-w)} H$



$\longrightarrow \; q \propto a^{\frac{3}{2}(1-w)}$

- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

- Late time behaviour
  - Dominated by a single wave function → same evolution as in single state case
  - Return to initial momentum $\dot{x}(\tau) \to x_0\,\omega$

FIRST APPROACH TO PERTURBATIONS

- Consider tensor perturbations in a flat FLRW spacetime with a radiation fluid (set $w = 1/3$)

$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + [\delta_{ij} + h_{ij}(\eta, \vec{x})]\,\mathrm{d}x^i \mathrm{d}x^j\right)$$

[Peter, Pinho, Pinto-Neto, ('05)]
[Peter, Pinho, Pinto-Neto, ('06)]

- Second order linear perturbative Hamiltonian for tensor modes: sum of the two polarisations and Fourier modes

$$\mathcal{H}_{\mathrm{FLRW}} = \mathcal{H}^{(0)} + \mathcal{H}^{(2)}$$

$$\mathcal{H}^{(2)} = \sum_{\vec{k}} \left(\mathcal{H}^{(2)}_{\vec{k},+} + \mathcal{H}^{(2)}_{\vec{k},\times}\right) \quad \text{with} \quad \mathcal{H}^{(2)}_{\vec{k},\lambda} = \pi^{(\lambda)}_{\vec{k}} \pi^{(\lambda)}_{-\vec{k}} + \left(k^2 - \frac{a''}{a}\right) \mu^{(\lambda)}_{\vec{k}} \mu^{(\lambda)}_{-\vec{k}}$$

$\lambda = +, \times$

conjugate momentum of $\mu^{(\lambda)}_{\vec{k}}$

$h_{ij} \propto \mu_{ij}/a$

- Use scale factor as given by Bohmian trajectories in perturbed Hamiltonian $\Psi_{\mathrm{pert}} = \Psi_{\mathrm{pert}}(a(\eta), \mu_k)$

- Canonical quantisation of the tensor perturbations leads to mode equation $\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$

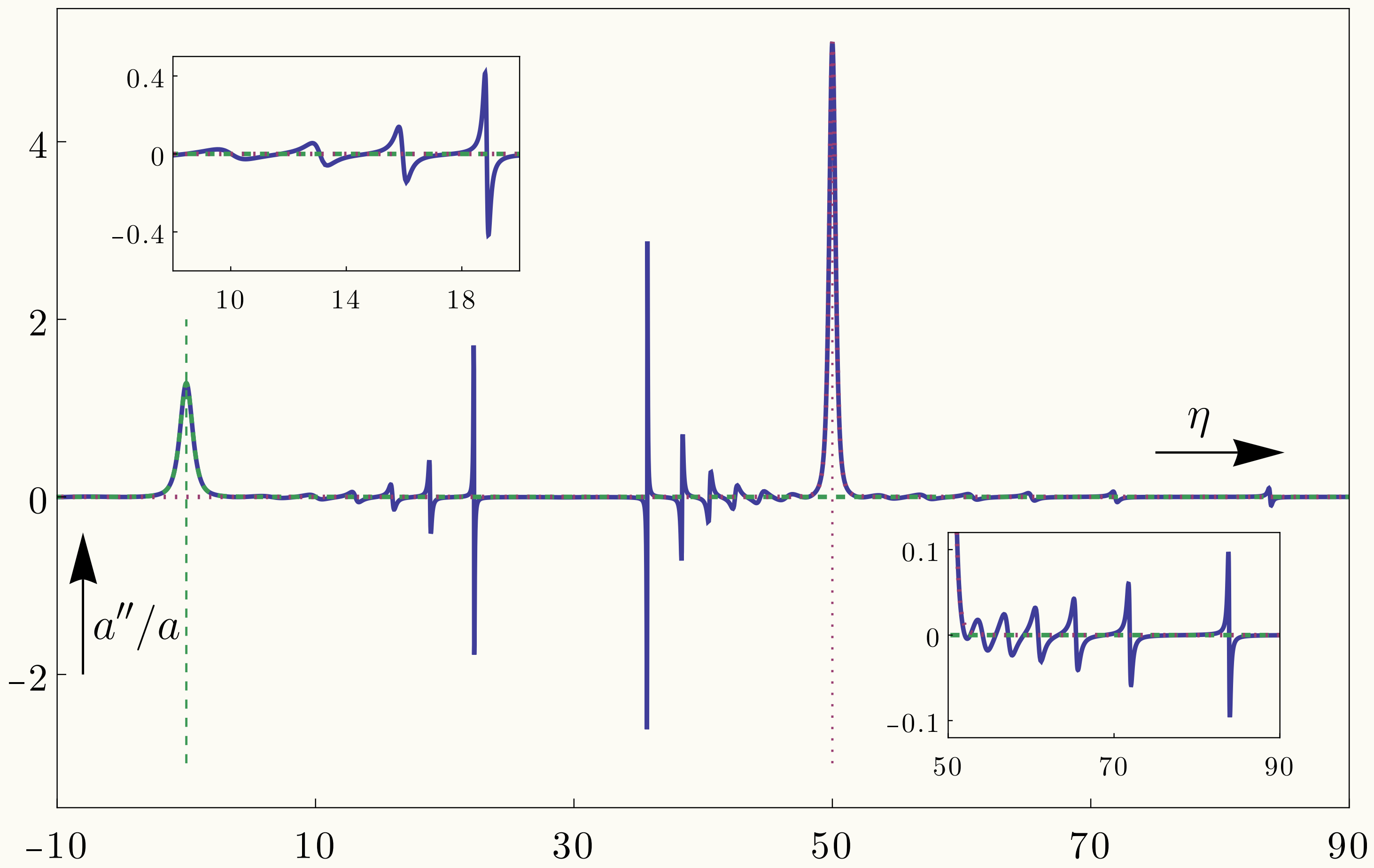
modified by trajectories

PERTURBATIONS FROM BIVERSE TRAJECTORIES

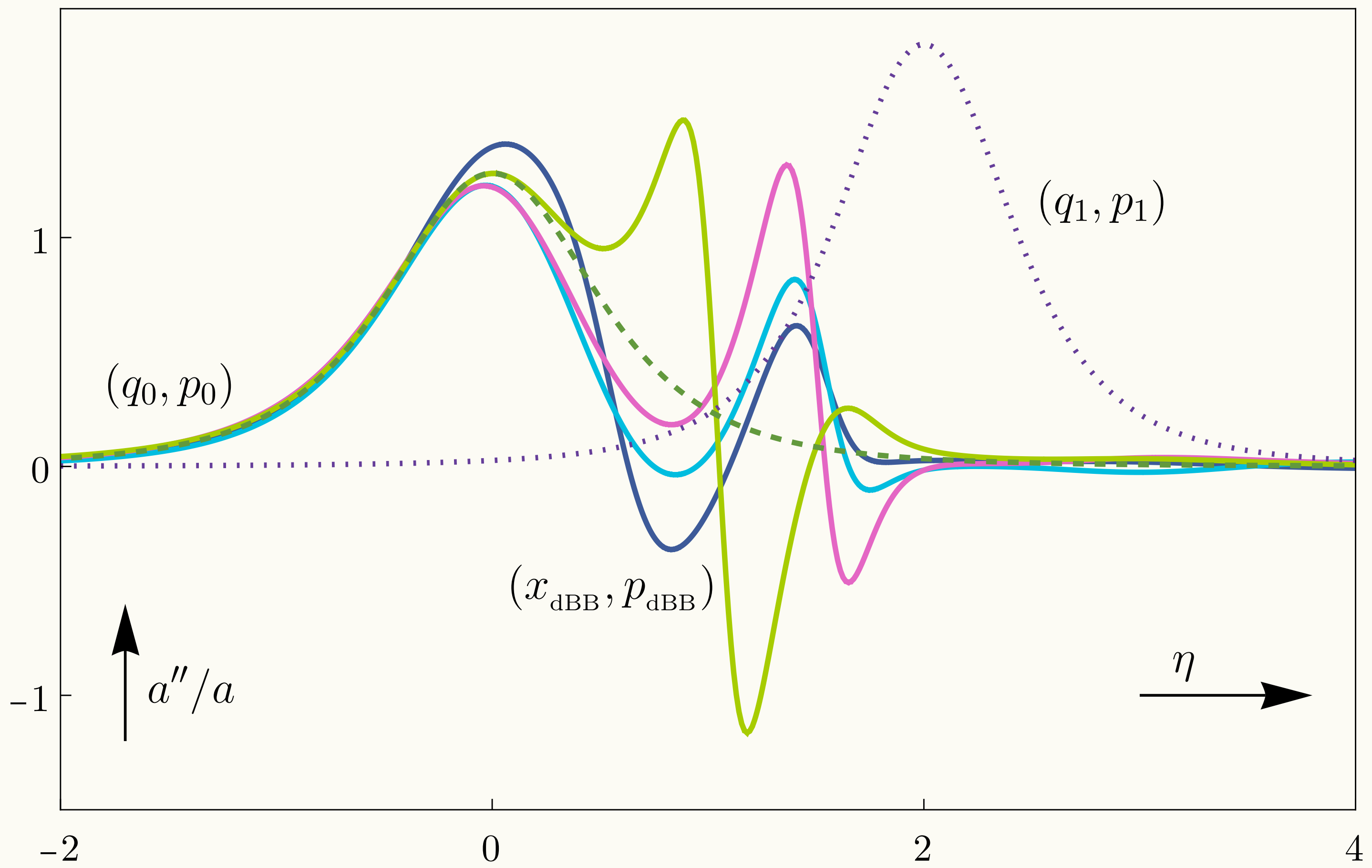
- Dynamics of perturbation modes $\mu_k'' + \left(k^2 - \frac{a''}{a}\right)\mu_k = 0$

