CMB + BAO + SN
Bayesian Model Comparison and Dataset Consistency

Lukas T. Hergt

Laboratoire de Physique des 2 infinis — Irène Joliot-Curie (IJCLab)
IN2P3, CNRS
<*lukas.hergt@ijclab.in2p3.fr*>

Synergistic Power of Combined Cosmological Observables

Objective: Put various tensions into perspective

Models

- Baseline: ΛCDM
  - Cold Dark Matter with cosmological constant Λ
  - flat ($\Omega_K = 0$)
  - 1 massive neutrino $m_\nu = 0.06\,\mathrm{eV}$
- Neutrinos $\Lambda\mathrm{CDM} + \sum m_\nu$
- Curvature $\Lambda\mathrm{CDM} + \Omega_K$
- Constant Dark Energy wCDM
- Evolving Dark Energy $w_0 w_a$CDM

Data/Likelihoods

- CMB and CMB lensing
  - **Planck** PR3 & PR4:

low-ℓ *TT*	low-ℓ *EE*	high-ℓ *TTTEEE*
Commander	SimAll (PR3)	**Plik** (PR3)
Commander	SimAll (PR3)	**CamSpec** (PR4)
Commander	Lollipop (PR4)	**Hillipop** (PR4)

- Baryon Acoustic Oscillations (BAO)
  - **SDSS DR12**
  - **SDSS DR16**
  - **DESI DR2**
- Supernovae (SN)
  - **Pantheon$^+$**
  - **DES y5**

The 3 Pillars of Bayesian Inference

Parameter Estimation

What do the data tell us about the parameters of a model?
(e.g. the size or age of a ΛCDM universe)

Bayes' Theorem:

$$P(\theta|D,M) = \frac{P(D|\theta,M)\times P(\theta|M)}{P(D|M)}$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

$$\mathcal{P}(\theta) = \frac{\mathcal{L}(\theta) \times \Pi(\theta)}{\mathcal{Z}} \propto \mathcal{L}(\theta) \times \Pi(\theta)$$

Model Comparison

How much does the data support one model compared to another?
(e.g. cosmology with a cosmological constant vs with a dynamic dark energy)

$$\frac{P(M_1|D)}{P(M_2|D)} = \frac{P(D|M_1) \times P(M_1)}{P(D|M_2) \times P(M_2)} = \frac{\mathcal{Z}_{M_1}}{\mathcal{Z}_{M_2}} \times \frac{P(M_1)}{P(M_2)}$$

$$\text{posterior odds} = \text{evidence ratio} \times \text{prior odds}$$

Occam's razor:

$$\log\mathcal{Z} = \langle \log\mathcal{L}\rangle_{\mathcal{P}} - \mathcal{D}_{\mathrm{KL}}$$

$$\text{log-evidence} = \text{fit} - \text{Occam penalty}$$

Dataset Consistency

Do different datasets A and B make consistent predictions for the same model?
(e.g. CMB vs BAO data)

H_0: datasets described by the same parameters

H_1: datasets described by distinct parameters

$$\mathcal{R} = \frac{\mathcal{Z}_{AB}}{\mathcal{Z}_A\mathcal{Z}_B},$$

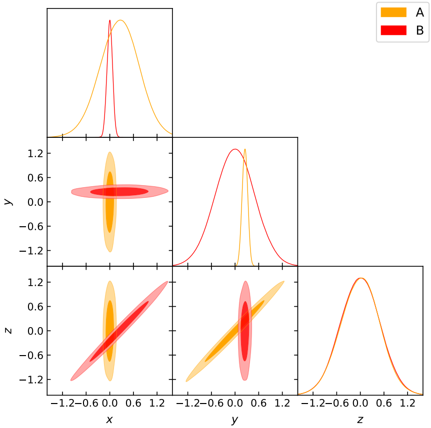
$$\mathcal{R} = \log\mathcal{S} - \mathcal{I},$$

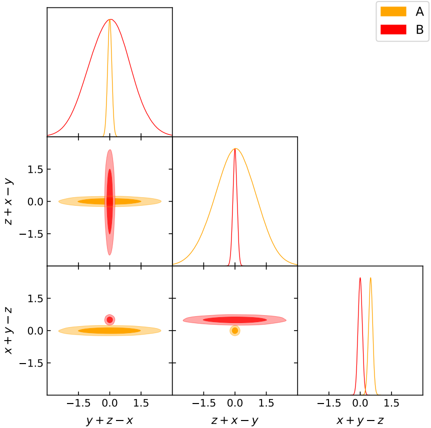
$$\log\mathcal{S} = \langle\log\mathcal{L}_{AB}\rangle_{\mathcal{P}_{AB}} - \langle\log\mathcal{L}_{A}\rangle_{\mathcal{P}_{A}} - \langle\log\mathcal{L}_{B}\rangle_{\mathcal{P}_{B}}$$

$$d - 2\log\mathcal{S} \sim \chi^2_d$$

Hidden Tensions

Handley & Lemos (2019)

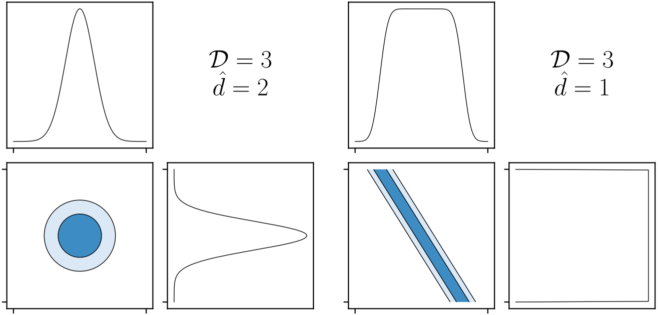




- Projections can be misleading. → Can miss tensions on marginalised parameters.
- Evidence ratio $\mathcal{R}$ and likelihood ratio $\mathcal{S}$ take **full dimensionality** into account, (estimated by the Gaussian Model Dimensionality $\hat{d}$).

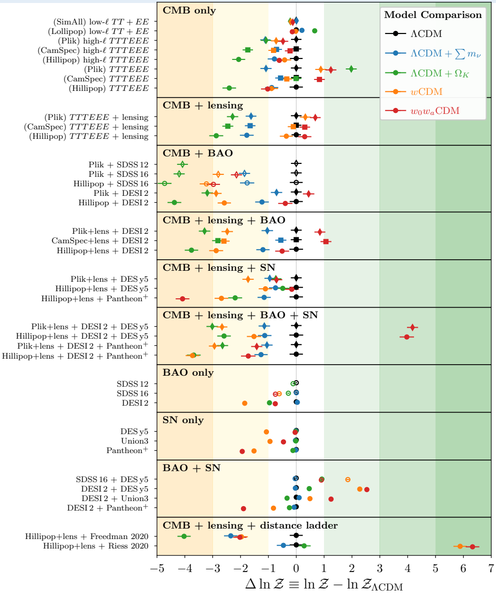
Gaussian Model Dimensionality $\hat{d}$

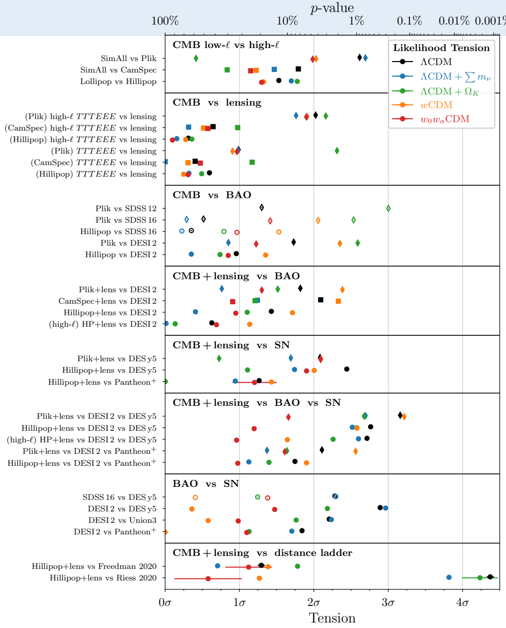
Handley & Lemos (2019)

