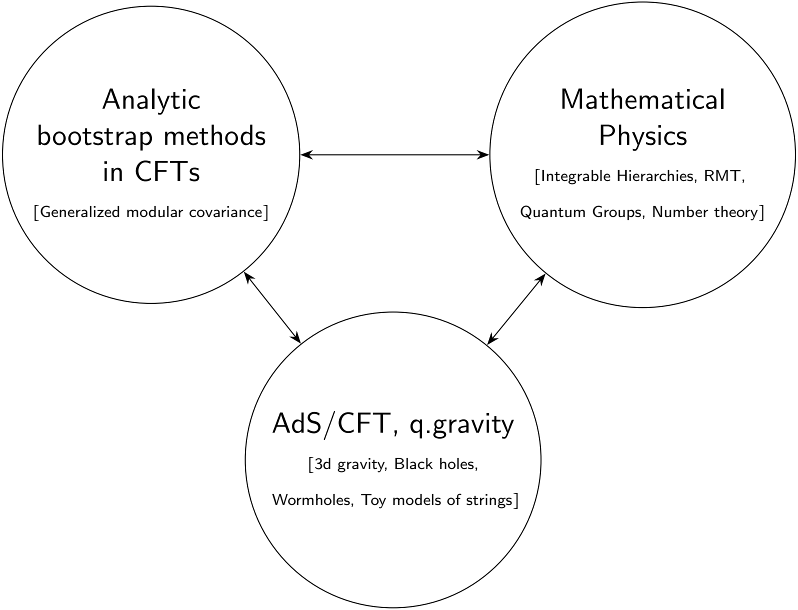
On Some Universal Results in 2D Conformal Field Theories

Ioannis Tsiares

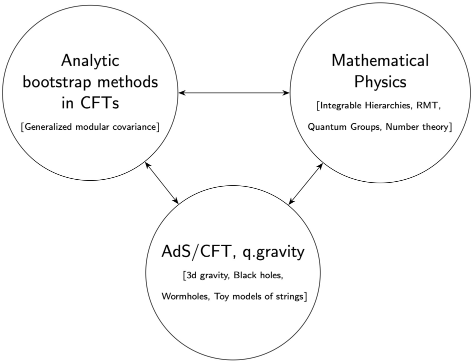
LPENS

FRIF Day, November 17th, 2025.

In a nutshell



Hopes and dreams

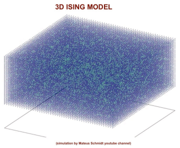


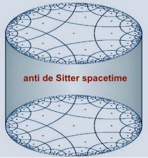
To understand the space of Conformal Field Theories using the **Conformal Bootstrap**, starting from two-dimensions and extending to higher dimensions.
In the process, learn something non-trivial about black holes and quantum gravity + learn some fun math!

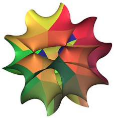
In more detail

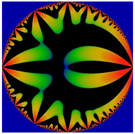
Conformal Field Theories

- QFTs with conformal symmetry. Fixed points of RG flow. Universality for different systems at criticality. Quantum gravity in AdS, ...









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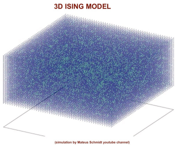

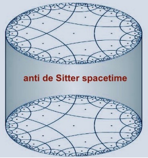

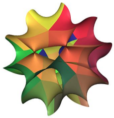

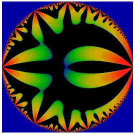

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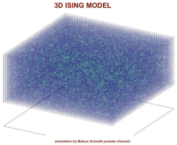

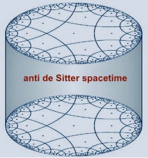

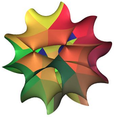
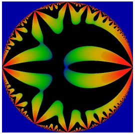

- Two dimensions are special. Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}\mathbb{1} \ , \qquad m, n \in \mathbb{Z}$$

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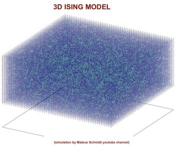

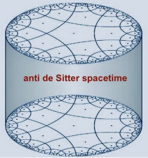

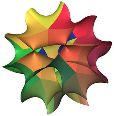
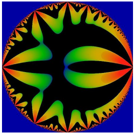

