

Credit: NASA/Swift Dana Berry



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UNIVERSITÉ
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NORMANDIE



IN2P3
Les deux infinis

GW and ultra-dense matter

Francesca Gulminelli – Université de
Caen-Normandie

5th MaNiTou summer school on gravitational waves,
June 29 – July 4, 2025

Lecture plan

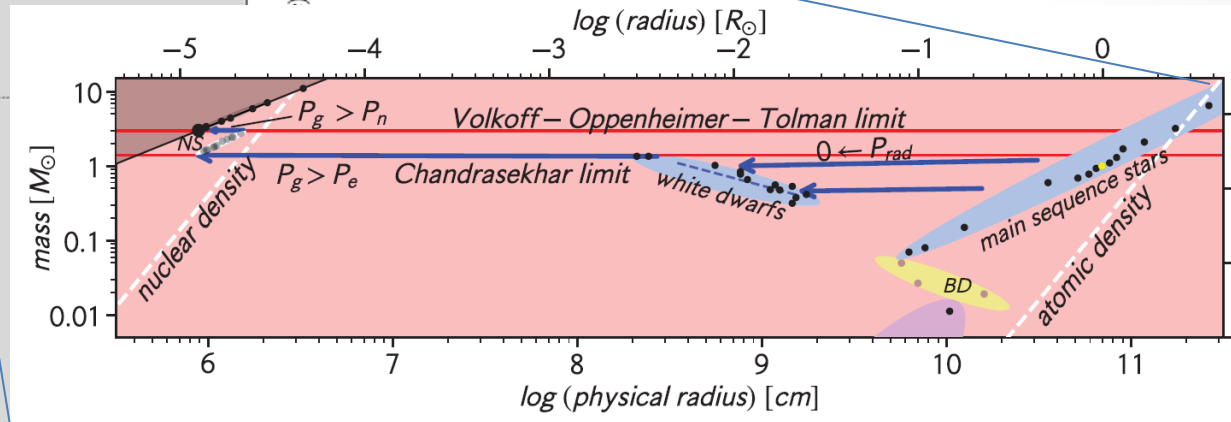
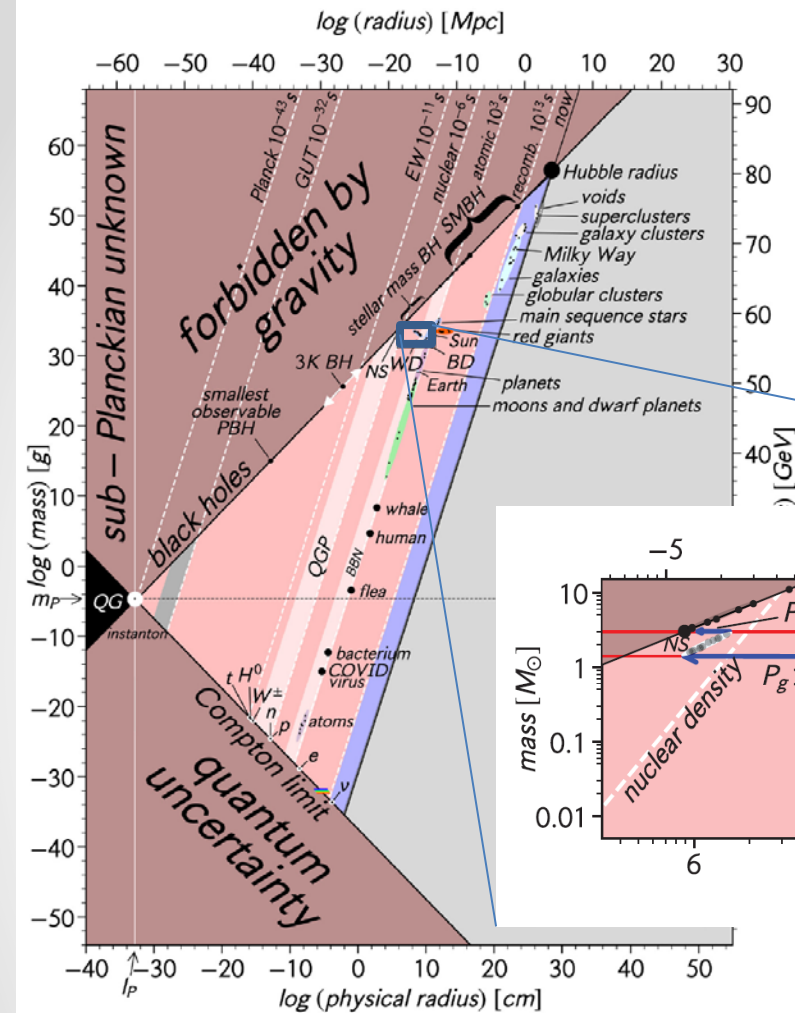
1. Introduction: ultra-dense matter in the universe

- a. The sites
- b. The signals
- c. The big questions

2. The gravitational wave probe for dense matter

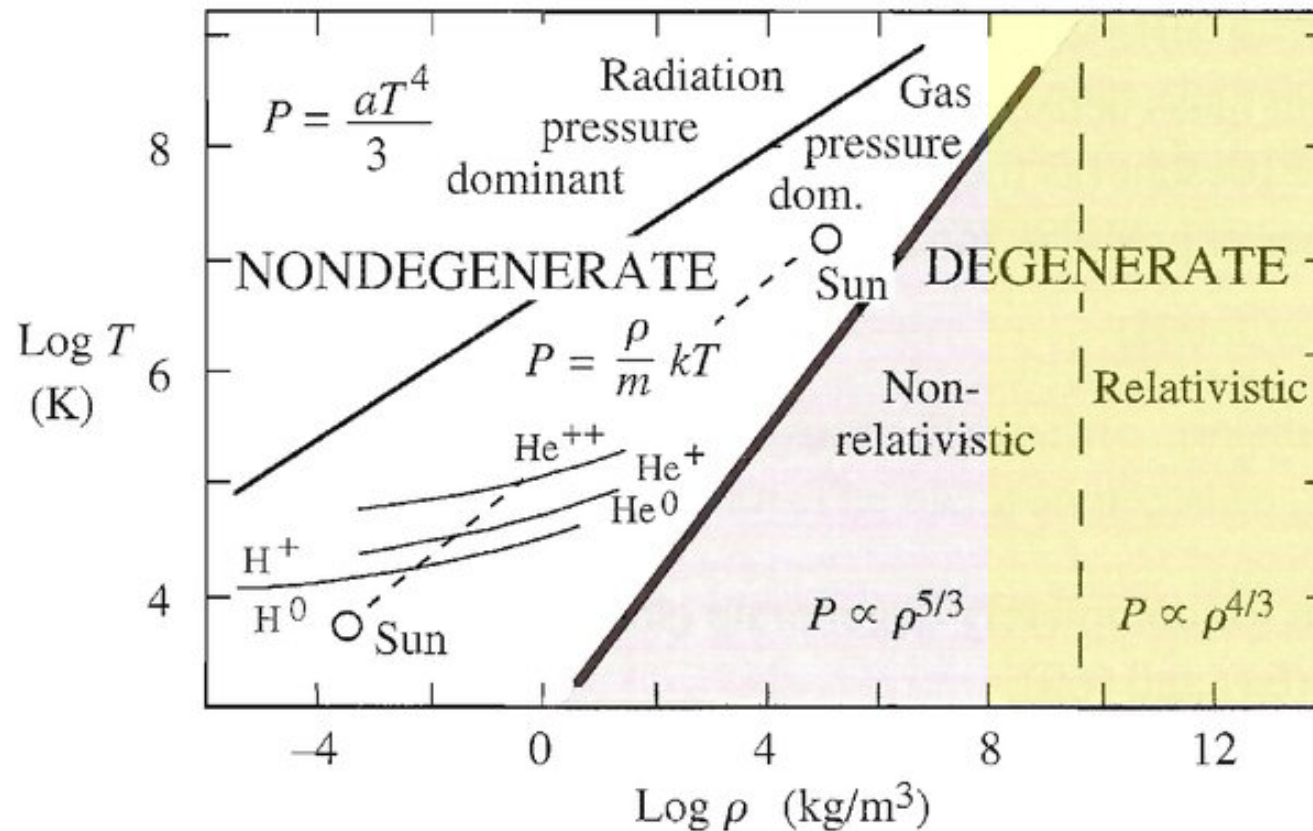
- a. Neutron star modelling and the equation of state (EoS)
- b. Observations
- c. EoS and observables: constraining the parameters

Ultra-dense matter



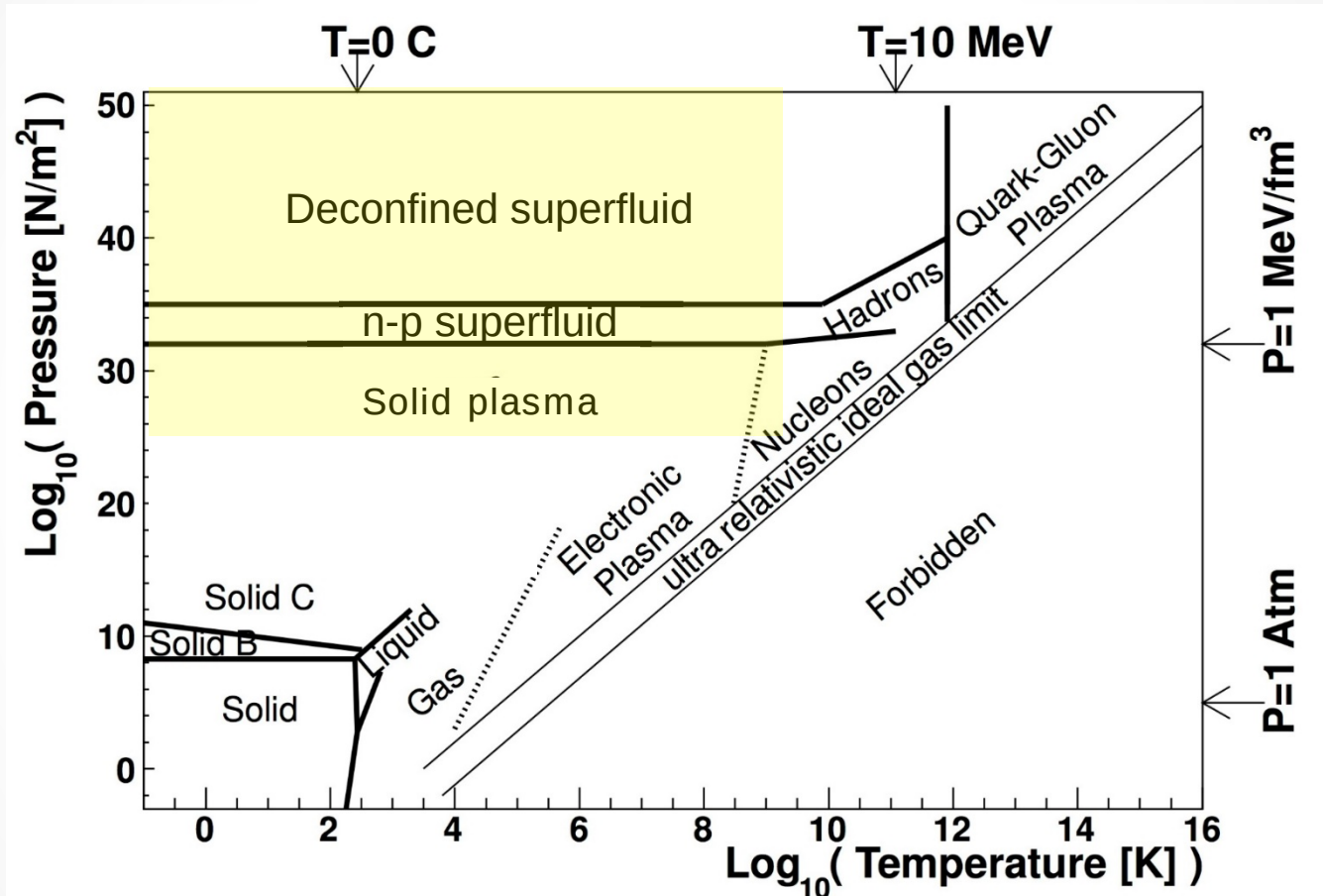
C.H.Lineweaver, V.M.Patel,
Am.J.Phys.91 (2023) 819

Ultra-dense matter



Degenerate and relativistic at all temperatures

Ultra-dense matter

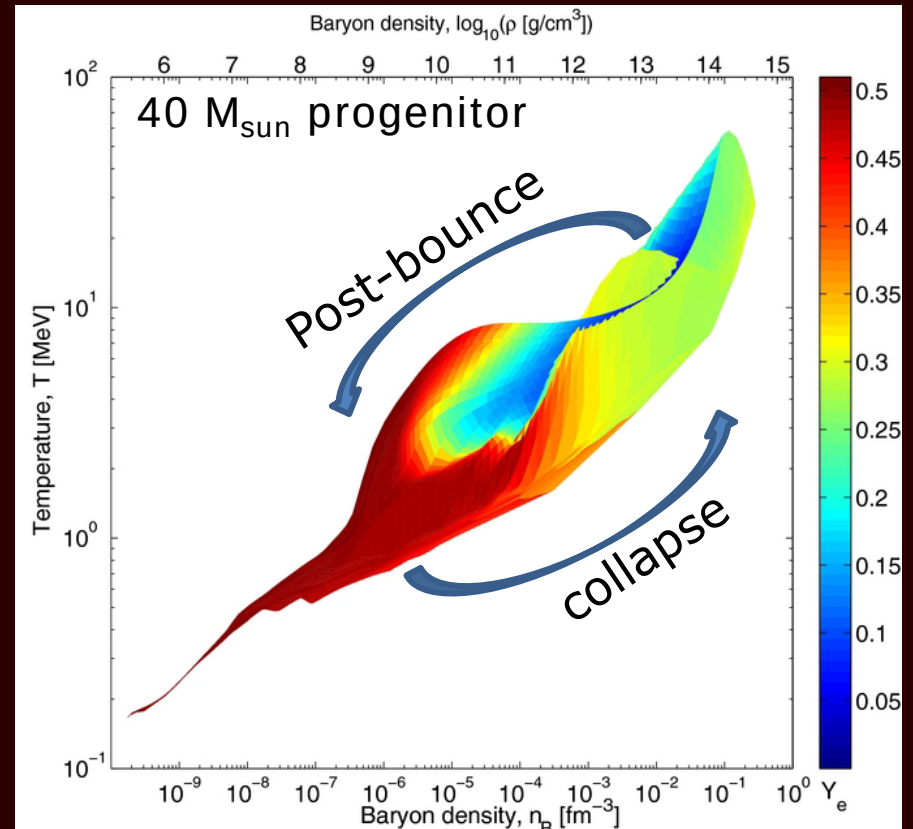
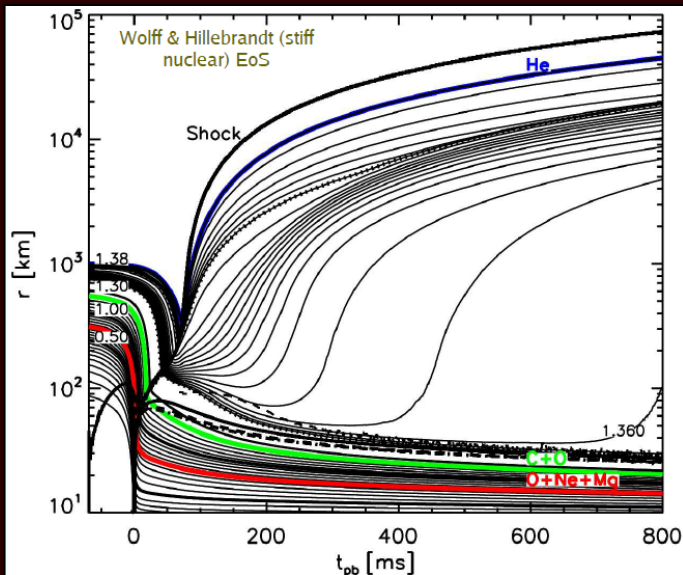


Exotic constituents, exotic phases

Supernova remnant
in Puppis A
MIPS+XMM
IR Credit: NASA)

Dense matter in the Universe: CCSN

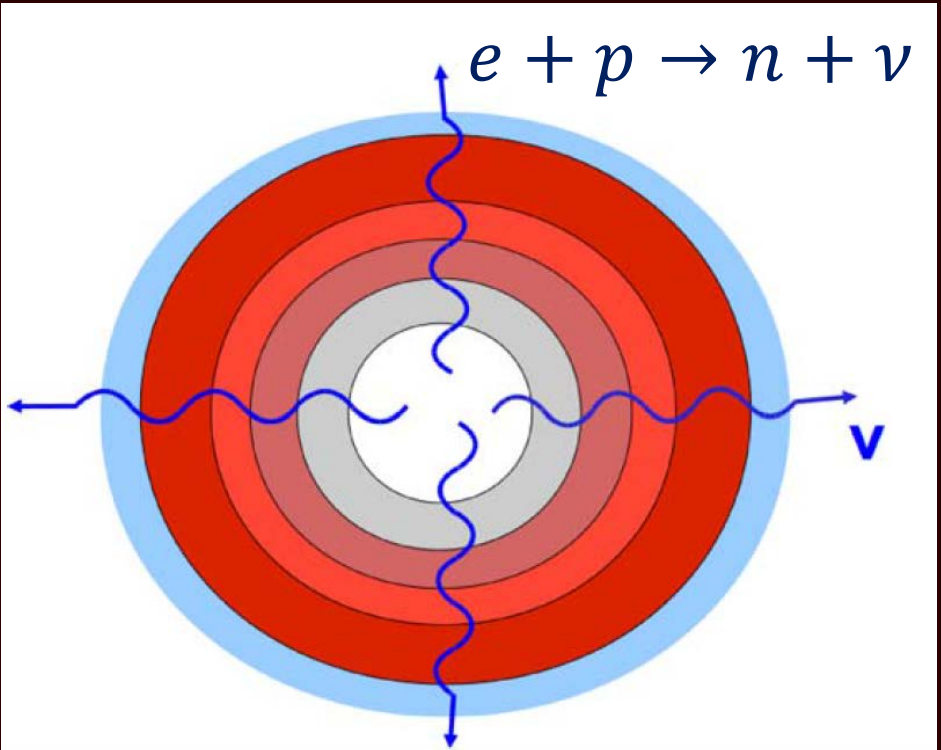
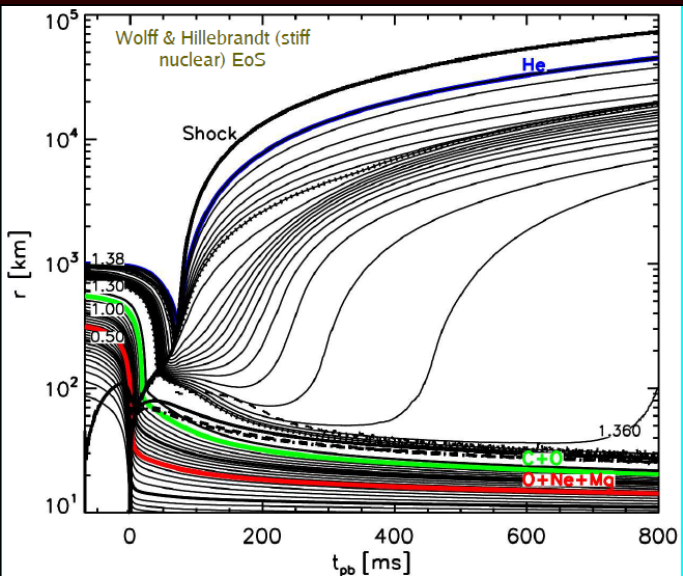
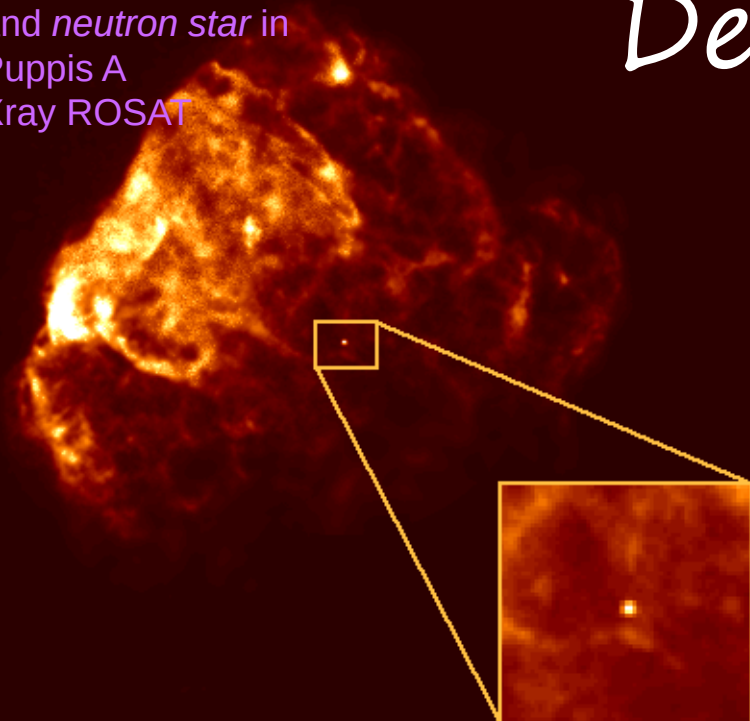
F.S.Kitaura et al, A&A 450 (06) 345



