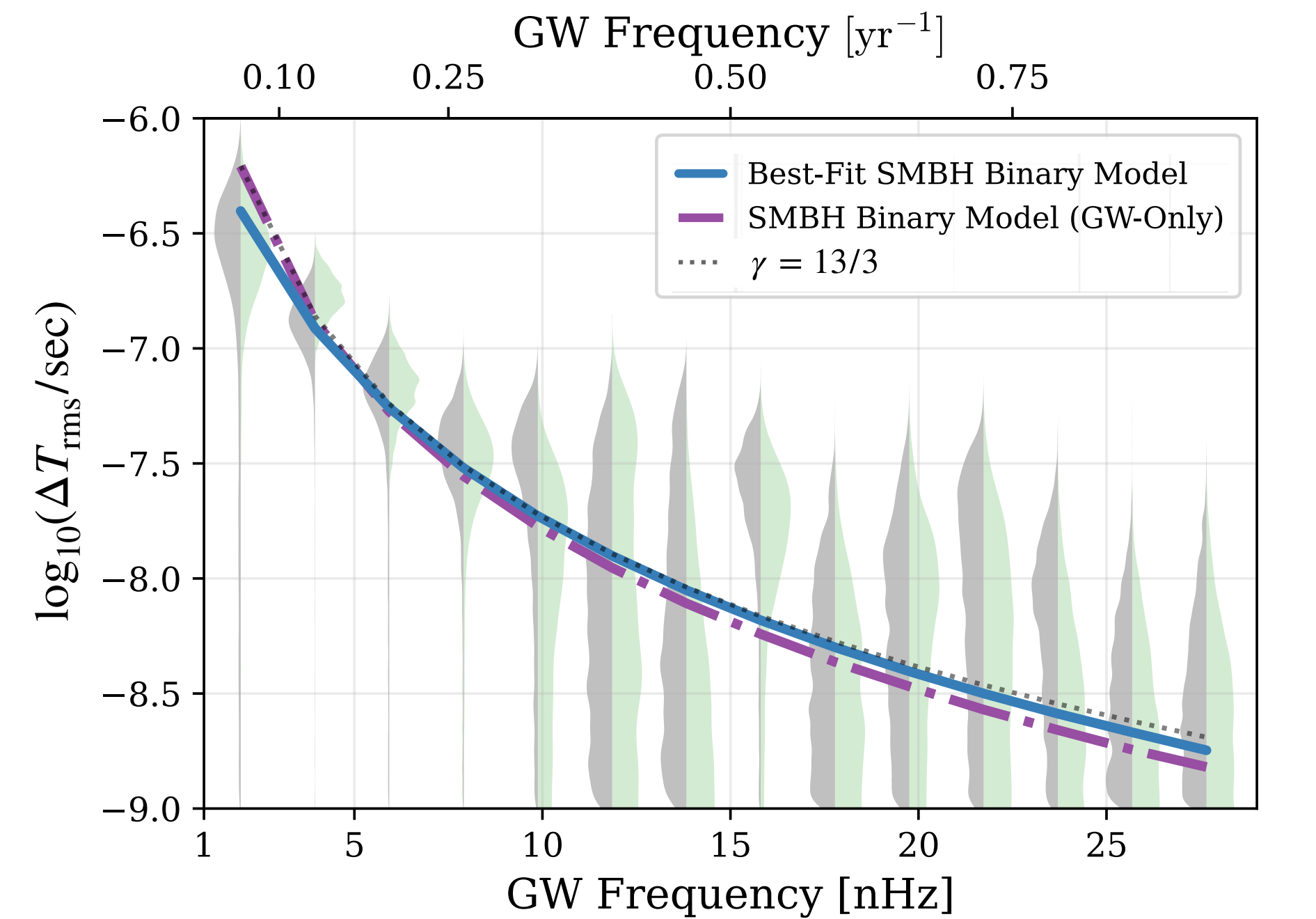
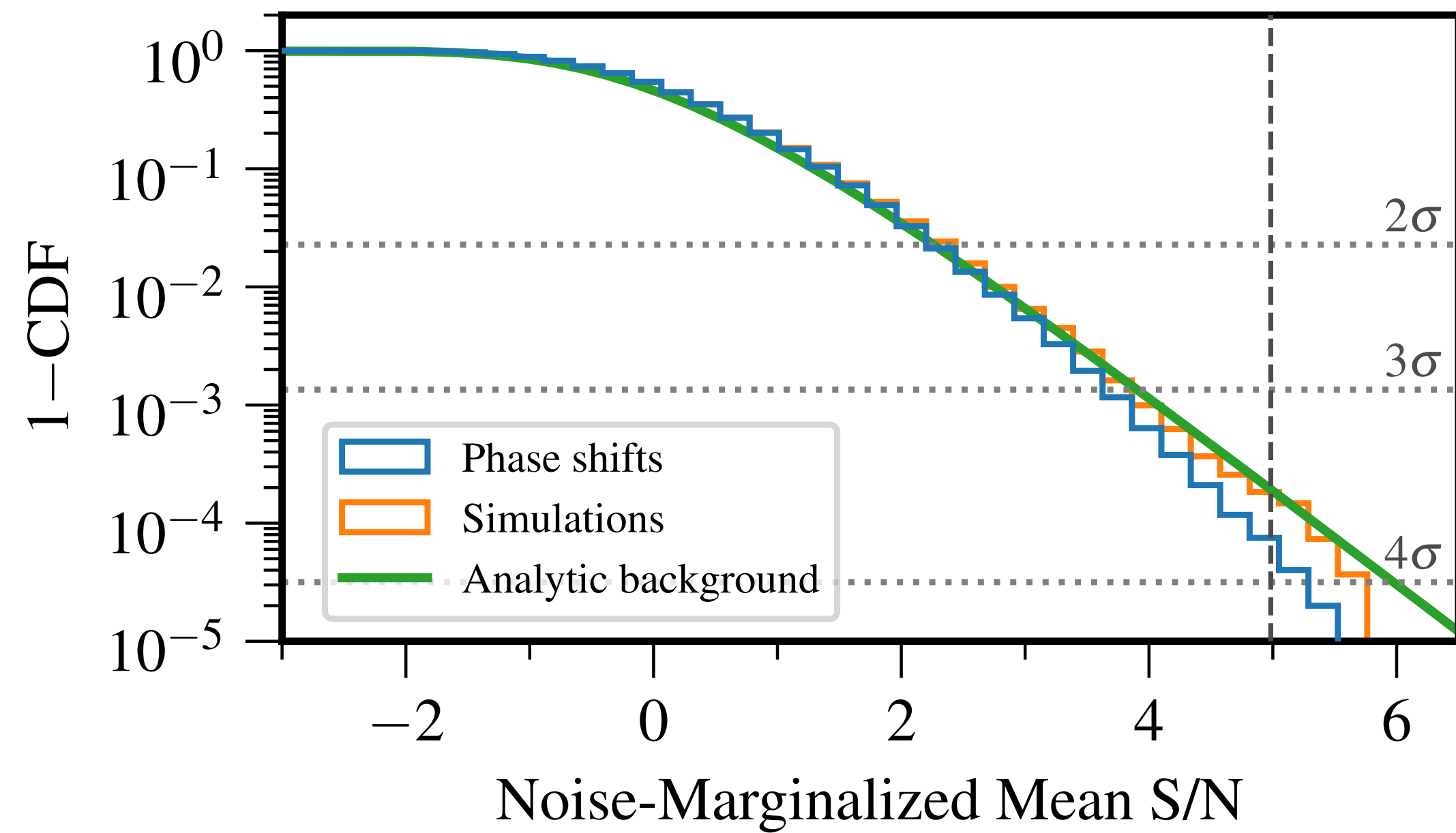
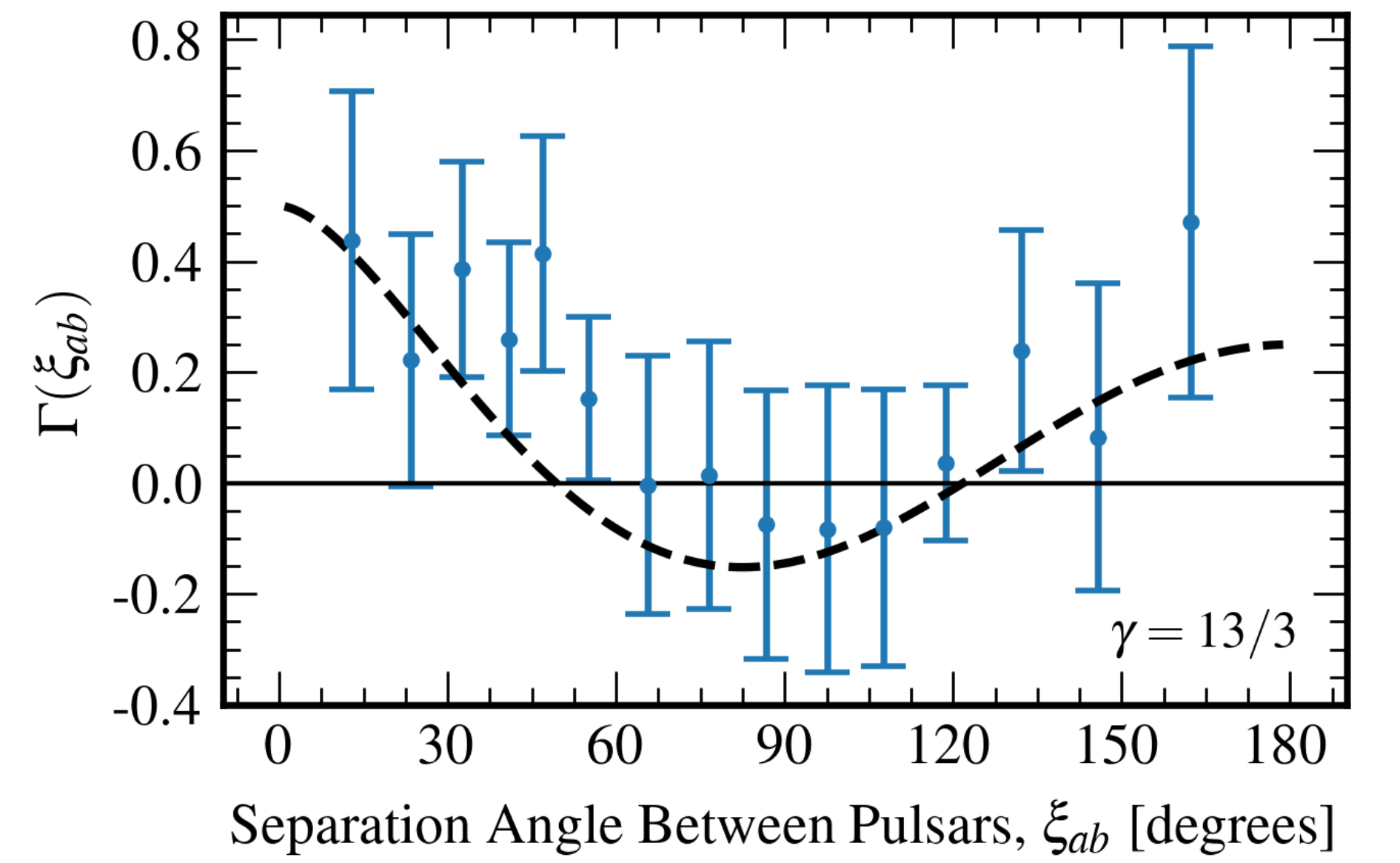
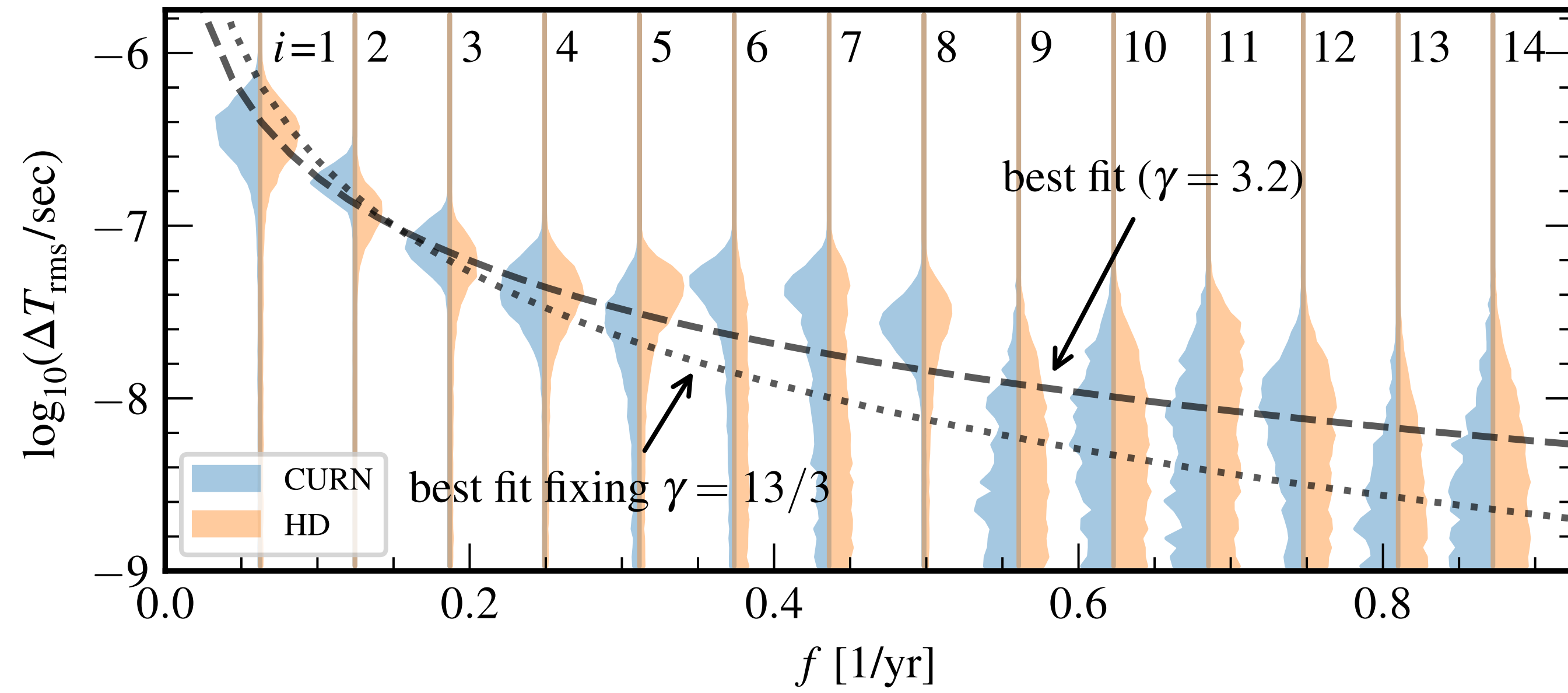
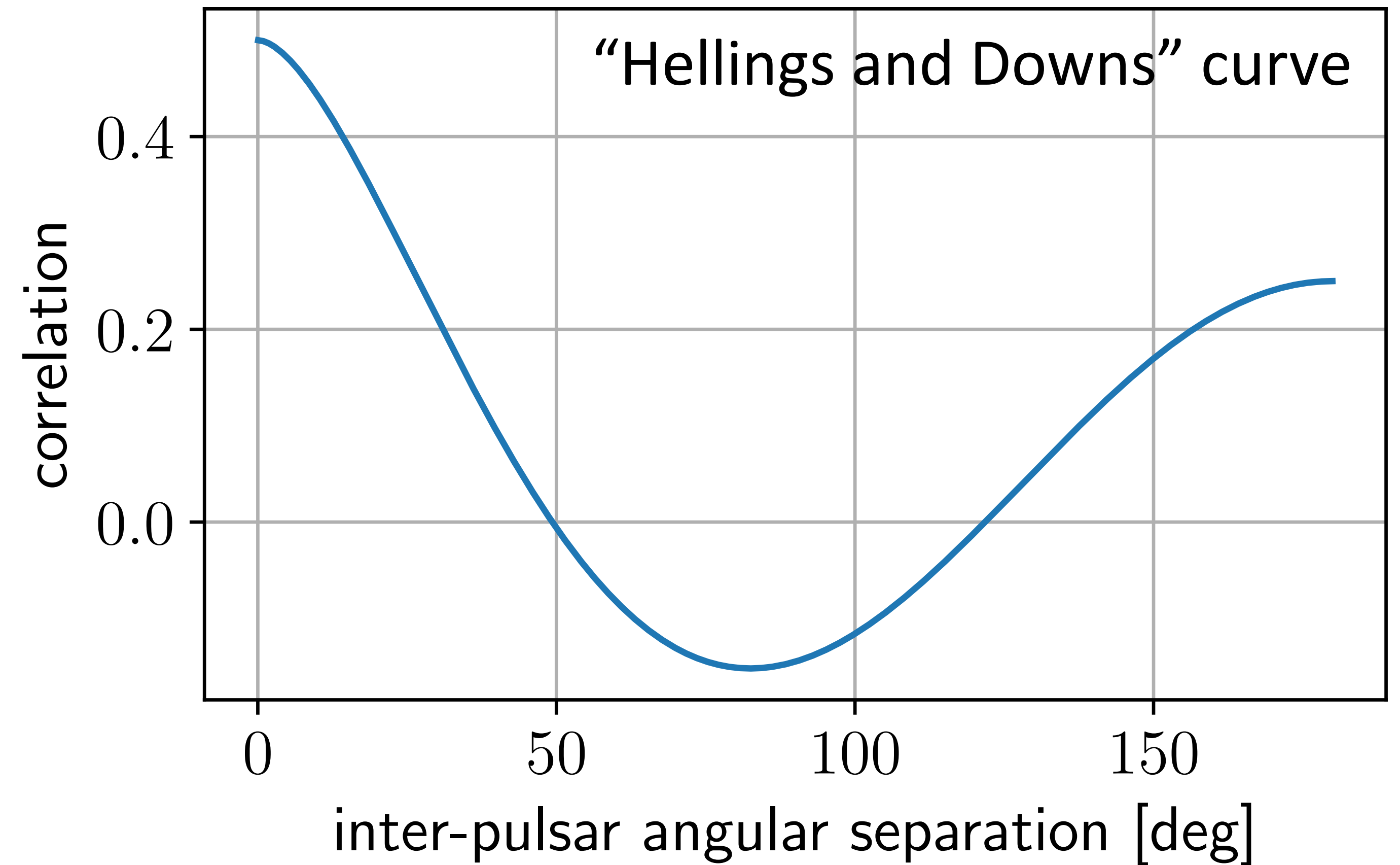
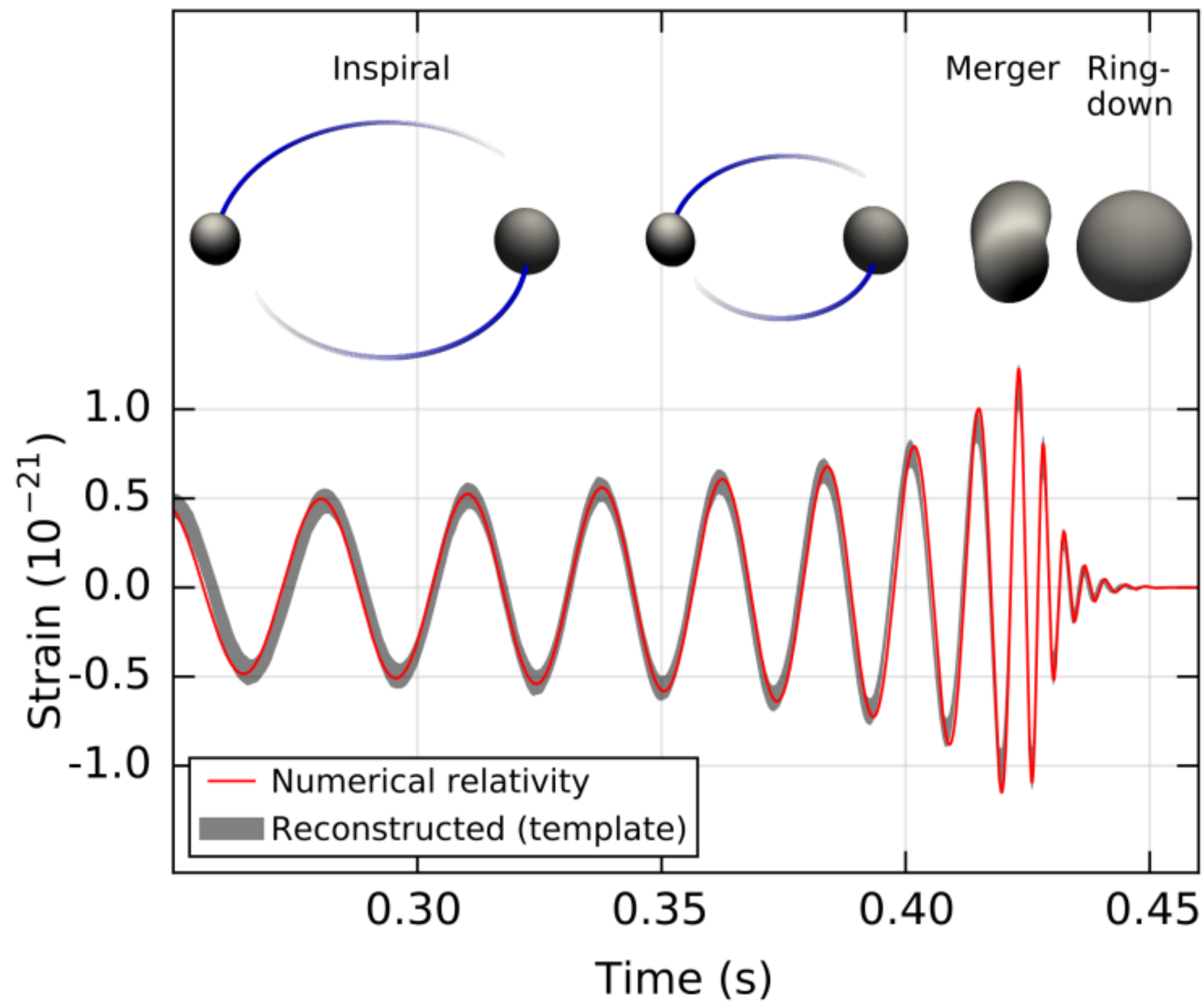


