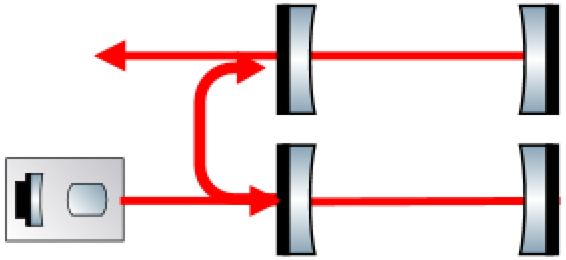
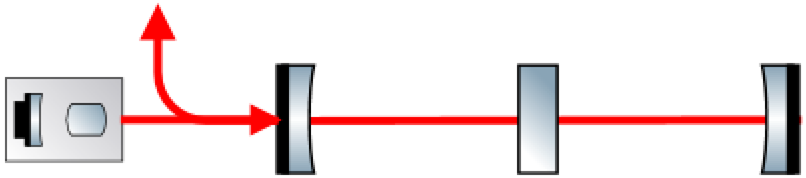
Two filter cavities vs coupled filter cavity for Frequency Dependent Squeezing

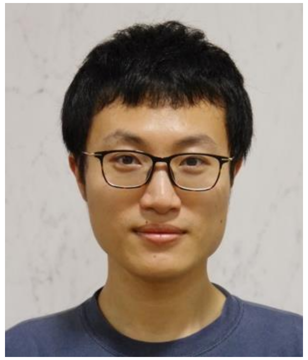




Jacques Ding (Laboratoire Astroparticule et Cosmologie)

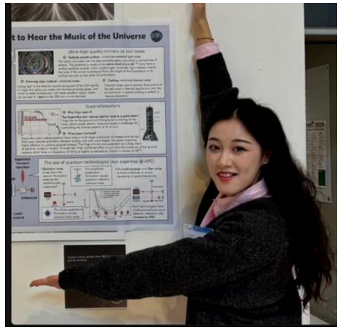
Eleonora Capocasa

Yuhang Zhao



Isander Ahrend

Fangfei Liu



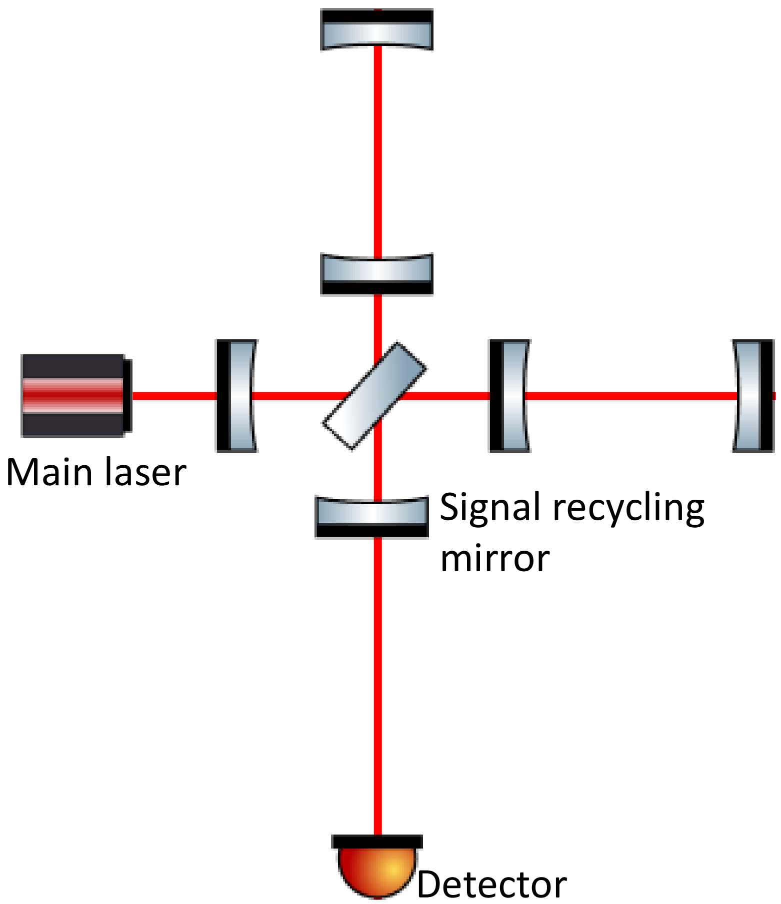
Matteo Barsuglia

APC, CNRS, FRANCE

Université Paris Cité

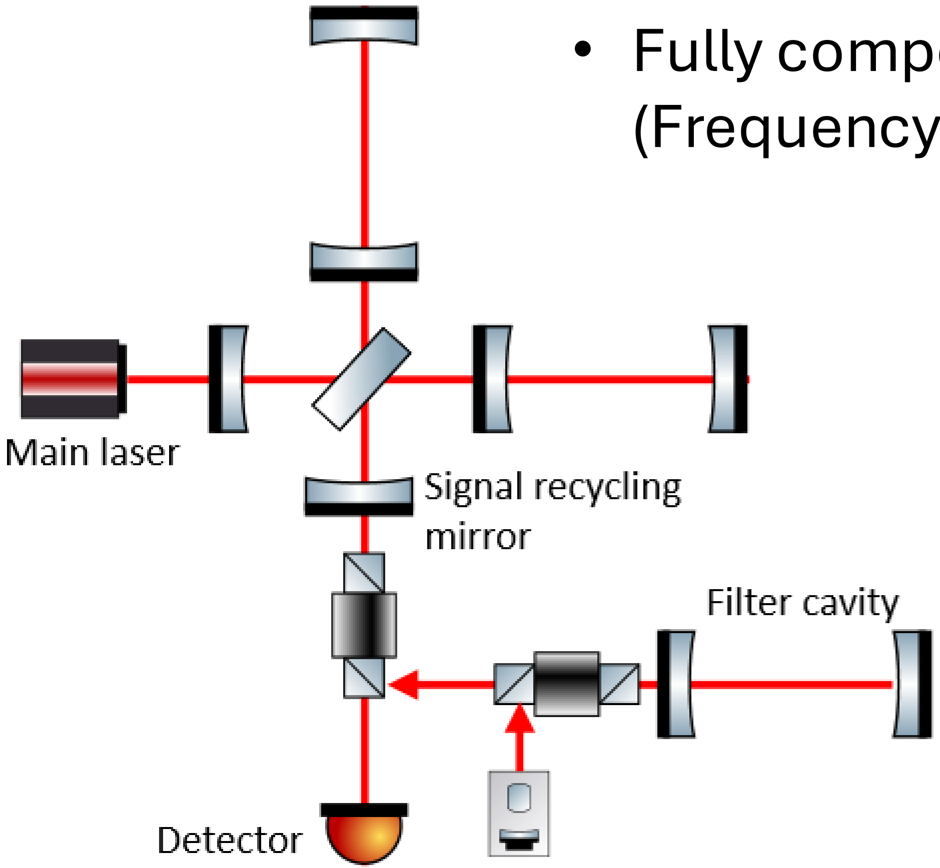
arXiv:2506.02222

Quantum noise in a tuned interferometer

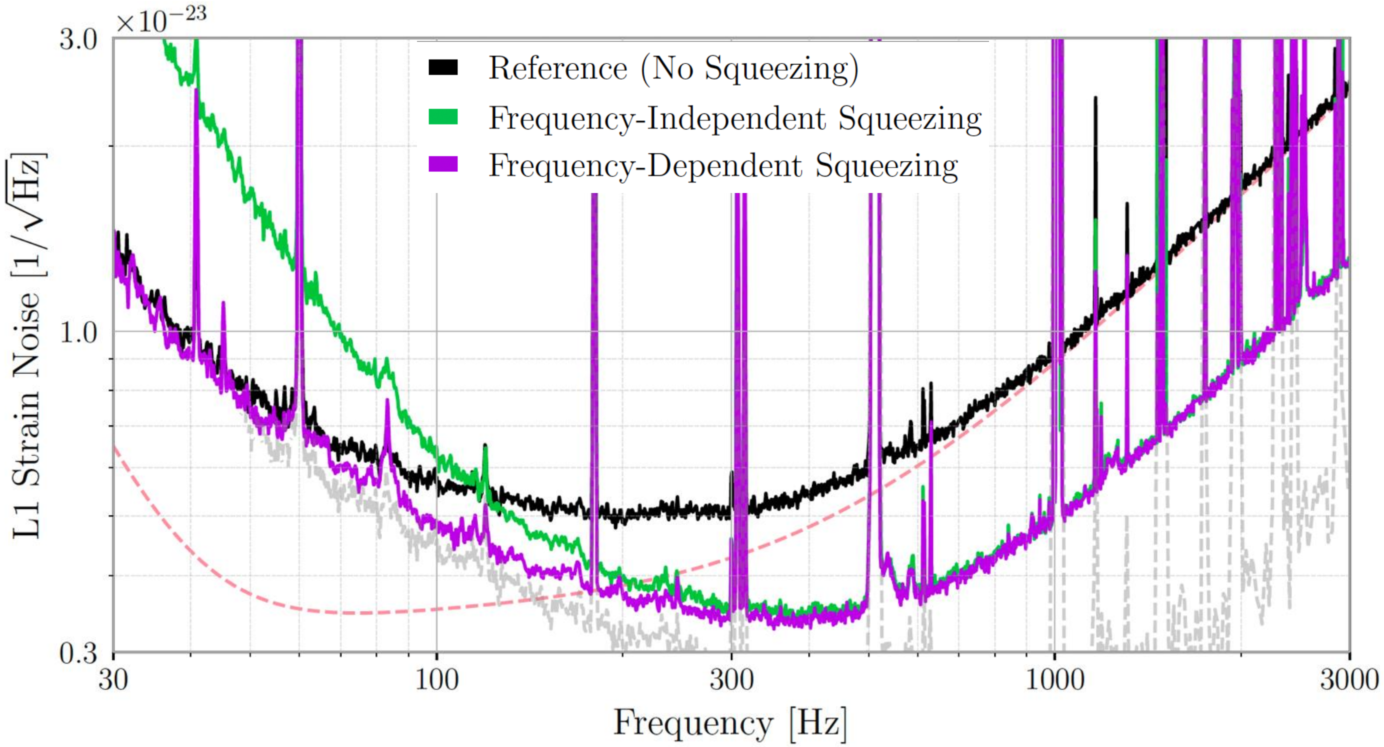


- Current interferometers operate in broadband configuration = tuned signal recycling
- Interferometer rotates quantum noise quadratures (radiation pressure & shot noise)

Quantum noise in a tuned interferometer

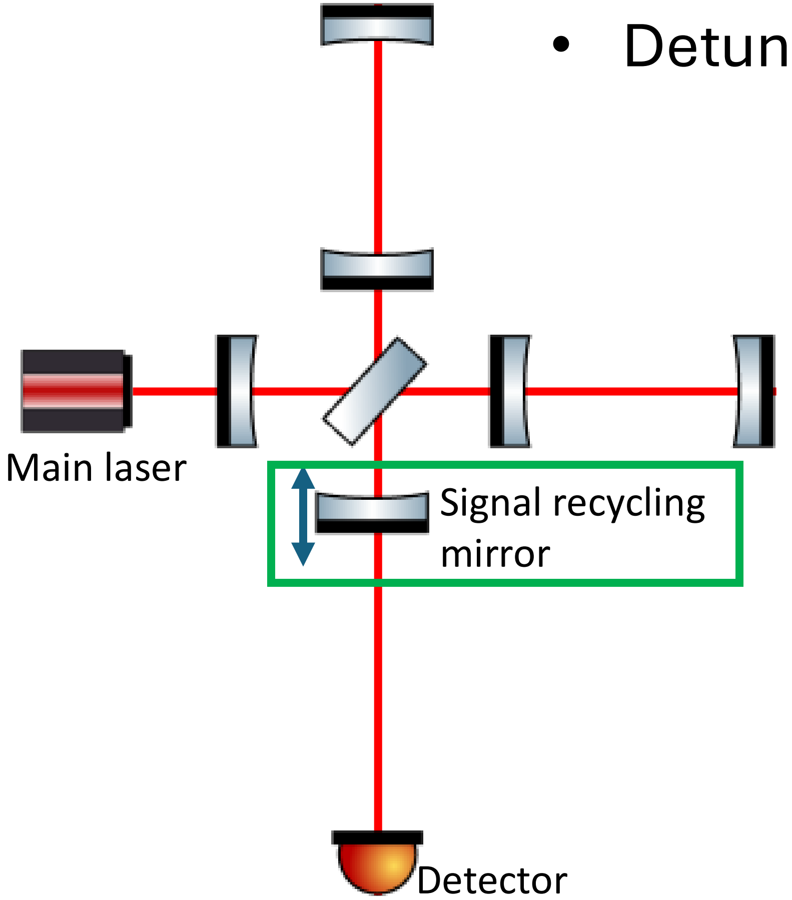


- Fully compensate the rotation using a single filter cavity (Frequency Dependent squeezing)



Physical Review X 13, 041021 (2023)

Quantum noise in a detuned interferometer



- Detuning SRM makes the detector more frequency-selective

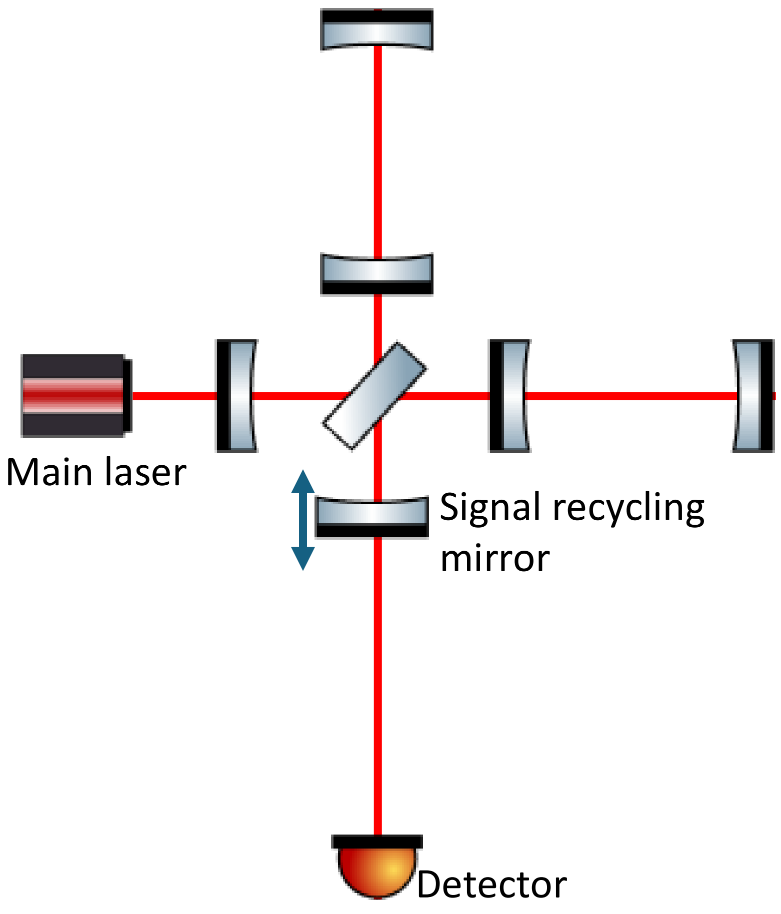
PHYSICAL REVIEW D, VOLUME 64, 042006

Quantum noise in second generation, signal-recycled laser interferometric gravitational-wave detectors