T.Fischer et al, 2011 ApJS 194 39

Supernova remnant
and neutron star in
Puppis A
Xray ROSAT

Dense matter in the Universe: PNS



Supernova remnant
and neutron star in
Puppis A
Xray ROSAT

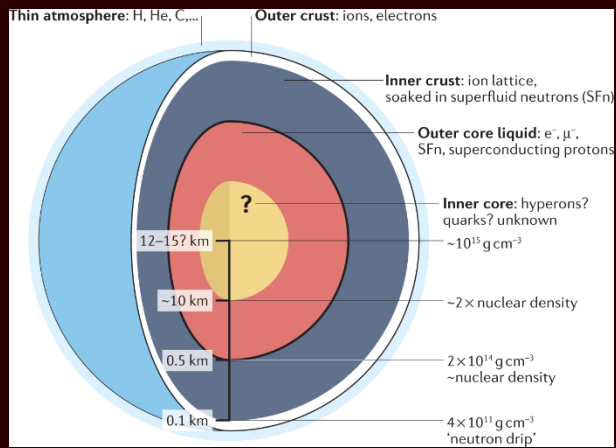
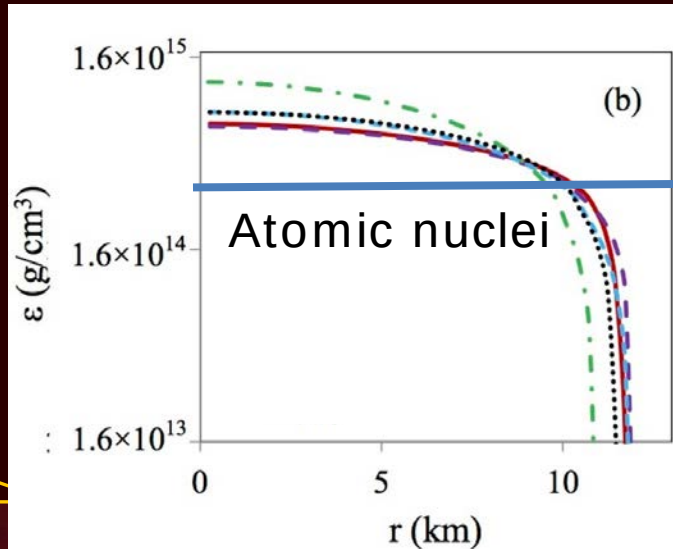
Dense matter in the Universe: NS

$y_e \cong 1/2$
 $T \sim 10^{12} K$

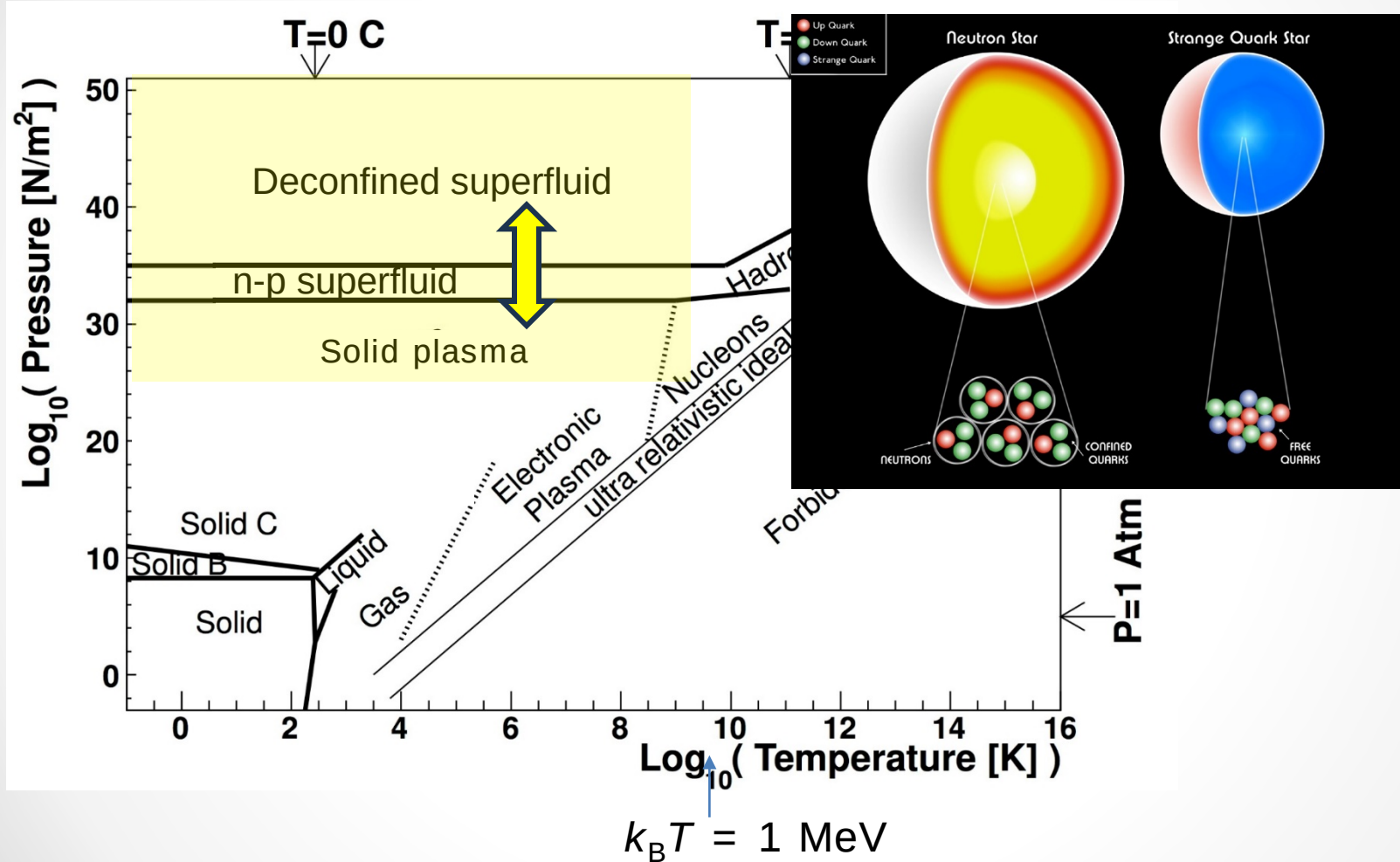
$y_e \cong 1/3$
 $T \sim 10^{11} K$



$y_e \cong 1/5$
 $T \sim 10^6 K$



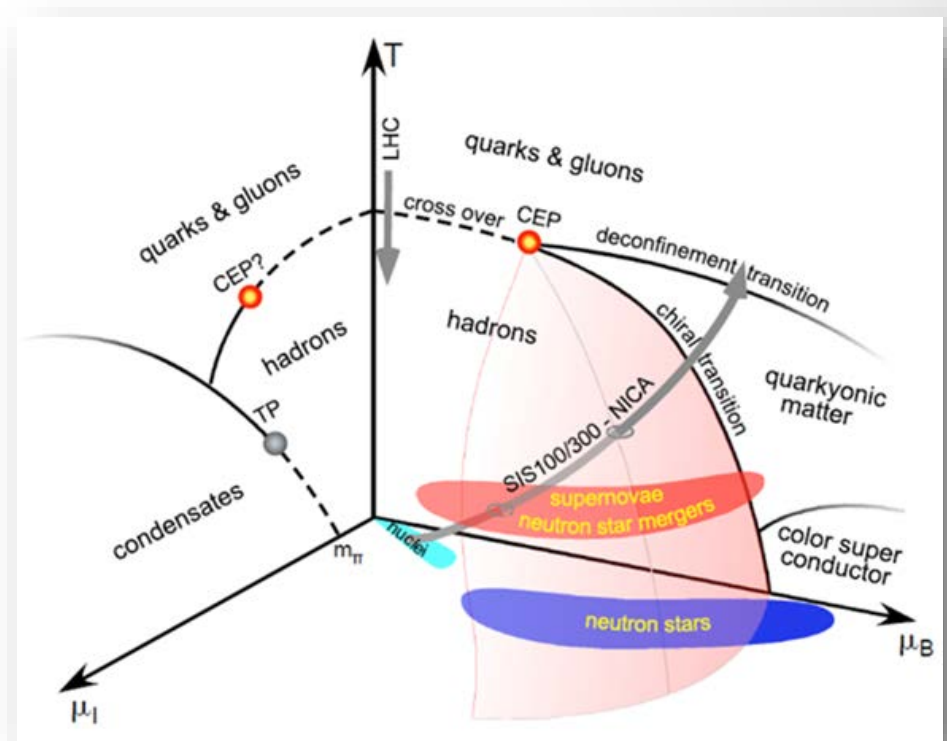
The phases of matter



Credits: CXC/M. Weiss

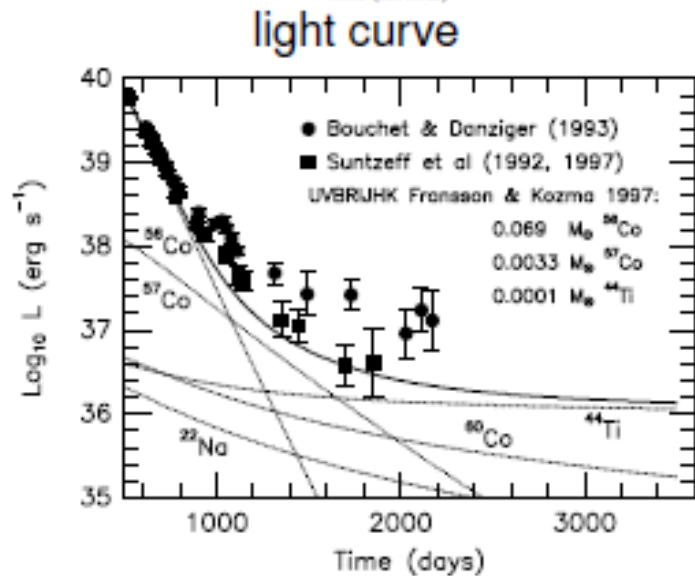
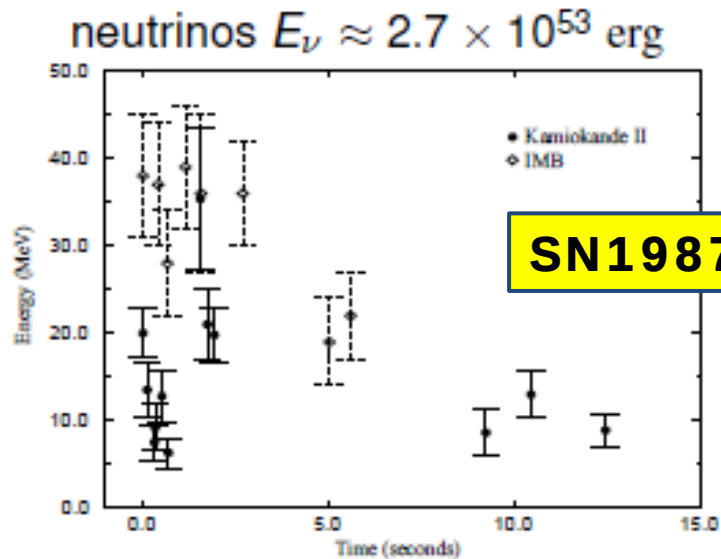
Dense matter: the Questions

1. What is the internal structure of the dense matter in neutron stars, supernova cores and mergers?
2. How does this structure reflect into the observable signals?
3. What can we learn on the underlying nuclear and hadronic physics?

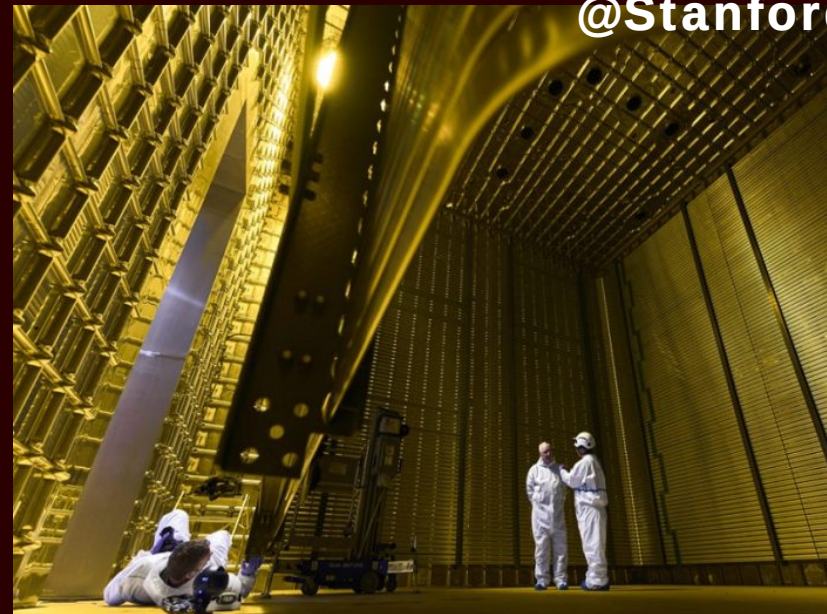


CCSN

Signals



**SuperKamioKande
@Kamioka**



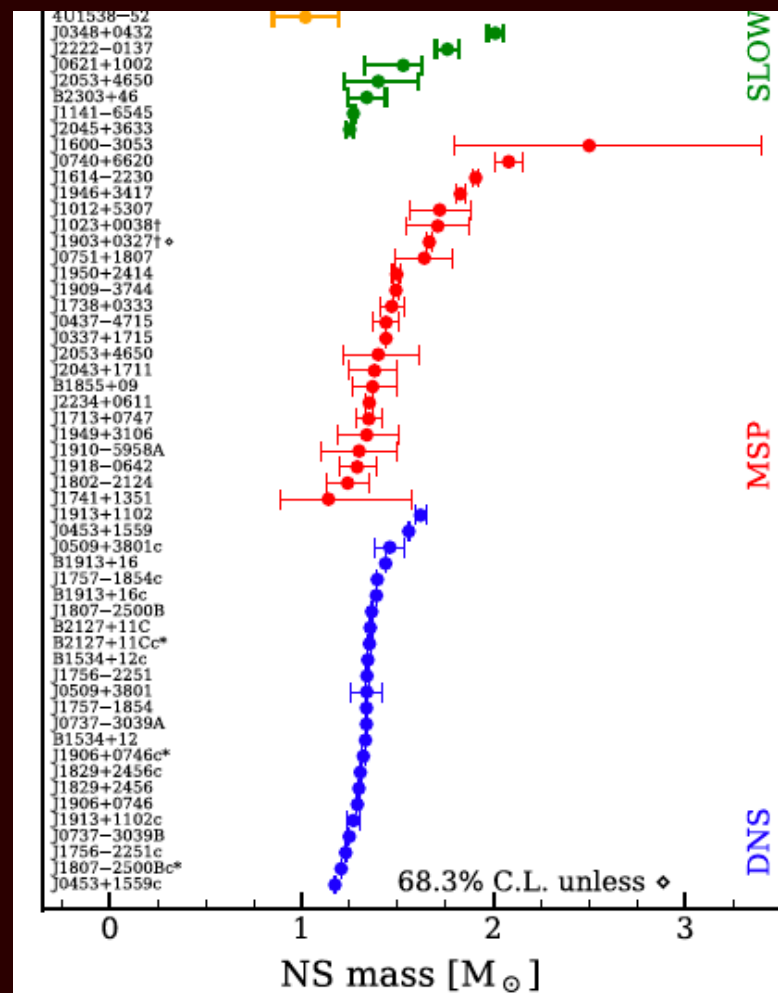
**DUNE
@Stanford**

Radio timing

Signals

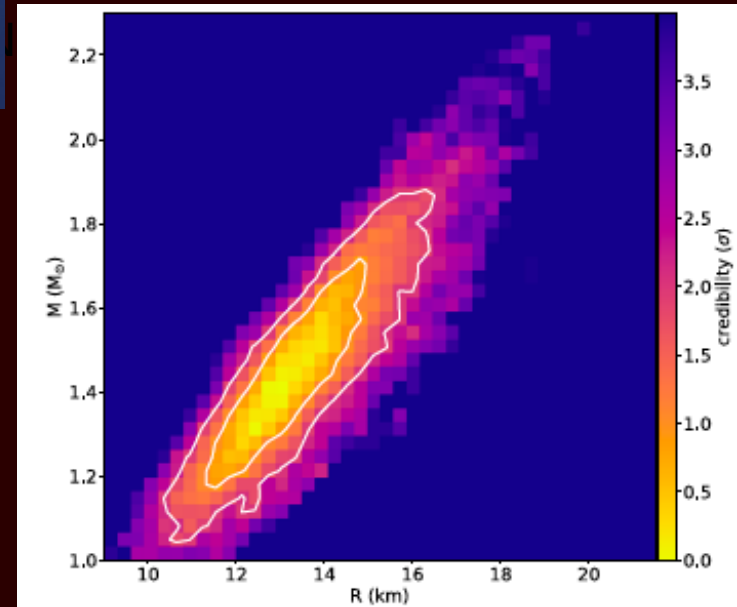
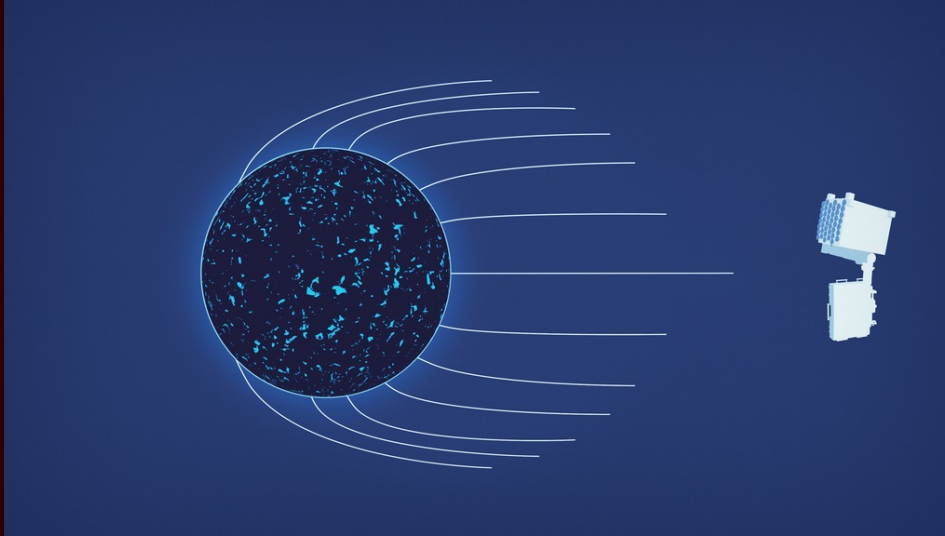


