Origin limits of large-charge correlators in planar N=4 SYM

Observables in gauge theory and gravity

IPhT, 10-12 December 2025

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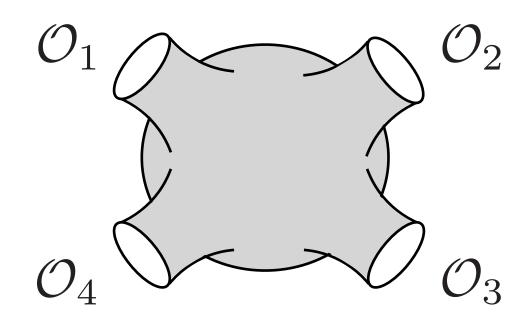
Work in progress with Thiago Fleury, Erkan Kaluç and Didina Serban

Motivation

- N = 4 SYM is a remarkable laboratory for exploring the dynamics of 4d massless gauge theories
- Symmetries open the way to new methods for computing correlation functions and amplitudes at both weak and strong coupling
- Key example: The duality between N=4 SYM and type IIB string theory on AdS5*S5
- This duality triggered the development of Integrability, enabling exact studies of observables in the large-N limit
- Key success: Spectrum of single-trace operators at finite 't Hooft coupling
- Ultimate goal: Correlation functions of arbitrary single-trace operators

Motivation

Compute correlation functions of single-trace operators in the large N limit



Simplest insertions are half-BPS operators $\mathcal{O}_i = \operatorname{Tr}\left[\phi(x_i,y_i)^{J_i}\right]$ with

with

Dimension given by R-charge $\Delta_i = J_i$

Correlators are finite functions of cross ratios $u_{ijkl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$ $v_{ijkl} = \frac{y_{ij}^2 y_{kl}^2}{y_{ik}^2 y_{jl}^2}$

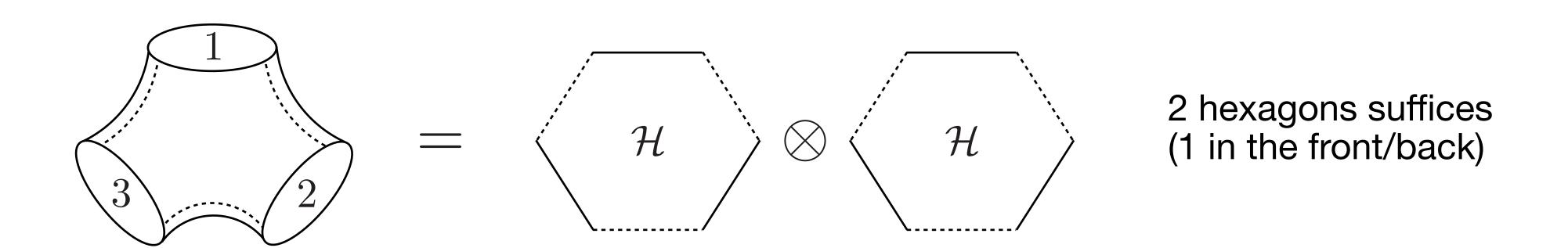
$$\phi(x_i, y_i) = \sum_{I=1}^6 y_i^I \phi^I(x_i)$$

$$\sum_{I=1}^6 y_i^I y_i^I = 0$$

Hexagonalization is a method for studying correlation functions at finite coupling $g^2=rac{g_{
m YM}^2N}{16\pi^2}$

Core idea: spheres with single-trace insertions can be triangulated using hexagons

Ex 1. Three-point function (structure constant)

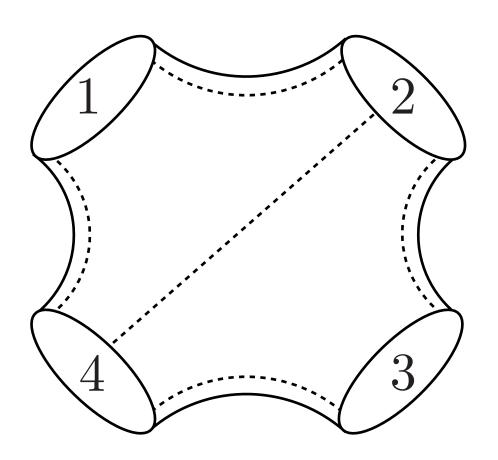


Cut open following Wick contractions between pairs of operators

Hexagonalization is a method for studying correlation functions at finite coupling $g^2 = \frac{g_{YM}^2 N}{16\pi^2}$

Core idea: spheres with single-trace insertions can be triangulated using hexagons

Ex 2. Four-point function



4 hexagons are needed (2 in the front/back)

Cut open following Wick contractions between pairs of operators

Motivation

Hexagonalization is a method for studying correlation functions at finite coupling $g^2 = \frac{g_{\rm YM}^2 N}{16\pi^2}$

Core idea: spheres with single-trace insertions can be triangulated using hexagons

Generally, break n-point functions into 2(n-2) hexagons

$$F_n \rightarrow \mathcal{H}_1 \otimes \mathcal{H}_2 \ldots \otimes \mathcal{H}_{2(n-2)}$$

As for triangulations of punctured spheres

Hexagons in some detail

Hexagon: made out of 3 physical edges and 3 mirror edges (cuts)