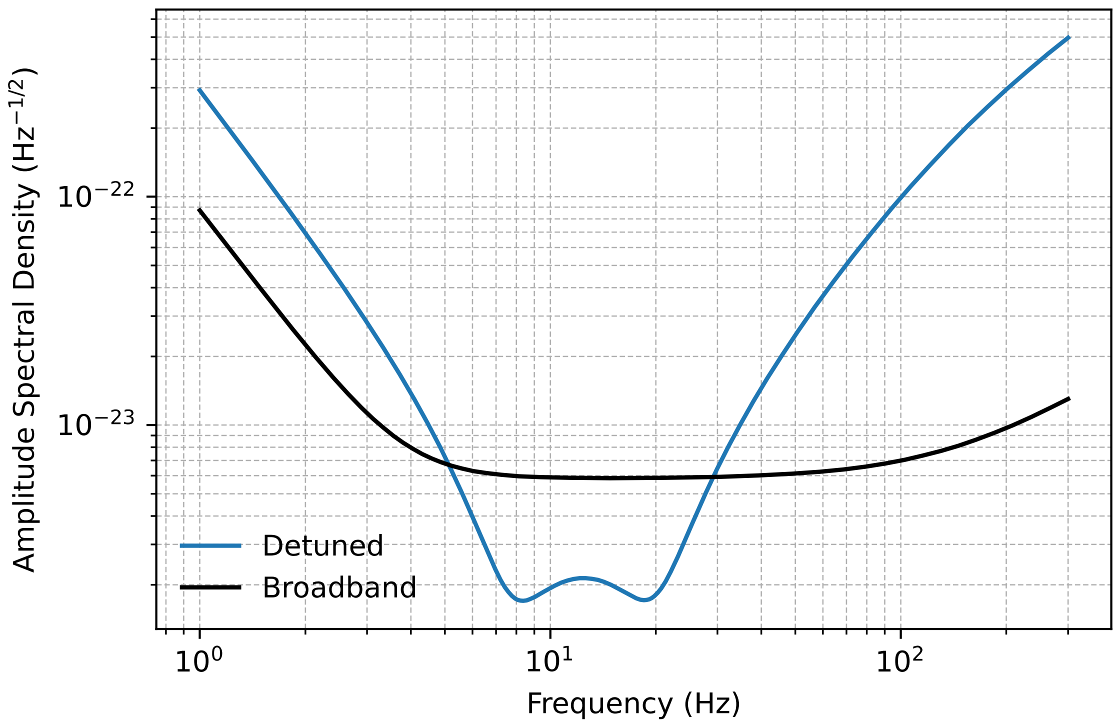
Alessandra Buonanno and Yanbei Chen
Theoretical Astrophysics and Relativity Group, California Institute of Technology, Pasadena, California 91125
(Received 6 February 2001; published 30 July 2001)

- Easy to say, hard to do (Caltech 40m)

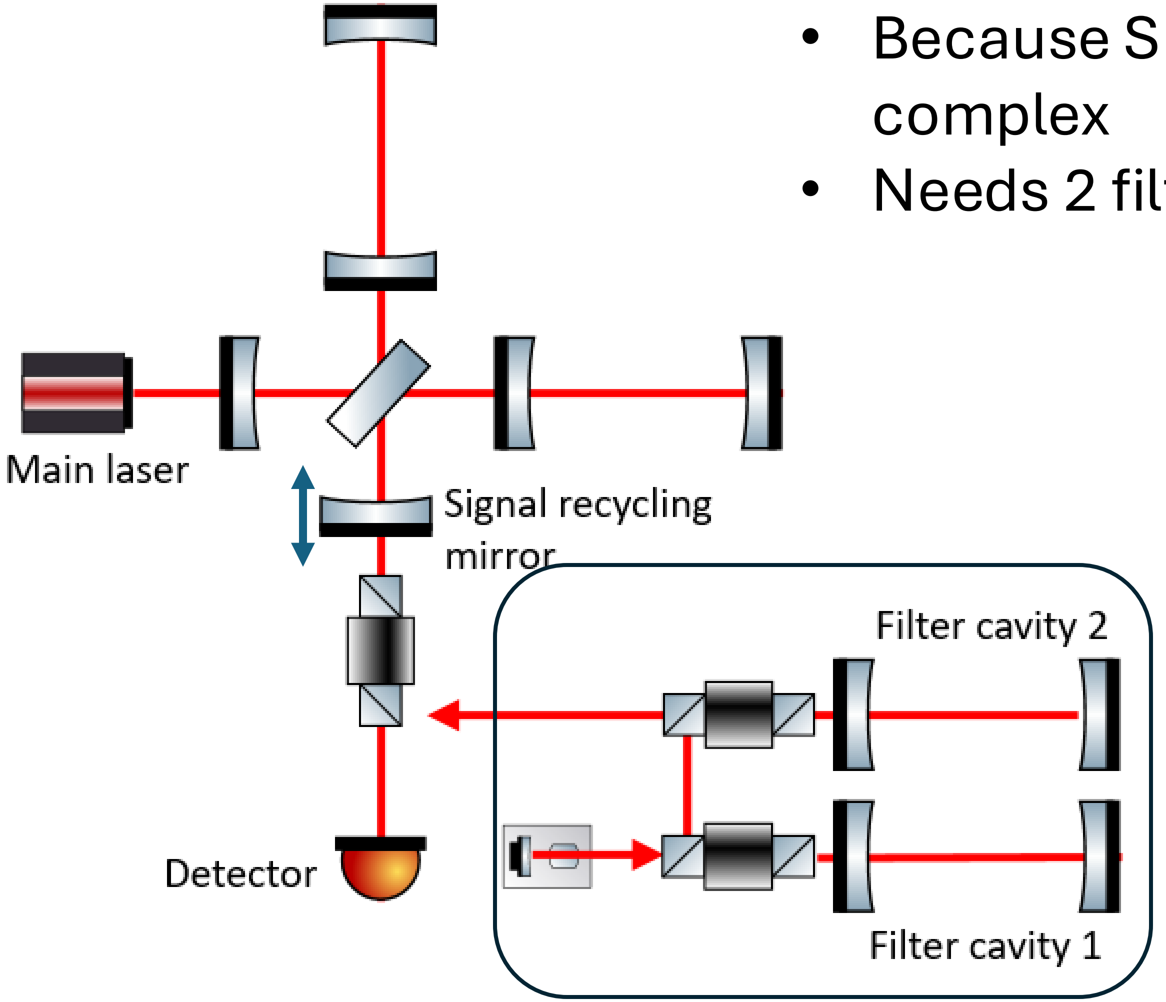
Quantum noise in a detuned interferometer



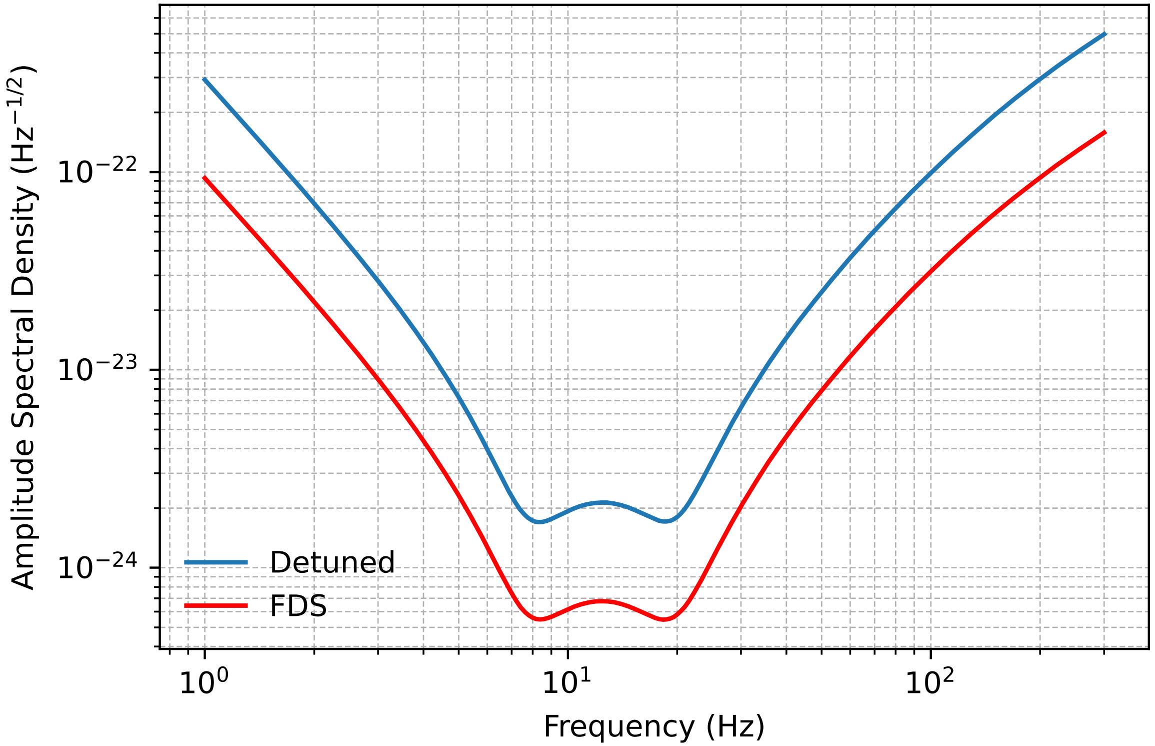
- Einstein Telescope Low Frequency for this talk (not official numbers):
  - 10 km arms, L *or* triangle
  - 1550 nm
  - 18 kW in arms
  - Detuned SR