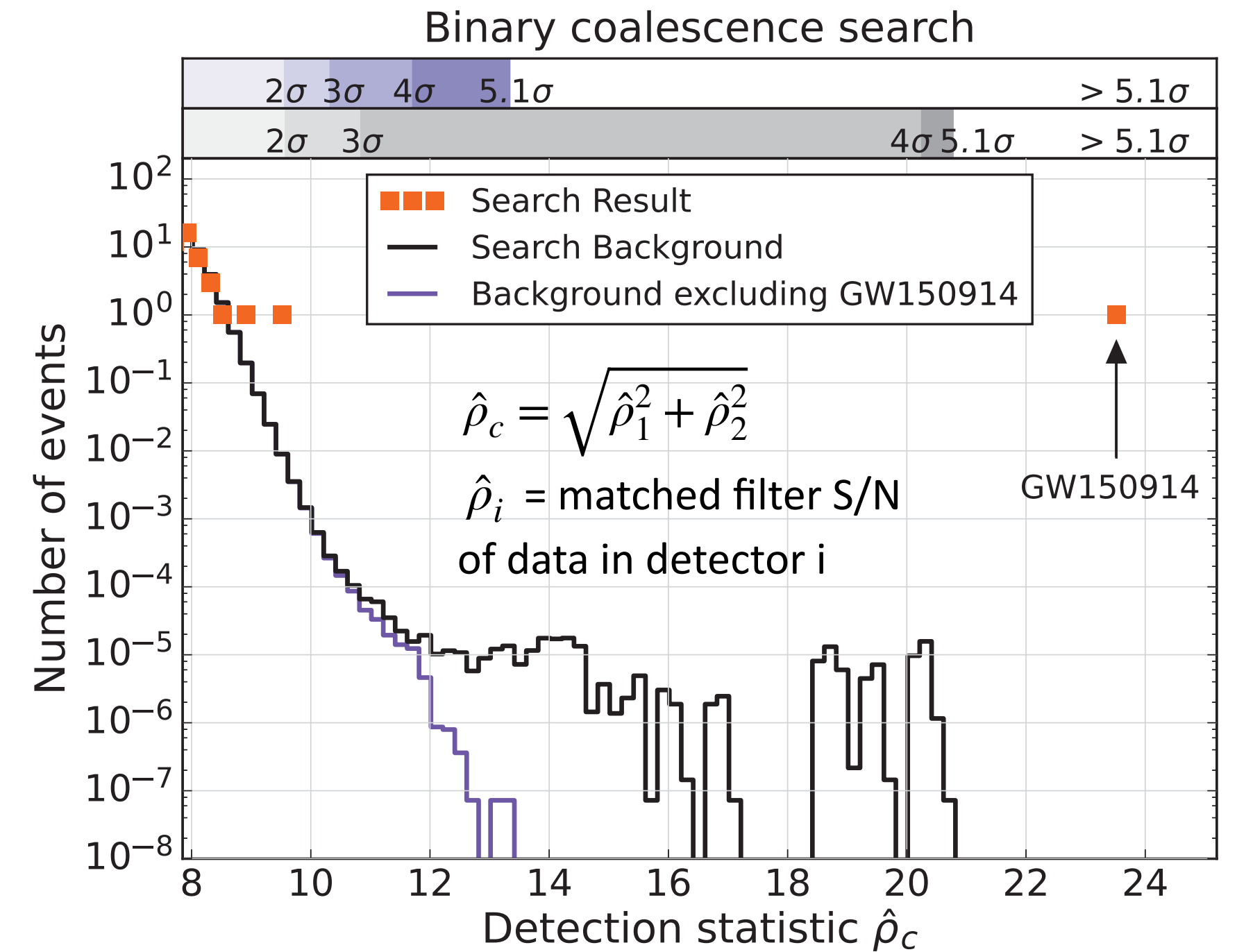
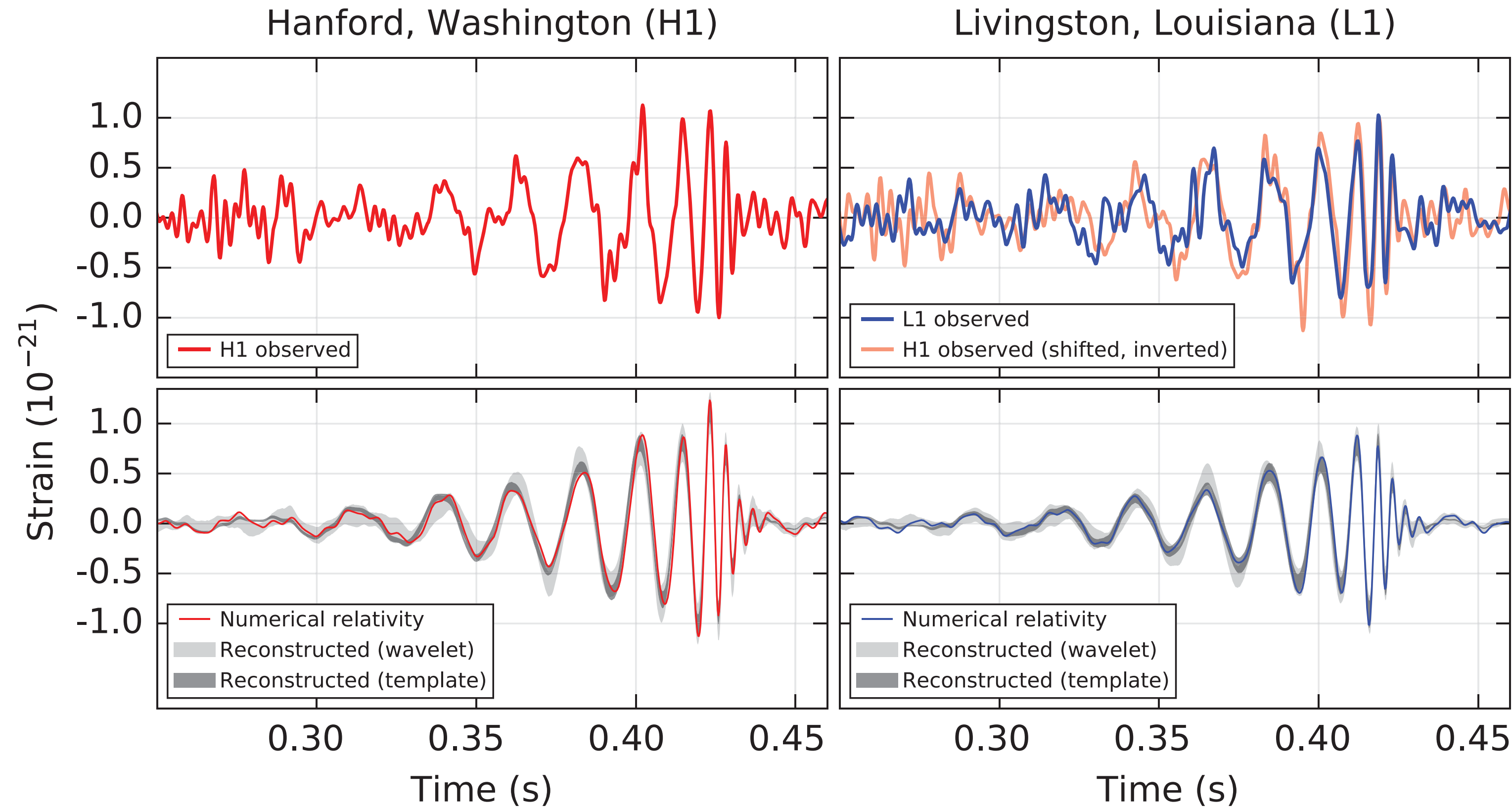


Part I. Theory and derivations

What plays the role of a binary “chirp” waveform for PTAs?



Why was GW150914 so convincing?



1. observed signal is consistent across detectors
2. observed signal agrees with predictions
3. observed signal is unlikely due to noise alone ($< 1/5$ million)

Table-top LIGO interferometer

Q: Is there an equivalent table-top demo for a pulsar timing array?