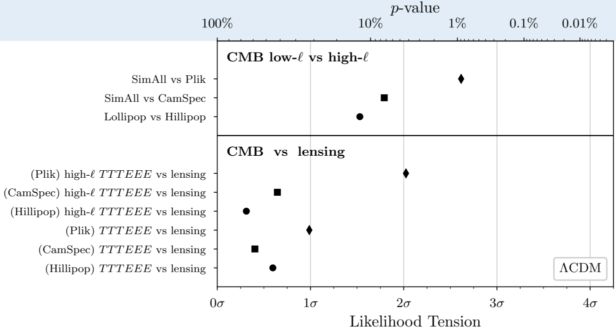


- **Same** number of sampling parameters.
- **Same** Kullback–Leibler divergence $\mathcal{D} = \left\langle \ln \frac{\mathcal{P}}{\Pi} \right\rangle_{\mathcal{P}}$ (i.e. same prior to posterior compression).
- **Different** dimensionality $\frac{\hat{d}}{2} = \left\langle (\ln \mathcal{L})^2 \right\rangle - \left\langle \ln \mathcal{L} \right\rangle^2$ (i.e. different number of *constrained* parameters).

Slightly Overwhelming Overview



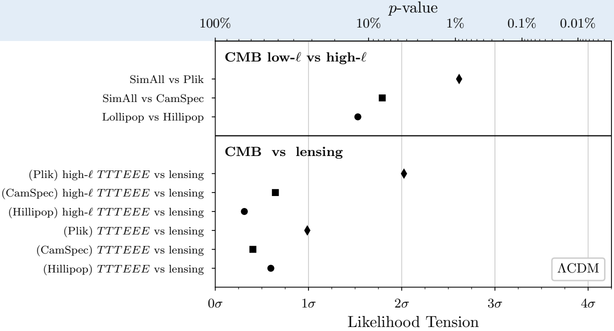



p-value
100%
10%
1%
0.1%
0.01%
CMB low-ℓ vs high-ℓ
SimAll vs Plik
SimAll vs CamSpec
Lollipop vs Hillipop
CMB vs lensing
(Plik) high-ℓ TTTEEE vs lensing
(CamSpec) high-ℓ TTTEEE vs lensing
(Hillipop) high-ℓ TTTEEE vs lensing
(Plik) TTTEEE vs lensing
(CamSpec) TTTEEE vs lensing
(Hillipop) TTTEEE vs lensing
ΛCDM
0σ
1σ
2σ
3σ
4σ
Likelihood Tension

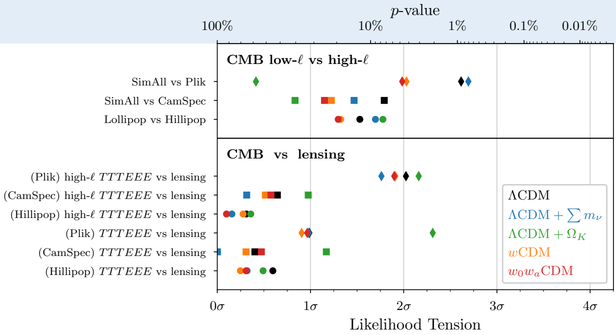
CMB + lensing

CMB: internal consistency

- Tensions reduce from Plik → CamSpec → Hillipop.
  - low-ℓ vs high-ℓ
  - CMB vs lensing



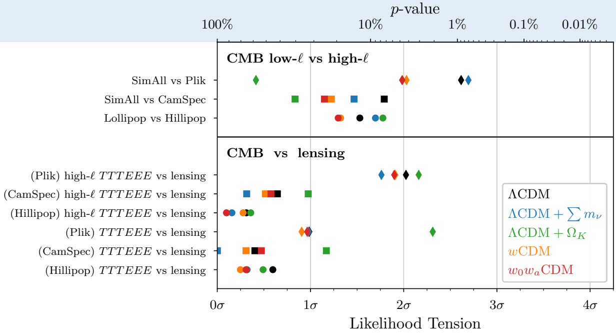
- Tensions reduce from Plik → CamSpec → Hillipop.
  - low-ℓ vs high-ℓ
  - CMB vs lensing

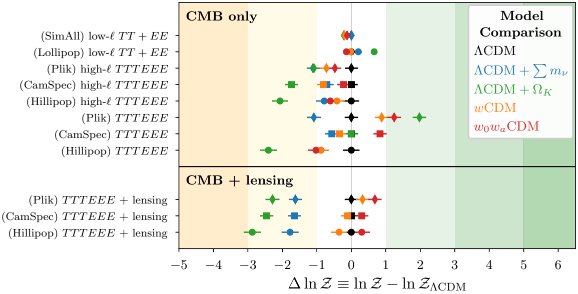


CMB: internal consistency

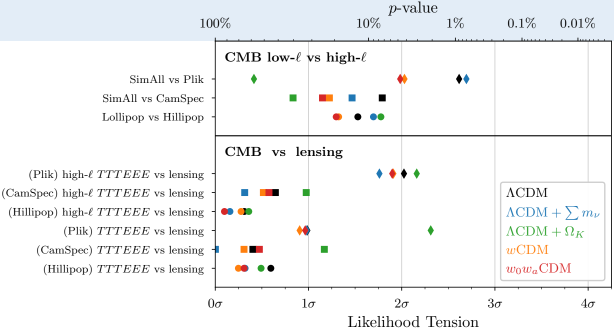
- Tensions reduce from Plik → CamSpec → Hillipop.
  - low-ℓ vs high-ℓ
  - CMB vs lensing

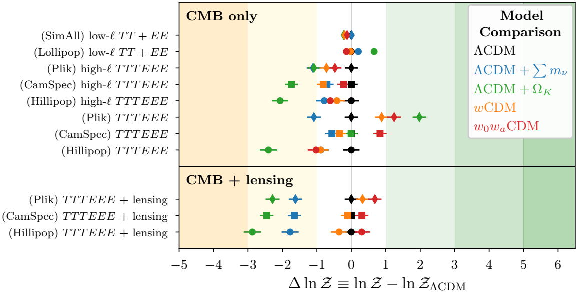
- No strong preference for any ΛCDM extension.

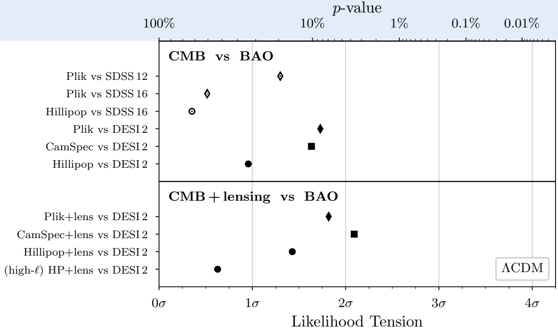




- Tensions reduce from Plik → CamSpec → Hillipop.
  - low-ℓ vs high-ℓ
  - CMB vs lensing
- No strong preference for any ΛCDM extension.
- **Curvature Tension** with Plik:
  - Slight tension between CMB and lensing for $\Lambda\mathrm{CDM} + \Omega_K$.
  - Slight preference for $\Lambda\mathrm{CDM} + \Omega_K$ (curvature).
  - Related to A_lens anomaly.