$h_{ij} \propto \mu_{ij}/a$ — modified by trajectories

$r = 2, \quad \Delta\tau = 50, \quad \rho = 1 \quad \& \quad \delta = 0$



$r = 1.2, \quad \Delta\tau = 2, \quad \rho = 0.2 \quad \& \quad \delta \in \{0, \pi/2, \pi, 3\pi/2\}$



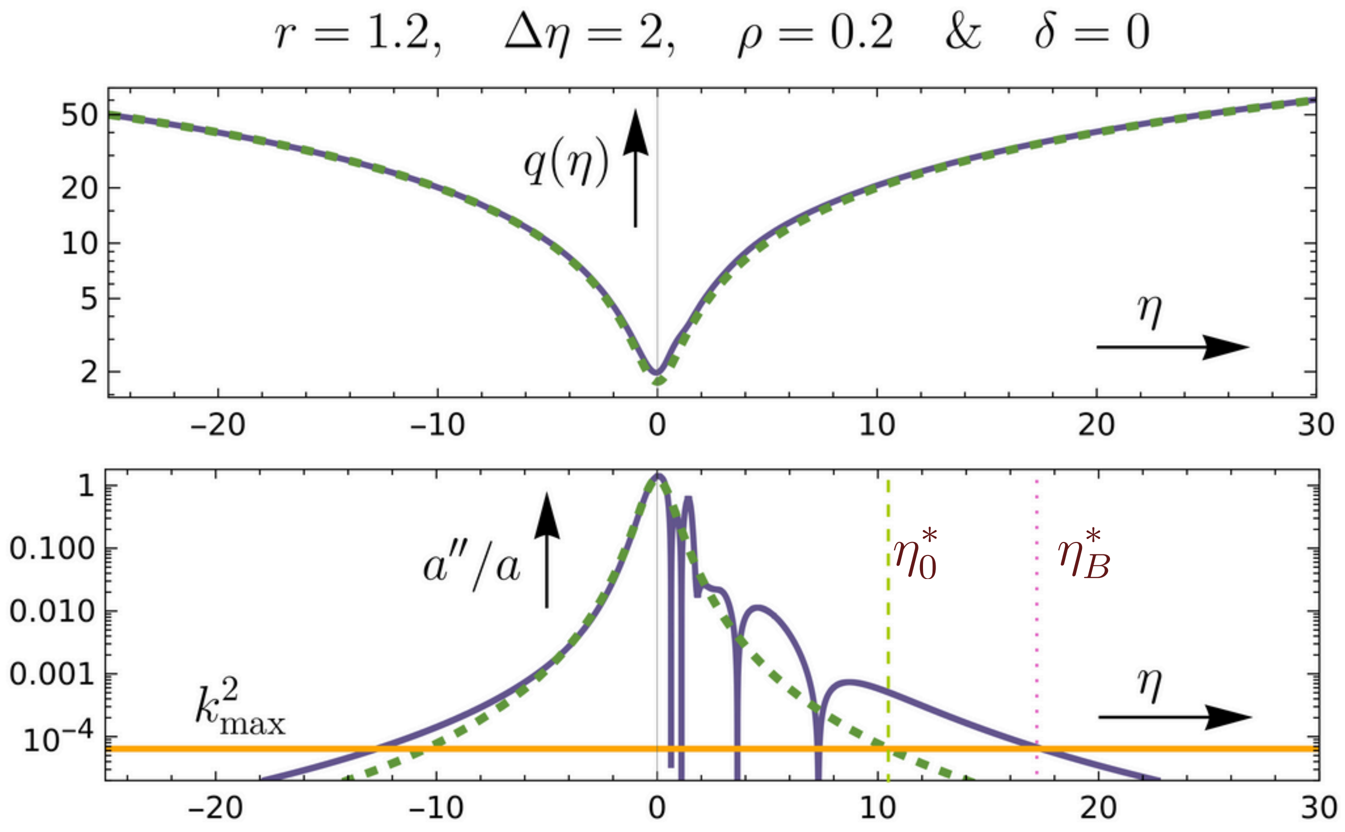
- Potential resulting from the biverse trajectories differs from single state case which would dictate perturbative dynamics in each universe separately if effective scale factor was obtained from projection onto a state

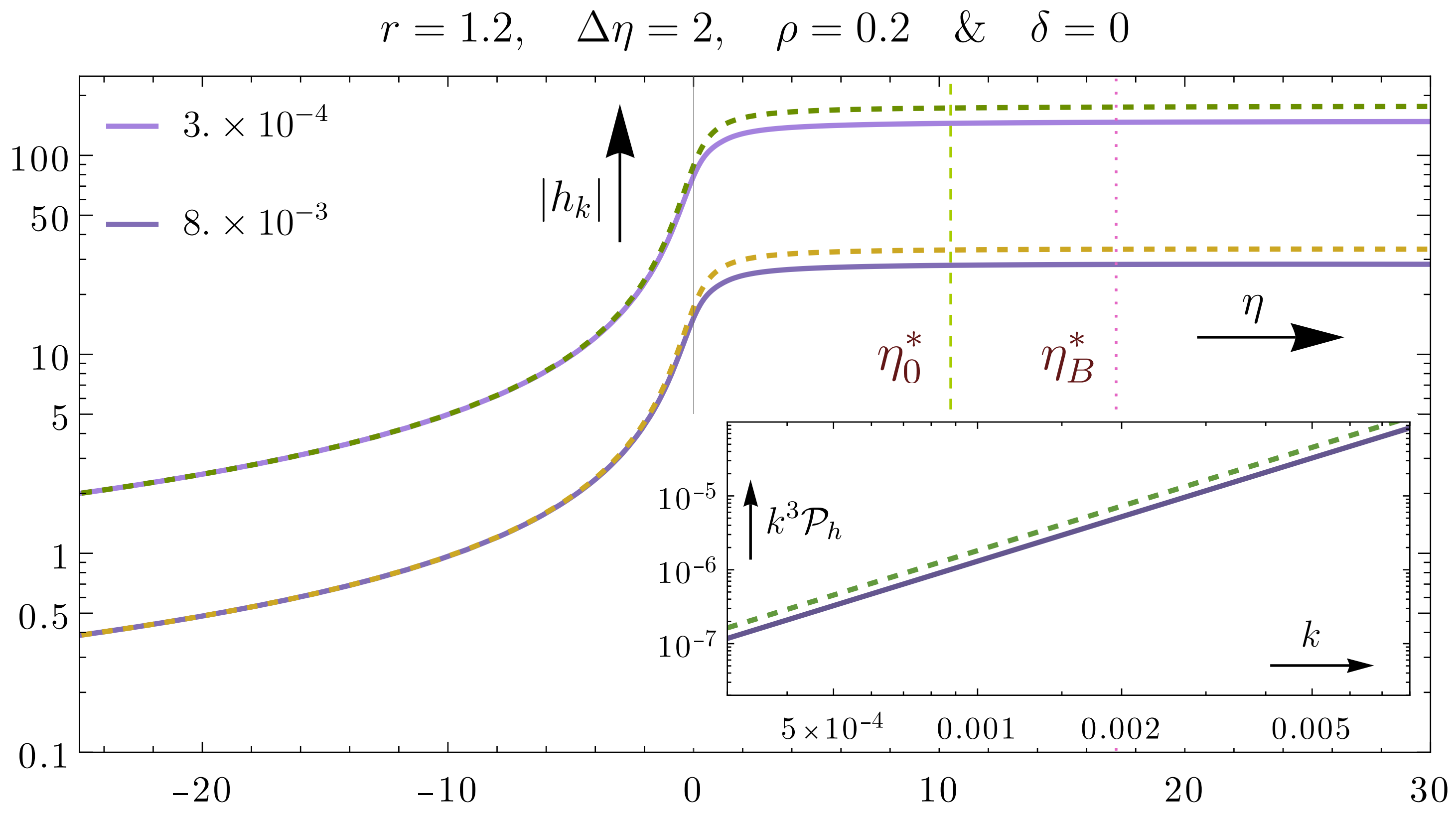
[Bergeron, Małkiewicz, Peter, ('24)]