Y: Metronome timing array

[AJP: Lam et al, 2018]

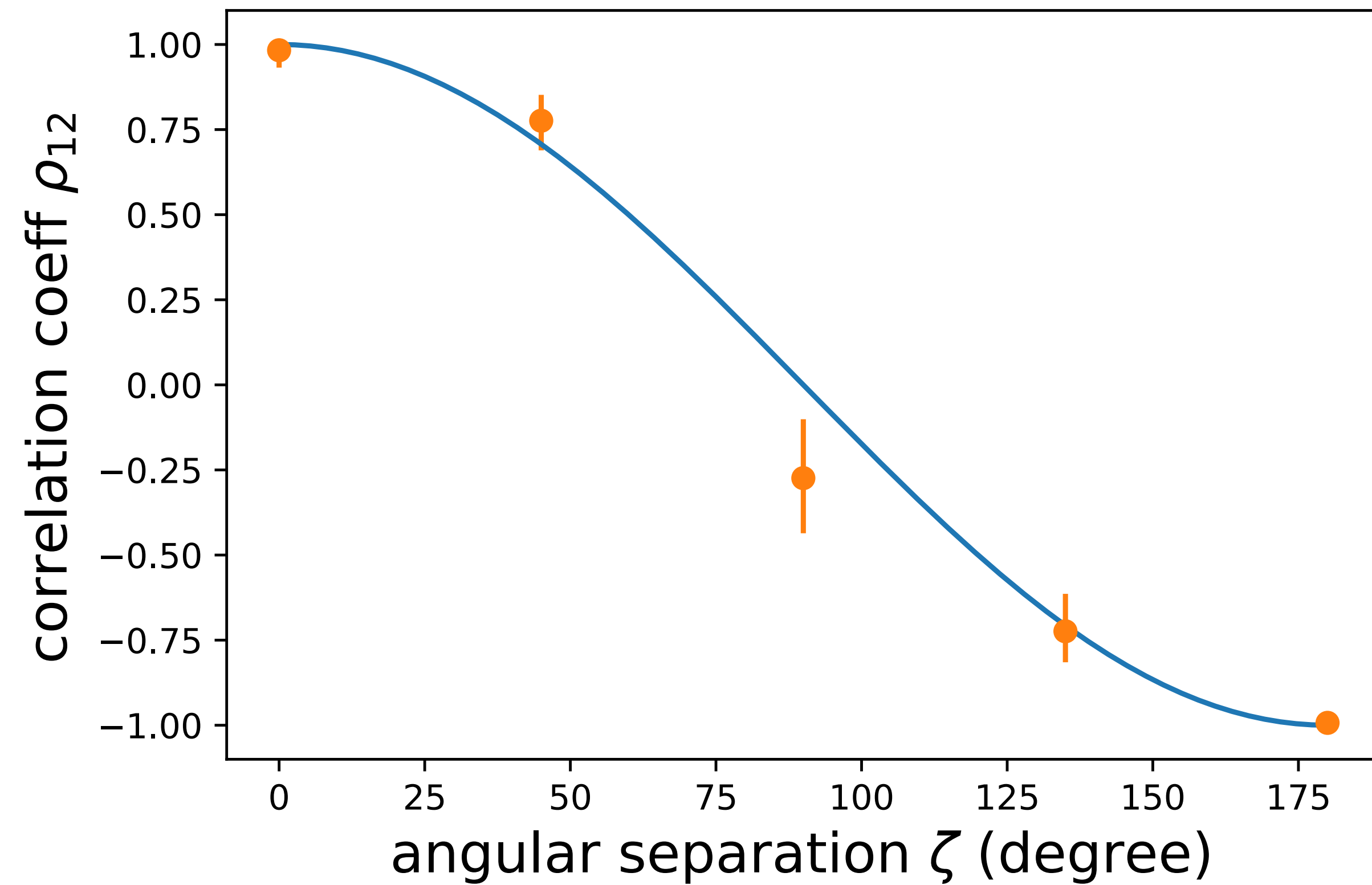
https://github.com/nanograv/tabletop_pta/

https://github.com/marcnormandin/tabletop_pta/

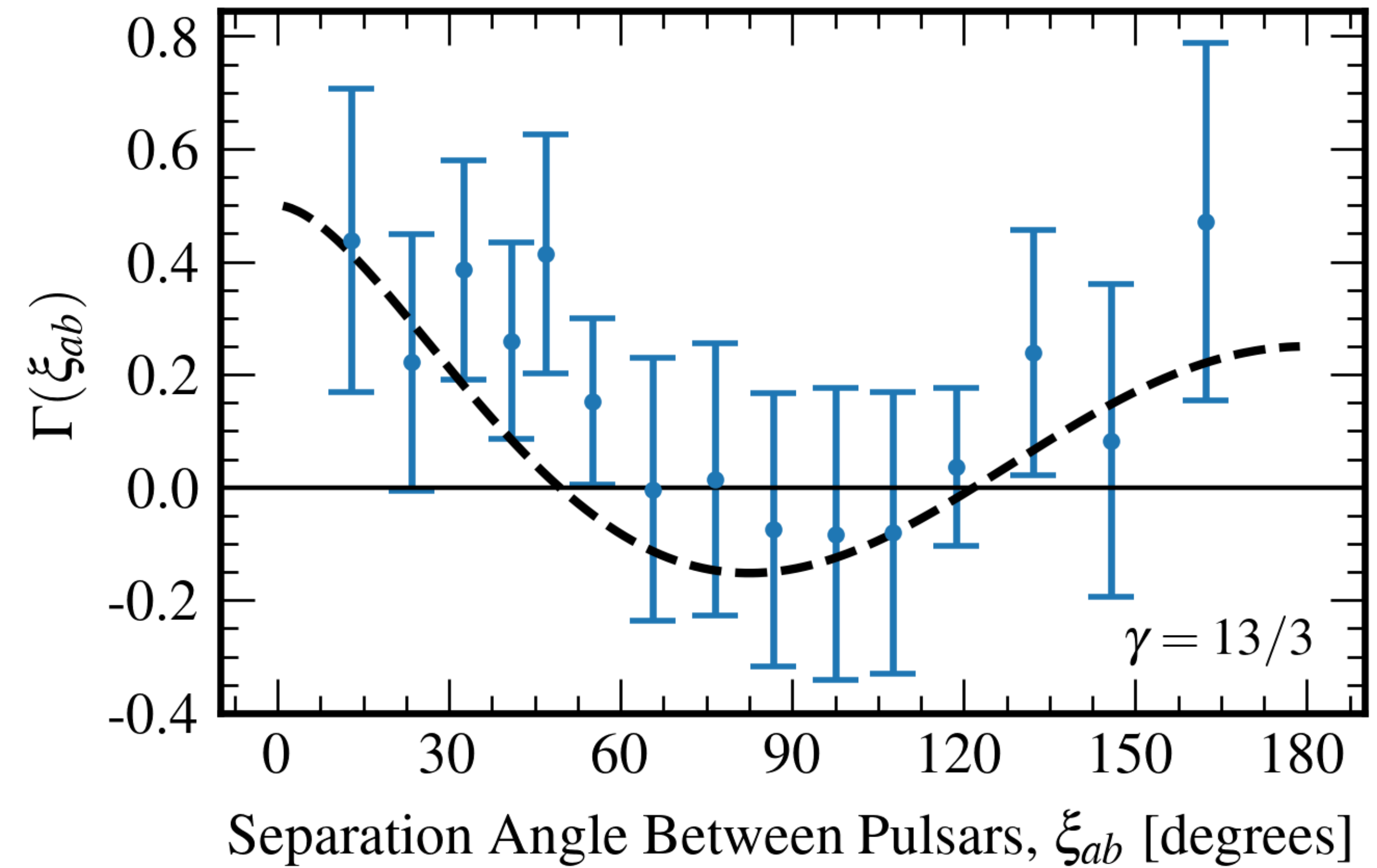
<https://github.com/josephromano/pta-demo/>

Measured and expected correlation

[sample metronome analysis]



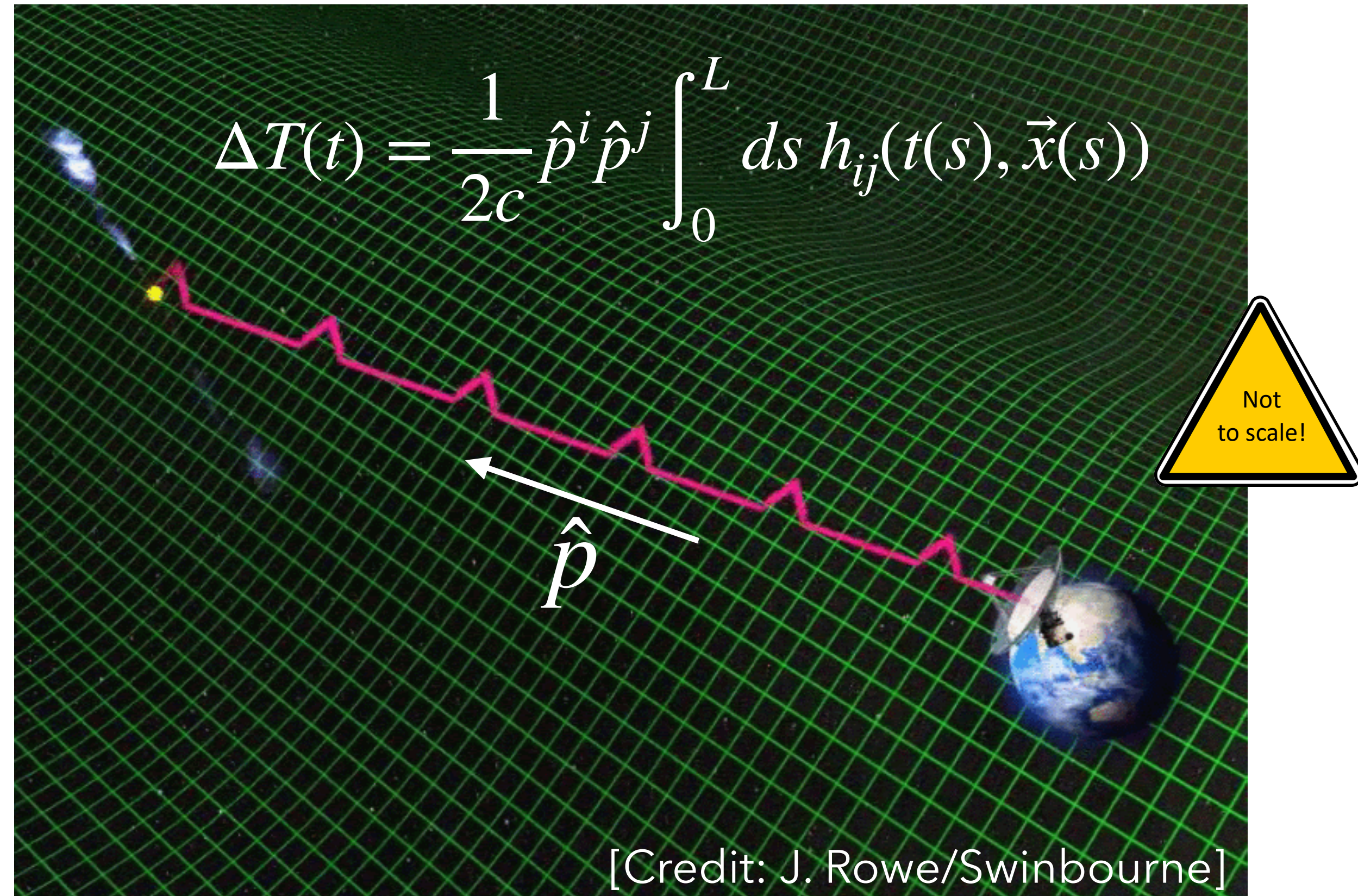
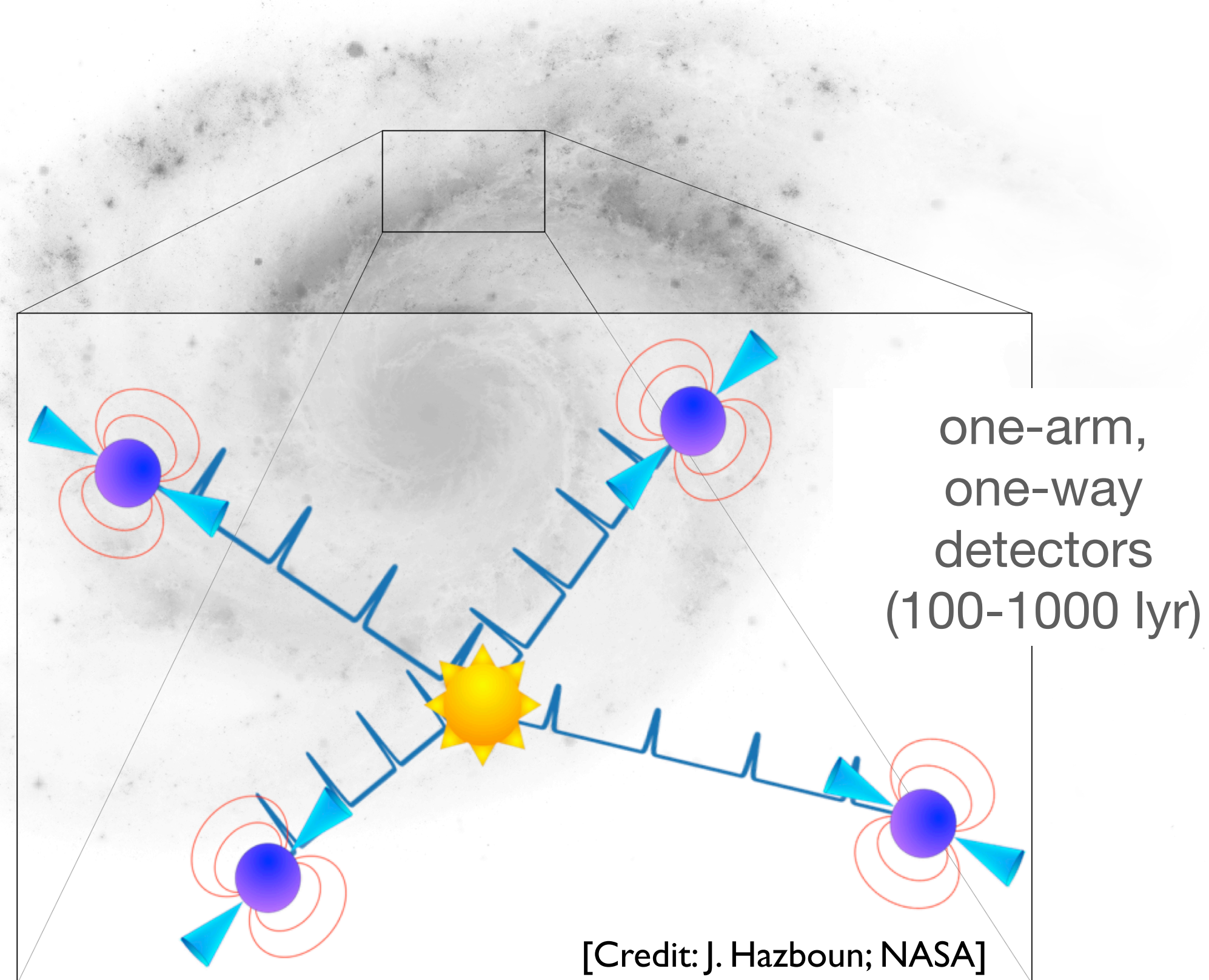
[NANOGrav 15-yr analysis]



Pulsar timing arrays (PTAs)

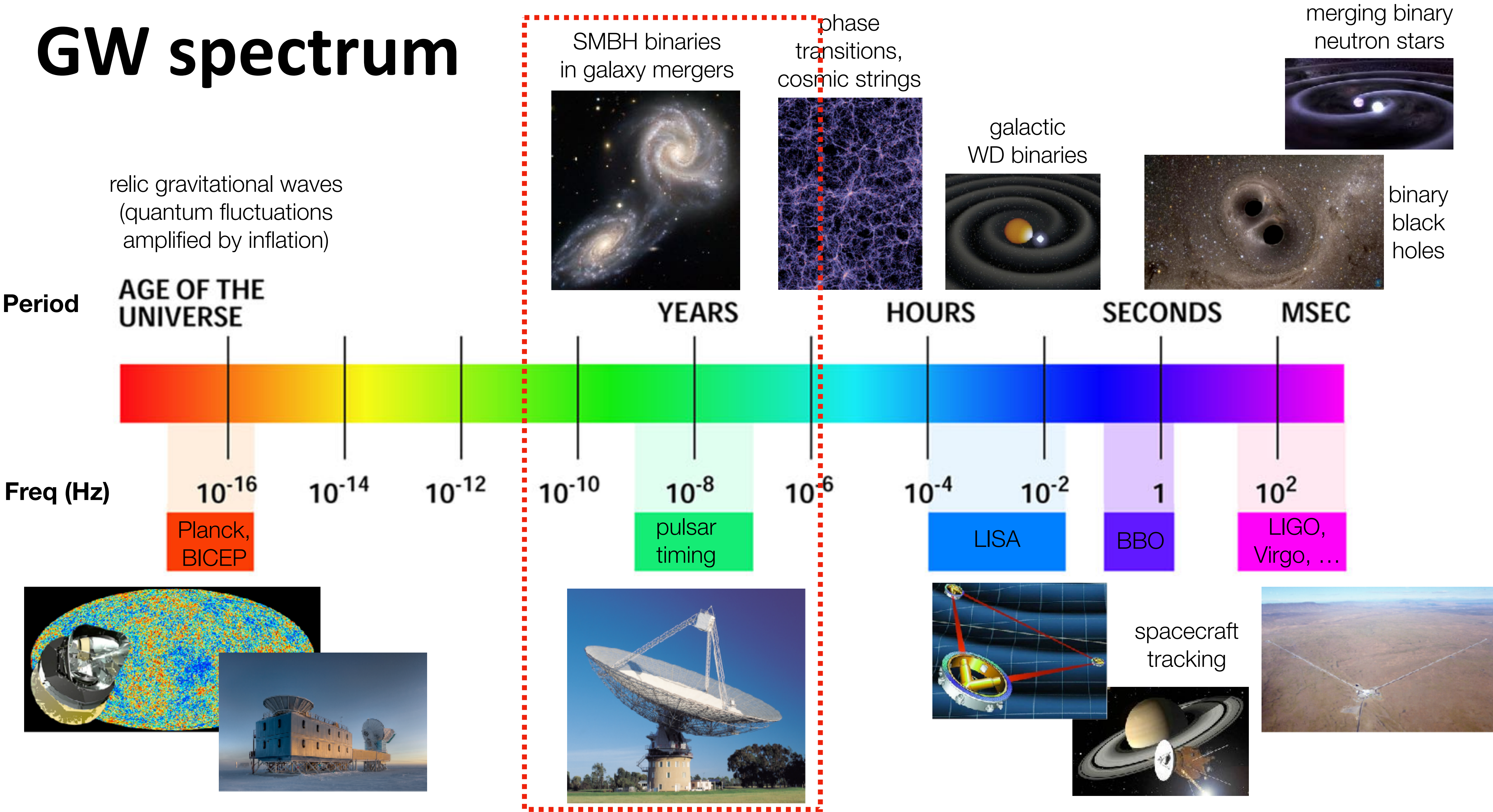
What is a pulsar timing array (PTA)?