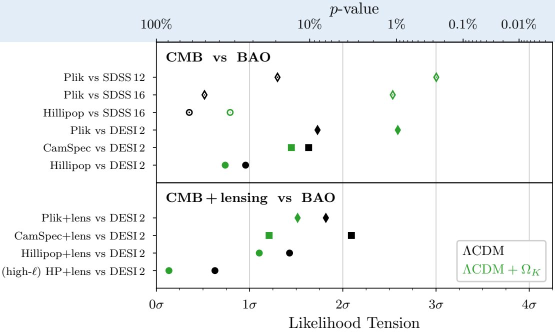




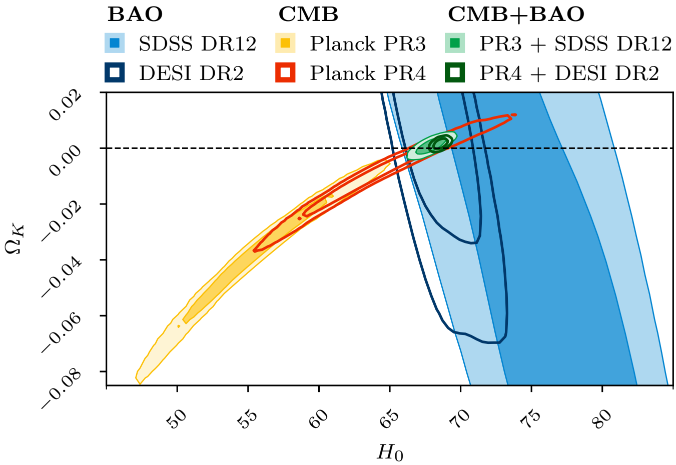

p-value
100%
10%
1%
0.1%
0.01%
CMB vs BAO
Plik vs SDSS 12
Plik vs SDSS 16
Hillipop vs SDSS 16
Plik vs DESI 2
CamSpec vs DESI 2
Hillipop vs DESI 2
CMB + lensing vs BAO
Plik+lens vs DESI 2
CamSpec+lens vs DESI 2
Hillipop+lens vs DESI 2
(high-ℓ) HP+lens vs DESI 2
ΛCDM
0σ
1σ
2σ
3σ
4σ
Likelihood Tension

CMB + BAO

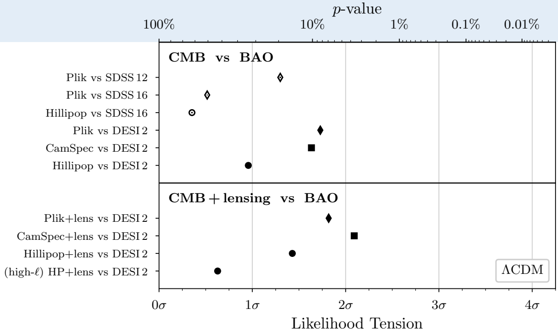
- **Curvature Tension** with Plik (ΛCDM + Ω_K):
  - 2.3σ tension between Plik and lensing.
  - 3σ tension between Plik and SDSS 12.
  - Reduces to 2.5σ with SDSS 16 or DESI 2.
  - Goes away when switching to CamSpec or Hillipop.



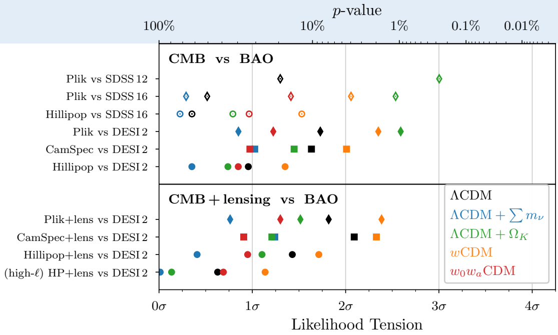
CMB + BAO


BAO
SDSS DR12
DESI DR2
CMB
Planck PR3
Planck PR4
CMB+BAO
PR3 + SDSS DR12
PR4 + DESI DR2
0.02
0.00
−0.02
−0.04
−0.06
−0.08
Ω_K
50
55
60
65
70
75
80
H_0

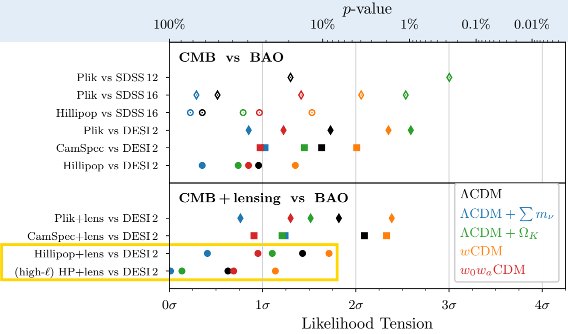
- **Curvature Tension** with Plik (ΛCDM + Ω_K):
  - 2.3σ tension between Plik and lensing.
  - 3σ tension between Plik and SDSS 12.
  - Reduces to 2.5σ with SDSS 16 or DESI 2.
  - Goes away when switching to CamSpec or Hillipop.
- Tensions reduce from Plik → Hillipop.
- BAO update from DESI DR2:
  - Hints of tension with CamSpec + lensing.
  - Goes away with Hillipop + lensing.



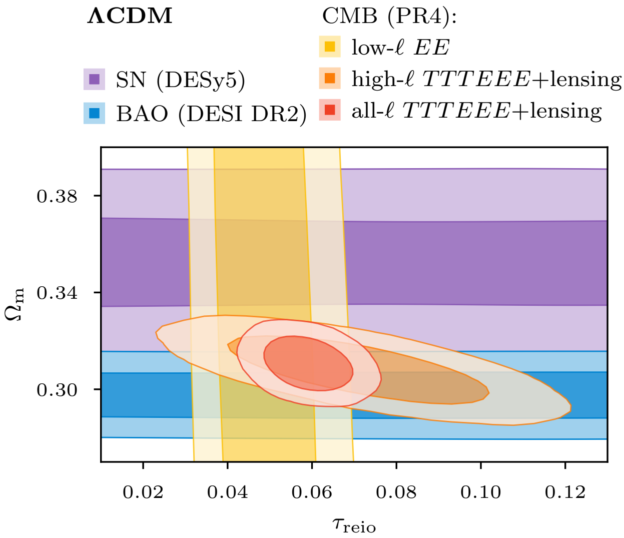
- **Curvature Tension** with Plik (ΛCDM + Ω_K):
    - 2.3σ tension between Plik and lensing.
    - 3σ tension between Plik and SDSS 12.
    - Reduces to 2.5σ with SDSS 16 or DESI 2.
    - Goes away when switching to CamSpec or Hillipop.
- Tensions reduce from Plik → Hillipop.
- BAO update from DESI DR2:
    - Hints of tension with CamSpec + lensing.
    - Goes away with Hillipop + lensing.