PERTURBATIVE DYNAMICS: EXAMPLE I

- Range of Fourier modes: $k \in (8 \times 10^{-4}, 3 \times 10^{-3})$
- Initial conditions: Bunch-Davies vacuum $\mu_k(\eta) = \frac{1}{\sqrt{k}} e^{-\mathrm{i}k(\eta-\eta_i)}$
- In potential dominated regime: $\mu_k(\eta) \approx a(\eta)\left(\frac{\mu_{k,0}}{a_0} + (\mu'_{k,0}a_0 - \mu_{k,0}a'_0)\int_{\eta_0}^{\eta} \frac{\mathrm{d}\eta}{a^2}\right)$

for single state: $\int \frac{\mathrm{d}\eta}{a^2} \propto \arctan((\eta-\eta_B)\omega)$





- Power spectrum $k^3\mathcal{P} = \frac{2k^3}{\pi^2}\left|\frac{\mu_k}{a}\right|^2_{\eta=\eta^*}$ taken at time when $V_{\mathrm{eff}}(\eta^*) = k^2_{\max}$

PERTURBATIVE DYNAMICS: EXAMPLE II

- Range of Fourier modes: $k \in (8 \times 10^{-4}, 3 \times 10^{-3})$
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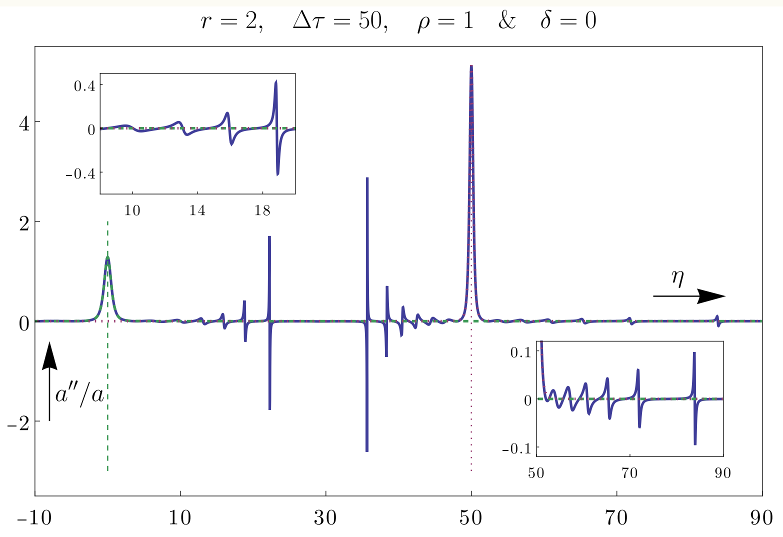