Galactic-scale GW detector



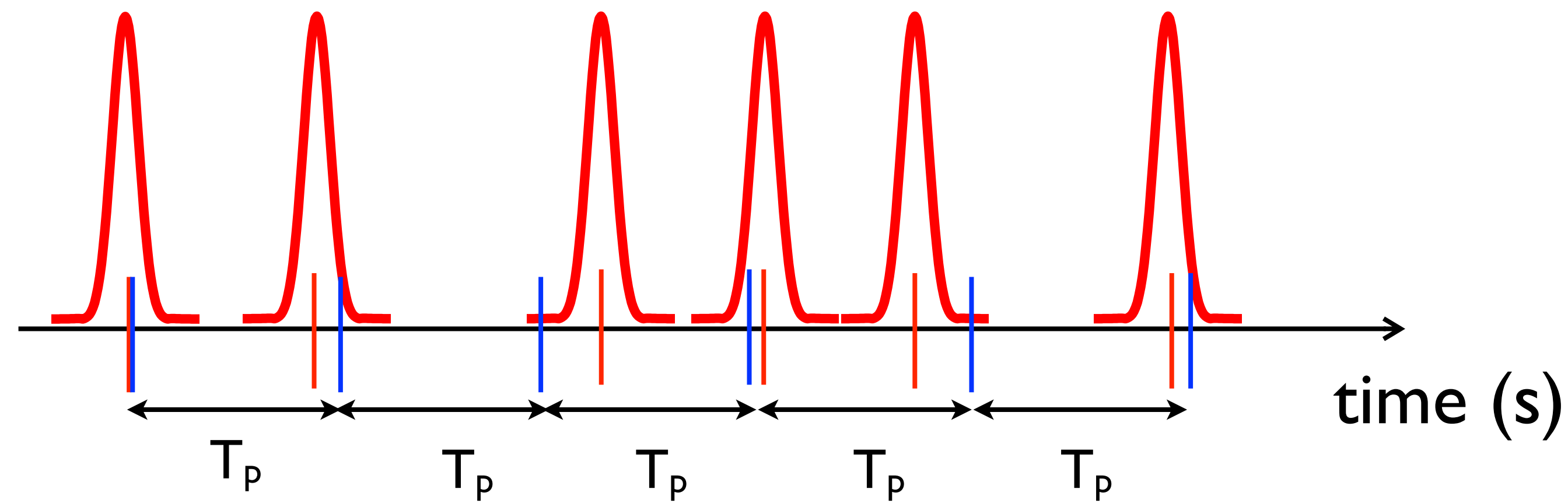
- GWs **perturb pulse arrival times** -> look for evidence of GWs in the **timing residuals**
- GW perturbations will be **correlated across pulsars** -> use this to **differentiate GWB from noise**

GW spectrum

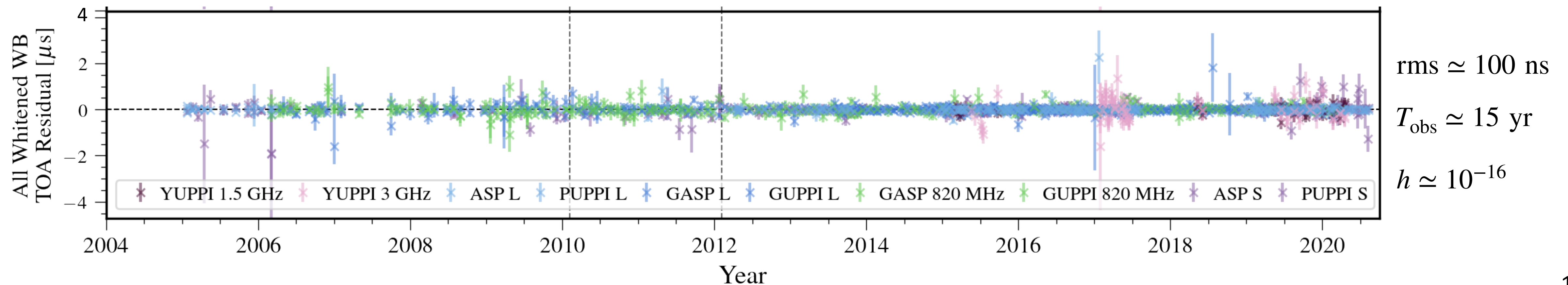


What are the data for PTA analyses?

timing residual = observed arrival – predicted arrival ← **timing model:** pulsar's spin period, period derivative, sky location, proper motion, ...
= unmodeled deterministic processes + noise sources + GW signals



PSR J1713+0747



How do GWs affect timing residuals?

- Perturbations to pulse arrival times:

$$\Delta T(t) = \frac{1}{2c} \hat{p}^i \hat{p}^j \int_0^L ds h_{ij}(t(s), \vec{x}(s))$$

$$t(s) = t - (L - s)/c, \quad \vec{x}(s) = (L - s)\hat{p}$$

- Doppler shift (“redshift/blueshift”) of pulse frequency :

$$Z(t) \equiv \frac{d\Delta T(t)}{dt} = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} \left[h_{ij}(t, \vec{0}) - h_{ij}(t - L/c, L\hat{p}) \right]$$

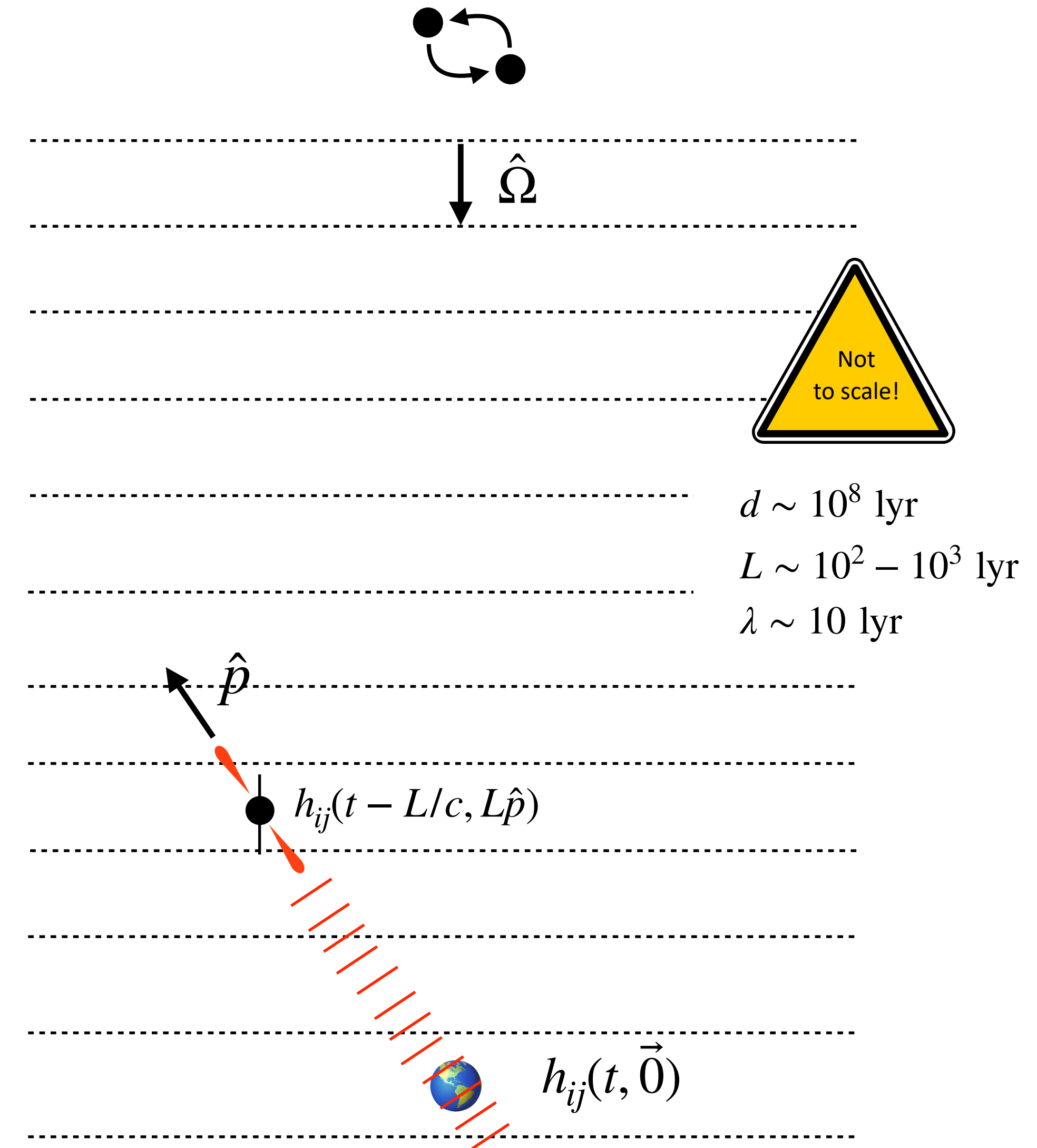
Earth term pulsar term

uses $h_{ij}(t, \vec{x}) = h_{ij}(t - \hat{\Omega} \cdot \vec{x}/c), \quad \frac{\partial h_{ij}}{\partial t} = \frac{c}{(1 + \hat{\Omega} \cdot \hat{p})} \frac{\partial h_{ij}}{\partial s}$

- In terms of polarizations $A = +, \times$:

$$Z(t) = \sum_{A=+, \times} \left[h^A(t) - h^A(t - L(1 + \hat{\Omega} \cdot \hat{p})/c) \right] F^A(\hat{\Omega})$$

$$F^A(\hat{\Omega}) = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} e_{ij}^A(\hat{\Omega}) \quad (\text{antenna pattern})$$



Can we detect GWs using data from a single pulsar?

No. Need to correlate data from multiple pulsars

- For expected correlations, can restrict to Earth-term contributions:

$$Z_a(t) = h^+(t)F_a^+(\hat{\Omega}) + h^\times(t)F_a^\times(\hat{\Omega}) \quad Z_b(t) = h^+(t)F_b^+(\hat{\Omega}) + h^\times(t)F_b^\times(\hat{\Omega})$$

$$F_a^A(\hat{\Omega}) = \frac{1}{2} \frac{\hat{p}_a^i \hat{p}_a^j}{1 + \hat{\Omega} \cdot \hat{p}_a} e_{ij}^A(\hat{\Omega}) \quad F_b^A(\hat{\Omega}) = \frac{1}{2} \frac{\hat{p}_b^i \hat{p}_b^j}{1 + \hat{\Omega} \cdot \hat{p}_b} e_{ij}^A(\hat{\Omega})$$

- Correlation is time-averaged product:

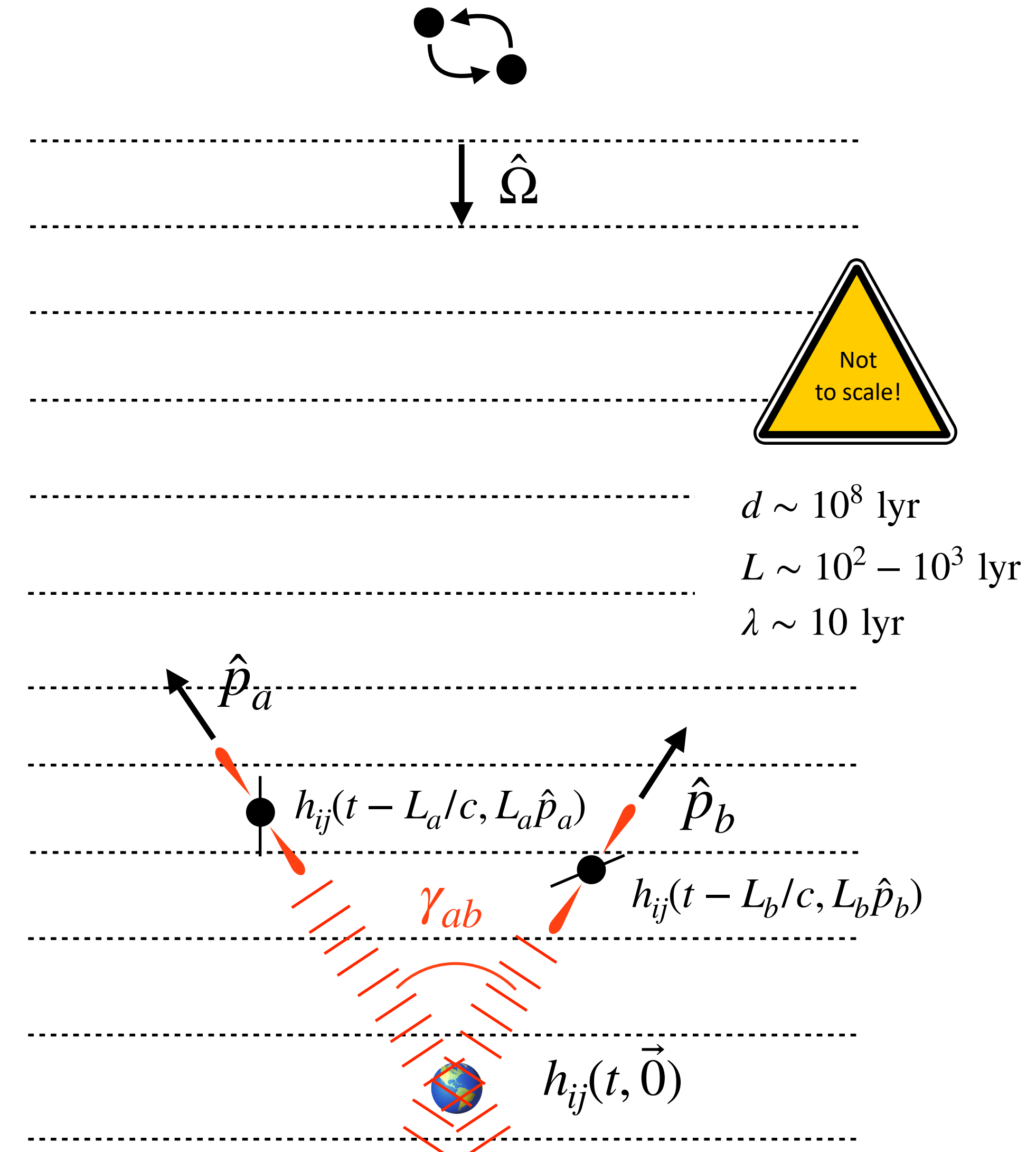
$$\begin{aligned} \rho_{ab} &\equiv \overline{Z_a(t)Z_b(t)} \equiv \frac{1}{T} \int_0^T dt Z_a(t)Z_b(t) \\ &= \overline{(h^+)^2} F_a^+(\hat{\Omega})F_b^+(\hat{\Omega}) + \overline{(h^\times)^2} F_a^\times(\hat{\Omega})F_b^\times(\hat{\Omega}) + \overline{h^+h^\times} \left(F_a^+(\hat{\Omega})F_b^\times(\hat{\Omega}) + F_a^\times(\hat{\Omega})F_b^+(\hat{\Omega}) \right) \\ &= F_a^+(\hat{\Omega})F_b^+(\hat{\Omega}) + F_a^\times(\hat{\Omega})F_b^\times(\hat{\Omega}) \quad (\text{unpolarized, unit amplitude}) \end{aligned}$$