- **Curvature Tension** with Plik (ΛCDM + Ω_K):
  - 2.3σ tension between Plik and lensing.
  - 3σ tension between Plik and SDSS 12.
  - Reduces to 2.5σ with SDSS 16 or DESI 2.
  - Goes away when switching to CamSpec or Hillipop.
- Tensions reduce from Plik $\rightarrow$ Hillipop.
- BAO update from DESI DR2:
  - Hints of tension with CamSpec + lensing.
  - Goes away with Hillipop + lensing.
  - Removing low-ℓ further reduces tension.
    $\Rightarrow$ **"τ-Tension"**

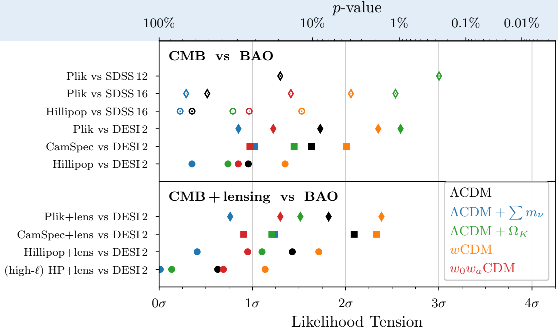


CMB + BAO



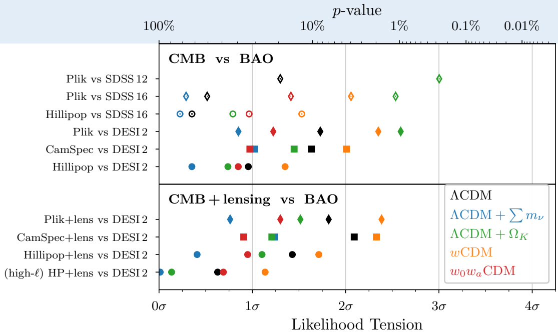
ΛCDM
SN (DESy5)
BAO (DESI DR2)
CMB (PR4):
low-ℓ EE
high-ℓ TTTEEE+lensing
all-ℓ TTTEEE+lensing
0.38
0.34
0.30
Ω_m
0.02
0.04
0.06
0.08
0.10
0.12
τ_reio

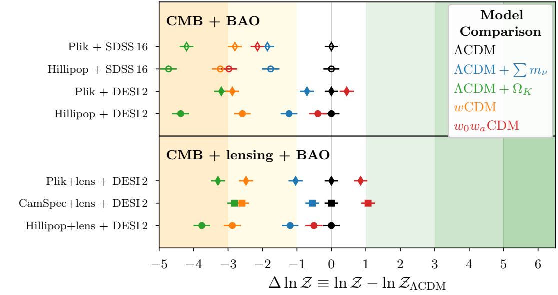
- **Curvature Tension** with Plik (ΛCDM + Ω_K):
  - 2.3σ tension between Plik and lensing.
  - 3σ tension between Plik and SDSS 12.
  - Reduces to 2.5σ with SDSS 16 or DESI 2.
  - Goes away when switching to CamSpec or Hillipop.
- Tensions reduce from Plik → Hillipop.
- BAO update from DESI DR2:
  - Hints of tension with CamSpec + lensing.
  - Goes away with Hillipop + lensing.
  - Removing low-ℓ further reduces tension.
    $\Rightarrow$ **"τ-Tension"**

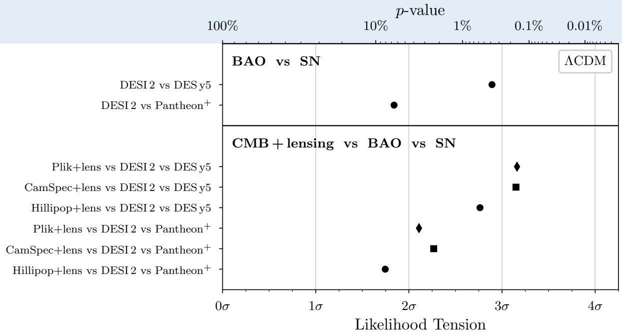


CMB + BAO

- **Curvature Tension** with Plik (ΛCDM + Ω_K):
  - 2.3σ tension between Plik and lensing.
  - 3σ tension between Plik and SDSS 12.
  - Reduces to 2.5σ with SDSS 16 or DESI 2.
  - Goes away when switching to CamSpec or Hillipop.
- Tensions reduce from Plik $\rightarrow$ Hillipop.
- BAO update from DESI DR2:
  - Hints of tension with CamSpec + lensing.
  - Goes away with Hillipop + lensing.
  - Removing low-ℓ further reduces tension. $\Rightarrow$ **“τ-Tension”**
- No preference for extensions.





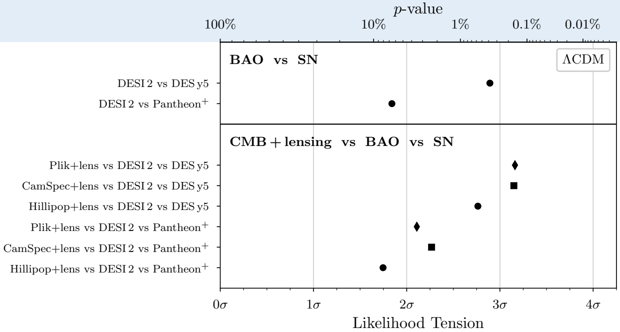


p-value
100%
10%
1%
0.1%
0.01%
BAO vs SN
ΛCDM
DESI 2 vs DES y5
DESI 2 vs Pantheon+
CMB + lensing vs BAO vs SN
Plik+lens vs DESI 2 vs DES y5
CamSpec+lens vs DESI 2 vs DES y5
Hillipop+lens vs DESI 2 vs DES y5
Plik+lens vs DESI 2 vs Pantheon+
CamSpec+lens vs DESI 2 vs Pantheon+
Hillipop+lens vs DESI 2 vs Pantheon+
0σ
1σ
2σ
3σ
4σ
Likelihood Tension

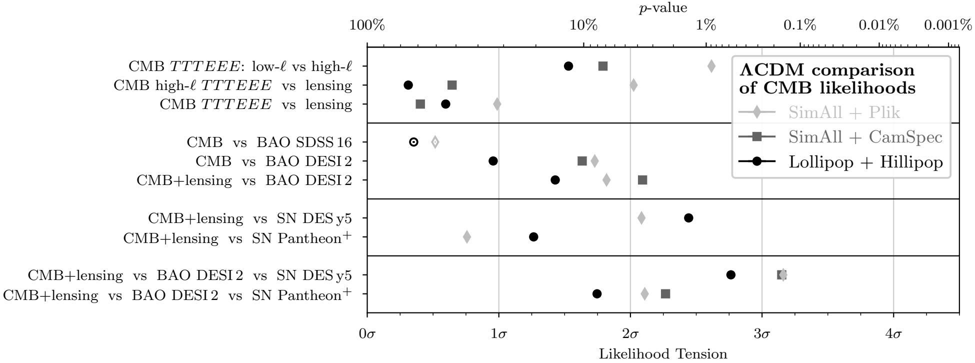
CMB + BAO + SN

CMB + BAO + SN

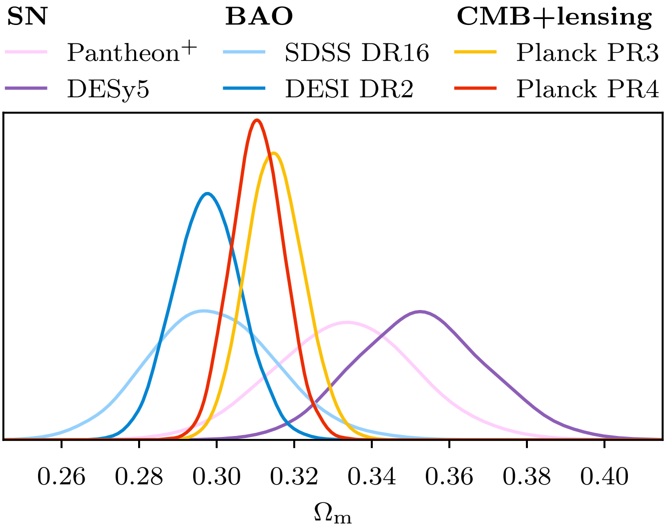
- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik $\to$ Hillipop ($\sim 0.5\sigma$).
  - from DES y5 $\to$ Pantheon$^+$ ($\sim 1\sigma$).