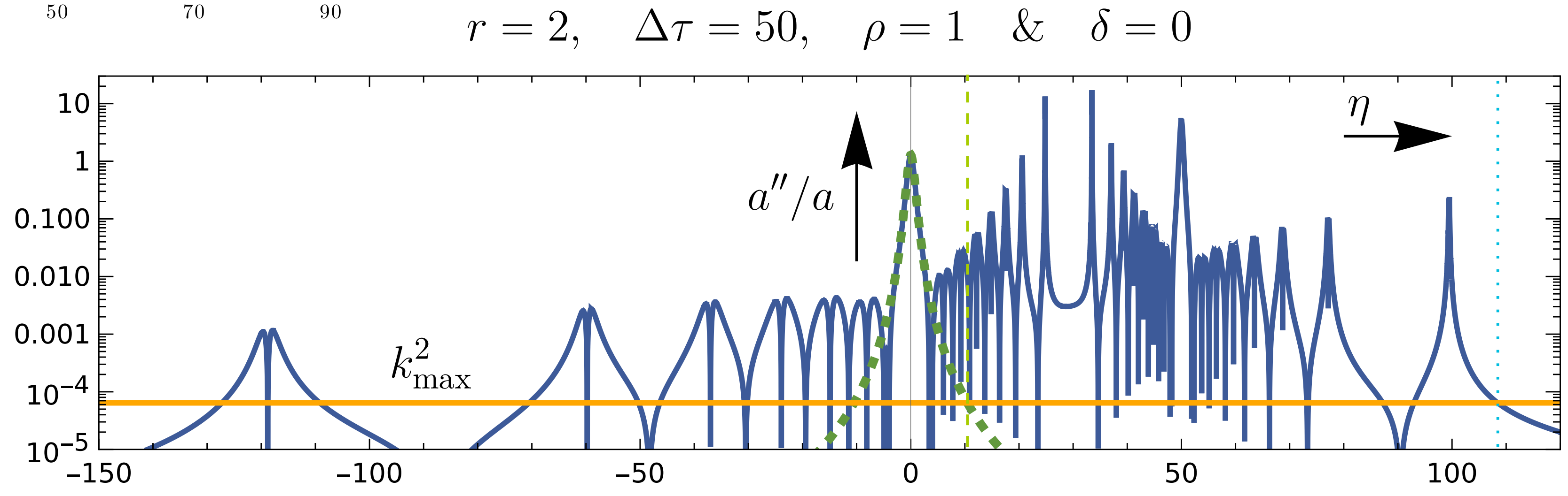

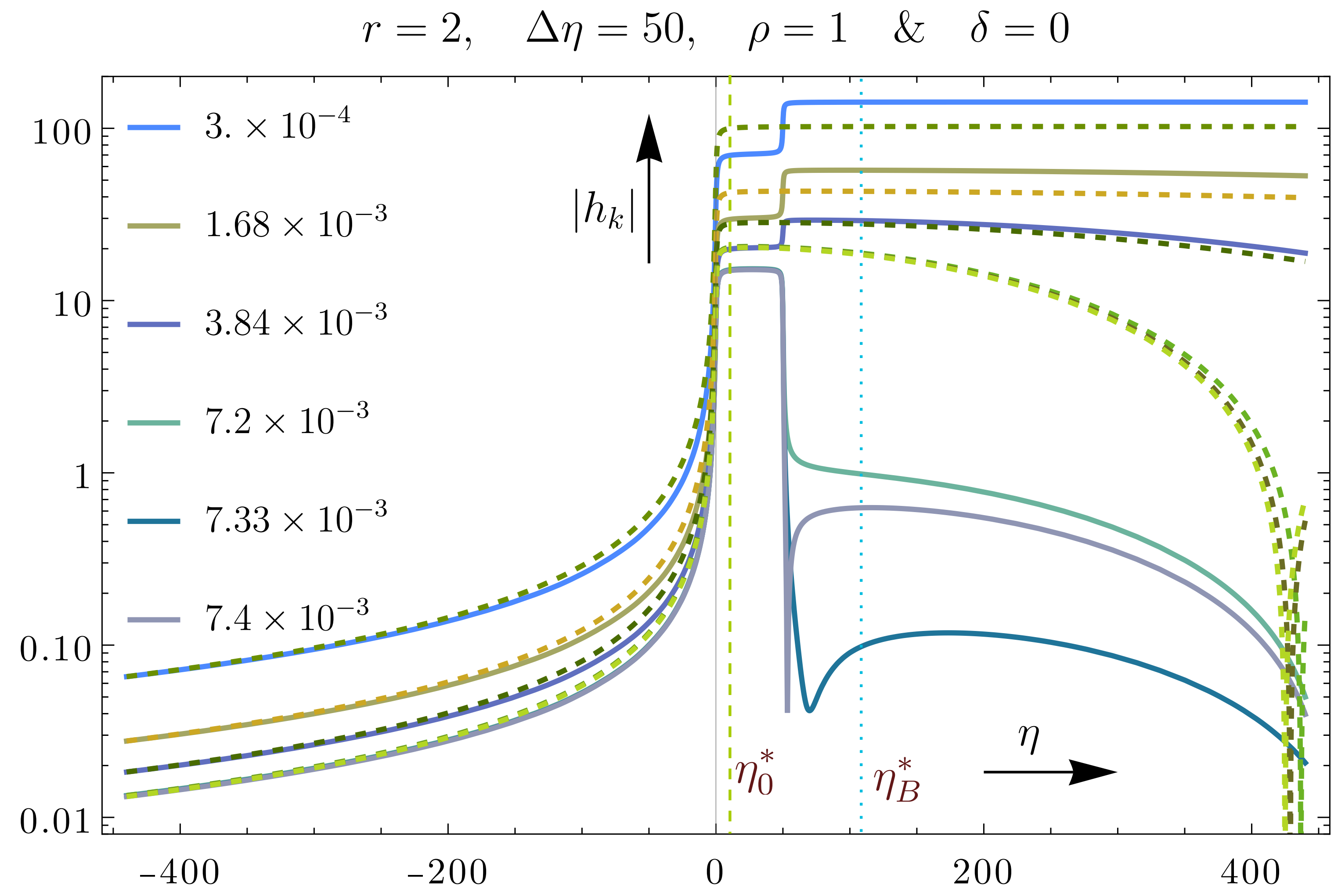

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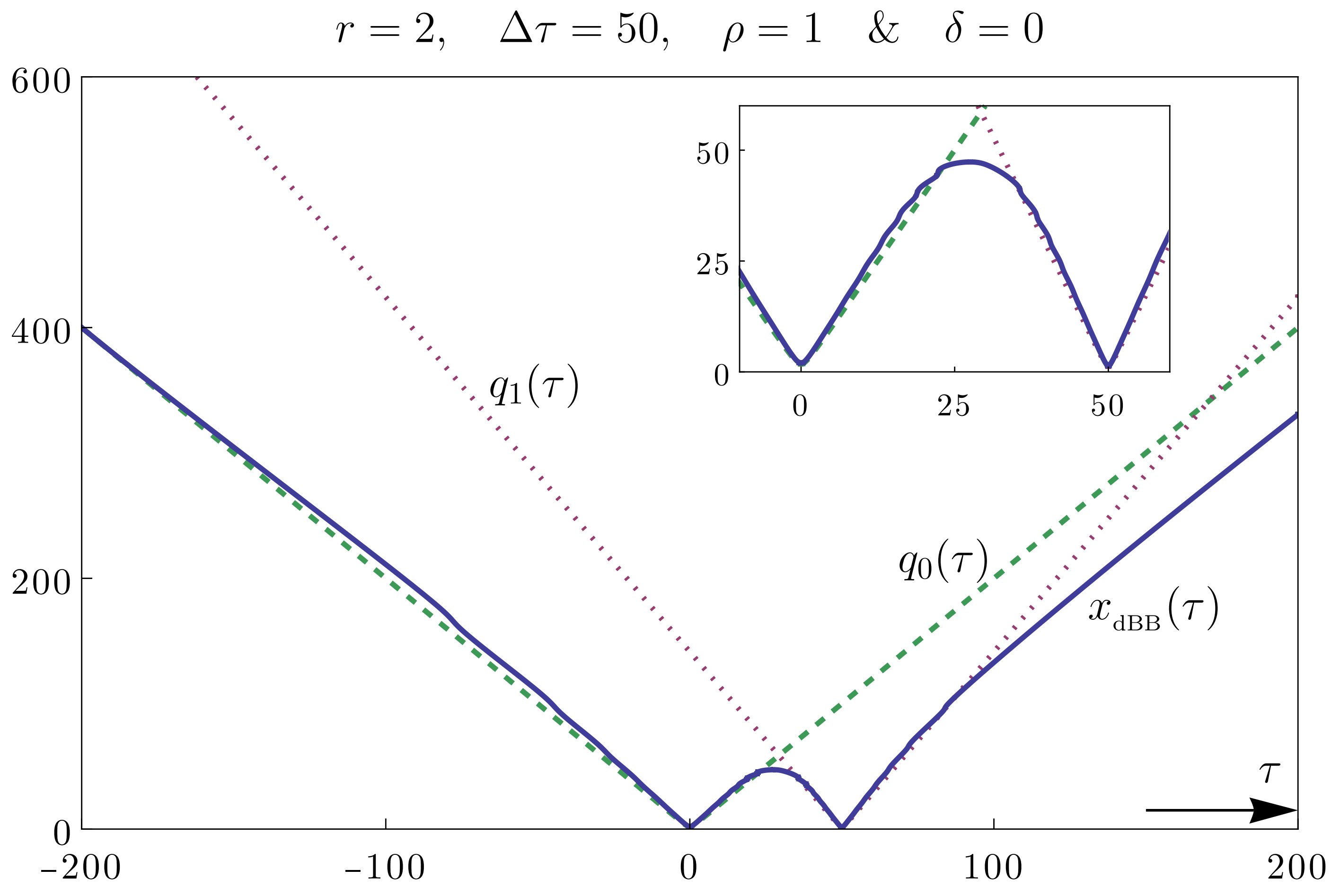
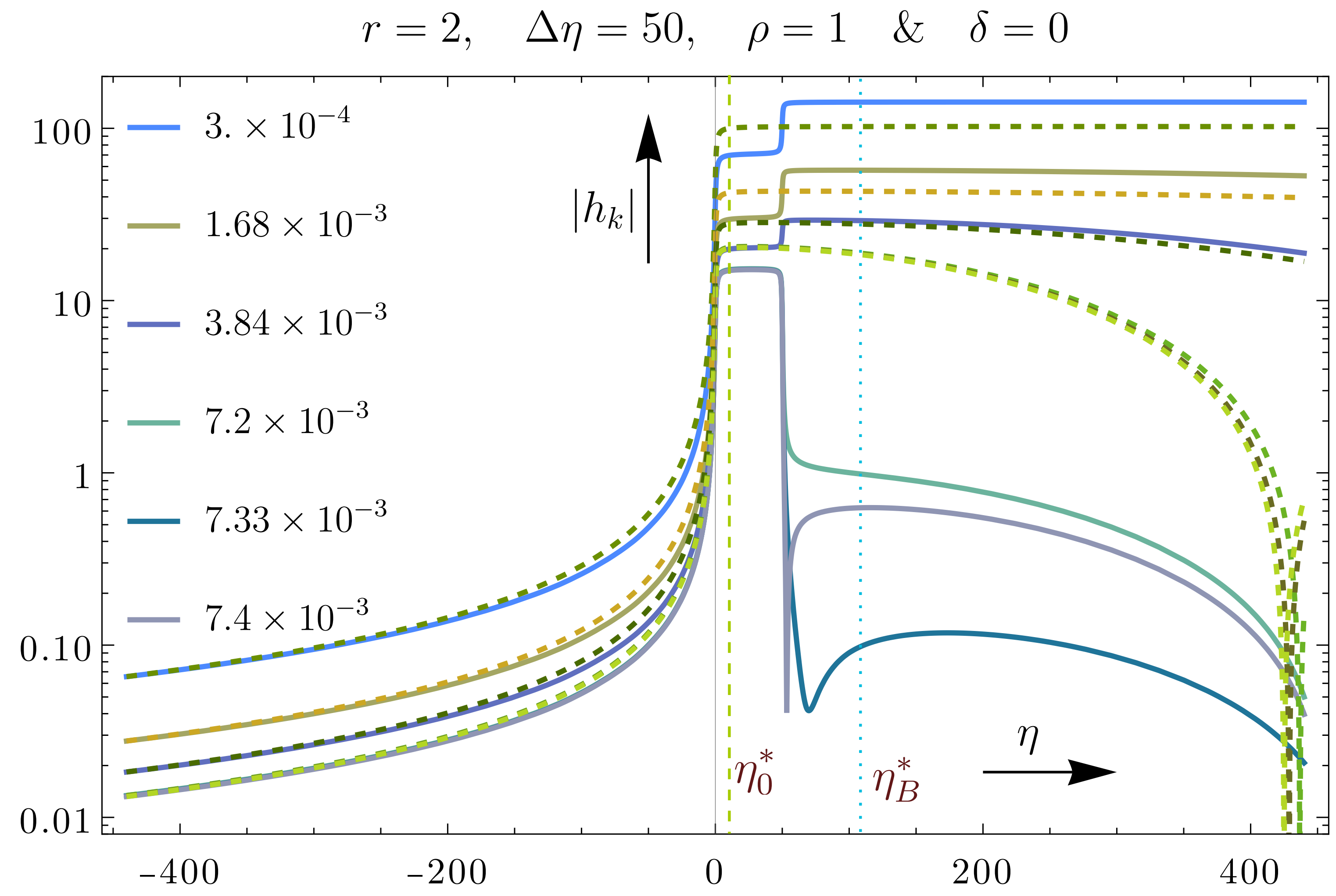
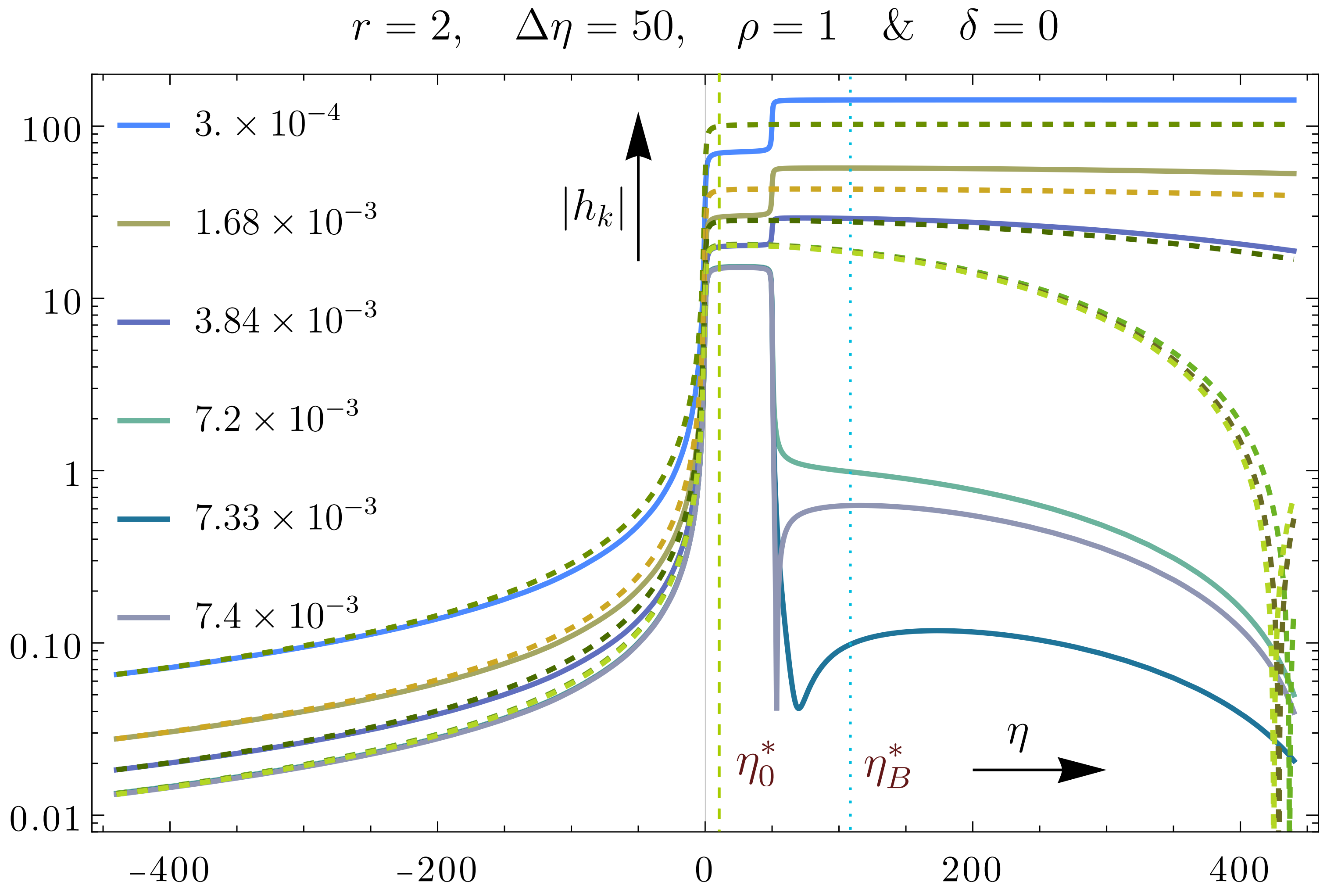
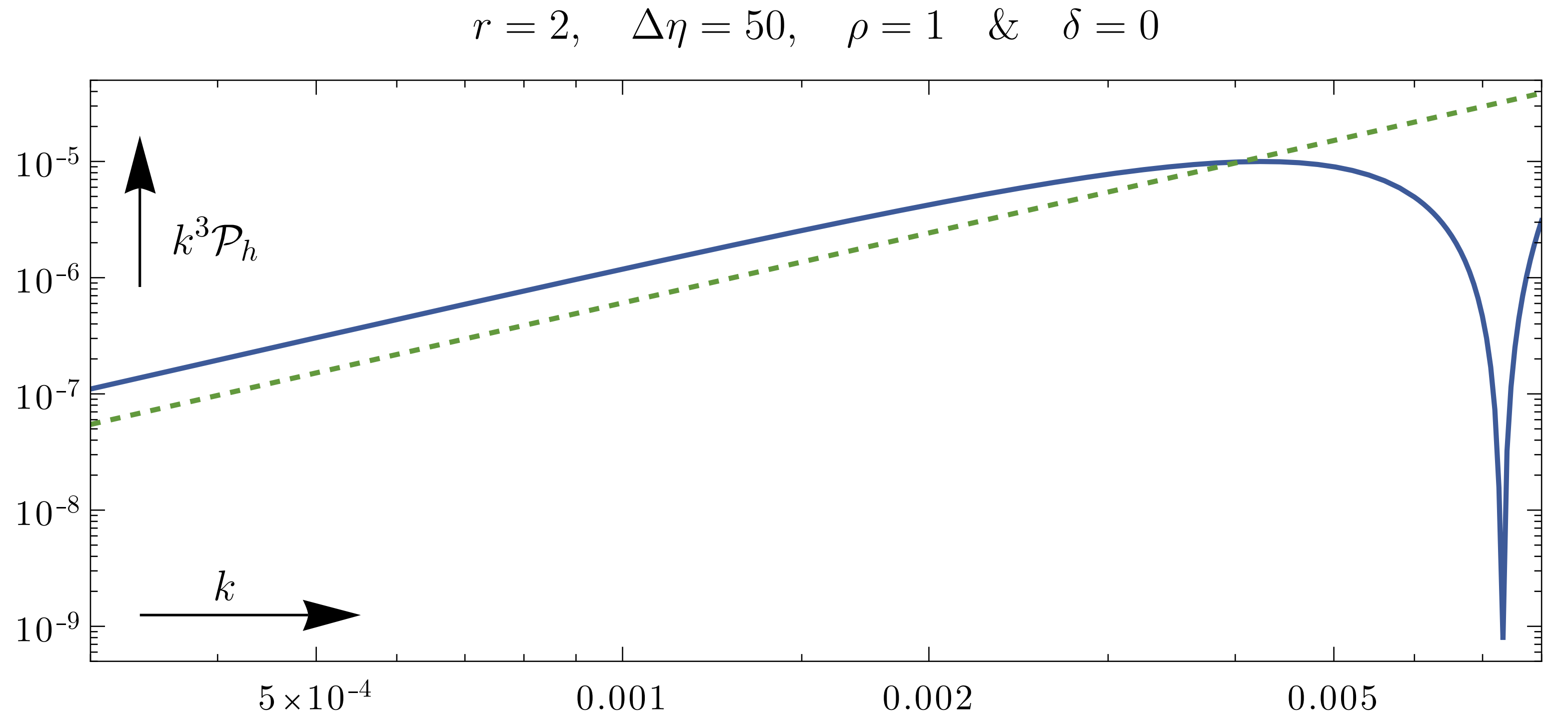