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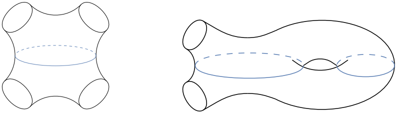
  Ubiquitous: 2d condensed matter systems, worldsheet string theory; number theory, random matrix theory, quantum groups, $\cdots$

Conformal Bootstrap Philosophy

We are usually interested in Euclidean correlation functions of local (primary) operators on (super) Riemann surfaces:



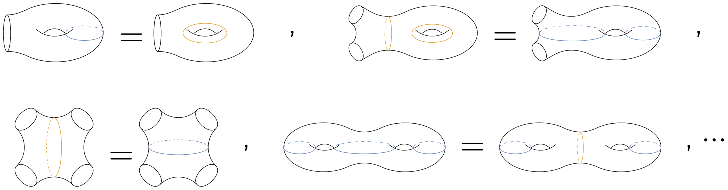
Using the power of the *Operator Product Expansion* (OPE), the basic **2d CFT data** at central charge c consist of:

- Dynamic: List of primary operators $\mathcal{O}_i$, along with scaling dimensions $\Delta_i = h_i + \overline{h}_i$ and spins $l_i = |h_i - \overline{h}_i|$, and their OPE coefficients C_{ijk}.
- Kinematic: *Virasoro conformal blocks*.

Question: How are these CFT data constrained from consistency conditions (e.g. associativity of OPE)? Are there any universal features that we can derive *analytically*?

Conformal Bootstrap Philosophy

- Generalized Modular Covariance on (super) *Riemann surfaces*:



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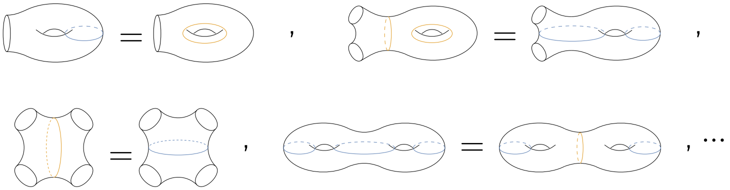


- Sphere 4-pt:

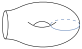
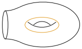
$$\sum_{h_s,\overline{h_s}} C_{12s}C_{34s}\,|\mathcal{V}_s^{(4-pt)}(z)|^2 = \sum_{h_t,\overline{h_t}} C_{14t}C_{23t}\,|\mathcal{V}_t^{(4-pt)}(1-z)|^2 \;.$$

- Torus 1-pt:

$$\sum_{h,\overline{h}} C_{h_0hh}\,|\mathcal{V}_s^{(torus\ 1-pt)}(-1/\tau)|^2 = \left(\tau^{h_0}\bar{\tau}^{\bar{h}_0}\right)\sum_{h,\overline{h}} C_{h_0hh}\,|\mathcal{V}_t^{(torus\ 1-pt)}(\tau)|^2 \;.$$

Generalized Modular Covariance in practice

- Famously, [Cardy,'86] used this philosophy to derive a universal spectral density at high energies (i.e. Bekenstein-Hawking entropy of BTZ black holes)

$\mathbb{1}$  $= \mathbb{1}$  $\Rightarrow \rho_{\text{spec.}}(h, \bar{h}) \approx \left| e^{4\pi\sqrt{\frac{(c-1)h}{24}}} \right|^2, \; h, \bar{h} \gg c.$

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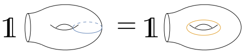

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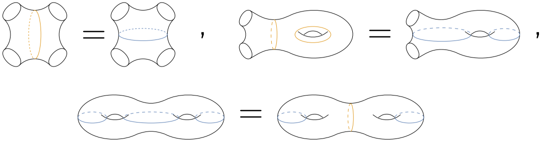

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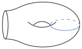
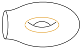
$\mathbb{1}$  $= \mathbb{1}$  $\Rightarrow \rho_{\text{spec.}}(h,\bar{h}) \approx \left| e^{4\pi\sqrt{\frac{(c-1)h}{24}}} \right|^2 , \;\; h,\bar{h} \gg c.$