Frequency dependent squeezing for ET-LF

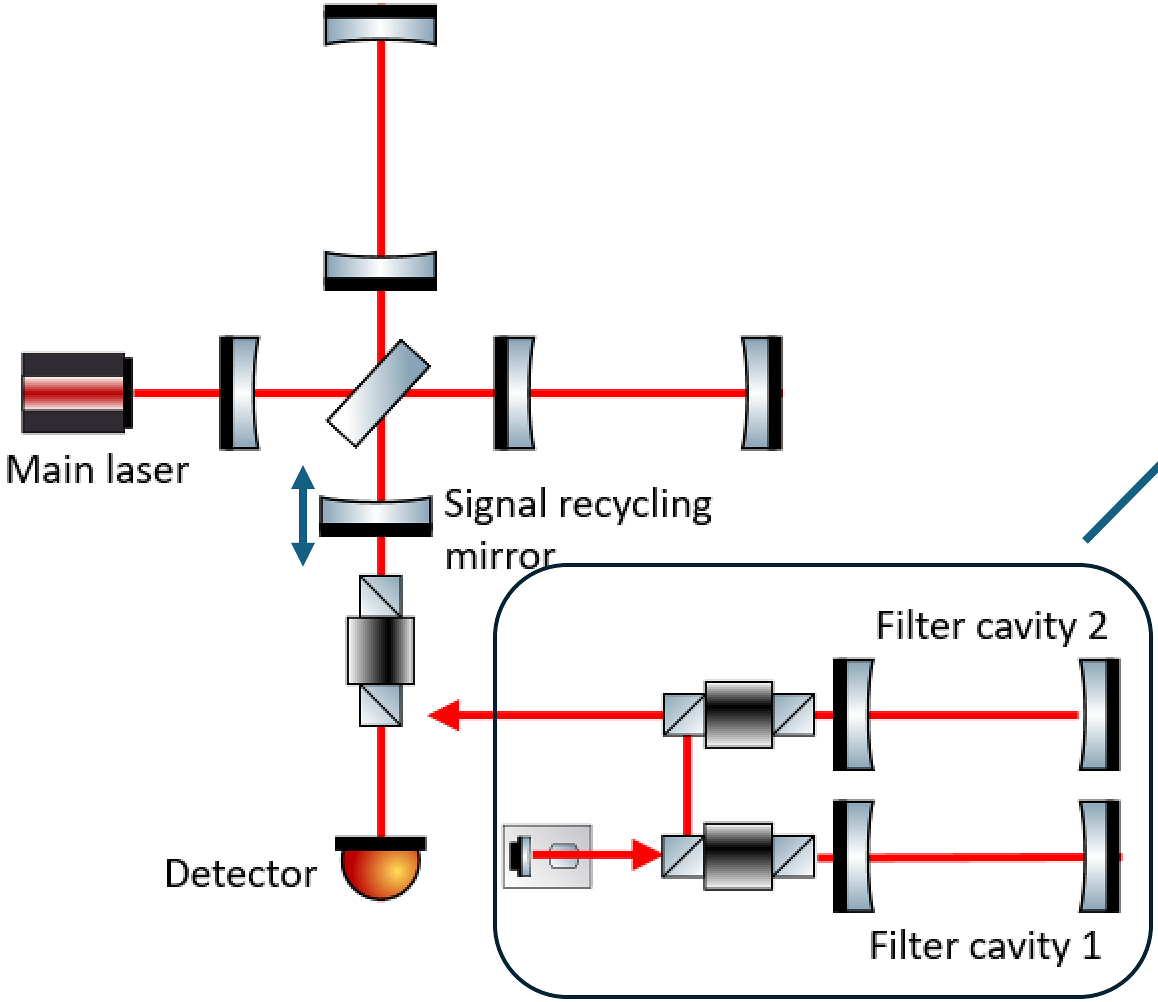


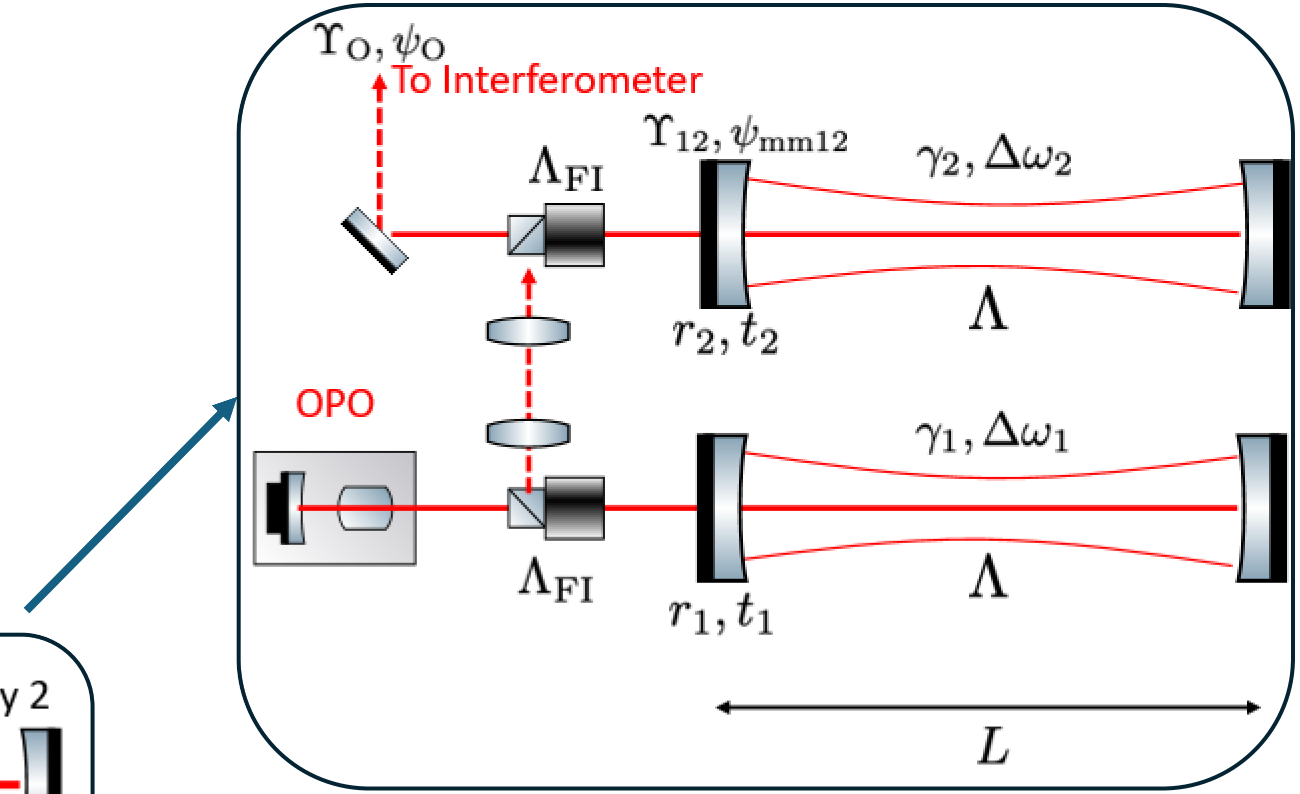
- Because SR detuned, quadrature rotation more complex
- Needs 2 filter cavities for FDS



Jacques Ding (ET-0156B-25, arXiv:2506.02222)

Frequency dependent squeezing for ET-LF





Is it possible to obtain the same frequency response with less optics?

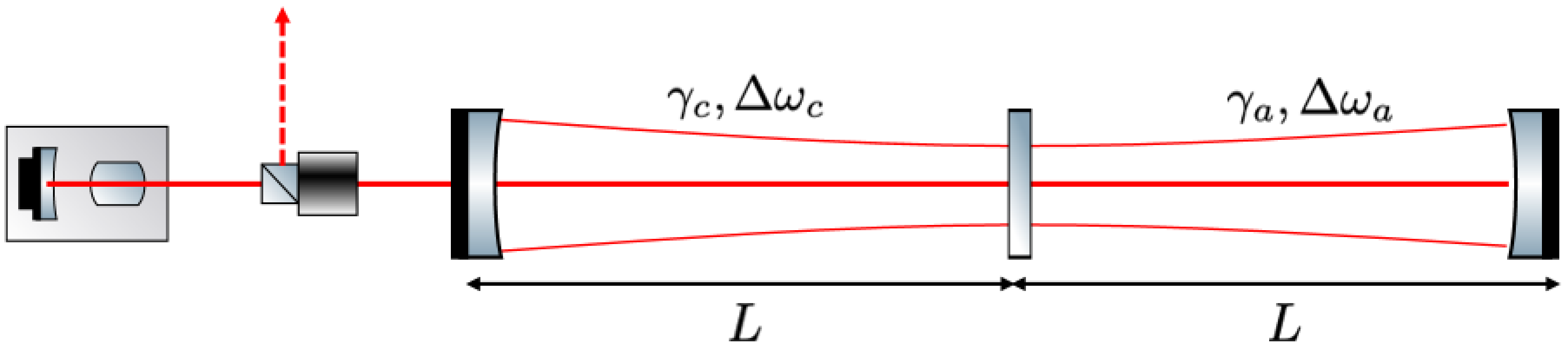
This talk

1. Introduce a Coupled Filter Cavity (CFC) alternative to 2FC

2. Compare the quantum performances of CFC with 2FC

3. Introduce a Single Filter Cavity (1FC) intermediary step to ET-LF design and compare its performances to CFC and 2FC

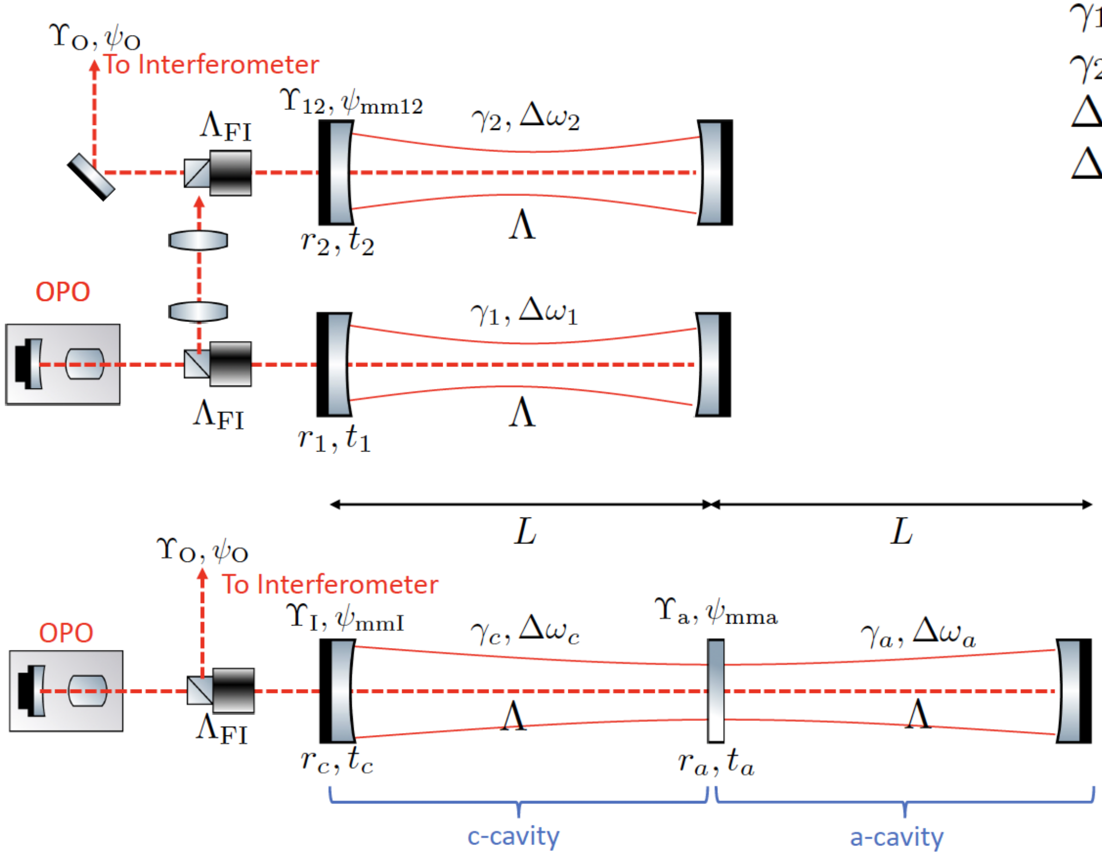
Coupled filter cavity



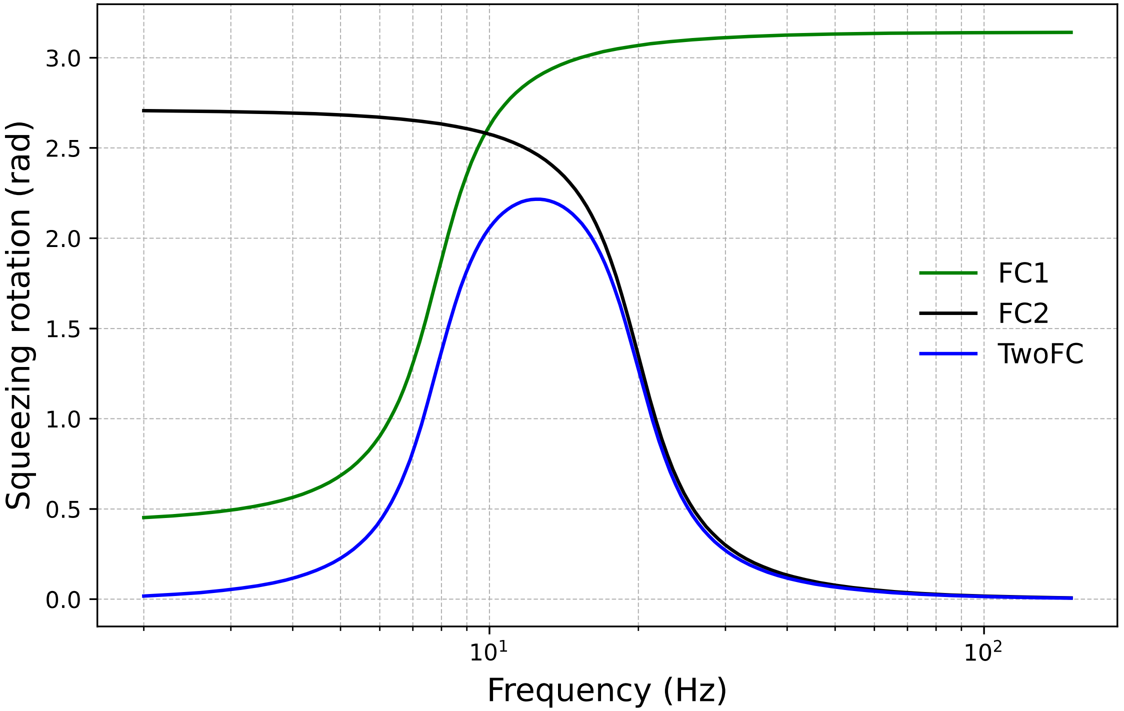
- One less Faraday
- One less mirror
- Less mode matching optics
- Same total footprint

Initially studied in Phys. Rev. D **101**, 082002 and Phys. Rev. D **110**, 082006, but no full quantum degradation analysis and issue of middle mirror transmissivity

Equivalence between two filter cavities and coupled filter cavity

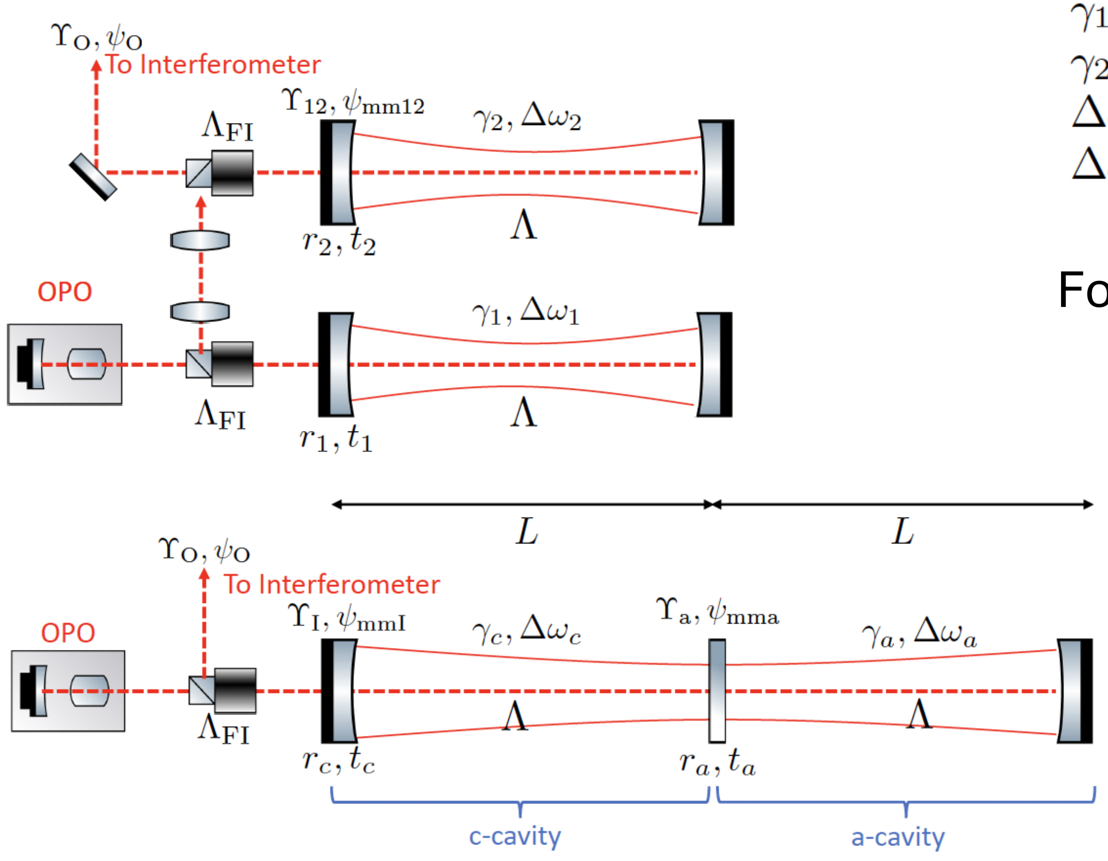


γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s



Jacques Ding (ET-0156B-25, arXiv:2506.02222)

Equivalence between two filter cavities and coupled filter cavity



γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s

For lossless systems: (Phys. Rev. D **110**, 08200)