SKA@ South Africa

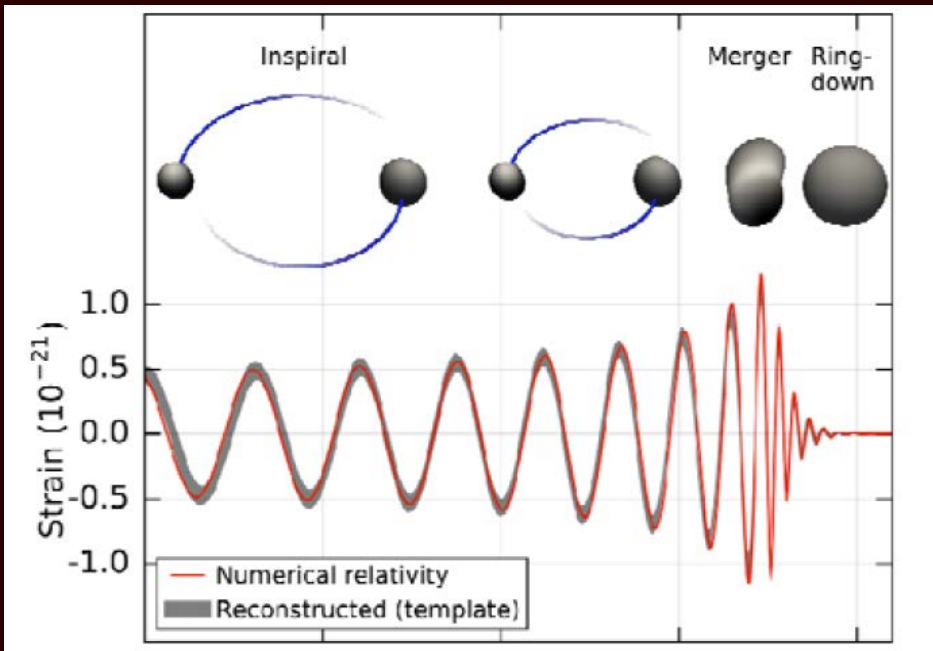


XR sources

Signals

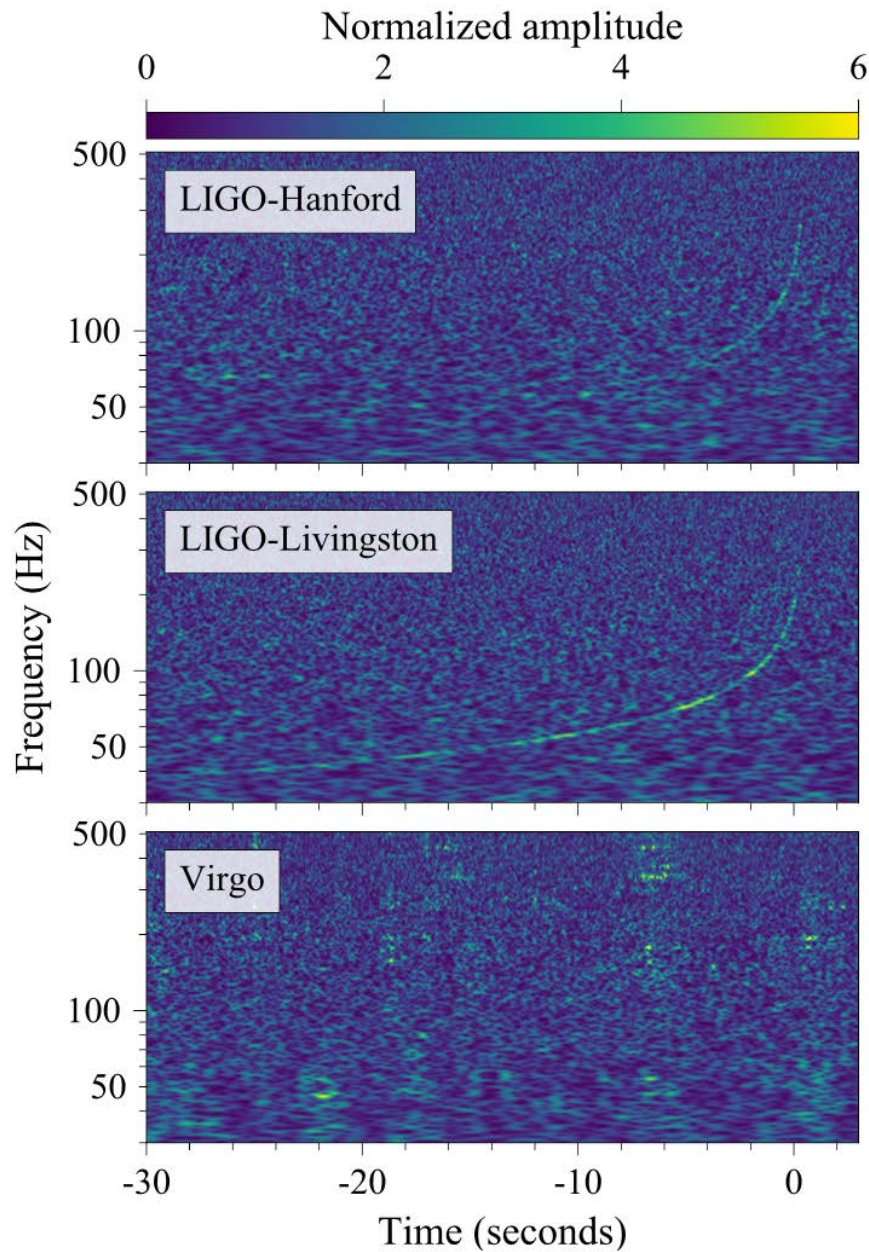


Signals



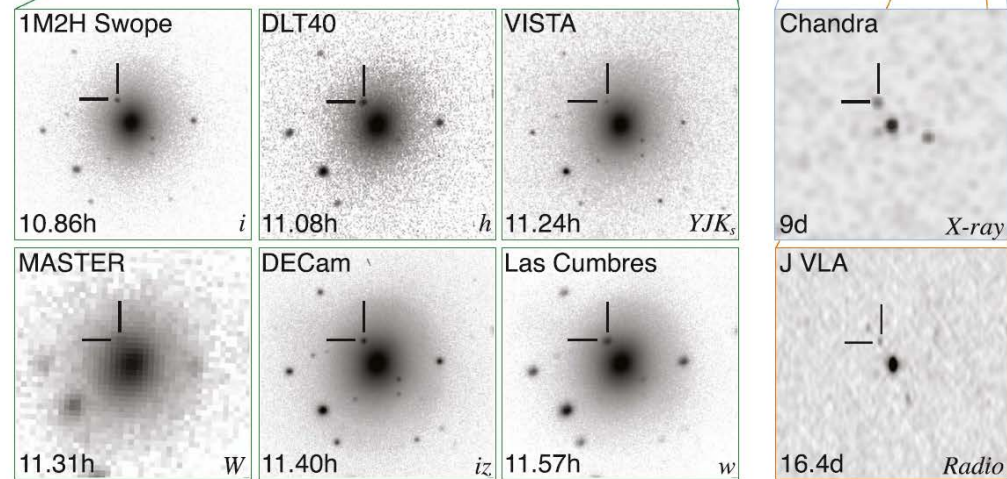
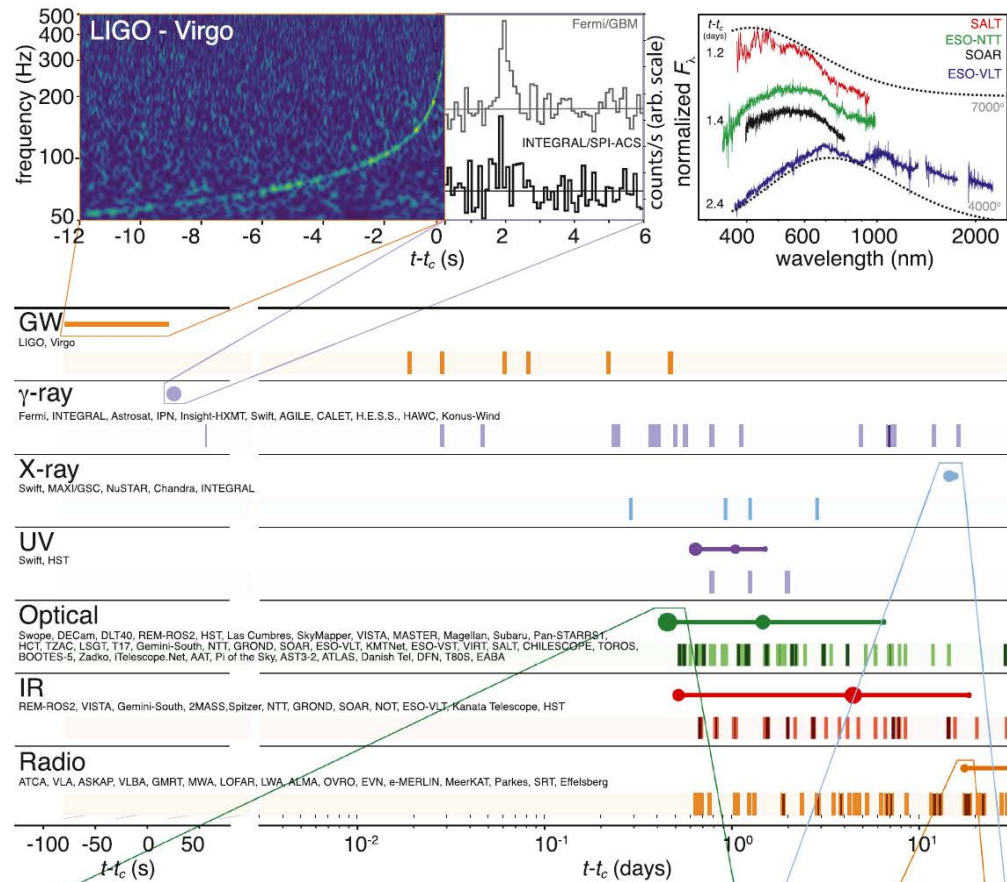
Mergers (CBC)

Signals

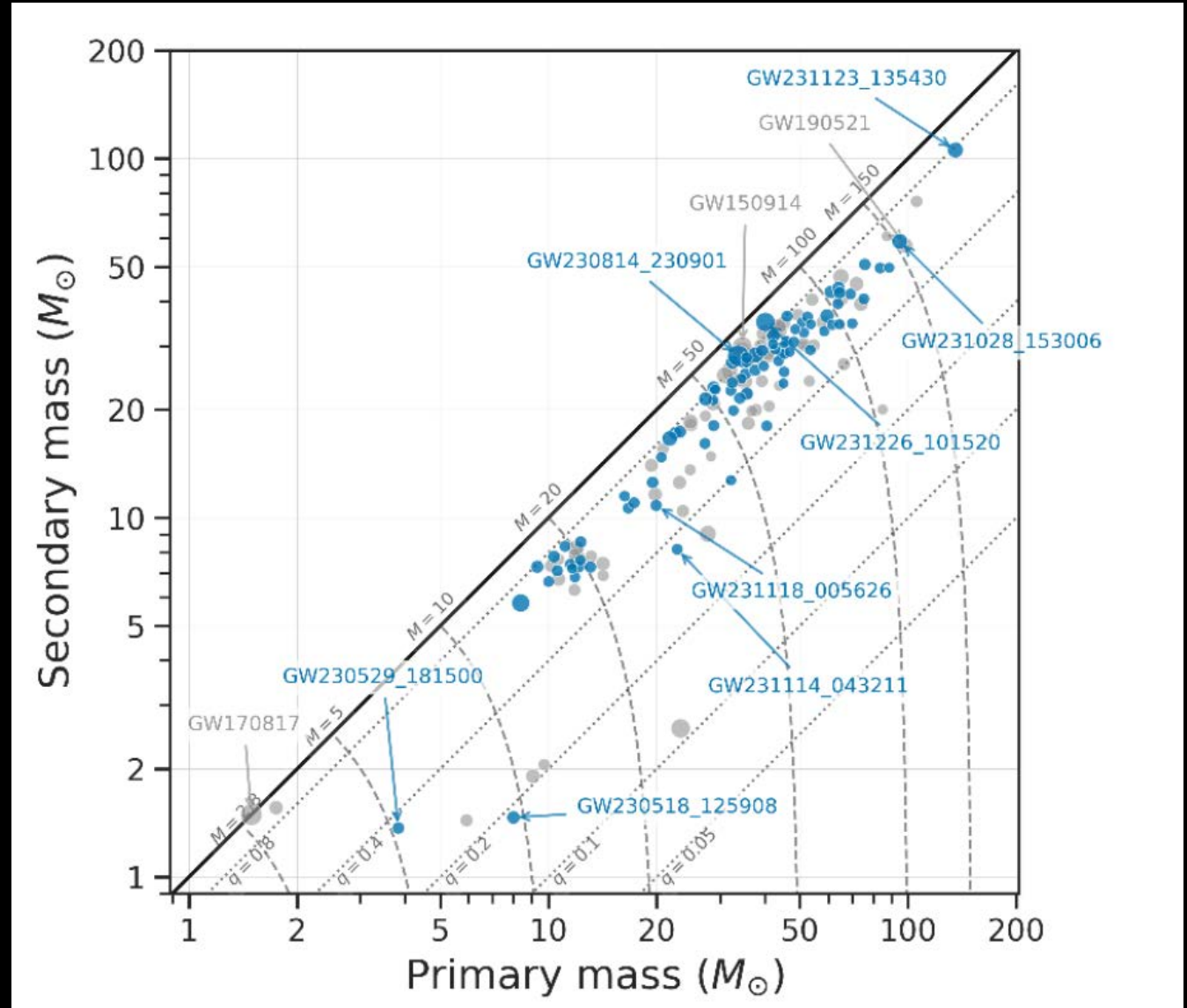


GW170817

Mergers (CBC)



10 years of LVK observations



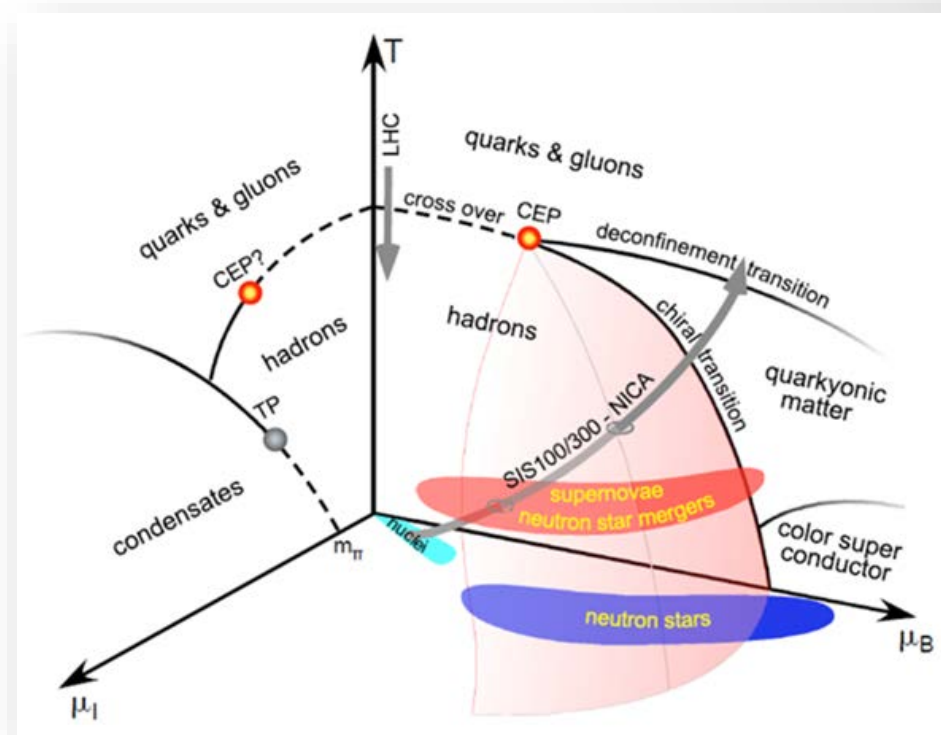
GWTC-4.0 LVK ArXiv:2508.18080

Lecture plan

1. Introduction: ultra-dense matter in the universe
 - a. The sites
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2. **The gravitational wave probe for dense matter**
 - a. **Neutron star modelling and the equation of state (EoS)**
 - b. **Observations**
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The message I would like to convey:

Gravitational waves, and particularly the inspiral GW signal from NS mergers, opens the possibility of a ***DIRECT*** measurement of the QCD phase diagram

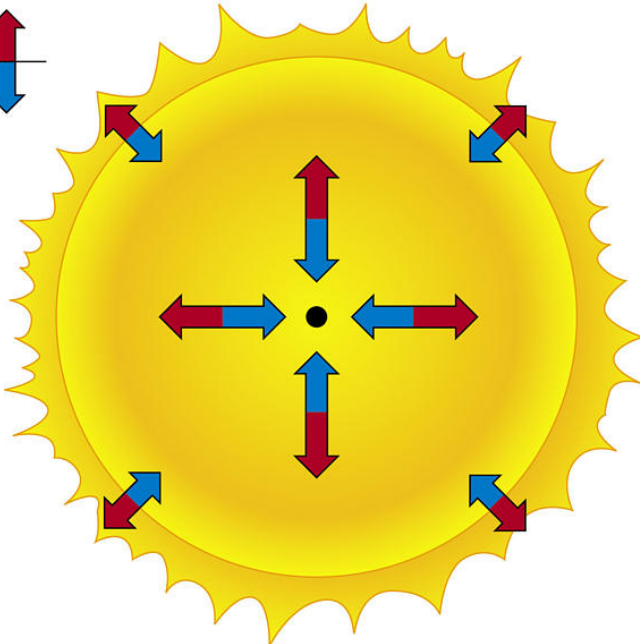


Modelling (Neutron) Stars: hydrostatics

- Self-gravitation => Tolman Oppenheimer Volkoff (1939):

$$\frac{dP(\epsilon)}{dr} = -\frac{G}{r^2} \left[\rho(r) + \frac{P(\epsilon)}{c^2} \right] \left[m(r) + 4\pi r^3 \frac{P(\epsilon)}{c^2} \right] \left[1 - \frac{2Gm(r)}{rc^2} \right]^{-1}$$