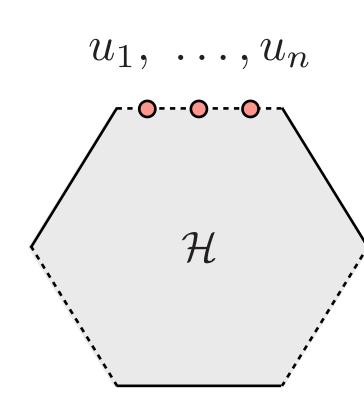
Their form factors describe absorption of magnons on edges

$$h(u_1, \dots, u_n) = \langle \mathcal{H} | \chi(u_1) \dots \chi(u_n) \rangle$$

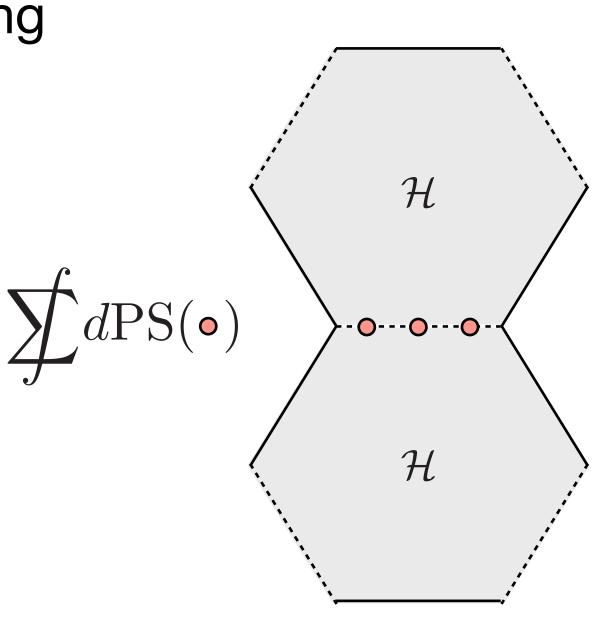
Stringent bootstrap constraints determine them at any coupling

Gluing: Attach hexagons across a common edge by summing over a complete basis of mirror magnons along that edge

It turns the computation of correlation functions into multichannel partition functions, with a sum over the multimagnon Fock space of each identified edge



Magnons =
Fluctuations
of open string



Disk correlators or polygons

Gluing leads to complicated sums/integrals over mirror magnons

The more edges we glue, the more complicated the sums become

In a 4-pt function: 6 cuts carry mirror magnons, which all interact on shared hexagons

Disk correlators or polygons

[Fleury,Komatsu]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Coronado][Fleury,Gonçalves]
[Bercini,Fernandes,Gonçalves]

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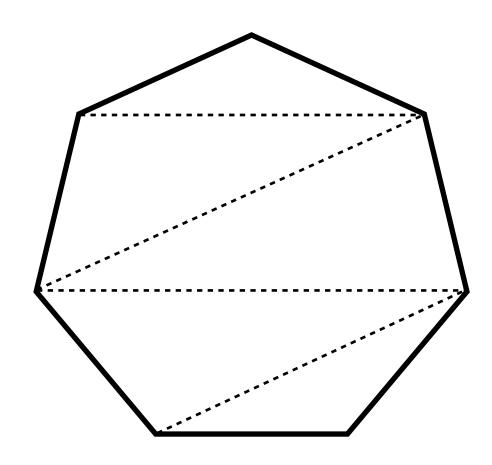
In a 4-pt function: 6 cuts carry mirror magnons, which all interact on shared hexagons

Seek simplification: large-charge correlators = **disk** correlators

No cycles, just hexagons glued together into a bigger polygon

Building blocks for correlation functions of (short) operators

Interesting connection with scattering amplitudes [Caron-Huot, Coronado]



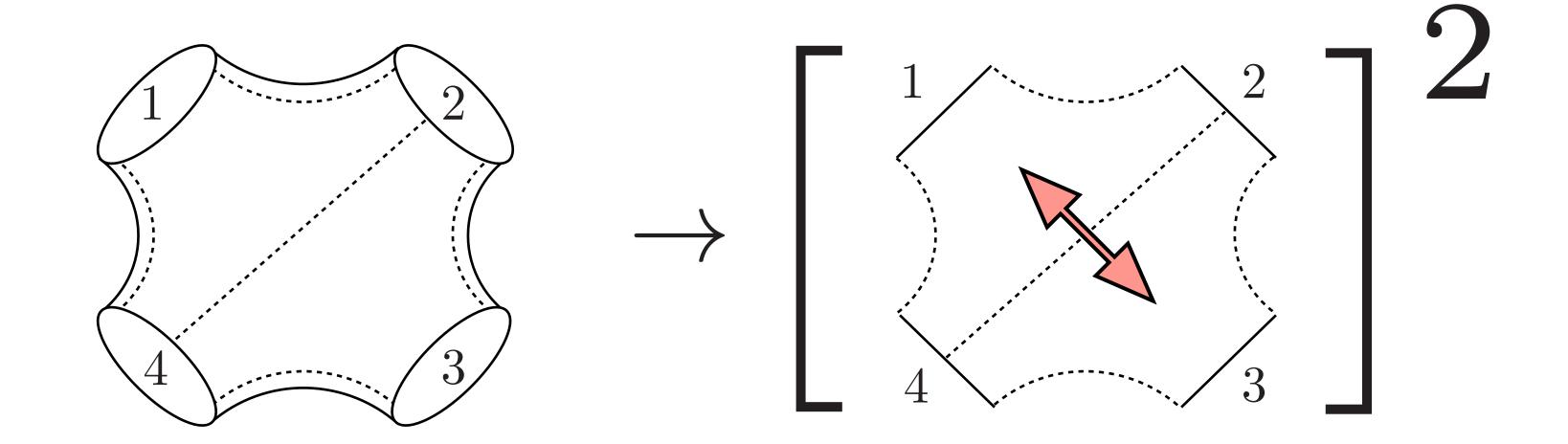
Ex. 7pt function covered by 5 hexagons

Toy model for exploring re-summation of hexagon expansion in various kinematics

Octagon

Prototype: The large-charge 4pt function or "Octagon"