- Hellings & Downs 1983:** fix pulsars; average over the GW source direction and polarization angle

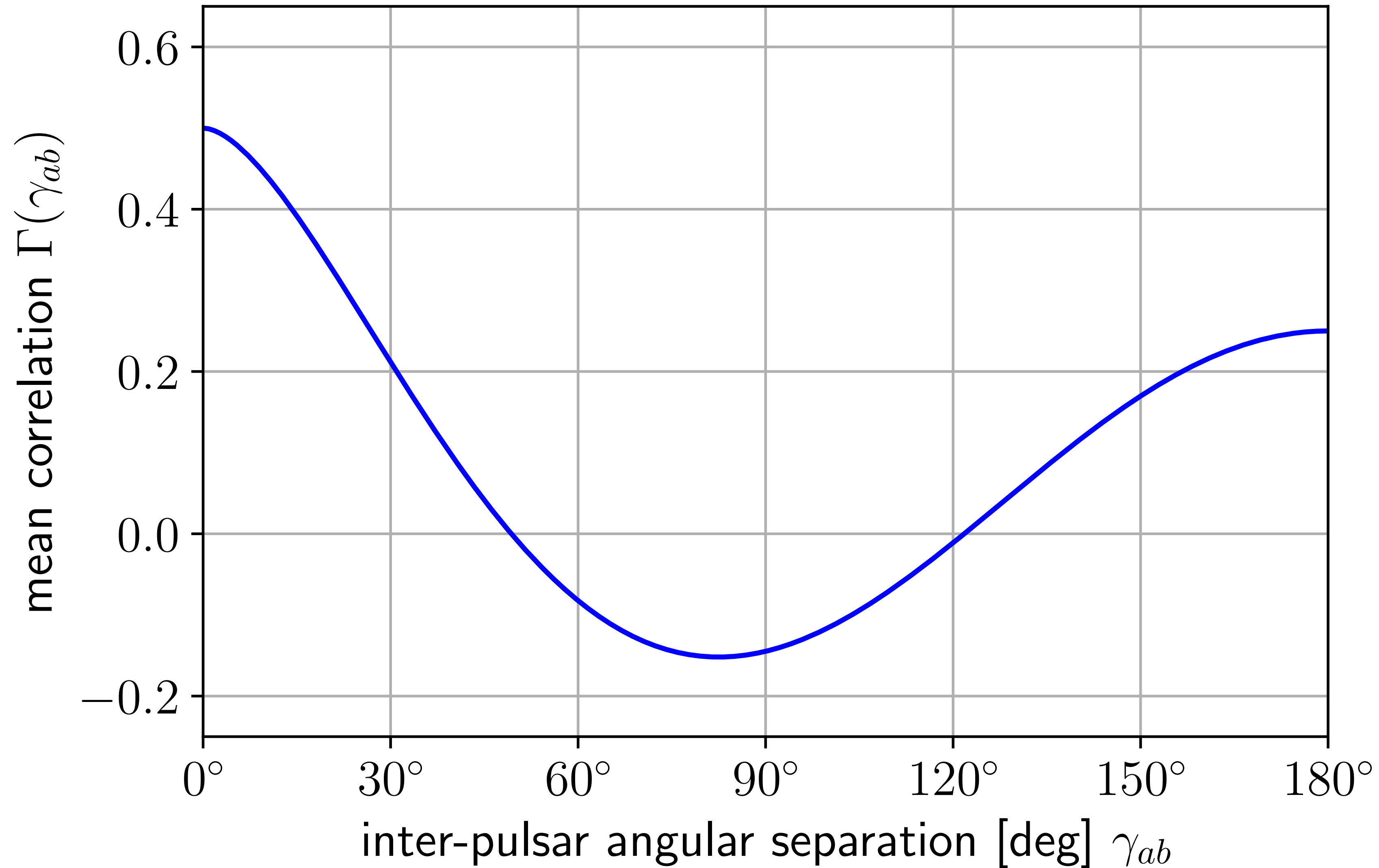
≡

Cornish & Sesana 2013: fix GW point source; average over all pulsar pairs separated by angle γ_{ab}

$$\langle \rho_{ab} \rangle_p = \langle \rho_{ab} \rangle_s = \frac{1}{2} - \frac{1}{4} \left(\frac{1 - \cos \gamma_{ab}}{2} \right) + \frac{3}{2} \left(\frac{1 - \cos \gamma_{ab}}{2} \right) \ln \left(\frac{1 - \cos \gamma_{ab}}{2} \right) \equiv \Gamma(\gamma_{ab})$$

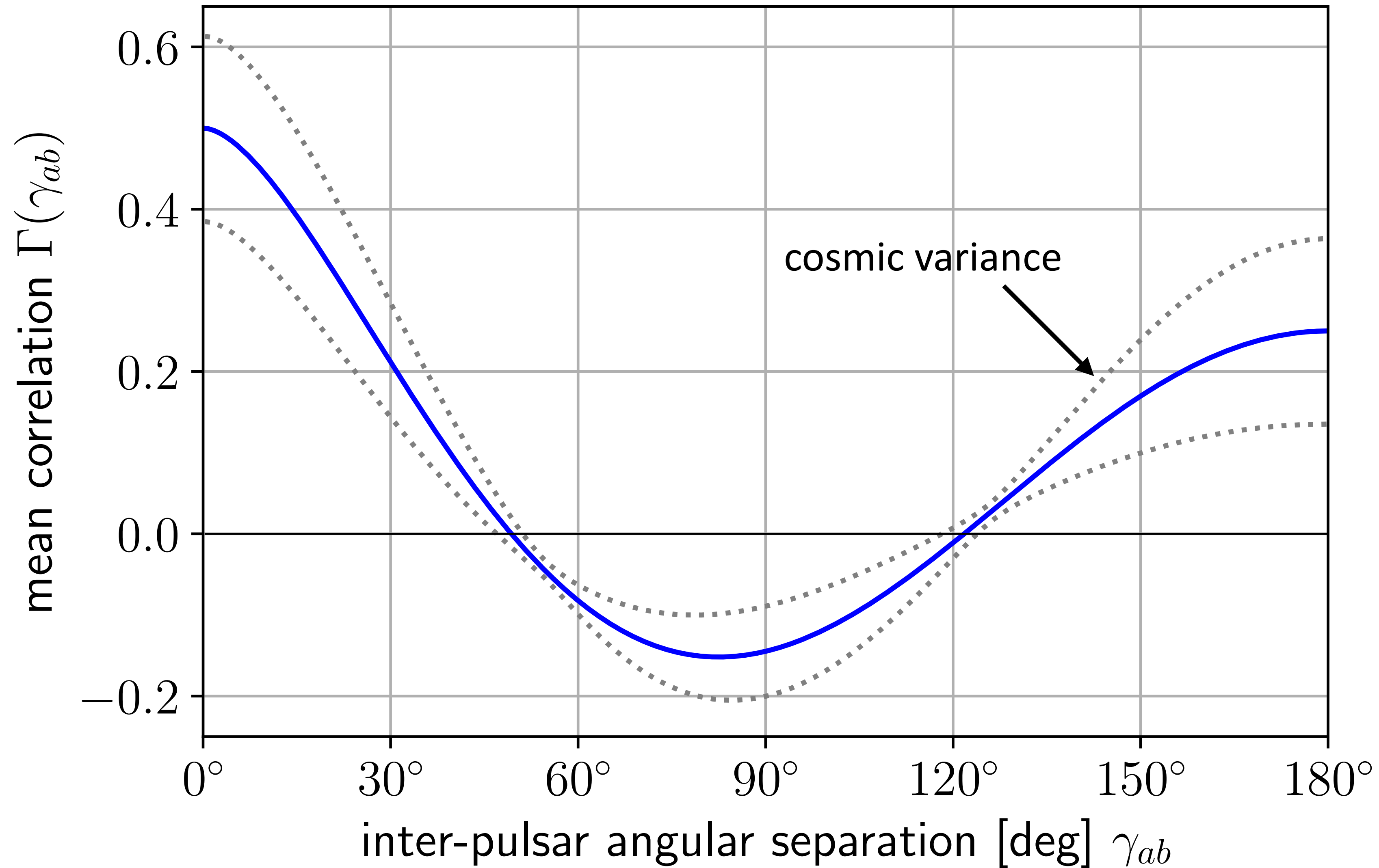


Hellings and Downs curve



Should we expect to recover the HD curve exactly?

Cosmic variance for interfering sources



Correlation calculation for interfering sources

- Two sources, **same frequency** (ignore polarization):

$$h_1(t) = A_1 \cos(2\pi f t + \phi_1), \quad h_2(t) = A_2 \cos(2\pi f t + \phi_2)$$

$$Z_a(t) = h_1(t)F_a(\hat{\Omega}_1) + h_2(t)F_a(\hat{\Omega}_2)$$

$$Z_b(t) = h_1(t)F_b(\hat{\Omega}_1) + h_2(t)F_b(\hat{\Omega}_2)$$

- Correlation:

$$\rho_{ab} = \overline{Z_a(t)Z_b(t)}$$

$$= \overline{h_1^2} F_a(\hat{\Omega}_1)F_b(\hat{\Omega}_1) + \overline{h_2^2} F_a(\hat{\Omega}_2)F_b(\hat{\Omega}_2) + \overline{h_1 h_2} \left(F_a(\hat{\Omega}_1)F_b(\hat{\Omega}_2) + F_a(\hat{\Omega}_2)F_b(\hat{\Omega}_1) \right)$$

$$= \frac{1}{2} A_1^2 F_a(\hat{\Omega}_1)F_b(\hat{\Omega}_1) + \frac{1}{2} A_2^2 F_a(\hat{\Omega}_2)F_b(\hat{\Omega}_2) + \frac{1}{2} A_1 A_2 \cos(\phi_1 - \phi_2) \left(F_a(\hat{\Omega}_1)F_b(\hat{\Omega}_2) + F_a(\hat{\Omega}_2)F_b(\hat{\Omega}_1) \right)$$

$$\langle \rho_{ab} \rangle_p = \frac{1}{2} \sum_j A_j^2 \Gamma(\gamma_{ab}) + \frac{1}{2} \sum_{j \neq k} A_j A_k \cos(\phi_j - \phi_k) \mu(\gamma_{ab}, \beta_{jk})$$

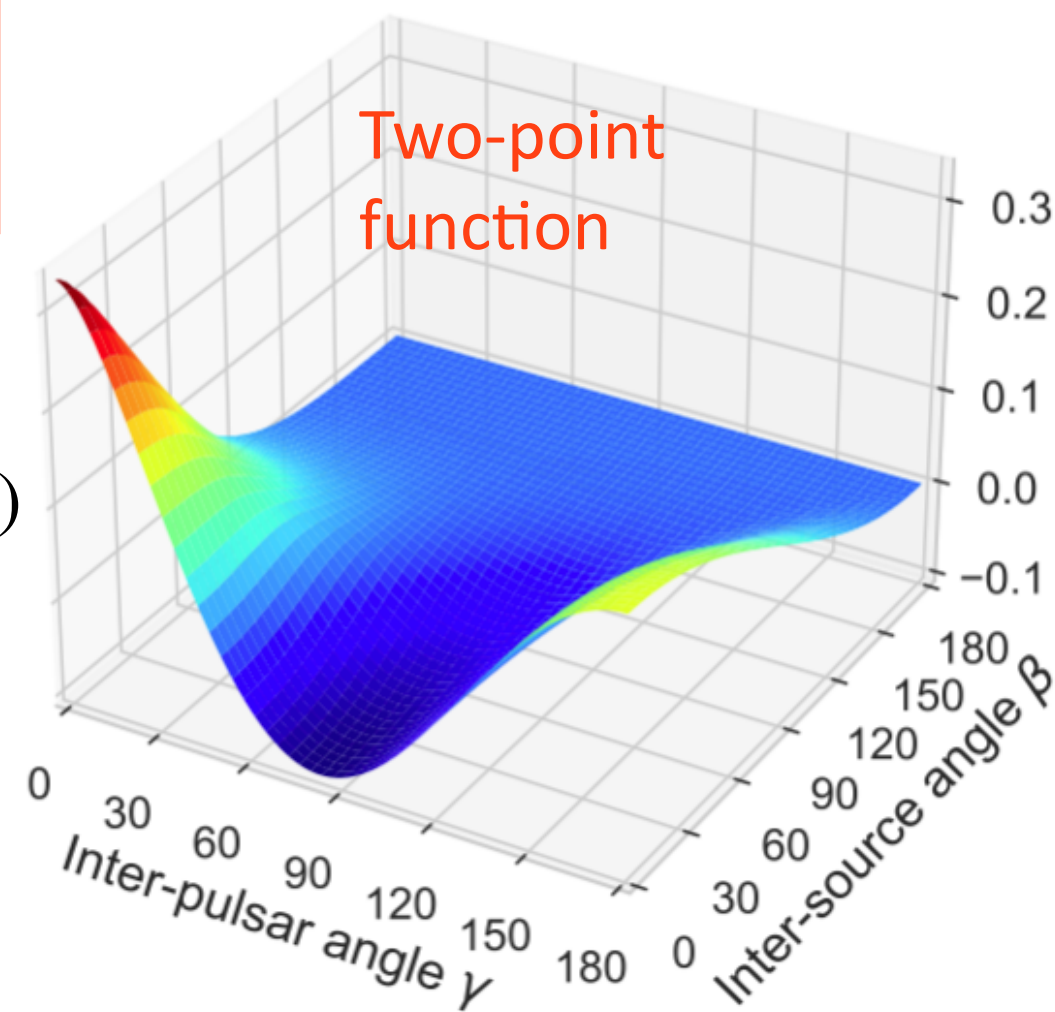
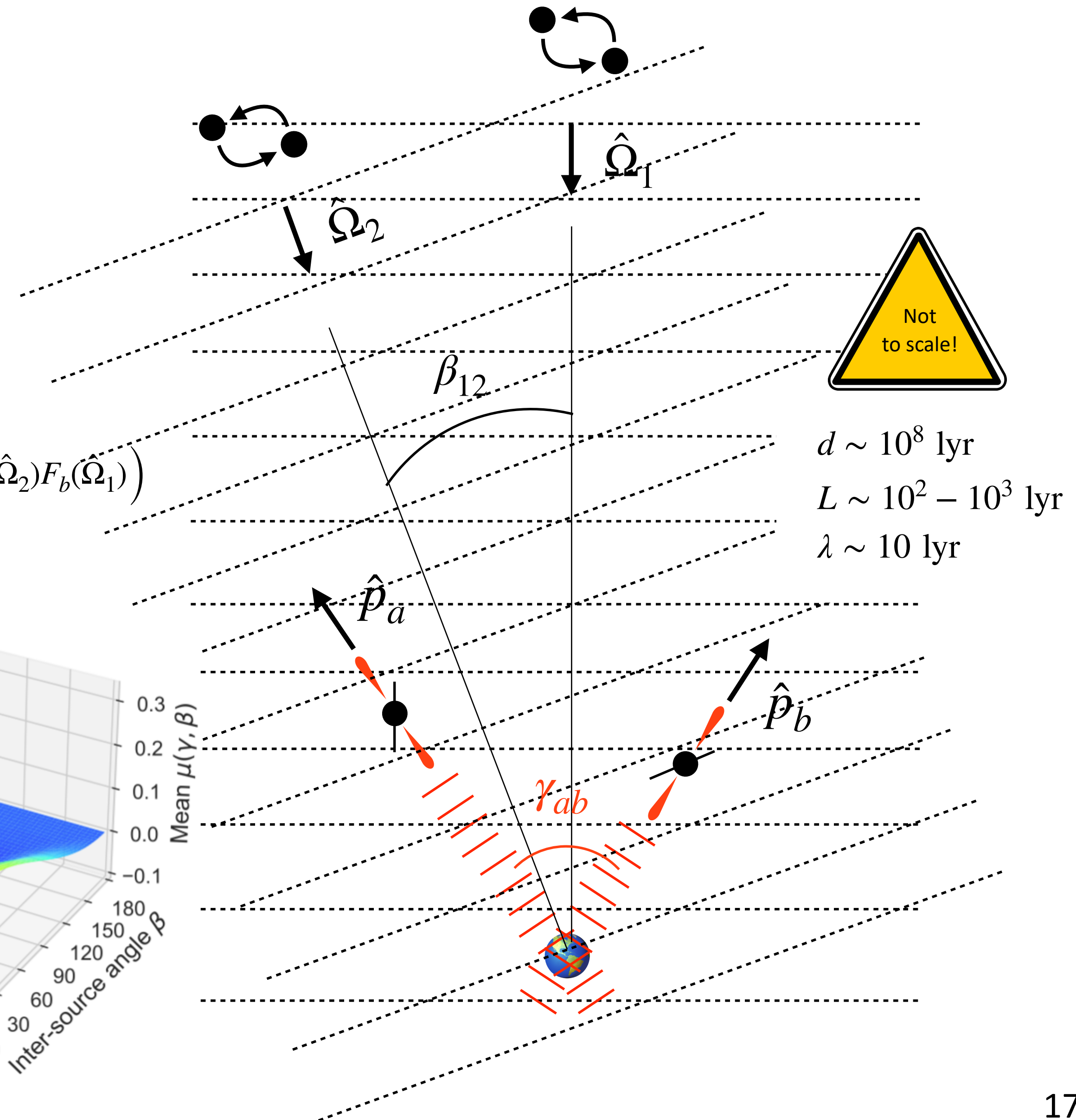
$$\mu(\gamma_{ab}, \beta_{jk}) \equiv \langle F_a^+(\hat{\Omega}_j)F_b^+(\hat{\Omega}_k) + F_a^\times(\hat{\Omega}_j)F_b^\times(\hat{\Omega}_k) \rangle_p$$