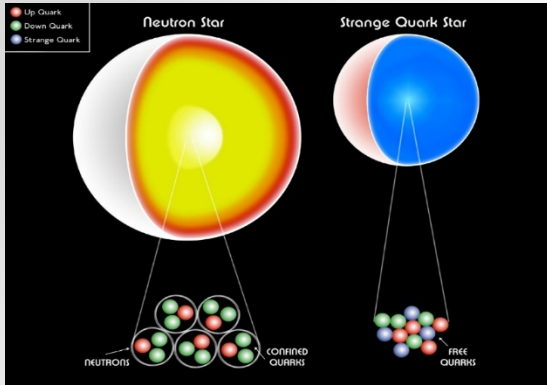
Pressure
out ↑
Gravity
in ↓



$$\begin{aligned} \forall \epsilon_c \\ R=r(P=0) \\ M=m(r=R) \end{aligned}$$

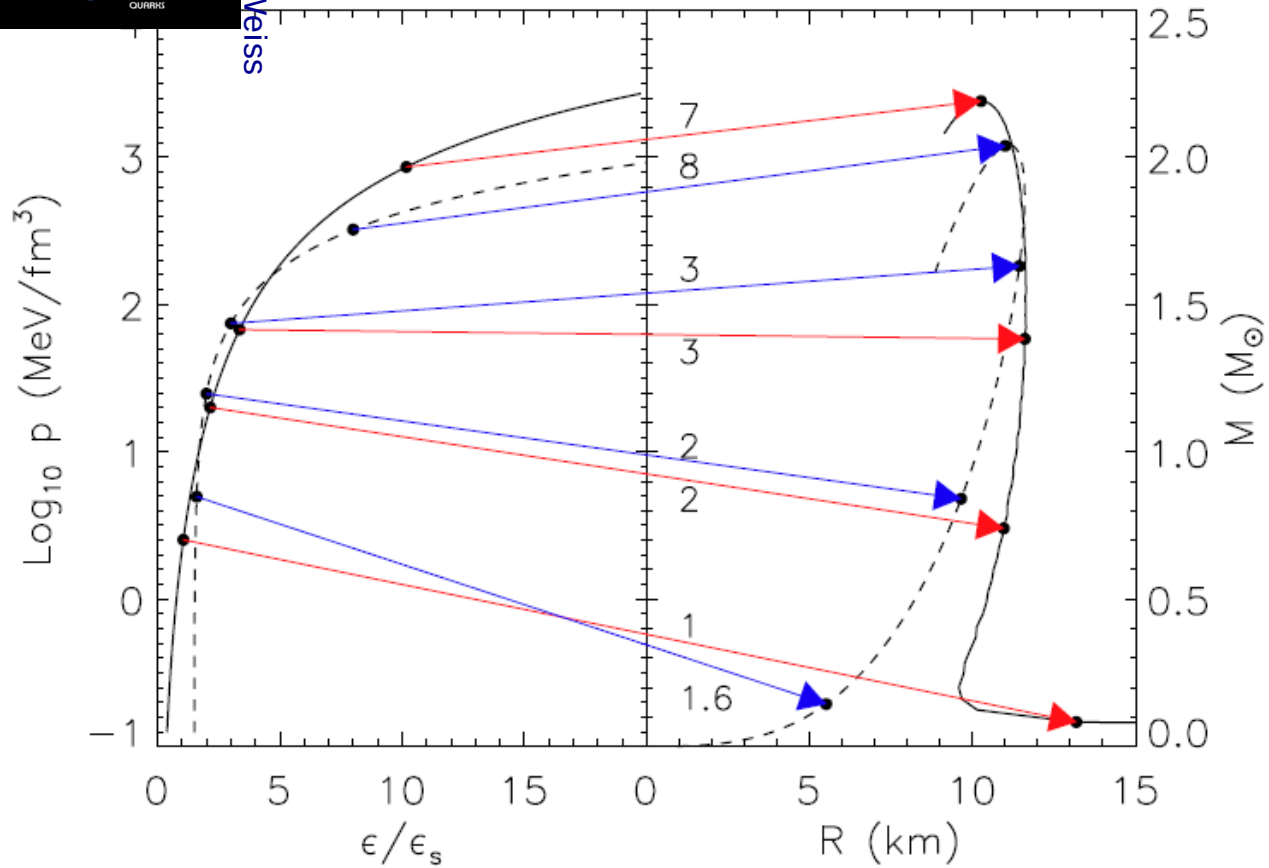
Mass and radius

**Needs ONLY $P(\epsilon)$
of NS matter**



Credits: CXC/M.Weiss

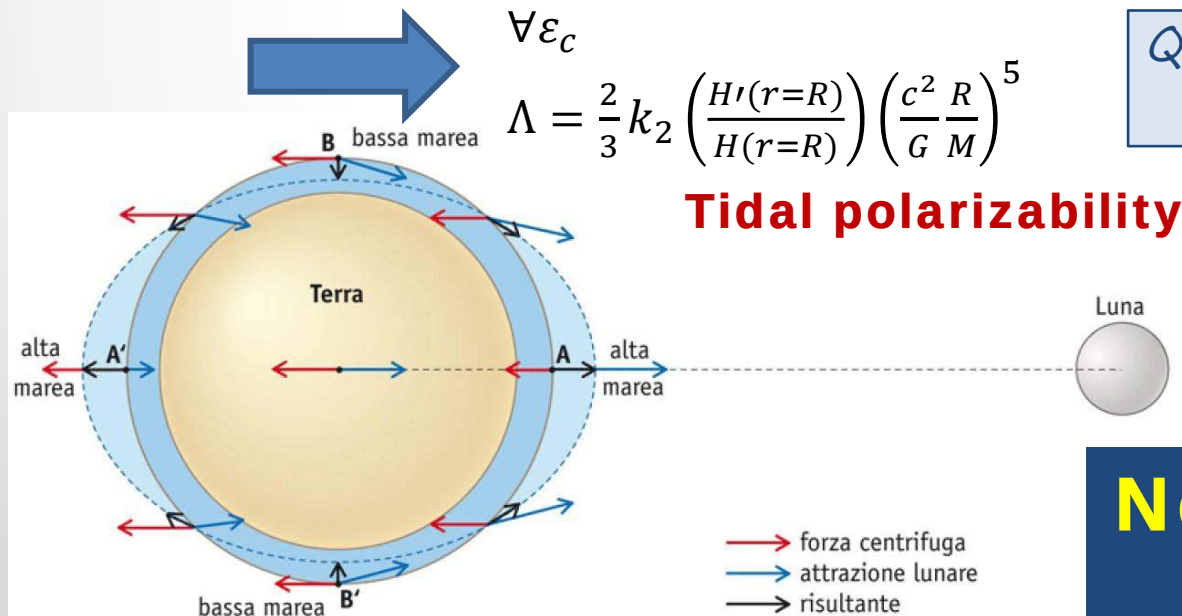
$$P(\varepsilon) \Leftrightarrow M(R)$$



Modelling (Neutron) Stars: hydrostatics

- Influence of a second body => Thorne and Campolattaro (1967):

$$\frac{d^2 H(r)}{dr^2} + \frac{dH(r)}{dr} \left[\frac{2}{r} + e^{\lambda(\mathbf{P}(\boldsymbol{\varepsilon}))} \left(\frac{2m(r)}{r^2} + 4\pi r(\mathbf{P}(\boldsymbol{\varepsilon}) - \varepsilon(r)) \right) \right] + H(r)Q(\mathbf{P}(\boldsymbol{\varepsilon})) = 0$$



Quadrupole
moment

Companion
tidal field

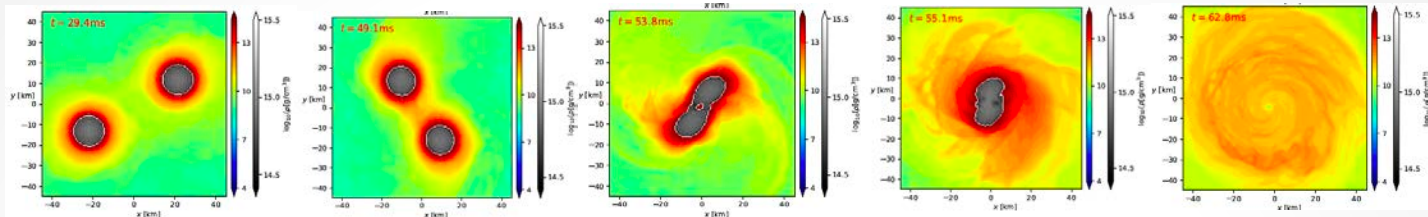
$$\bar{\bar{Q}}^{(i)} = -\Lambda_i \bar{\bar{\varepsilon}}^{(j)}$$

LIGO
Livingston

**Needs ONLY $P(\boldsymbol{\varepsilon})$
of NS matter**

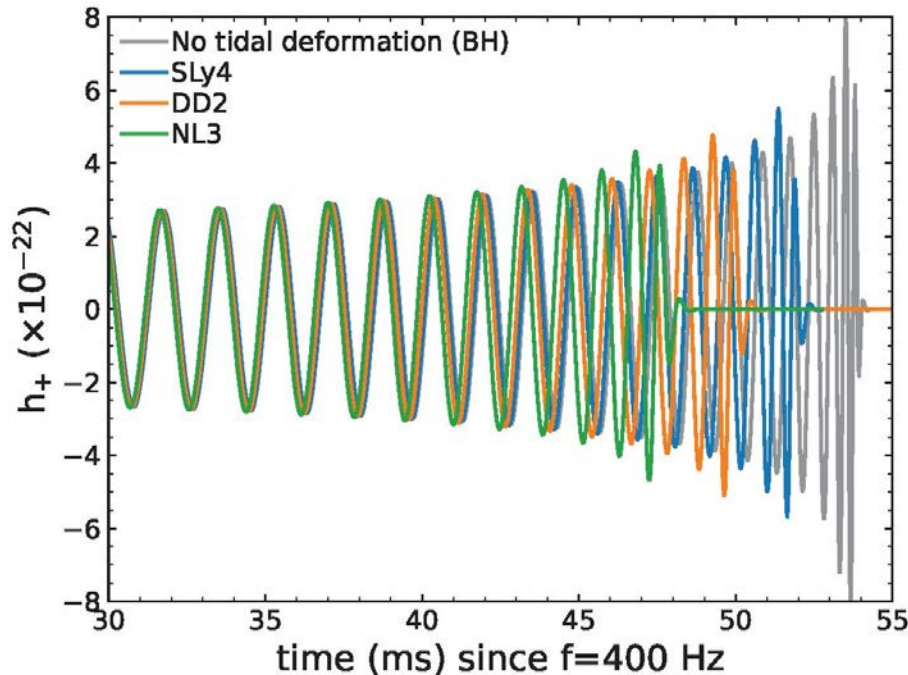


Extraction of the tidal effects



Gieg et al., Particle 2, 23 (2019)

SEOBNRV4T_SURROGATE waveform,
Courtesy J.Read



R.Wade et al., PRD89(2014)103012

$$\delta\psi_{\text{tidal}} = \frac{3}{128\eta x^{5/2}} \left[\left(-\frac{39}{2} \tilde{\Lambda} \right) x^5 + \left(-\frac{3115}{64} \tilde{\Lambda} + \frac{6595}{364} \sqrt{1-4\eta\delta\tilde{\Lambda}} \right) x^6 \right]$$

LIGO
Livingston

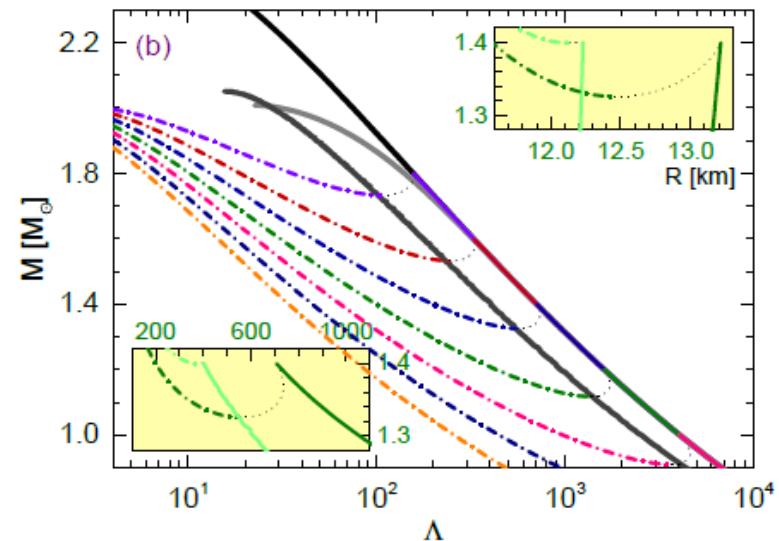
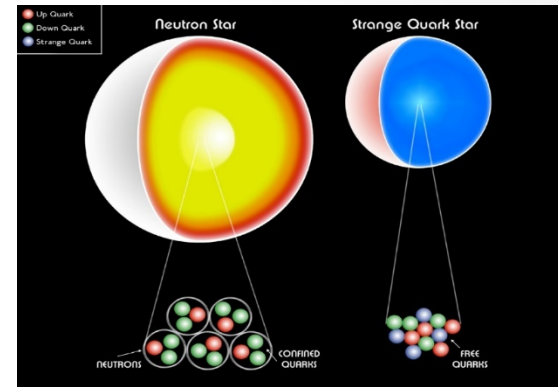
$$(\Lambda_1, \Lambda_2) \leftrightarrow (\tilde{\Lambda}, \delta\tilde{\Lambda})$$

Tidal effects: modification of the gravitational waveform wrt a GR point-particle calculation, due to the star deformation just before merging

...a brief summary

$$\langle \theta \rangle \Leftrightarrow P(\varepsilon)$$

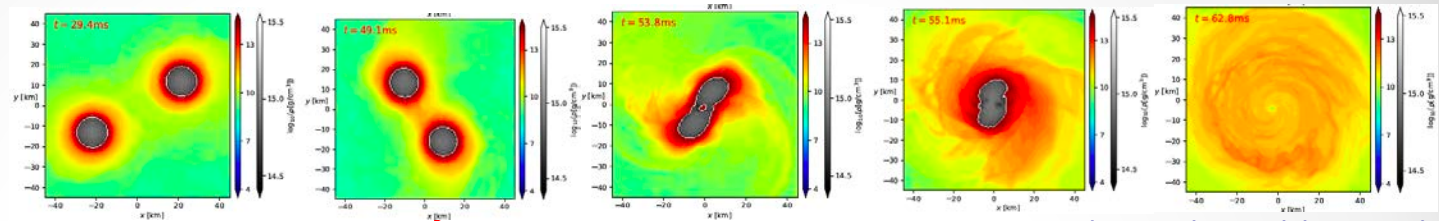
- GR imposes a 1-to-1 correspondence between the dense matter EoS and static properties of NS ($M(R)$ - $M(\Lambda)$)
- Different compositions $\Rightarrow$ different $M(\Lambda) \Rightarrow$ different gravitational signals!
- Systematics due to the astrophysical modelling in principle under control



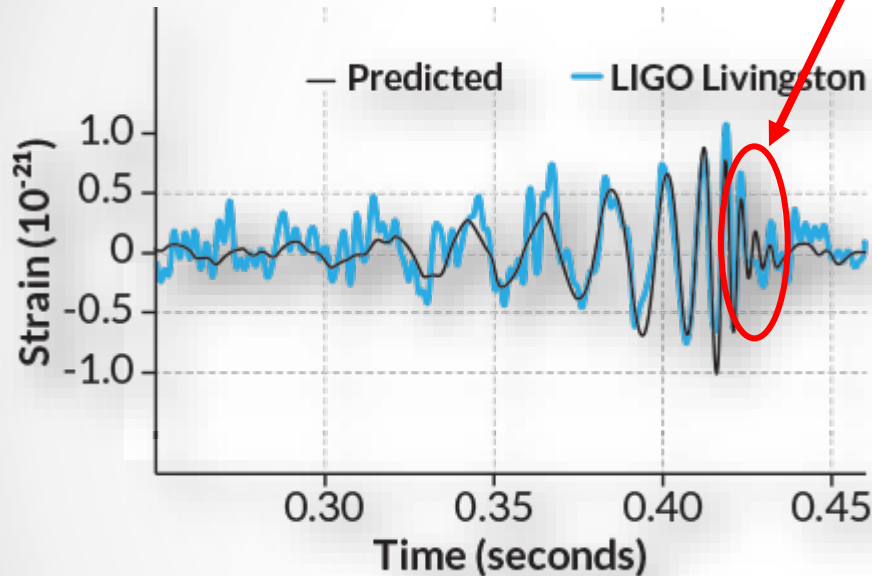
J.J.Li, A.Sedrakian, M.Alford,
PRD101 (2020) 063022

Problem 1 : the Λ info is hard to measure

$$\langle \theta \rangle \leftrightarrow P(\varepsilon)$$

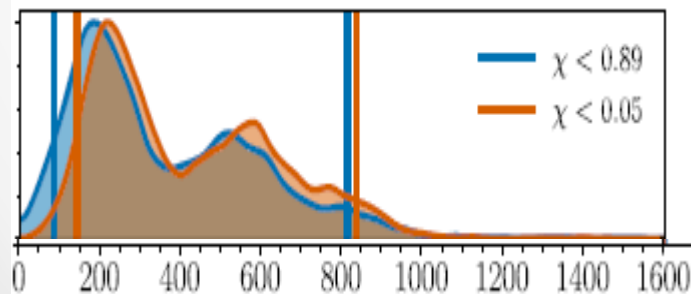


Gieg et al., Particle 2, 23 (2019)



R.Wade et al., PRD89(2014)103012

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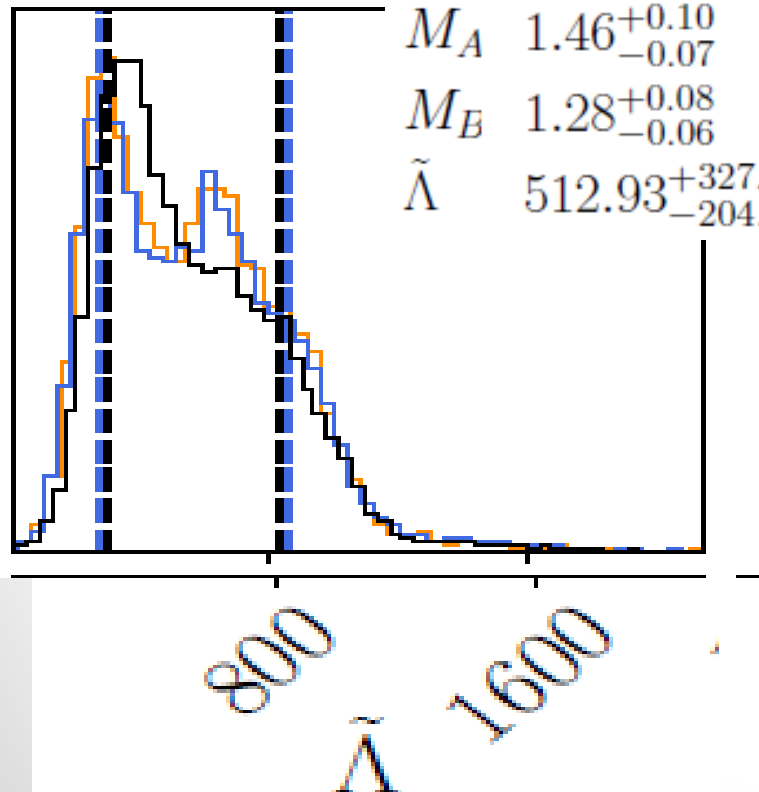


LVC, PRX 9 (2019), 011001

$$(\Lambda_1, \Lambda_2) \leftrightarrow (\tilde{\Lambda}, \delta\tilde{\Lambda})$$

=> Only the combined $\tilde{\Lambda}$ is measurable, and with error bars

The waveform modelling

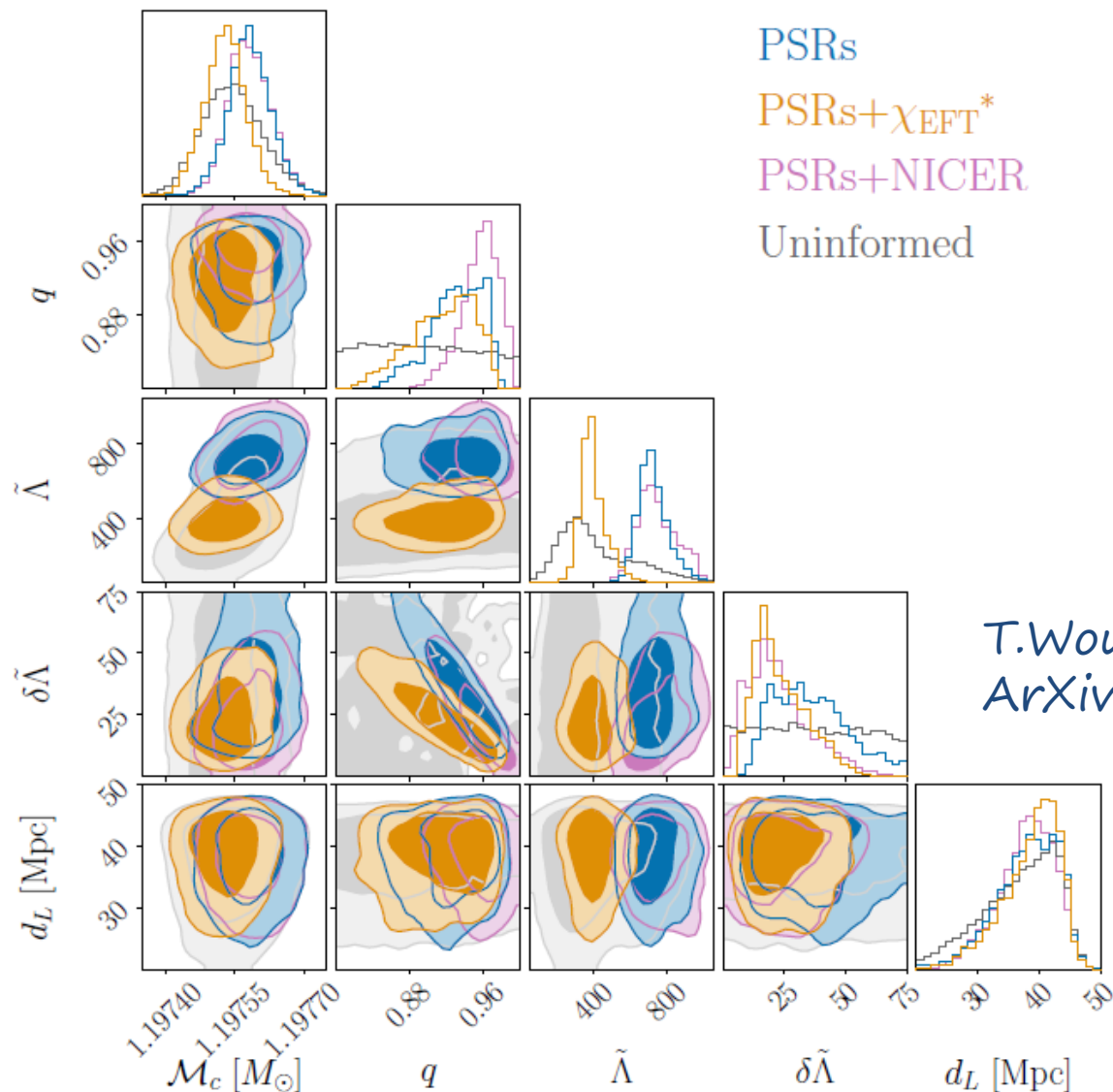


	IMRPhenomXP	IMRPhenomPv2	IMRPhenomXP
	_NRTidalv3	_NRTidalv2	_NRTidalv2
M_A	$1.46^{+0.10}_{-0.07}$	$1.46^{+0.09}_{-0.07}$	$1.46^{+0.09}_{-0.07}$
M_E	$1.28^{+0.08}_{-0.06}$	$1.28^{+0.06}_{-0.07}$	$1.28^{+0.06}_{-0.07}$
$\tilde{\Lambda}$	$512.93^{+327.53}_{-204.15}$	$552.11^{+308.49}_{-269.56}$	$544.14^{+305.67}_{-260.55}$