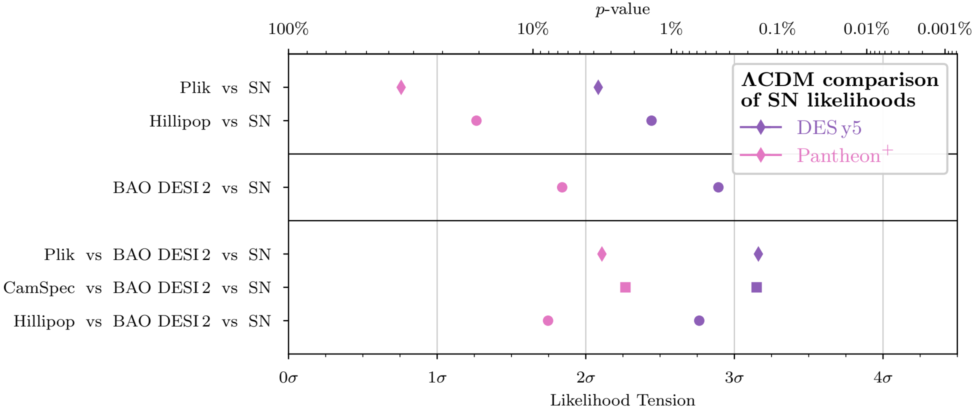


CMB + BAO + SN



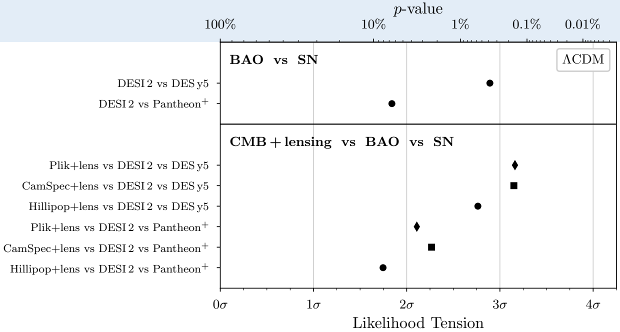
p-value
100%
10%
1%
0.1%
0.01%
0.001%
CMB $TTTEEE$: low-ℓ vs high-ℓ
CMB high-ℓ $TTTEEE$ vs lensing
CMB $TTTEEE$ vs lensing
CMB vs BAO SDSS 16
CMB vs BAO DESI 2
CMB+lensing vs BAO DESI 2
CMB+lensing vs SN DES y5
CMB+lensing vs SN Pantheon$^+$
CMB+lensing vs BAO DESI 2 vs SN DES y5
CMB+lensing vs BAO DESI 2 vs SN Pantheon$^+$
ΛCDM comparison of CMB likelihoods
SimAll + Plik
SimAll + CamSpec
Lollipop + Hillipop
0σ
1σ
2σ
3σ
4σ
Likelihood Tension


SN
Pantheon+
DESy5
BAO
SDSS DR16
DESI DR2
CMB+lensing
Planck PR3
Planck PR4
0.26
0.28
0.30
0.32
0.34
0.36
0.38
0.40
Ω_{m}


p-value
100%
10%
1%
0.1%
0.01%
0.001%
Plik vs SN
Hillipop vs SN
BAO DESI 2 vs SN
Plik vs BAO DESI 2 vs SN
CamSpec vs BAO DESI 2 vs SN
Hillipop vs BAO DESI 2 vs SN
ΛCDM comparison of SN likelihoods
DES y5
Pantheon+
0σ
1σ
2σ
3σ
4σ
Likelihood Tension

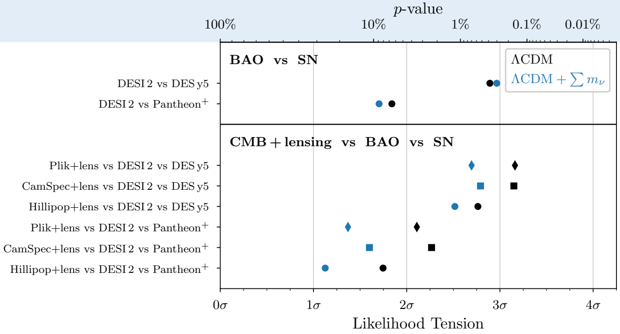
CMB + BAO + SN

- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik $\rightarrow$ Hillipop ($\sim 0.5\sigma$).
  - from DES y5 $\rightarrow$ Pantheon$^+$ ($\sim 1\sigma$).

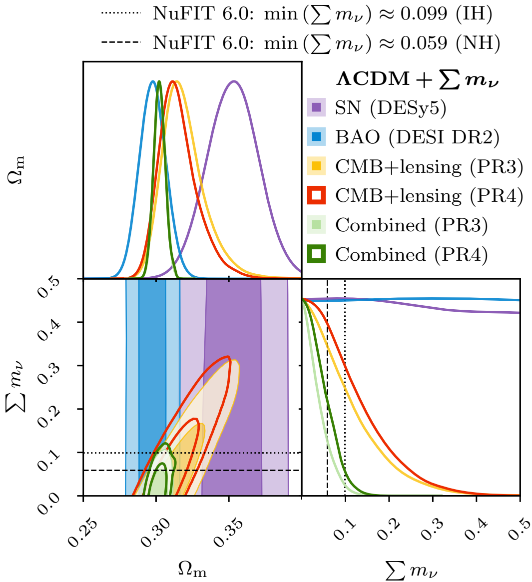


CMB + BAO + SN

- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik $\rightarrow$ Hillipop ($\sim 0.5\sigma$).
  - from DES y5 $\rightarrow$ Pantheon$^+$ ($\sim 1\sigma$).
- Tensions reduce from ΛCDM $\rightarrow$ ΛCDM$+\sum m_\nu$ (~ 0.25 to 0.5σ).

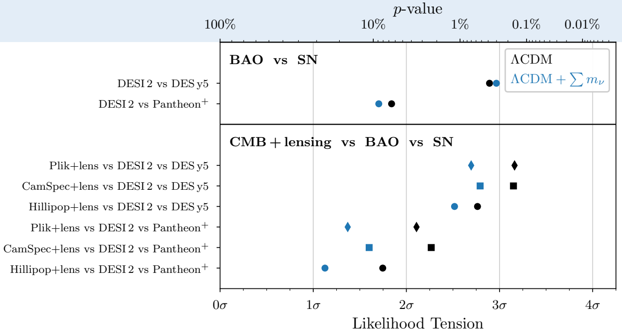


Neutrinos

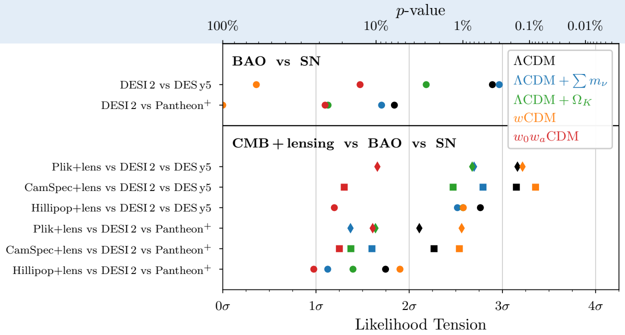


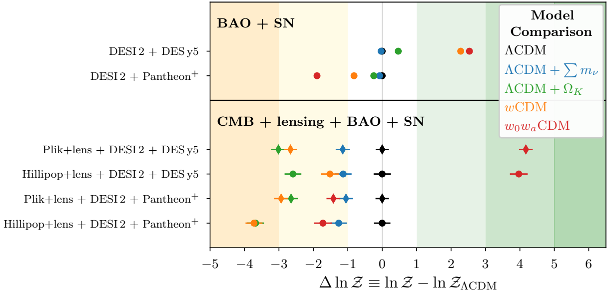
NuFIT 6.0: $\min(\sum m_\nu) \approx 0.099$ (IH)
NuFIT 6.0: $\min(\sum m_\nu) \approx 0.059$ (NH)
$\boldsymbol{\Lambda\mathrm{CDM} + \sum m_\nu}$
SN (DESy5)
BAO (DESI DR2)
CMB+lensing (PR3)
CMB+lensing (PR4)
Combined (PR3)
Combined (PR4)
Ω_m
$\sum m_\nu$
0.0
0.1
0.2
0.3
0.4
0.5
0.25
0.30
0.35
0.1
0.2
0.3
0.4
0.5
Ω_m
$\sum m_\nu$

- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik → Hillipop ($\sim 0.5\sigma$).
  - from DES y5 → Pantheon$^+$ ($\sim 1\sigma$).
- Tensions reduce from ΛCDM → ΛCDM+ $\sum m_\nu$ ($\sim$ 0.25 to 0.5σ).