- Cosmic variance:

$$\mu_{ab} \equiv \Gamma(\gamma_{ab}) + \frac{1}{N} \sum_{j \neq k} \cos(\phi_j - \phi_k) \mu(\gamma_{ab}, \beta_{jk}) \quad (\text{unit amplitude})$$

$$\langle \mu_{ab} \rangle_s = \Gamma(\gamma_{ab}), \quad \sigma_{\text{cosmic}}^2(\gamma_{ab}) = \langle (\mu_{ab} - \langle \mu_{ab} \rangle_s)^2 \rangle_s$$

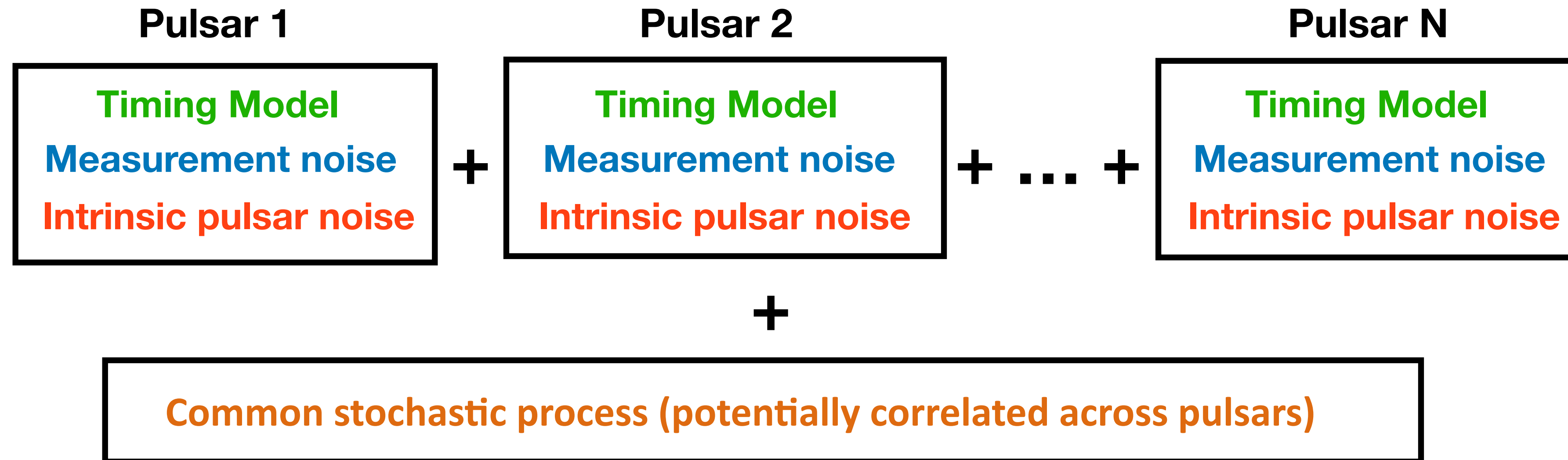
$$\sigma_{\text{cosmic}}^2(\gamma) = \frac{1}{4} \int_0^\pi d\beta \sin \beta \mu^2(\gamma, \beta)$$



How does one analyze PTA data to search for GWs?

Analysis methods

- Bayesian inference:



- (i) construct multivariate-Gaussian likelihood functions for null (CURN) and signal (HD) hypotheses
- (ii) calculate posterior distributions for parameter estimation and / or Bayes factors for model comparison

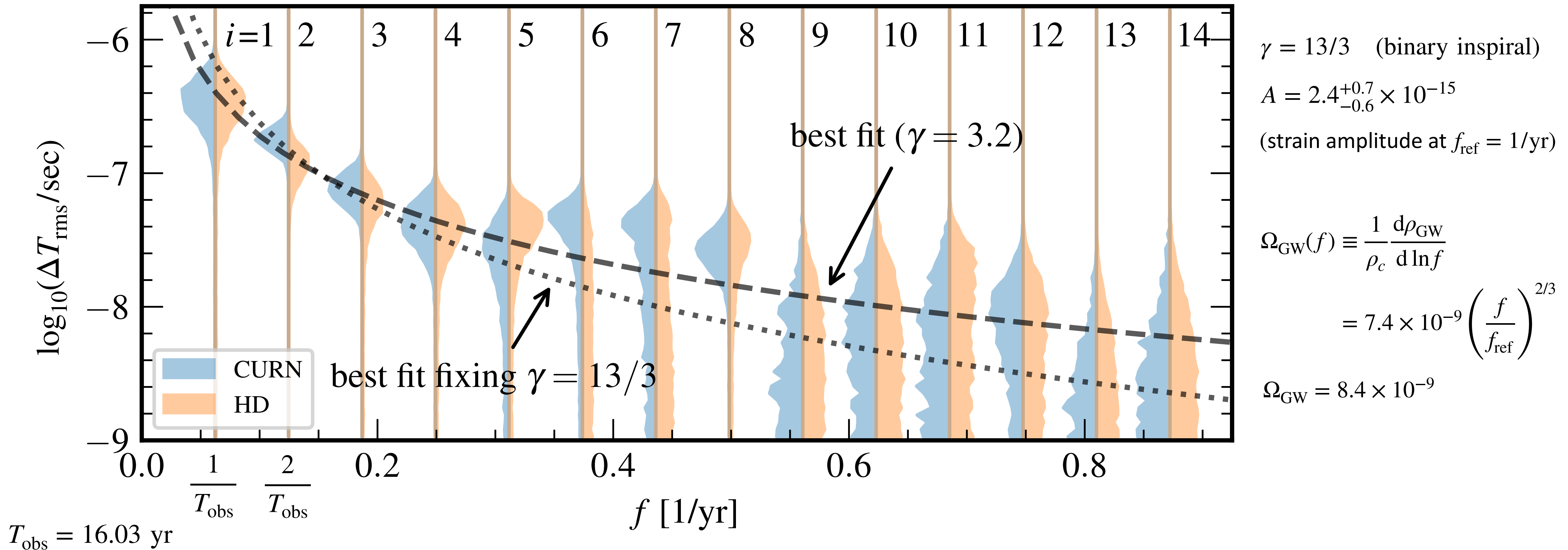
- Frequentist detection statistic:

$$S/N = \frac{\sum_{a < b} \rho_{ab} \Gamma_{ab} / \sigma_{ab,0}^2}{\sqrt{\sum_{c < d} \Gamma_{cd}^2 / \sigma_{cd,0}^2}}$$

- (i) inner-product of measured and expected correlations (**matched filter**)
- (ii) compare observed value to null distribution to assess detection significance

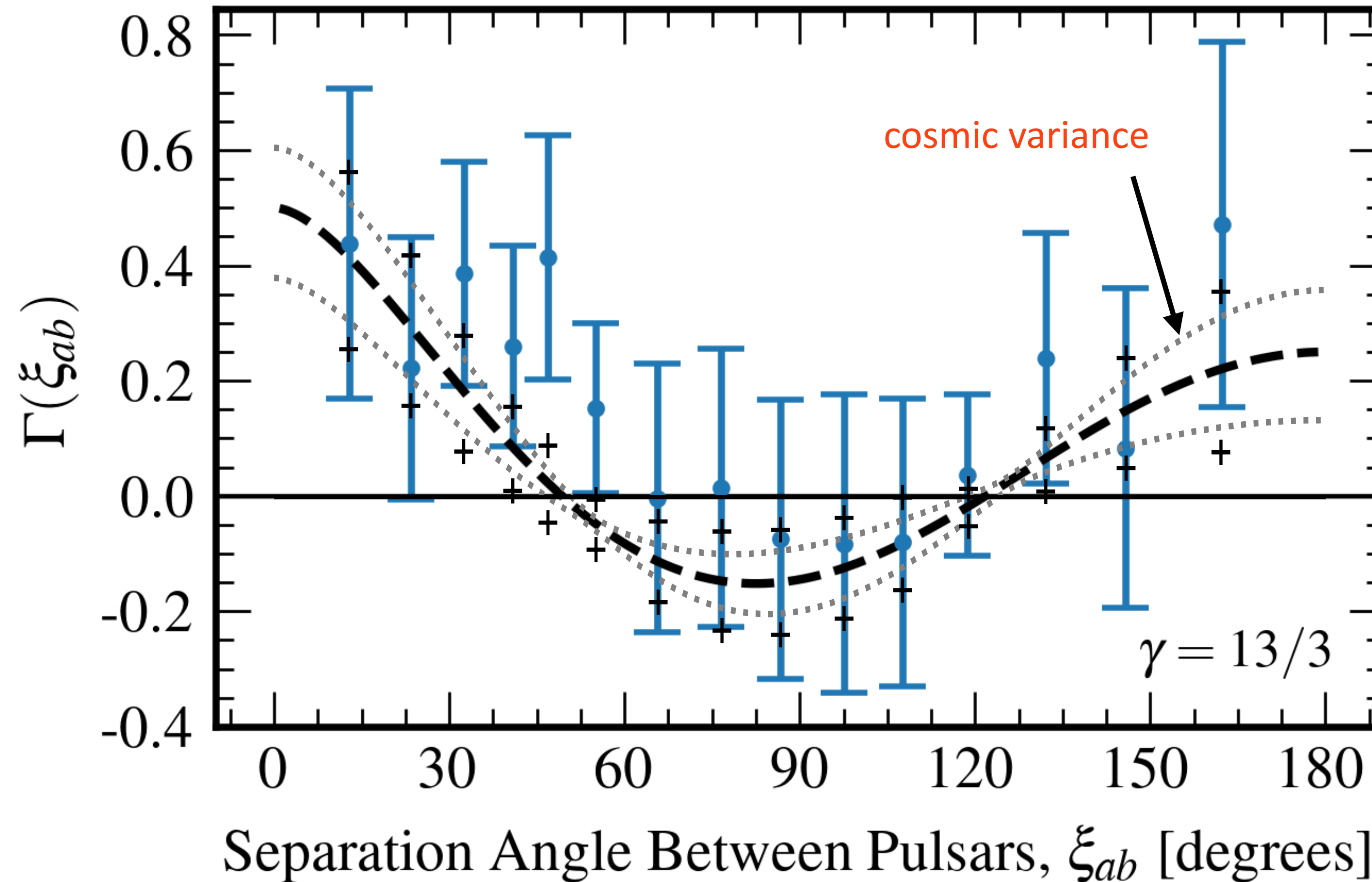
Part II. Plots from NANOGrav 15-yr papers

NANOGrav's observed common power spectrum



-consistent with predictions from SMBH binaries (and many other source models)

NANOGrav's observed correlations



$$\frac{67(67 - 1)}{2} = 2211 \text{ distinct pairs}$$

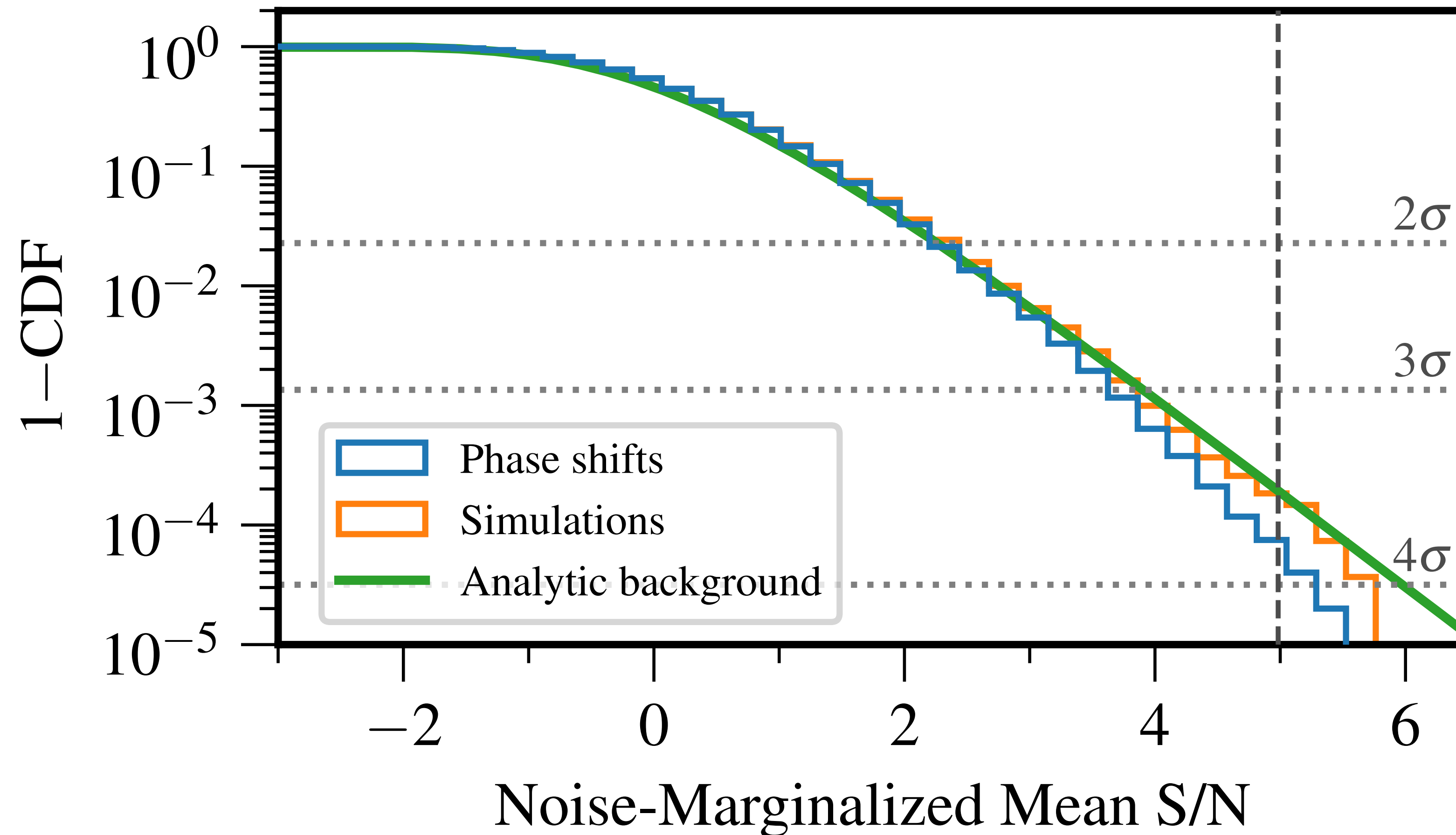
$$\frac{2211}{15} \approx 150 \text{ pairs per bin}$$