A.Abac et al, arXiv:2311.074562

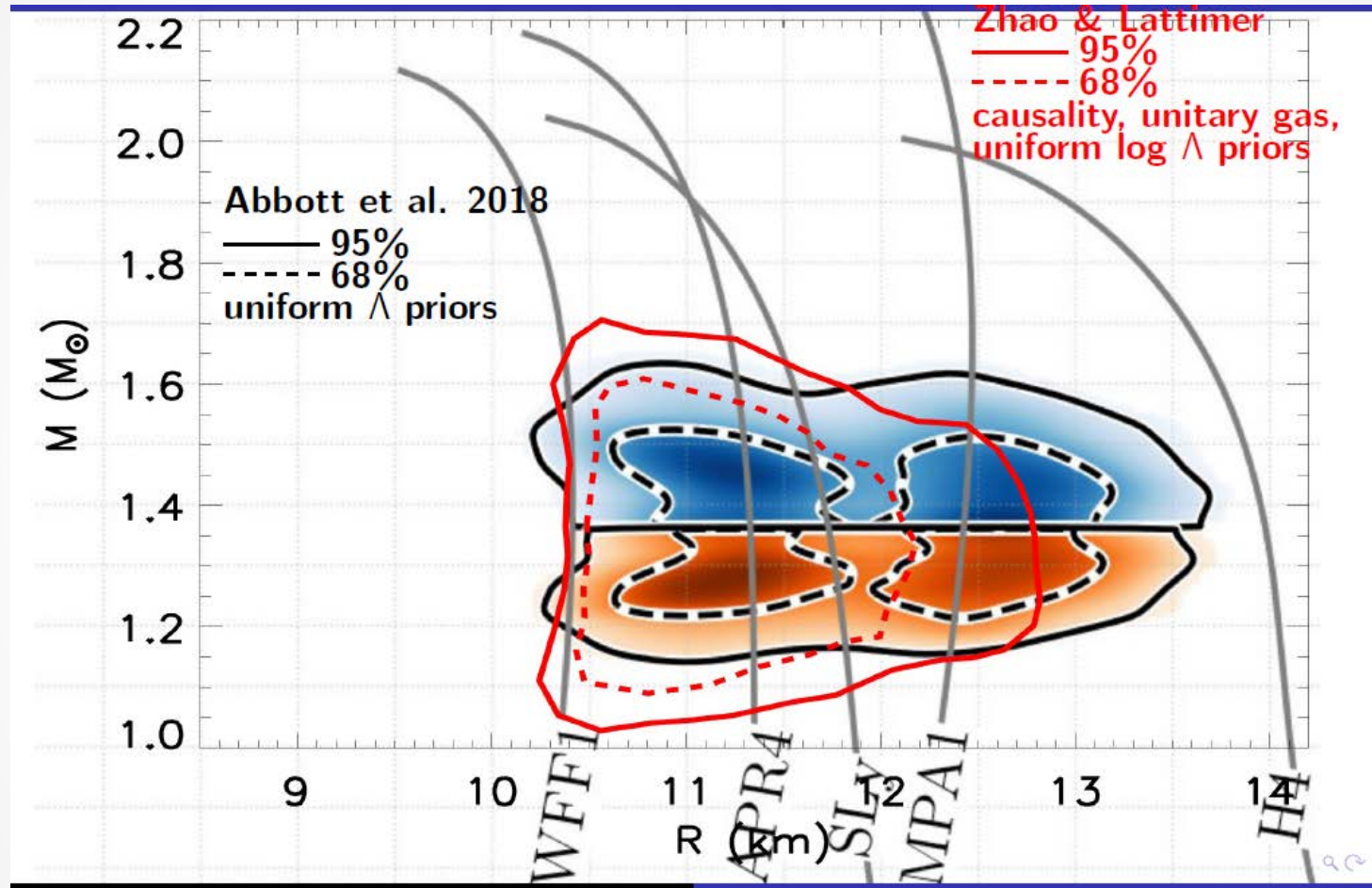
The prior choice

$$P(\vec{\theta}|\vec{d}) = \frac{\mathbf{P}(\vec{\theta}) P(\vec{d}|\vec{\theta})}{P(\vec{d})}$$



T.Wouters et al,
ArXiv:2511.22987

The prior choice



H.Zhao & J.Lattimer, 2026

Problem II : no ab-initio theory of dense matter

$$\langle \theta \rangle \Leftrightarrow \mathbf{P}(\varepsilon)$$

=> Density Functional Approach

$\varepsilon_{tot} = \varepsilon_B + \varepsilon_L$ (baryons and leptons decoupled, leptons free FG)

- **NS core:** $n_q(r) = n_q$ ($\forall q$ constituent)

- **Effective single particles:** $e_q(k) = \sqrt{m_q^{*2} + k^2} + V_q(n_q, n_{q'})$

$\Rightarrow m_q^*, V_q$ from a phenomenological Hamiltonian (Skyrme, Gogny, M3Y..) or Lagrangian (RMF)

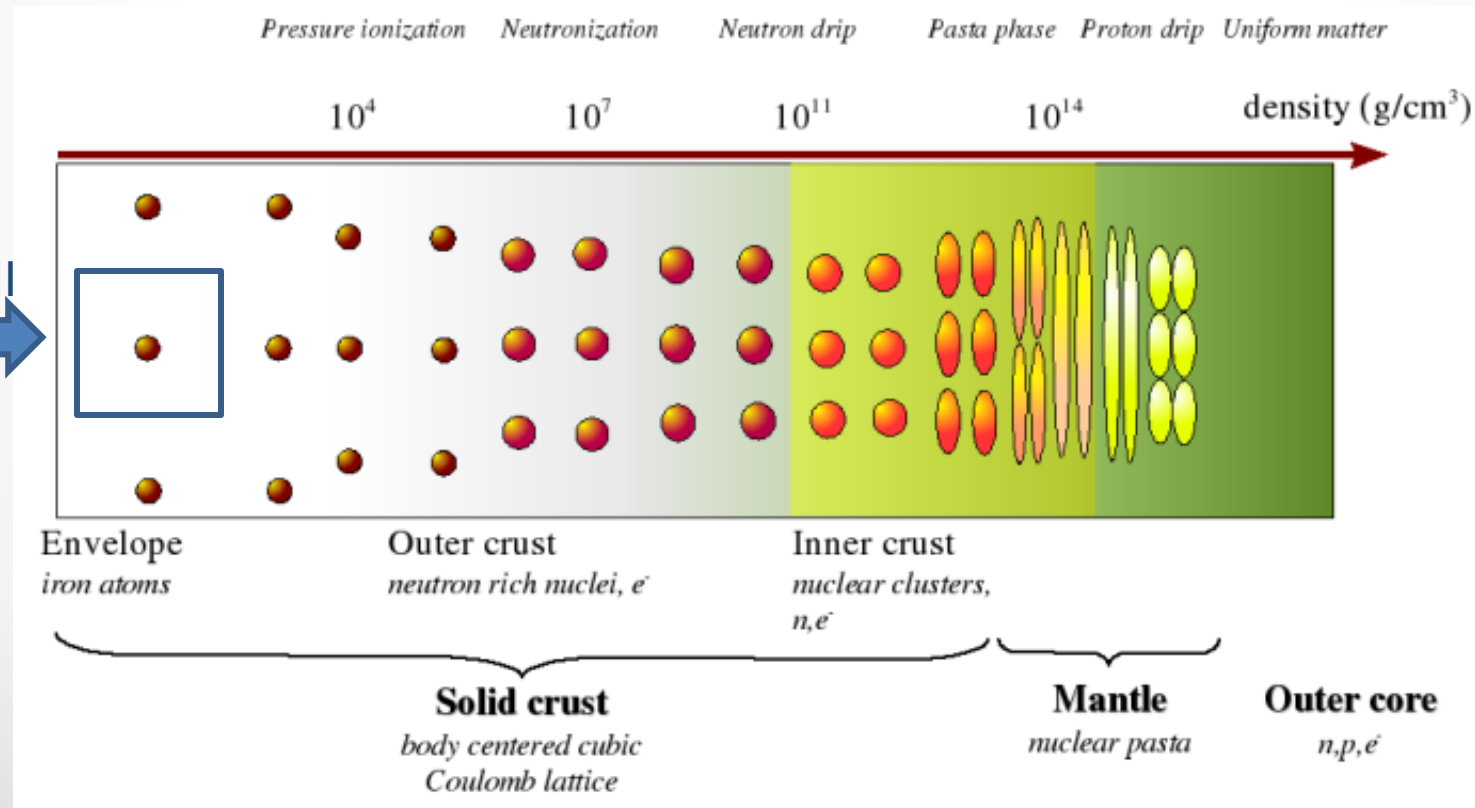
$\Rightarrow$ Parameters fitted on nuclear data and/or ab-initio calculations

$\Rightarrow e(n_B, n_L, n_S) \quad P(\varepsilon) = -n_B^2 \left. \frac{\partial e}{\partial n_B} \right|_{\mu_L=0, \mu_S=0},$

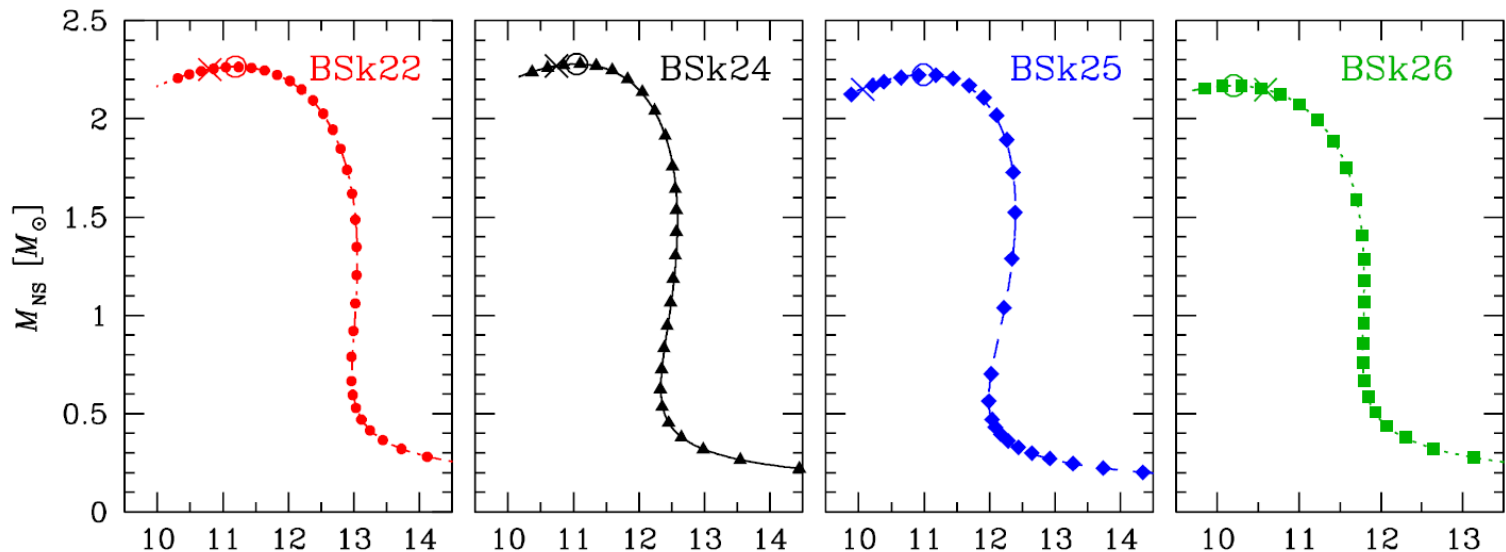
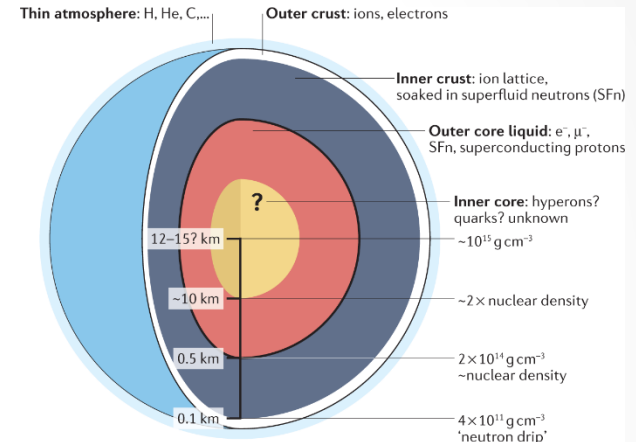
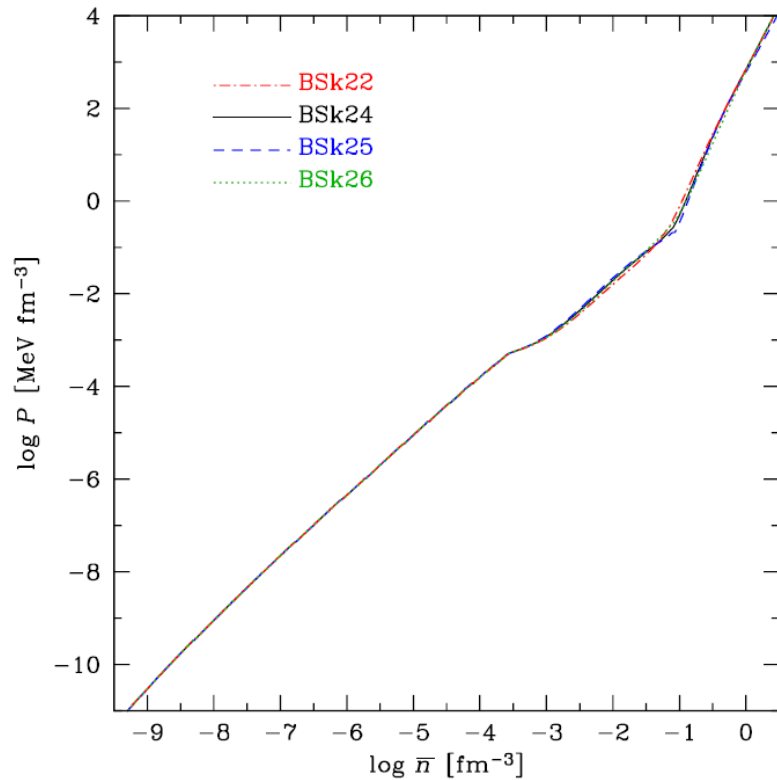
Theoretical evaluation of the Equation of State

=> Density Functional Approach

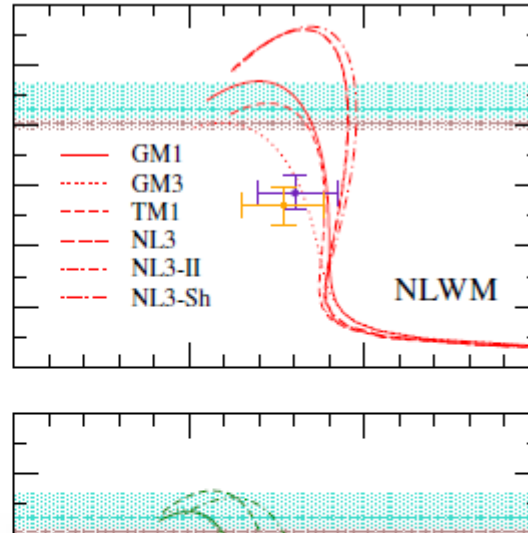
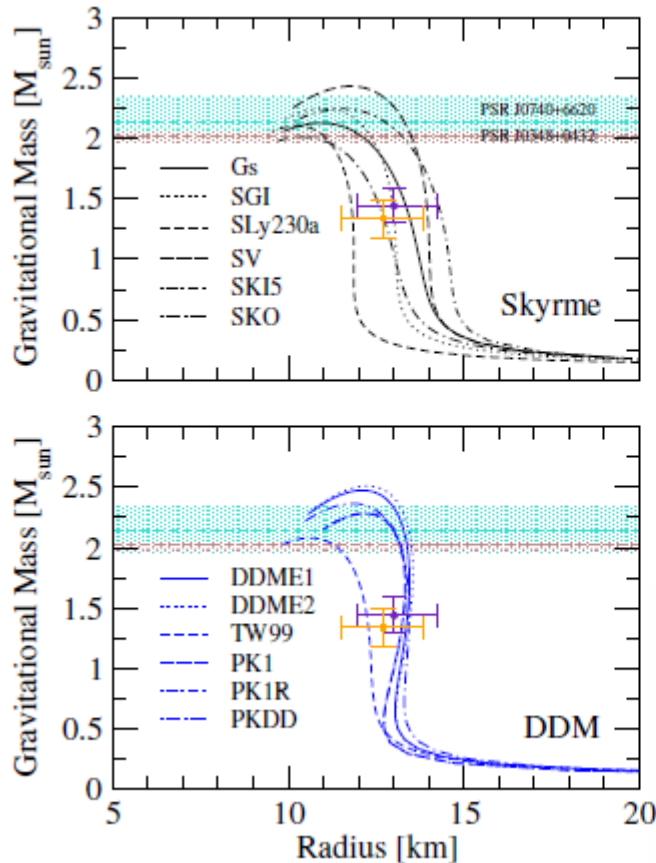
- **NS crust:** $q=n,p$ variational calculation of the single particle wavefunctions => $n_q(r)$ within the Hartree-Fock-(Bogoliubov) theory



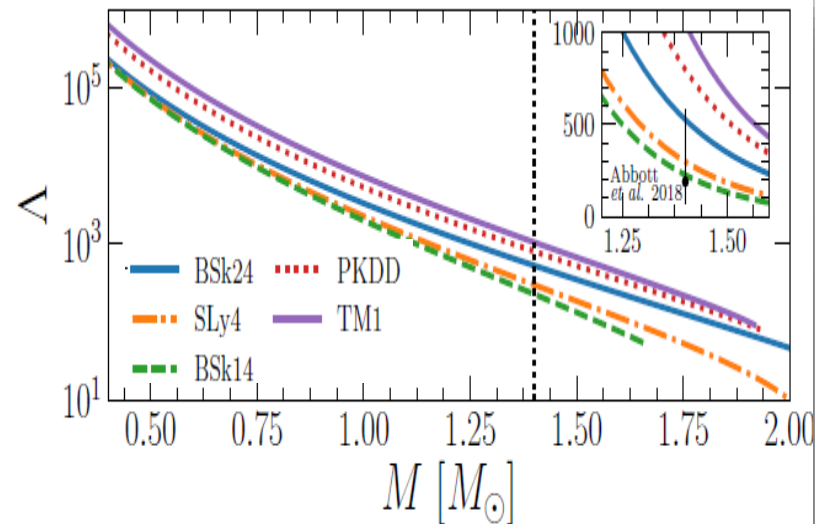
An example: the Brussels-Montreal functionals BSk