PERTURBATIVE DYNAMICS: EXAMPLE II

- Power spectrum $k^3\mathcal{P} = \frac{2k^3}{\pi^2}\left|\frac{\mu_k}{a}\right|^2_{\eta=\eta^*}$ taken at time when $V_{\text{eff}}(\eta^*) = k^2_{\text{max}}$
  - Lowering of spectral index





CONCLUSION

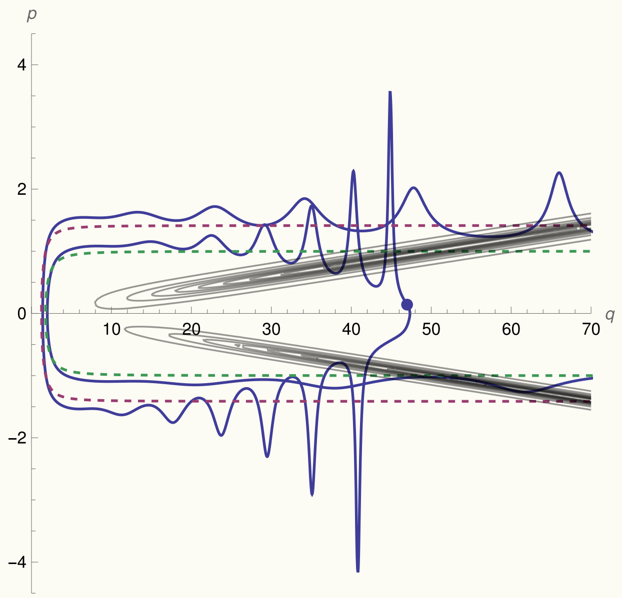
... and next steps

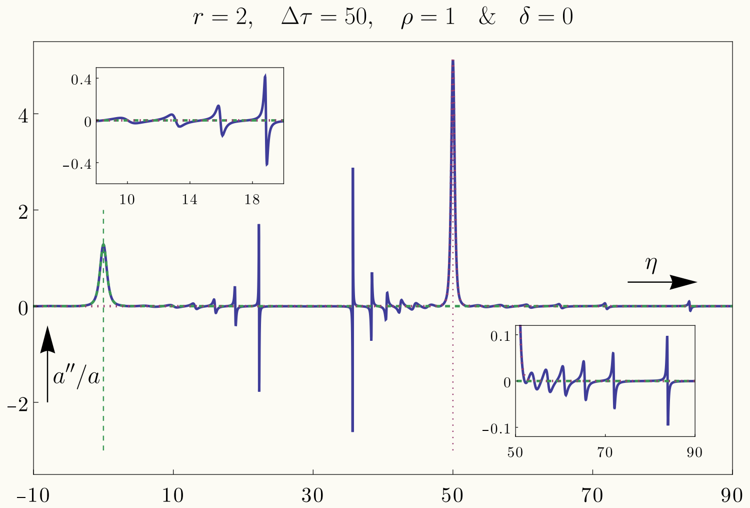
CONCLUSION

- Affinely quantise an FLRW spacetime to obtain bouncing trajectories
    - trajectories assign unambiguous value to the scale factor at all times