- weighted averages of measured correlations ρ_{ab} in each bin
- includes contributions from GW-induced covariances

$$C_{ab,cd} \equiv \langle \rho_{ab} \rho_{cd} \rangle - \langle \rho_{ab} \rangle \langle \rho_{cd} \rangle$$

-correlations follow the pattern expected for a GW background

NANOGrav's detection confidence

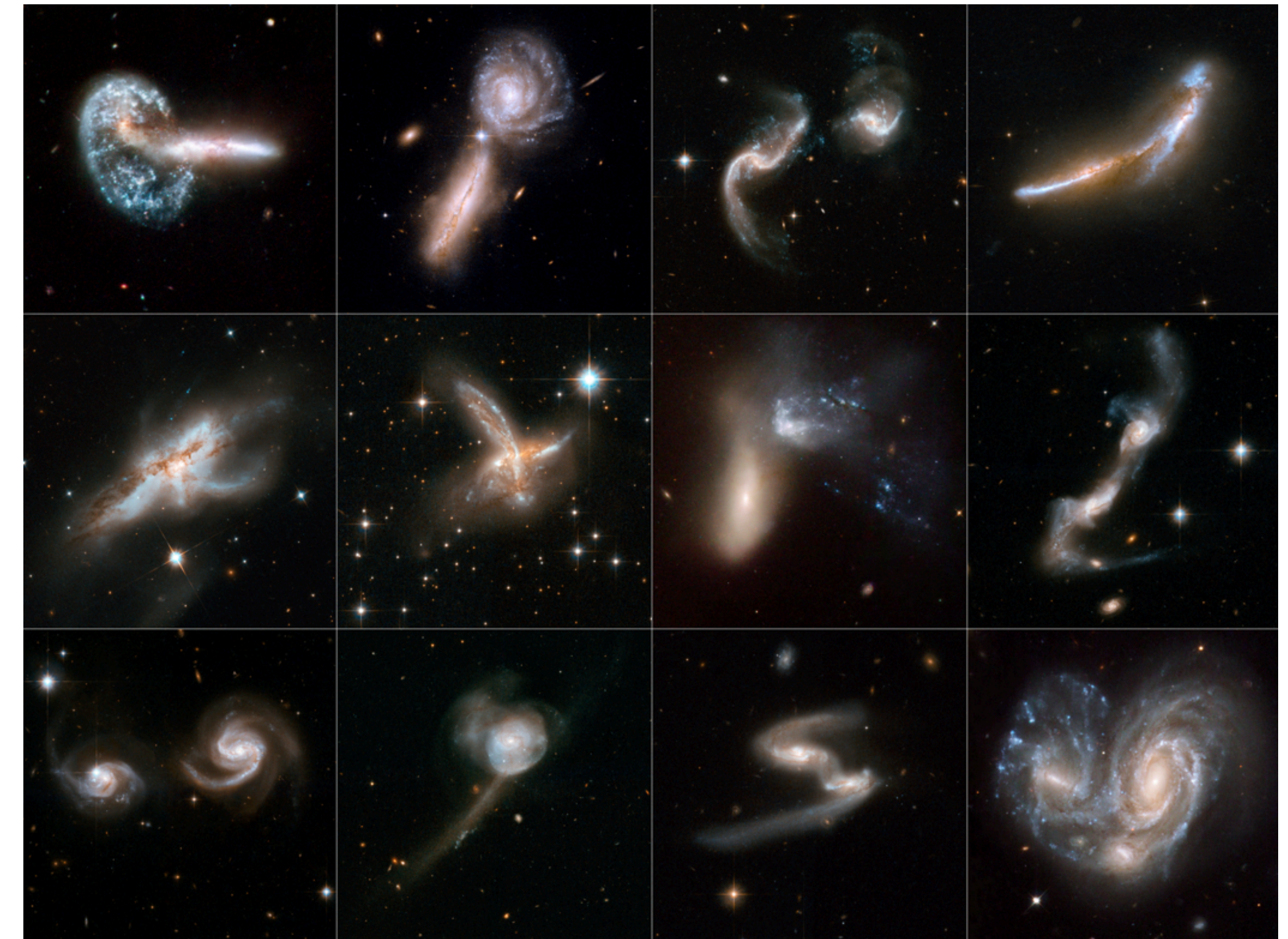
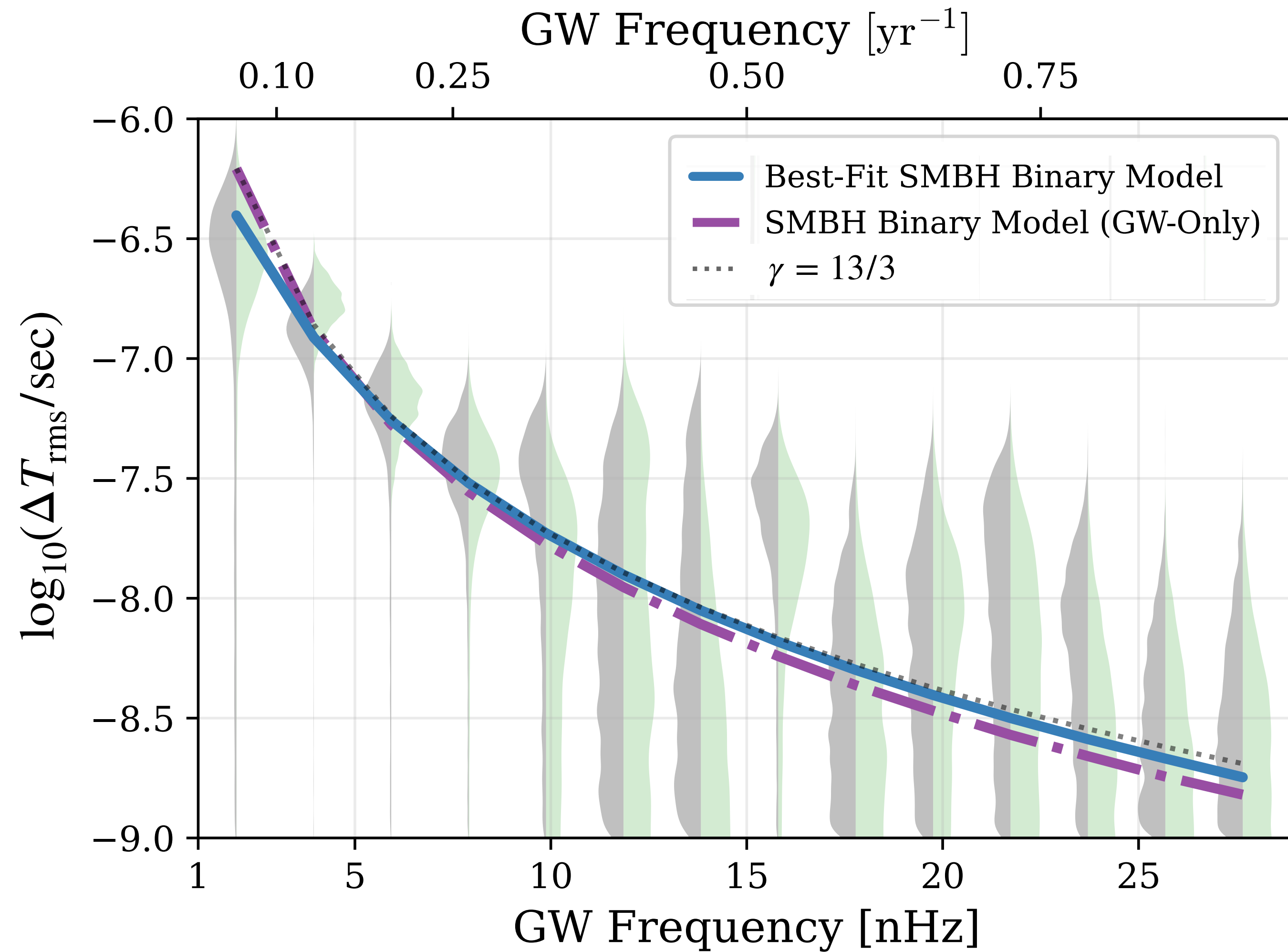


$$S/N = \frac{\sum_{a < b} \rho_{ab} \Gamma_{ab} / \sigma_{ab,0}^2}{\sqrt{\sum_{c < d} \Gamma_{cd}^2 / \sigma_{cd,0}^2}}$$

- inner product of measured and expected correlations (“**matched filter**” statistic)
- null distribution has zero mean, unit variance; but is not Gaussian

-unlikely due to noise alone (prob $\approx 1/10,000$) $\rightarrow$ “evidence for”

Possible astrophysical interpretation



pairs of inspiraling supermassive black holes
(masses $\sim 10^9 M_{\odot}$; millions of such binaries)

- environmental interactions remove GW power at low freqs, better fitting data

Updated analysis

Stochastic gravitational-wave background search using data from five pulsar timing arrays

Wang-Wei Yu * and Bruce Allen †

*Max Planck Institute for Gravitational Physics (Albert Einstein Institute),
Leibniz Universität Hannover, Callinstrasse 38, D-30167, Hannover, Germany*

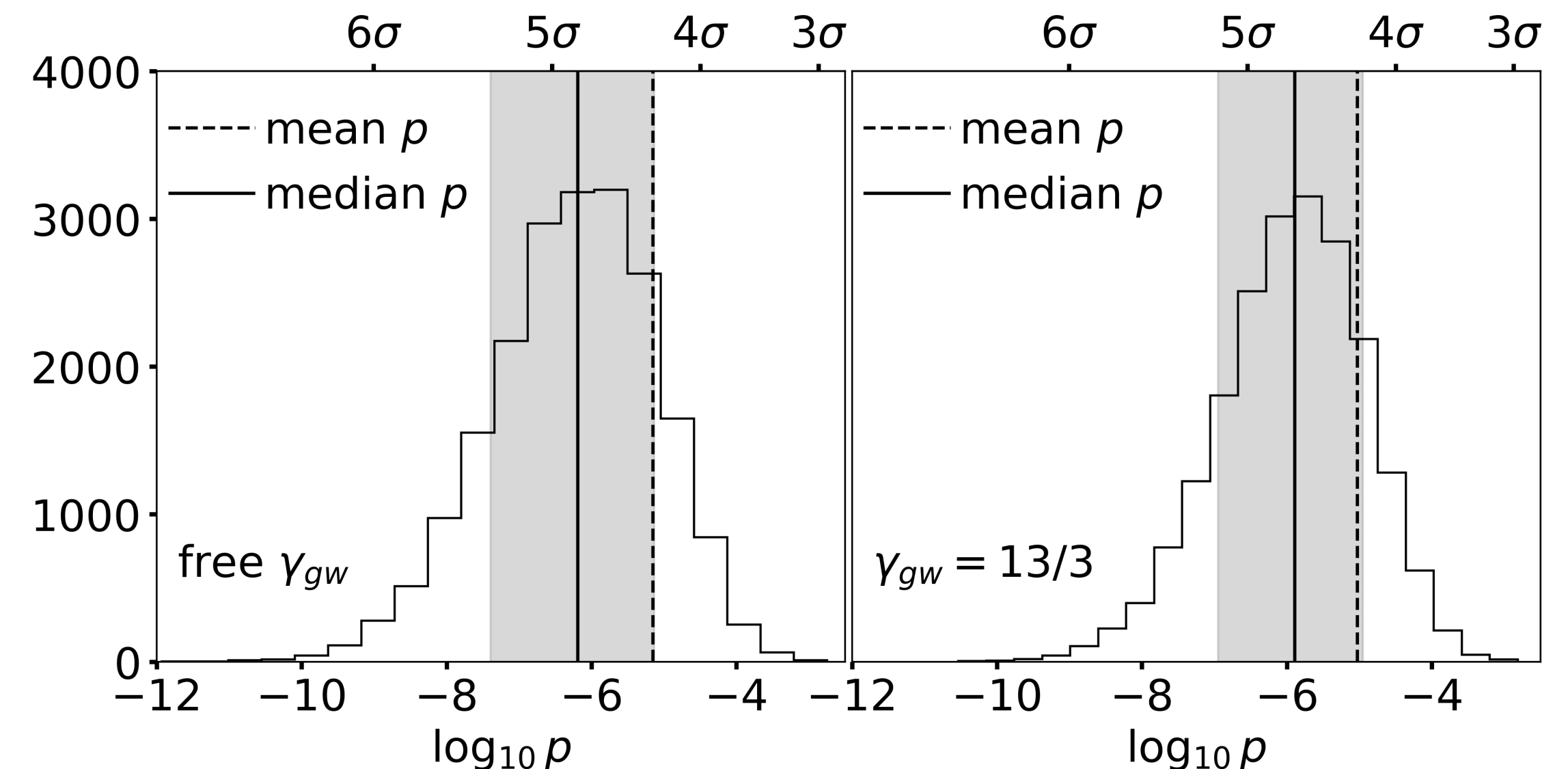
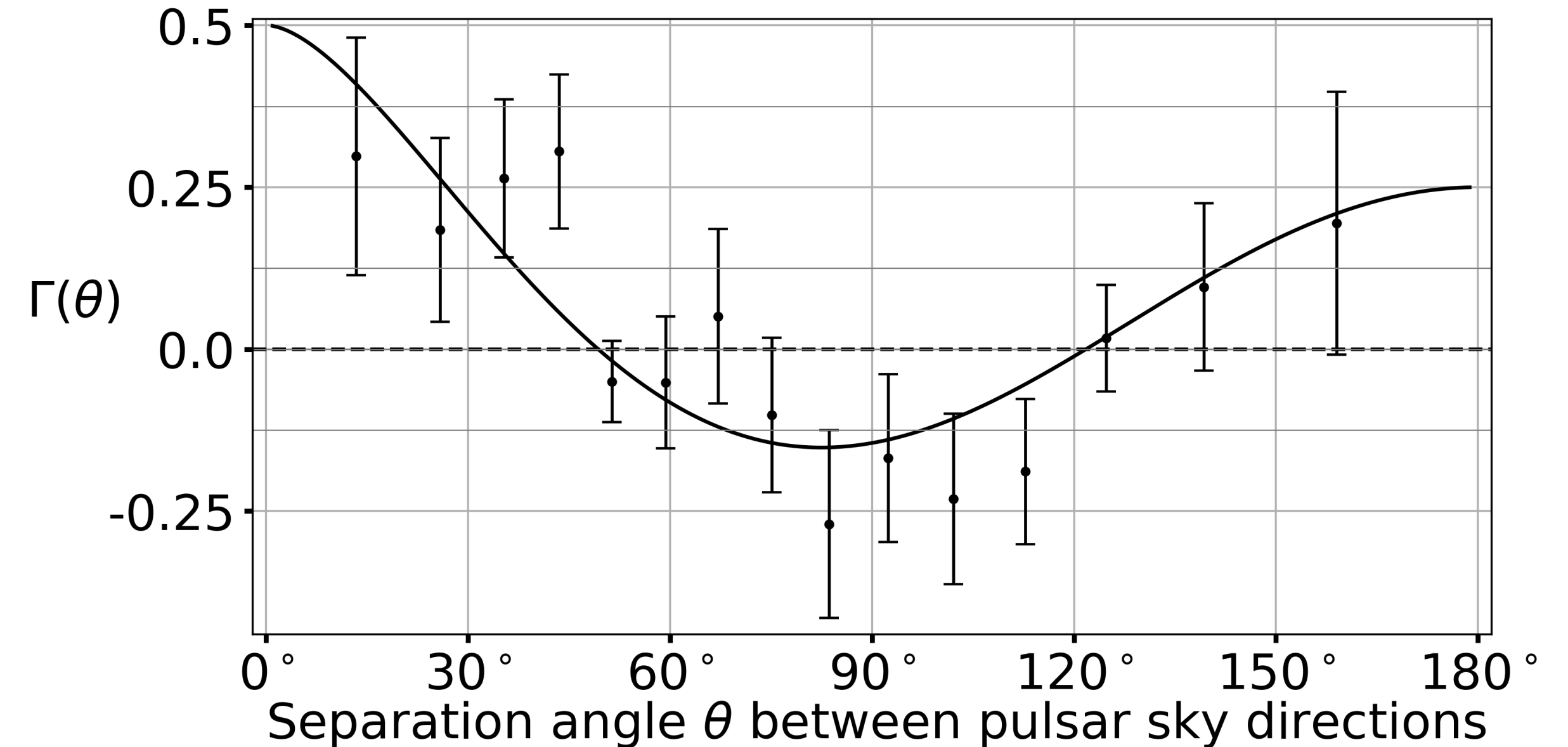
(Dated: April 29, 2026)

Using public pulse time-of-arrival data from five pulsar timing arrays (PTAs), we search for a stationary, isotropic, and unpolarized nHz stochastic gravitational-wave background (SGWB). This analysis is more sensitive than previous individual PTA searches because the combined 121-pulsar dataset is about four times larger than any single PTA's. For pulsars observed by multiple PTAs, we employ a new “direct combination” method to merge their astrophysical models and data. This avoids the challenge of reconciling different PTA timing models to obtain a single “best” model. A central result of our analysis is the posterior distribution of the amplitude A_{gw} and exponent γ_{gw} of the putative SGWB energy-density spectrum, modeled as a power law in frequency. While these results are consistent with a nonzero SGWB amplitude A_{gw} , the statistical significance-assessed via a Bayesian odds ratio and noise-marginalized false-alarm probabilities (p -values) for three detection statistics-remains below the conventional 5σ threshold for a confident detection. The inter-pulsar timing-residual correlation, reconstructed as a function of angle θ between the pulsar lines of sight, matches the Hellings and Downs (HD) prediction.