[Coronado] [Kostov,Petkova,Serban] [Belitsky,Korchemsky]



Assume we can take bridge lengths (R-charges flowing) between two consecutive operators to infinity

No magnons can flow across infinite bridges and four-point function factorizes into two octagons - each containing a single sum over mirror magnons

Octagon

Prototype: Large-charge 4pt function or "Octagon"

[Coronado] [Kostov,Petkova,Serban] [Belitsky,Korchemsky]

It can be computed exactly and given as a Fredholm determinant of a Bessel kernel

$$\mathbb{O} = \det (1 - \mathbf{K})$$

See Alessandro's and Gwenael's talks

Caveat: Hard to generalize to higher polygons (higher-point functions)

Null limit and Origins

Exact results may still exist for higher points in special kinematics

[Coronado] [Kostov,Petkova,Serban] [Belitsky,Korchemsky]

Null square limit where the two cross ratios vanish $U, V \rightarrow 0$

[Fleury,Komatsu][Fleury,Gonçalves]
[Bargheer,Caetano,Fleury,Komatsu,Vieira]
[Bercini,Fernandes,Gonçalves]
[Crisanti,Eden,Gottwald,Mastrolia,Scherdin]

Large-charge correlator exhibits a Sudakov-type behavior

$$\log \mathbb{O} = -\frac{\Gamma_{\text{oct}}}{32} \log^2 (UV) + \dots$$

with coefficient exactly known

$$\Gamma_{\rm oct} = \frac{2}{\pi^2} \log \cosh (2\pi g)$$

Similar observations were made for scattering amplitudes in so-called Origin limits
[BB,Dixon,Liu,Papathanasion

Goal: Explore Origin limits with many vanishing cross ratios for large-charge correlators

Plan

Origins for polygons

Hexagon approach and all-order conjecture

Light-like limit of polygons

Conclusion

Origins for polygons

2d geometry

Focus on correlation functions in 2d kinematics

Kinematic data for the polygon can be given in terms of 2-component spinors $\{(\lambda_i, \bar{\lambda}_i), i = 1, \dots, n\}$

Defined projectively from the left- and right-moving coordinates of the vertices in $\mathbb{R}^{1,1}$

Equivalently, one may package this information inside the Grassmannian $\operatorname{Gr}(2,n)/T$

$$\begin{pmatrix} 1 & 1 & \dots & 1 & 1 & 1 & 0 \\ x_1 & x_2 & \dots & x_{n-3} & 0 & 1 & 1 \end{pmatrix}$$

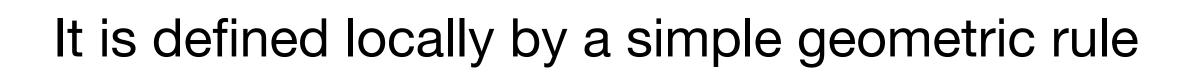
Distances:
$$x_{ij}^2 = (x_i - x_j)^2 \propto \langle ij \rangle \times [ij]$$

where $\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^{\alpha} \lambda_j^{\beta}$ and similarly for [ij]

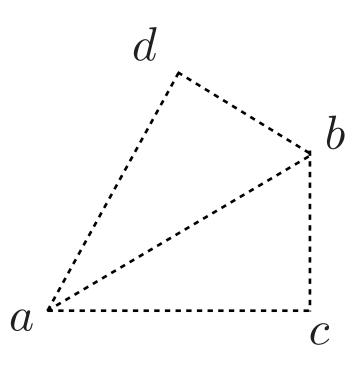
Cluster coordinates

Each triangulation comes with a natural set of so-called cluster coordinates

Each coordinate is associated with a cut (or chord) in the triangulation



$$z_{ab} = \frac{\langle ad \rangle \langle cb \rangle}{\langle ac \rangle \langle bd \rangle}$$

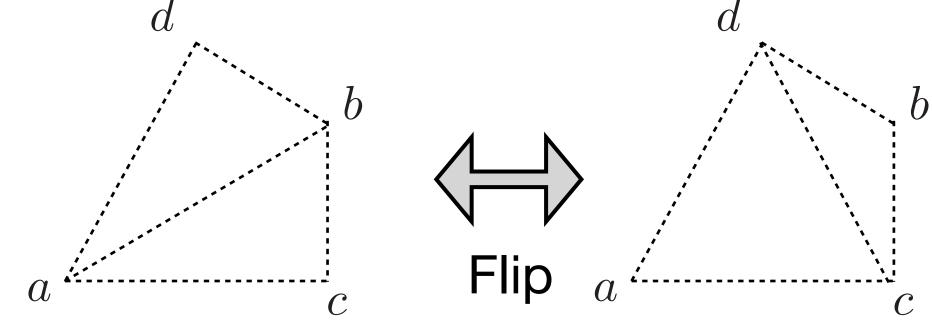


These coordinates show up naturally in the hexagonalization, where they determine the weights of the mirror magnons

[Fleury,Komatsu]