- Trajectories for a universe in a superposition introduce distinct features in the scale factor evolution

- These features affect the evolution of perturbations and thereby the tensor power spectrum





→ ***Next:*** detailed study of perturbations, scalar perturbation, and perturbative trajectories

Quantum cosmology offers a unique scenario where quantum trajectories could lead to different phenomenological implications

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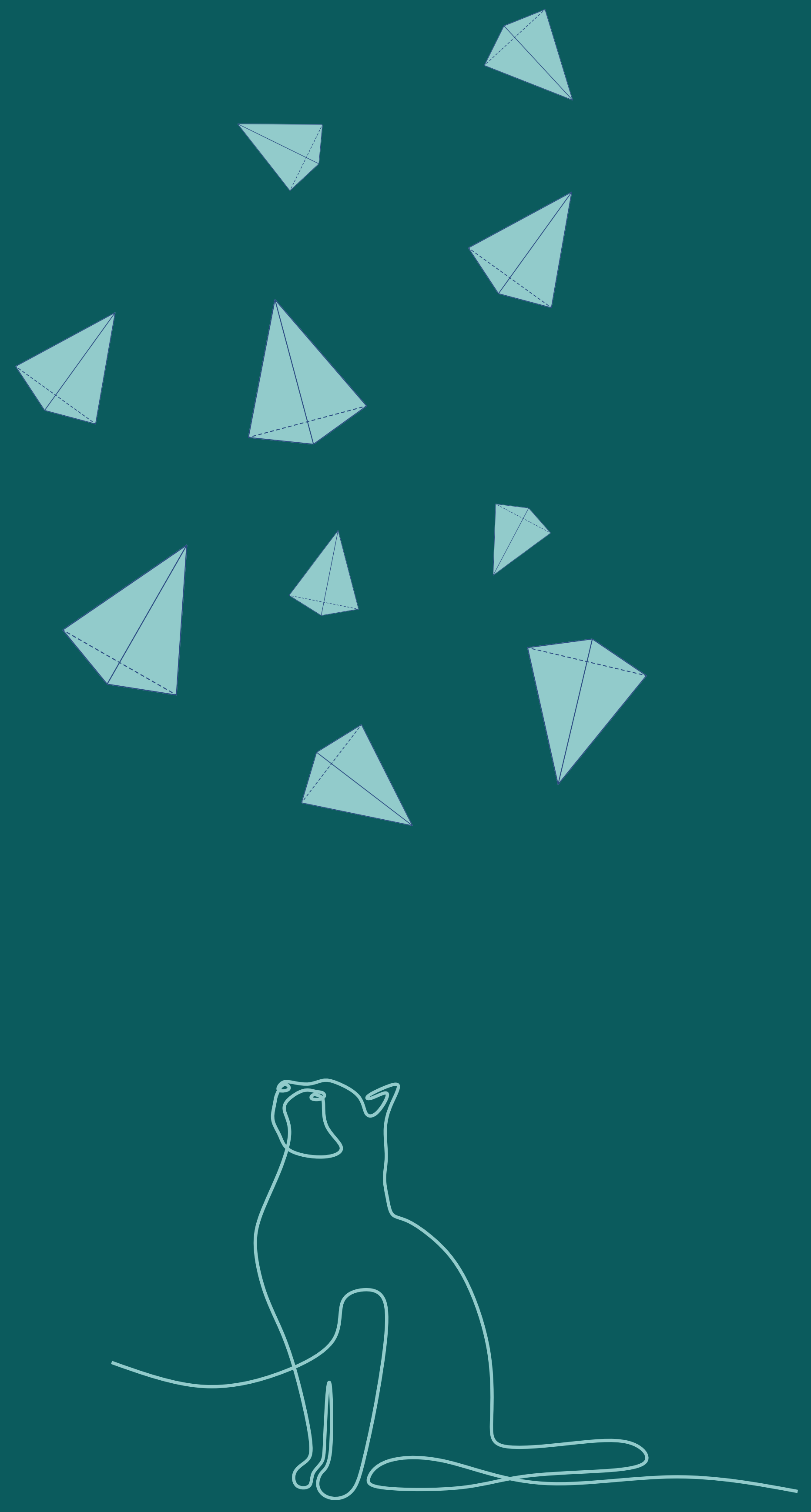
THANK YOU!

Questions...?

BACKUP



THE CHOICE OF STATE

- Use semiclassical state $|\psi\rangle = e^{-\mathrm{i}\phi(\tau)}|q(\tau), p(\tau)\rangle$
  - Satisfies the Schrödinger equation $\hat{\mathcal{H}}_{\mathrm{FLRW}}\psi - \mathrm{i}\partial_\tau\psi = 0$
  - Follows dynamics generated by the semiclassical Hamiltonian $\mathcal{H}_{\mathrm{sem}} = p^2 + \dfrac{K}{q^2}$

$$q(\tau) = q_B\sqrt{1+\omega^2(\tau-\tau_B)^2} \qquad p(\tau) = \frac{1}{2}\dot{q}(\tau) = \frac{q_B\,\omega^2(\tau-\tau_B)}{2\sqrt{1+\omega^2(\tau-\tau_B)^2}}$$

  - Specific choice of **fiducial state** $\langle x|q(\tau), p(\tau)\rangle = \dfrac{1}{\sqrt{q(\tau)}}\exp\left(\mathrm{i}\dfrac{p(\tau)}{2q(\tau)}x^2\right)\Phi_n\left(\dfrac{x}{q(\tau)}\right)$

[Bergeron, Gazeau, Małkiewicz, Peter ('23)]

Note: use different representation

$$V(q,p)\psi(x) = \frac{1}{\sqrt{q}}\exp\left(\mathrm{i}\frac{p}{2q}x^2\right)\psi\left(\frac{x}{q}\right)$$

- Wave function of the universe: $\psi_a(x) = \langle x|\psi_a\rangle$ with $x \propto a^{\frac{3}{2}(1-w)}$

$n \in \mathbb{N}$

$\nu^2 \geq 1$

$$\psi_a(x) = \sqrt{\frac{2\;n_a!}{\Gamma(\nu+n_a+1)}}\left(\frac{\xi_{\nu,n_a}-\mathrm{i}q_ap_a}{\xi_{\nu,n_a}+\mathrm{i}q_ap_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}\xi_{\nu,n_a}^{\frac{\nu+1}{2}}\frac{x^{\nu+1/2}}{q_a^{\nu+1}}L_{n_a}^{\nu}\left(\xi_{\nu,n_a}\frac{x^2}{q_a^2}\right)\exp\left(-\frac{1}{2}(\xi_{\nu,n_a}-\mathrm{i}\,q_ap_a)\frac{x^2}{q_a^2}\right)$$