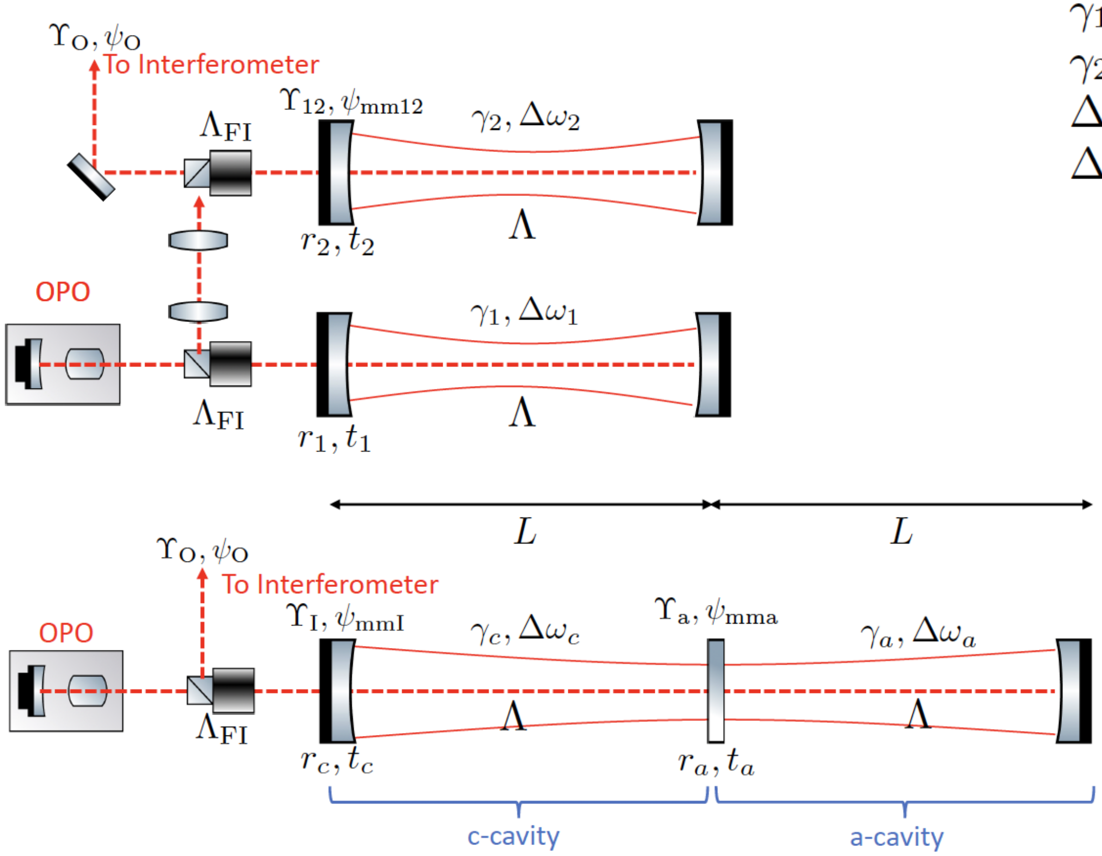
$$\gamma_c = \gamma_1 + \gamma_2$$

$$\gamma_a = \frac{\gamma_1\gamma_2}{2\nu_c}\left[1 + \left(\frac{\Delta\omega_1 - \Delta\omega_2}{\gamma_1 + \gamma_2}\right)^2\right]$$

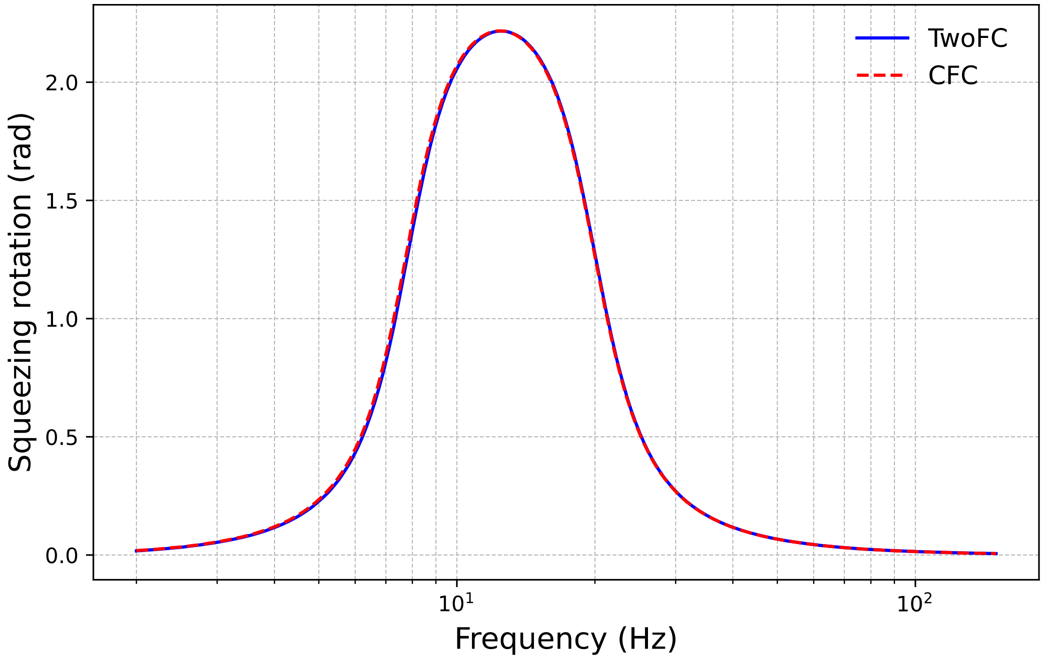
$$\Delta\omega_a = \frac{\Delta\omega_1\gamma_2 + \Delta\omega_2\gamma_1}{\gamma_1 + \gamma_2}$$

$$\Delta\omega_c = \frac{\Delta\omega_1\gamma_1 + \Delta\omega_2\gamma_2}{\gamma_1 + \gamma_2}$$

Equivalence between two filter cavities and coupled filter cavity

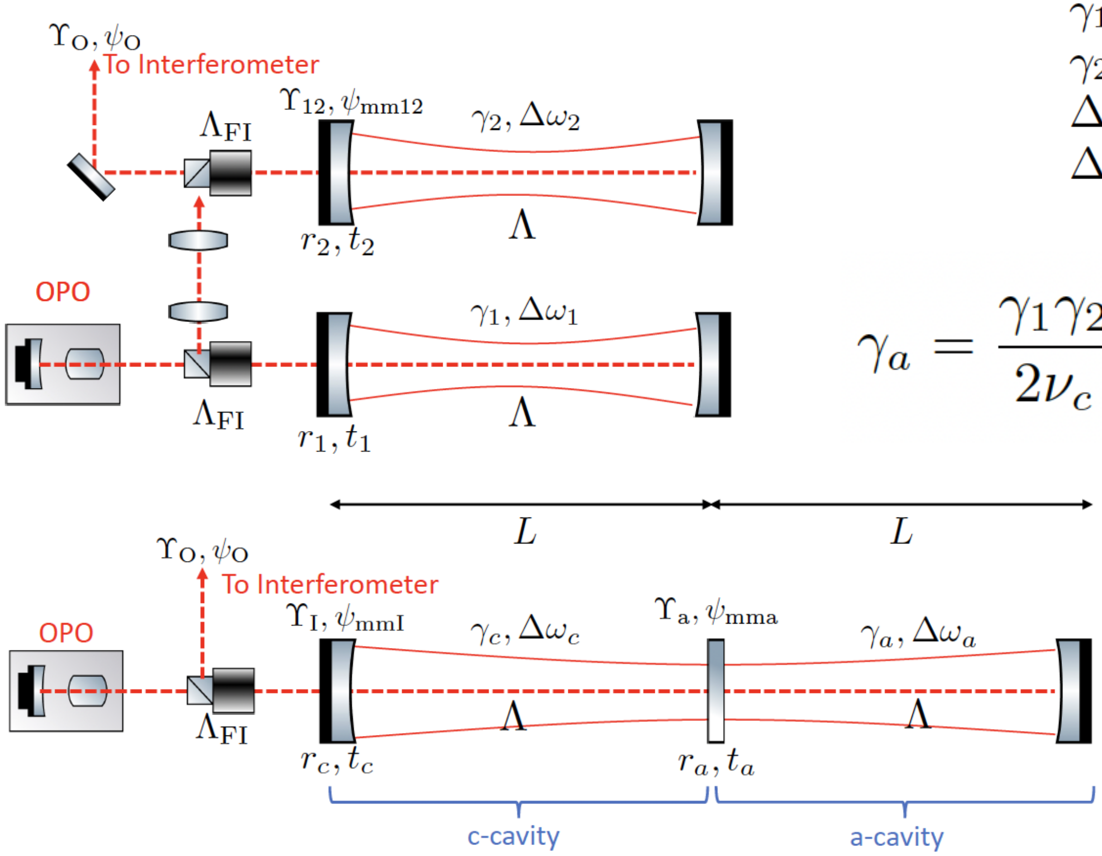


γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s



Jacques Ding (ET-0156B-25, arXiv:2506.02222)

Equivalence between two filter cavities and coupled filter cavity



γ_1	1st FC linewidth	$2\pi \times (4.26)$ rad/s
γ_2	2nd FC linewidth	$2\pi \times (1.65)$ rad/s
$\Delta\omega_1$	1st FC detuning	$2\pi \times 19.51$ rad/s
$\Delta\omega_2$	2nd FC detuning	$2\pi \times (-7.65)$ rad/s

$$\gamma_a = \frac{\gamma_1\gamma_2}{2\nu_c}\left[1 + \left(\frac{\Delta\omega_1 - \Delta\omega_2}{\gamma_1 + \gamma_2}\right)^2\right] = \frac{cT_a}{4L}$$

$$T_a \propto L^2$$

$$\text{for } L = 1 \text{ km}, T_a = 0.27 \text{ ppm}$$ (!)

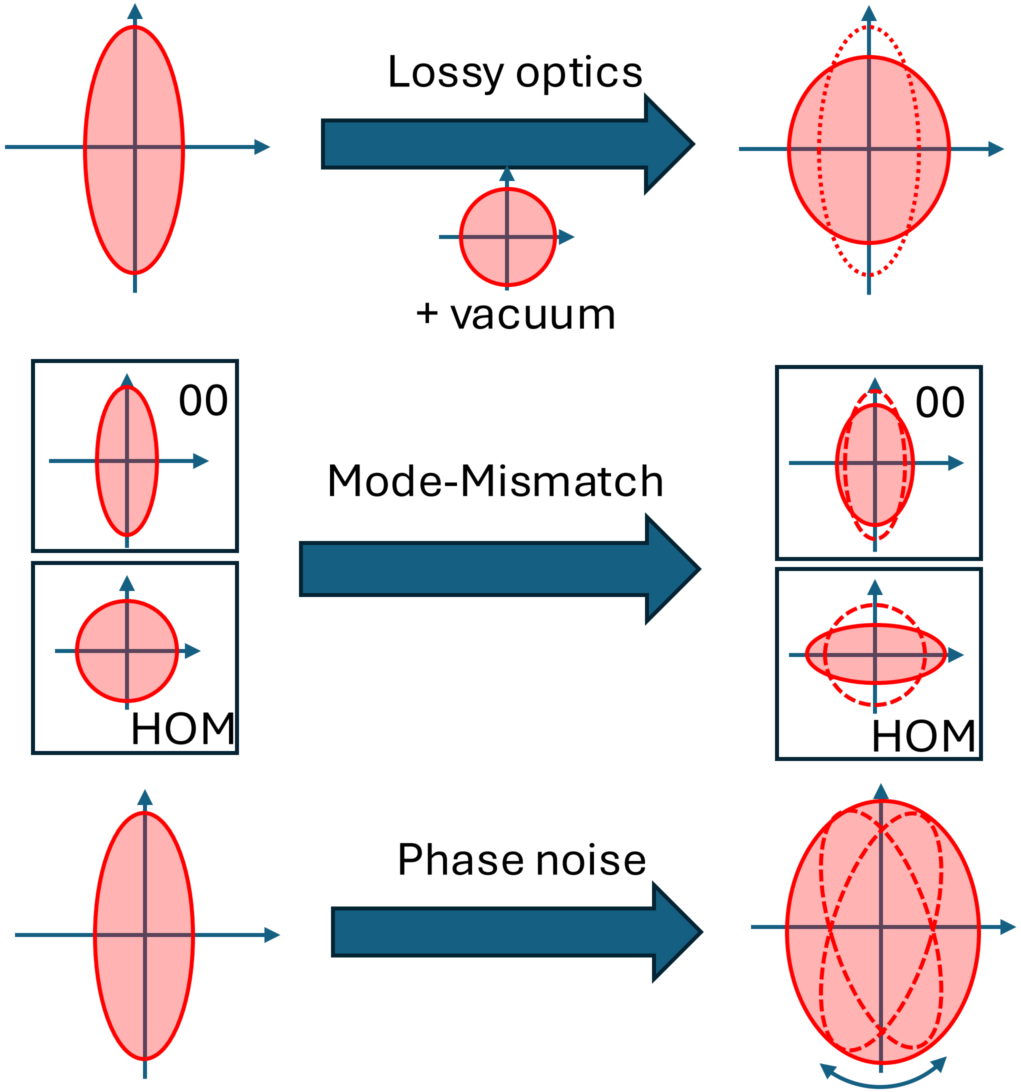
$$\text{for } L = 5 \text{ km}, T_a = 6.75 \text{ ppm}$$

- Compatible with current (future?) coatings
- **CFC only feasible for long enough cavities**

Physical Review D 104, 062006 (2021)
Physical Review D 90, 062006 (2014)

Squeezing degradation sources

- **Loss**: coupling to vacuum
- **Mode mismatch**: possible coupling between squeezing and antisqueezing through higher-order modes
- **Phase noise**: Technical, also couples squeezing to antisqueezing