Mutations map the cluster coordinates of one triangulation to those of a neighboring one,

corresponding to a local flip of an edge



Boundary and Origins

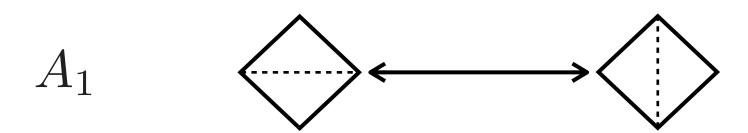
Exchange graph of the cluster algebra A_{n-3} for n-gon

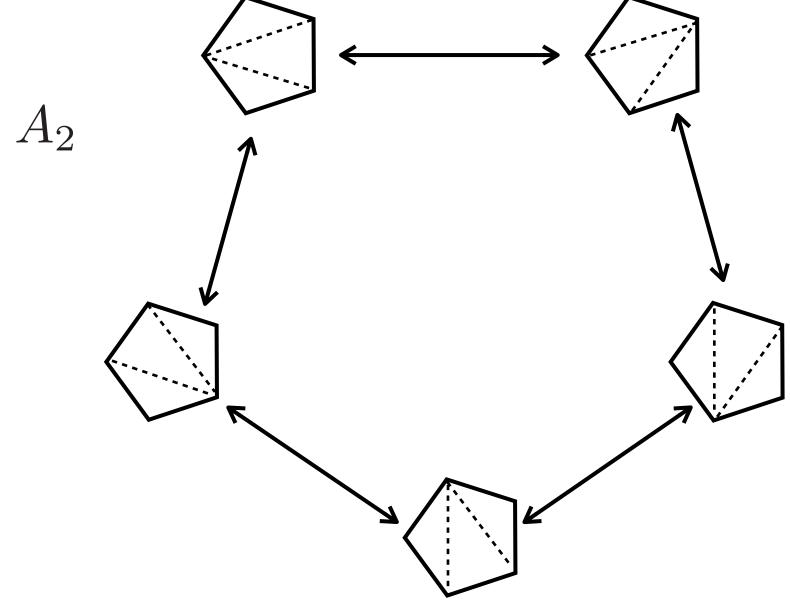
The interior of the diagram corresponds to the positive region, defined as the set of points reached by assigning positive values to the cluster coordinates

Boundaries represent limits where certain cluster coordinates vanish or become large

Cluster Origin: A vertex can be identified with the origin of the cluster coordinate system defined by the associated triangulation

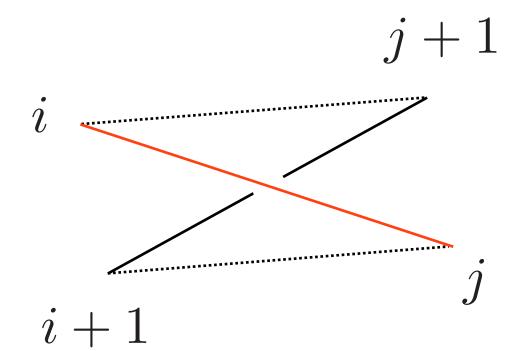
Similar to Origins in [BB,Dixon,Liu,Papathanasiou]





Boundaries are where correlators can develop singularities, and cluster algebras provide an algebraic framework to systematically explore these boundary regions

Binary geometry



To every (internal) chord attach a cross ratio

$$u_{ij} = \frac{\langle i, j+1 \rangle \langle i+1, j \rangle}{\langle i, j \rangle \langle i+1, j+1 \rangle}$$

Multiplicatively independent but **overcomplete** basis of $\frac{n(n-3)}{2}$ cross ratios in 2d

Relations take the elegant form u_{ij} +

$$u_{ij} + \prod_{(kl) \text{ crossing } (ij)} u_{kl} = 1$$

[Arkani-Hamed,He,Lam]
[Arkani-Hamed,He,Lam,Thomas]
[Brown]

with product running over all chords crossing ij

$$u_{ij} \to 0 \qquad \Rightarrow \qquad u_{kl} \to 1$$

It follows that

Cross ratios are bounded in (0,1) over positive regions

Each **facet** of the boundary corresponds to a limit where one cross ratio approaches zero, while its crossing-related counterparts approach one

Vertices on the boundary correspond to intersections of (n-3) facets. Each such vertex can be labeled by a binary sequence indicating which (n-3) cross ratios vanish (set to 0) and which complementary ones are set to 1

Origin limits

In our case, we have both a left and a right polygon to parametrize

$$u_{ij}^{\text{Left}} = \frac{\langle i, j+1 \rangle \langle j, i+1 \rangle}{\langle i, j \rangle \langle i+1, j+1 \rangle} \qquad u_{ij}^{\text{Right}} = \frac{[i, j+1][j, i+1]}{[i, j][i+1, j+1]}$$

Physical cross ratios are products of left and right cross ratios

$$U_{ij} = u_{ij}^{\text{Left}} \times u_{ij}^{\text{Reft}}$$

The number of vanishing cross ratios is **maximized** when the sets of internal chords in the left and right triangulations are mutually disjoint

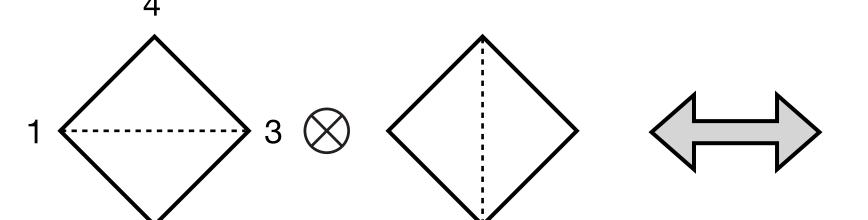
In that case, 2(n - 3) cross ratios approach zero, while the remaining ones approach one