Combined analysis using data from 5 PTAs: 121 pulsars

Bayes factor (HD vs CURN): ~ 26000 vs ~ 200 (NG 15-yr)

Mean p -value for standard detection statistic: 4.3σ



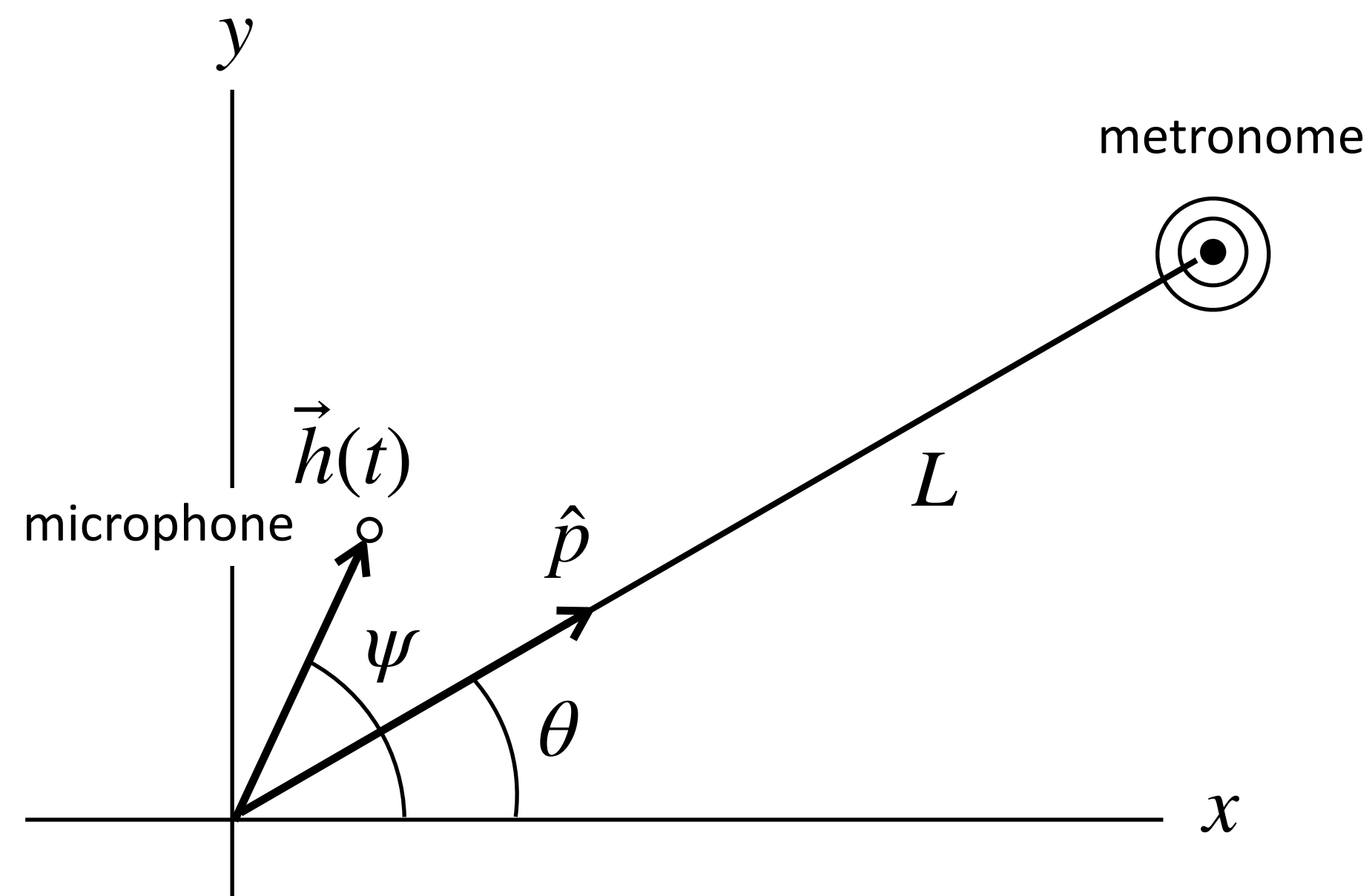
GW150914, etc	PTA observation
deterministic / transient signal	stochastic / persistent signal
waveforms & coincidence	power spectra & cross-correlations
single binary black hole merger	combined signal from a population of approx monochromatic inspiraling binaries
stellar mass black holes (1 - 100 solar masses)	supermassive black holes (10^9 solar masses)
laser interferometers with km-scale arms	galactic-scale detector using msec pulsars, with “arm” lengths ~ 100 - few x 1000 light-years
audio frequencies (10's - 1000 Hz)	nanoherzt frequencies (10^{-9} - 10^{-7} Hz) [periods: decades -> months]
GW wavelength $\gg$ arm length	GW wavelength $\ll$ arm length
“detection of ...” (>5 sigma)	“evidence for ...” (3-4 sigma)

extra slides

Metronome timing array

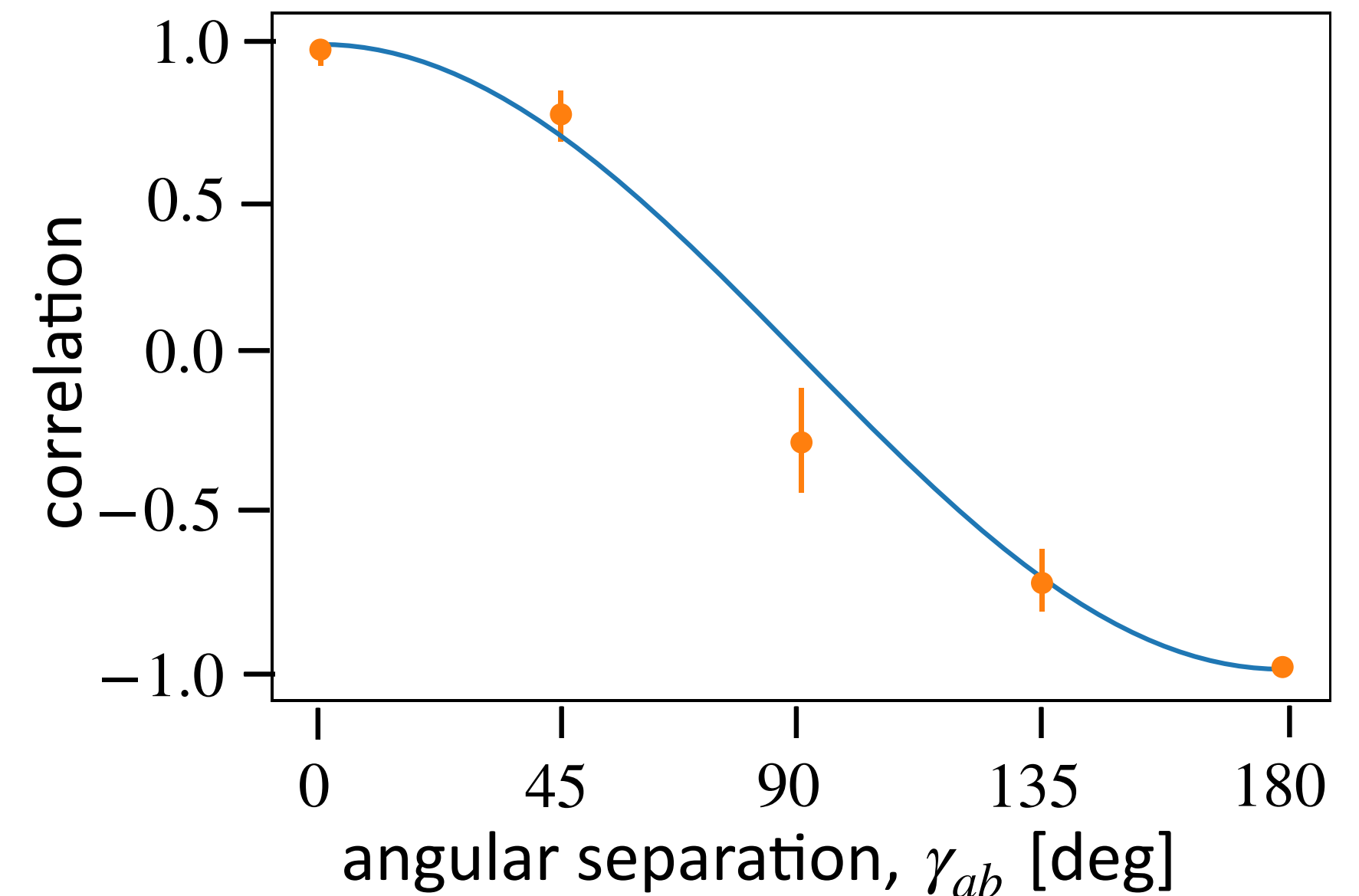
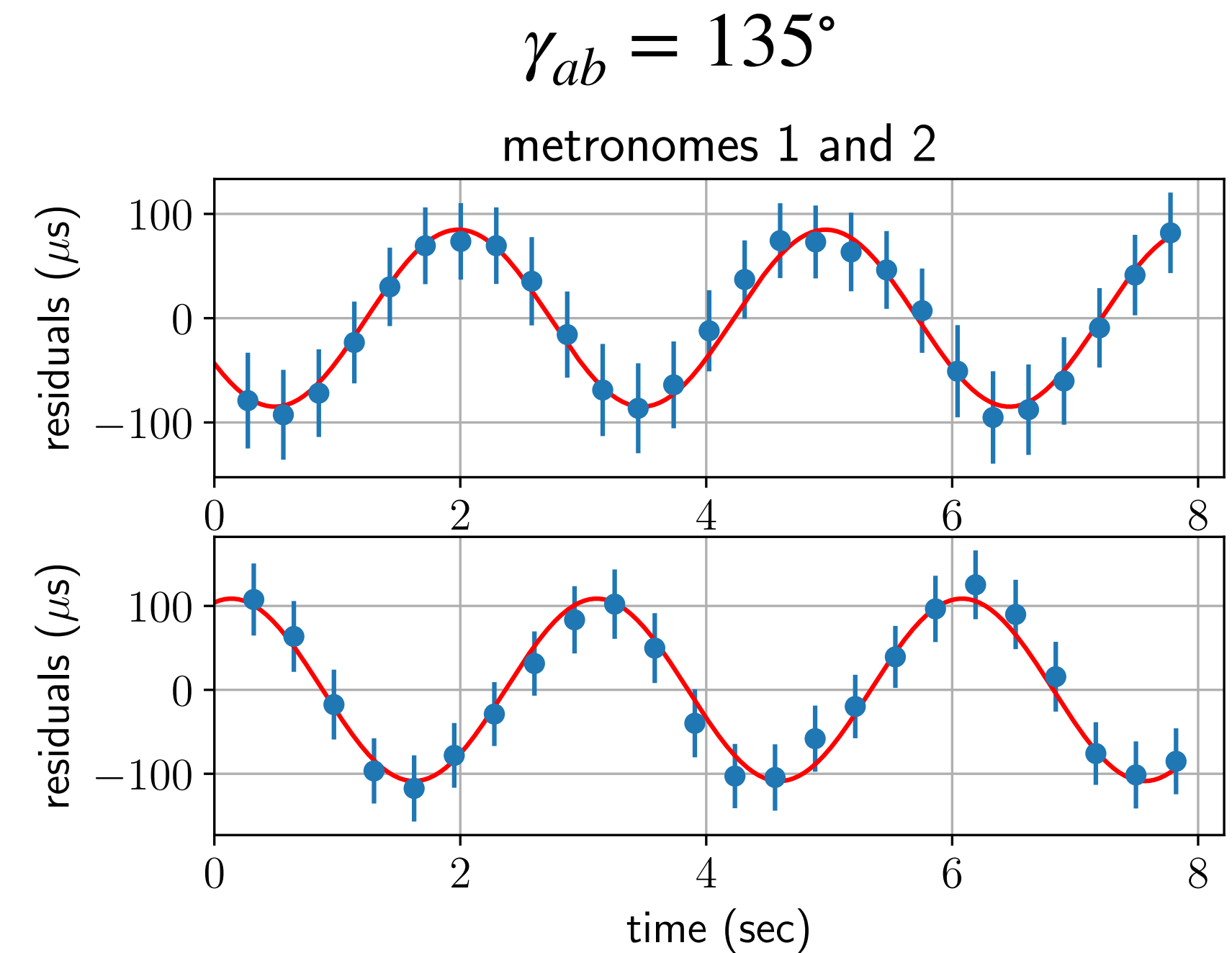
[AJP: Lam et al, 2018]

<https://github.com/josephromano/pta-demo>



$$\Delta T(t) = \frac{\Delta L(t)}{c_s} \simeq -\frac{\hat{p} \cdot \vec{h}(t)}{c_s}$$

Unif circular motion: $\Delta T_{a,b}(t) = -\frac{A}{c_s} \cos(2\pi f_0 t + \phi_0 - \theta_{a,b})$



Metronome demo numbers

- $c_s = 340 \text{ m/s}$
- amplitude microphone motion: $A \approx 5 \text{ cm}$
- period of microphone motion: $T_{\text{gw}} = 2 \text{ sec}$
- amplitude microphone motion: $\approx 5 \text{ cm}$
- 184 bpm: $T_{\text{pulse}} = 0.3261 \text{ sec}$
- 208 bpm: $T_{\text{pulse}} = 0.2885 \text{ sec}$

Pulsar timing numbers

$f \sim 1/\text{few weeks to } 1/10 \text{ years } (10^{-7} \text{ Hz to } 10^{-9} \text{ Hz})$

$\lambda \sim 0.1 \text{ to } 10 \text{ yr } (\text{GW wavelength})$

$L \sim \text{few } 100 - 1000 \text{ yr } (\text{distance to pulsars})$

$\lambda \ll L$ (short-wavelength limit)

sensitivity $\sim \sigma_{\text{rms}}/T_{\text{obs}} \sim 100 \text{ ns}/10 \text{ yr} \sim 10^{-15}$



can detect changes $\sim 10 \text{ km}$ in the position
of a pulsar at a distance of $\sim 1000 \text{ yr}$