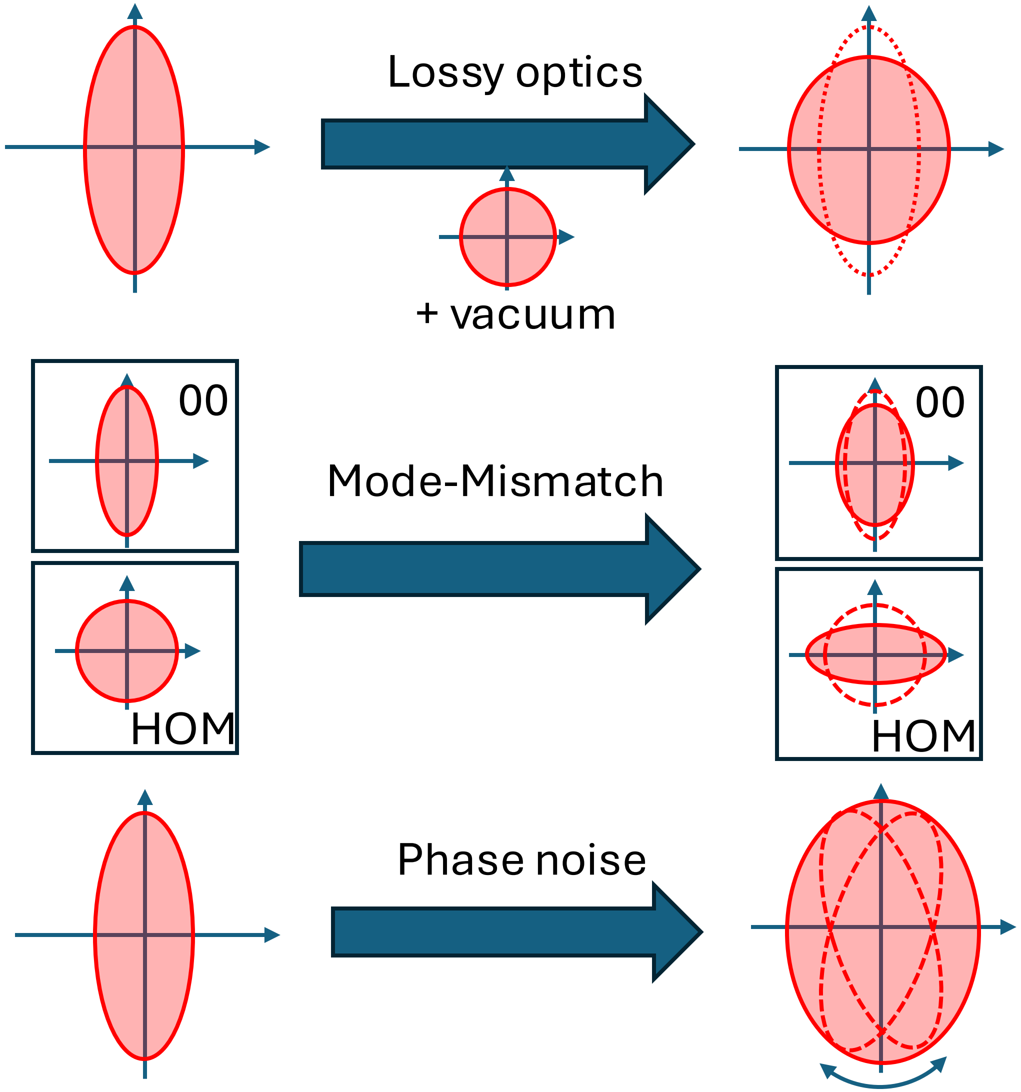
Physical Review D 104, 062006 (2021)
Physical Review D 90, 062006 (2014)

Squeezing degradation sources

Figures of merit $\bar{S} = e^{-2r}$

- **Efficiency:**
  $\bar{S} = \eta e^{-2r} + 1 - \eta$
- **Dephasing:**
  $\bar{S} = (1-\Xi)e^{-2r} + \Xi\, e^{2r}$
- **Misphasing:**
  $\bar{S} = e^{-2r}\cos^2\Delta\theta_D + e^{2r}\sin^2\Delta\theta_D$
- Total squeezing PSD:

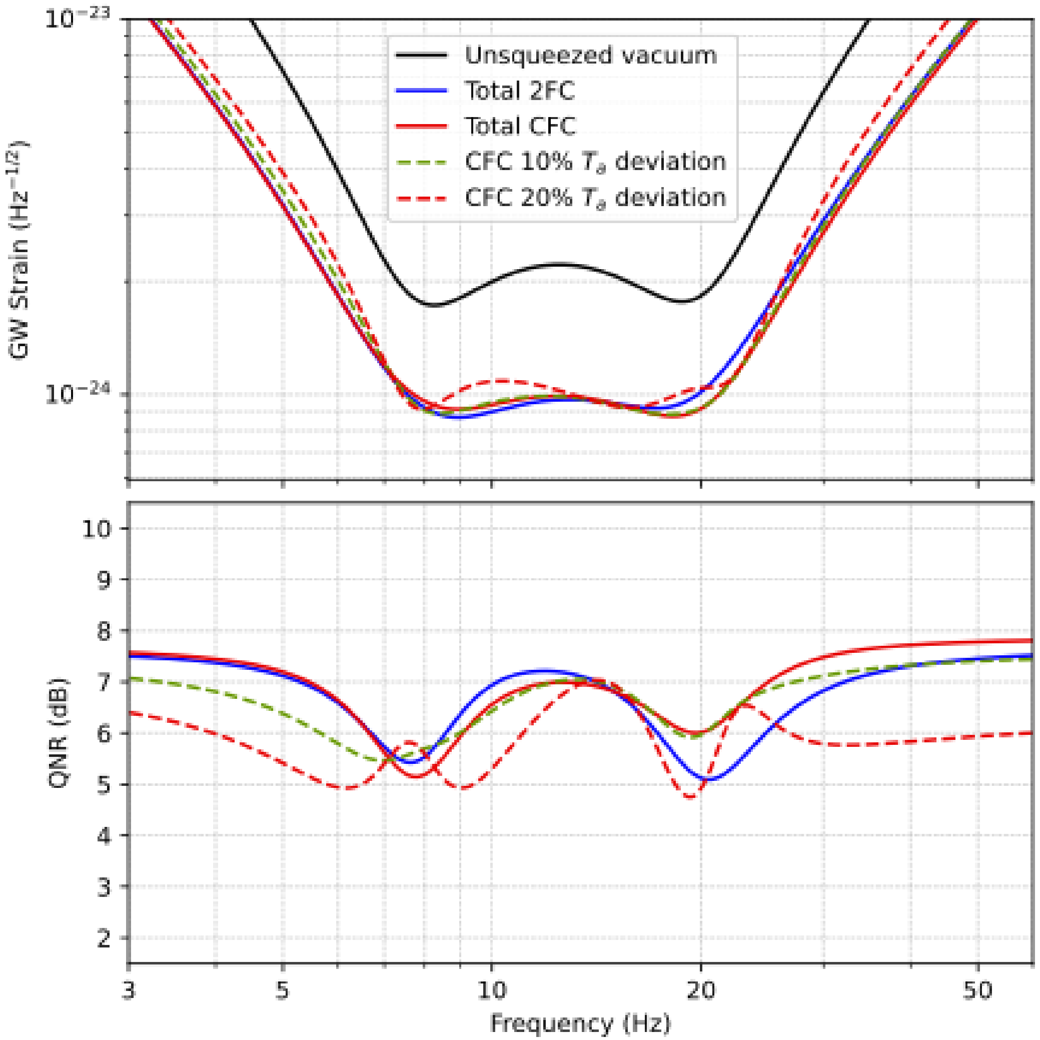
$$\bar{S}[\Omega] = \eta[\Omega]\{[(1-\Xi[\Omega])e^{-2r} + \Xi[\Omega]e^{2r}]\cos^2(\Delta\theta_D[\Omega]) + [(1-\Xi[\Omega])e^{2r} + \Xi[\Omega]e^{-2r}]\sin^2(\Delta\theta_D[\Omega])\} + 1 - \eta[\Omega]$$



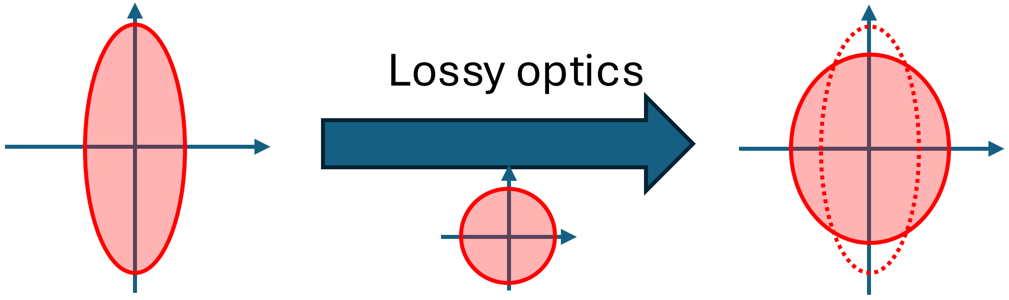
Example for misphasing: error on middle mirror transmissivity

- What if the middle mirror has 10% or 20% manufacturing error on transmission value? (±1 ppm)

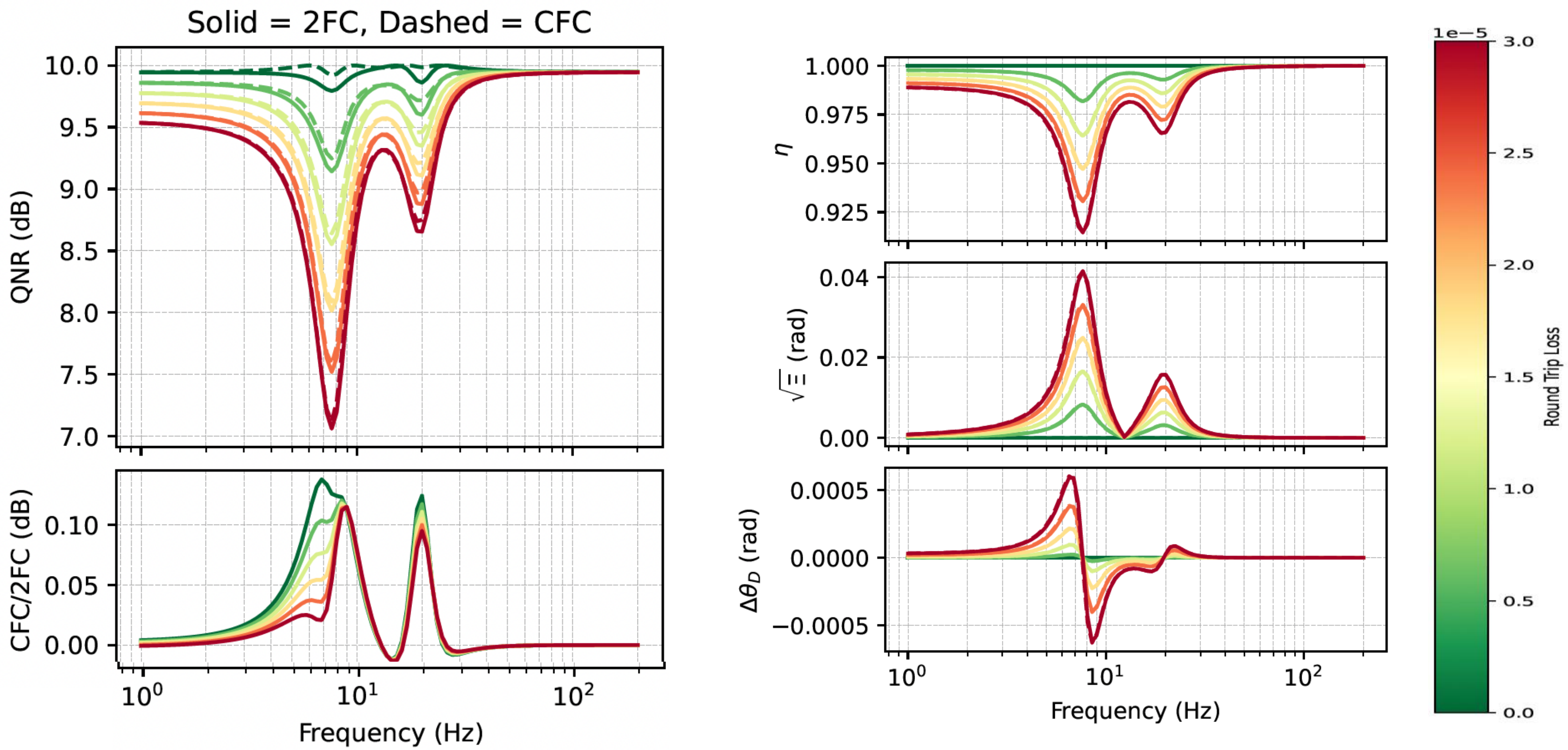
- Squeezing is degraded (mis-rotation) but impact can be mitigated
  - Can be partially compensated using other degrees of freedom (detunings)
  - Can be fully compensated using thermal controls



Comparing Round Trip Loss



- Mathematically, if all cavity losses equal, the **2FC-CFC equivalence still holds**



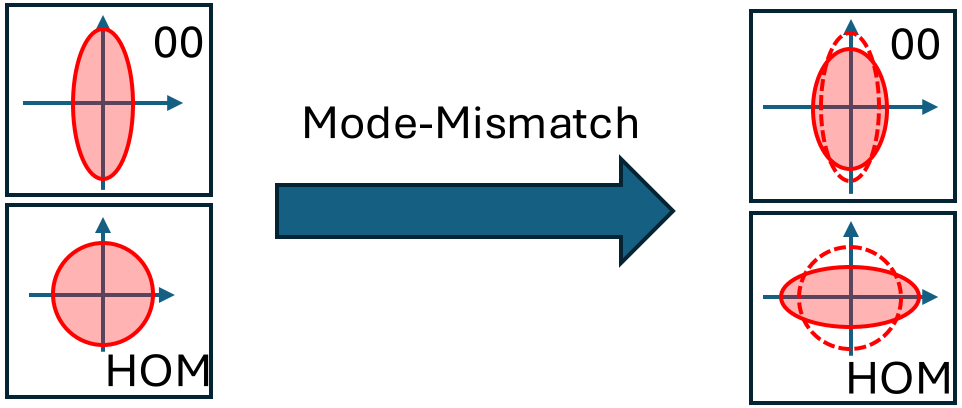
- Mainly degrading efficiency
- $T_a = 6.75$ ppm $\ll 30$ ppm $=$ RTL

But squeezing is never fully lost inside of the second subcavity (non-trivial resonance interplay)

Jacques Ding (ET-0156B-25, arXiv:2506.02222)

Modeling mode-mismatch

- Either matches to very high order modes, modelled by loss
- Or matches to nearby modes in a coherent way



$$\hat{a}_{00}^{\mathrm{out}} = \sqrt{1-\Upsilon}\,\hat{a}_{00}^{\mathrm{in}} + \sqrt{\Upsilon}\,e^{i\psi_{mm}}\,\hat{a}_{mm}^{\mathrm{in}}$$

Amplitude of mismatch

Phase of mismatch (e.g. waist size, waist position…)

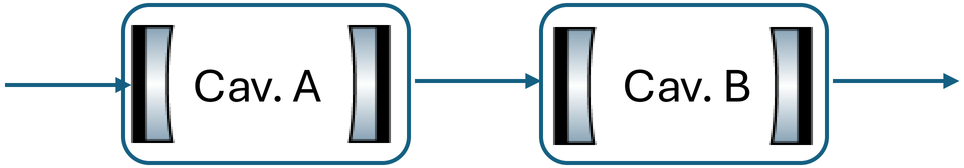
$$\begin{bmatrix} \hat{a}_{00}^{\mathrm{out}} \\ \hat{a}_{\mathrm{mm}}^{\mathrm{out}} \end{bmatrix} = \boldsymbol{U}(\Upsilon, \psi_{mm}) \begin{bmatrix} \hat{a}_{00}^{\mathrm{in}} \\ \hat{a}_{\mathrm{mm}}^{\mathrm{in}} \end{bmatrix}$$

where

$$\boldsymbol{U}(\Upsilon, \psi_{mm}) = \begin{bmatrix} \sqrt{1-\Upsilon} & -\sqrt{\Upsilon}e^{i\psi_{mm}} \\ \sqrt{\Upsilon}e^{-i\psi_{mm}} & \sqrt{1-\Upsilon} \end{bmatrix}$$

Addition of mode mismatches

- Assume two consecutive mode mismatches:



$$\begin{bmatrix} \hat{a}_{00}^{\text{out}} \\ \hat{a}_{\text{mm}}^{\text{out}} \end{bmatrix} = \boldsymbol{U}(\Upsilon_B, \psi_{mmB})\boldsymbol{U}(\Upsilon_A, \psi_{mmA}) \begin{bmatrix} \hat{a}_{00}^{\text{in}} \\ \hat{a}_{\text{mm}}^{\text{in}} \end{bmatrix}$$

- Equivalent single mismatch?