Problem 11 : no ab-initio theory of dense matter

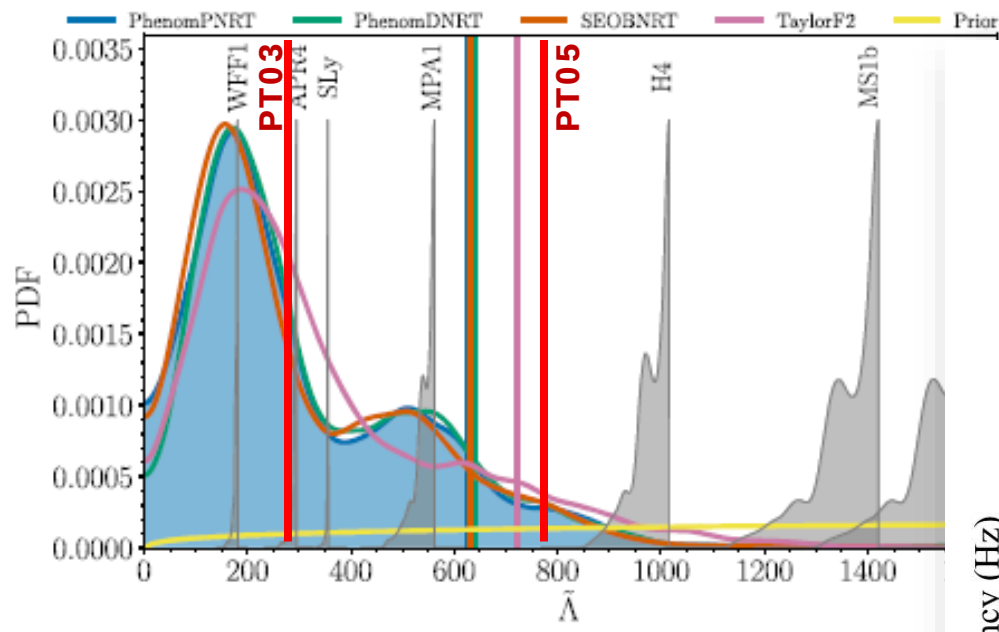


- **Model dependence: choice of functional form (Lagrangian versus Hamiltonian) and extrapolations lead to different predictions**



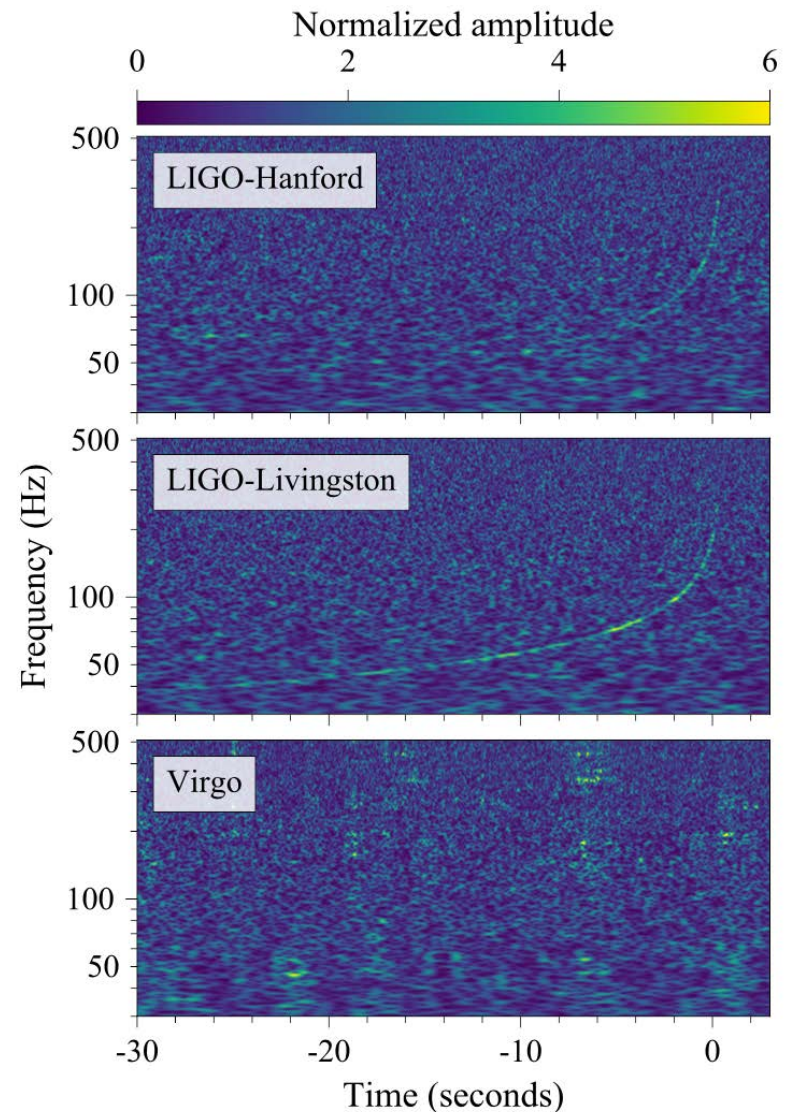
F.Burgio, I.Vidana,
Universe 2020, 6,
119

Present status: GW170817

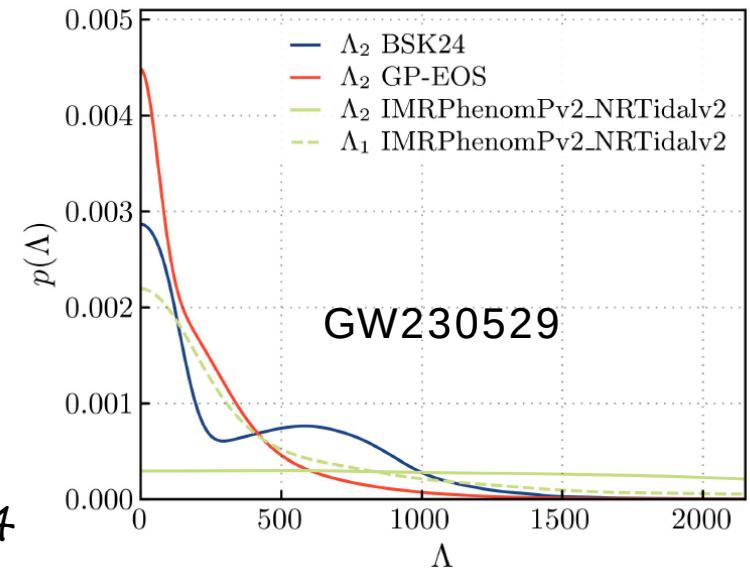
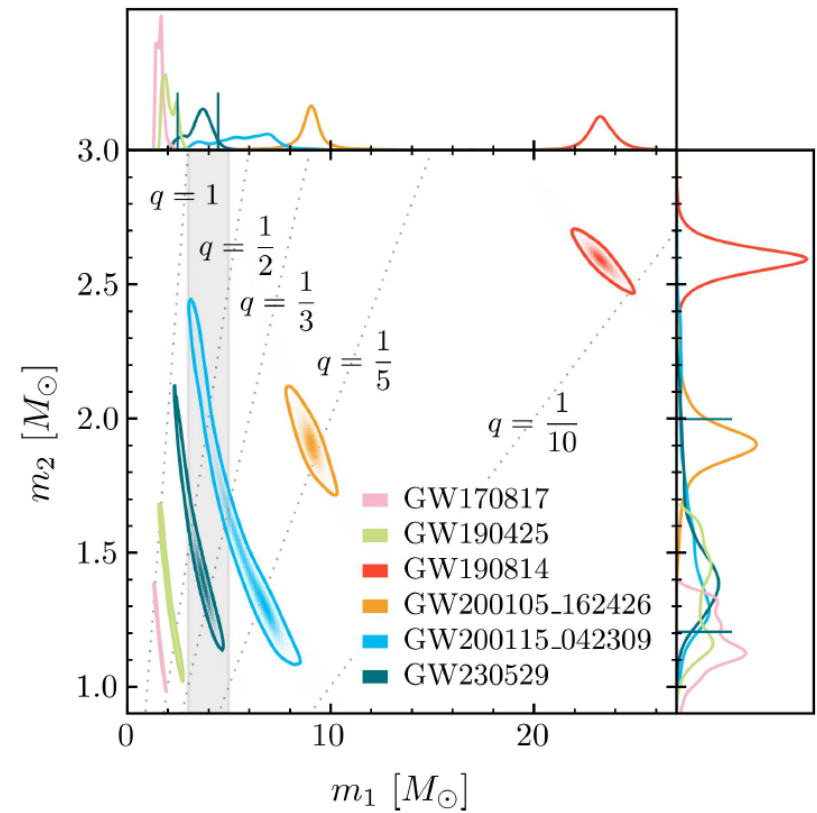
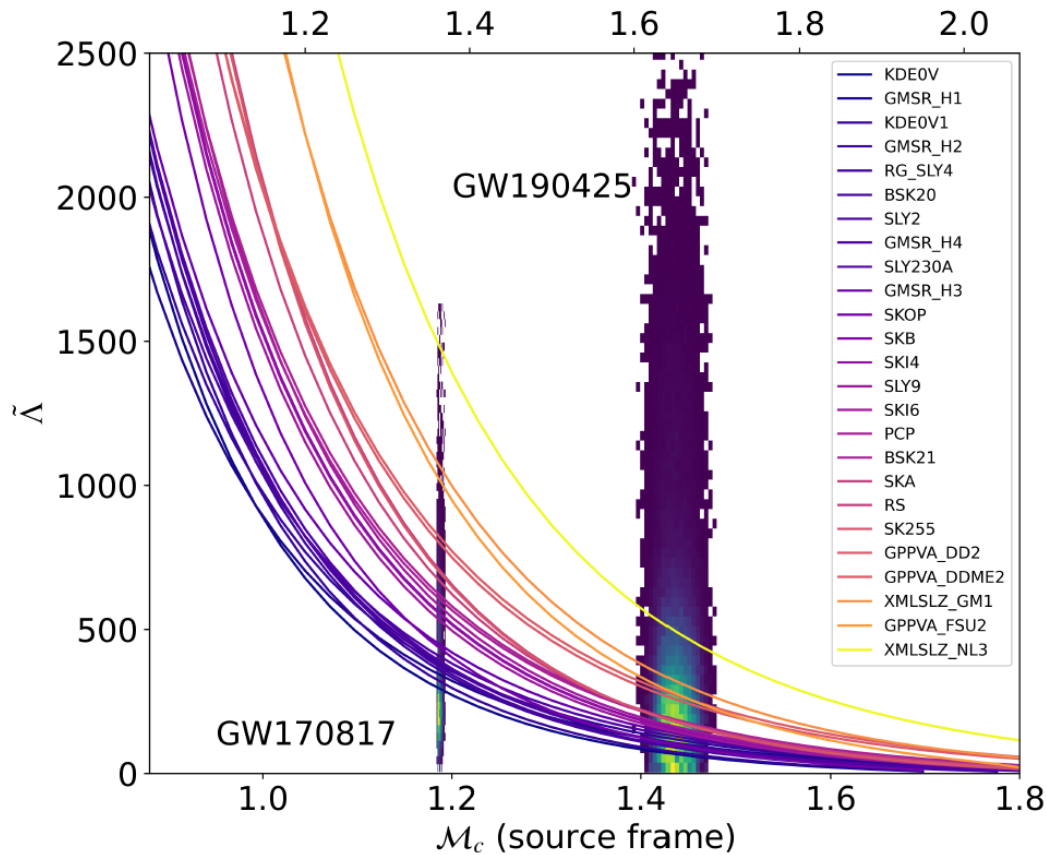


LVC, APJL2017 & PRX9 2019

- The observation cannot discriminate

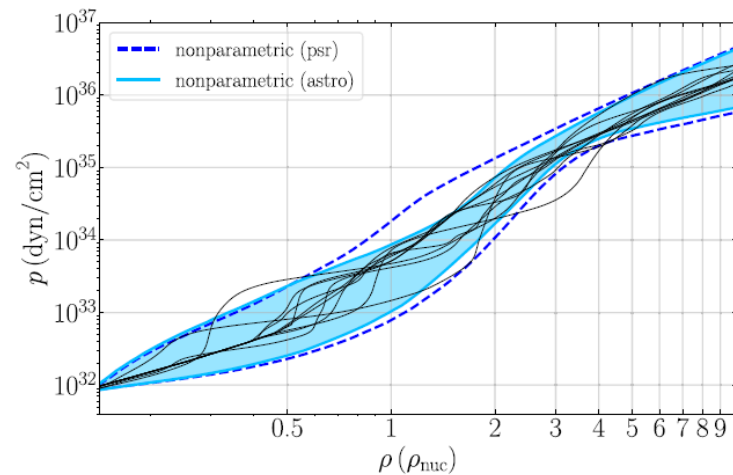
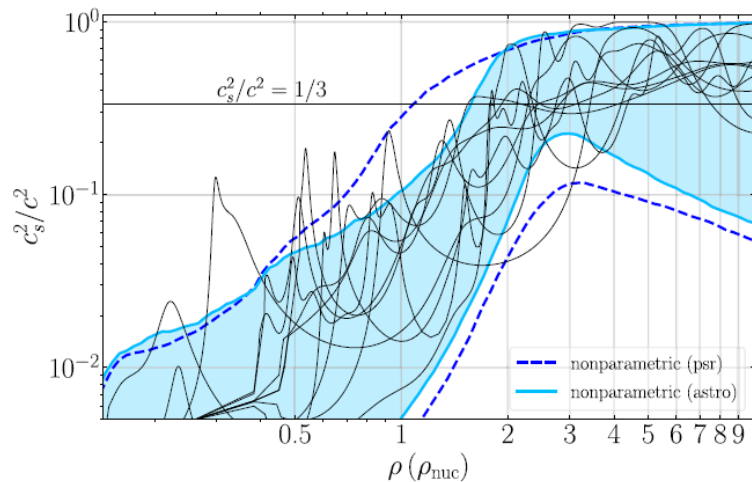


GW info after GW170817



A. Abac et al, ApJL 2024

The inversion strategy



- Agnostic EoS only respecting causality and thermodynamical stability
- Example: non-parametric EoS from Gaussian Process

$$\phi(p) = \log \left(\frac{c^2}{c_s^2} - 1 \right) = N \left(\mu(p_i), K(p_i, p_j) \right)$$

- Bayesian inference

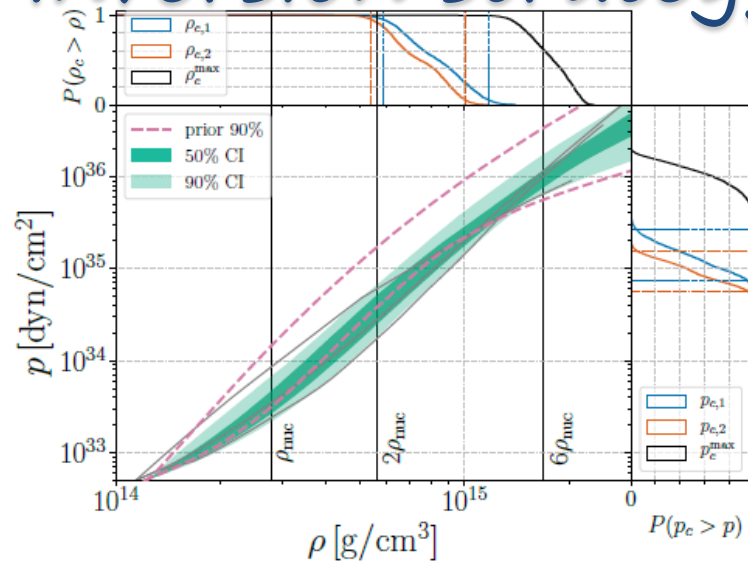
$$P(EoS | \vec{\theta}) = \frac{P(EoS) \prod_i P(\theta_i | EoS)}{P(\vec{\theta})}$$

- θ_1 . max.mass (radio)
- θ_2 . tidal polarisability (GW)
- θ_3 . radius (X-ray)

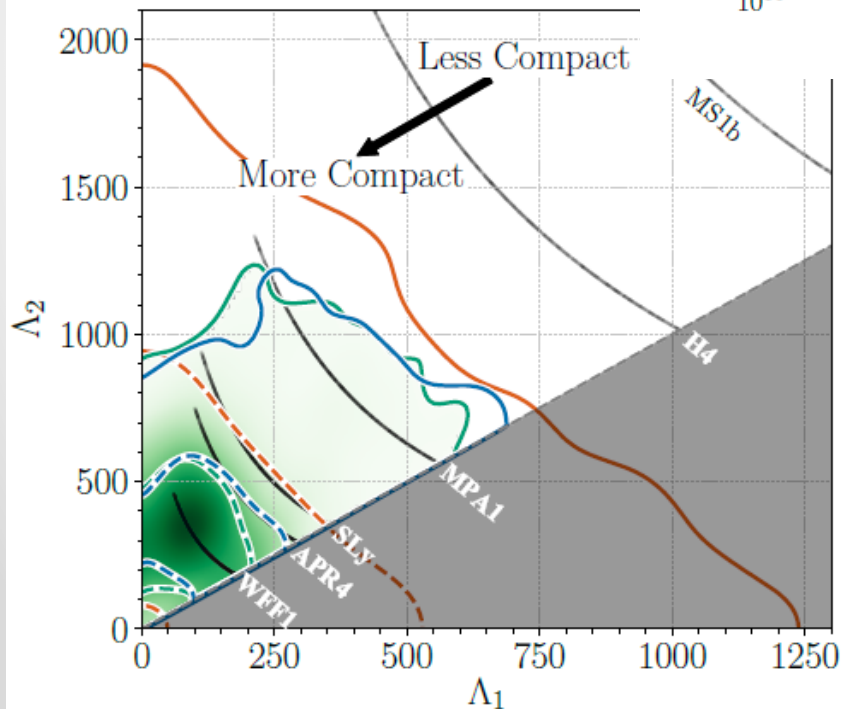
P. Landry and R. Essick, PRD 99, 084049 (2019)

I. Legred et al PRD105, 043016 (2022)

The inversion strategy



A. Abbott et al, PRL 2018



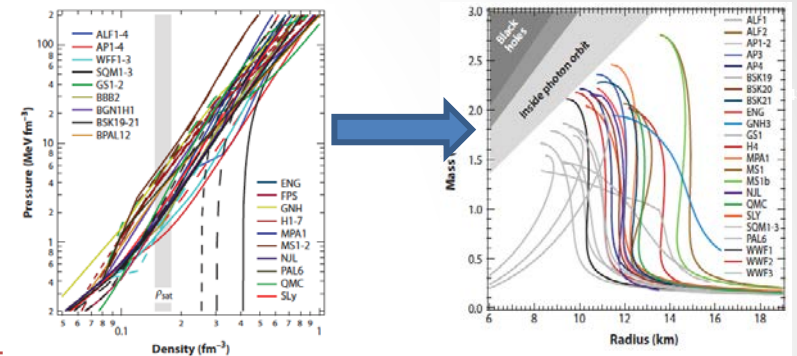
- (Almost) model independent evaluation of the EoS
- Still, we do not learn much about dense matter
- We do not exploit our nuclear physics knowledge either

...a brief summary $\langle \vec{\theta} \rangle \Leftrightarrow P(\varepsilon)$

The nuclear physicist viewpoint: $e(n_n, n_p) \Rightarrow P(\varepsilon) \Rightarrow \langle \vec{\theta} \rangle$

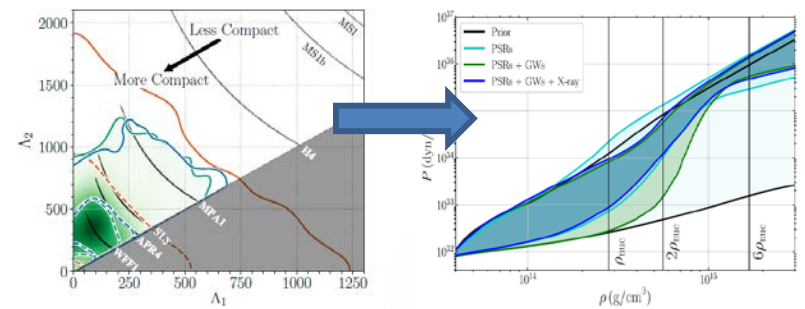
- Controlled dof, hypotheses and approximations, exp info included
- Still, the predictive power is limited

Ozel & Freire, ARAA 54, 401 (2016)



The astrophysicist viewpoint: $\langle \vec{\theta} \rangle \Rightarrow P(\varepsilon)$

- Non-parametric representation: model independent evaluation of the EoS
- Still, we do not learn much about dense matter



A nuclear-astrophysicist viewpoint:
meeting in the middle...