We call such configurations Origins or Origin limits, in analogy with scattering amplitudes

Examples

Origin limits

4-point function



 $U_{13}, U_{24} \to 0$

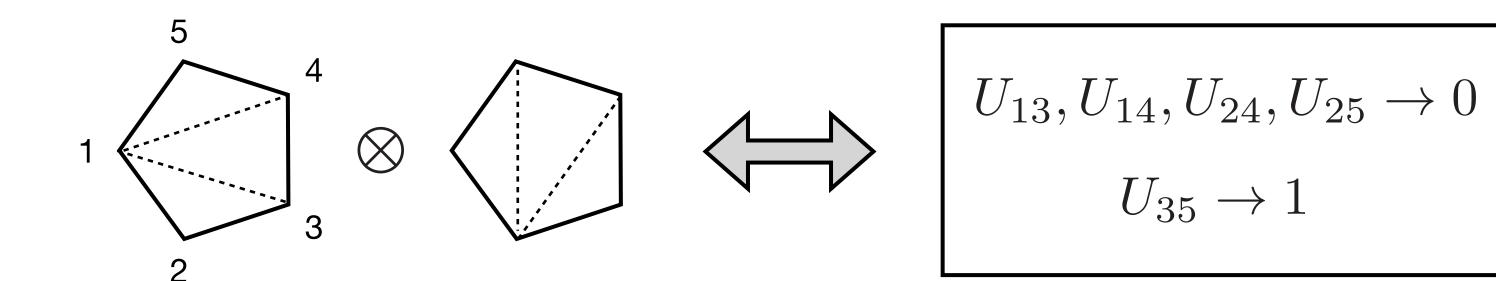
Left

Right

Same as null square limit studied in [Coronado] [Kostov,Petkova,Serban] [Belitsky,Korchemsky]

5-point function

Comments



- 1) Other choices are possible here, but all give cyclic images of the limit above
- 2) At most 4 cross ratios can simultaneously approach zero. This is the maximal number allowed by the kinematics in 2d. (In contrast, in four dimensions, there is more freedom: the five cross ratios are independent and, in principle, can all be taken to zero simultaneously.)

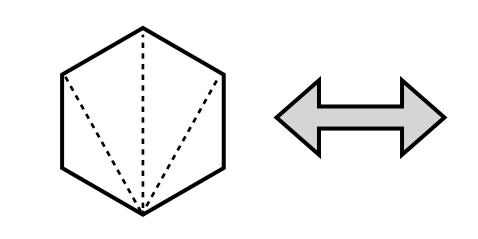
Examples

Origin limits

6-point function

Ex. 1

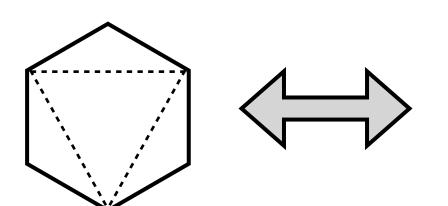
6 4 3



 $U_{13}, U_{24}, U_{15}, U_{26}, U_{14}, U_{25} \rightarrow 0$ $U_{35}, U_{46}, U_{36} \rightarrow 1$

Ex. 2

6 4 3



 $U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26} \rightarrow 0$ $U_{14}, U_{25}, U_{36} \rightarrow 1$

Comments

- 1) There are a few inequivalent choices in total (up to cyclic shifts), so not all configurations are the same here
- 2) Ex. 2 is particularly interesting: it corresponds to the limit where all boundaries become light-like

Origin classes for hexagon

Define $(u_1, u_2, u_3, u_4, u_5, u_6; v_1, v_2, v_3) = (U_{13}, U_{24}, U_{35}, U_{46}, U_{15}, U_{26}; U_{14}, U_{25}, U_{36})$

Inequivalent classes of Origins are given by

Origin Class	u_1	u_2	u_3	u_4	u_5	u_6	v_1	v_2	v_3
O_1	0	0	1	0	0	1	0	0	1
O_2	0	1	0	0	1	0	0	0	1
O_3	0	0	1	0	1	0	0	0	1
O_4	1	0	0	0	0	1	0	0	1
O_5	0	0	1	0	0	0	0	1	1
O_6	0	0	0	0	0	0	1	1	1

Images are obtained by acting on them with dihedral group

Orbits with 6,3,12,6,6,1 elements for O1 to O6 respectively

Hexagon approach and all-order conjecture

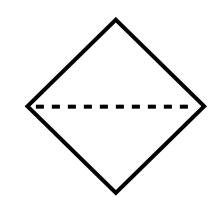
One-loop exploration

One-loop hexagonalization can be performed in closed form

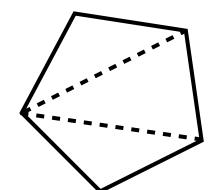
[Fleury,Komatsu] [Bargheer, Caetano, Fleury, Komatsu, Vieira]

Building block is given by the Bloch-Wigner function

$$\mathcal{M}(z) = \frac{z + \bar{z} - \text{R-charge}}{2(z - \bar{z})} \left(2\text{Li}_2(-z) - 2\text{Li}_2(-\bar{z}) + \log\left(\frac{1+z}{1+\bar{z}}\right) \log\left(z\bar{z}\right) \right)$$



Ex. 4-pt
$$\mathcal{M}(z_1) + \mathcal{M}\left(\frac{1}{z_1}\right)$$



Ex. 5-pt
$$\mathcal{M}(z_1) + \mathcal{M}(z_2(1+z_1)) + \mathcal{M}\left(\frac{1+z_2}{z_1z_2}\right) + \mathcal{M}\left(\frac{1+z_2(1+z_1)}{z_1}\right) + \mathcal{M}\left(\frac{1}{z_2}\right)$$