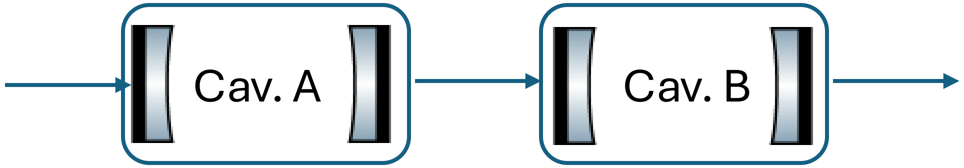
$$\begin{bmatrix} \hat{a}_{00}^{\text{out}} \\ \hat{a}_{\text{mm}}^{\text{out}} \end{bmatrix} = \boldsymbol{U}(\Upsilon', \psi'_{mm}) \begin{bmatrix} \hat{a}_{00}^{\text{in}} \\ \hat{a}_{\text{mm}}^{\text{in}} \end{bmatrix}$$

- Not simply a sum of mismatches

$$\begin{aligned} \Upsilon' =& \Upsilon_A + \Upsilon_B - 2\Upsilon_A\Upsilon_B \\ &+ 2\sqrt{\Upsilon_A(1-\Upsilon_B)\Upsilon_B(1-\Upsilon_A)}\cos(\psi_A - \psi_B) \end{aligned}$$

Addition of mode mismatches

- Assume two consecutive mode mismatches:



$$\begin{bmatrix} \hat{a}_{00}^{\text{out}} \\ \hat{a}_{\text{mm}}^{\text{out}} \end{bmatrix} = \boldsymbol{U}(\Upsilon_B, \psi_{mmB})\boldsymbol{U}(\Upsilon_A, \psi_{mmA}) \begin{bmatrix} \hat{a}_{00}^{\text{in}} \\ \hat{a}_{\text{mm}}^{\text{in}} \end{bmatrix}$$

- Equivalent ?

$$\begin{bmatrix} \hat{a}_{00}^{\text{out}} \\ \hat{a}_{\text{mm}}^{\text{out}} \end{bmatrix} = \boldsymbol{U}(\Upsilon', \psi'_{mm}) \begin{bmatrix} \hat{a}_{00}^{\text{in}} \\ \hat{a}_{\text{mm}}^{\text{in}} \end{bmatrix}$$

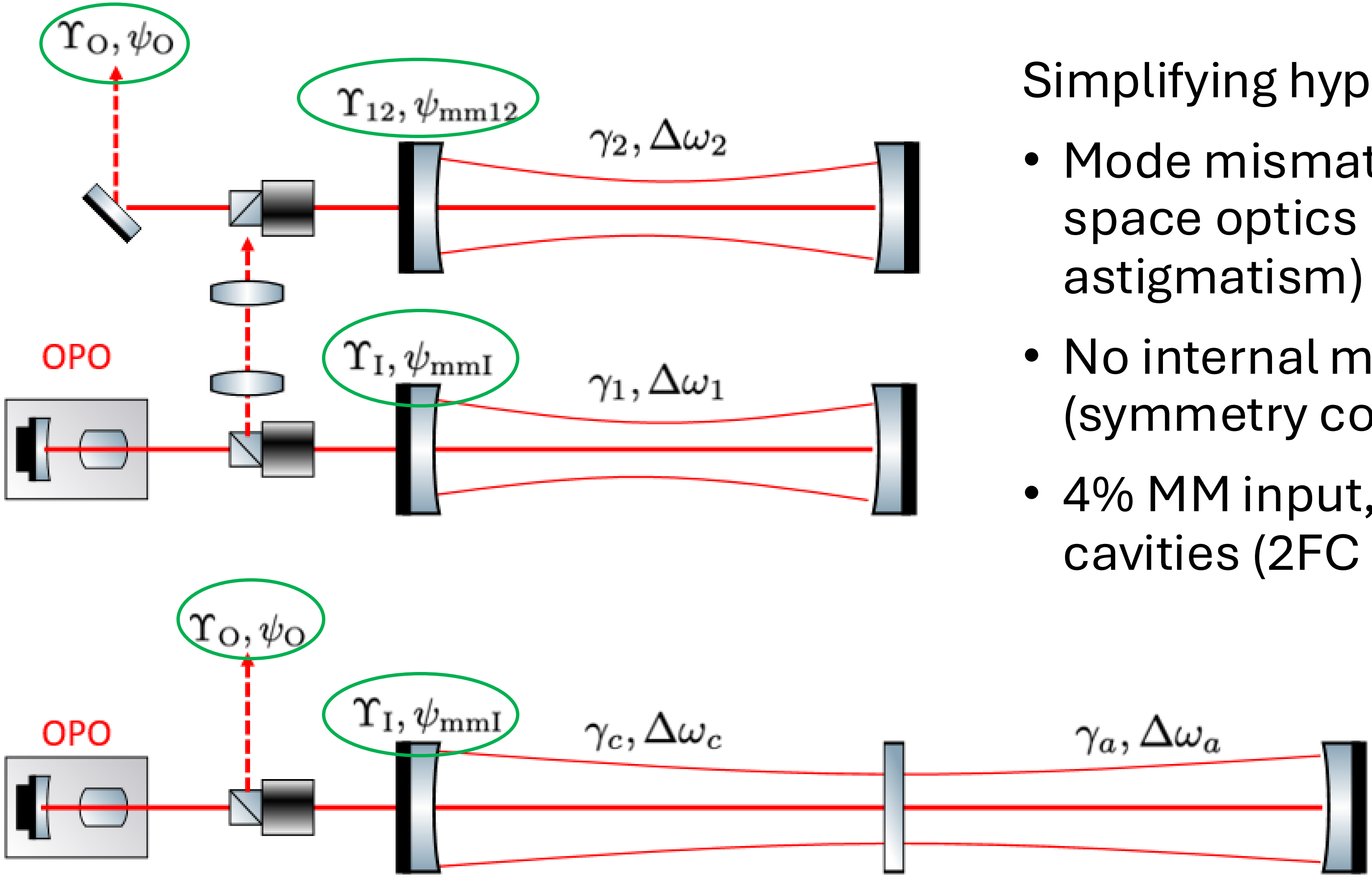
- Not simply a sum of mismatches

$$\Upsilon' = \Upsilon_A + \Upsilon_B - 2\Upsilon_A\Upsilon_B + 2\sqrt{\Upsilon_A(1-\Upsilon_B)\Upsilon_B(1-\Upsilon_A)}\cos(\psi_A - \psi_B)$$

- Example: $1\% + 3\% \in [0.5\%\,, 7\%]$

Cascading coherent mismatches easily destroys squeezing

Comparing mode-mismatch

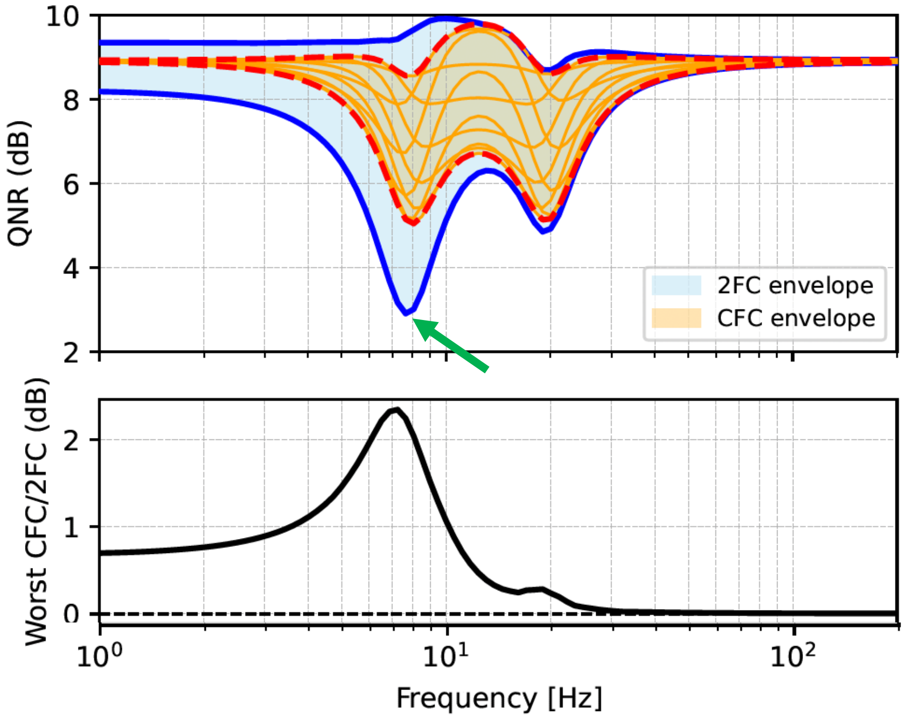


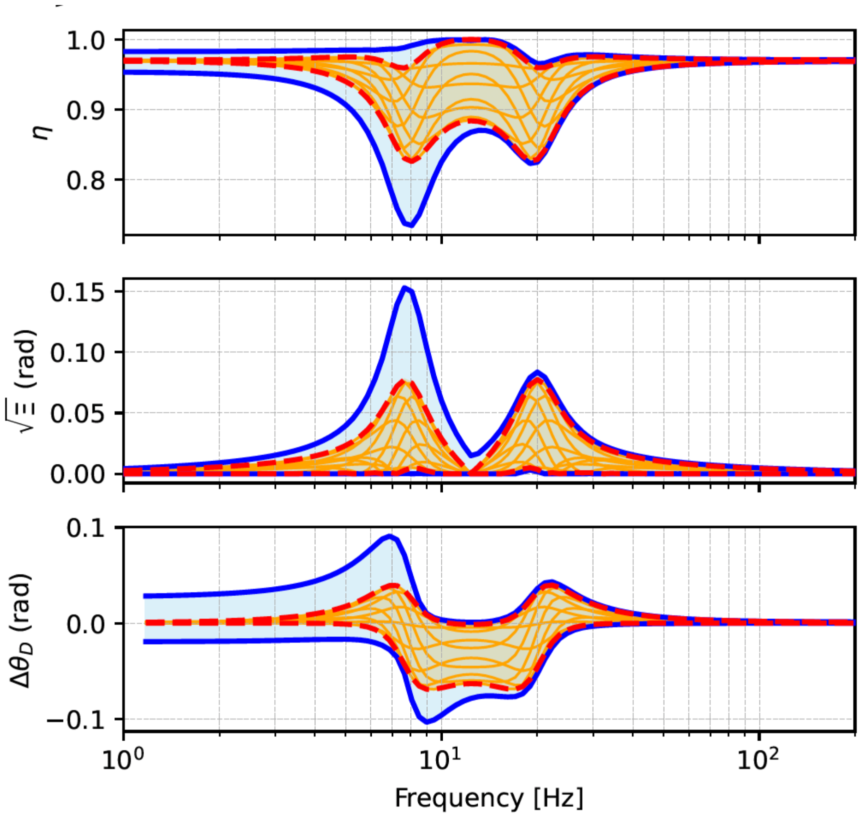
Simplifying hypotheses:

- Mode mismatch coming from free space optics (clipping, uncompensated astigmatism)
- No internal mode mismatch in CFC (symmetry considerations)
- 4% MM input, 3% output, 1% between cavities (2FC only)

Comparing mode-mismatch

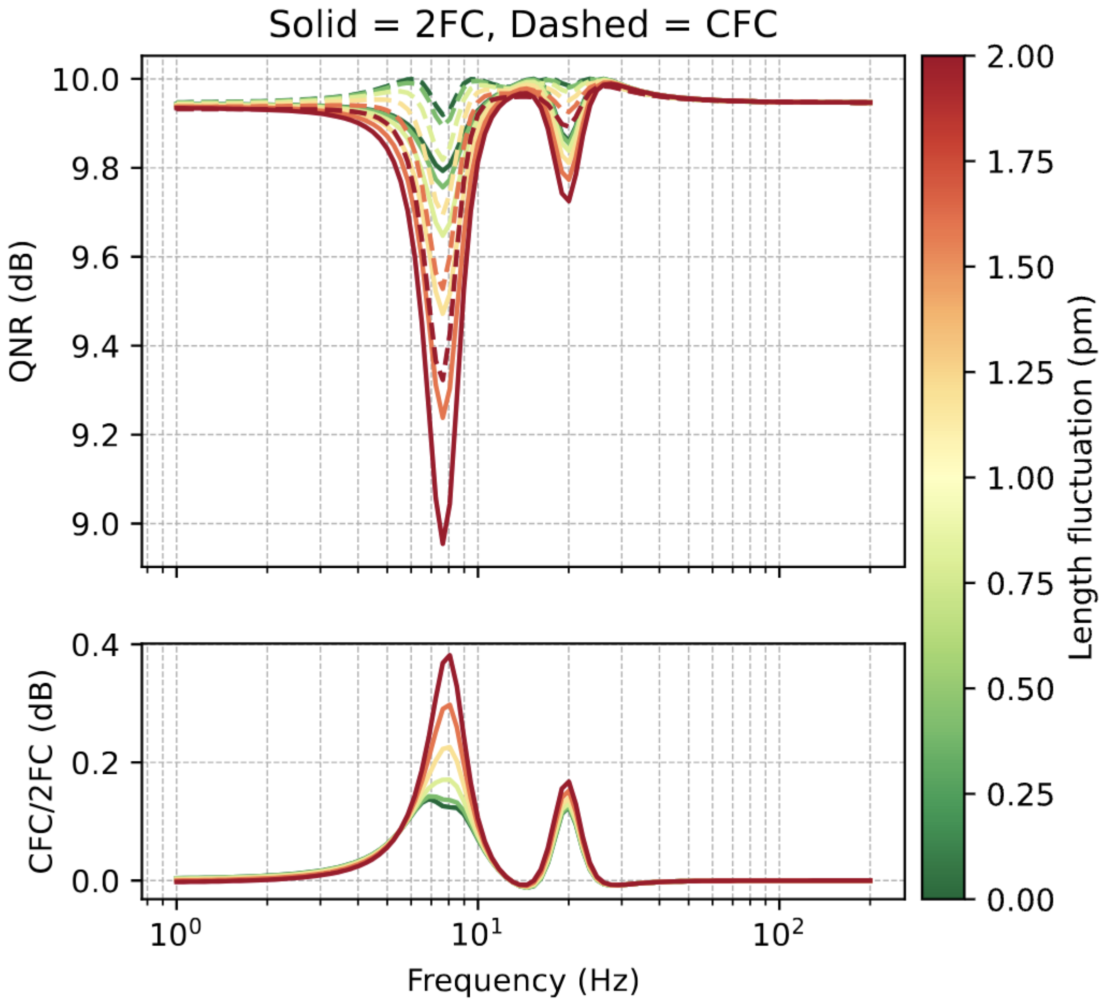
- 4% MM input, 3% output, 1% between cavities (2FC only)