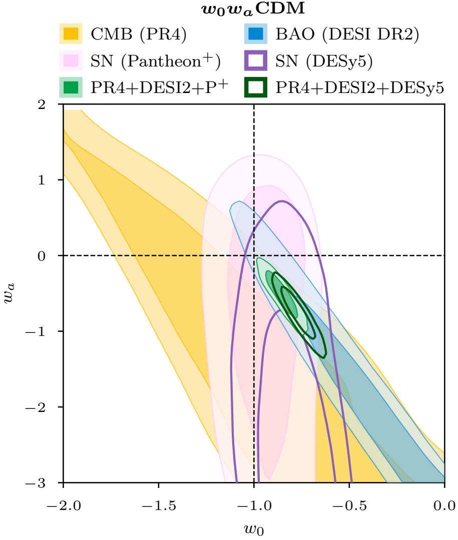


- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik → Hillipop ($\sim 0.5\sigma$).
  - from DES y5 → Pantheon$^+$ ($\sim 1\sigma$).
- Tensions reduce from ΛCDM → ΛCDM+$\sum m_\nu$ ($\sim$ 0.25 to 0.5σ).
- Tensions reduce from ΛCDM → wCDM or $w_0 w_a$CDM ($\sim$ 1 to 2σ).
- Preference for wCDM and $w_0 w_a$CDM:
  - Preference only in combination with DES y5.
  - No preference with Pantheon$^+$.

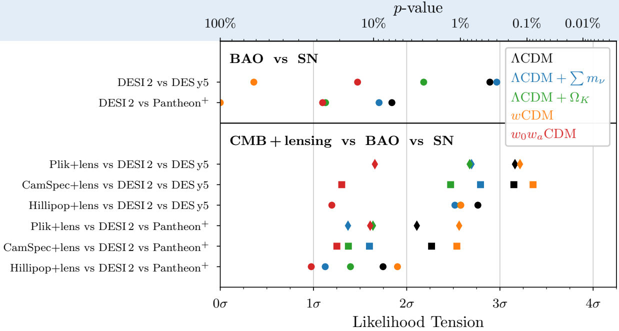


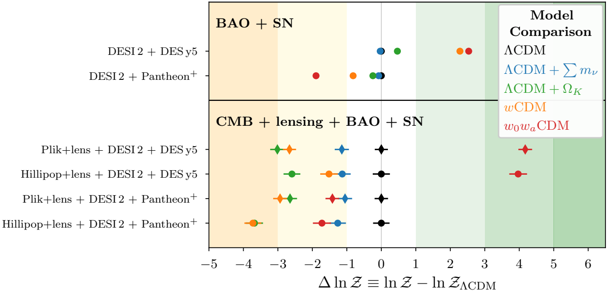


CMB + BAO + SN


$w_0 w_a$CDM
CMB (PR4)
BAO (DESI DR2)
SN (Pantheon$^+$)
SN (DESy5)
PR4+DESI2+P$^+$
PR4+DESI2+DESy5
2
1
0
−1
−2
−3
−2.0
−1.5
−1.0
−0.5
0.0
w_a
w_0

- $\sim 3\sigma$ tension with DES y5 data.
- Tensions reduce across models:
  - from Plik → Hillipop ($\sim 0.5\sigma$).
  - from DES y5 → Pantheon$^+$ ($\sim 1\sigma$).
- Tensions reduce from ΛCDM → ΛCDM+$\sum m_\nu$ (~ 0.25 to 0.5σ).
- Tensions reduce from ΛCDM → wCDM or $w_0 w_a$CDM (~ 1 to 2σ).
- Preference for wCDM and $w_0 w_a$CDM:
  - Preference only in combination with DES y5.
  - No preference with Pantheon$^+$.





Conclusion

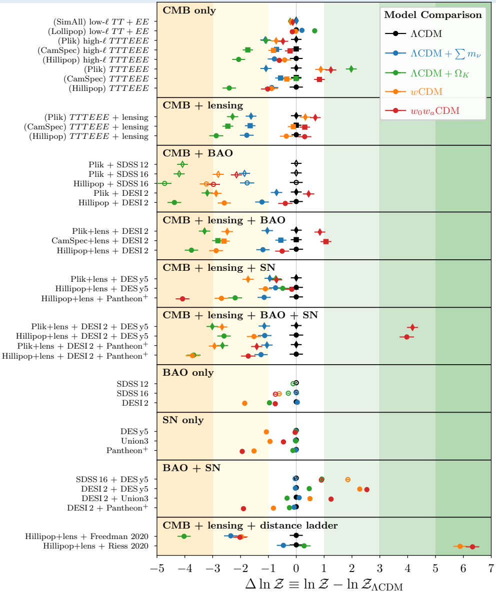
- Tensions vary with choice of CMB likelihood:
  - 0.5σ to 1.5σ.
  - Trend: tensions reduce from Plik $\rightarrow$ CamSpec $\rightarrow$ Hillipop.
  - Note: tensions might tighten for ACT & SPT.
- Tensions and Model preference vary with choice of SN likelihood:
  - $\sim -1\sigma$ going from DES y5 $\rightarrow$ Pantheon$^+$.
  - Model comparison switches from "strongly preferred" to "disfavoured".
- ΛCDM still going strong for majority of data combinations.

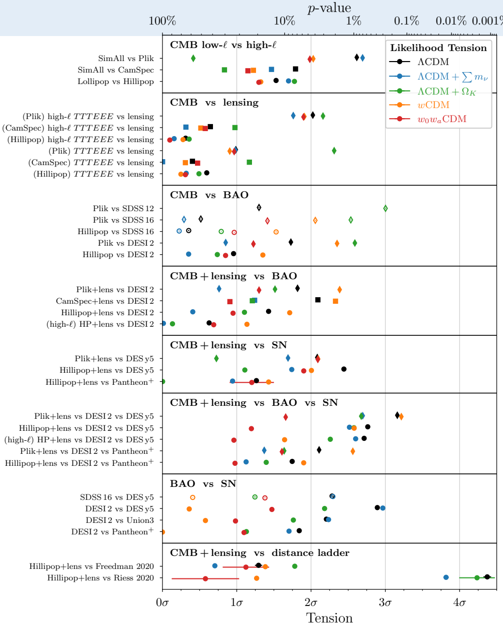
- Curvature tension?
  - Model preference for $\Lambda\mathrm{CDM} + \Omega_K$.
  - Tension between Plik and lensing, or Plik and BAO for $\Lambda\mathrm{CDM} + \Omega_K$.
  - Both model preference and tension go away with CamSpec and Hillipop.
- τ tension?
  - Exclusion of low-ℓ *EE* reduces tension between CMB and BAO.
  - Irrelevant when combining CMB, BAO *and* SN.
- Preference for $w_0 w_a$CDM?
  - Only specifically for Plik + DESI + DESy5.