General EoS modelling: the syllabus

- An agnostic (parametric) representation ~~$P(\varepsilon)$~~ $e_{\vec{X}}(n_n, n_p)$: the variation of the parameter set $\vec{X}$ allows reproducing the different nuclear models and interpolating among them ~ 15 parameters – RMF and EDF versions
- The X_i variation explores the equation of state space compatible with the hypothesis of a matter of neutrons and protons

$$P(\varepsilon) = -n^2 \left. \frac{\partial e(n_n, n_p)}{\partial n} \right|_{\mu_L=0} \quad \text{and also}$$

$y_e(n)|_{\mu_L=0}, P(n, y_e, T)$ for the SN and merger simulations

A. Steiner et al ApJ 2010

A. Bulgac et al 2016

J. Margueron et al PRC 2018

Y. Lim, J.W. Holt, PRL 2018

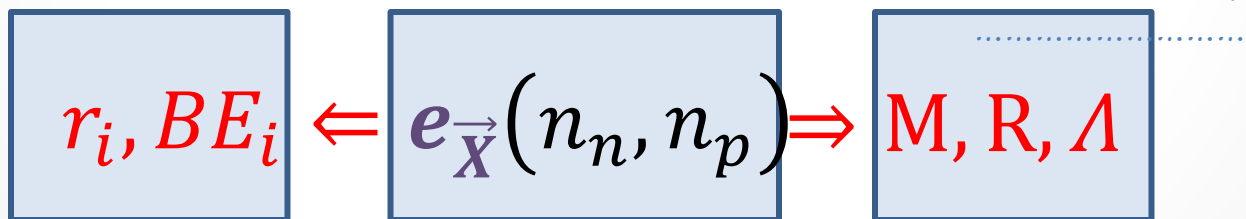
C. Mondal et al, PRC 2022

P. Char et al, PRD 2023

Y. Lim, A. Schwenk PRC (2024)

G. Montefusco et al, A&A 2025

H. Koehn et al, PRX 2025



Laboratory
observables

Nuclear model

Astronomical
observables

Bayesian Inference

$$P(EoS|\vec{\theta}) = \frac{P(EoS) \prod_i P(\theta_i|EoS)}{P(\vec{\theta})} \quad (\text{MCMC or Nested Sampling})$$

θ_1 . nuclear structure data

θ_2 . ab-initio theory (chiral)

θ_3 . HI collisions



Nuclear physics

θ_4 . max.mass (radio)

θ_5 . tidal polarisability (GW)

θ_6 . radius (X-ray)

θ_7 . multi-messenger



Astrophysics

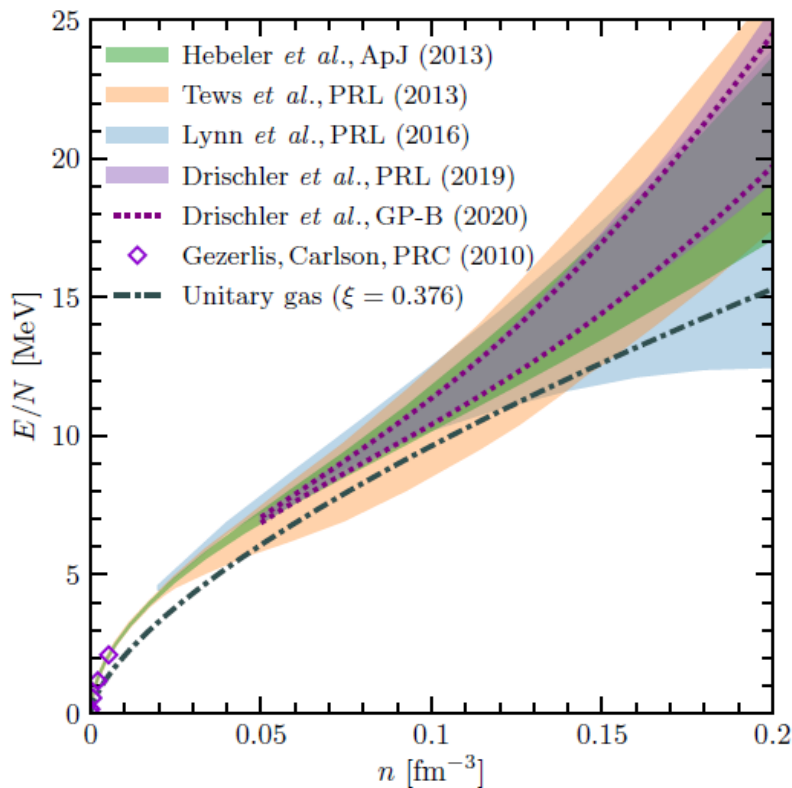
(3) PSR J0348+0432 $M=2.01\pm0.04 M_{\odot}$

(4) GW170817 $\tilde{\Lambda}(M)$ LVK

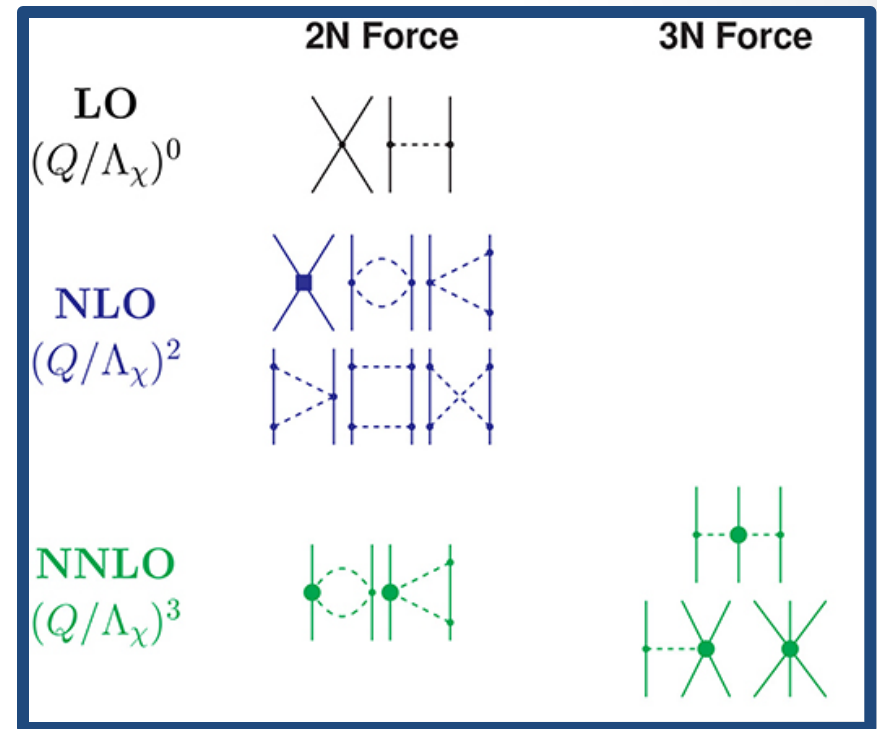
(5) PSRJ0030+0451, PSRJ0740+6620, PSR10030-0451, PSRJ0614-3329 NICER

Ab-initio nuclear theory

- interaction from χ -EFT, different many body methods (MBPT, AFMC)
- Diagrammatic expansion : controlled truncation errors
- Moment expansion! Only valid at low density



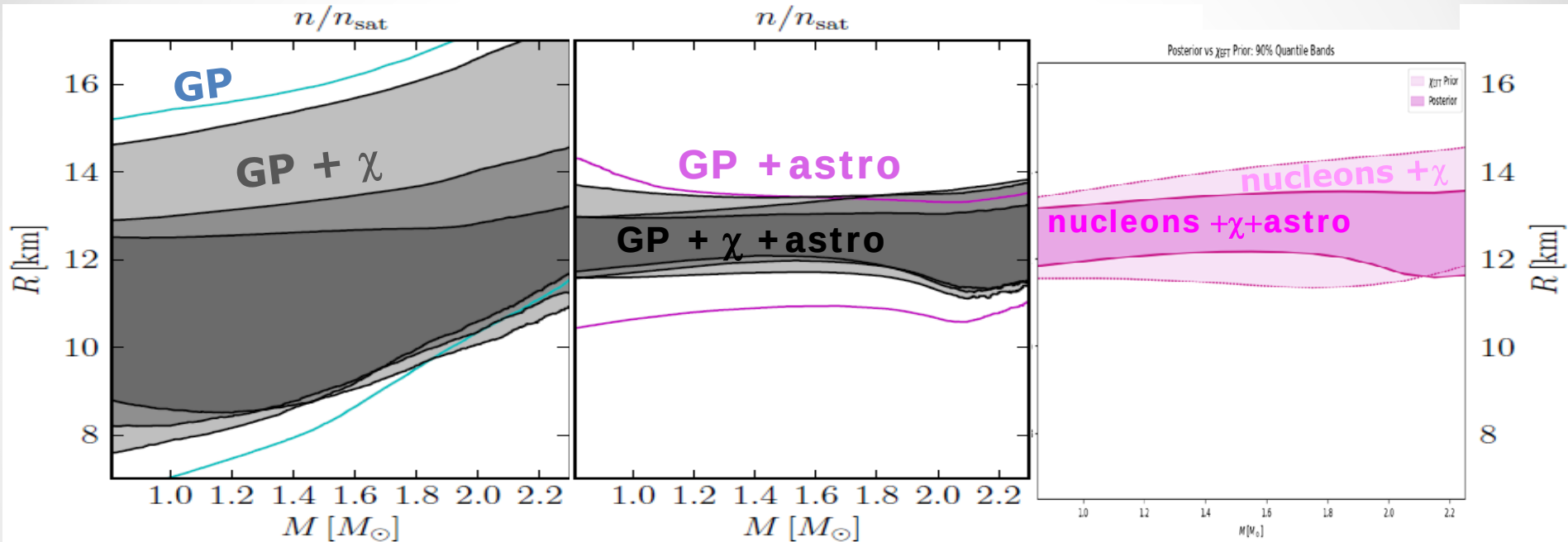
Machleidt R., Int J Mod Phys. (2017)



Agnostic versus nucleonic

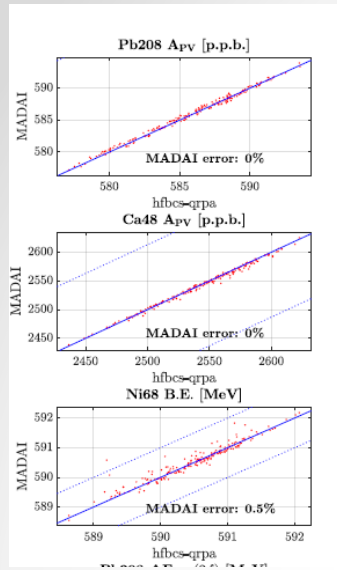
R.Essick et al, PRC 2020

G.Montefusco et al, arxiv:2604.00196

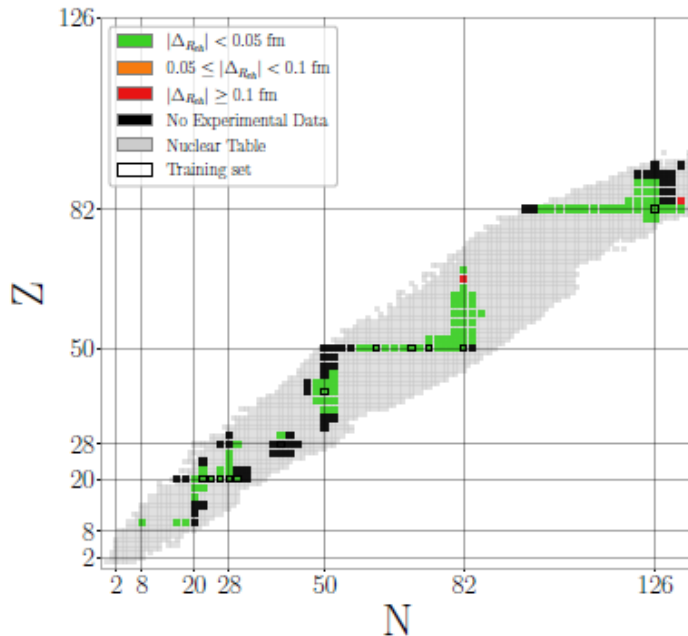


- Prior distributions are larger with agnostic models (GP)
- Distributions are very comparable with chiral (χ)+astro $\Rightarrow$ the nucleonic models do cover all the parameter space compatible with the observations ! $\Rightarrow$ the observations are compatible with a pure nucleonic composition

Laboratory experiments



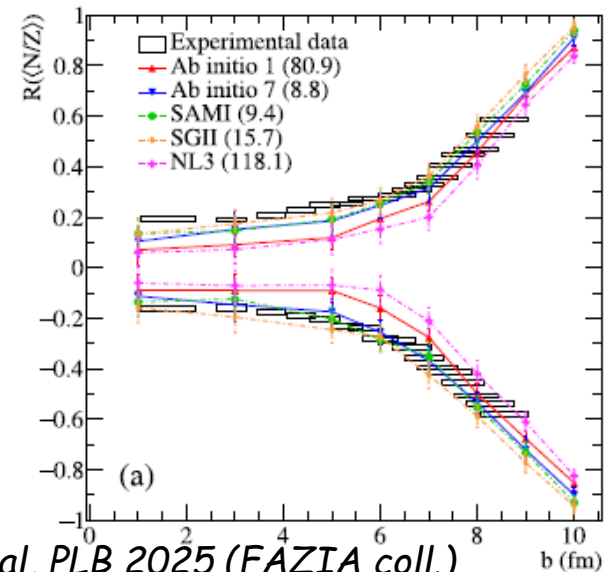
madai.phy.duke.edu



P.Klausner et al, PRC 2024

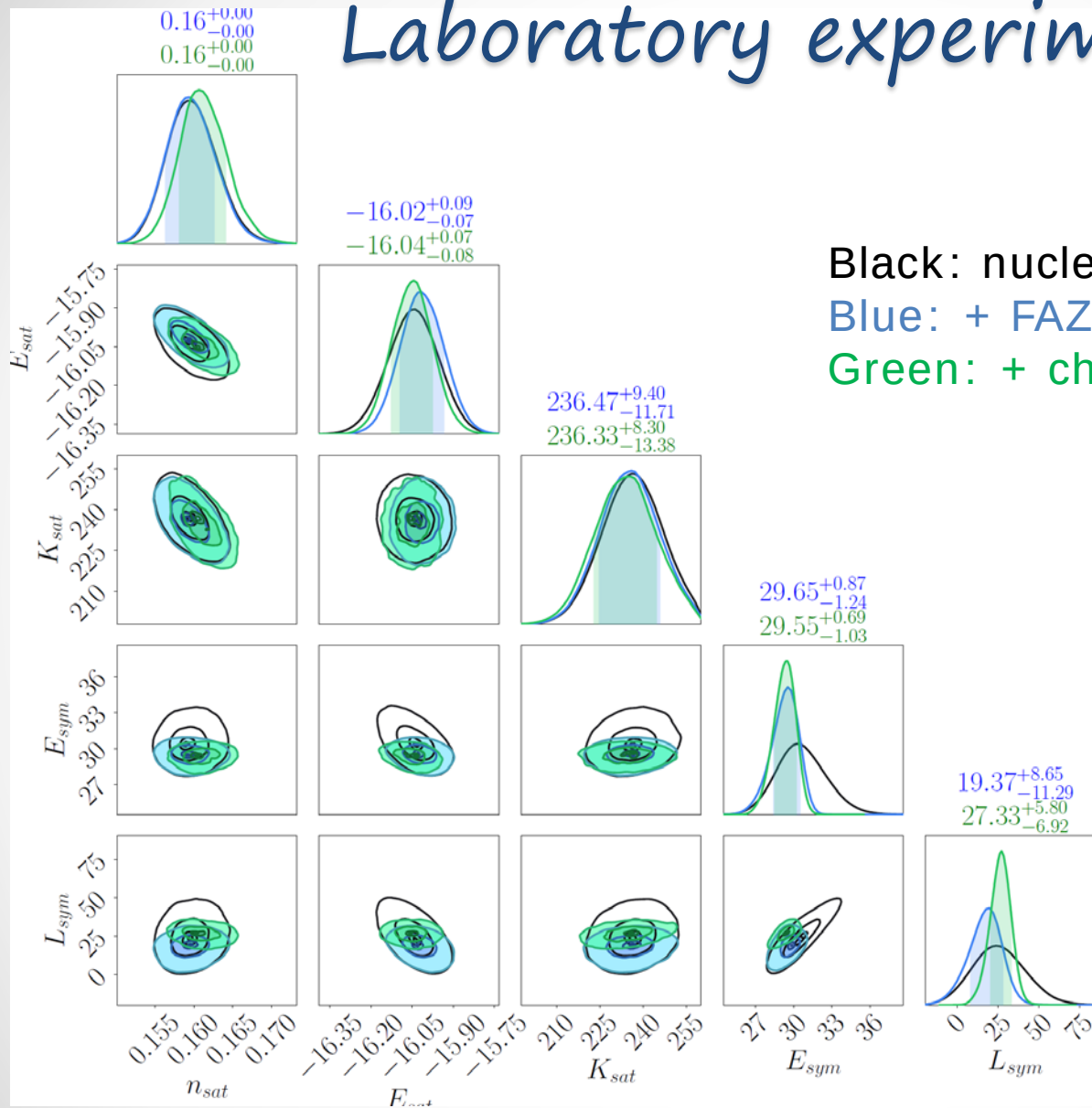
	$B.E.$ [MeV]	R_{ch} [fm]	ΔE_{SO} [MeV]	E_{GMR}^{IS} [MeV]
Pb208	1636.4 ± 2.0	5.50 ± 0.05	2.02 ± 0.50	13.5 ± 0.5
Ca48	416.0 ± 2.0	3.48 ± 0.05	1.72 ± 0.50	
Ca40	342.1 ± 2.0	3.49 ± 0.05		
Ni56	484.0 ± 2.0			
Ni68	590.4 ± 2.0			
Sn100	825.2 ± 2.0			
Sn132	1102.8 ± 2.0	4.65 ± 0.05		
Zr90	783.9 ± 2.0	4.27 ± 0.05		17.7 ± 0.5
	α_D [fm ³]	$m(1)$ [MeV fm ²]	A_{PV} (p.p.b.)	E_{GQR}^{IS} [MeV]
Pb208	19.60 ± 0.60	961 ± 22	550 ± 18	10.9 ± 0.5
Ca48	2.07 ± 0.22		2668 ± 113	

+ Isospin Transport Ratio $^{58,64}\text{Ni} + ^{58,64}\text{Ni}$



C.Ciampi et al, PLB 2025 (FAZIA coll.)