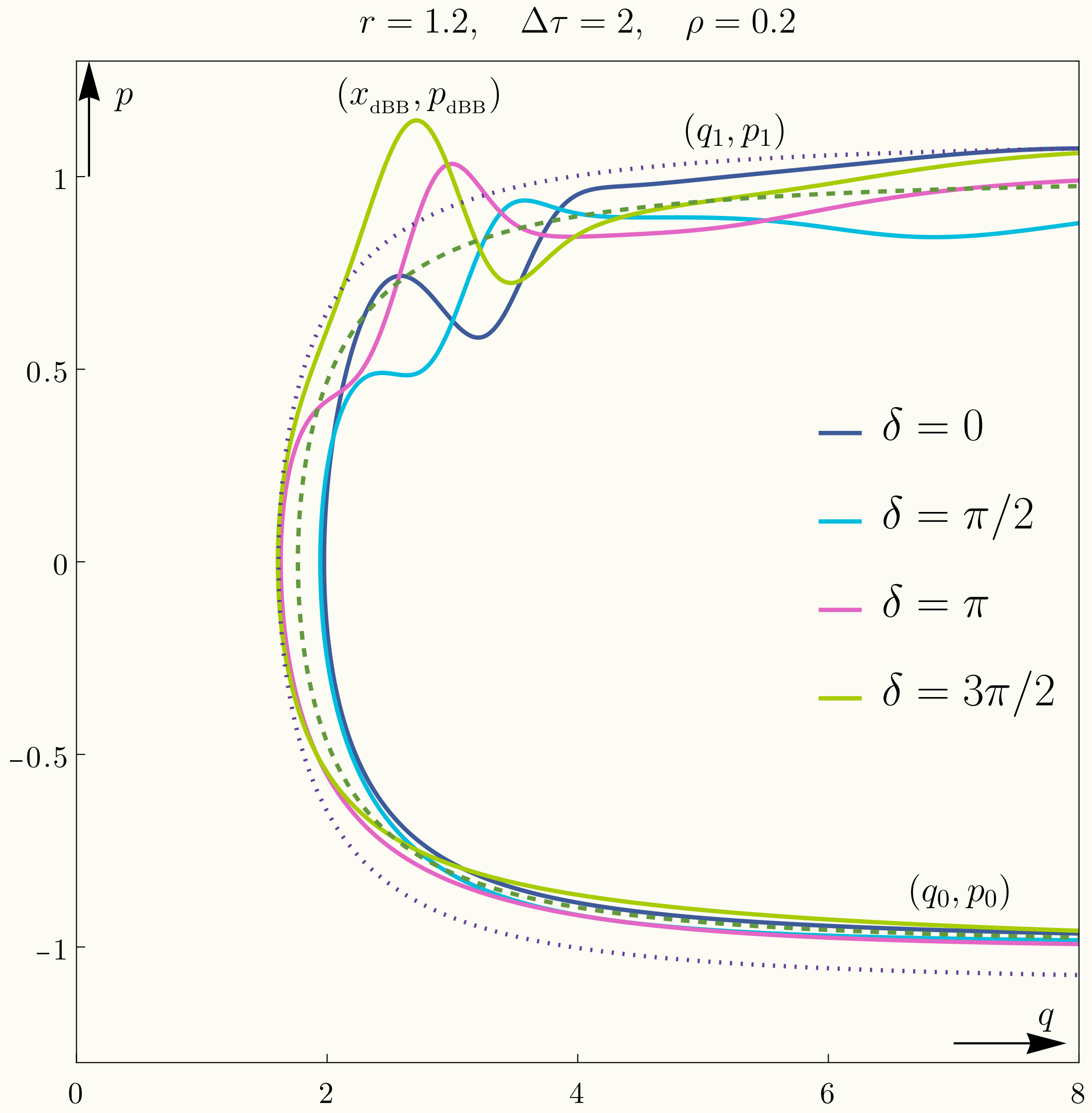
Phase: $e^{-\mathrm{i}\phi_a(\tau)} = \left(\dfrac{\xi_{\nu,n_a}-\mathrm{i}q_ap_a}{\xi_{\nu,n_a}+\mathrm{i}q_ap_a}\right)^{\frac{1}{2}(2n_a+\nu+1)}$

$$\xi_{\nu,n} = \left(\frac{n!}{\Gamma(n+\nu+1)}\int_0^\infty y^{\nu+\frac{1}{2}}\left(L_n^{(\nu)}(y)\right)^2 e^{-y}\mathrm{d}y\right)^2$$

UNIVERSE IN A SUPERPOSITION

- Phase space portraits and wave functions for the biverse $\Psi = \mathcal{N}(\psi_0 + \rho e^{i\delta}\psi_1)$

$p \propto a^{\frac{3}{2}(1-w)} H$ ↑



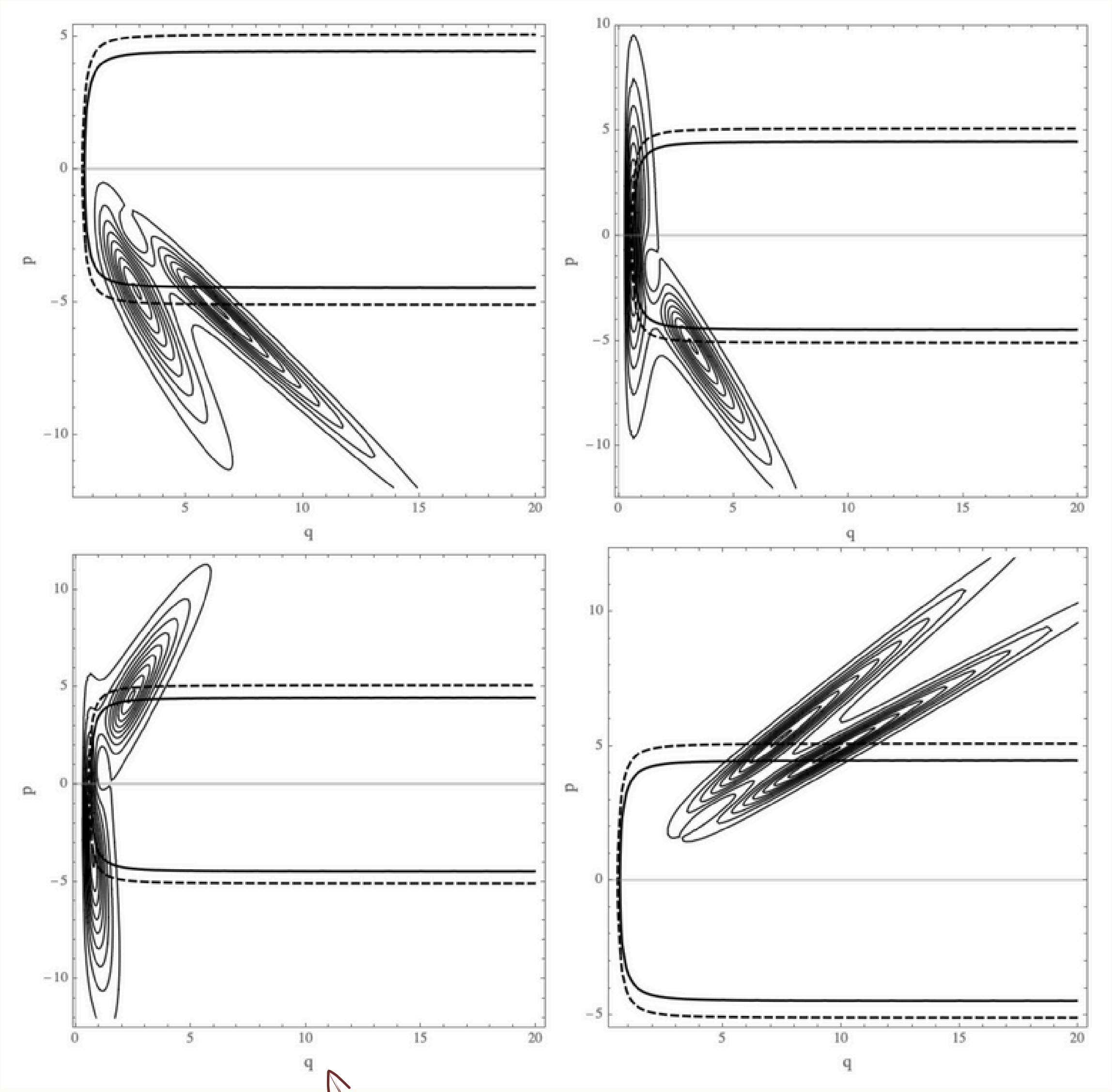
→ $q \propto a^{\frac{3}{2}(1-w)}$

- Trajectories highly dependent on initial conditions, but generally exhibit features that differ from single state trajectories

COMPARISON

[Bergeron, Małkiewicz, Peter, ('24), arXiv: 2405.09307]

- Biverse $\Psi = \mathcal{N}(\psi_0 + \alpha\,\psi_1)$
- Our universe: projection on a single wave function