Should we drop ΛCDM and make $w_0 w_a$CDM our new Cosmic Standard Model?

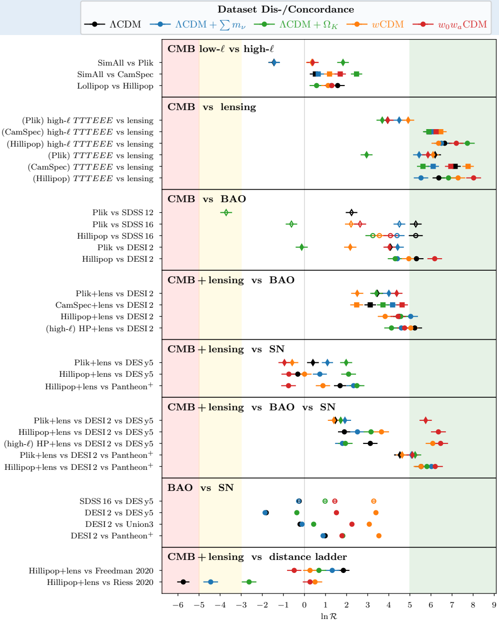
- "Strong claims need strong evidence."
- Singling out specific dataset/likelihood combinations not enough.
- Evidence needs to be sufficiently strong independent of dataset/likelihood choice.

Slightly Overwhelming Overview





Slightly Overwhelming Overview



The 3 Pillars of Bayesian Inference

Parameter Estimation

What do the data tell us about the parameters of a model?
(e.g. the size or age of a ΛCDM universe)

Bayes' Theorem:

$$P(\theta|D,M) = \frac{P(D|\theta,M)\times P(\theta|M)}{P(D|M)}$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

$$\mathcal{P}(\theta) = \frac{\mathcal{L}(\theta) \times \Pi(\theta)}{\mathcal{Z}}$$

$$\propto \mathcal{L}(\theta) \times \Pi(\theta)$$

Model Comparison

How much does the data support one model compared to another?
(e.g. cosmology with a cosmological constant vs with a dynamic dark energy)

$$\frac{P(M_1|D)}{P(M_2|D)} = \frac{P(D|M_1) \times P(M_1)}{P(D|M_2) \times P(M_2)}$$

$$= \frac{\mathcal{Z}_{M_1}}{\mathcal{Z}_{M_2}} \times \frac{P(M_1)}{P(M_2)}$$

$$\begin{matrix}\text{posterior}\\ \text{odds}\end{matrix} = \begin{matrix}\text{evidence}\\ \text{ratio}\end{matrix} \times \begin{matrix}\text{prior}\\ \text{odds}\end{matrix}$$

Occam's razor:

$$\log \mathcal{Z} = \langle \log \mathcal{L} \rangle_{\mathcal{P}} - \mathcal{D}_{\mathrm{KL}}$$

$$\text{log-evidence} = \text{fit} - \text{Occam penalty}$$

Dataset Consistency

Do different datasets A and B make consistent predictions for the same model?
(e.g. CMB vs BAO data)

H_0: datasets described by the same parameters

H_1: datasets described by distinct parameters

$$\mathcal{R} = \frac{\mathcal{Z}_{AB}}{\mathcal{Z}_A \mathcal{Z}_B},$$

$$\mathcal{R} = \log \mathcal{S} - \mathcal{I},$$

$$\log \mathcal{S} = \langle \log \mathcal{L}_{AB} \rangle_{\mathcal{P}_{AB}} - \langle \log \mathcal{L}_{A} \rangle_{\mathcal{P}_{A}} - \langle \log \mathcal{L}_{B} \rangle_{\mathcal{P}_{B}}$$

$$d - 2\log \mathcal{S} \sim \chi^2_d$$

Occam's Razor [arXiv:2102.11511]

- Bayesian inference quantifies Occam's Razor:
  - *"Frustra fit per plura quod potest fieri per pauciora"* — William of Occam
    ("It is futile to do with more things that which can be done with fewer")
  - *"Accept the simplest explanation that fits the data."* — David MacKay
- Consider the Evidence $\mathcal{Z} \equiv P(D|M)$:
  Parameter estimation $\rightarrow$ normalisation constant
  Model comparison $\rightarrow$ critical update factor from model prior to model posterior
- Properties of the evidence: rearrange Bayes' theorem for parameter estimation

$$\mathcal{P}(\theta) = \frac{\mathcal{L}(\theta)\Pi(\theta)}{\mathcal{Z}} \qquad \Rightarrow \qquad \log \mathcal{Z} = \log \mathcal{L}(\theta) - \log \frac{\mathcal{P}(\theta)}{\Pi(\theta)}$$

- Evidence is composed of a "goodness of fit" term and "Occam Penalty"

- RHS true for all θ. Take max likelihood point θ_{ML}:

$$\log \mathcal{Z} = \log \mathcal{L}_{\text{max}} - \log \frac{\mathcal{P}(\theta_{ML})}{\Pi(\theta_{ML})}$$

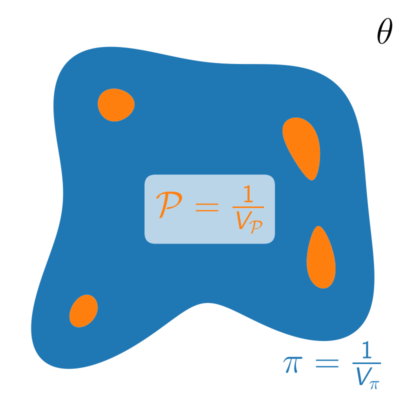
- Be more Bayesian and take posterior average to get the "Occam equation"

$$\boxed{\log \mathcal{Z} = \langle \log \mathcal{L} \rangle_{\mathcal{P}} - \mathcal{D}_{\text{KL}}}$$

- Natural regularisation which penalises models with too many parameters.

Kullback–Leibler divergence $\mathcal{D}_{\mathrm{KL}}$

Prior and posterior volumes



- The KL divergence between prior Π and posterior $\mathcal{P}$ is defined as:

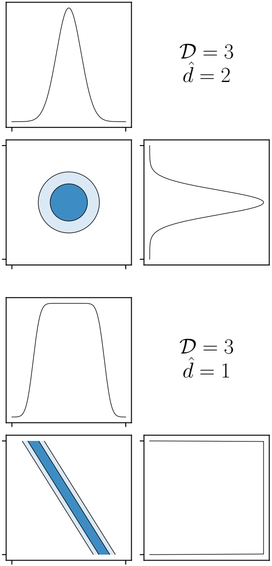
$$\mathcal{D}_{\mathrm{KL}} = \left\langle \log \frac{\mathcal{P}}{\Pi} \right\rangle_{\mathcal{P}} = \int \mathcal{P}(\theta) \log \frac{\mathcal{P}(\theta)}{\Pi(\theta)} d\theta.$$

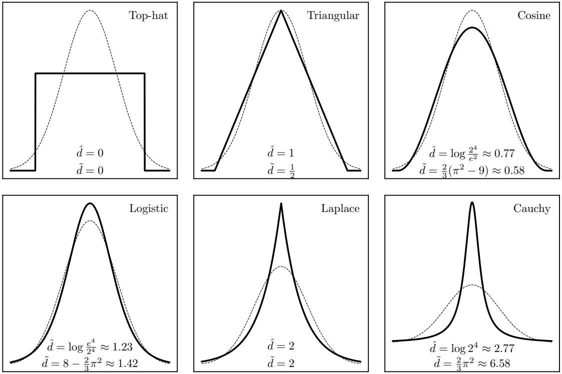
- Whilst not a distance, $\mathcal{D} = 0$ when $\mathcal{P} = \Pi$.
- Occurs in the context of machine learning as an objective function for training functions.
- In Bayesian inference it can be understood as a log-ratio of “volumes”:

$$\mathcal{D}_{\mathrm{KL}} \approx \log \frac{V_\Pi}{V_\mathcal{P}}.$$

(this is exact for top-hat distributions).