Find that the result is quadratic in logarithms in the Origin limits (NB: R-charge drops out)

Hexagon approach

Method

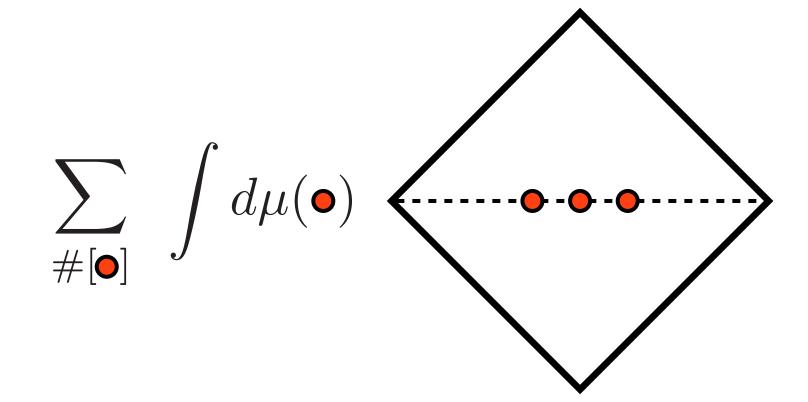
Identify states that dominate and truncate the hexagon mirror sums

Use tricks to simplify the analysis (partial re-summation, Sommerfeld-Watson transform)

Prototypal example: 4pt function

Revisiting octagon

Four-point function



Single sum of mirror magnon integrals

In limit of interest only states of maximal spin contribute: abelian sector spanned by derivatives

Truncating the hexagon series, one verifies agreement with the general formula up to high orders

$$\log \mathcal{C}_4 \approx -\frac{1}{32} \Gamma_{\alpha=0}(g) \log^2 (U_{13} U_{24}) + \frac{1}{32} \Gamma_{\alpha=\pi/2}(g) \log^2 (U_{13} / U_{24})$$

[Coronado] [Kostov,Petkova,Serban] [Belitsky,Korchemsky]

with

$$\Gamma_{\alpha=0} = \Gamma_{\rm oct} = \frac{2}{\pi^2} \log \cosh (2\pi g)$$
 and $\Gamma_{\alpha=\pi/2} = 4g^2$

Not better than approach based on Fredholm determinant, but extends to higher polygons

Hexagon approach

Method

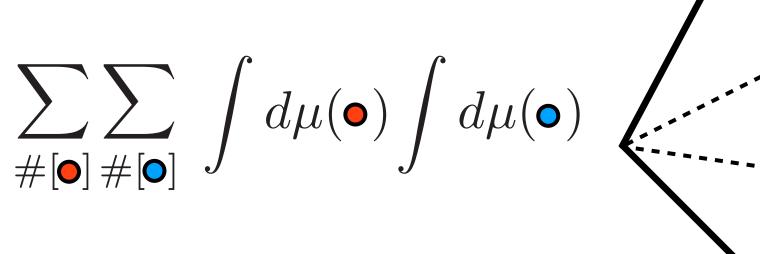
Identify states that dominate and truncate the hexagon mirror sums

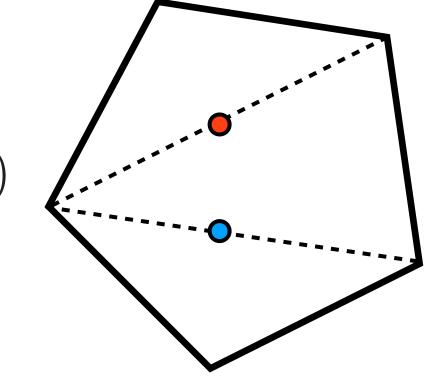
Use tricks to simplify the analysis (partial re-summation, Sommerfeld-Watson transform)

Prototypal example: 4pt function 5pt function?

5-point function

Two-cut interactions





All Origins should give the same thing up to cyclic shifts

Simplest choice corresponds to a double abelian truncation with derivatives on one cut and conjugate derivatives on the other

$$\log \mathcal{C}_5 \approx g^2 \left(\log U_1 \log U_2 + \log U_4 \log U_5 + \log U_1 \log U_5\right) + \mathcal{O}\left(g^4\right)$$

in limit $U_1, U_2, U_4, U_5 \to 0 \ (U_3 \to 1)$

Exponentiation & quadratic in logarithms at higher loops (5 loops at least) - like 4pt function

Result not expressible in terms of the octagon anomalous dimension

Structure very similar to Origins in scattering amplitudes

Hexagon approach

Method

Identify states that dominate and truncate the hexagon mirror sums

Use tricks to simplify the analysis (partial re-summation, Sommerfeld-Watson transform)

- ✓ Prototypal example: 4pt function
- 5pt function (log simple through at least 3-loops)

Higher polygons? Possible to perform similar checks for a family of higher n-gon Origins

However, for a general Origin, it is not always clear which states dominate

Details depend on the chosen Origin limit and the triangulation used in the computation

General formula?