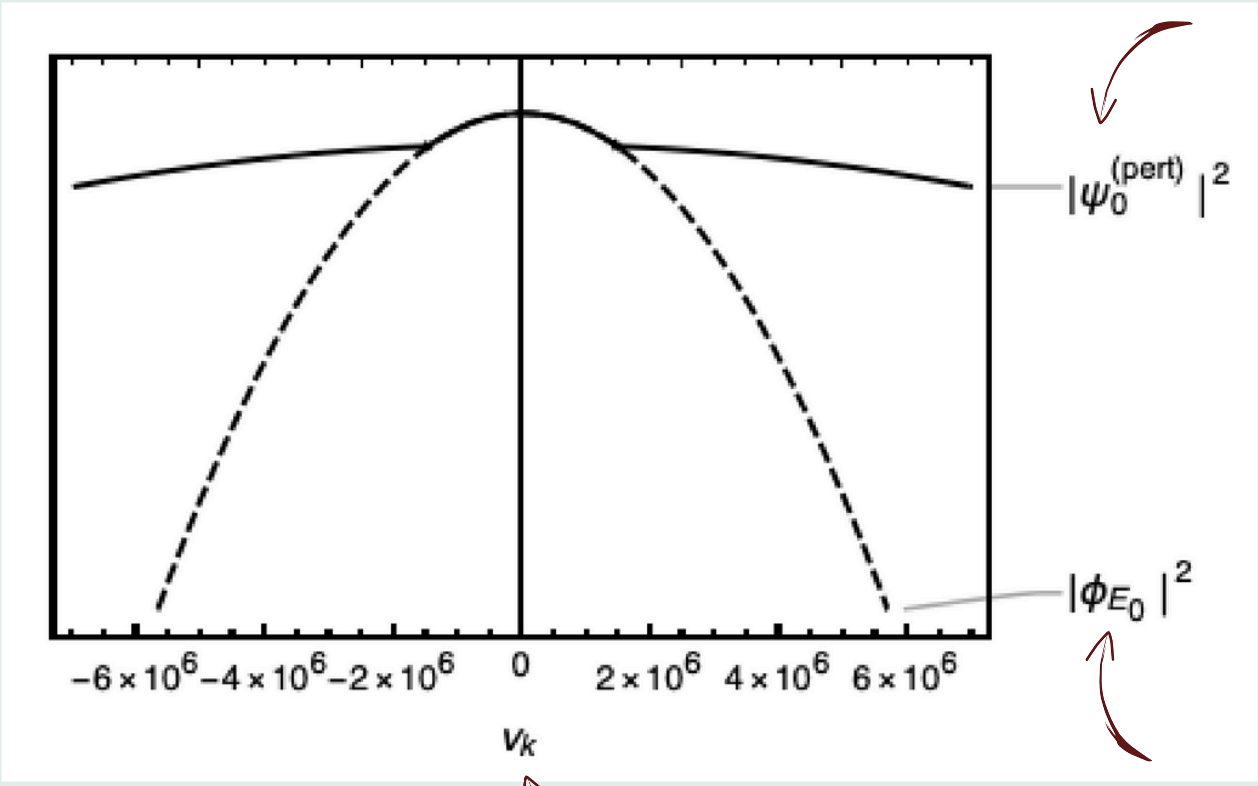


$\propto H$

$\propto a$

- Entanglement of perturbations between states

total perturbations



perturbation

single state $\Psi = \psi_0$

QUANTUM CORRECTED SCALE FACTOR

- Quantum corrected scale factor has the same dynamics as a semiclassical trajectory → recover GR dynamics at late times

$$x(\tau) \to x_0\, \omega\, \tau \Rightarrow a \propto t^{2/(3(1+w))} \quad \text{with} \quad x \propto a^{\frac{3}{2}(1-w)} \quad \text{and} \quad N \mathrm{d}\tau = \mathrm{d}t$$

  - Initial condition related to the expansion rate of the universe $\dot{x}(\tau) \to x_0\, \omega$

- Quantum potential $Q(x,\tau) = \frac{2\xi_\nu(\nu+1)}{q^2} - \frac{\xi_\nu^2 x^2}{q^4} - \frac{\nu^2 - \frac{1}{4}}{x^2}$ gives $\ddot{x} = -2\frac{\partial}{\partial x}\left(\frac{\nu^2 - \frac{1}{4}}{x^2} + Q(x,\tau)\right) = \frac{4\xi_\nu^2}{q(\tau)^4}x$

- Hubble rate from trajectories
  - FLRW with perfect fluid for $\partial_x S = \text{const.}$

$$H^2 = \left(\frac{\dot{a}}{Na}\right)^2 = \frac{4}{9(1-w)^2}\frac{\dot{x}^2}{N^2 x^2} \propto \frac{(\partial_x S)^2}{a^{3(1+w)}} \propto \frac{(\partial_a S)^2}{a^4}$$

- Acceleration
  - FLRW for $\ddot{x} \to 0$

$$\dot{H} = \frac{2}{3(1-w)}\frac{\ddot{x}}{Nx} - \frac{3}{2}(1+w)NH^2$$

QUANTISATION $\mathcal{H} \to \hat{\mathcal{H}}$

- Quantisation based on the Weyl-Heisenberg group $x \in \mathbb{R}$, $p \in \mathbb{R}$
  - $U(q,p)\psi(x) = e^{\mathrm{i}p(x-q/2)}\psi(x-q)$

- Here: $x \geq 0$, $p \in \mathbb{R}$ $\longrightarrow$ use affine group $U(q,p)\psi(x) = \frac{e^{ipx}}{\sqrt{q}}\psi\left(\frac{x}{q}\right)$

- Quantisation map $\hat{A}_f = \mathcal{N} \int_{\mathbb{R}\times\mathbb{R}^+} \mathrm{d}p\mathrm{d}q\, |q,p\rangle\, f(p,q)\, \langle q,p|$ where $|q,p\rangle = U(q,p)|\psi_0\rangle$ (fiducial state)

- Quantisation of FLRW Hamiltonian $\mathcal{H} \propto p^2 + p_T$ leads to a repulsive potential

$$\hat{A}_{p^2}\,\psi = -\partial_x^2\,\psi + \frac{K}{x^2}\psi \longrightarrow \hat{\mathcal{H}} \propto \hat{p}^2 + \frac{K}{\hat{q}^2} + \hat{p}_T$$

($\hat{p}_T$: time evolution; $\frac{K}{\hat{q}^2}$: introduces a bounce)

with

$$\hat{p}\,\psi = -\mathrm{i}\partial_x\psi$$
$$\hat{q}\,\psi = x\,\psi$$
$$\hat{p}_T\psi = -\mathrm{i}\partial_T\psi$$

[Klauder, ('99)] [Bergeron, Dapor, Gazeau, Małkiewicz, ('14)]

EXAMPLE: DOUBLE SLIT EXPERIMENT

- Weak measurements – reconstruct trajectories

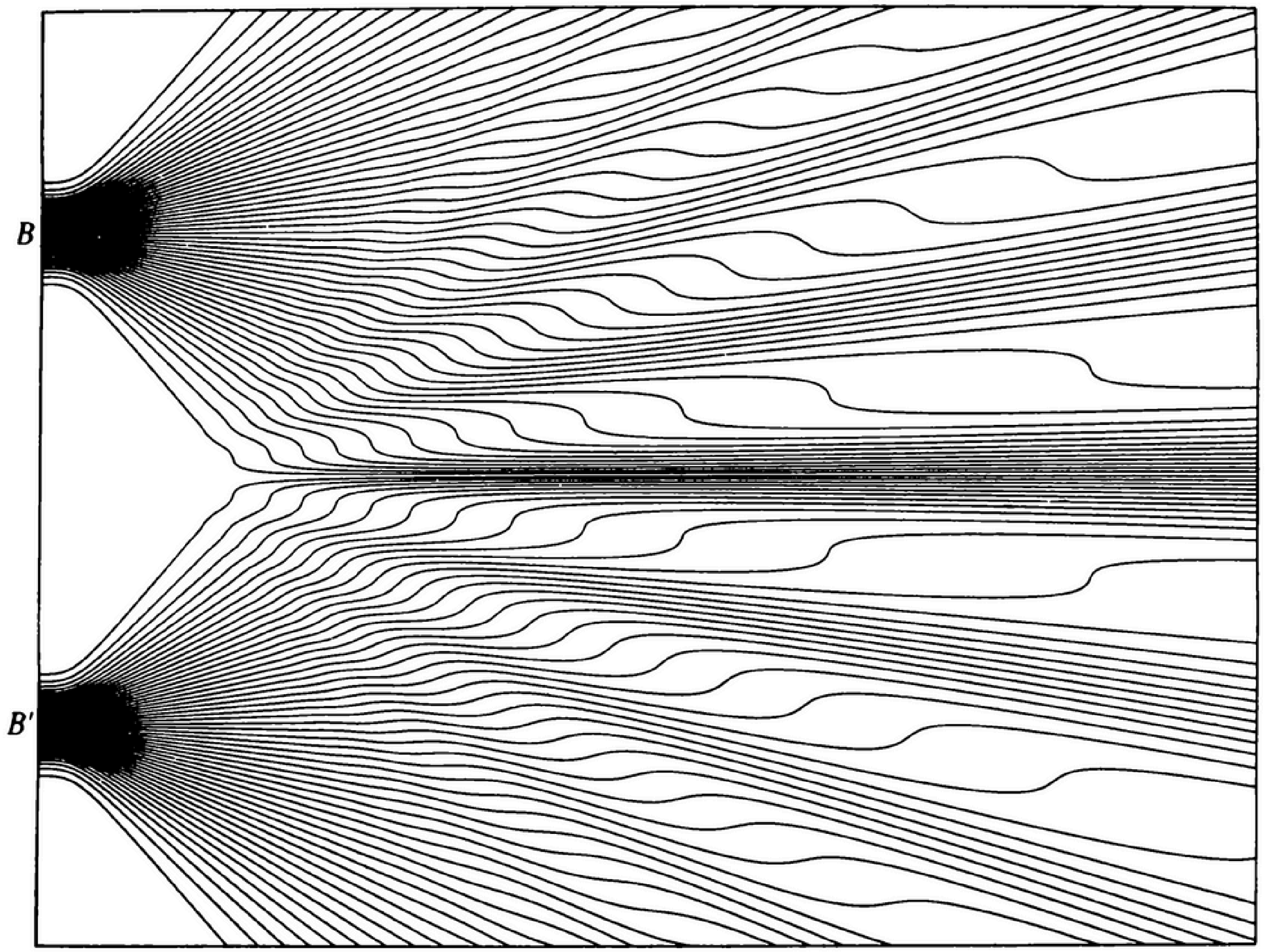


Fig. 5.7 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at each slit. The probability density is proportional to the number of lines per unit length in the *y*-direction (from Philippidis *et al.* (1982)).