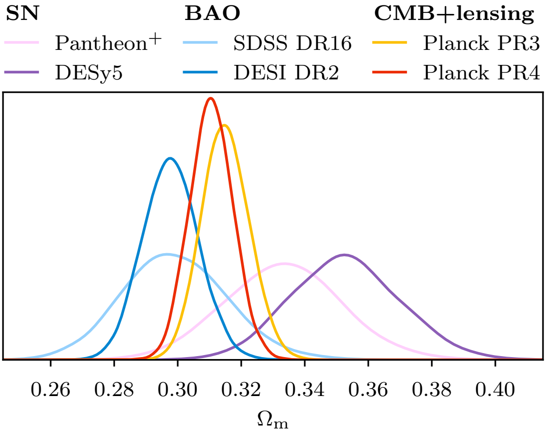
Bayesian Model Dimensionality

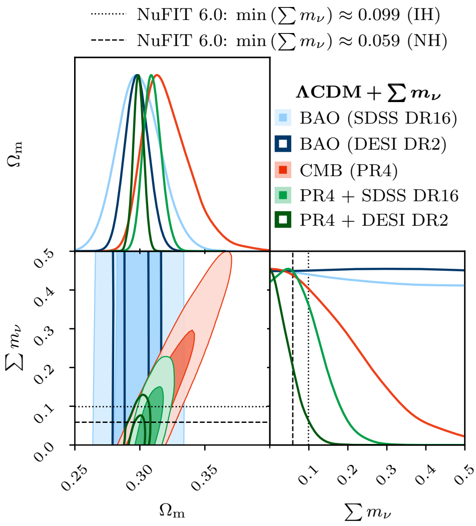




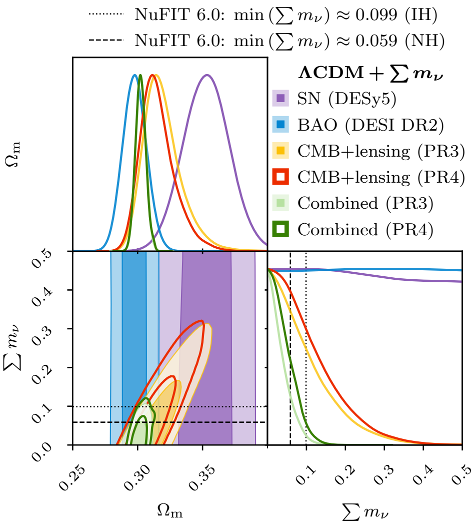
Handley & Lemos (2019)

Matter Density



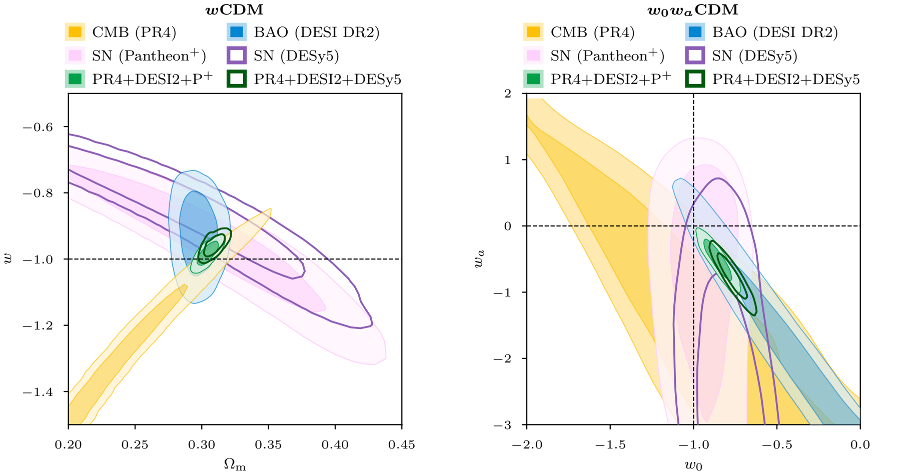


NuFIT 6.0: $\min(\sum m_\nu) \approx 0.099$ (IH)
NuFIT 6.0: $\min(\sum m_\nu) \approx 0.059$ (NH)
ΛCDM + $\sum m_\nu$
BAO (SDSS DR16)
BAO (DESI DR2)
CMB (PR4)
PR4 + SDSS DR16
PR4 + DESI DR2
Ω_m
$\sum m_\nu$

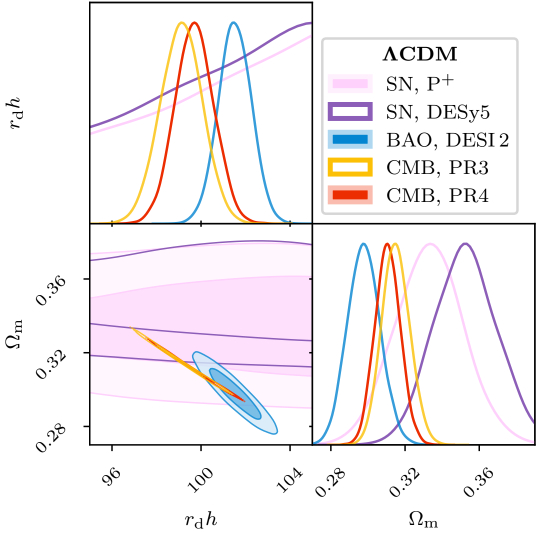


NuFIT 6.0: $\min(\sum m_\nu) \approx 0.099$ (IH)
NuFIT 6.0: $\min(\sum m_\nu) \approx 0.059$ (NH)
ΛCDM + $\sum m_\nu$
SN (DESy5)
BAO (DESI DR2)
CMB+lensing (PR3)
CMB+lensing (PR4)
Combined (PR3)
Combined (PR4)
Ω_m
$\sum m_\nu$

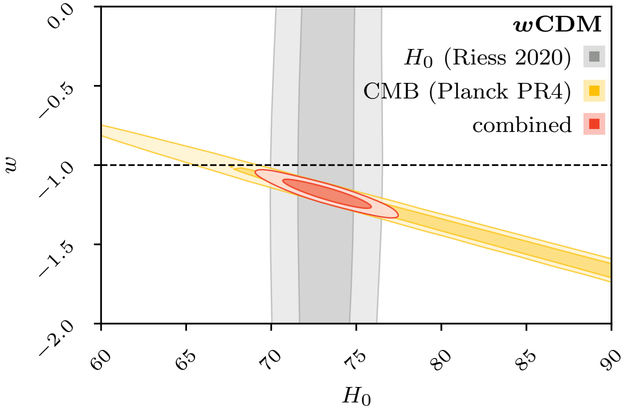
Constant and Evolving Dark Energy



$r_\mathrm{d} H_0$


ΛCDM
SN, P+
SN, DESy5
BAO, DESI 2
CMB, PR3
CMB, PR4
$r_\mathrm{d}h$
Ω_m
0.36
0.32
0.28
96
100
104
0.28
0.32
0.36
$r_\mathrm{d}h$
Ω_m

H_0 tension



wCDM
H_0 (Riess 2020)
CMB (Planck PR4)
combined
w
H_0
0.0
−0.5
−1.0
−1.5
−2.0
60
65
70
75
80
85
90