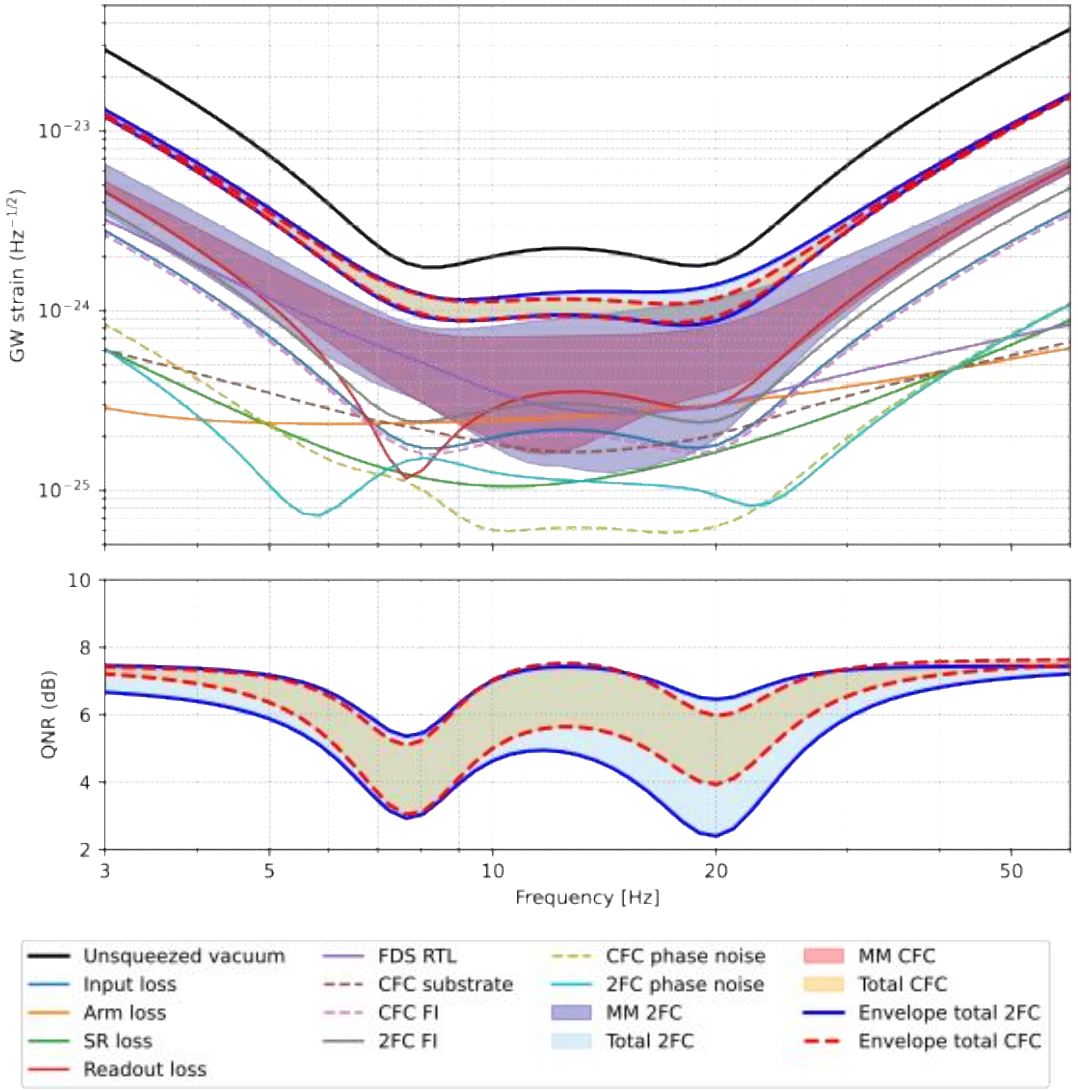
- Significant dephasing at resonance frequencies
- **1% extra MM ⇒ 2 dB lost**
- CFC better on this set of MM params but hard to generalize (how to measure intra-cavity mismatch?)

Comparing length noise



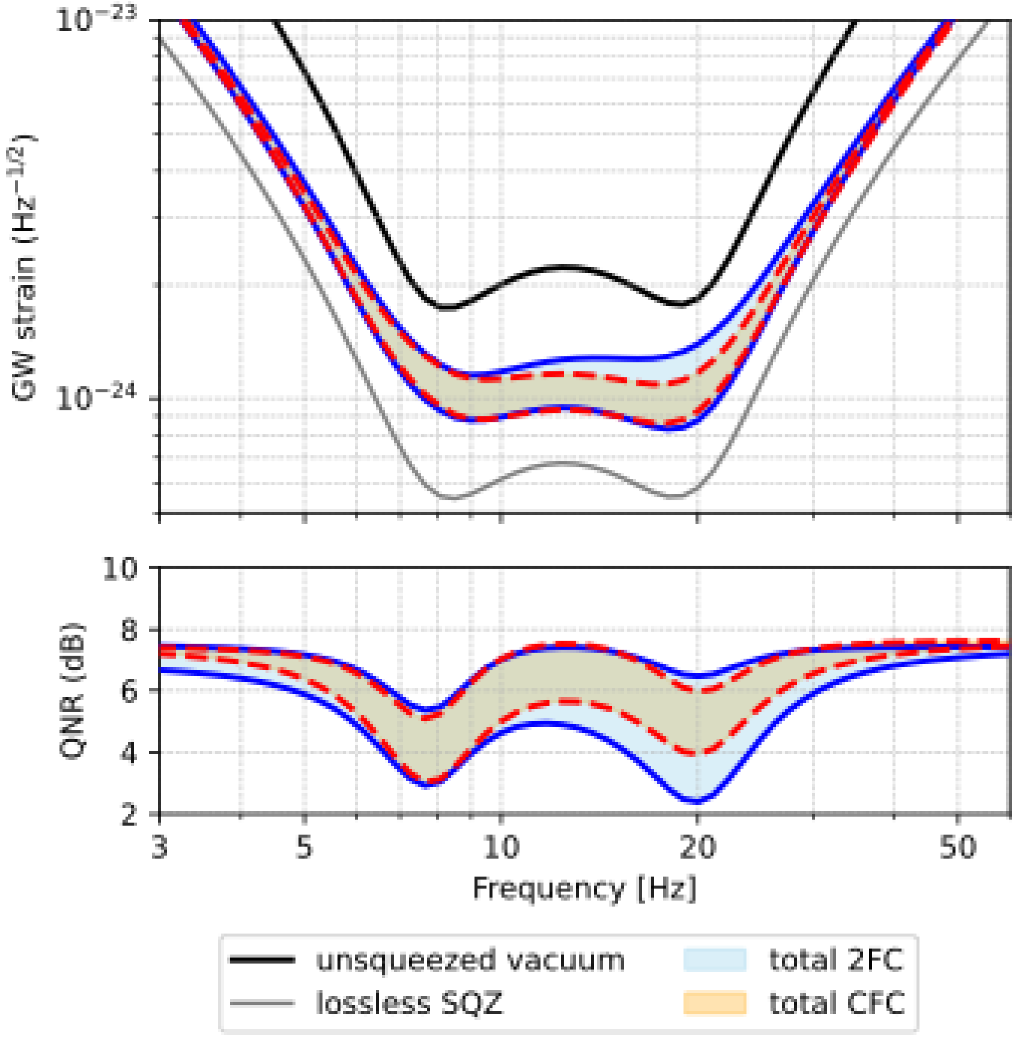
- Cavity length fluctuations due to imperfect controls
- Realistic ~pm for single filter cavities (Virgo/LIGO)
- **CFC somewhat better**
- But actual control of coupled cavity to be further investigated.

Full budget on FDS



- Add all loss sources in the FDS system + lossy interferometer
- Add extra Faraday isolator losses to 2FC

Full budget on FDS

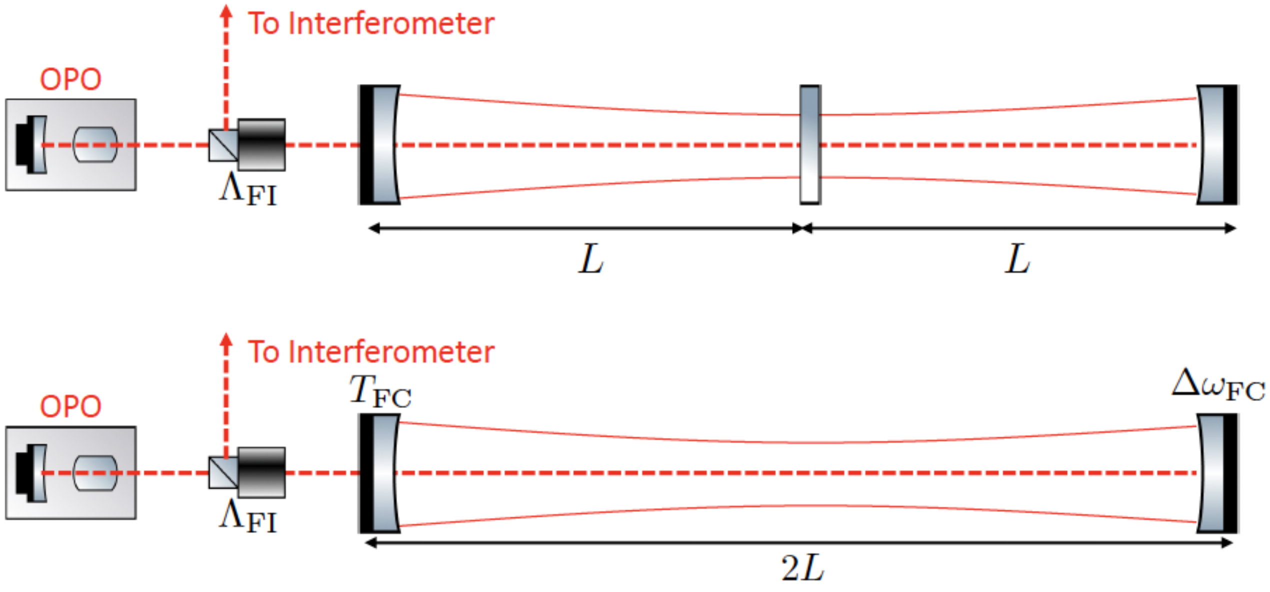


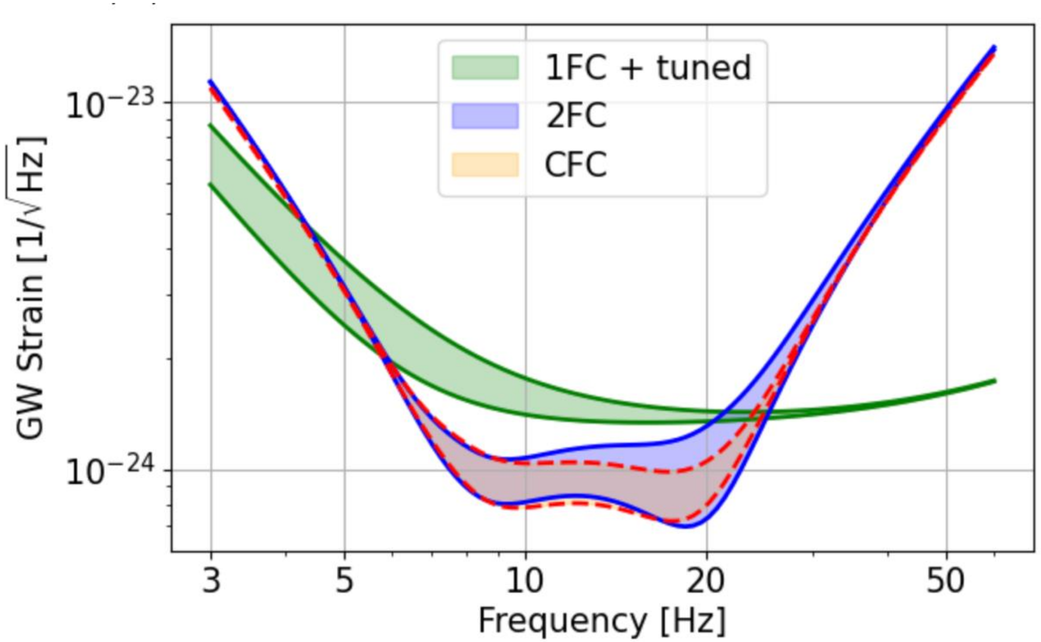
- Add all loss sources in the FDS system + lossy interferometer
- Add extra Faraday isolator losses to 2FC

- Degradation phenomenon dominated by mode matching

- **CFC more robust to degradation**
- **CFC performs overall marginally better than 2FC**