General conjecture I

Drawing inspiration from scattering amplitudes and generalizing the observations made for the 4-point function, we are led to conjecture that the disk correlation functions exponentiate in any Origin limit and that its exponent is a quadratic polynomial in the logarithms of the cross ratios

[BB,Dixon,Liu,Papathanasiou]
[Coronado]
[Belitsky,Korchemsky]
[Kostov,Petkova,Serban]

More precisely, we expect the disk n-point function to take the form

[BB,Fleury,Kaluç,Serban - in progress]

$$\log \mathcal{C}_n \approx -\frac{1}{2} \oint_{C_n} \frac{(z - 1/z)dz}{2\pi i z} \mathcal{G}(z, g) \times \mathcal{S}_n(z, \{\log U_{ij}\})$$

- 1) \mathcal{G} is a known function of the coupling constant (tilted cusp anomalous dimension)
- 2) S_n is rational in z and quadratic in log U's (string integrand)
- 3) The contour of integration is chosen such as to enclose the poles of S_n

Tilted cusp

Following the structure observed in scattering amplitude, we set

[BB,Dixon,Liu,Papathanasiou]

$$\mathcal{G}(z,g) = \Gamma_{\alpha}(g)$$
 with $z = -e^{2i\alpha}$

 $\Gamma_{\alpha}(g)$ is the **tilted** cusp anomalous dimension Γ_{cusp}

One-parameter deformation of the cusp anomalous dimension

$$\Gamma_{\alpha} = 4g^2 - 16\zeta_2 \cos^2 \alpha g^4 + 32\zeta_4 \cos^2 \alpha (3 + 5\cos^2 \alpha) g^6 + \dots$$

with same zeta-structure as for the cusp but dressed with trigonometric numbers

All-order expression follows from deforming the BES equation for cusp

[Beisert, Eden, Staudacher]

$$\Gamma_{\alpha}(g) = 4g^2(1 + \mathbb{K}(\alpha))_{11}^{-1}$$

In particular
$$\Gamma_{\rm cusp} = \Gamma_{\alpha=\pi/4}$$
 $\Gamma_{\rm oct} = \Gamma_{\alpha=0}$

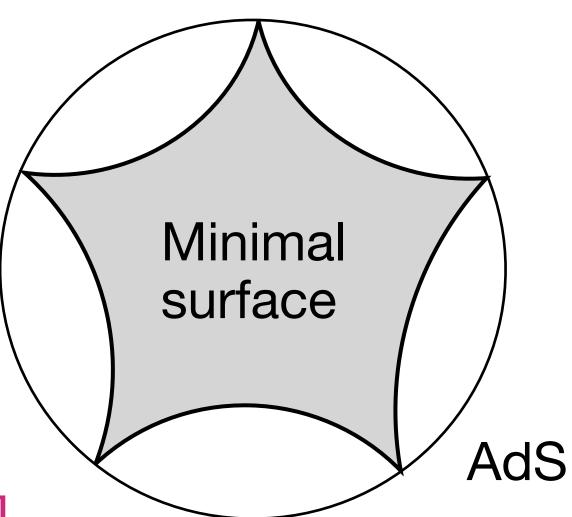
$$\mathbb{K}_{ij} = 2j(-1)^{ij+j} \int_{0}^{\infty} \frac{dt}{t} \frac{J_i(2gt)J_j(2gt)}{e^t - 1}$$

$$\mathbb{K}(\alpha) = 2\cos\alpha \begin{bmatrix} \cos\alpha \mathbb{K}_{\circ\circ} & \sin\alpha \mathbb{K}_{\circ\bullet} \\ \sin\alpha \mathbb{K}_{\bullet\circ} & \cos\alpha \mathbb{K}_{\bullet\bullet} \end{bmatrix}$$

String integrand

At strong coupling one expects correlation functions to be described by minimal surfaces in AdS

Precise definition is still lacking for the **disk** correlators on the string theory side



[Bargheer, Coronado, Vieira]

But a lot of progress can be made from integrability using hexagons

In particular, a description akin to the one obtained for scattering amplitudes can be derived for the disk n-point functions

It expresses the logarithm of the correlation function as the free energy of a system of TBA equations

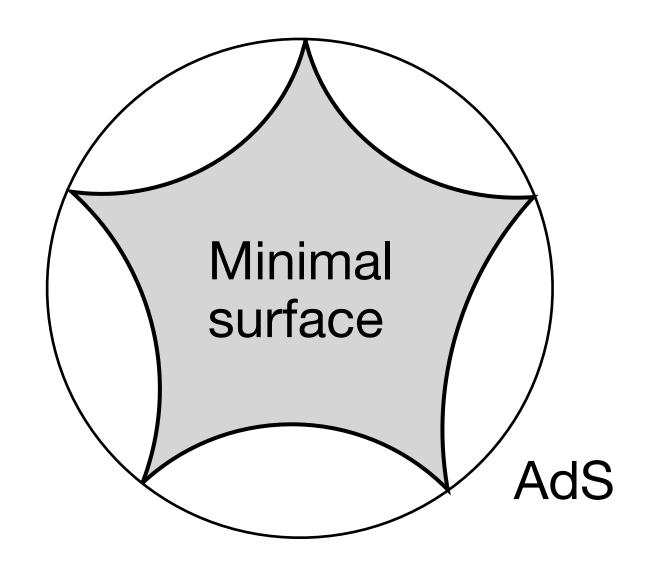
[BB,Kaluç,Serban - in progress]

String integrand

Y functions = collective fields that re-sum excitations in Origin limit

They obey a simple system of (linearized) TBA equations

$$\log Y_i(z) = \frac{1}{1 - K_{ij}(z)} \cdot I_j(z)$$



with z related to the spectral parameter and with known kernels and source terms

The free energy then follows from the string integrand

$$S_n(z) = \sum_i \log I_i(1/z) \log Y_i(z)$$

Generically there is (n-3) Y functions to consider for an n-point Origin limit

General conjecture II

In all cases the poles are on the unit circle and in most cases they are simple

$$\log \mathcal{C}_n \approx \sum_{\alpha} \Gamma_{\alpha}(g) \times P_{\alpha}(\{\log U_{ij}\})$$

where the set of alpha's and associated polynomials are determined by the string integrand