Laboratory experiments



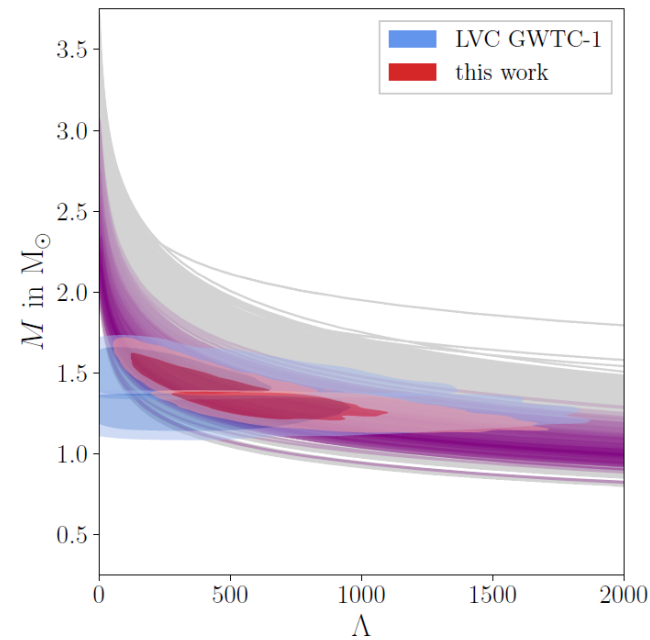
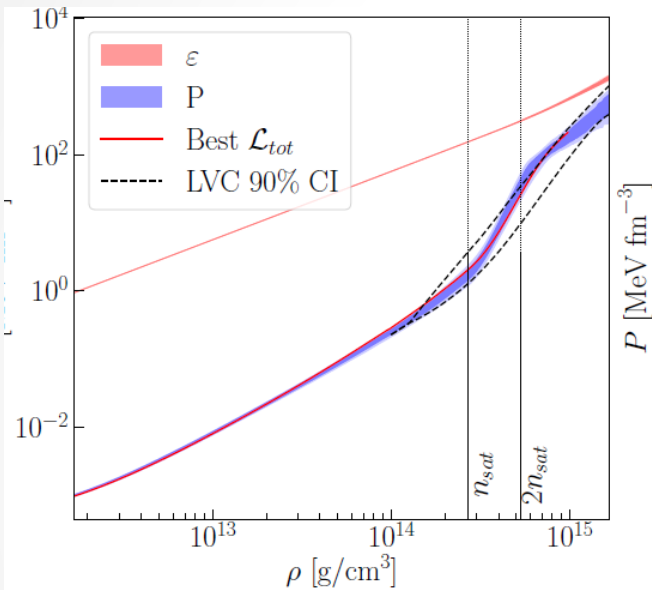
Black: nuclear structure only

Blue: + FAZIA data

Green: + chiral EFT

Agnostic versus nucleonic

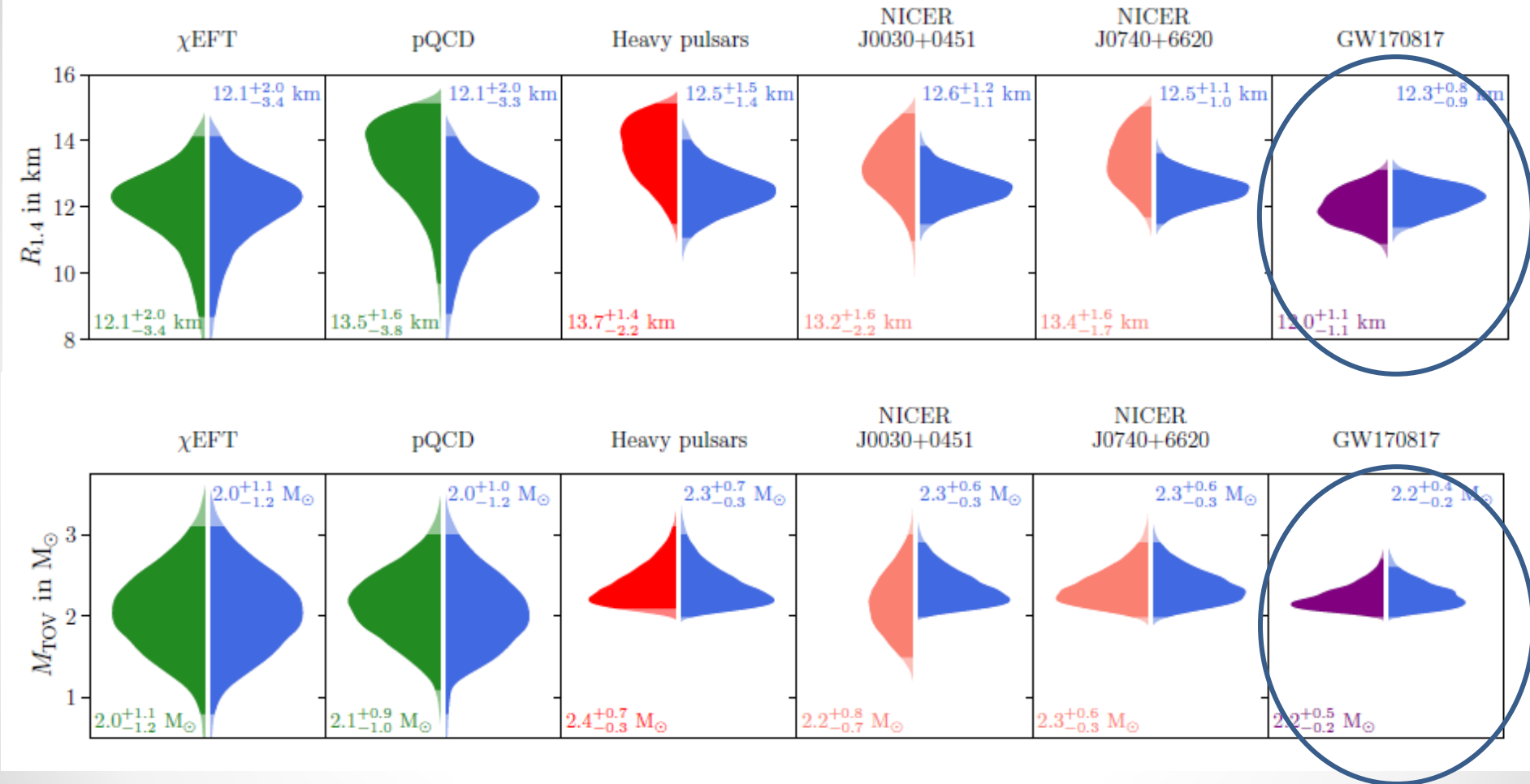
P.Klausner et al, ArXiv 2025



H.Koehn et al, PRX 2025

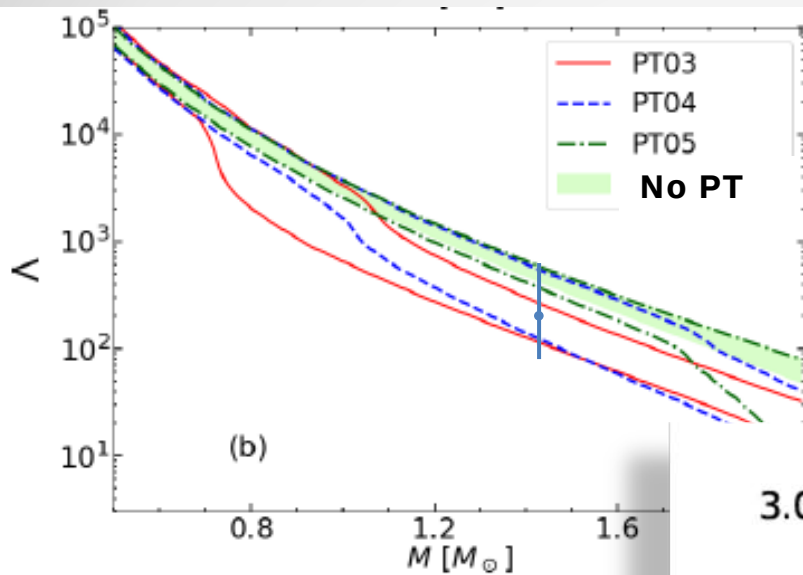
- Distributions get shrunk when nuclear experiments are included => potential challenge of the nucleonic hypothesis!

The importance of the GW signal



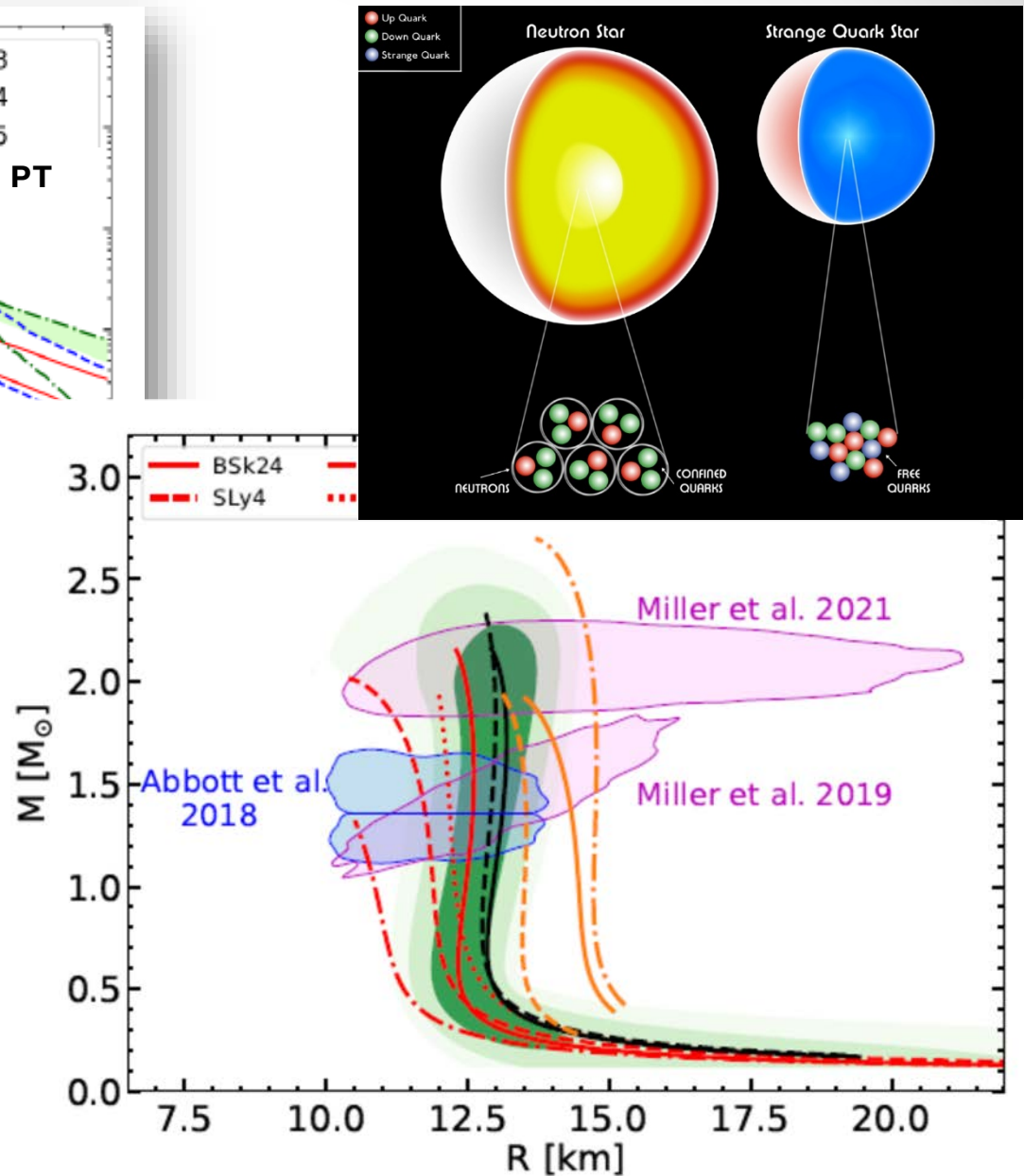
H.Koehn et al, PRX 2025

Quarks in the core of neutron stars ?

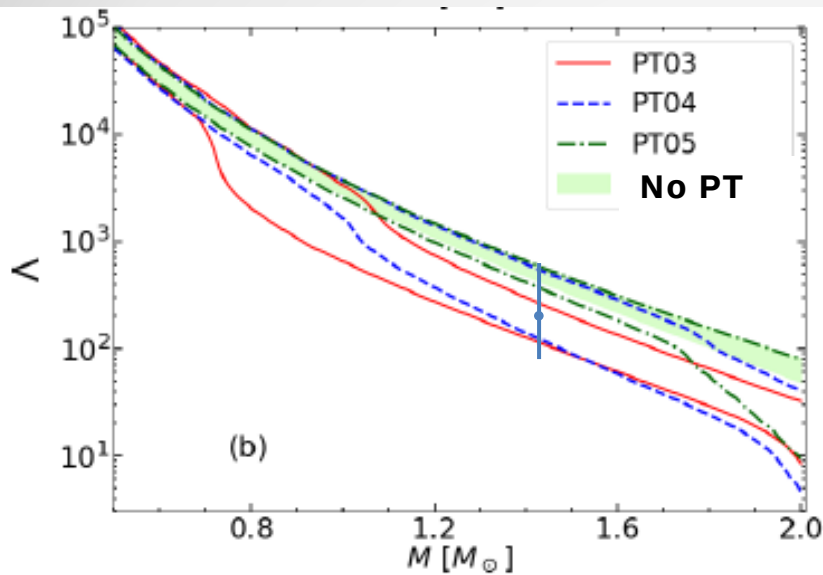


H.Dinh Thi et al, A&A 2021

- The observations are compatible with a pure nucleonic composition



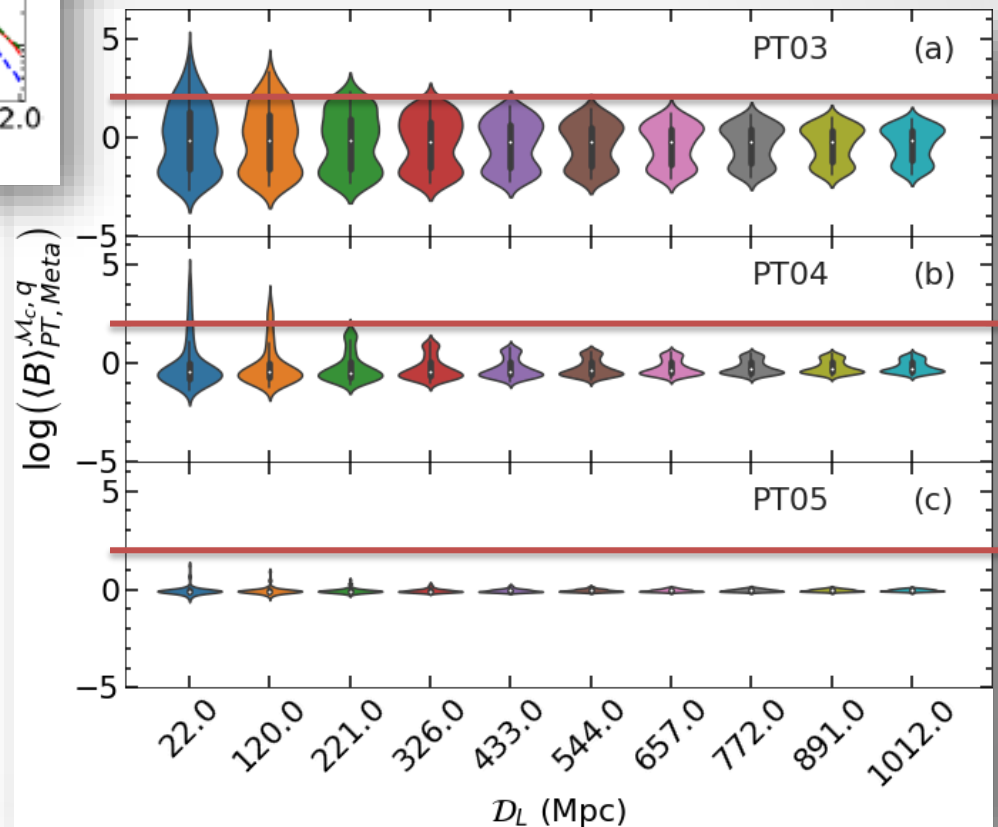
Quarks in the core of neutron stars ?



- Even with AdV+ sensitivity, only a very close detection would allow identifying deconfined matter, and only if the quark core is large

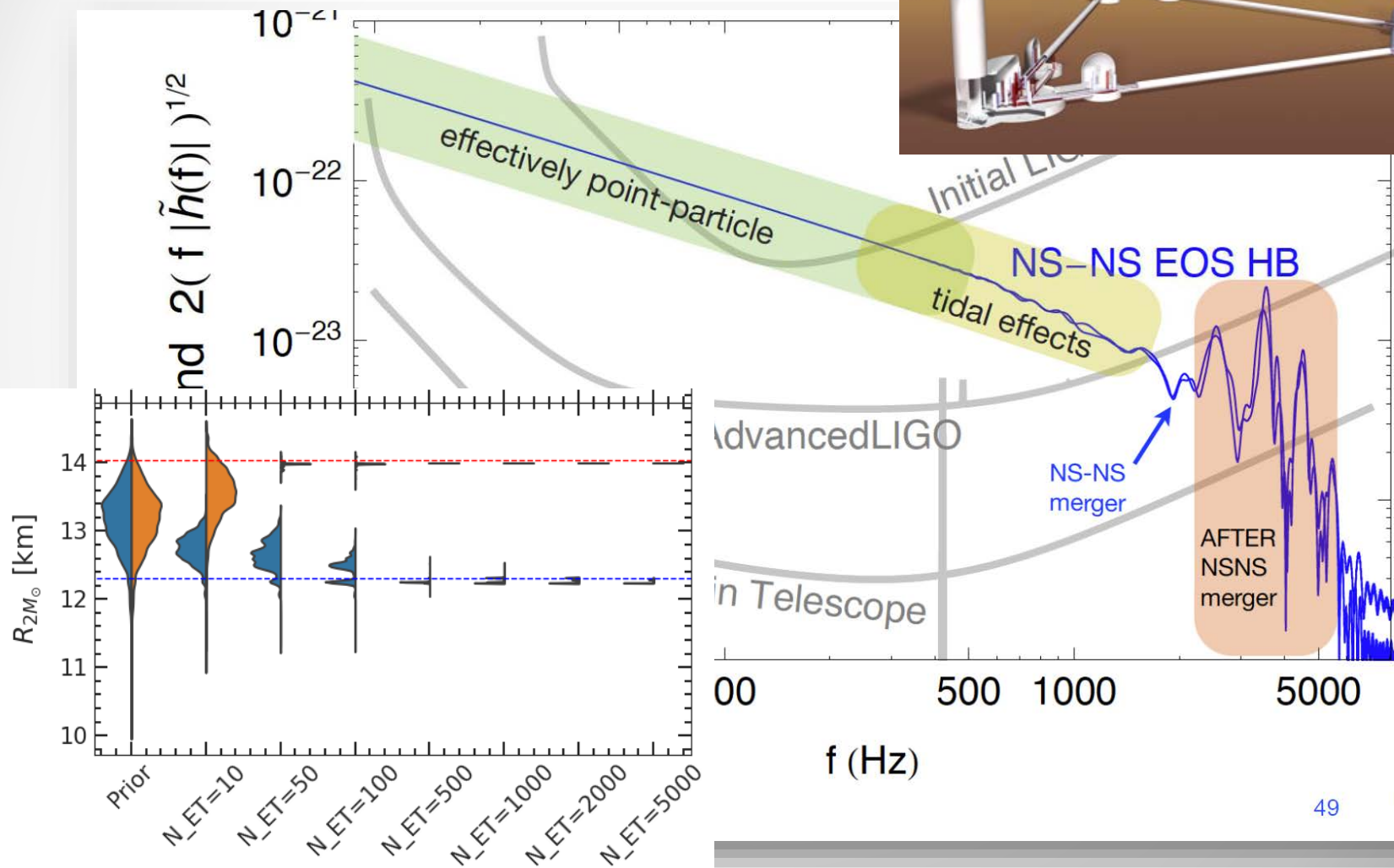
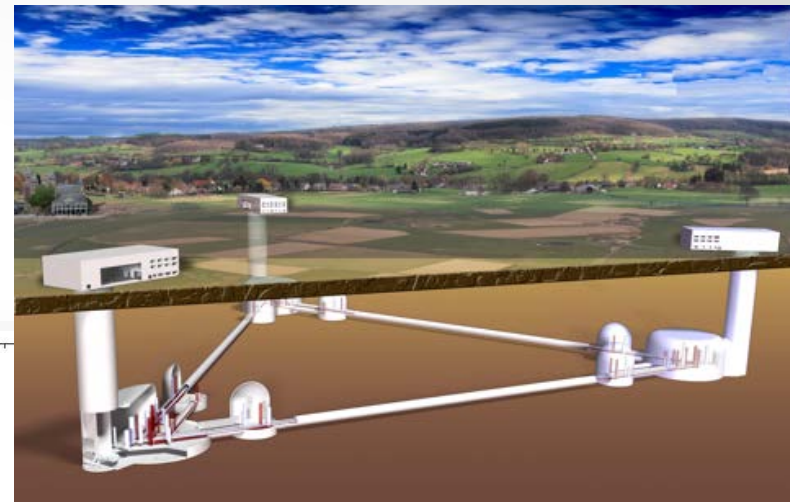
- Need to reduce the uncertainties!
 - New nuclear observables
 - Multiple detections with better SNR => Einstein Telescope, Cosmic Explorer

C.Mondal et al, MNRAS 2023



The Einstein Telescope project

F.Iacovelli, C.Mondal et al, PRD 2023



Summary & Conclusions

- The Equation of State can be univocally mapped to the static properties of NS $\Leftrightarrow$ to the GW signals from NS mergers
- No ab-initio model of nuclear matter for all densities! But Bayesian techniques allow controlled extrapolations of low density constraints from nuclear theory and experiments
 - => No present indication of exotic degrees of freedom**
- Relatively tight observable prediction within the nucleonic hypothesis
 - => A 1st order phase transition to deconfined matter can potentially be detected with 3G interferometers**