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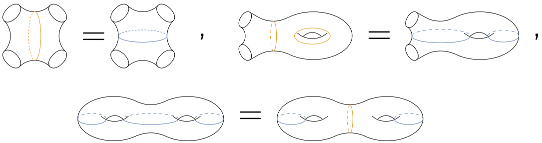

we showed analogous **universal** result for heavy OPE coeff's at finite c (i.e. universal dynamics in BTZ backgrounds)

$$\Rightarrow \;\; \rho_{\text{OPE}^2}(h_i,\overline{h_i},h_j,\overline{h_j},h_k,\overline{h_k}) \approx C_0(P_i,P_j,P_k)C_0(\bar{P}_i,\bar{P}_j,\bar{P}_k)$$

where $C_0(P_i,P_j,P_k) = \frac{\Gamma_b(2Q)\prod_{\pm\pm\pm}\Gamma_b(\frac{Q}{2}\pm iP_i \pm iP_j \pm iP_k)}{\sqrt{2}\Gamma_b(Q)^3\prod_{\alpha\in\{i,j,k\}}\Gamma_b(Q+2iP_\alpha)\Gamma_b(Q-2iP_\alpha)}$ (c.f. Liouville CFT!)

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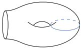
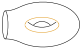
$\mathbb{1}$  $= \mathbb{1}$  $\Rightarrow \rho_{\text{spec.}}(h,\bar{h}) \approx \left| e^{4\pi\sqrt{\frac{(c-1)h}{24}}} \right|^2 , \ \ h,\bar{h} \gg c.$

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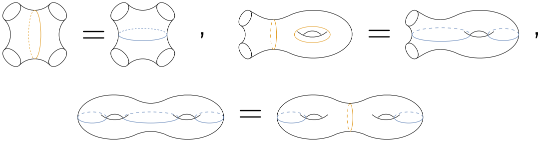

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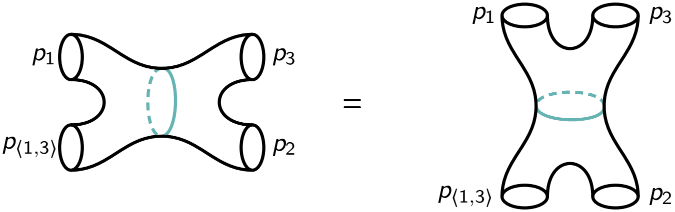
- Also analogous results in 2d BCFTs in [T.Numasawa, **I.T.**'22].

Generalized Modular Covariance in practice v2

- Sometimes one is able to find a 2D CFT explicitly!

Analyticity + **Degenerate representations** + Crossing Symmetry.



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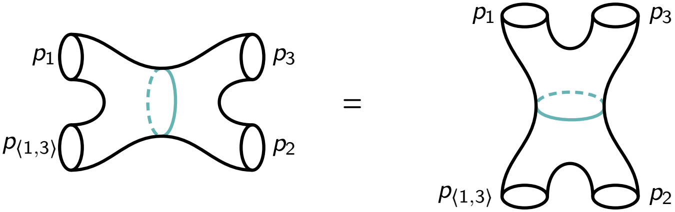

In [B.Mühlmann, V.Narovlansky, **I.T.**'25] we found a new $\mathcal{N}=1$ 2d CFT at $c_s \leq 1$ called:

$\mathcal{N}=1$ **timelike Liouville theory**.

We provided explicit expressions for the structure constants in the Neveu–Schwarz and Ramond sectors that solve the bootstrap equations.

Current work

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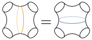
Let's return to the case of the four-point functions: 

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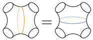
Let's return to the case of the four-point functions: 

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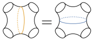
Let's return to the case of the four-point functions: 

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One then gets

$$\boxed{\rho_{\text{t-channel}}(h,\overline{h}) = \sum\!\!\!\!\!\!\!\int_{h',\overline{h'}} \mathbf{F}_{h,h'}\mathbf{F}_{\overline{h},\overline{h'}}\ \rho_{\text{s-channel}}(h',\overline{h'})}$$

where ρ is an appropriate *distribution*.

Current work

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- Surprisingly, even though the Virasoro blocks are not known in closed form, **F** is known exactly! Related to rep.theory of a particular quantum group.

- Intricate connections with 3D TQFT called Virasoro TQFT, wormholes and ensemble average of CFTs, towards understanding the fine-grained BTZ black hole spectrum.

Related questions that I'd like answers to

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- How is chaos and ETH (as inspired by black hole physics) consistent with Generalized Modular Covariance in 2d CFTs?
- Is there a consistent quantum theory of *pure* 3d gravity?
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