Ex. I. Simplest example is given by the 4-point function: pole at $z^2=1$

$$\log \mathcal{C}_4 \approx -\frac{1}{32} \Gamma_{\alpha=0}(g) \log^2 (U_{13}U_{24}) + \frac{1}{32} \Gamma_{\alpha=\pi/2}(g) \log^2 (U_{13}/U_{24})$$

[Coronado]
[Kostov,Petkova,Serban]
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where the set of alpha's and associated polynomials are determined by string integrand

Ex. II. For 5-point function we have poles at $z^2(1-z^2)=1$ resulting in

$$\log C_5 \approx \Gamma_{\pi/12}(g) P_1(\{\log U_{ij}\}) + \Gamma_{5\pi/12}(g) P_2(\{\log U_{ij}\})$$

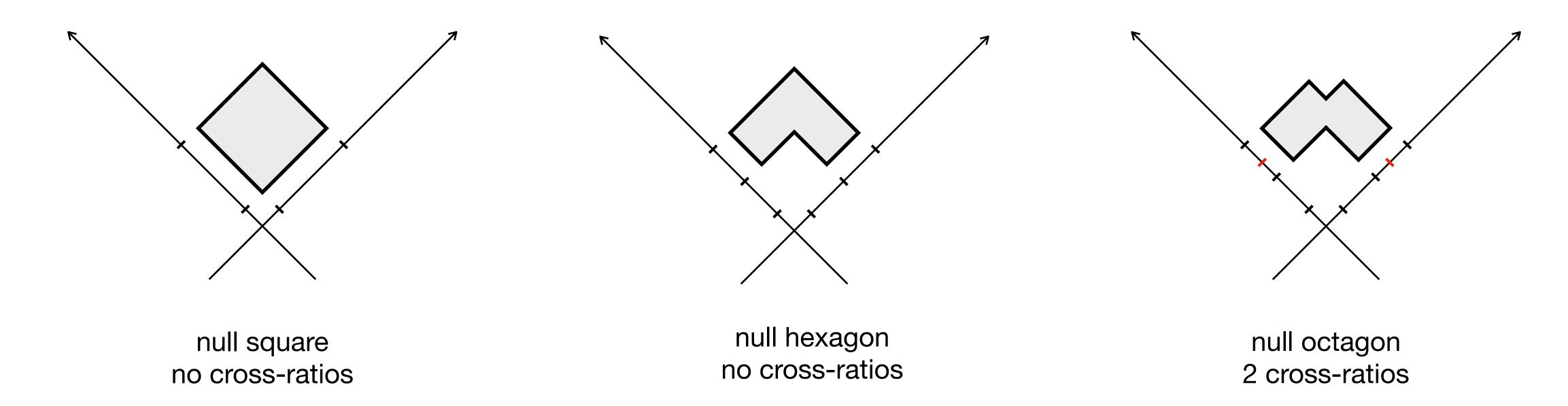
with
$$P_{1,2} = \frac{1}{2}P_{\text{one-loop}} \pm \frac{1}{2\sqrt{3}}(\log U_1^2 + \log U_2^2 + \log U_4^2 + \log U_5^2 + \log U_1 \log U_4 + \log U_2 \log U_5)$$

Evaluation at weak coupling in perfect agreement with previous data through 3 loops (at least)!

Light-like limit of polygons

Null polygons in two dimensions

Consider limits where consecutive vertices are **null** separated (n even)



Interesting due to their possible connection with Wilson loops and scattering amplitudes

Origin limits when n = 4 and 6 (null square and null hexagon are unique in 2d)

For $n \ge 8$ we have (n-6) cross ratios - General structure?

Structuring the null polygons

Not Origin limits for higher n... **Yet** null kinematics *is* cornered by Origins = soft-collinear limits

Drawing inspiration from IR divergences of amplitudes, we may assume that these Origins control the logarithmic divergent part of the null polygons

$$\log C_n = \text{Quad-Log} + R_n$$

Remaining object should be finite and function of the finite cross ratios of the null polygon

However: examples from n = 4 and 6 reveal that the form of this divergent part is **not** universal!

E.g., for n = 4 it is controlled by the octagon dimension, while for n = 6 it also involves $\alpha = \pi/3, \pi/6$

At general n, we expect alpha's associated with n-th roots of unity to play a role

Quid of the finite part?

Does it have anything to do with the remainder function of a null polygonal WLs?

Conclusion

Planar disk correlation functions admit interesting logarithmic behaviors in Origin limits

Classification is possible in 2d using cluster algebra and U-coordinates

Behaviors at Origins may be studied using hexagons

Checks of exponentiation to higher loops for a number of Origins

Consistent with all-order formula based on tilted cusp and string integrands

Outlook

How general is that? Can we prove the formula to all loops for some Origins?

Classification of TBA solutions and precise dictionary?

Generalization to 4d? New Origins? Cluster algebras?

Hexagons are toy models for amplitudes: simpler structures and geometry (at least in 2d)

We may be able to prove results more rigorously

What about light-like limit? Can we study finite parts in these limits? Relation to pentagon OPE?

Relation to other limits and approaches (Stampedes, Large spin/OPE)