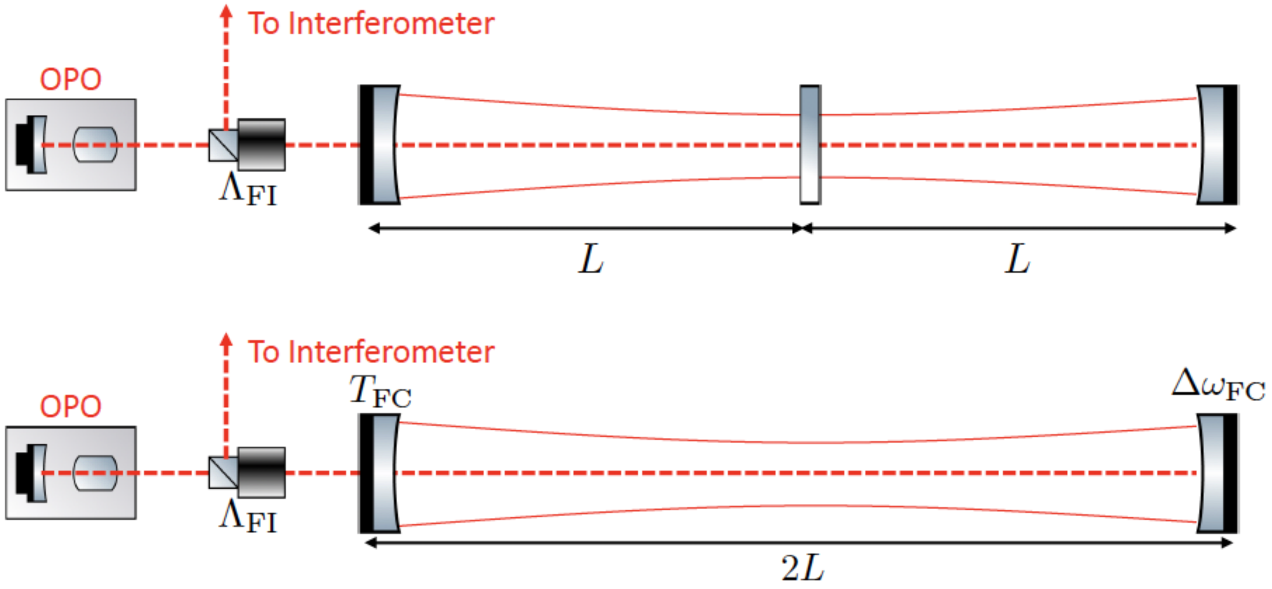
Path to CFC: tuned ET with single FC

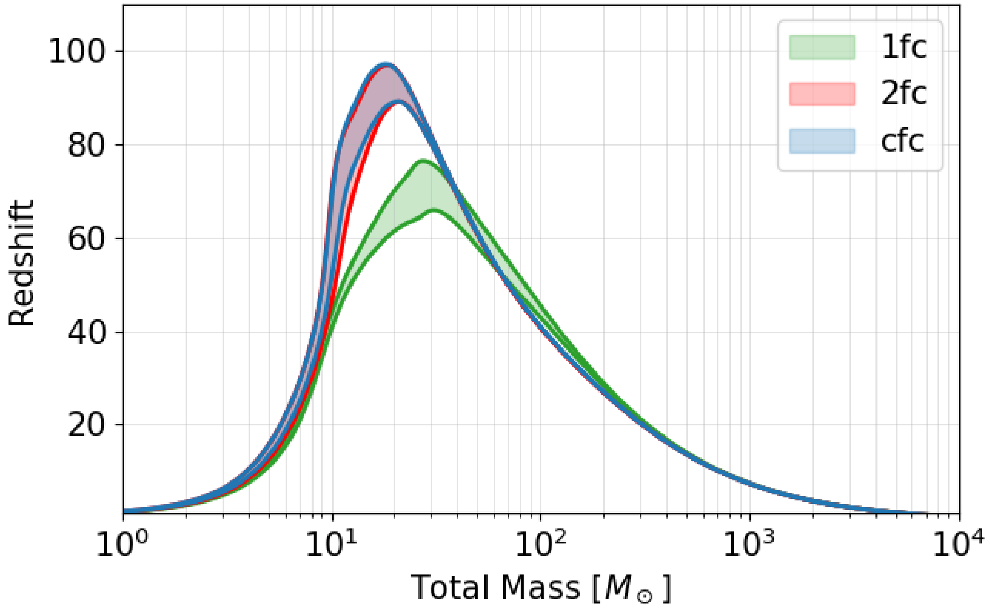




Parameter	Physical meaning	Value
T_{FC}	FC input mirror transmissivity	0.37 %
$\Delta\omega_{FC}^{10km}$	FC detuning	4.20 Hz
r_{tuned}	Injected squeezing	12 dB
T_{SRM}	SR mirror transmissivity	44 %

Path to CFC: tuned ET with single FC



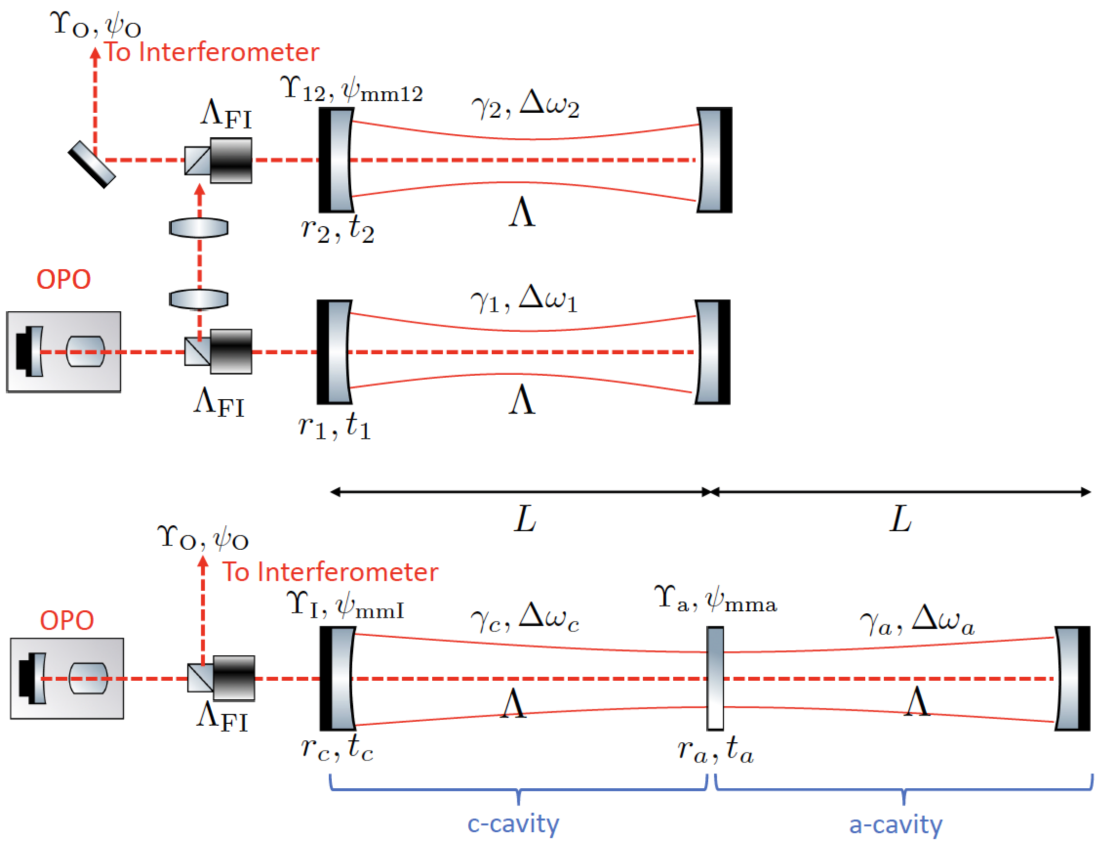


Parameter	Physical meaning	Value
T_{FC}	FC input mirror transmissivity	0.37 %
$\Delta\omega_{FC}^{10km}$	FC detuning	4.20 Hz
r_{tuned}	Injected squeezing	12 dB
T_{SRM}	SR mirror transmissivity	44 %

Current LIGO redshift ~1

{1FC (10km) + tuned ET-LF} attractive for new science and a priori easier to commission

Takeaways



- Theoretical equivalence between 2FC and CFC also holds when losses are considered
- Constraint on middle mirror transmission can be attained if cavities are ~5 km long each
- Non-linear addition of mode mismatches
- CFC performs similarly to 2FC
- Viable path to CFC through 1FC+Tuned ET-LF
- Controls of CFC need to be further investigated

Squeezing degradation

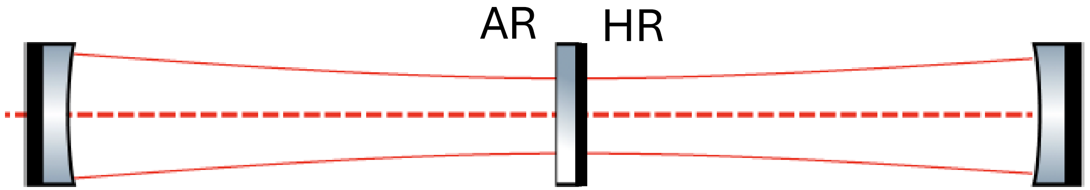
Physical Review D 104, 062006 (2021)
Physical Review D 90, 062006 (2014)
(...)

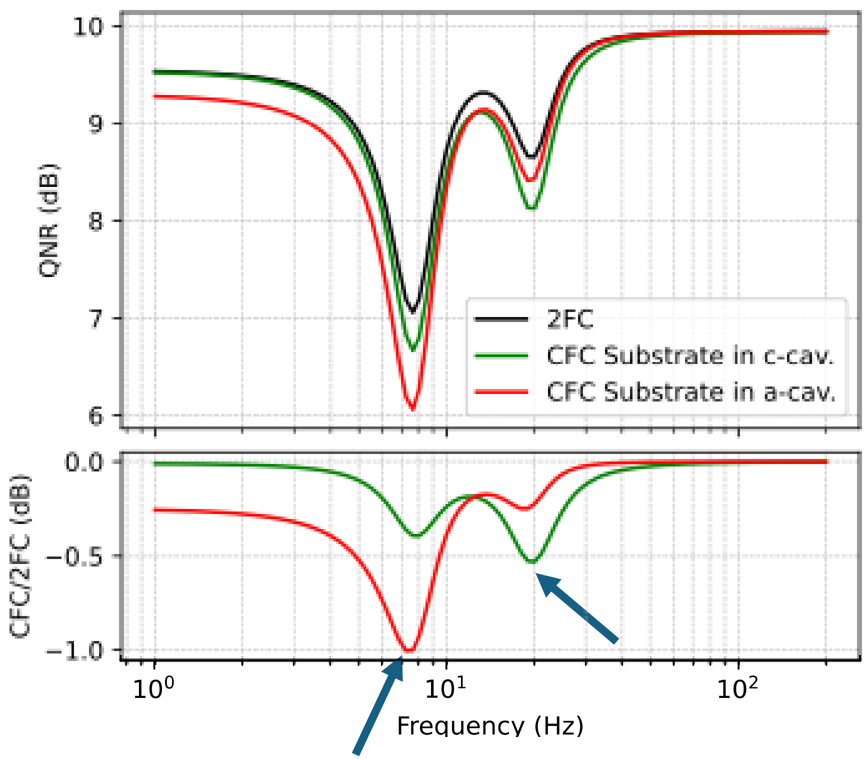
- Squeezing degradation sources:
  - **Loss**: coupling to vacuum
  - **Mode mismatch**: possible coupling between squeezing and antisqueezing
  - **Phase noise**: Technical, also couples squeezing to antisqueezing
- Figures of merit: $\bar{S} = e^{-2r}$
  - Efficiency: $\bar{S} = \eta e^{-2r} + 1 - \eta$
  - Dephasing: $\bar{S} = (1 - \Xi)e^{-2r} + \Xi\, e^{2r}$
  - Misphasing: $\bar{S} = e^{-2r} \cos^2 \Delta\theta_D + e^{2r} \sin^2 \Delta\theta_D$

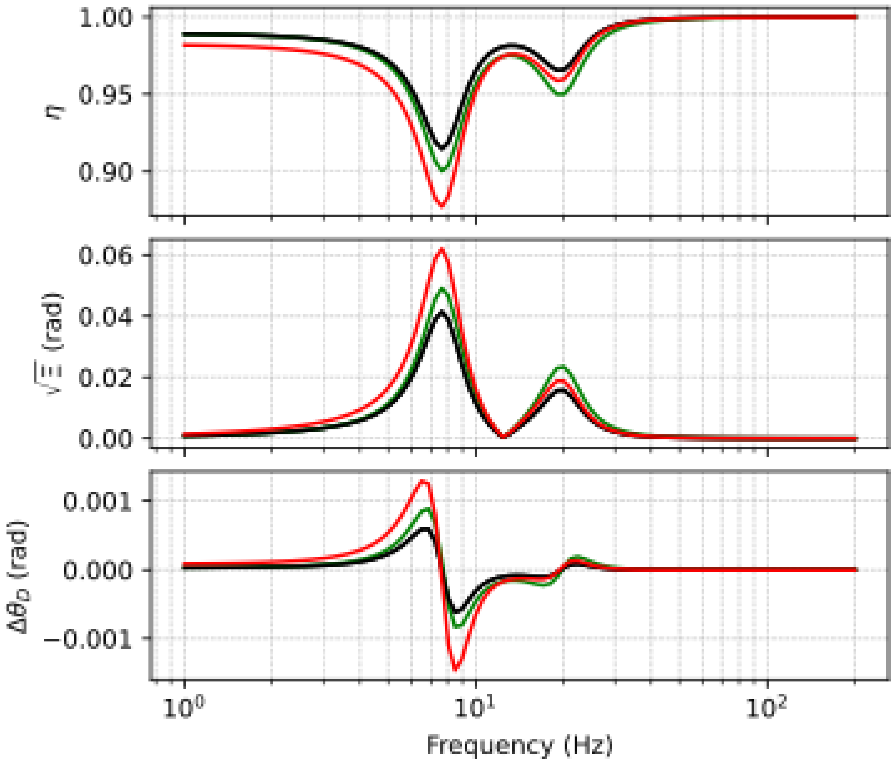
$$\bar{S}[\Omega] = \eta[\Omega]\{[(1-\Xi[\Omega])e^{-2r} + \Xi[\Omega]e^{2r}] \cos^2(\Delta\theta_D[\Omega]) + [(1-\Xi[\Omega])e^{2r} + \Xi[\Omega]e^{-2r}] \sin^2(\Delta\theta_D[\Omega])\} + 1 - \eta[\Omega]$$

Comparing Round Trip Loss

- Adding substrate loss from middle mirror
  (~1 ppm/cm, 5 cm thick mirror)







- Planar middle mirror (no wedge)
- Serves as an etalon for fine transmission tuning
- No substantial extra degradation due to substrate loss

Jacques Ding (ET-0156B-25, arXiv:2506.02222)